The Parton-Hadron Transition in Structure Functions and Moments

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Science & Technology Review
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- Introduction: QCD and the Strong Nuclear Force
- The Parton-Hadron Transition in Moments of Structure Functions
- Quark-Hadron Duality: How Local is the Transition?
- Applications and Theoretical Understanding
- Summary

How are the Nucleons Made from Quarks and Gluons?

How do we understand QCD in the confinement regime?

A) What are the spatial distributions of u, d, and s quarks in the hadrons?

B) What is the excited state spectrum of the hadrons, and what does it reveal about the underlying degrees of freedom?

C) What is the QCD basis for the spin structure of the hadrons?

Q^2 evolution of structure functions and their moments
Extended GDH sum rule, Bjorken sum rule

D) What can other hadron properties tell us about 'Strong QCD'?

Inclusive Resonance Electroproduction and Quark-Hadron Duality
Q^2 evolution of structure functions and their moments

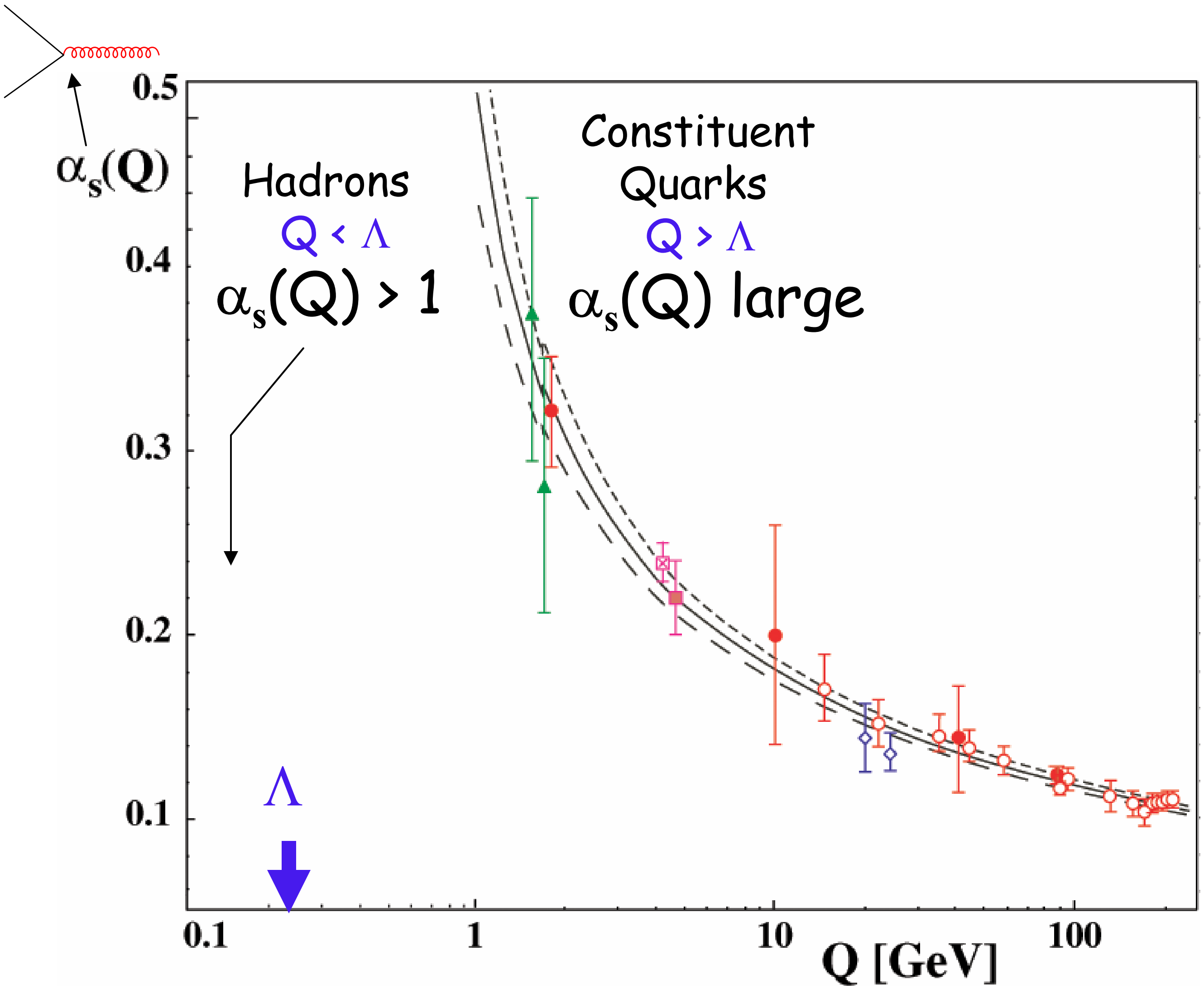
QCD and the Strong Nuclear Force

QCD has the most bizarre properties of all the forces in nature

- **Confinement:**
  - **restoring force between quarks at large distances equivalent to 10 tons, *no matter how far apart***
- **Asymptotic freedom:**
  - **quarks feel almost no strong force when closer together**

QCD (+ electro-weak) in principle describes all of nuclear physics - at all distance scales - *but how does it work?*

QCD and the Parton-Hadron Transition



One parameter, Λ_{QCD},
~ Mass Scale or Inverse Distance Scale where $\alpha_s(Q) = 1$

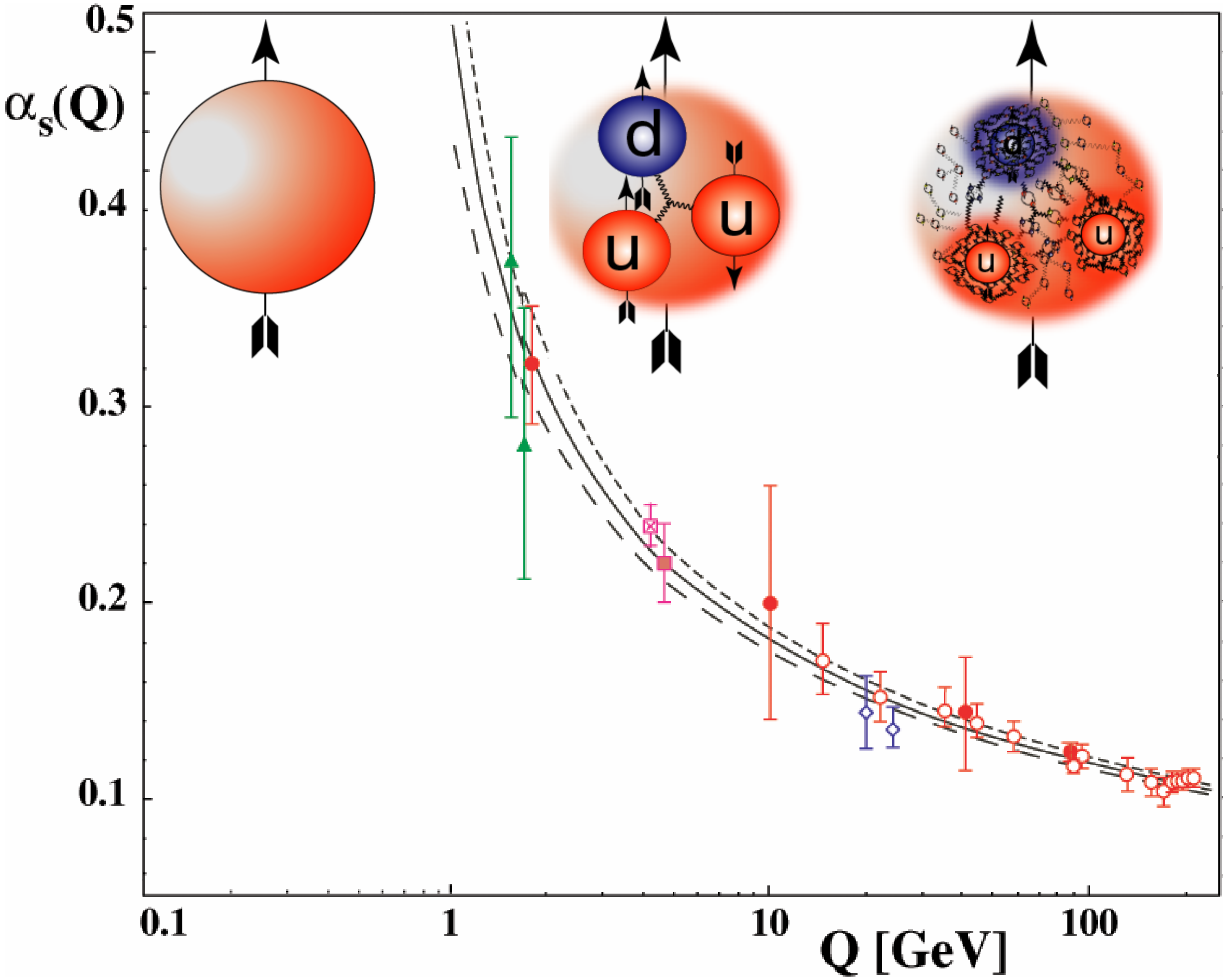
"Separates" Confinement and Perturbative Regions

Mass and Radius of the Proton are (almost) completely governed by

$\Lambda_{QCD} \approx 213$ MeV

Asymptotically Free Quarks
Q >> Λ
$\alpha_s(Q)$ small

QCD and the Parton-Hadron Transition



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"Separates" Confinement
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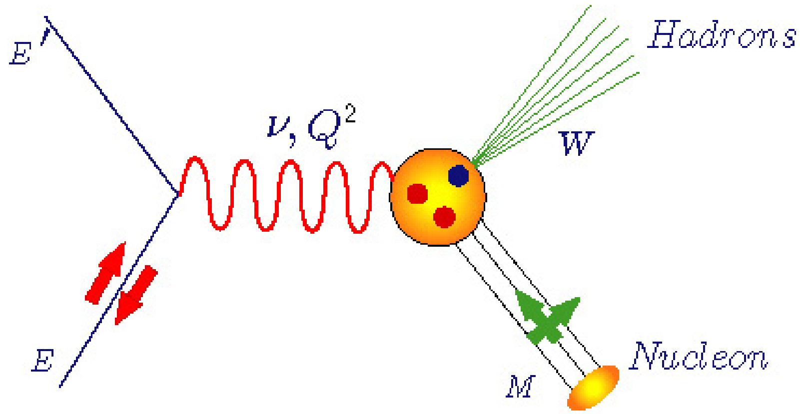
Inclusive Electron Scattering – Formalism



Q^2 : Four-momentum transfer
x : Bjorken variable (=$Q^2/2M\nu$)
ν : Energy transfer
M : Nucleon mass
W : Final state hadronic mass

U $$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Uparrow + \uparrow\Uparrow) = \frac{8\alpha^2\cos^2(\theta/2)}{Q^4}\left[\frac{F_2(x,Q^2)}{\nu} + \frac{2F_1(x,Q^2)}{M}\tan^2(\theta/2)\right]$$

- Unpolarized structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$, or $F_T(x,Q^2)$ [$=2xF_1(x,Q^2)$] and $F_L(x,Q^2)$, to separate by measuring $R = \sigma_L/\sigma_T$
- Polarized structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$

L $$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Uparrow - \uparrow\Uparrow) = \frac{4\alpha^2}{MQ^2}\frac{E'}{\nu E}\left[(E+E'\cos\theta)g_1(x,Q^2) - \frac{Q^2}{\nu}g_2(x,Q^2)\right]$$

T $$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Rightarrow - \uparrow\Rightarrow) = \frac{4\alpha^2\sin\theta}{MQ^2}\frac{E'^2}{\nu^2 E}\left[\nu g_1(x,Q^2) + 2E g_2(x,Q^2)\right]$$

The Parton-Hadron Transition in Structure Functions and Moments

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QCD and Moments

- Moments of the Structure Function $M_n(Q^2) = \int_0^1 dx\ x^{n-2}F(x,Q^2)$

Let $F(x,Q^2) = xf(x)$ be a probability density distribution, describing the velocity distribution in a dilute gas

$M_1 = 1$

$M_2 = \langle x \rangle$ average velocity

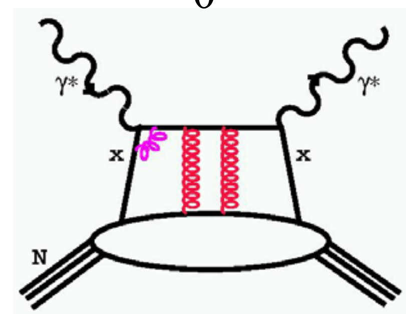
$M_3 = \langle x^2 \rangle$ variance

etc.

