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S-wave charmed mesons in lattice NRQCD

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Heavy-light mesons can be studied using the $1/M$ expansion of NRQCD, provided the heavy quark mass is sufficiently large. Calculations of the S-wave charmed meson masses from a classically and tadpole-improved action are presented. A comparison of $O(1/M)$, $O(1/M^2)$ and $O(1/M^3)$ results allows convergence of the expansion to be discussed. It is shown that the form of discretized heavy quark propagation must be chosen carefully.

1. INTRODUCTION

Nonperturbative strong dynamics is typified by the mass scale Λ_{QCD}. Interactions involving a very heavy quark of mass M can be studied systematically by expanding in Λ_{QCD}/M. Upon truncation of the expansion at some order, the resulting effective theory is not renormalizable and requires a momentum cutoff.

If the required regularization is performed via a space-time lattice, then the cutoff is proportional to the inverse lattice spacing $1/a$. A useful effective theory must satisfy

$$\Lambda_{QCD} \ll 1/a \lesssim M, \tag{1}$$

so the cutoff is large enough to include the bulk of the nonperturbative dynamics in the low-energy effective theory, but small enough that the truncation of the expansion remains sensible.

Is the charm quark heavy enough for a useful lattice effective theory? The present work addresses this question through a study of the masses of S-waves charmed mesons using quenched lattice NRQCD.[1] Calculations are performed at two lattice spacings near 0.22 fm and 0.26 fm, and in each case results are given separately at $O(1/M)$, $O(1/M^2)$ and $O(1/M^3)$ in the effective theory. This work is an extension of results that have been reported previously[2], and further details can be found in that paper. Other authors have considered NRQCD up to $O(1/M^2)$[3].

2. ACTION

The lattice action has three terms: gauge action, light quark action and heavy quark action. The entire action is classically and tadpole-improved with the tadpole factor defined by

$$U_0 = \left\langle \frac{1}{3}\mathrm{ReTr}U_{pl} \right\rangle^{1/4}. \tag{2}$$

The gauge action includes a sum over 1×2 rectangular plaquettes as well as 1×1 elementary plaquettes. For light fermions, the Sheikholeslami-Wohlert action[4] is used with the clover coefficient set to its tadpole-improved value. The heavy quark action is NRQCD.

A discretization of the NRQCD action leads to the following Green's function propagation[1]:

$$G_1 = \left(1-\frac{aH_0}{2n}\right)^n \frac{U_4^\dagger}{U_0} \left(1-\frac{aH_0}{2n}\right)^n \delta_{\vec{x},0}, \tag{3}$$

$$G_{\tau+1} = \left(1-\frac{aH_0}{2n}\right)^n \frac{U_4^\dagger}{U_0} \left(1-\frac{aH_0}{2n}\right)^n \times(1-a\delta H)G_\tau \ , \quad \tau>0, \tag{4}$$

where "n" should be chosen to stabilize the numerics, and the Hamiltonian is

$$H = H_0 + \delta H, \tag{5}$$

$$\delta H = \delta H^{(1)} + \delta H^{(2)} + \delta H^{(3)} + O(1/M^4), \tag{6}$$

$$H_0 = \frac{-\Delta^{(2)}}{2M}, \tag{7}$$

$$\delta H^{(1)} = -\frac{c_4}{U_0^4}\frac{g}{2M}\sigma\cdot\tilde{\mathbf{B}} + c_5\frac{a^2\Delta^{(4)}}{24M}, \tag{8}$$

$$\delta H^{(2)} = \frac{c_2}{U_0^4}\frac{ig}{8M^2}(\tilde{\mathbf{\Delta}}\cdot\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\cdot\tilde{\mathbf{\Delta}}) - c_6\frac{a(\Delta^{(2)})^2}{16nM^2} - \frac{c_3}{U_0^4}\frac{g}{8M^2}\sigma\cdot(\tilde{\mathbf{\Delta}}\times\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\times\tilde{\mathbf{\Delta}}), \quad (9)$$

