FEW BODY PHYSICS: THEORY

JLab Users Group Symposium and Annual Meeting
11-13 June, 2003
dedicated to the memory of Nathan Isgur

Franz Gross

JLab and W&M

Outline:

Introduction

I: The *NN* interaction and the nuclear force

- Deuteron form factors
- Deuteron photo and electrodisintegration

II: The *NNN* interaction and correlations

- ^{3}He electrodisintegration

III: What have we learned?

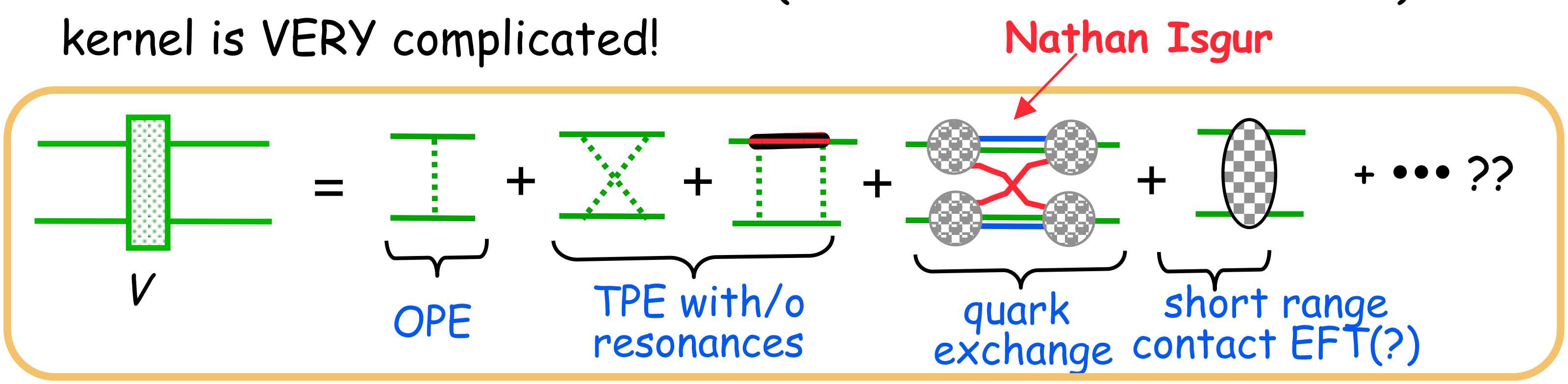
IV: What is left to be done?

Introduction: JLab's mission

- The JLab scientific mission is to
  - understand how hadrons are constructed from the quarks and gluons of QCD;
  - understand the QCD basis for the nucleon-nucleon force; and
  - to explore the limits of our understanding of nuclear structure
    - high precision
    - short distances
    - the transition from the nucleon-meson to the QCD description
- Few Body physics addresses the last two of these scientific missions
- when applied to the quark sector (not discussed in this talk) it also applies (approximately) to the first mission
- theory and experiment are a partnership

Introduction: the Few-Body point of view

- ALL degrees of freedom are treated explicitly; no "averages", precise solutions
- Problems are solved in sequence:
  - two-body problem first
  - then the three-body problem using results from the two-body problem
  - . . .
  - the A-body problem uses results from the solutions of A-1 and fewer bodies
- the starting point for the *NN* problem is the *NN* force, which is a "two nucleon irreducable" kernel (*i.e.* with no two nucleon cuts)-the kernel is VERY complicated!



Recent developments (in hadronic sector -- not discussed here)

- One pion exchange now well established by
  - chiral effective field theory
  - direct comparison with data
- Effective field theory provides an organization principle for low momentum interactions
  - two pion exchange now understood to work very well
- low energy three body calculations by Glockle (and others) establish the correctness of the extension from 2N to 3N
- OPE plus exchange of vector and scalar effective "mesons" provides a very successful phenomenology for scattering up to lab energies of 350 MeV
- Off-shell effects can substitute for higher order $NN\square^{n}$ point interactions

I. The *NN* interaction and the nuclear force

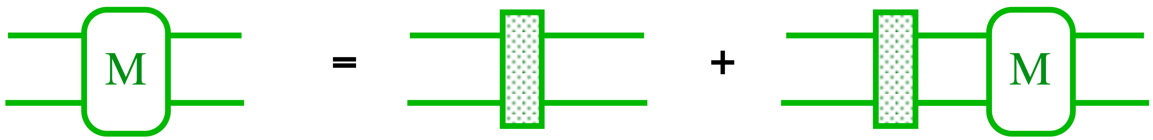
Deuteron form factors

Deuteron photodisintegration

Deuteron electrodisintegration

Theory overview (two body scattering)

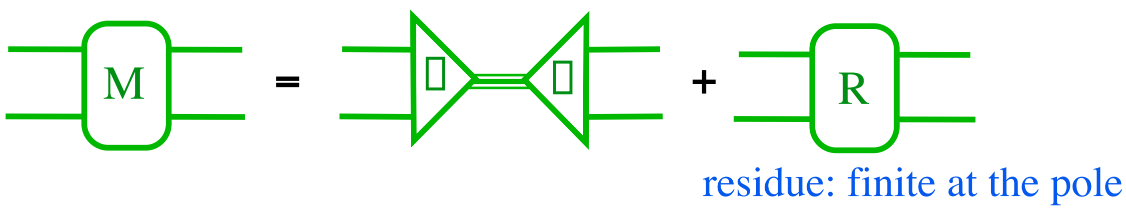
- The two-body scattering amplitude is constructed by summing the irreducable two-body kernel *V* (the *NN* "force" or the *NN* "potential") to all orders. The solution is non-perturbative.
- The sum is obteined by solving the relativistic integral equation



there are several choices for the two nucleon propagator

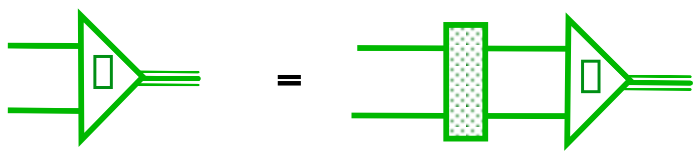
the covariant spectator theory has been developed locally

- if a bound state exists, there is a pole in the scattering amplitude



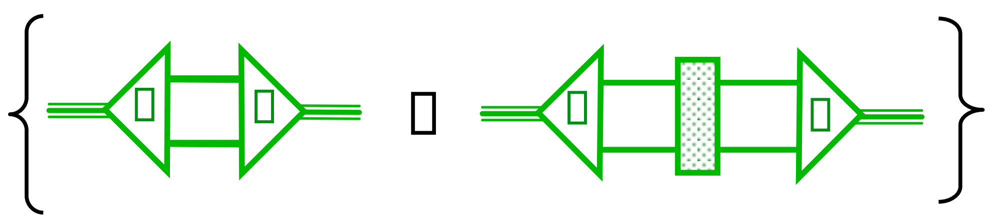
Theory overview (two body bound state)

- the equation for the bound state vertex function is obtained from the scattering equation near the bound state pole



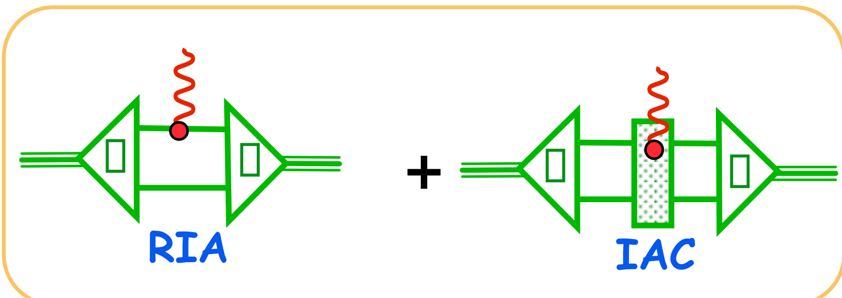
- the (covariant) bound state normalization condition follows from examination of the residue of the bound state pole