In QCD the moments are also dependent on Q^2 due to Q^2 evolution of probability density distributions $\rightarrow$ $\langle x \rangle$, $\langle x^2 \rangle$ vary with Q^2

QCD and the Operator-Product Expansion

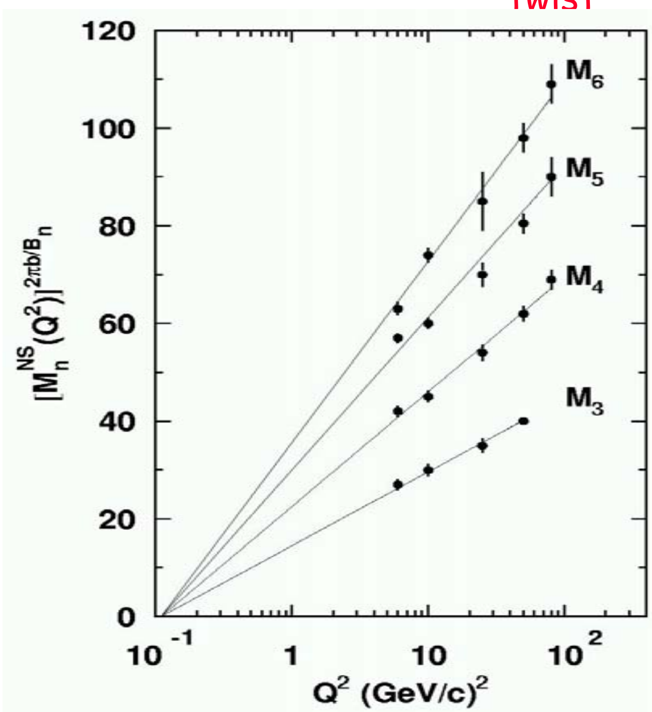
- Moments of the Structure Function $M_n(Q^2) = \int_0^1 dx\ x^{n-2} F(x,Q^2)$
- Operator Product Expansion



$$M_n(Q^2) = \sum_{k=1}^{\infty} (nM_0^2/\ Q^2)^{k-1}\ B_{nk}(Q^2)$$

higher twist

logarithmic dependence

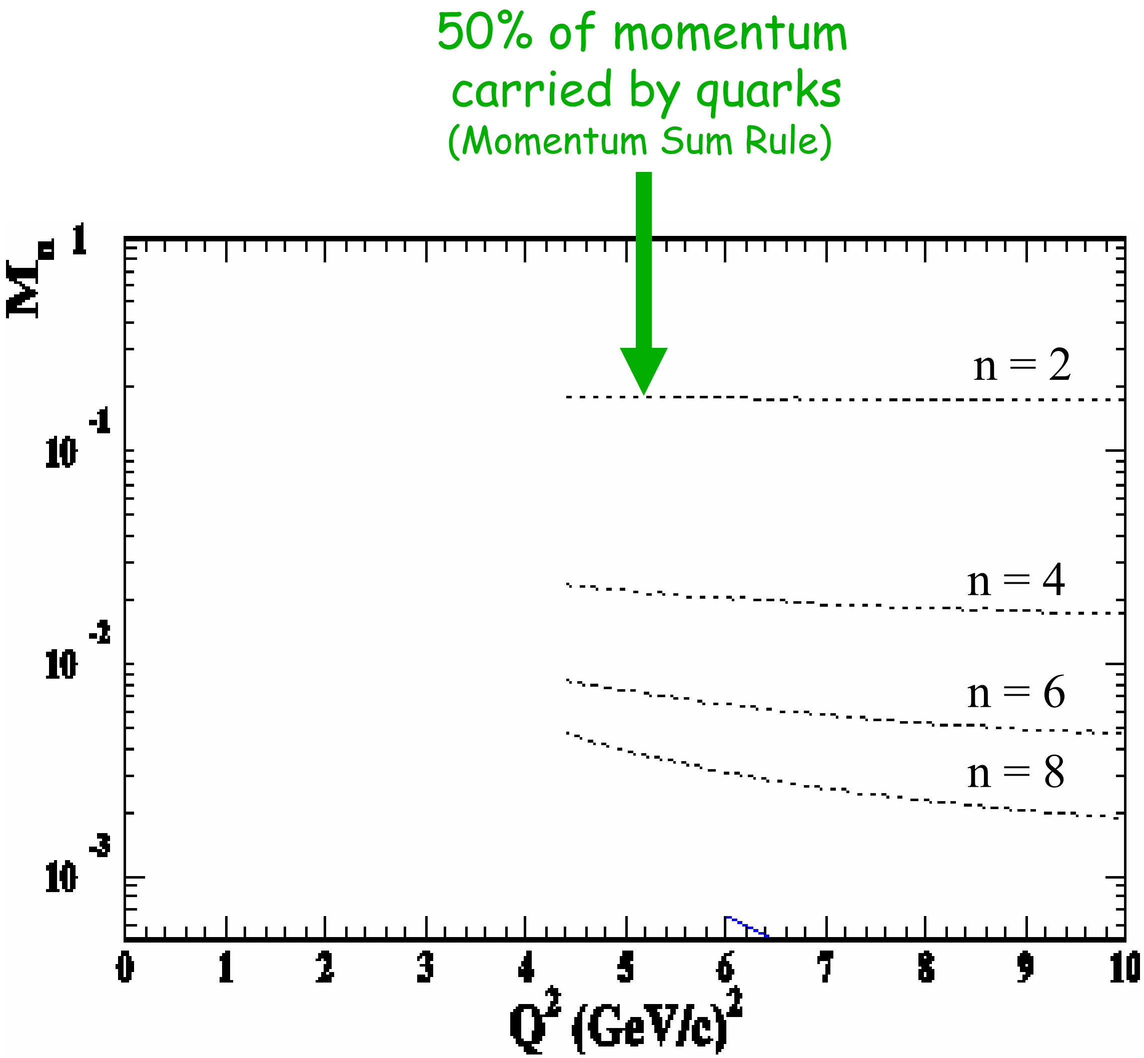


At High Q^2 : $\ln(Q^2)$ dependence of moments one of first "proofs" of QCD
→ Λ(QCD)

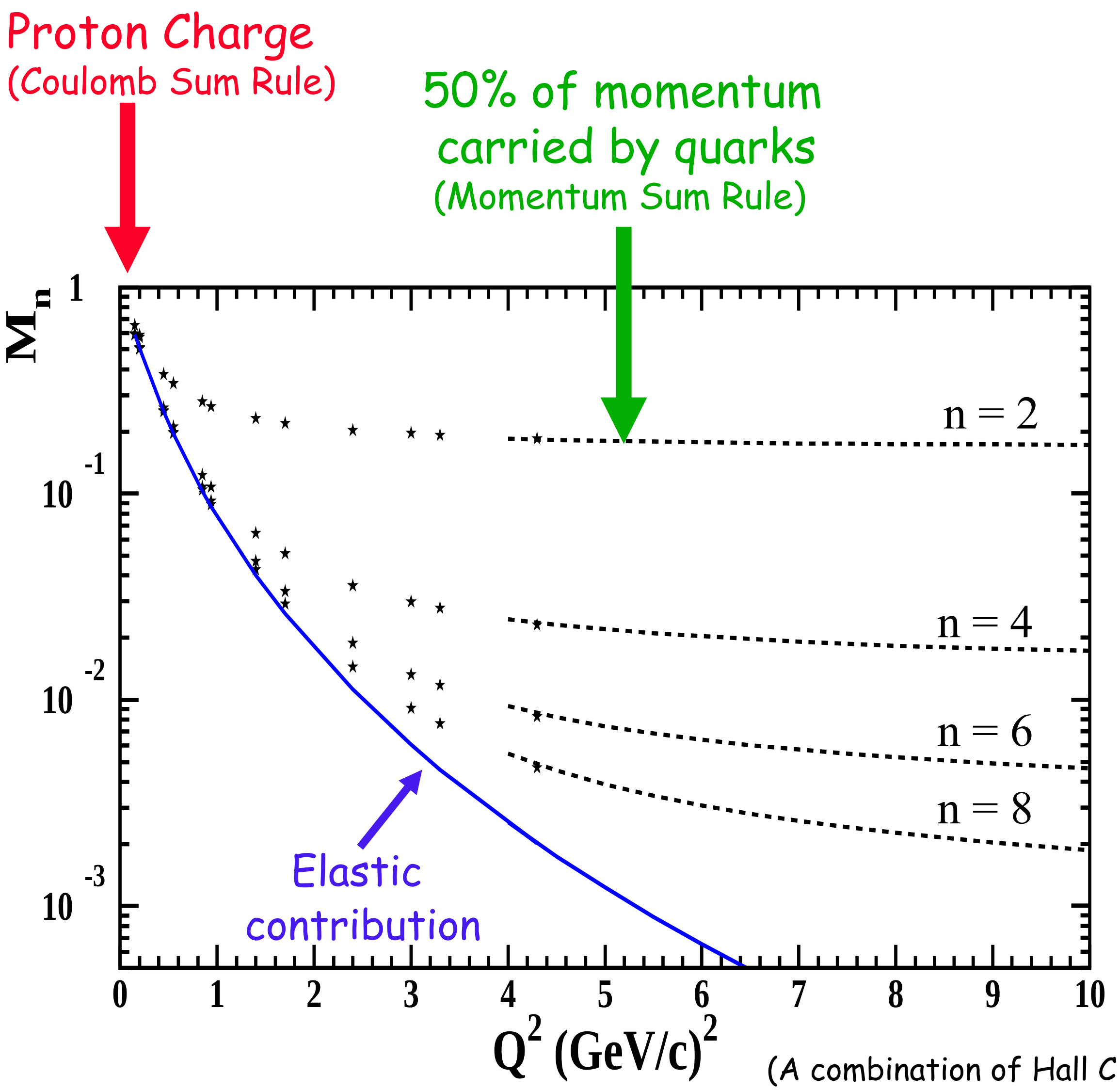
At Low Q^2 : Unique regime for JLab to determine $(1/Q^2)^m$
→ Higher Twist effects

Lowest moments are calculable in Lattice QCD (LQCD): least computer intensive!

