Sum Rules and Spin Structure

- Baldin Sum Rule
  - Hall C: E94-110
- Q^2-evolution of GDH integral
  - Hall A: E94-010, E97-110
  - Hall B: E91-023, E93-009, E03-006
- Spin Structure at "large" $x_{Bjorken}$
  - Hall A: E99-117, E97-103
  - Hall B: E91-023, E93-009

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Extended Baldin Sum Rule

$Q^2 = 0$, photoproduction

GDH Sum Rule

$$-\frac{\kappa^2}{4} = \frac{M^2}{2\pi e^2}\int_{\nu_0}^{\infty}\frac{\sigma_{1/2}-\sigma_{3/2}}{\nu}d\nu$$

Baldin Sum Rule

$$(\alpha+\beta)_N = \frac{1}{4\pi^2}\int_{\nu_0}^{\infty}\frac{\sigma_{1/2}+\sigma_{3/2}}{\nu^2}d\nu$$

$Q^2 > 0$, electroproduction

Extended GDH Sum Rule

$$-\frac{\kappa^2}{4} = \frac{M^2}{\pi e^2}\int_{\nu_0}^{\infty}\frac{K}{\nu}\frac{\sigma_{TT}}{\nu}d\nu$$

$$= \frac{2M^2}{Q^2}\int_0^{x_0} g_1\,dx$$

Extended Baldin Sum Rule*

$$(\alpha+\beta)_N = \frac{1}{2\pi^2}\int_{\nu_0}^{\infty}\frac{K}{\nu}\frac{\sigma_T}{\nu^2}d\nu$$

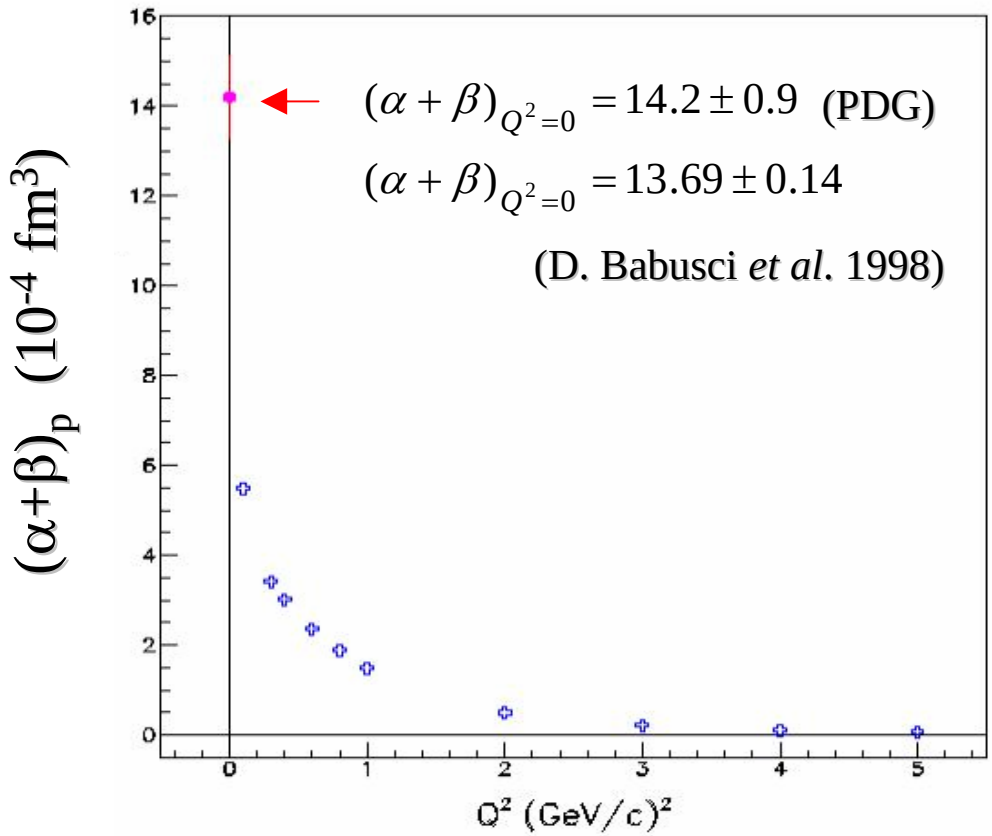
$$= \frac{e^2 M}{\pi Q^4}\int_0^{x_0} 2xF_1\,dx$$

Need L/T separated data!

where κ : anomalous magnetic moment of the nucleon
α, β : electric and magnetic polarizabilities, respectively
ν_0 : pion photoproduction threshold

* D. Drechsel, B. Pasquini, M. Vanderhaeghen hep-ph/0212124 Dec 2002

The extended Baldin Sum Rule for the proton



Hall C E94-110
Preliminary!

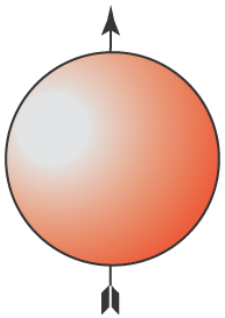
- Extended Baldin Integral goes smoothly from $Q^2 > 5$ GeV^2 to real photon point

Why is $I^{GDH}(Q^2)$ interesting?

One of the few opportunities to "zoom out" from tiny length scales (DIS) to large length scales

$$I^{GDH}(Q^2)$$

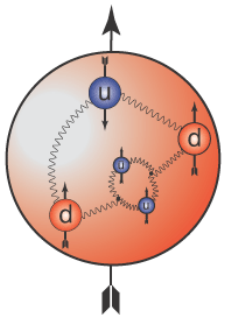
$$-\frac{2\pi^2\alpha}{M^2}\kappa^2 \xleftarrow{\text{Chiral pert. theory}} 2\pi^2\alpha S_1(0;Q^2) \xrightarrow{\text{Higher twist corrections}} \frac{16\pi^2\alpha}{Q^2}\Gamma_1$$



Non-perturbative, confinement

GDH

PQCD

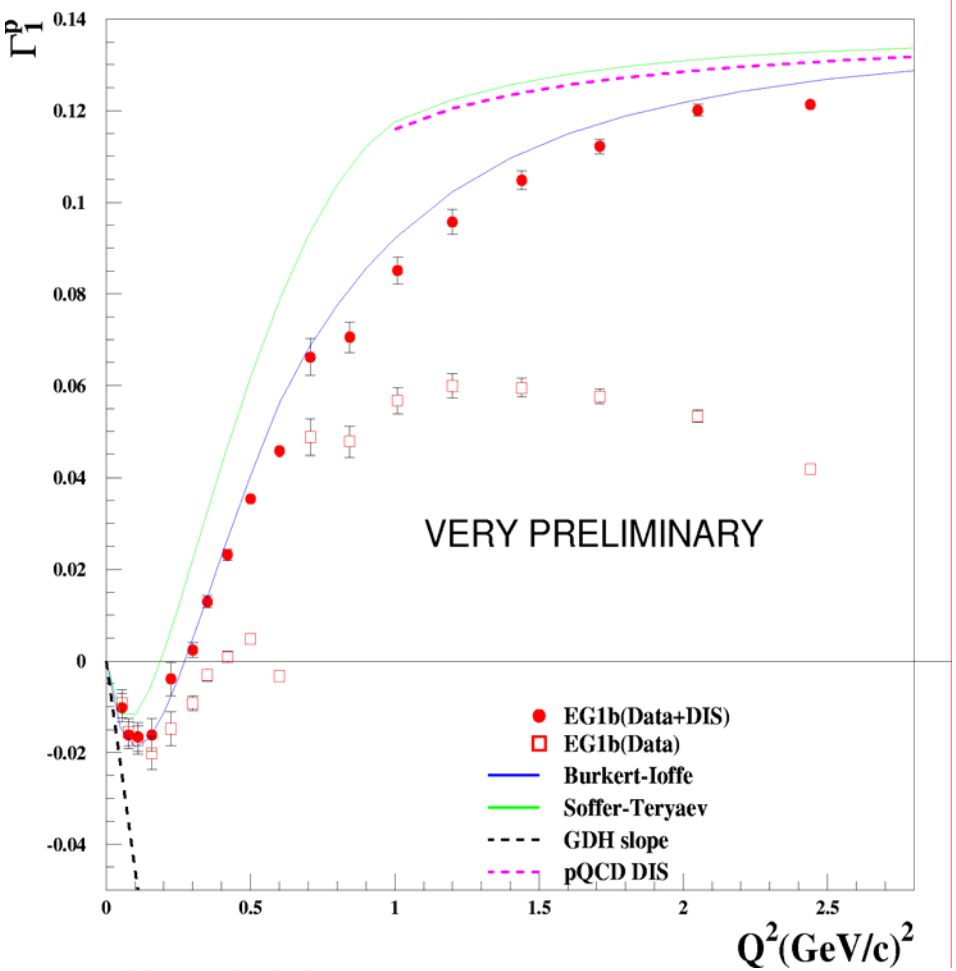


OPE, Bjorken, Ellis-Jaffe

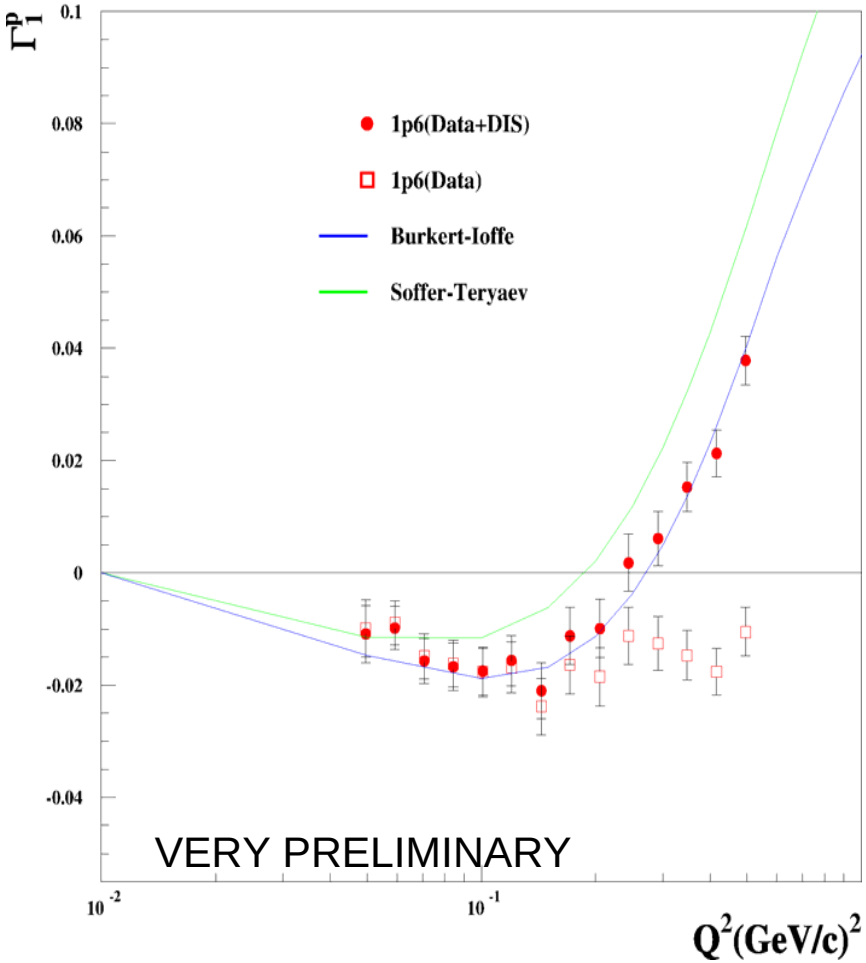
Many of the underlying assumptions are the same as those being tested in high-energy spin-structure tests