$$\delta H^{(3)} = -c_1\frac{(\Delta^{(2)})^2}{8M^3} - \frac{c_7}{U_0^4}\frac{g}{8M^3}\left\{\tilde{\Delta}^{(2)}, \sigma\cdot\tilde{\mathbf{B}}\right\} - \frac{c_9}{U_0^8}\frac{ig^2}{8M^3}\sigma\cdot(\tilde{\mathbf{E}}\times\tilde{\mathbf{E}} + \tilde{\mathbf{B}}\times\tilde{\mathbf{B}}) - \frac{c_{10}}{U_0^8}\frac{g^2}{8M^3}(\tilde{\mathbf{E}}^2 + \tilde{\mathbf{B}}^2) - c_{11}\frac{a^2(\Delta^{(2)})^3}{192n^2M^3} + \text{purely quantum effects}, \quad (10)$$

$$\tilde{E}_i = \tilde{F}_{4i}, \quad (11)$$

$$\tilde{B}_i = \frac{1}{2}\epsilon_{ijk}\tilde{F}_{jk}. \quad (12)$$

A tilde denotes removal of the leading discretization errors. Classically, the coefficients c_i are all unity, and their nonclassical corrections will not be discussed in this work.

It should be noted that the separation of H_0 and δH in Eq. (4) is not unique. The heavy quark propagation of Eq. (4) uses a simple linear approximation to the true exponential dependence on δH, while using a better-than-linear approximation for H_0. The present work will report on a generalization of this choice.

3. RESULTS

All data presented here correspond to a charmed meson with a light quark mass that is roughly twice the strange quark mass. More extensive results, including bottom mesons, can be found in Ref. [2]. The data sample includes 400(300) gauge field configurations at $\beta = 6.8(7.0)$ corresponding to $a \approx 0.26\text{fm}(0.22\text{fm})$. All plots include bootstrap errors from 1000 ensembles.

Fig. 1 shows the simulation energy (as read from the plateau of an effective mass plot) of the $^1S_0$ meson. Notice that at both lattice spacings the $O(1/M^3)$ contribution is twice as large as the $O(1/M^2)$ contribution, and that this large effect is dominated by the term containing c_{10} in the Hamiltonian, Eq. (10). This term is unique because it is the only term up to $O(1/M^3)$ which contains a nonzero vacuum expectation value. The vacuum value can be calculated from our gauge field configurations, and as shown in Fig. 1 the simulation energy displays a very pleasing $1/M$ expansion after removal of the vacuum value.

The mass difference between $^3S_1$ and $^1S_0$ mesons is shown in Fig. 2. When the vacuum expectation value is not removed from the Hamiltonian, the $O(1/M^3)$ contribution is twice as large as the $O(1/M^2)$ contribution in magnitude, due to large effects from the terms containing c_7 and c_{10}. Fig. 2 indicates that the c_{10} effect is entirely due to the vacuum expectation value.

This very substantial dependence of the spin splitting on the vacuum value should be disturbing, since the c_{10} term is spin-independent. Apparently the vacuum value is so large that it destabilizes heavy quark propagation and thereby introduces a spurious effect unless the vacuum value is removed from the Hamiltonian. To support this claim, we have redone the calculation after subtracting the c_{10} term from δH and adding it to H_0 in Eq. (4). When this is done, the triangles of Fig. 2 are reproduced regardless of whether the vacuum value is subtracted from the Hamiltonian. This is the physically-expected result.

Still, Fig. 2 contains a large $O(1/M^3)$ contribution dominated by the term containing c_7. Although this term does not contain a nonzero vacuum expectation value, one wonders if the heavy quark propagation might be unstable for this term as well, unless a better-than-linear approximation is used for the c_7 term in Eq. (4).