Moments of F_2^p @ Low Q^2



Moments of F_2^p @ Low Q^2



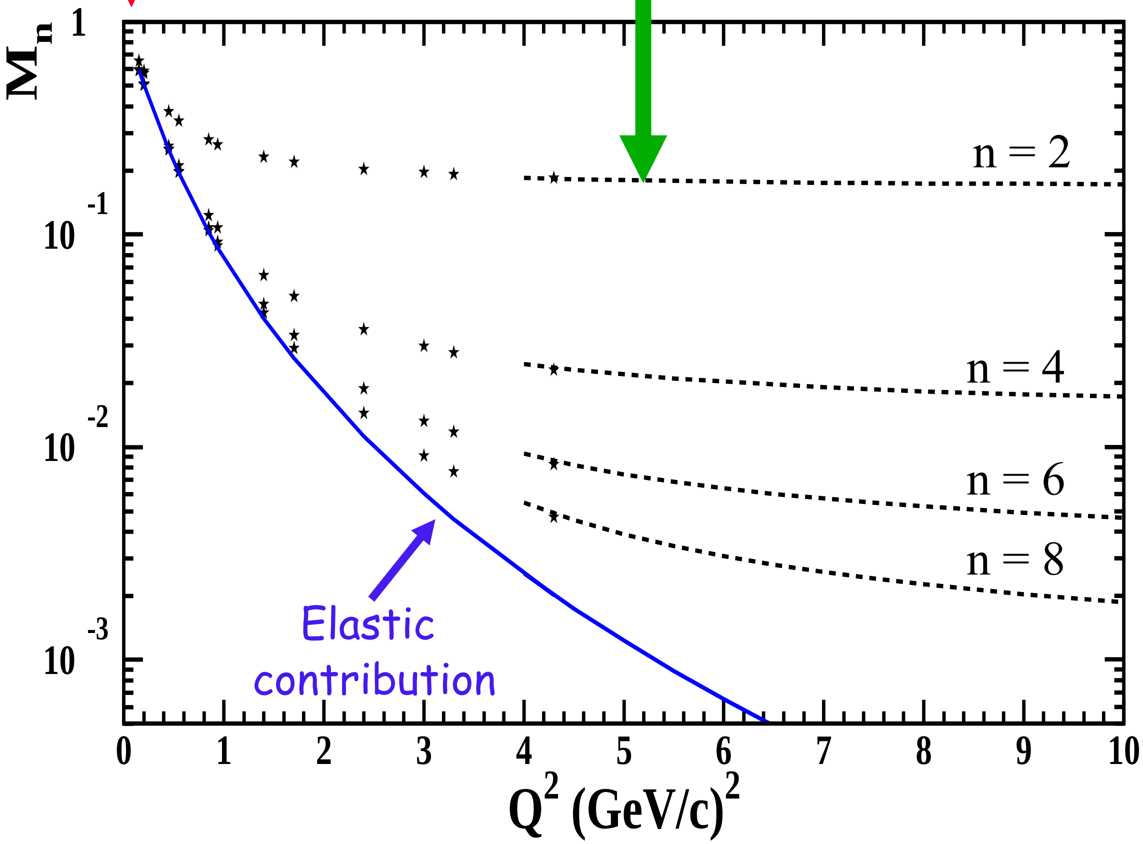
(A combination of Hall C + CLAS data presented by VB to constrain the Constituent Quark radius, here only Hall C data are shown)

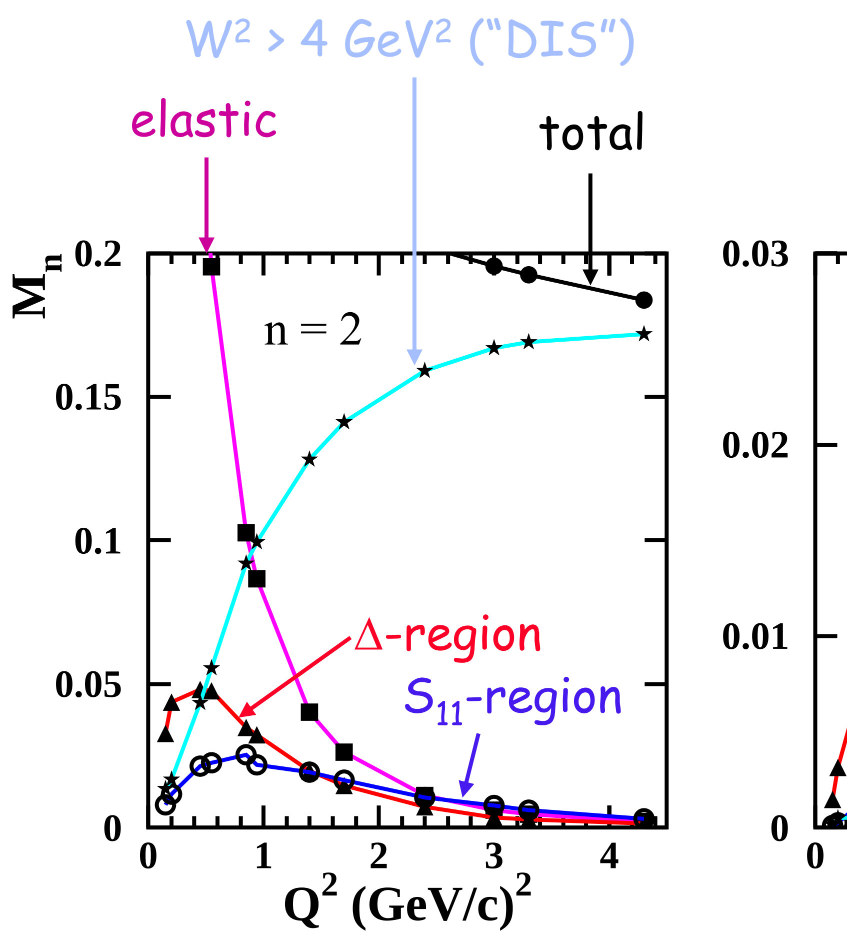
Moments of F_2^p @ Low Q^2

Proton Charge
(Coulomb Sum Rule)

50% of momentum
carried by quarks
(Momentum Sum Rule)

@ $Q^2 = 2\ (GeV/c)^2$ 30% of M_2
comes from the resonance region

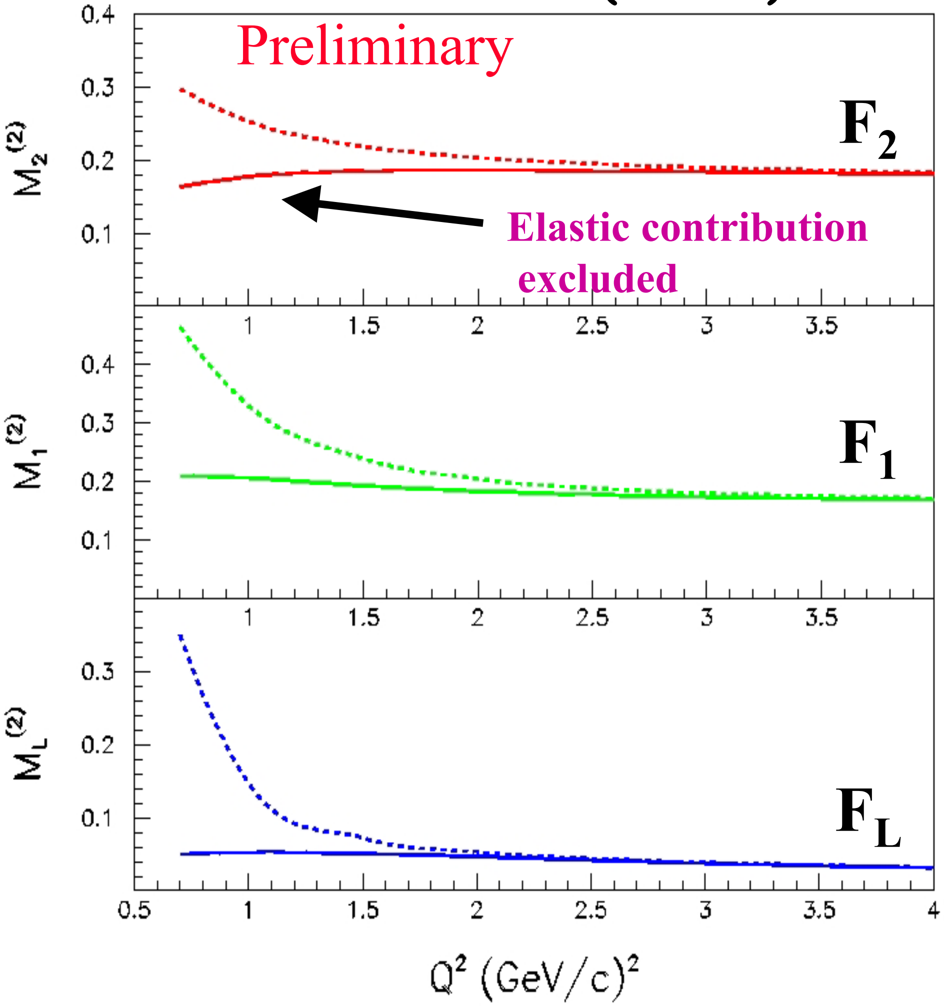




n = 2 Moments of F_2, F_1 and F_L: $M_n(Q^2) = \int_0^1 dx\, x^{n-2} F(x,Q^2)$

DIS: SLAC fit to F_2 and R

Resonances: E94-110 (Hall C) fit



Elastic Contributions

$$F_1^{EL} = G_M^2\, \delta(x-1)$$

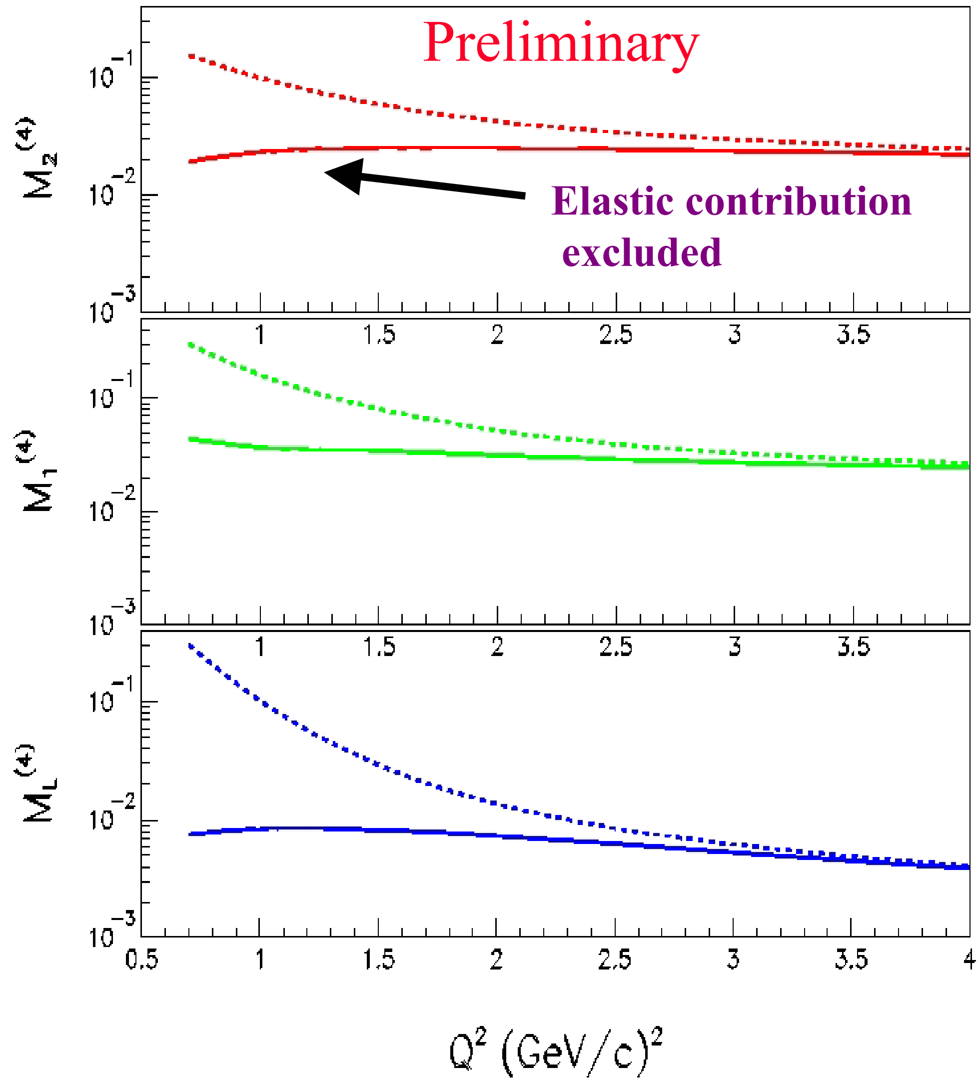
$$F_2^{EL} = \frac{(G_E^2 + \tau G_M^2)\delta(x-1)}{1+\tau}$$

$$\tau = Q^2/4M_p^2$$

$$F_L^{EL} = G_E^2\, \delta(x-1)$$

Flat Q^2 dependence → small higher twist! - not true for contributions from the elastic peak (bound quarks)

n = 4 Moments of F_2, F_1 and F_L



Neglecting elastic contribution, n = 4 moments have only a small Q^2 dependence as well.

Momentum sum rule

$$M_L^{(n)} = \alpha_s(Q^2)\{ \frac{4M_2^{(n)}}{3(n+1)} + \frac{2c\int dx\, xG(x,Q^2)\}}{(n+1)(n+2)}$$

Gluon distributions!