CLAS - First Moment $\Gamma_{1p}(Q^2)$

$$\Gamma_1^p = \int_0^1 g_1^p(x)dx = \int_{0.001}^{x_{cut}} g_1^p(x)dx + \int_{x_{cut}}^1 g_1^p(x)dx$$



$\Gamma_1^p(Q^2)$ at low Q^2



CLAS - First moment Γ_1 for the deuteron

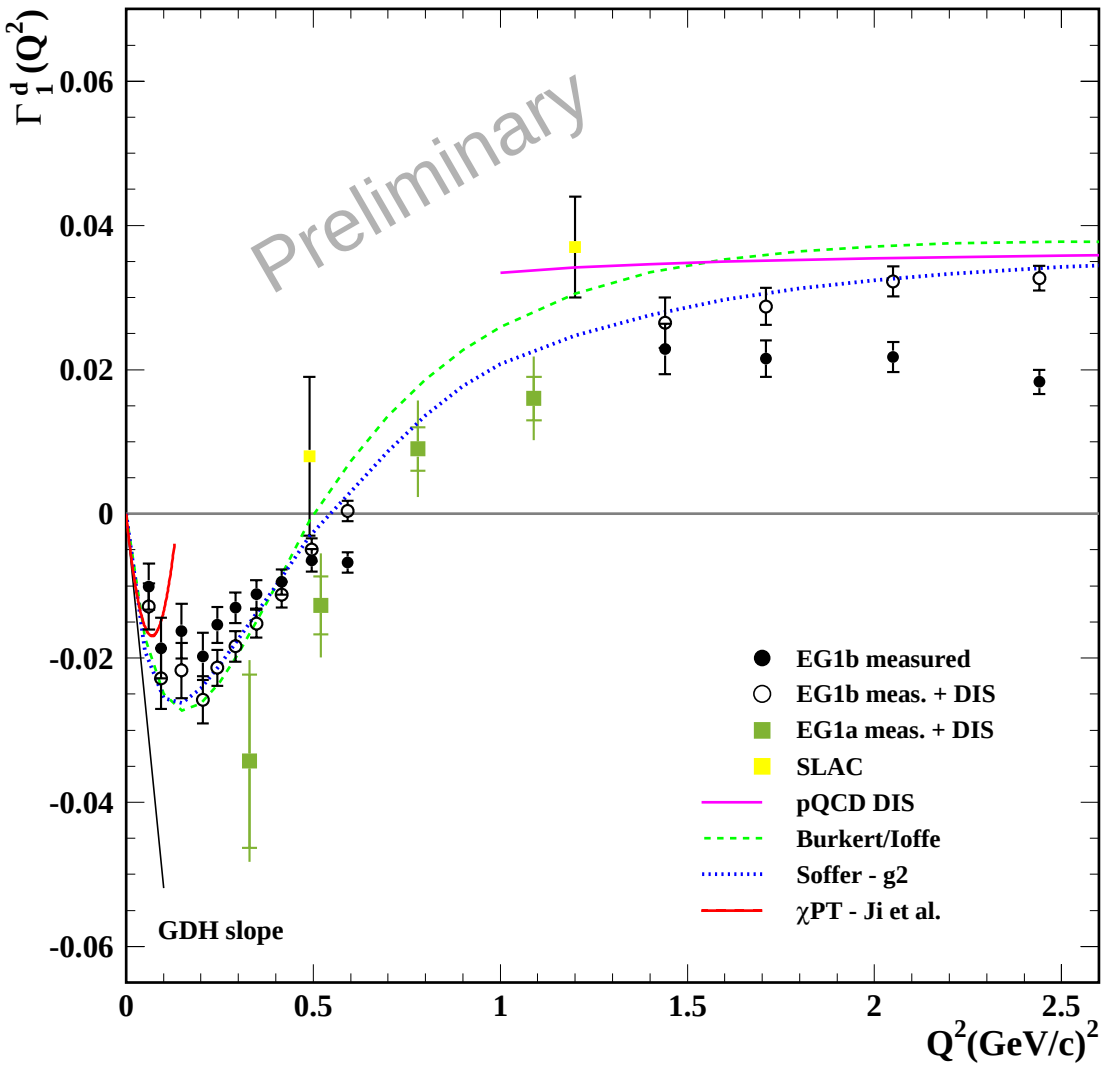
$$\Gamma_1 = \underbrace{\int_{x=0.001}^{x_{min}} g_1 dx}_{\text{DIS (unmeasured)}} + \int_{x_{min}}^{x=1} g_1 dx$$

DIS (unmeasured)
Parameterization of world data

Phenomenological Models
•Q^2 parameterizations

Burkert/Ioffe
➢Resonance contribution pion electroproduction analysis

Soffer/Teryaev
➢Interpolation of the integral $\int (g_1+g_2)dx$

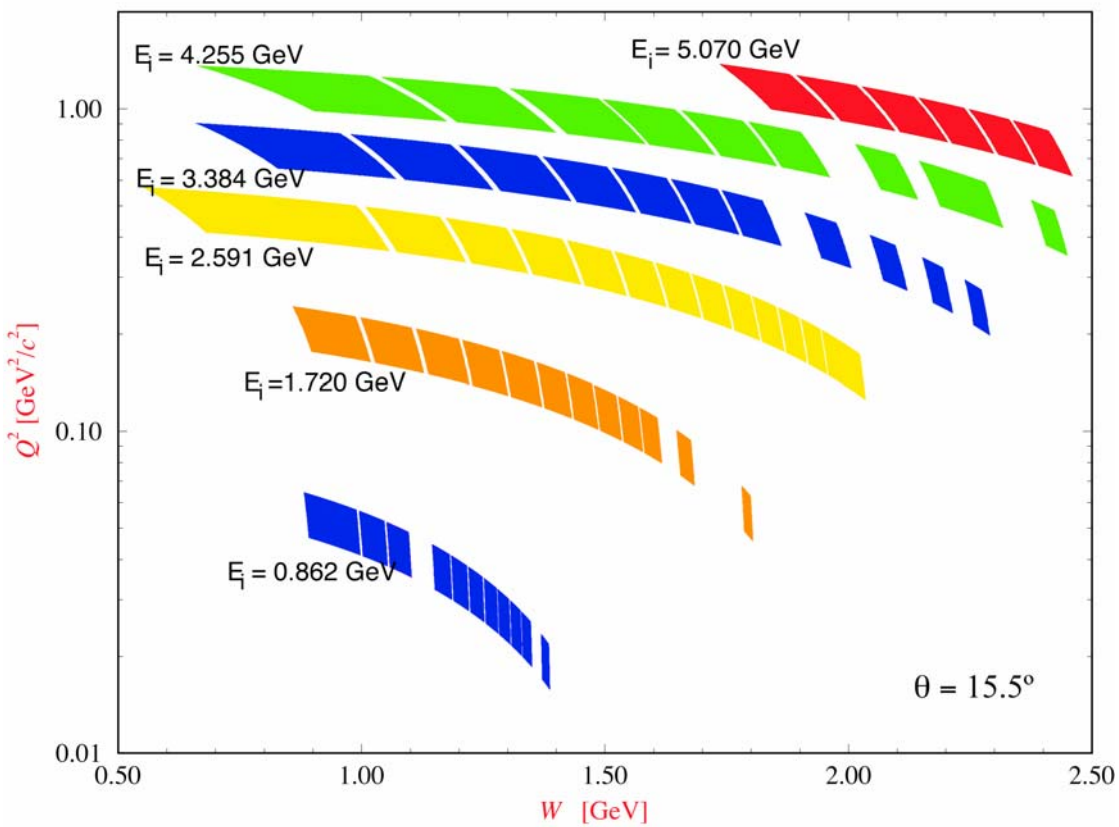


Q^2-Evolution of the Gerasimov-Drell-Hearn Integral

$$I_{GDH} = \int_{\nu_{in}}^{\infty} \left(\sigma_{1/2}(\nu,Q^2) - \sigma_{3/2}(\nu,Q^2)\right)\frac{d\nu}{\nu} = -2\int_{\nu_{in}}^{\infty} \sigma_{TT}\frac{d\nu}{\nu} = -\frac{8\pi^2\alpha}{MK}\int_{\nu_{in}}^{\infty}\left(g_1 - \frac{Q^2}{\nu^2}g_2\right)\frac{d\nu}{\nu}$$

- Longitudinal and transverse target polarization allows separation of g_1 and g_2
- Kinematic coverage sufficient to integrate to W ≈ 2 GeV