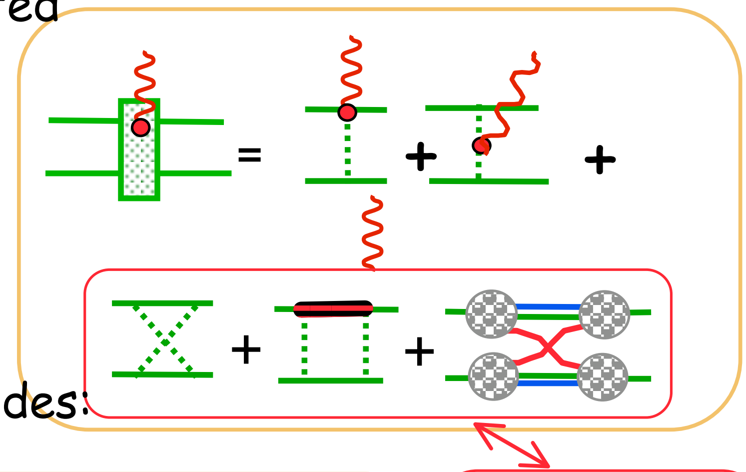
$$1 = \frac{d}{dM_d^2}$$



Theory overview (2 body currents)

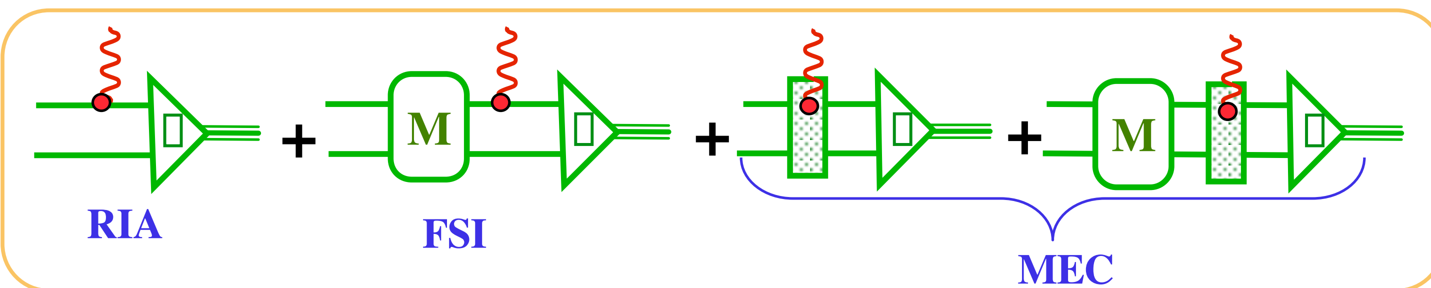
- Gauge invariant, two-body currents can then be constructed from the scattering theory. Only a finite number of amplitudes are needed:*
- there are two amplitudes for elastic scattering, which are gauge invariant if the IAC is properly constructed





IAC: photon must couple to all charged particles inside of *V*

- inelastic scattering requires four amplitudes:

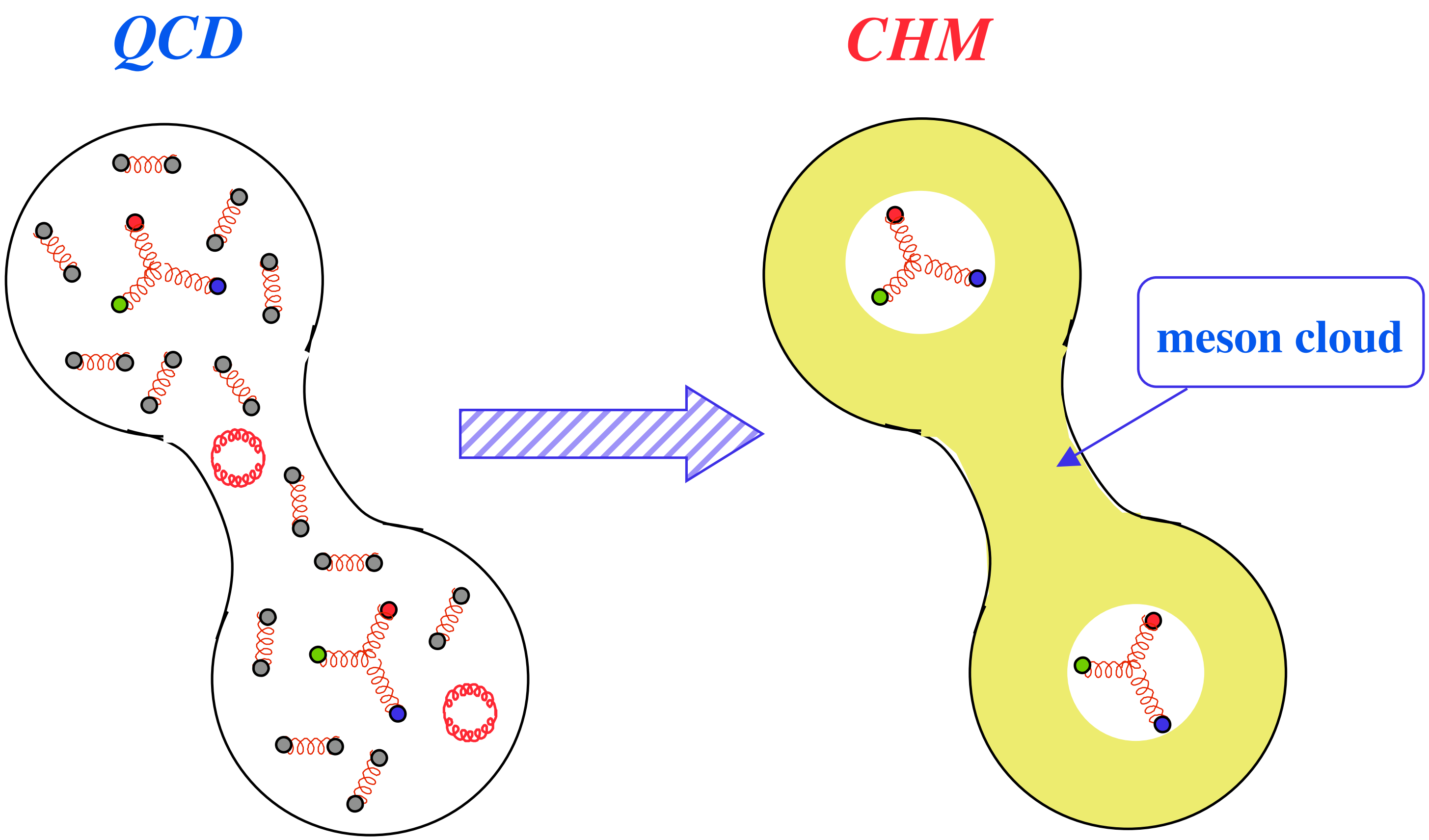


*FG and D.O. Riska

Theory overview (definition of the CHM)

- The previous discussion defines the Consistent Hadronic Model (CHM) of Few Body Physics
- Assumptions of the CHM
  - nuclei are not fundamental particles: they arise from the *NN* interaction.
  - the physics is non-perturbative: not describable by a few selected diagrams
  - nucleons and mesons are composite systems of quarks: their structure cannot be calculated within the CHM (this is a major shortcoming)
  - consistency: many body forces, currents, and final state interactions must all be based on the same dynamics
- Implications
  - the current operator is constrained by the *NN* interaction and current conservation
  - three body forces are constrained by two body dynamics
  - ambiguities exist because of the composite nature of the nucleon and mesons

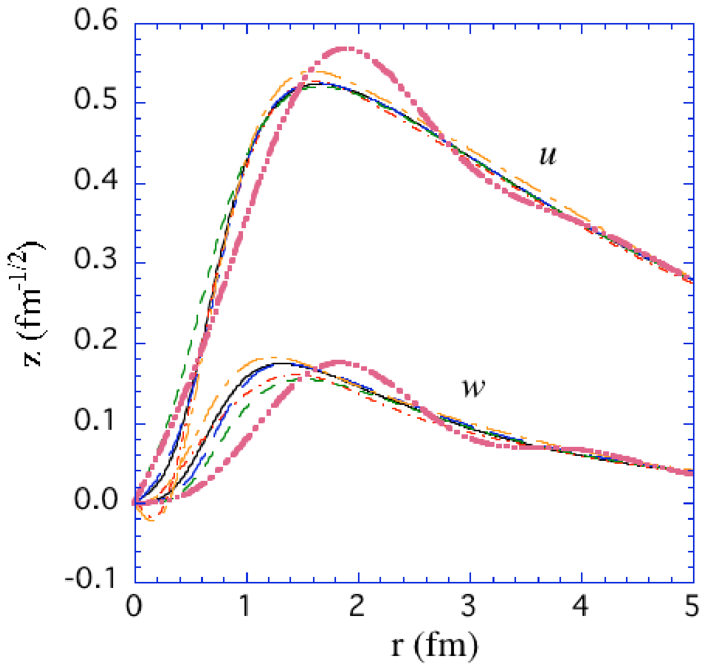
Pictures: the *CHM* is an effective theory of *QCD*