This is only at leading twist and for zero proton mass

⇒ Must remove non-zero proton mass effects from data to extract moment of $xG(x,Q^2)$

⇒ Work in progress

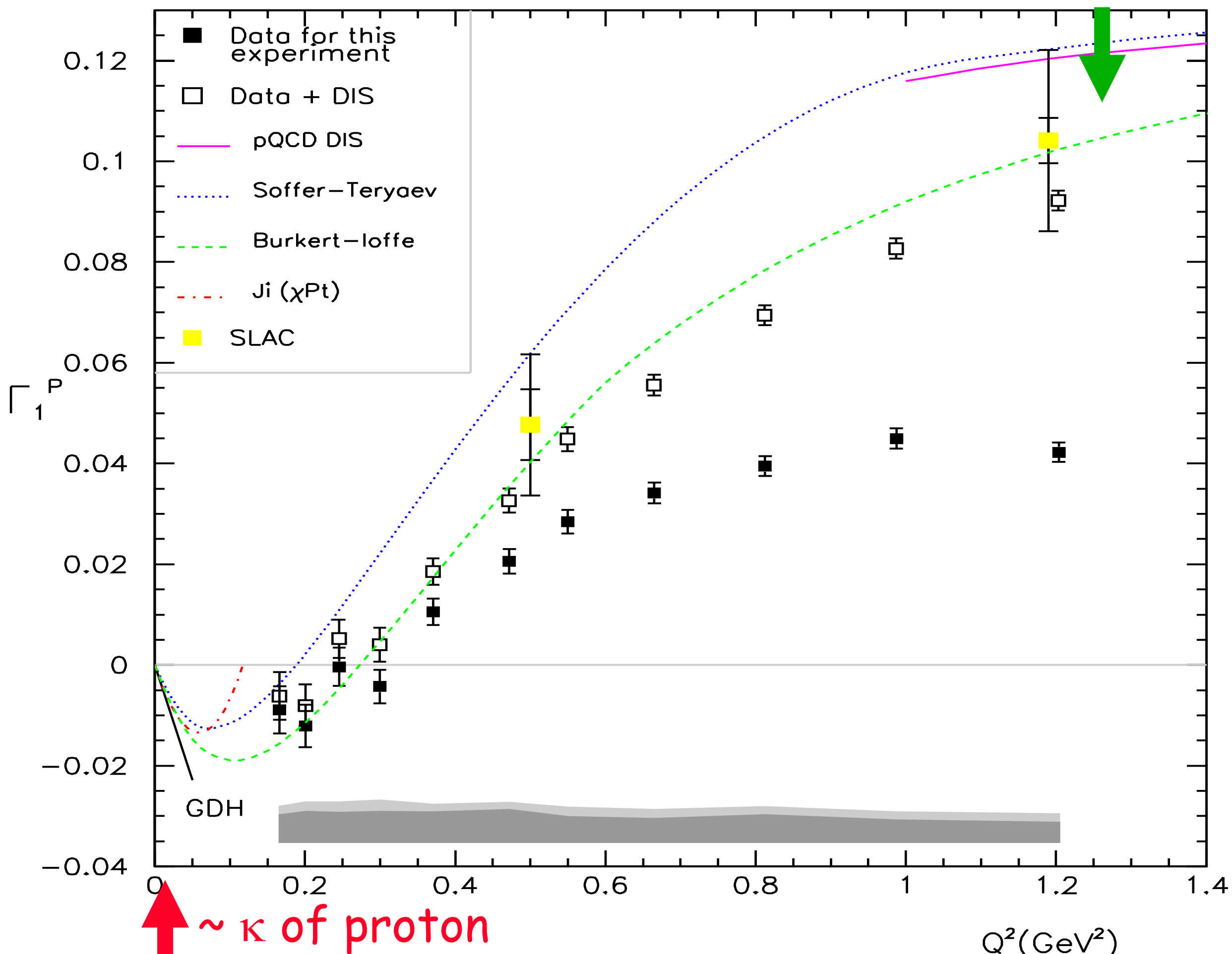
Moments of g_1^p ($=\Gamma_1^p$)

Elastic not included in Moment as shown →

- With Elastic included no zero crossing, and Q^2 dependence far smoother

30% of Spin carried by quarks
(Ellis-Jaffe Sum Rule)

CLAS EG1 Data



~ κ of proton
(GDH Sum Rule)

Moments of g_1^p $(=\Gamma_1^p)$

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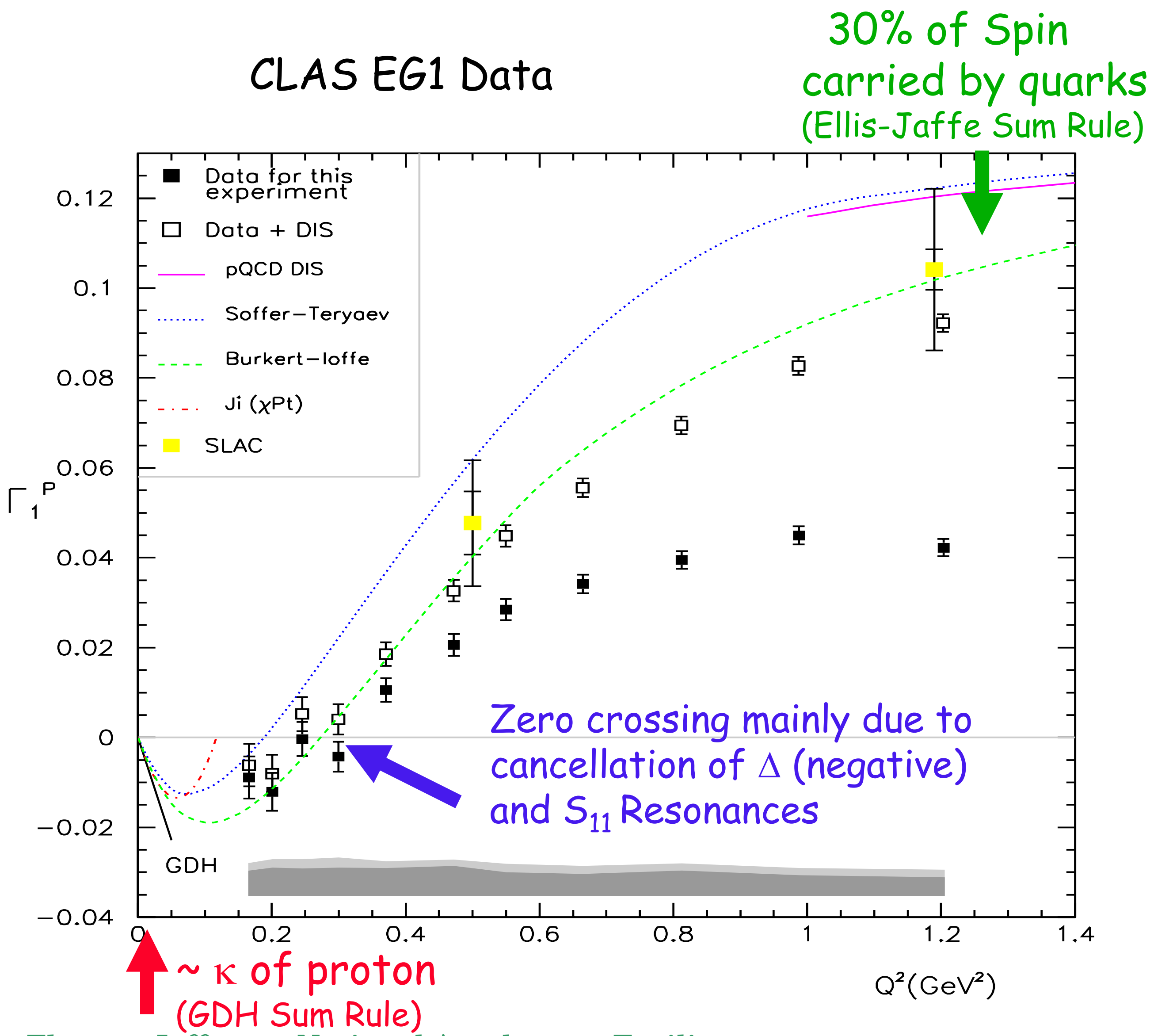

Moments of g_1^p (=Γ_1^p)

CLAS EG1 Data

30% of Spin carried by quarks
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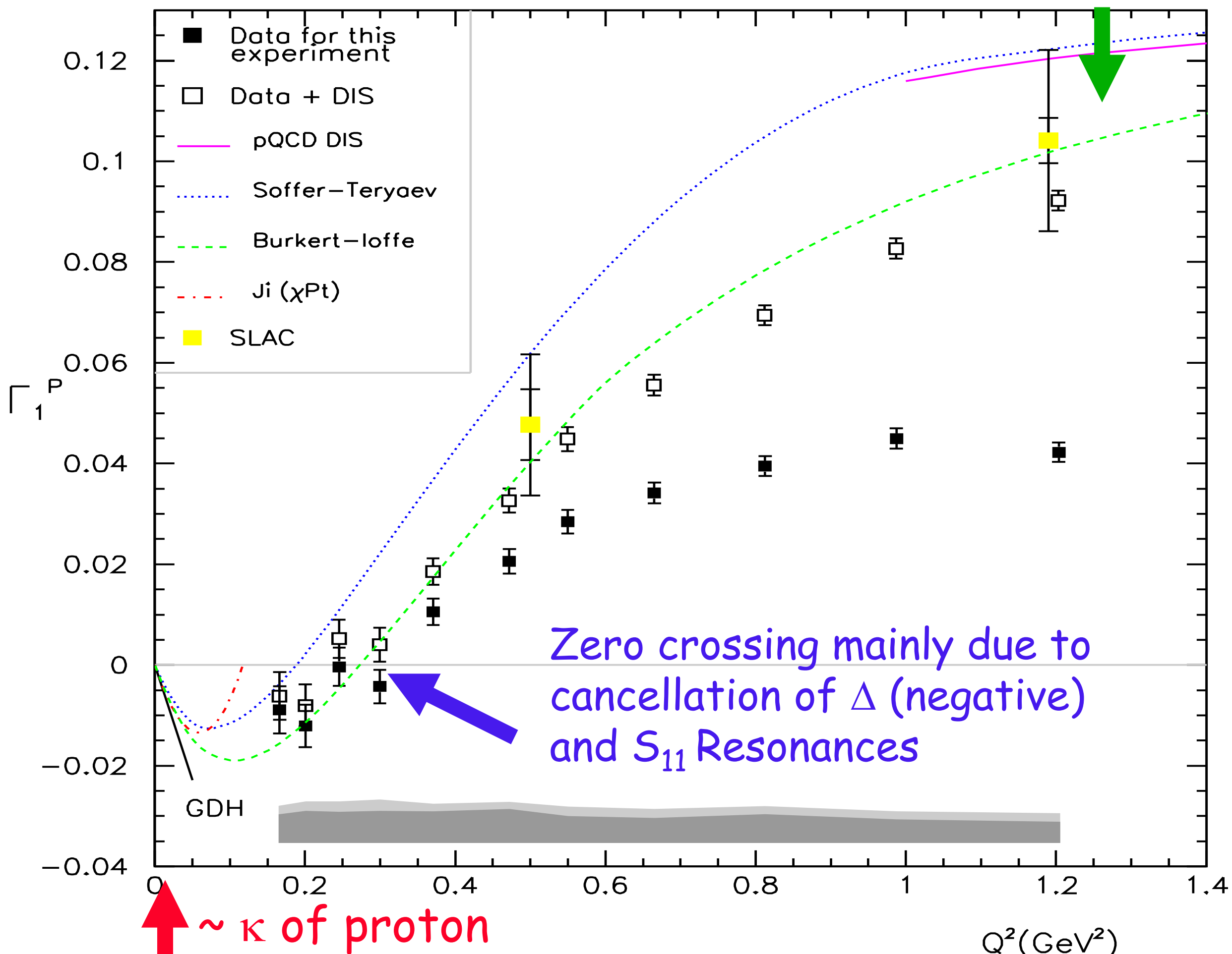
$$\frac{\sigma^{\downarrow\Uparrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\downarrow\Uparrow} + \sigma^{\uparrow\Uparrow}} = \sigma_{1/2} - \sigma_{3/2}$$

SU(6) unbroken: $M_p = M_\Delta$
Δ ground state of $\sigma_{3/2}$

In this scenario the Δ is a δ function and all higher twist → it plays the same role as the elastic in F_2