Kinematic coverage of JLab E94-010 Experiment



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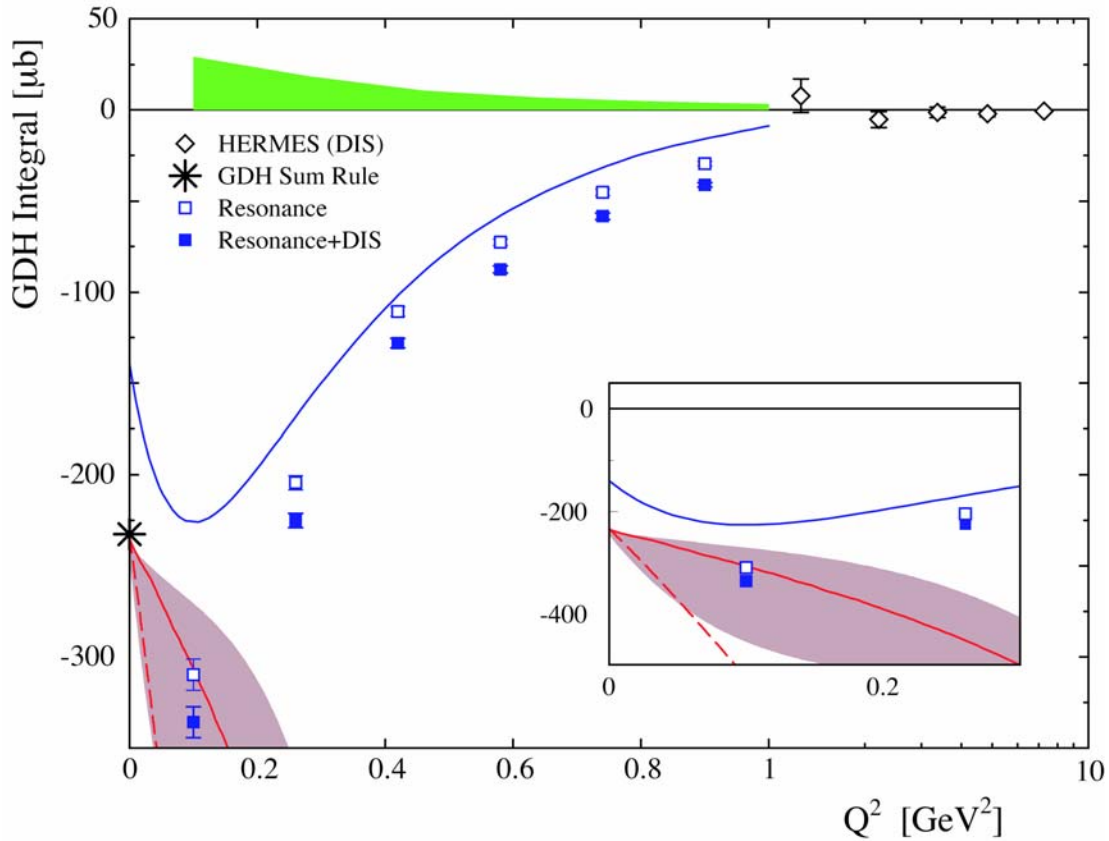
Hall A E94-010

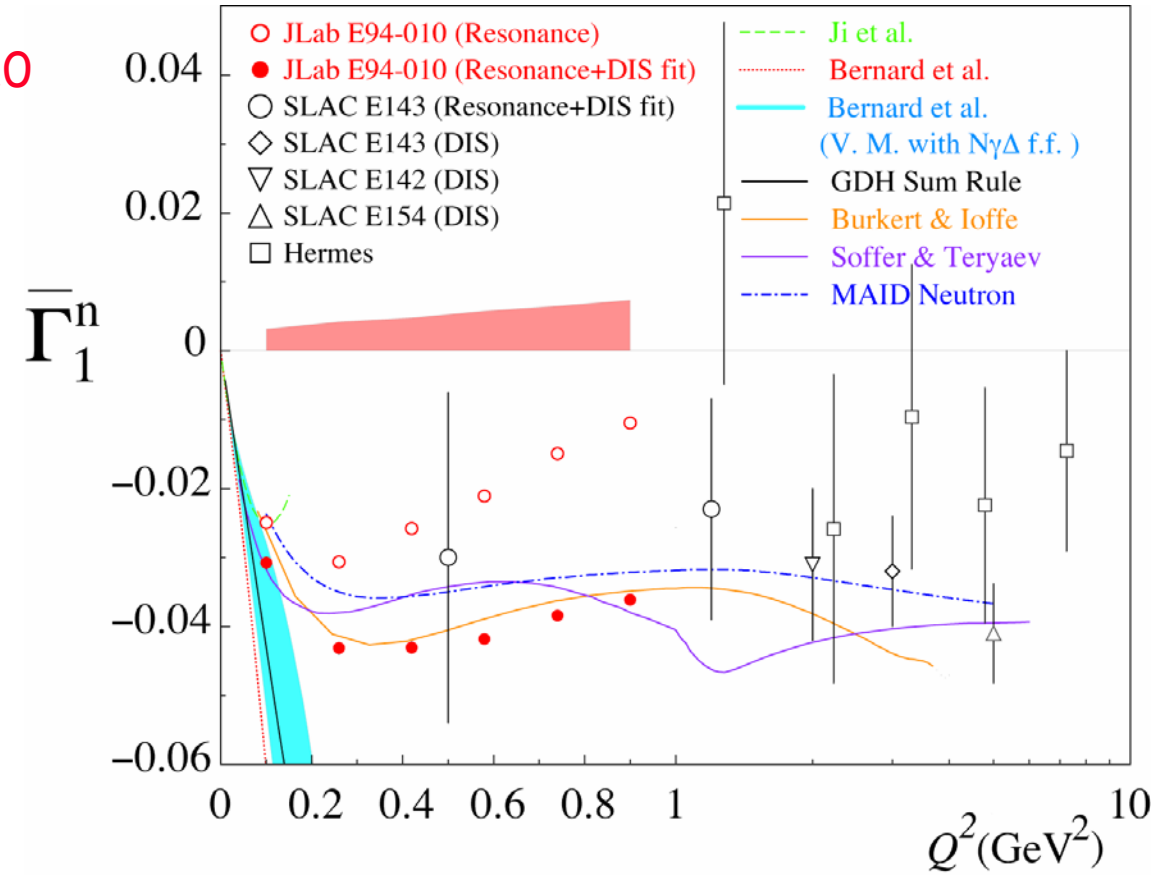
- Nuclear medium corrections from Ciofi degli Atti and Scopetta
- Compared to calculations by Drechsel et al. which neglect contributions from DIS and by Ji and Bernard based on Chiral Perturbation Theory (band shows uncertainty in contribution from Δ-resonance)



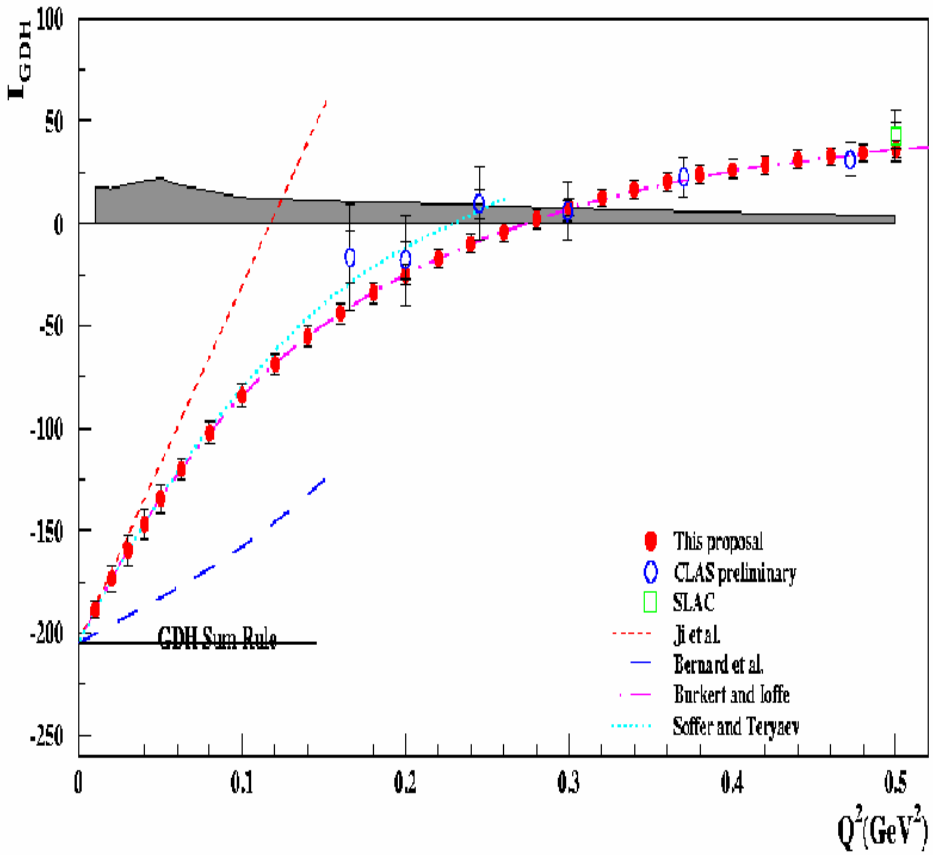
Q^2-Evolution of the GDH Integral (cont.)

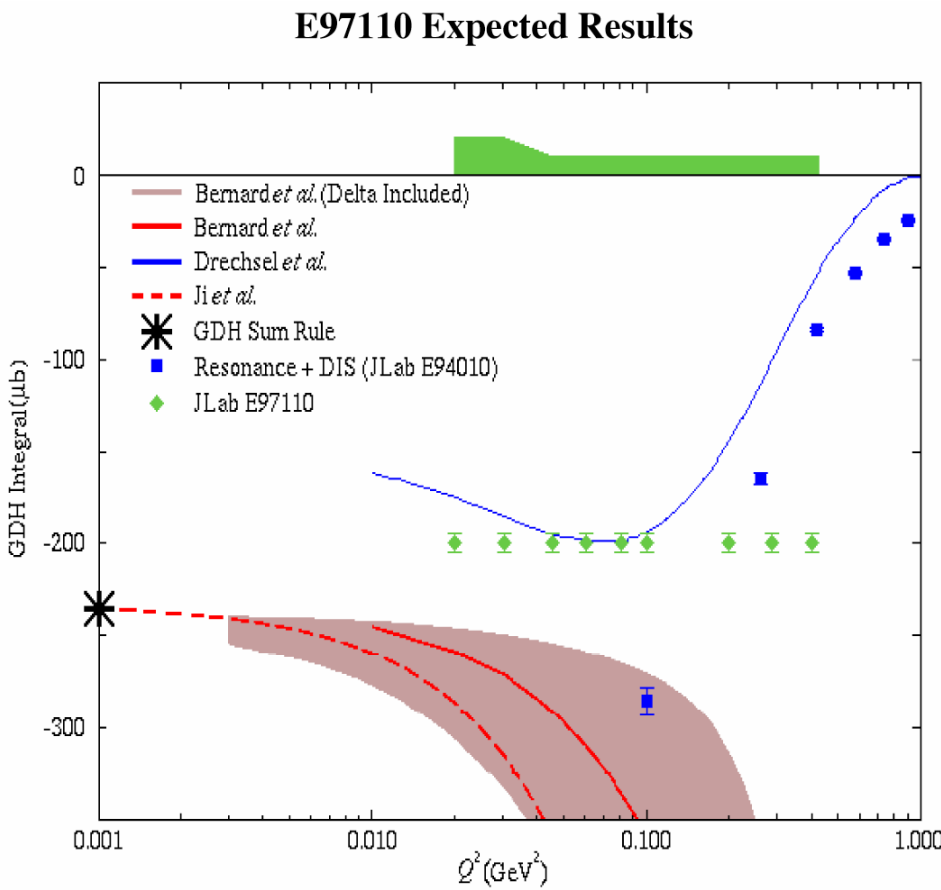
$$\Gamma_1^{p/n} = \int_0^1 g_1^{p/n}(x_{Bj},Q^2)dx_{Bj} \propto \int_0^1 (A_{//} + \alpha A_\perp)F_1(x_{Bj},Q^2)dx_{Bj}$$