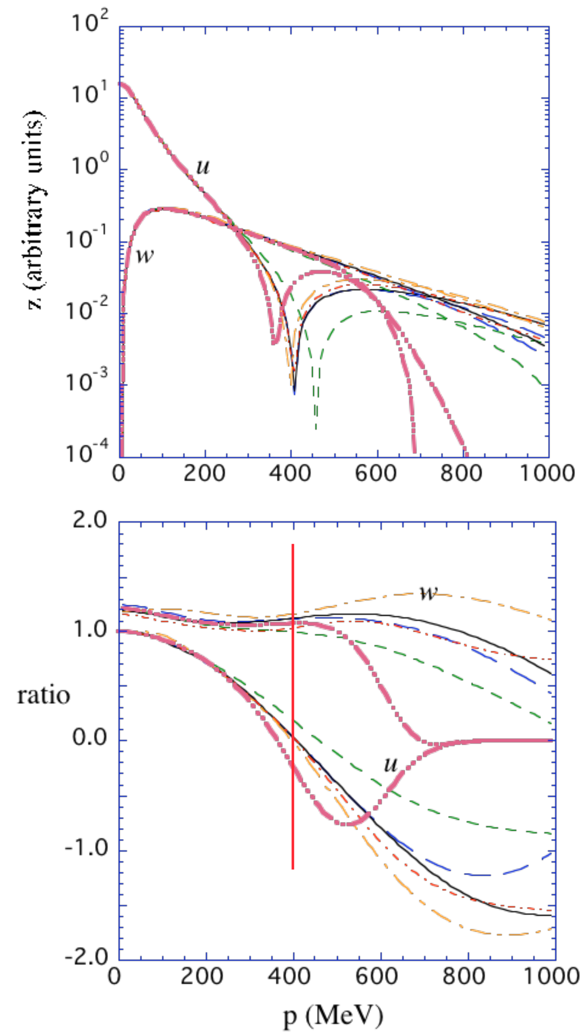


Applications of CHM to the deuteron form factors

Deuteron wave functions

Six models:
Argonne V18 (black), Paris (blue), CDBonn (green), IIB (red), W16 (orange), Idaho (pink)





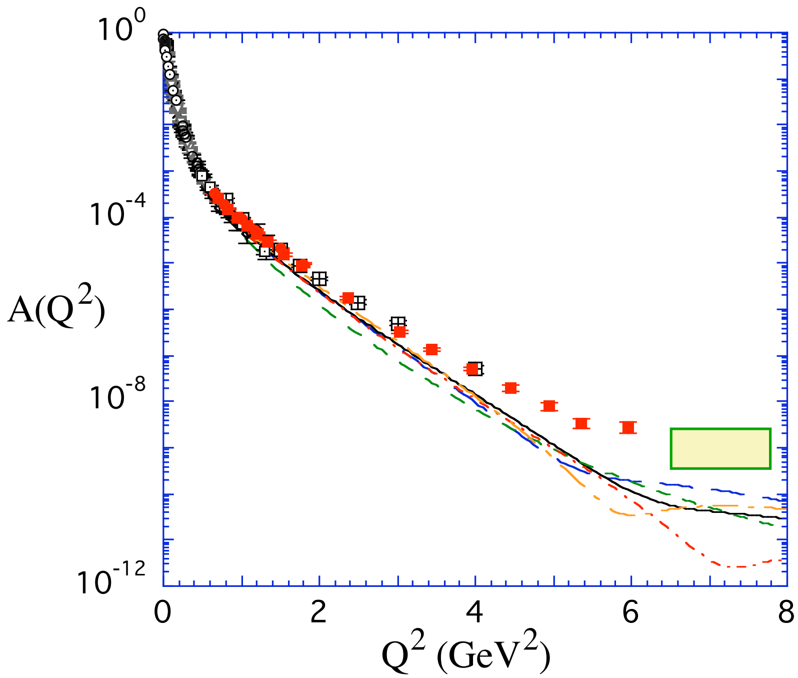
All very close up to 500 MeV
(except CDBonn and Idaho)

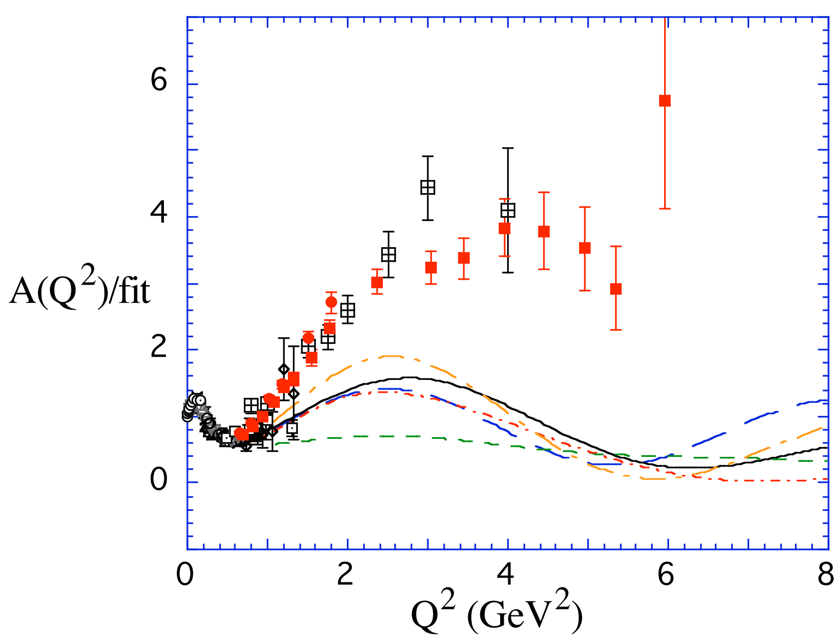
local wave functions are the same!

Nonrelativistic models fail* at Q^2 beyond 1 GeV^2

*(by a factor of 10)

But, a 15% to 20% change in effective Q^2 is a factor of 10





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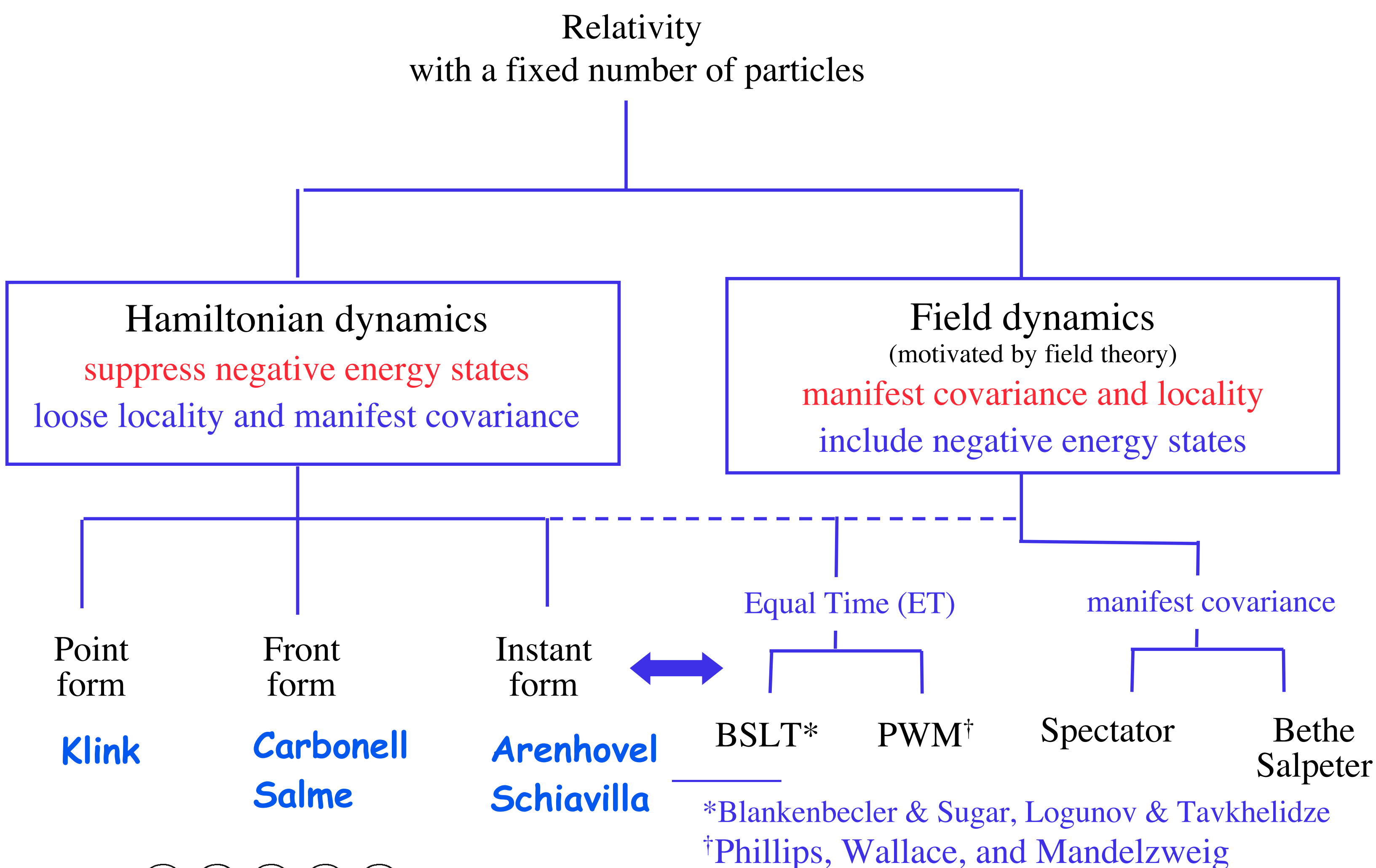
A relativistic theory is needed for JLab physics: and there are many choices:



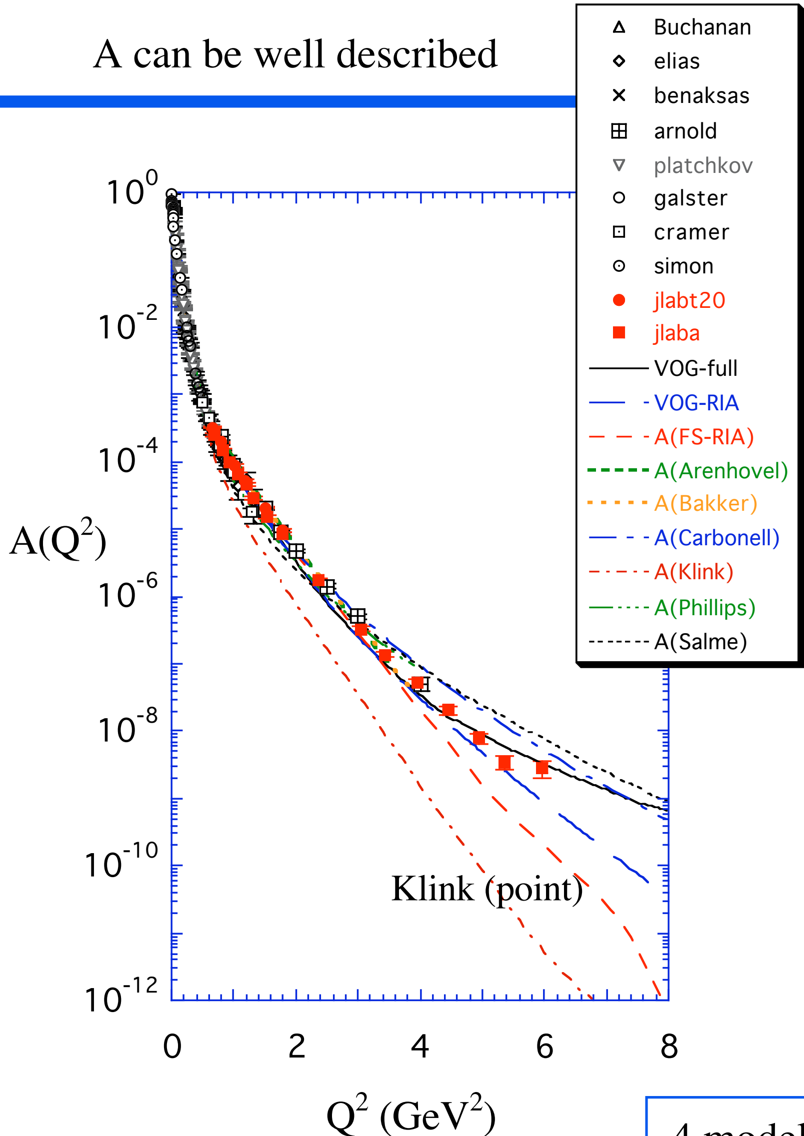
Comparison: Relativistic calculations of deuteron form factors*

- Field dynamics
  - VODG - Van Orden, Devine, and FG, PRL 75, 4369(1995).
    Manifestly covariant spectator theory
  - Phillips - Phillips, Wallace, and Devine, PRC 58, 2261 (1998).
    Equal time formalism
- Hamiltonian dynamics
  - Arenhovel - Arenhovel, Ritz, and Wilbois, PRC 61, 034002 (2000).
    instant-form with (v/c) expansion
  - Schiavilla - Schiavilla and Pandharipande (PRC 66, to be published)
    instant-form without (v/c) expansion
  - Carbonell - Carbonell and Karmanov, EPJ A6, 9 (1999).
    front-form averaged over the light cone direction
  - Salme - Lev, Pace, and Salme, PRC 62, 064004-1 (2000).
    front-form
  - Klink - Allen, Klink, and Polyzou, PRC 63. 034002 (2001).
    point-form

*See R. Gilman and FG, J. Phys. G: Nucl. Part. Phys. 28, R37-R116 (2002)

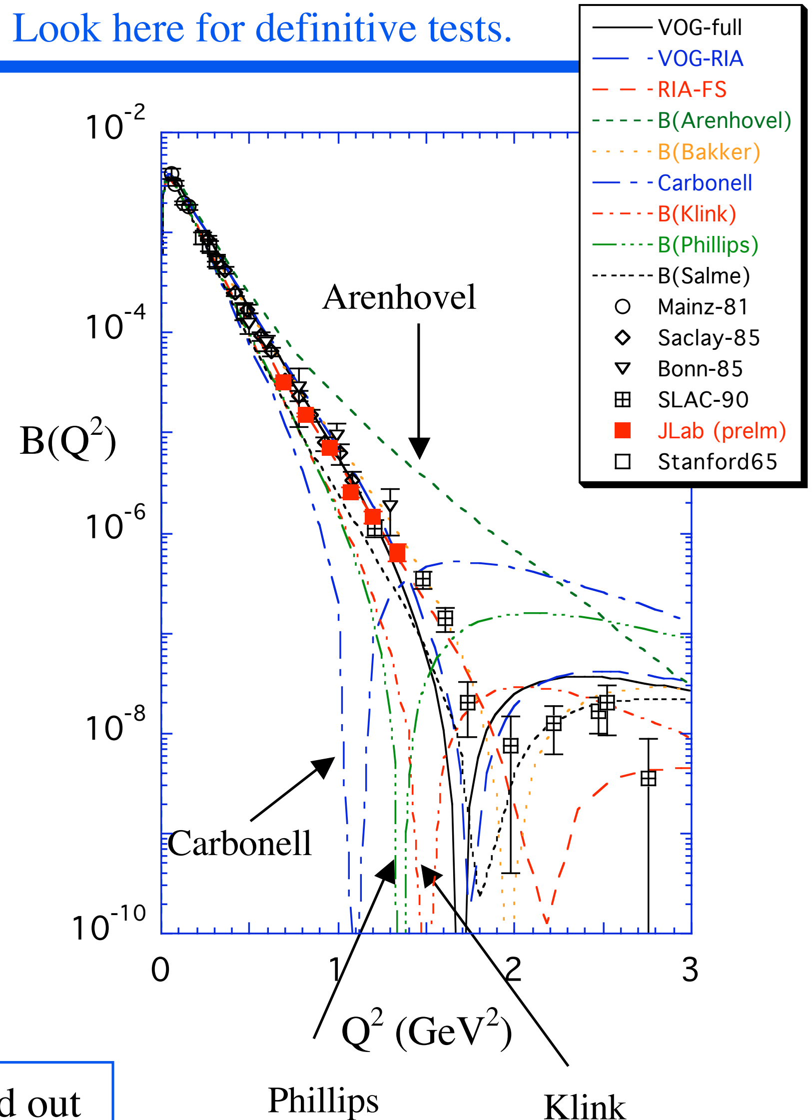
At larger Q^2

A can be well described



B is VERY sensitive

Look here for definitive tests.