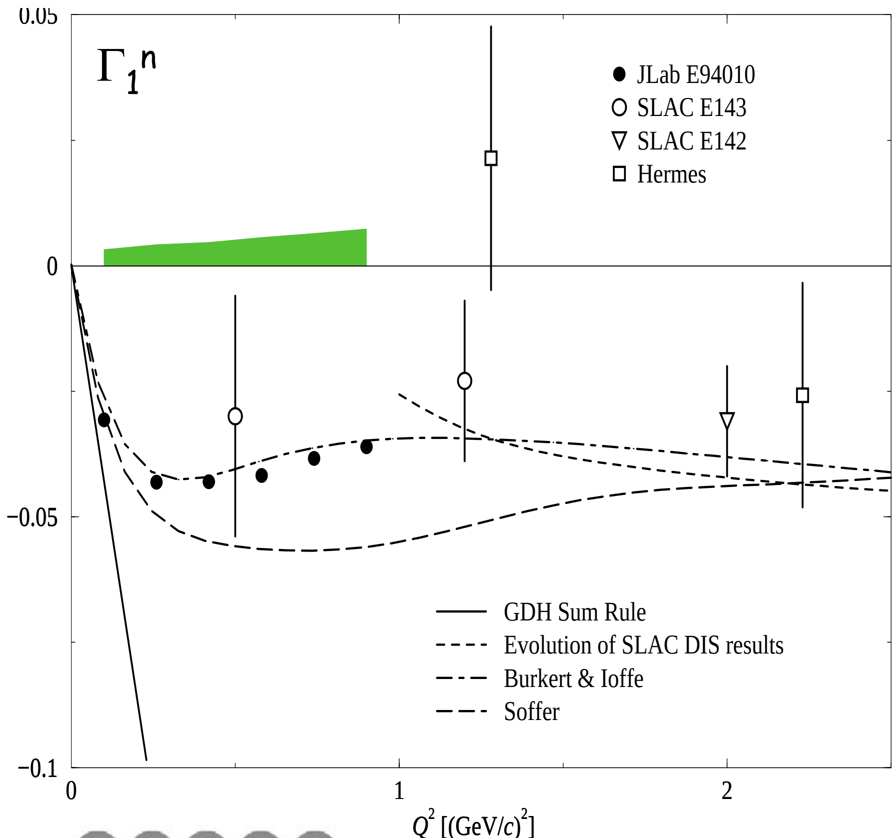
Zero crossing mainly due to cancellation of Δ (negative) and S_{11} Resonances

~ κ of proton
(GDH Sum Rule)

Moments of g_1^n and g_1^p-g_1^n

Hall A $^3\vec{He}(\vec{e},e')$ to extract g_1^n and its moment Γ_1^n (E94-010)

- Similar ideas as with proton
- Here, whole region negative

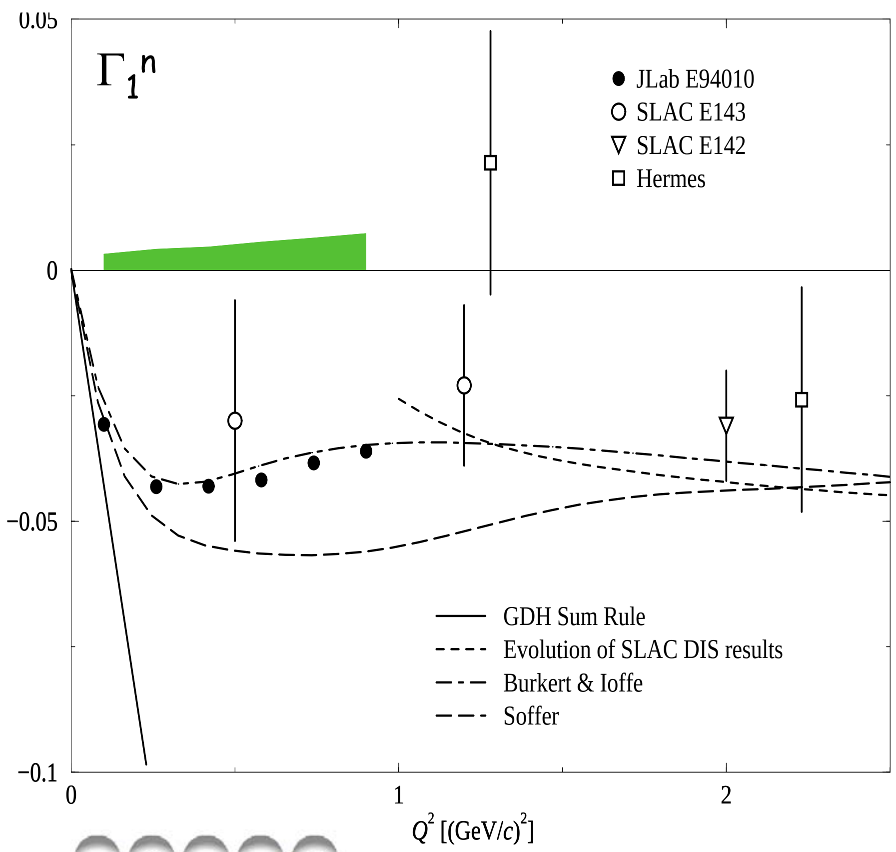


Moments of g_1^n and $g_1^p - g_1^n$

Hall A $^3\vec{He}(\vec{e},e')$ to extract g_1^n and its moment Γ_1^n

- Similar ideas as with proton
- Here, whole region negative

Combine Hall A g_1^n with Hall B g_1^p data



Moments of g_1^n and $g_1^p-g_1^n$

Hall A $^3\vec{He}(\vec{e},e')$ to extract g_1^n and its moment Γ_1^n

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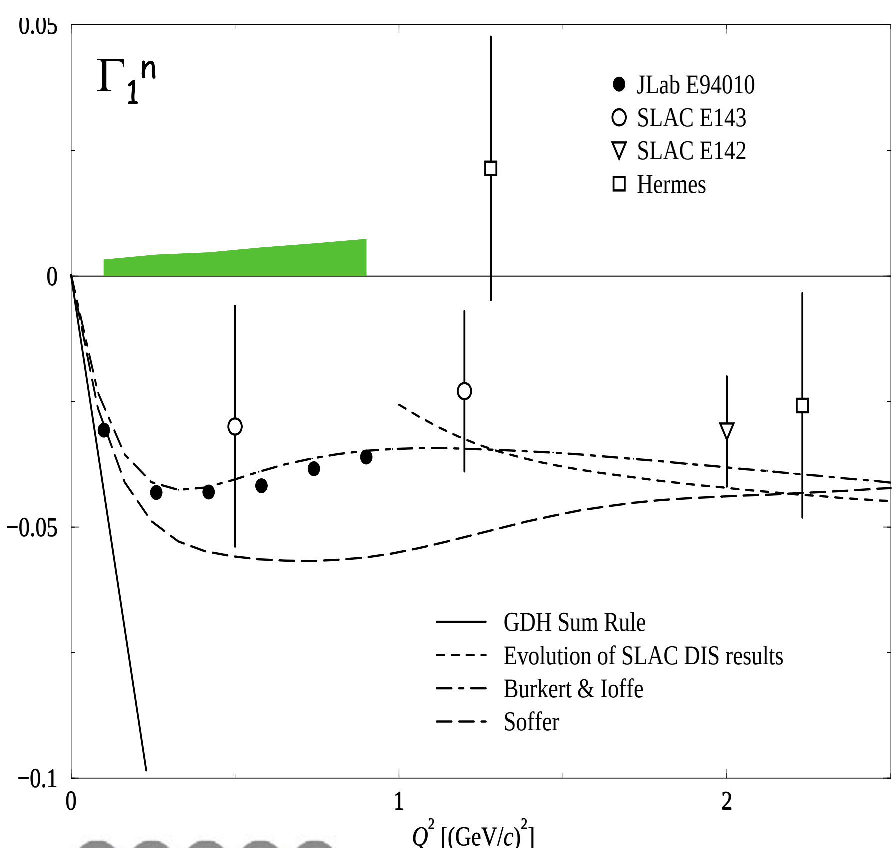
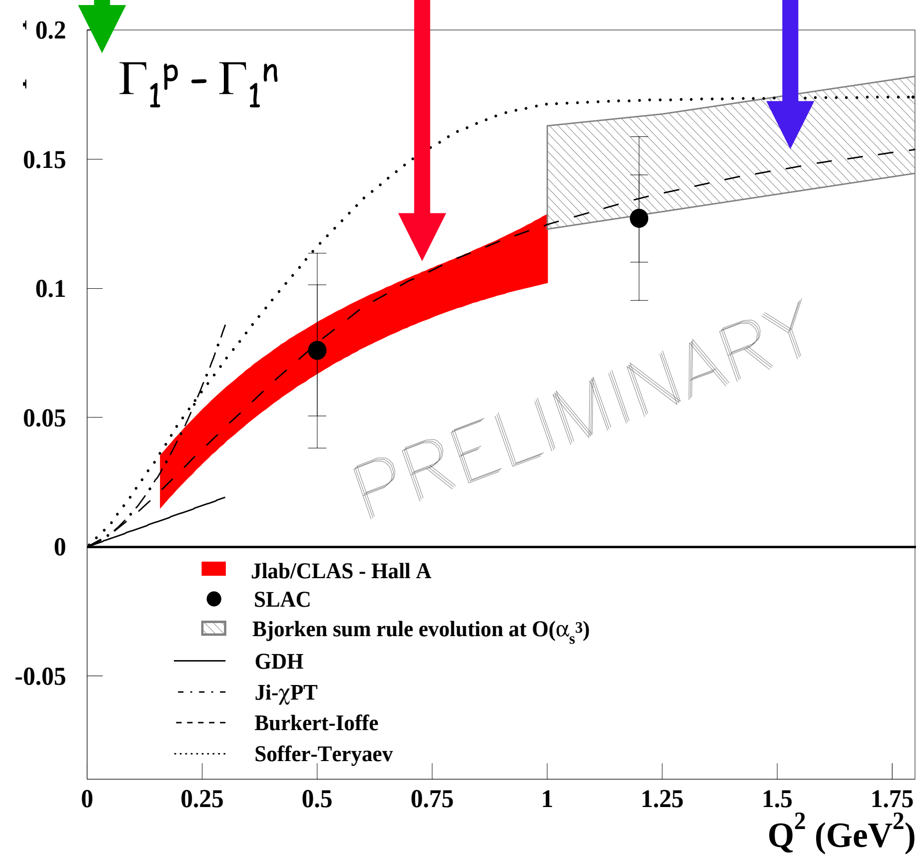
Combine Hall A g_1^n with Hall B g_1^p data

Chiral Perturbation Theory (χPT)

Bjorken Sum Rule
(Verification of QCD)





Summary I - Moments

Resonances are an integral part of the Structure Function Moments at Low Q^2

(Note: QCD deals with the Moments and does not care what contributes to the moments!)