Hall A E94-010



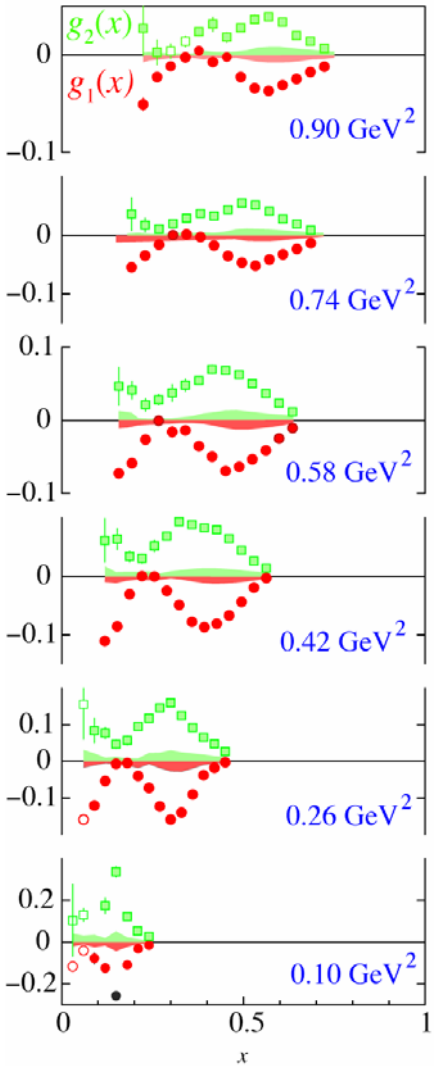
Q^2-Evolution of the GDH Integral (extensions)





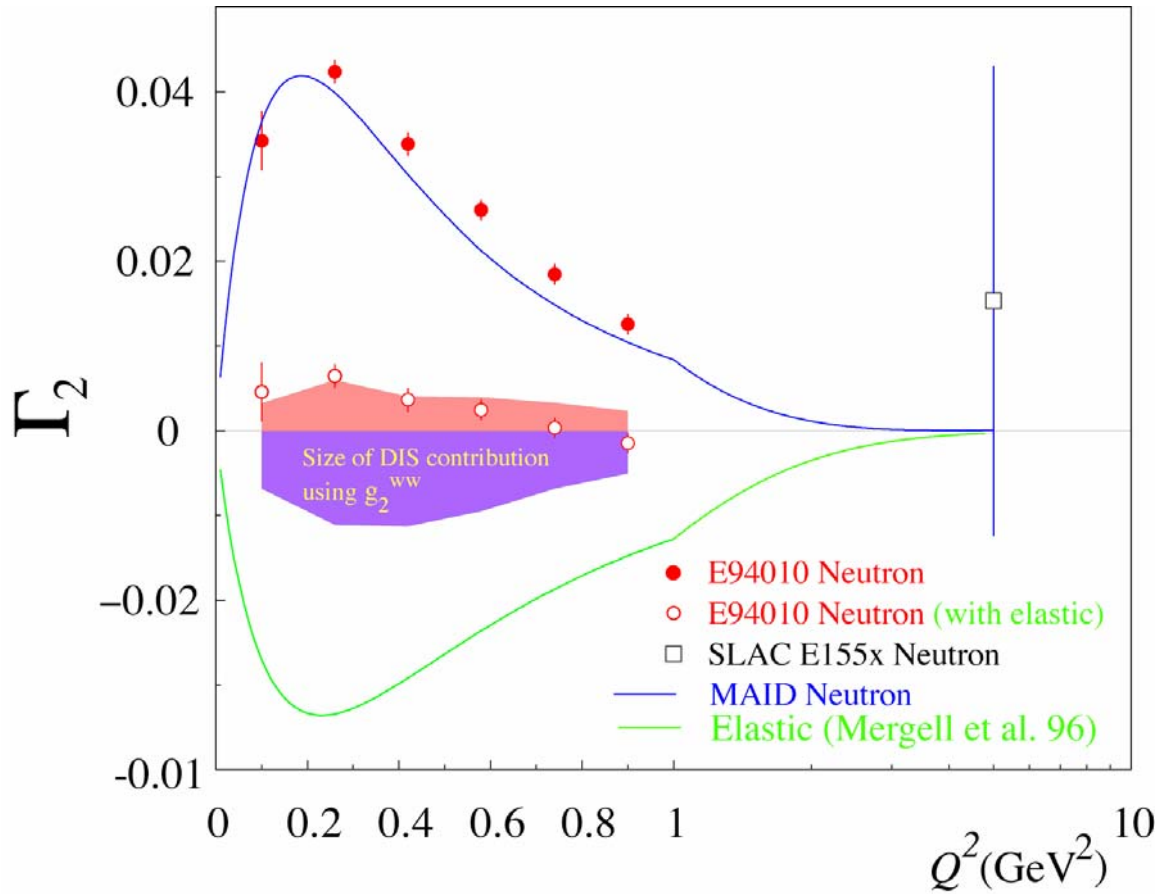
- E97-110 (Hall A, completed) and E03-006 (Hall B) will extend Q^2-range down to 0.02 GeV^2

Burkhardt-Cottingham Sum Rule



Hall A E94-010

$$\Gamma_2^{p,n}(Q^2) = \int_0^1 g_2^{p,n}(x,Q^2)dx = 0$$



Higher Twist Effects

- g_2 allows access to "higher twist" effects (quark-gluon correlations)
- E97-103 made accurate measurements of g_2 at different Q^2-values, but analysis not completed

$$\Gamma_1(Q^2) = \int_0^1 g_1(x,Q^2)dx = \mu_2 + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \ldots\ldots$$

leading twist higher twist

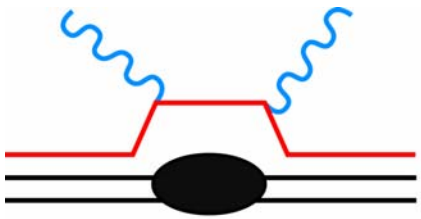
$$\mu_4(Q^2) = \frac{1}{9} M[a_2(Q^2) + 4d_2(Q^2) - 4f_2(Q^2)]$$

twist-2 twist-3 twist-4

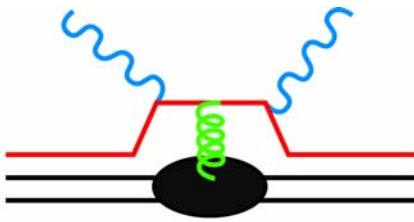
$$g_2(x,Q^2) = g_2^{WW}(x,Q^2) + \bar{g}_2(x,Q^2)$$

$$g_2^{WW}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 g_1(y,Q^2)\frac{dy}{y}$$