4 models ruled out

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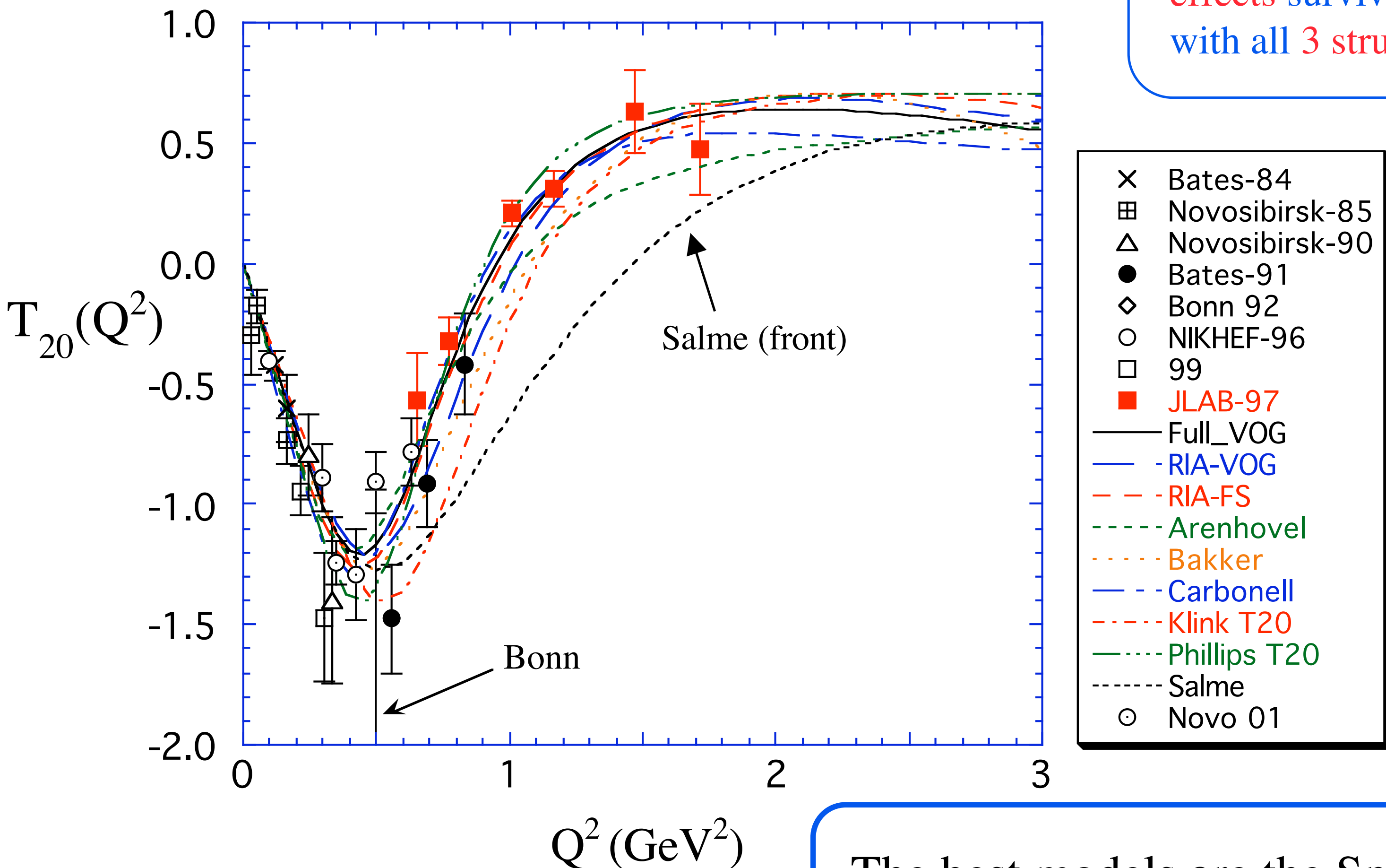
T_{20} is also well described by most models

only models with complete currents and full relativistic effects survive comparison with all 3 structure functions!



The best models are the Spectator, and instant form calculation of Schiavilla

A final touch; using the Spectator theory !

- A precise description of all the form factors can be obtained by exploiting the off-shell freedom of the current operator
- To conserve current, the current operator must satisfy the WT identity

$$q_\mu j_N^\mu(p',p) = S^{-1}(p) - S^{-1}(p')$$

- The spectator models use a *nucleon form factor, h(p)*. This means that the nucleon propagator can be considered to be dressed