Fig. 3 shows the effect of subtracting all of δH from its present location in Eq. (4) and putting the full Hamiltonian in place of H_0. That is,

$$G_{\tau+1} = \left(1 - \frac{aH}{2n}\right)^n \frac{U_4^\dagger}{U_0}\left(1 - \frac{aH}{2n}\right)^n G_\tau, \quad (13)$$

and we choose $G_0 \equiv \delta_{\vec{x},0}$. To maintain classical improvement, the following $O(1/M^3)$ term must be added to the Hamiltonian:

$$\delta H_{\text{new}} = -\frac{a}{4n}\left\{\left(H_0 + \delta H^{(1)}\right), \delta H^{(2)}\right\}. \quad (14)$$

Simulations with $n = 5$ and $n = 7$ are indistinguishable. At both lattice spacings, the large c_7 effect is found to be robust, and the contribution from c_9 tends to increase. No discussion of the spin-independent terms containing c_1 and c_{11} is presented here.

4. DISCUSSION

Instabilities can arise in the NRQCD expansion for charmed mesons due to the presence of a large vacuum expectation value. A better-than-linear approximation to heavy quark propagation is valuable for ensuring stability.

Substantial effects on spin splitting were found from $O(1/M^3)$ terms in the action. However further study, for example, of alternative definitions for the tadpole factor or of perturbative improvement for the NRQCD coefficients is needed before one can reach a definitive conclusion about the convergence of the NRQCD expansion for charmed mesons.

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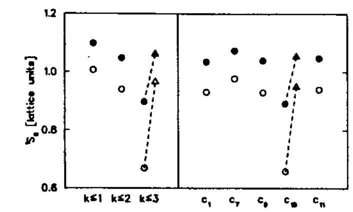

Figure 1. The simulation energy of a ground state charmed meson at rest, up to $O(1/M^k)$. Solid(open) symbols denote data at $\beta = 6.8(7.0)$. Triangles are produced by subtracting the vacuum expectation value from the c_{10} term. To the right of the vertical line, the effect of adding each $O(1/M^3)$ term to the $O(1/M^2)$ Hamiltonian is shown individually.

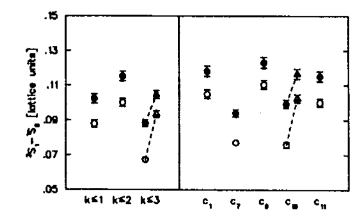

Figure 2. The spin splitting of S-wave charmed mesons, with notation as in Fig. 1.

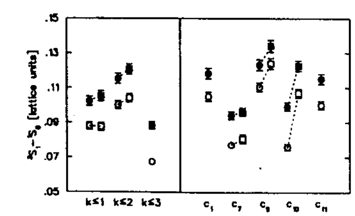

Figure 3. The data (circles) of Fig. 2, which use Eq. (4), are compared to data (squares) obtained from Eq. (13). The vacuum expectation value is not removed.

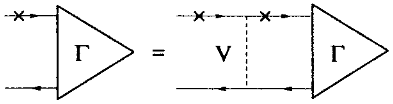

FIG. 1. The Gross equation. The × indicate that the particle is on-shell.

his is the Gross equation (see Fig. 1) and the discussion shows that this is the ost natural relativistic equation obtained from a generalization of the nonrela- ristic Schrödinger equation for confined particles.

In a previous application of the Gross equation [2] the kernel (2.3), with $\to -q^2$, was used. This choice is undesirable for use with the two channel :rsion of this equation, where the mass shell constraints which fix q_0 introduce ngularities for non-zero momentum transfers (i.e. $q^2 = 0$, but $q^\mu \neq 0$). To orrect this problem the kernel $V_A(q)$ is defined to be

$$V_A(q) \equiv -8\pi\sigma \frac{1}{q^4 + (P\cdot q)^4/P^4} \tag{3.5}$$

dvantages of this form are many: **i)** singularities are restricted to $q^\mu = 0$, **ii)** teraction strength *does not* depend on the bound state momentum P in the st frame, **iii)** it has the correct nonrelativistic dependence on $\vec{q}^{\,2}$, and **iv)** the traviolet regularization used previously [2] is no longer needed.