$$d_2^n(Q^2) = \int_0^1 x^2\left[2g_1^n(x,Q^2) + 3g_2^n(x,Q^2)\right]dx$$



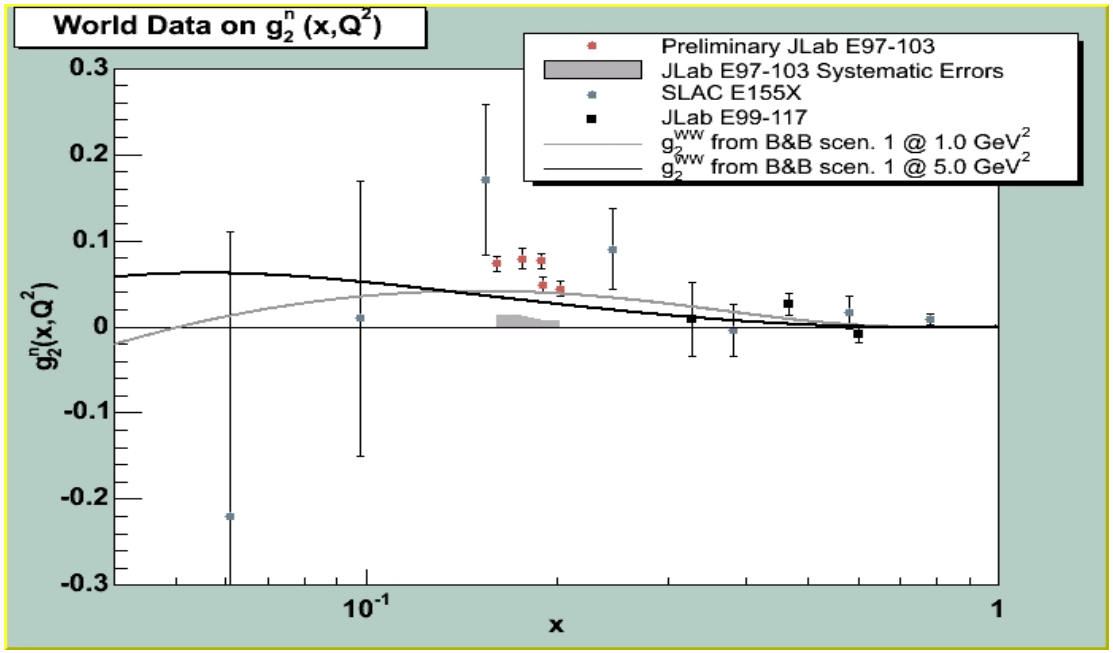
twist–2



twist–3

Jefferson Lab Hall A Experiment E97-103

T. Averett, W. Korsch (spokespersons) K. Kramer (Ph.D. student)

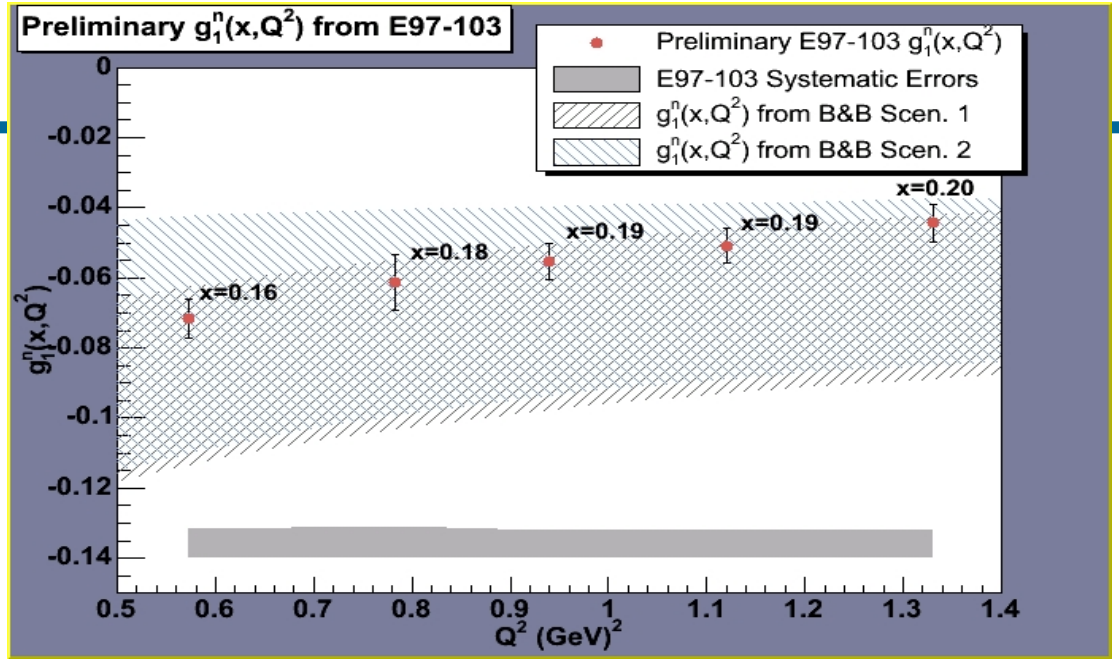


- Inclusive DIS of polarized electrons from a polarized ^{3}He target
- Precision g_2^n data covering $0.57 < Q^2 < 1.34$ GeV^2 at $x \sim 0.2$
- Direct comparison to *twist-2* g_2^{ww} prediction using world g_1^n data
- Quantitative measurement of higher twist effects provides information on nucleon structure beyond simple parton model (e.g. quark-gluon correlations)

JLab E97-103

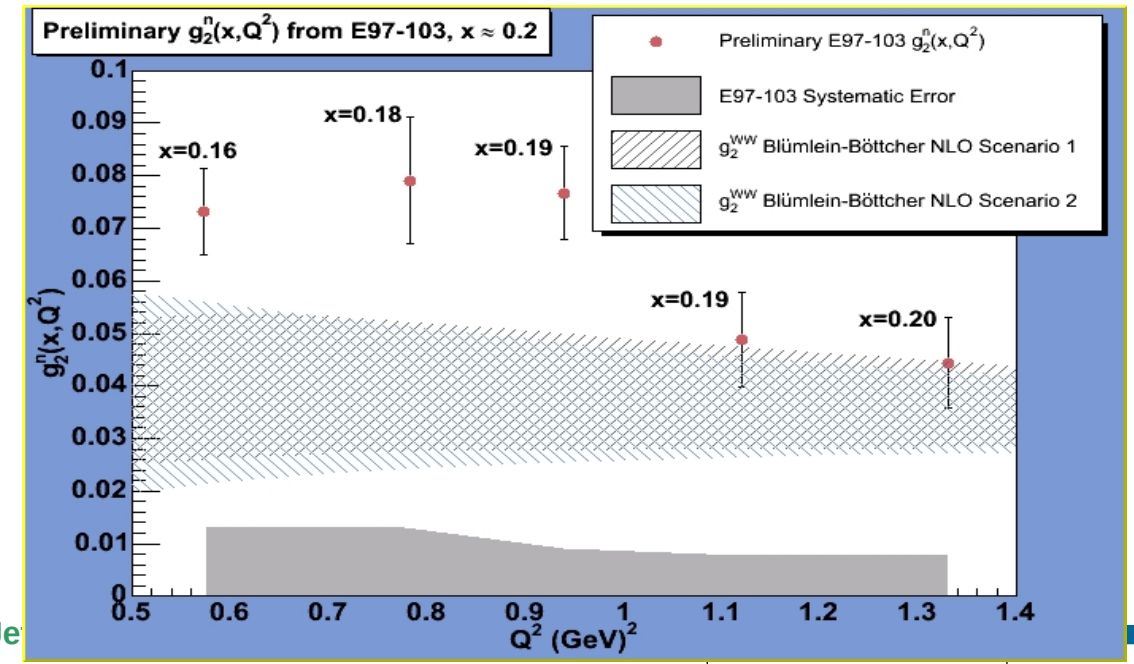
Preliminary Results

- Measured g_1^n agree with NLO fit to world data, evolved to our Q^2



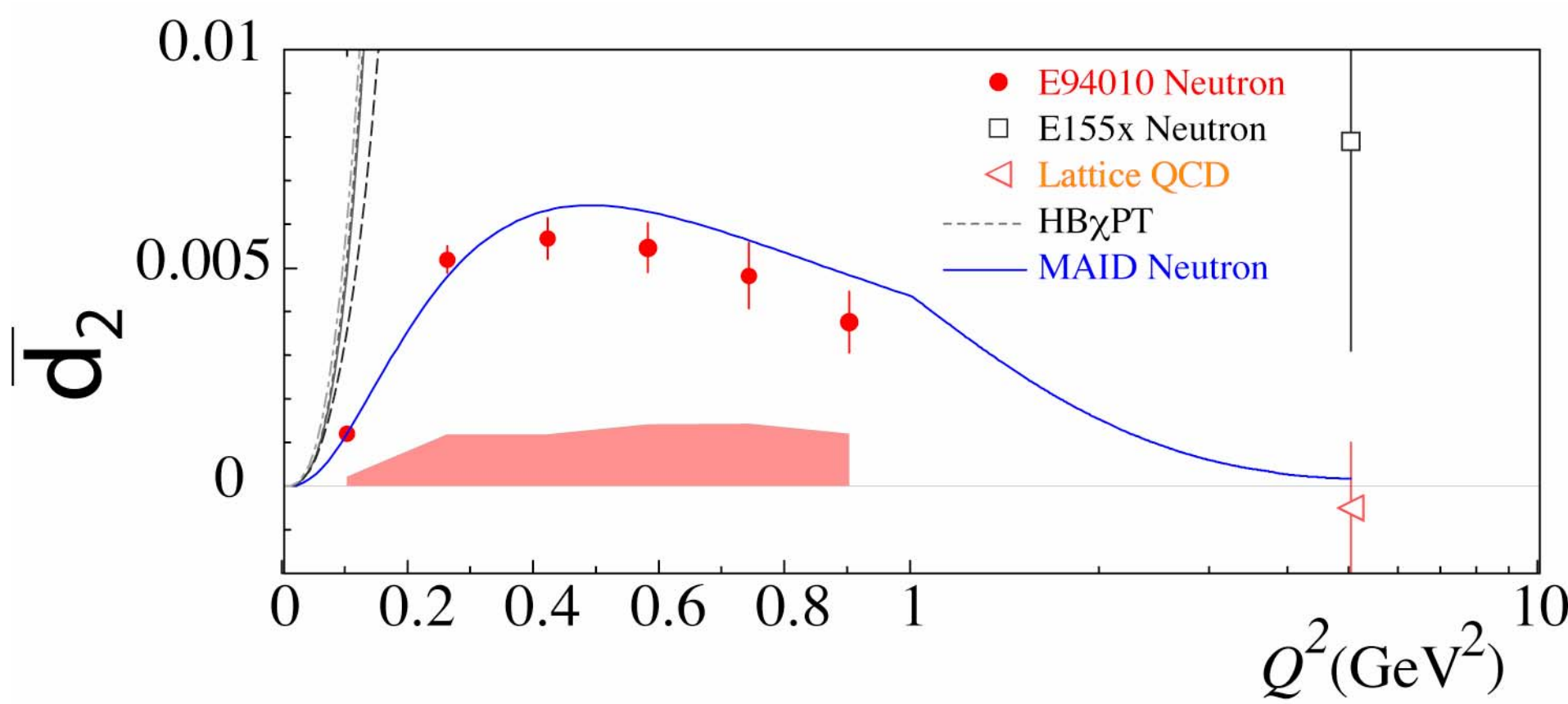
- Measured g_2^n consistently higher than g_2^{ww} at low Q^2.

- E97-103 improved precision of g_2^n by an order of magnitude



Higher Twist Effects (cont.)

Hall A E94-010

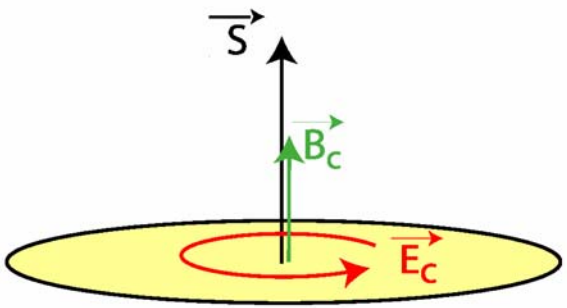


Color Polarizabilities

How does the gluon field respond when a nucleon is polarized?