The Structure Function Moments have a smooth behavior as a function of Q^2, and in fact pick up almost uniquely the quark-quark interactions in the SU(6) ground states

This seems to indicate that the parton-hadron transition occurs in a local and small region, with only few resonances

→ "Quark-Hadron Duality"

→ Let us examine the structure functions themselves rather than their moments

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Quark-Hadron Duality

complementarity between quark and hadron descriptions of observables

At high enough energy:

Hadronic Cross Sections averaged over appropriate energy range

Perturbative Quark-Gluon Theory

$$\sum_{\text{hadrons}} = \sum_{\text{quarks+gluons}}$$

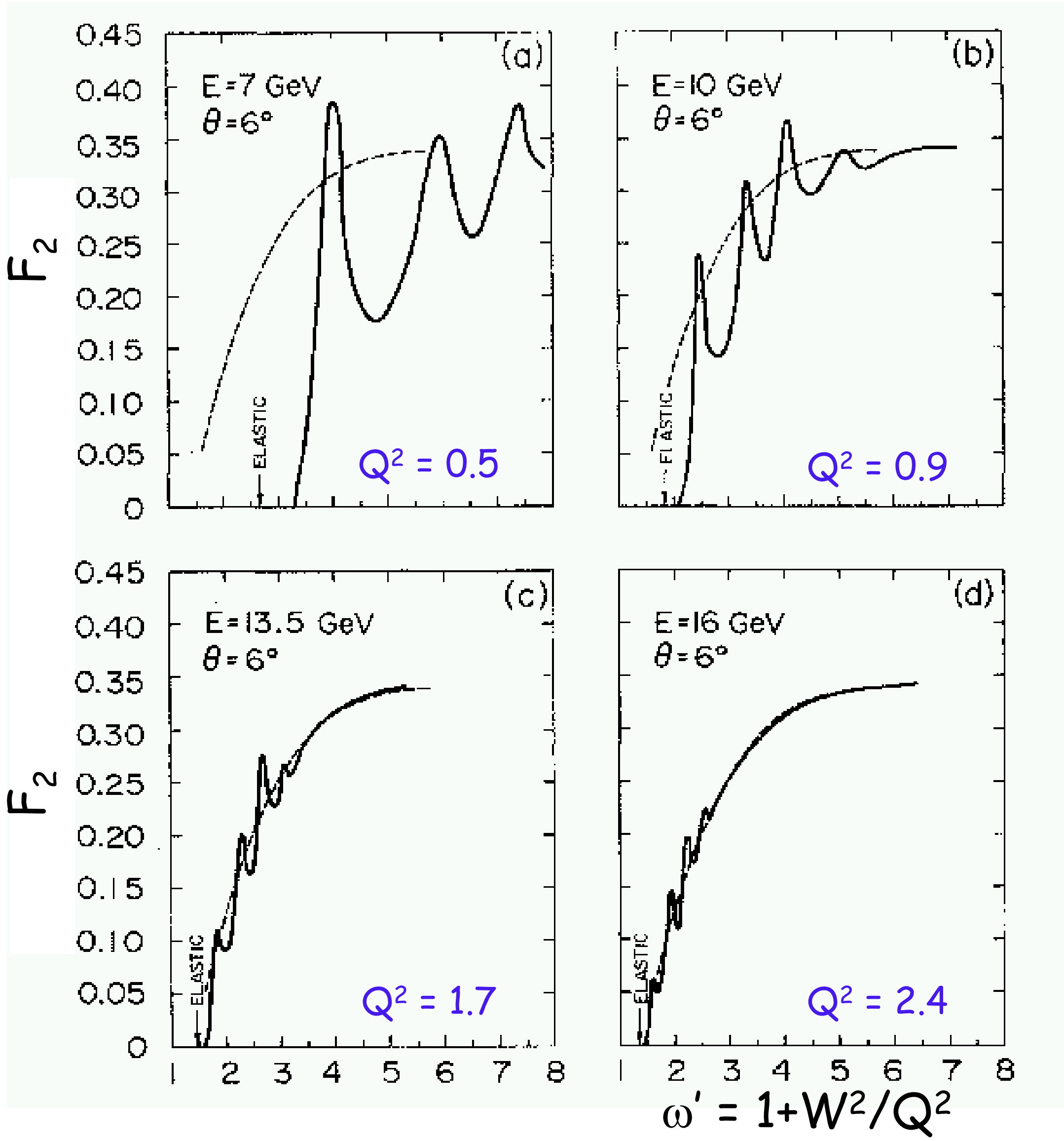
Can use either set of complete basis states to describe physical phenomena

But why also in limited local energy ranges?

Duality in the F_2 Structure Function

First observed ~1970 by Bloom and Gilman at SLAC by comparing resonance production data with deep inelastic scattering data

- *Integrated F_2 strength in Nucleon Resonance region equals strength under scaling curve. Integrated strength (over all ω') is called Bloom-Gilman integral*

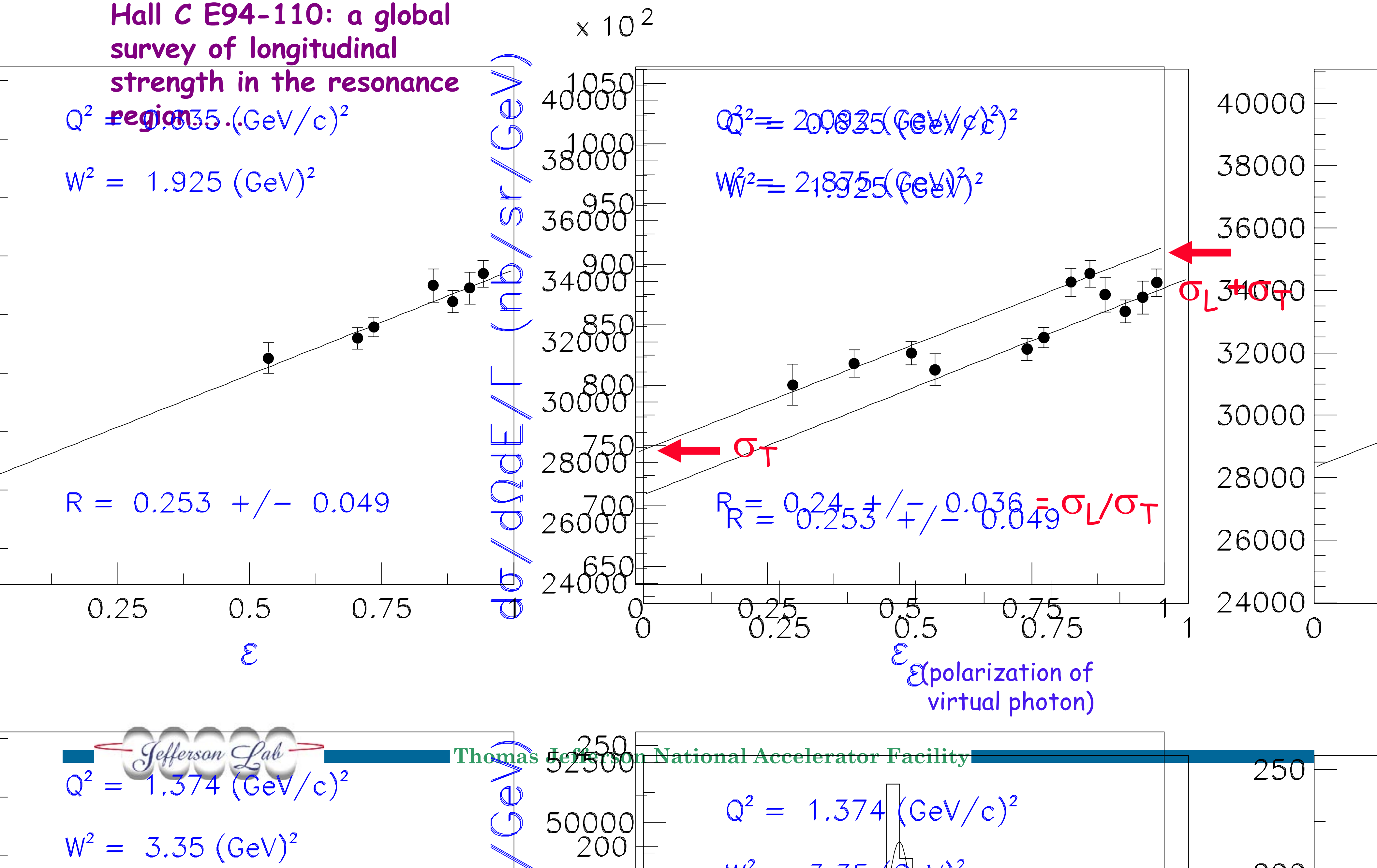


Shortcomings:

- Only a single scaling curve and no Q^2 evolution (Theory inadequate in pre-QCD era)
- No σ_L/σ_T separation → F_2 data depend on assumption of $R = \sigma_L/\sigma_T$
- Only moderate statistics

Rosenbluth Separations

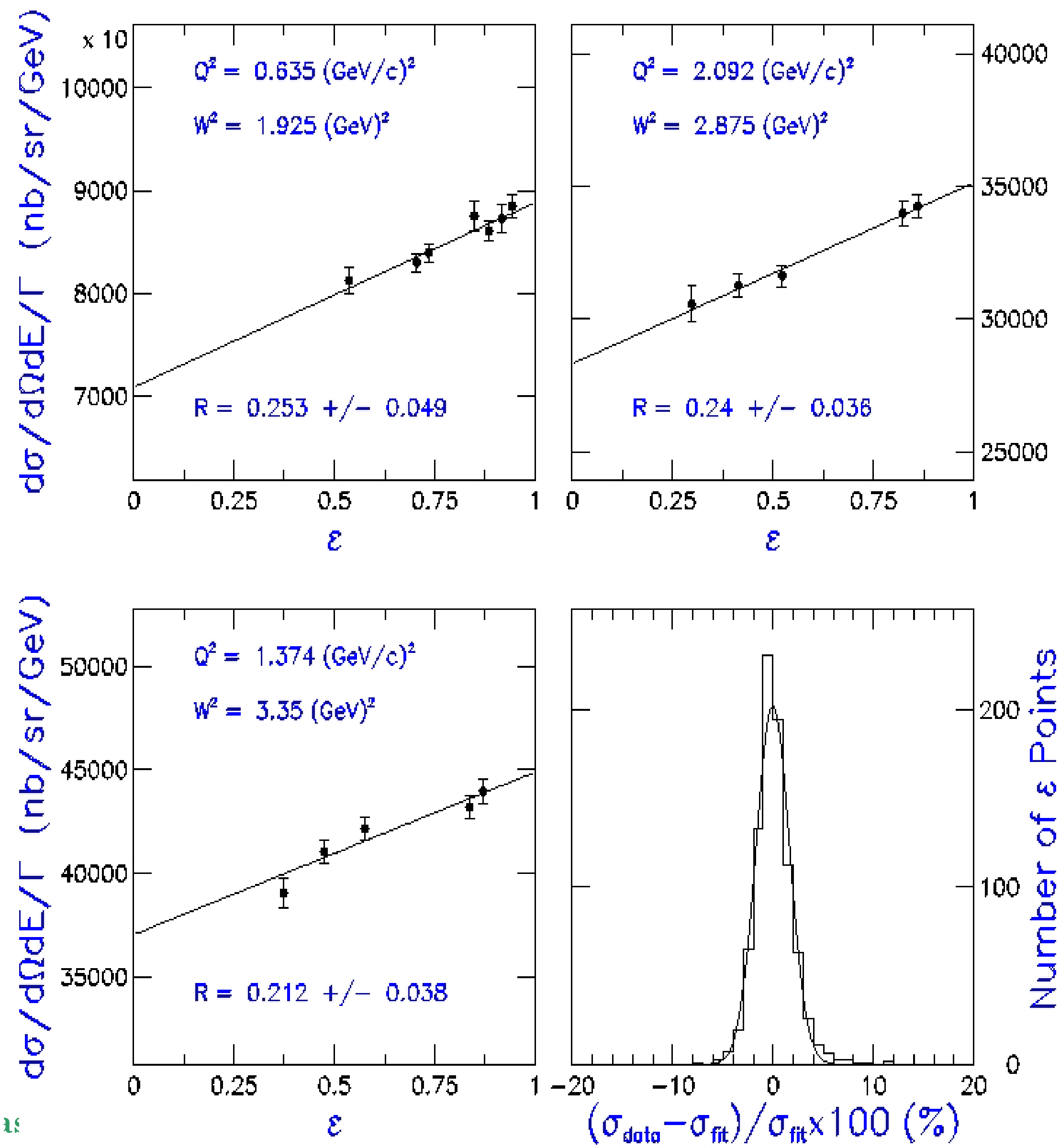
Hall C E94-110: a global survey of longitudinal strength in the resonance region.