$$S(p) = \frac{h^2(p)}{m - \not{p}} = \frac{h^2(p)}{\Lambda_-(p)}$$

- one solution (the simplest) is

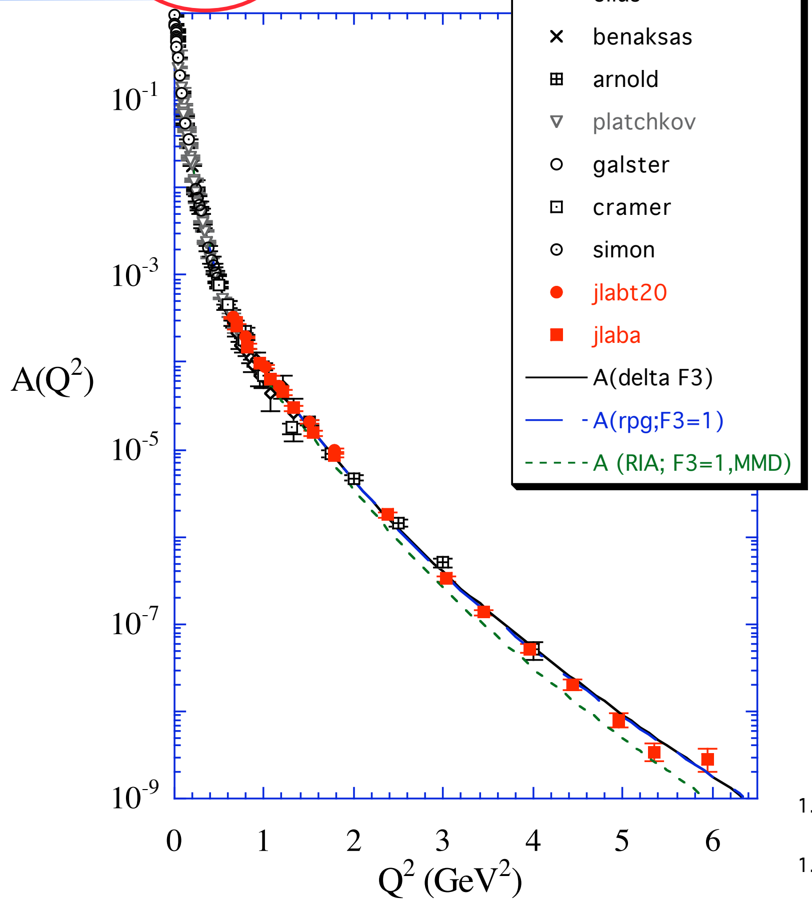
off-shell effects

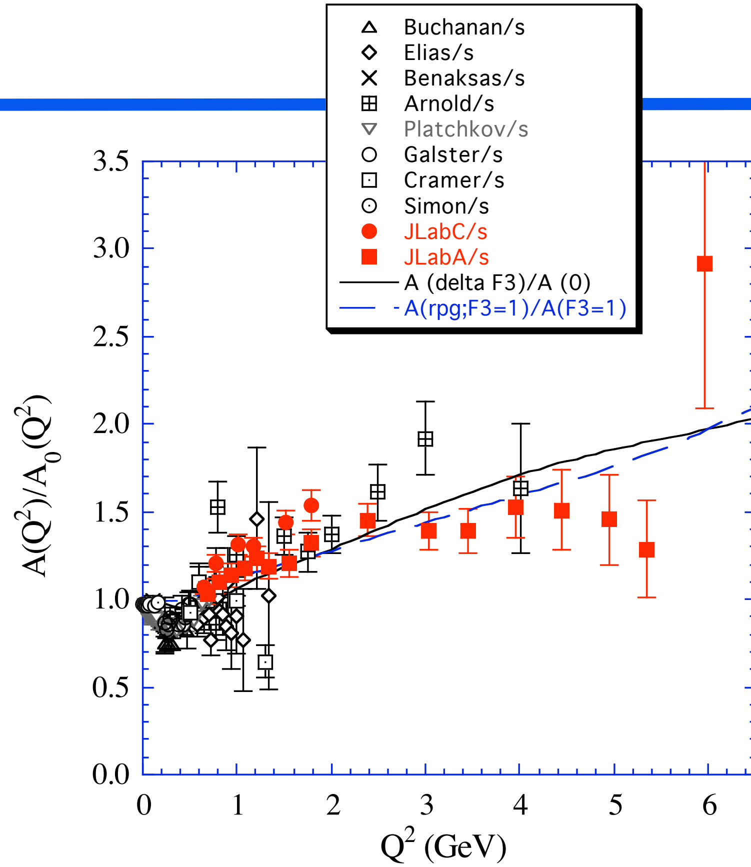
$$j^\mu(p',p) = F_0 \left[F_1\,\gamma^\mu + F_2\,\frac{i\sigma^{\mu\nu}q_\nu}{2m} \right] + G_0\,F_3\,\Lambda_-(p')\,\gamma^\mu\,\Lambda_-(p)$$

$$F_0 = \frac{h(p)}{h(p')}\left[\frac{m^2 - p'^2}{p^2 - p'^2}\right] - \frac{h(p')}{h(p)}\left[\frac{m^2 - p^2}{p^2 - p'^2}\right] \qquad G_0 = \left[\frac{h(p')}{h(p)} - \frac{h(p)}{h(p')}\right]\frac{4m^2}{p^2 - p'^2}$$

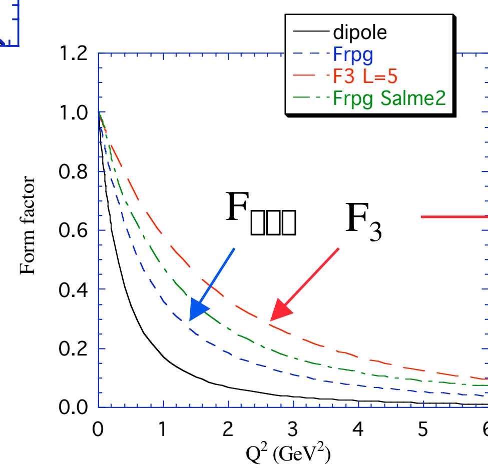
- $F_3(Q^2)$ is unknown, except $F_3(0)=1$. EXPLOIT THIS FREEDOM
- compare the F_3 choice with the $\rho\pi\gamma$ current

A(Q²)





Choice of a "hard" F_3 is sufficient for an excellent fit!



$$F_3(Q^2) = \frac{1}{\left(1 + Q^2/5\right)^3}$$

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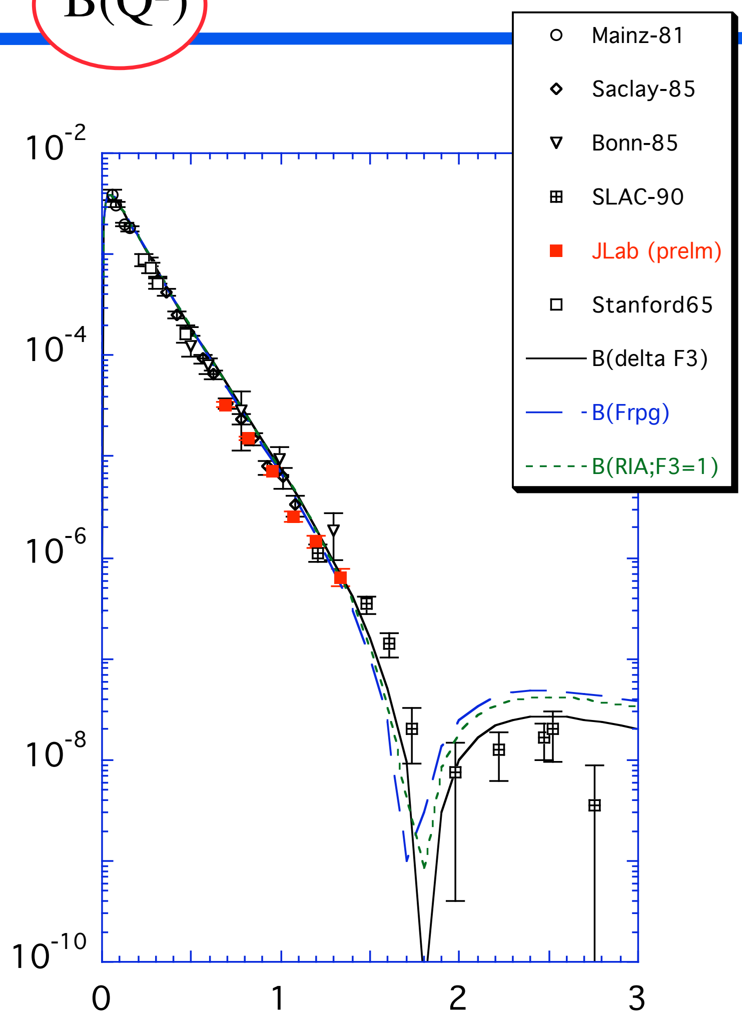
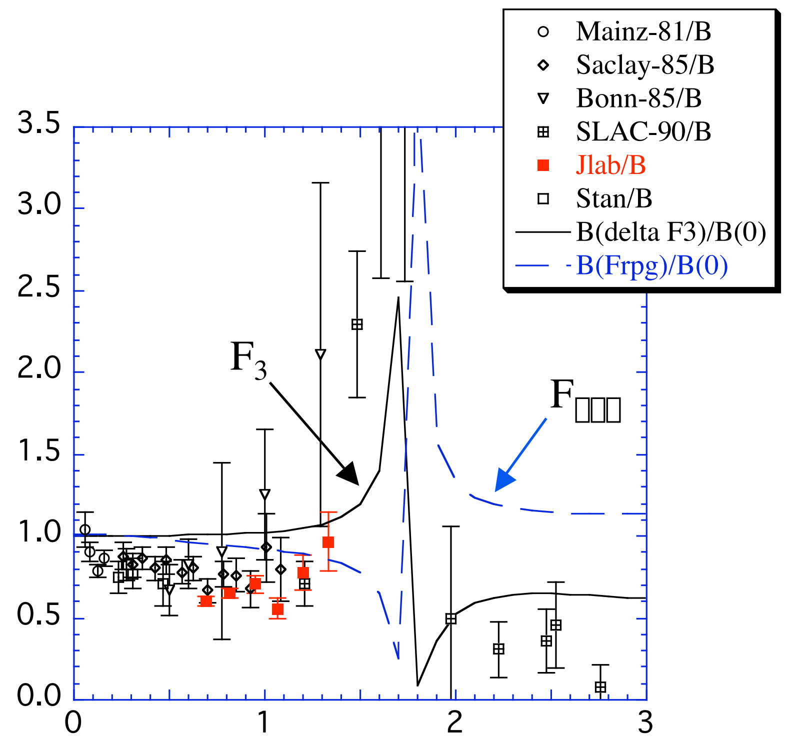
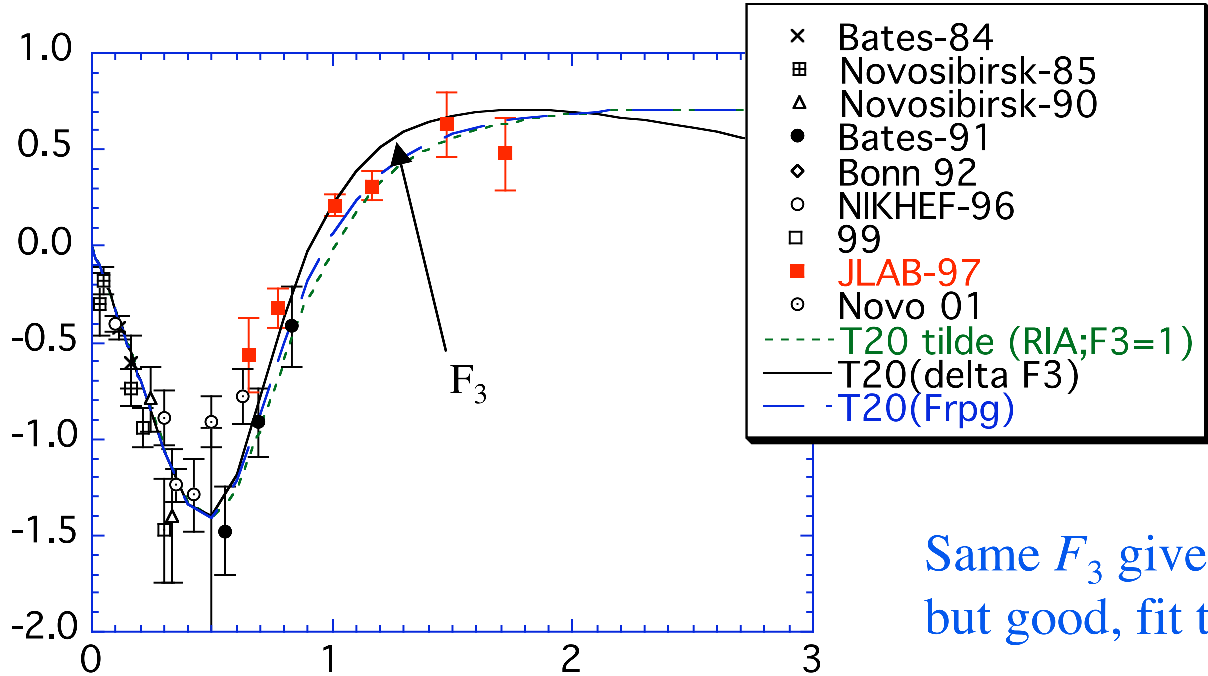
$B(Q^2)$