Define color electric and magnetic polarizabilities (in nucleon restframe):

$$\chi_{B,E}\, 2M^2 \vec{S} = \left\langle PS \middle| \vec{O}_{B,E} \middle| PS \right\rangle$$



where

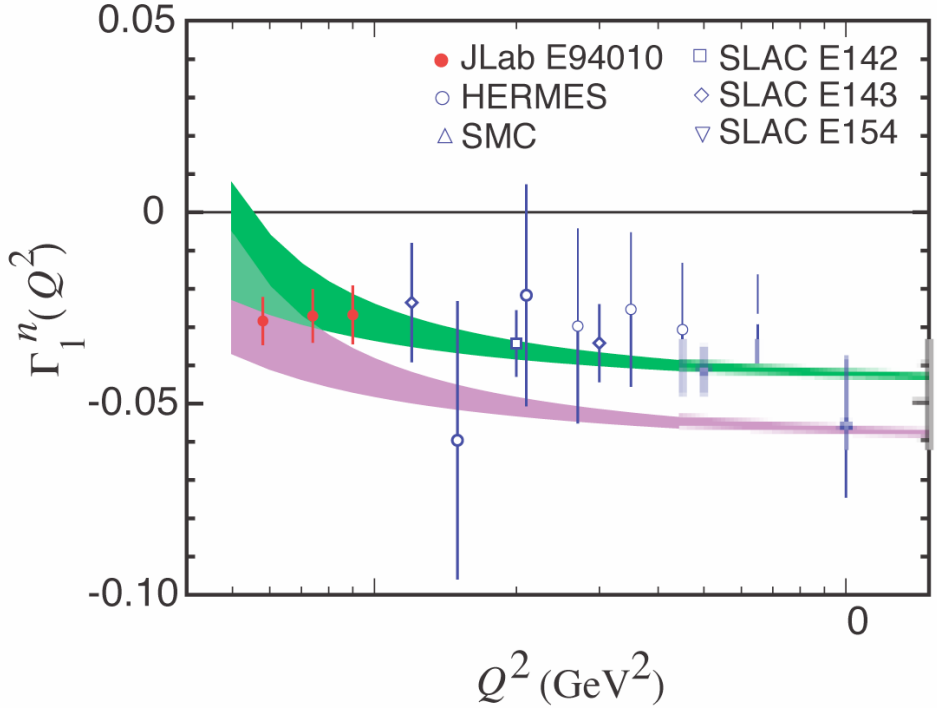
$$\vec{O}_B = \psi^* g \vec{B} \psi$$

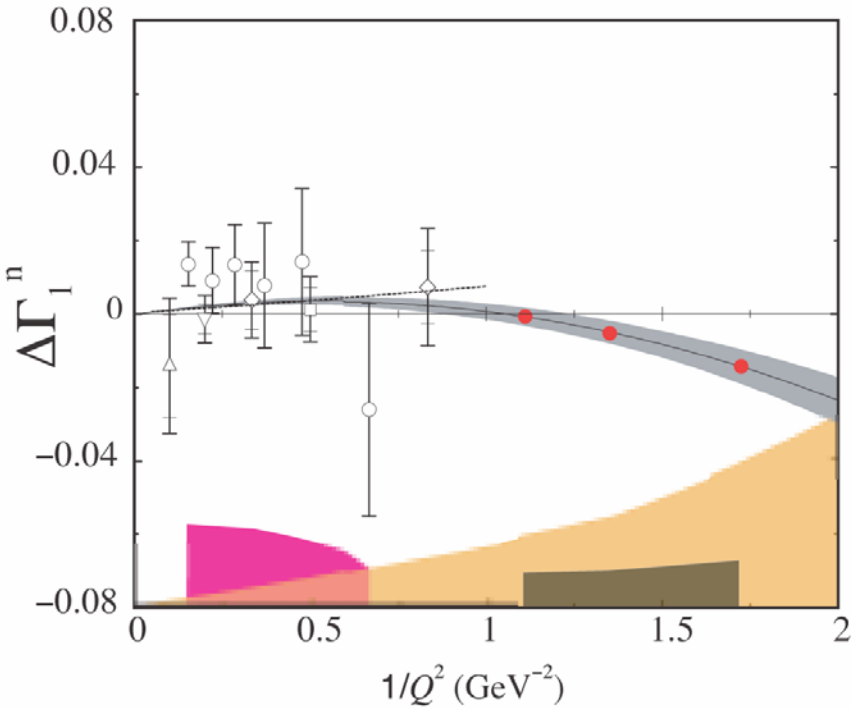
$$\vec{O}_E = \psi^* \vec{\alpha} \times g \vec{E} \psi$$

$$\boxed{\begin{aligned} d_2 &= \left(2\chi_B + \chi_E\right)/8 \\ f_2 &= \left(\chi_E - \chi_B\right)/2 \end{aligned}}$$

d_2 and f_2 represent the response of the color fields to the nucleon polarization

Color Polarizabilities (cont.)



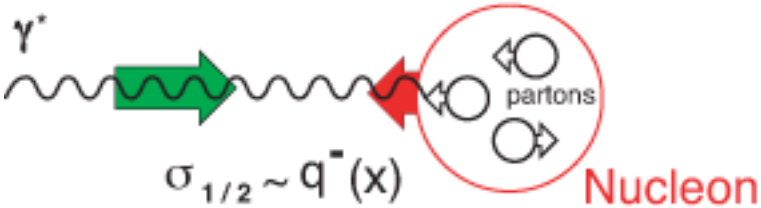


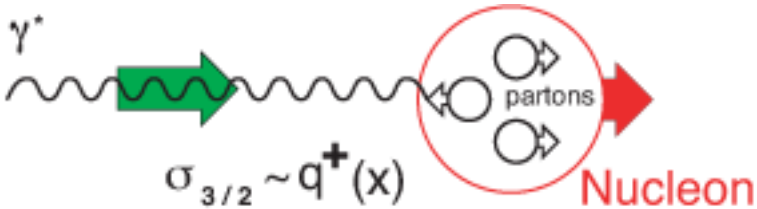
$$f_2^n = 0.034 \pm 0.043$$
$$\mu_6^n = (-0.019 \pm 0.017) M^4$$

$$\chi_E^n = 0.033 \pm 0.029$$
$$\chi_B^n = -0.001 \pm 0.016$$

Subtract leading twist curves from data
Two-parameter fir to higher-twist residuals

Spin Structure in Deep Inelastic Scattering





Partonic Interpretation

x = fraction of nucleon momentum carried by struck quark

$q^{+/-}$ quark helicity parallel/antiparallel to photon helicity

NO simple partonic picture of g_2

$$x = \frac{Q^2}{2M\nu}$$

$$q(x) = q^+(x) + q^-(x)$$

$$F_1(x) = \frac{1}{2}\sum_{flavor} e_f^2 q_f(x) \qquad F_2(x) = xF_1(x)$$

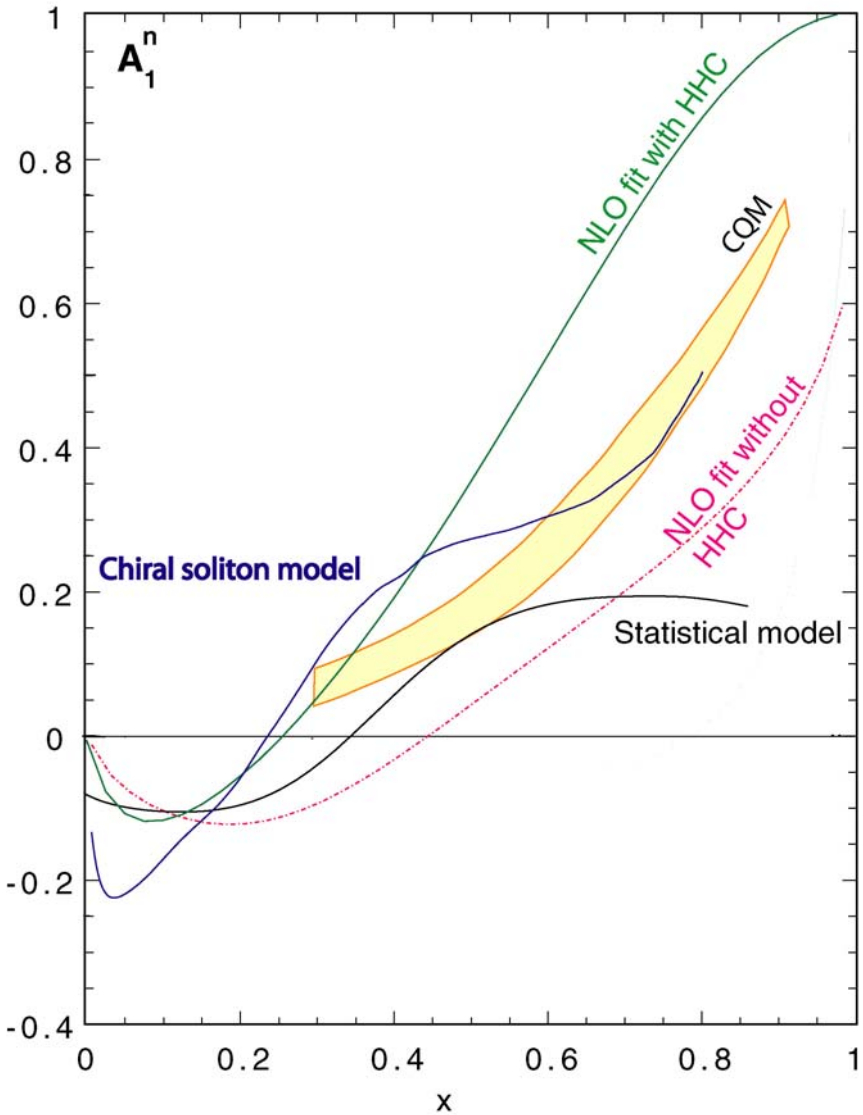
$$\Delta q(x) = q^+(x) - q^-(x) \qquad A_1(x) = \frac{g_1(x)}{F_1(x)}$$

$$g_1(x) = \frac{1}{2}\sum_{flavor} e_f^2 \Delta q_f(x) \qquad g_2(x) = 0$$

Valence Quark Region

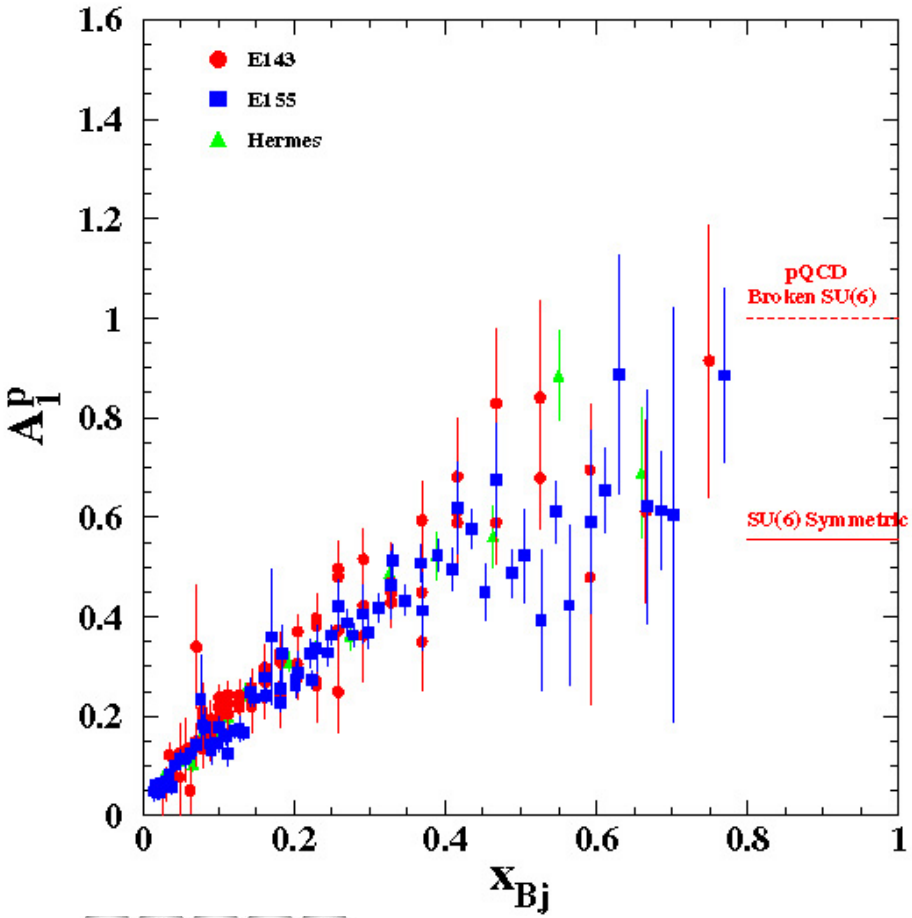
- SU(6) symmetry:
  - $A_1^p = 5/9$ $A_1^n = 0$ $d/u=1/2$
  - $\Delta u/u = 2/3$ $\Delta d/d = -1/3$,
- Broken SU(6) via scalar diquark dominance
  - A_1^p 1 A_1^n 1 d/u 0
  - $\Delta u/u$ → 1 → $\Delta d/d$ → -1/3
- Broken SU(6) via helicity conservation
  - A_1^p 1 A_1^n 1 d/u 1/5
  - $\Delta u/u$ 1 $\Delta d/d$ 1