IV. PROOF OF CONFINEMENT

It is sufficient to consider the case when the constant term $C = 0$. If the mass : the bound state $\mu > 2m$, then there exists a value of three momentum $|\vec{p}| = p_c$ hen both quarks can be on-shell. In this case $(P-p)^2 = \mu^2 - 2\mu E_{p_c} + m^2 = m^2$, ıd the subtraction term in Eq. (3.4) appears to be singular. This singularity is ɔt cancelled by the first term, and hence, if the equation is to have a solution, ıe vertex function must be zero at p_c. In particular, as $|\vec{p}| \to p_c \pm \epsilon$,

$$(P-p)^2 - m^2 \to \mp \frac{2\mu p_c\, \epsilon}{E_{p_c}}\,, \tag{4.1}$$

ıd the vertex function would have an infinite discontinuity at p_c unless it were :ro there. We conclude that $\Phi(p) = 0$, when $2E_p = \mu$ (as illustrated in Fig. 2).

A numerical confirmation of this result is shown in Fig. 3. In the actual ılculations to be presented in an upcoming paper, the bound state equation is reraged over positive and negative energy contributions and symmetrized by icking up the pole contributions of both constituents. The average over positive

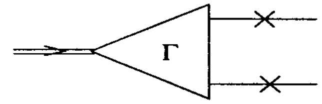

FIG. 2. When both constituents are on-shell, the vertex function vanishes.

and negative energy contributions leads to a two channel equation with a two component vertex function. In Fig. 3 the solid line is the (large) component in which the off-shell quark has positive energy and the dashed line is the (small) component in which the off-shell quark has negative energy. Since a physical decay must produce two positive energy quarks, only the large component must have the "confinement" node. Fig. 3 shows the vertex functions for an excited pion with mass $\mu = 1.2$ GeV composed of two quarks with masses $m = 0.34$ MeV. The large vertex function must have *two* nodes: one due to the excitation and another at $p^2 = 0.244$ GeV^2, exactly where both particles are on-shell.

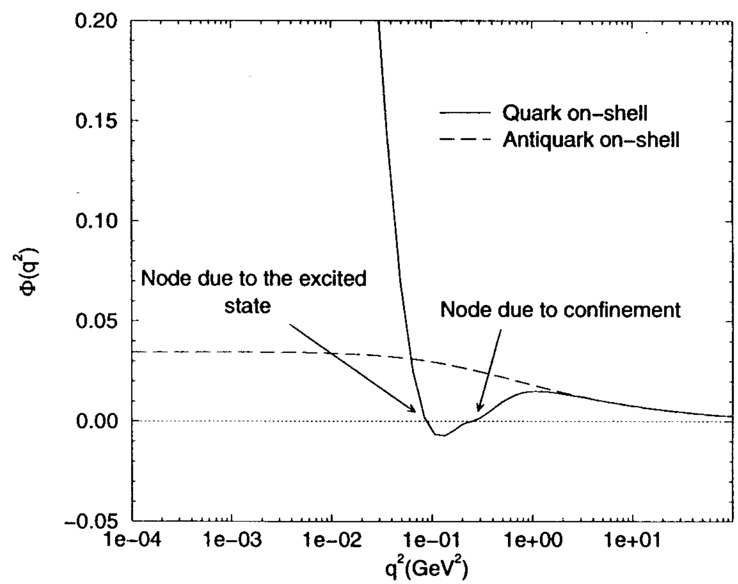

FIG. 3. The pion 1st excited state Gross vertex functions are shown. The first node is due to the excited state. The second node assures that the bound state does not decay.

V. CONCLUSIONS

Γhe relativistic equation obtained from a generalization of the nonrelativistic :ödinger equation for confined particles is of the Gross equation type rather ı the Bethe-Salpeter equation type. The confinement mechanism arises not ı the lack of quark mass poles, but through the vanishing of the vertex func- when both quarks are on their positive energy mass-shell. Because of this ìnement mechanism, the vertex functions (ground or excited state) have one ıtional node if the bound state is heavy enough.

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