Same F_3 also works for $B(Q^2)$

$T_{20}(Q^2)$



Same F_3 gives a different, but good, fit to T_{12}!

The Spectator theory, with a suitable F_3 , can explain the elastic electron deuteron scattering data!

What have we learned from the deuteron form factors?

- This reaction is the simplest possible two body process to study
  - the I=0 exchange currents are small (in the relativistic spectator theory)
  - BUT, in other models, there must be large two-body currents
  - the initial and final state are "known"
  - the results are insensitive to coupling to excited nucleon channels because "left hides right"
- This data has profoundly stimulated the development of relativistic few body physics
- The CHM using nucleon degrees of freedom can explain the data out to $Q^2 \approx 6$ (GeV)^2, provided some new physics is added:
  - new off-shell nucleon form factor, F_3
  - or some missing IAC (from the energy dependence of the high energy *NN* scattering, or from the □□□ exchange current)

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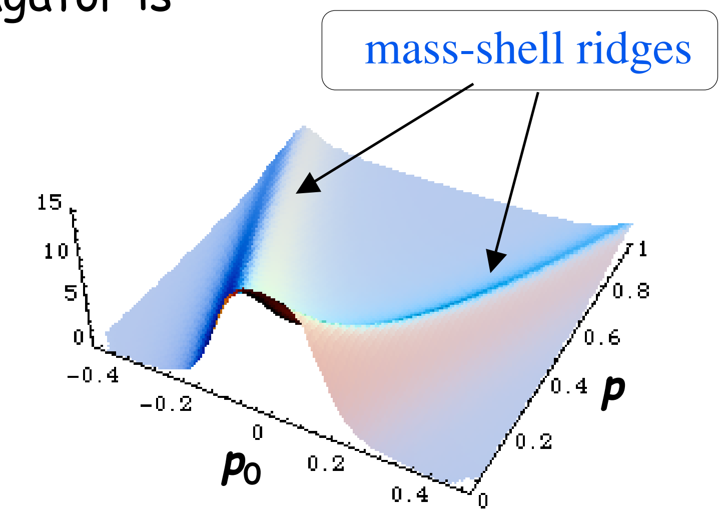
Why does the CHM work for the deuteron form factors?

- ★ The relativistic two-body propagator peaks when one of the two nucleons is on-mass shell. The 2-body propagator is

$$G(p_0,p)=\frac{1}{\left(E_p^2-\left(\tfrac{1}{2}M+p_0\right)^2-i\epsilon\right)\left(E_p^2-\left(\tfrac{1}{2}M-p_0\right)^2-i\epsilon\right)}$$

with $E_p=\sqrt{m^2+p^2}$

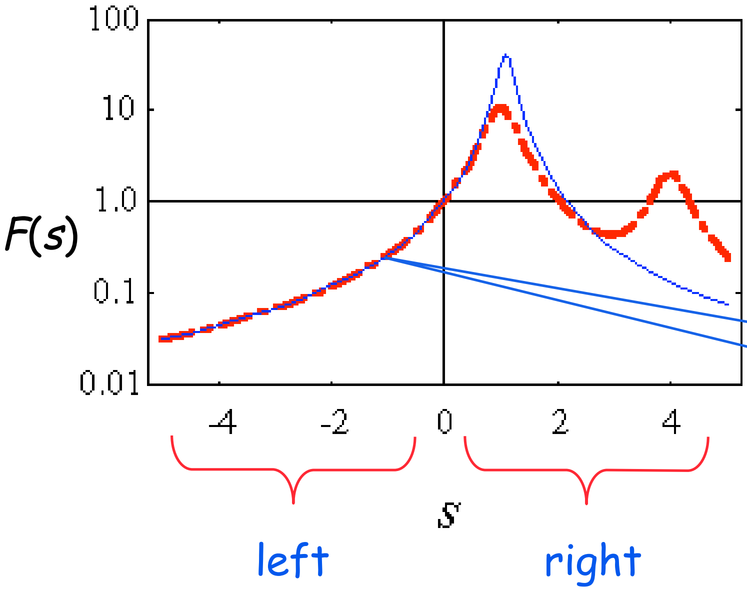


- ★ If we take one particle on-shell (as in the covariant spectator theory), then the mass of the other is

$$p^2_{\text{off-shell}}=\left(P-p_{\text{on-shell}}\right)^2=M\left(M-2E_p\right)+m^2\simeq m^2-M\left(BE\right)$$

- ★ the mass of the off-shell particle is on the "left hand side" of the p^2 axis:

BUT: "Left hides right"



★ Compare the "left-hand-side" of two resonance structures

★ Under certain conditions they are indistinguishable

in this case, the two functions agree on the left-hand side to 1%!

$$F_1(s) = \frac{1.033^2 + 0.03}{(1.033 - s)^2 + 0.03}$$

$$F_2(s) = \frac{1.1\left(1 - \frac{0.2}{16.1}\right)}{(1 - s)^2 + 0.1} + \frac{0.2}{(4 - s)^2 + 0.1}$$

★ LESSON:

THE RIGHT-HAND NUCLEON RESONANCE STRUCTURE CANNOT BE INFERRED UNIQUELY FROM THE LEFT-HAND STRUCTURE

★ The deuteron form factors do not "see" the resonances

Study of deuteron photodisintegration

100's of channels excited in photodisintegration at 4 GeV

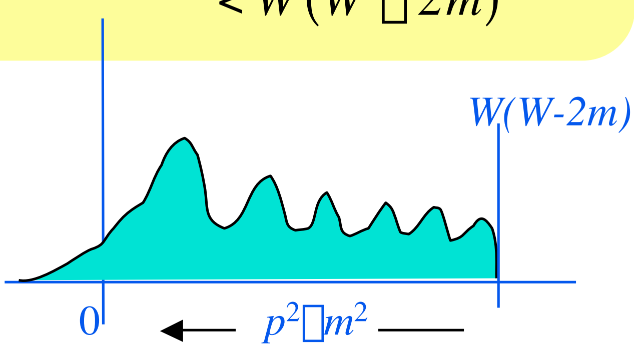
numbers of N*N* channels that are excited