Note that $\Delta q/q$ as x--> 1 is more sensitive to spin-flavor symmetry breaking effects than A_1

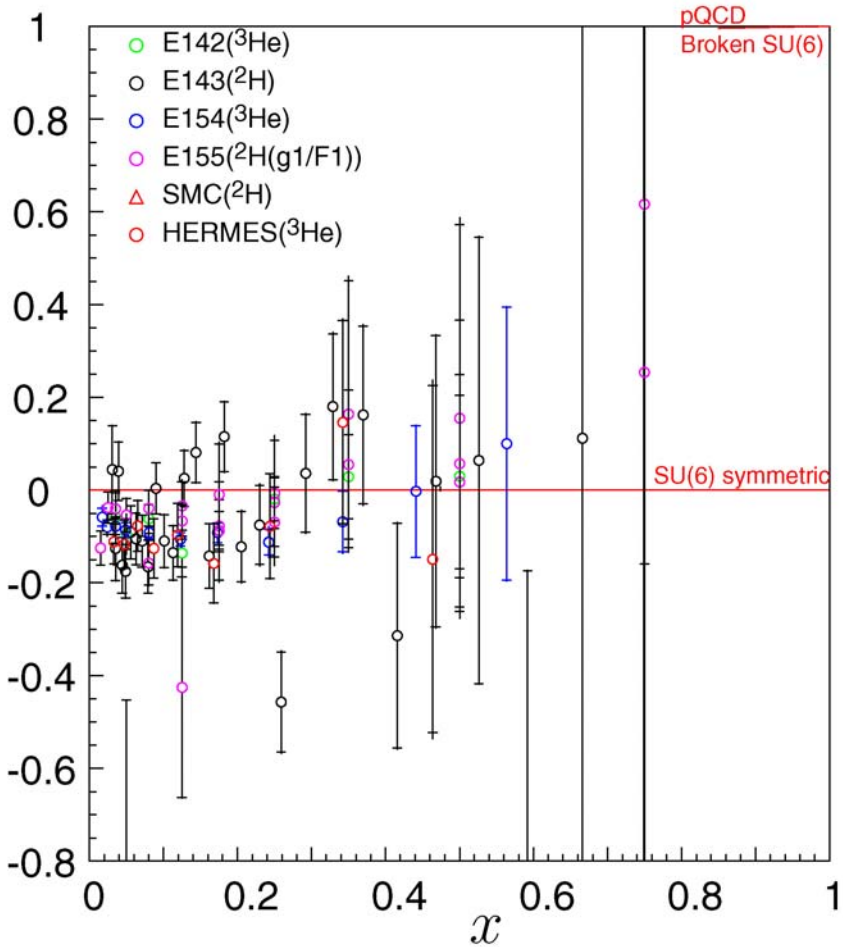


World Data for A_1

Proton

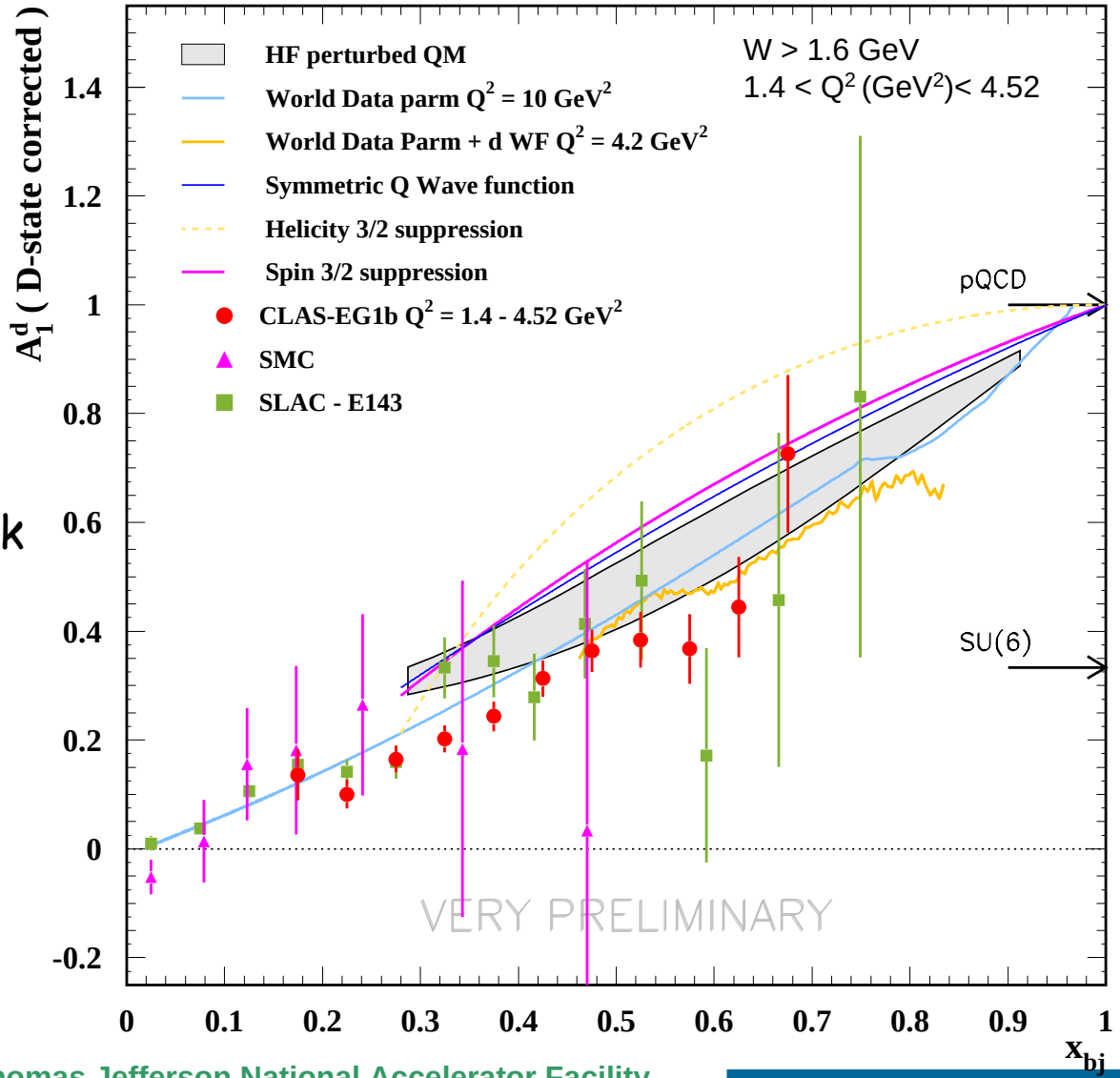


Neutron



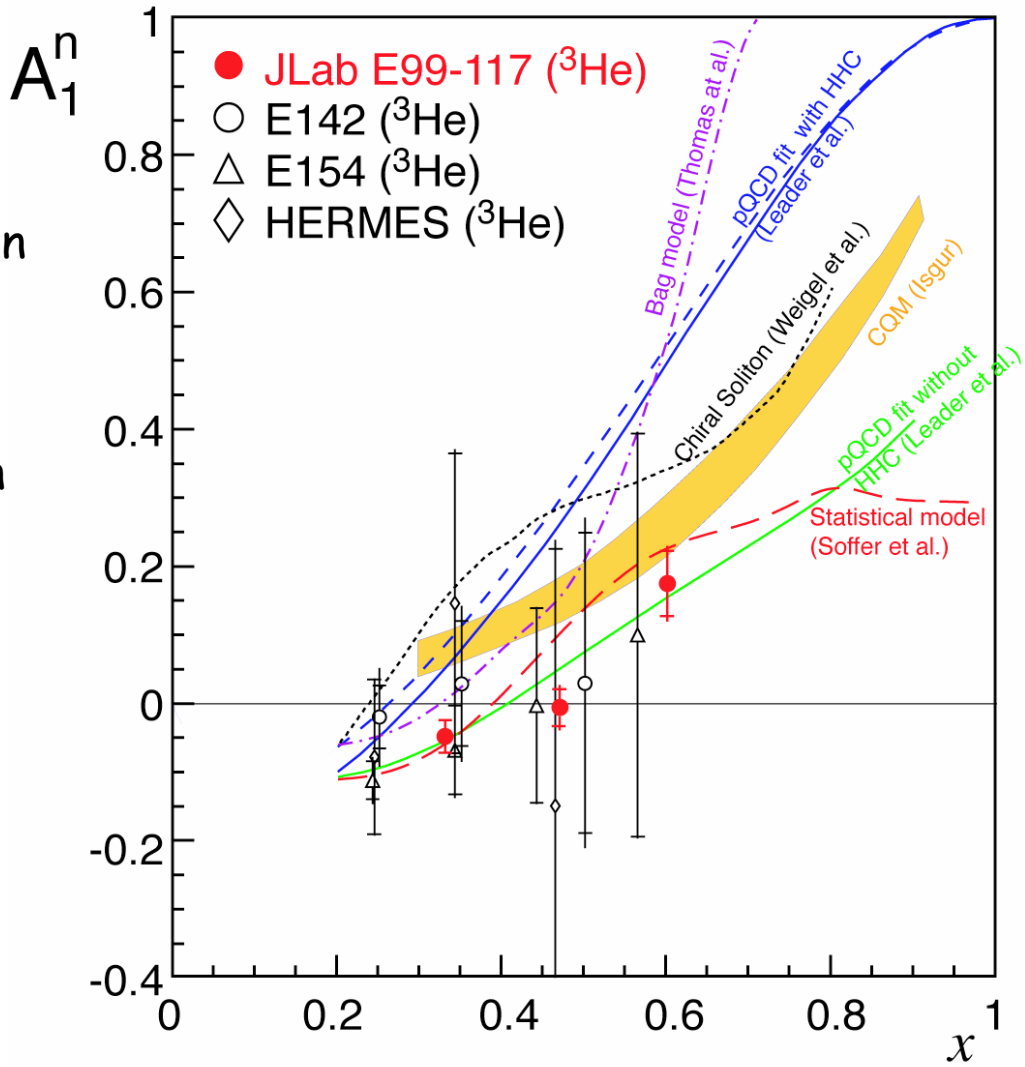
Virtual photon asymmetry $A_1(x)$ for the deuteron