Rosenbluth Separations

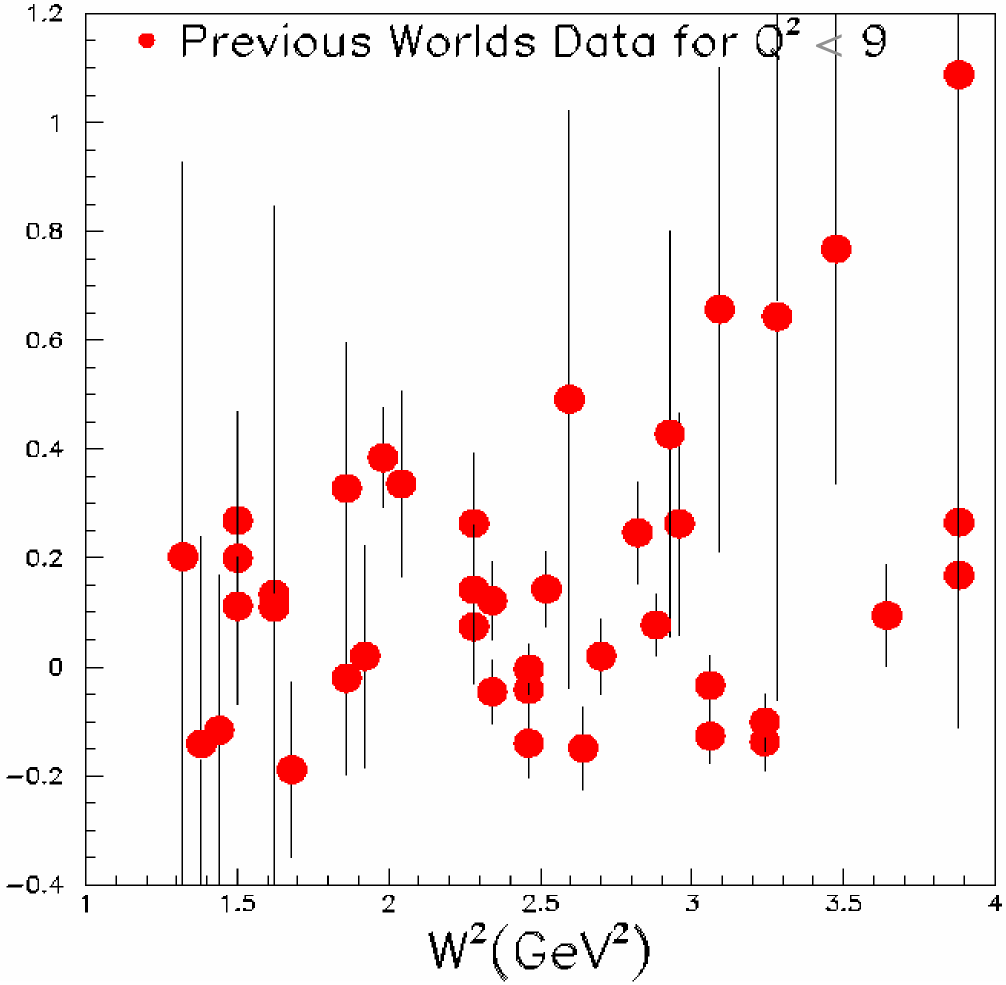
Hall C E94-110: a global survey of longitudinal strength in the resonance region……

- Spread of points about the linear fits is Gaussian with $\sigma \sim 1.6$ % consistent with the estimated point-point experimental uncertainty (1.1-1.5%)
  - a systematic "tour de force"



World's L/T Separated Resonance Data

$R = \sigma_L/\sigma_T$



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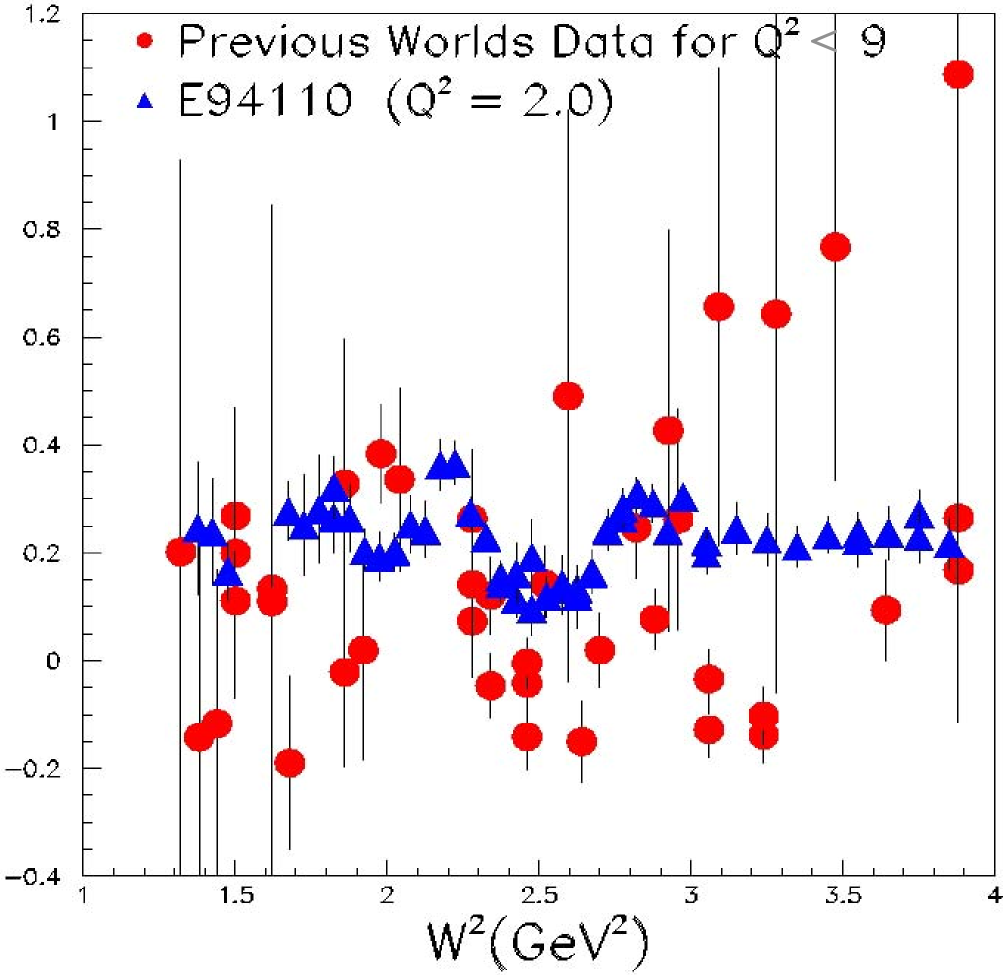


Now able to study the Q^2 dependence of individual resonance regions!

Clear resonant behaviour can be observed!

Use R to extract

F_2, F_1, F_L

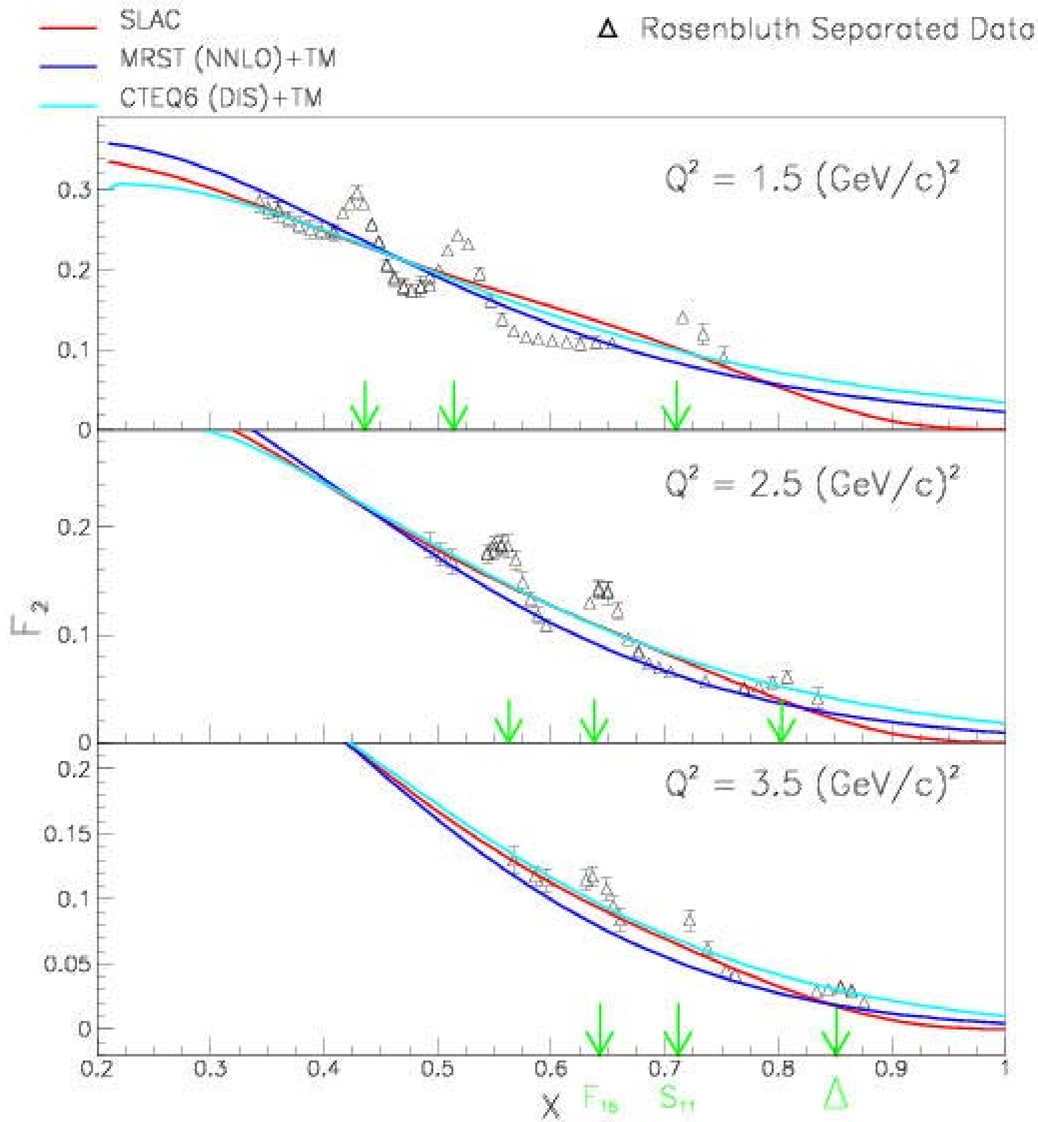
Duality in the F_2 Structure Function

Now can truly obtain F_2 structure function data, and compare with DIS fits or QCD calculations/fits (CTEQ/MRST)

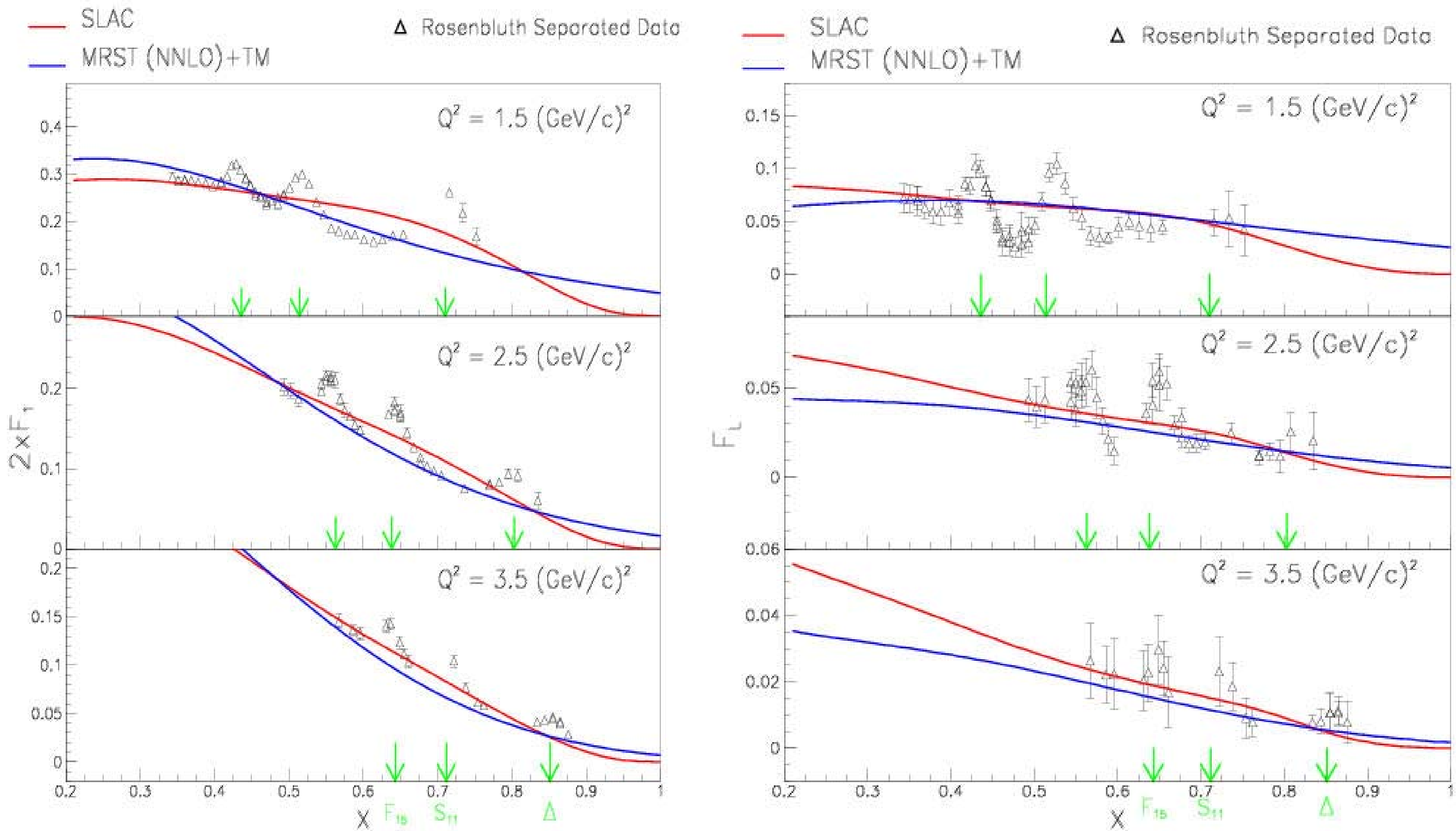
Use Bjorken x instead of Bloom-Gilman's ω'