- pQCD
  - Minimal gluon exchanges
  - Spectator pair have opposite helicities
- SU(6)
- Hyperfine perturbed QM
  - makes S=1 pairs more energetic than S=0 pairs
  - At large x the struck quark carries the spin of the nucleon

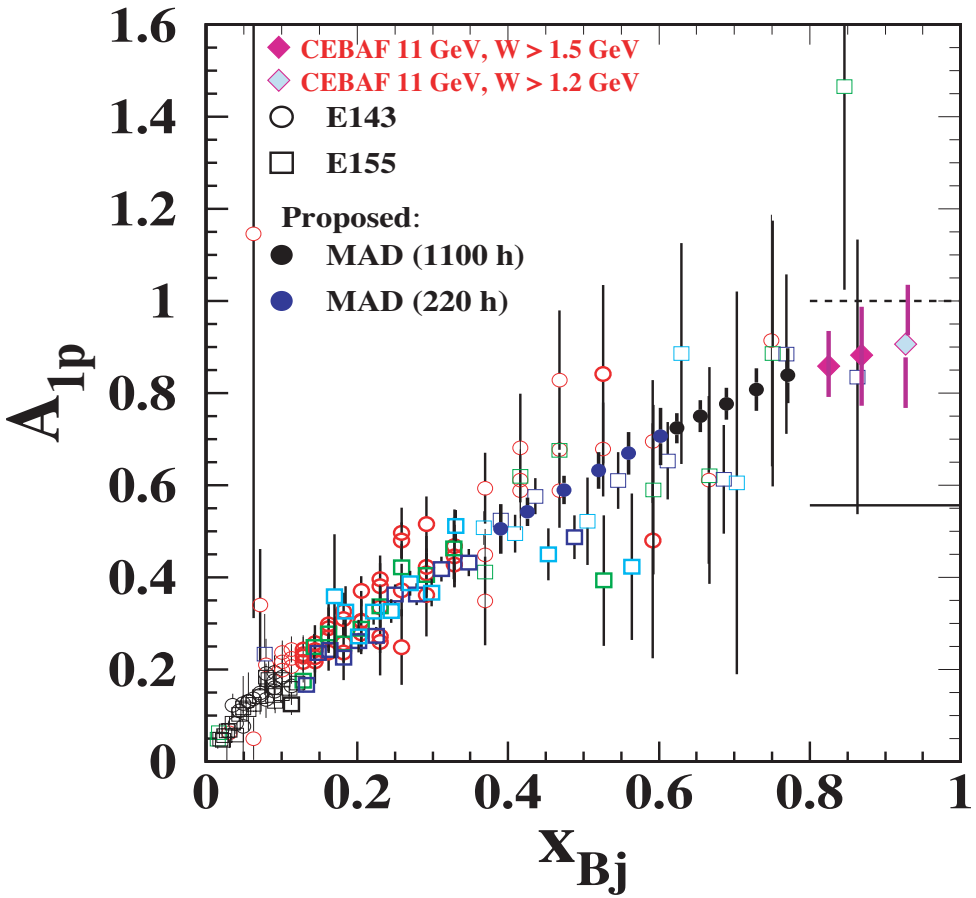


New Hall A data on A_1^n (E99-117)

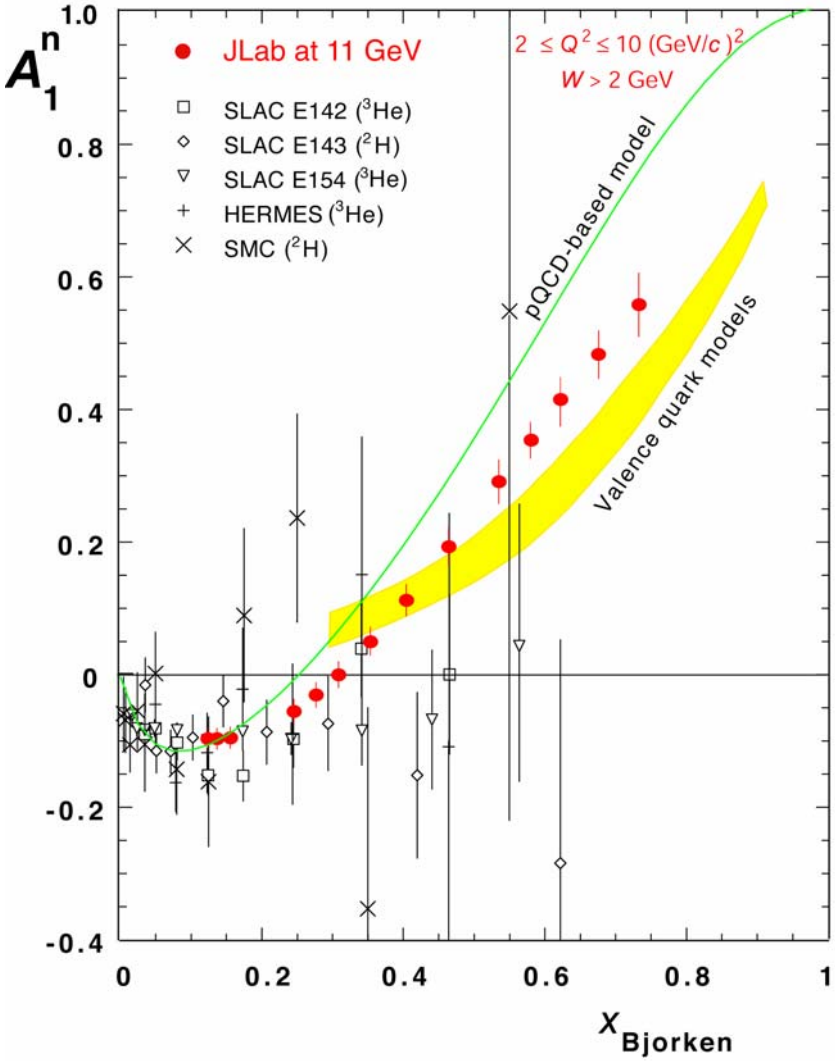
Data provide first indication that A_1^n deviates from 0 at large x, but are clearly at variance with pQCD prediction assuming Hadron Helicity Conservation



Projected 11 GeV Hall A data on A_1



JLab-12



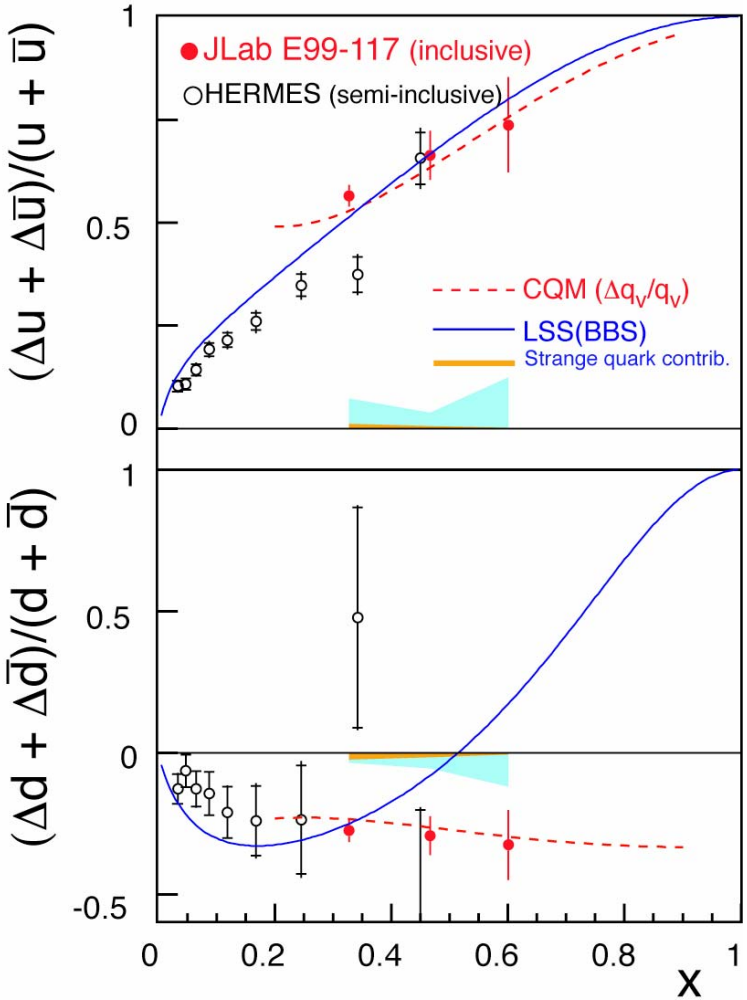
Helicity-Flavor Distributions

$$A_1^n = \frac{\Delta u + 4\Delta d}{u + 4d} \qquad A_1^p = \frac{4\Delta u + \Delta d}{4u + d}$$

$$\frac{\Delta u}{u} = \frac{4}{15} A_1^p \left(4 + \frac{d}{u}\right) - \frac{1}{15} A_1^n \left(1 + 4\frac{d}{u}\right)$$

$$\frac{\Delta d}{d} = \frac{4}{15} A_1^n \left(4 + 1/\frac{d}{u}\right) - \frac{1}{15} A_1^p \left(1 + 4/\frac{d}{u}\right)$$

Use d/u ratio from F_2 on proton and neutron



Summary

- Very active experimental program on nucleon sum rules and spin structure thanks to development of polarized beam (> 100 μA, > 75 %) and polarized targets
- $\Gamma_{1,2}$, d_2 Q^2-evolution of GDH integral
- A_1^n first accurate measurements at intermediate x-values