- Bjorken Limit: Q^2, $\nu \to \infty$
- Empirically, DIS region is where logarithmic scaling is observed: $Q^2 > 5$ GeV^2, $W^2 > 4$ GeV^2
- Duality: Averaged over W, logarithmic scaling observed to work also for $Q^2 > 0.5$ GeV^2, $W^2 < 4$ GeV^2, resonance regime

  (note: $x = Q^2/(W^2-M^2+Q^2)$)
- JLab results: Works quantitatively to better than 10% at such low Q^2



Duality in F_T and F_L Structure Functions



Duality works well for both F_T and F_L above Q^2 ~ 1.5 (GeV/c)^2

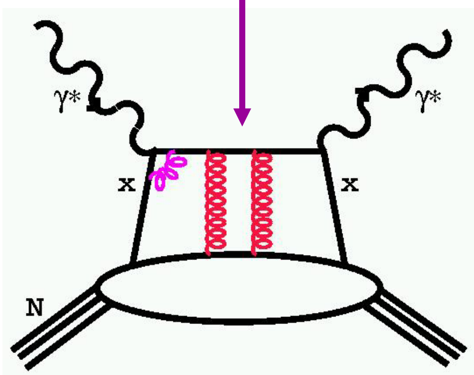
QCD and the Operator-Product Expansion

- Moments of the Structure Function $M_n(Q^2) = \int_0^1 dx\ x^{n-2} F(x,Q^2)$

  If n = 2, this is the Bloom-Gilman duality integral!
- Operator Product Expansion

$$M_n(Q^2) = \sum_{k=1}^{\infty} (nM_0{}^2/\ Q^2)^{k-1}\ B_{nk}(Q^2)$$

higher twist   logarithmic dependence



- Duality is described in the Operator Product Expansion

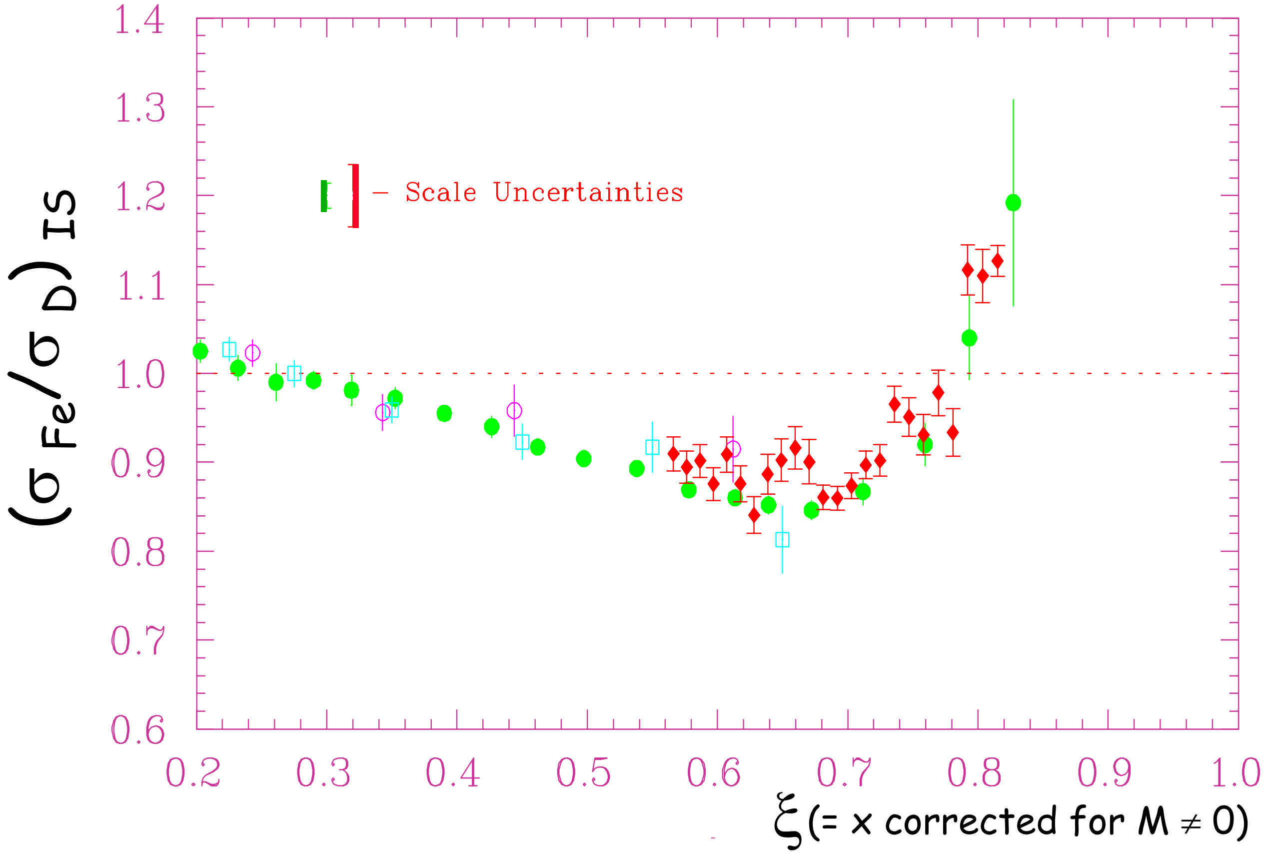
  as higher twist effects being small or canceling

DeRujula, Georgi, Politzer (1977)

Duality more easily established in Nuclei

EMC Effect
Fe/D

Resonance
Region Only

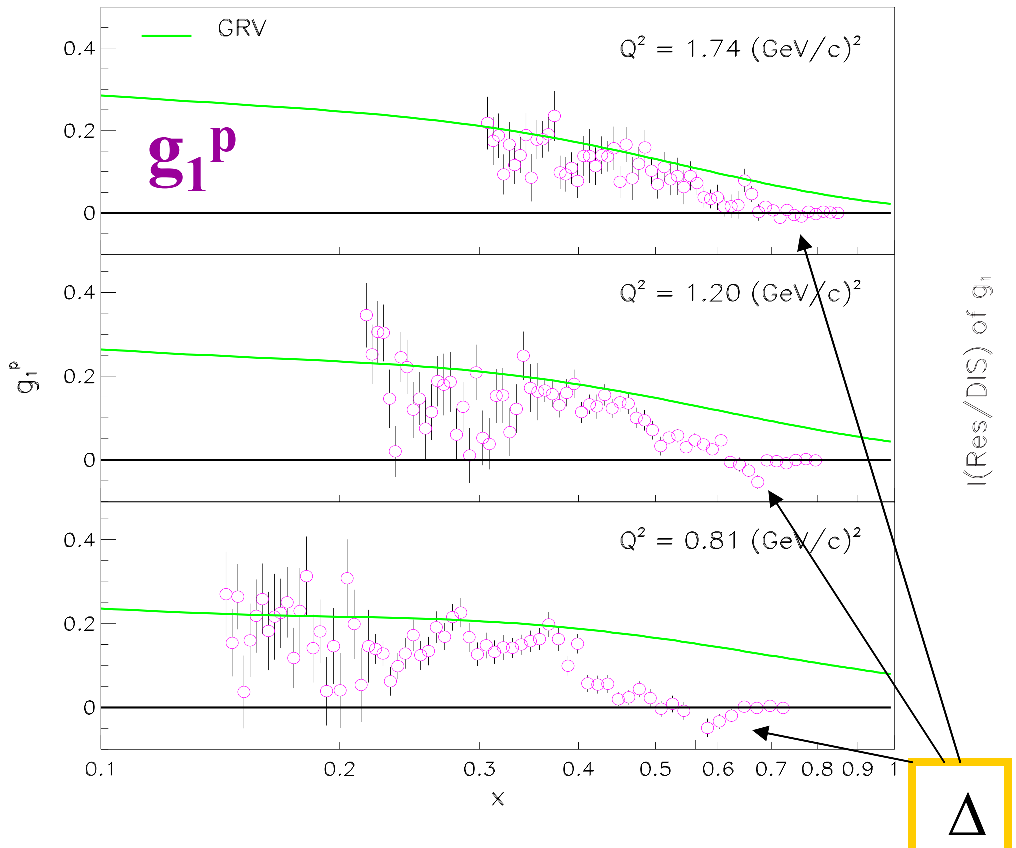


Nucleons have Fermi motion in a nucleus

The nucleus does the averaging for you!

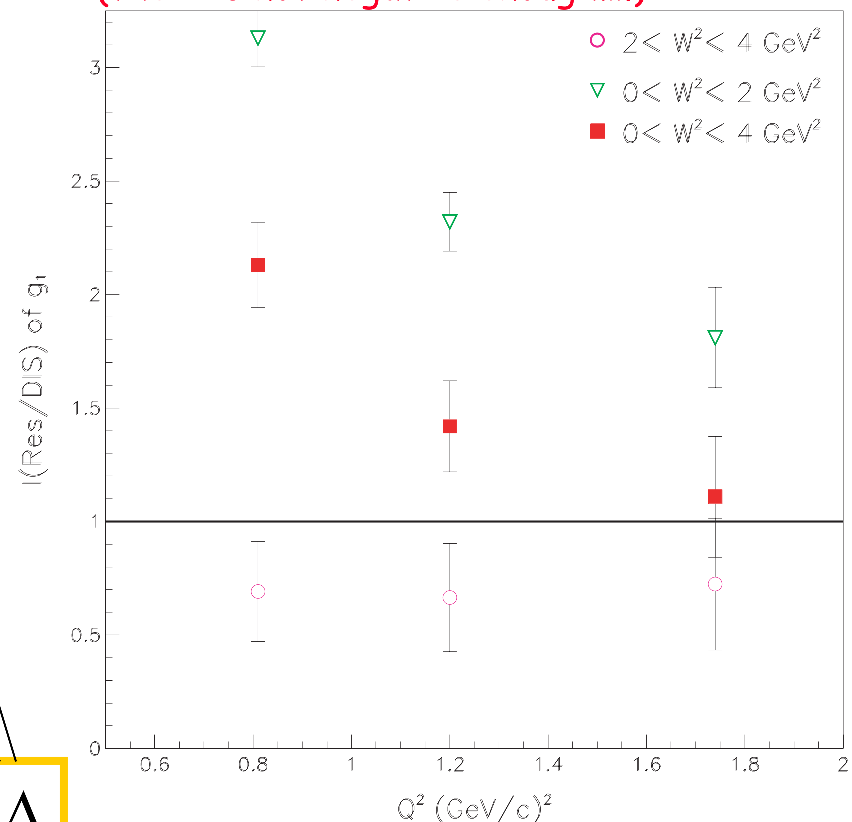
… but tougher in Spin Structure Functions

CLAS EG1



Pick up effects of both N and Δ

(the Δ is not negative enough….)



CLAS: N-Δ transition region turns positive at Q^2 = 1.5 (GeV/c)²

Elastic and N-Δ transition cause most of the higher twist effects