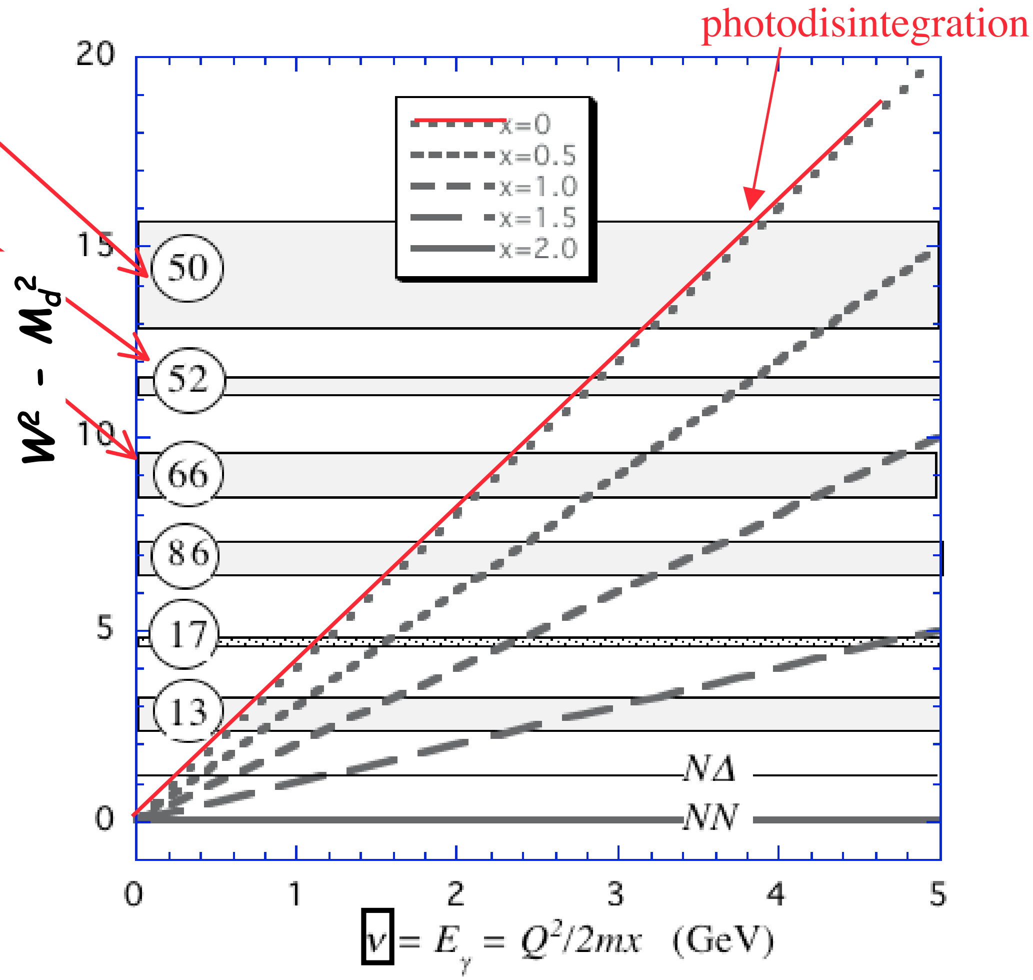
a total of 286 channels composed of two well established resonances!

off-shell mass

cm 3-momentum

$$p^2 - m^2 = W^2 - 2WE_p$$
$$< W(W - 2m)$$





IN DEUTERON PHOTODISINTEGRATION, THE "RIGHT-HAND" RESONANCES ARE EXPOSED

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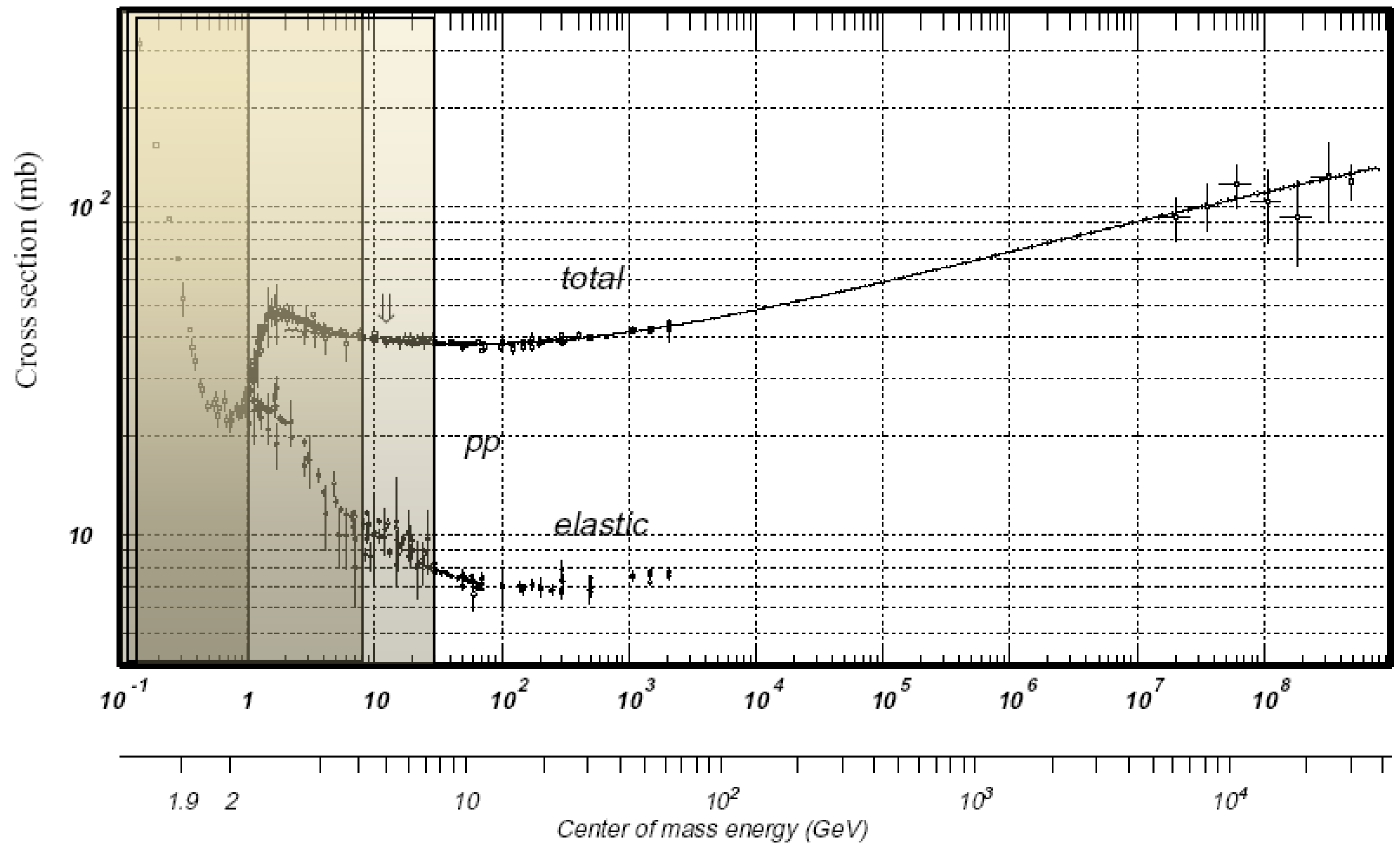
total *NN* cross sections



12 Gev photons

High energy photodisintegration probes deep into the inelastic region

High energy *NN* scattering must be treated explicitly

- Schwamb, Arenhövel, and collaborators: "conventional" models with □ resonances (not intended to explain the high energy data)
- H. Lee: "conventional" model with □ and P_{11} (Roper) resonances
- Bonn (Kang, *et. al.*): all established resonances with m < 2 GeV and J ≤ 5/2
- pQCD (Brodsky, Hiller, and others): predicts $s^{\square 11}$ fall off and hadron helicity conservation (HHC)
- Quark Exchange model (Frankfurt, Miller, Sargsian, and Strikman): uses the quark exchange diagram to relate *□d to NN*
- Quark Gluon String model (Kondratyuk, Grishina, *et. al.*): relate to Reggie pole description of *NN* scattering

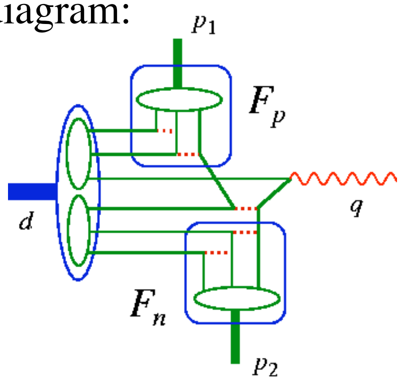
Smooth, scaling-like behavior at high energies

Conventional models fail (so far)

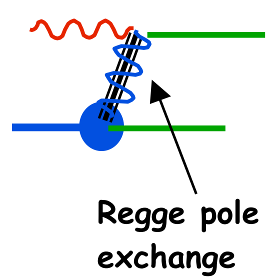
Quark-interaction models:

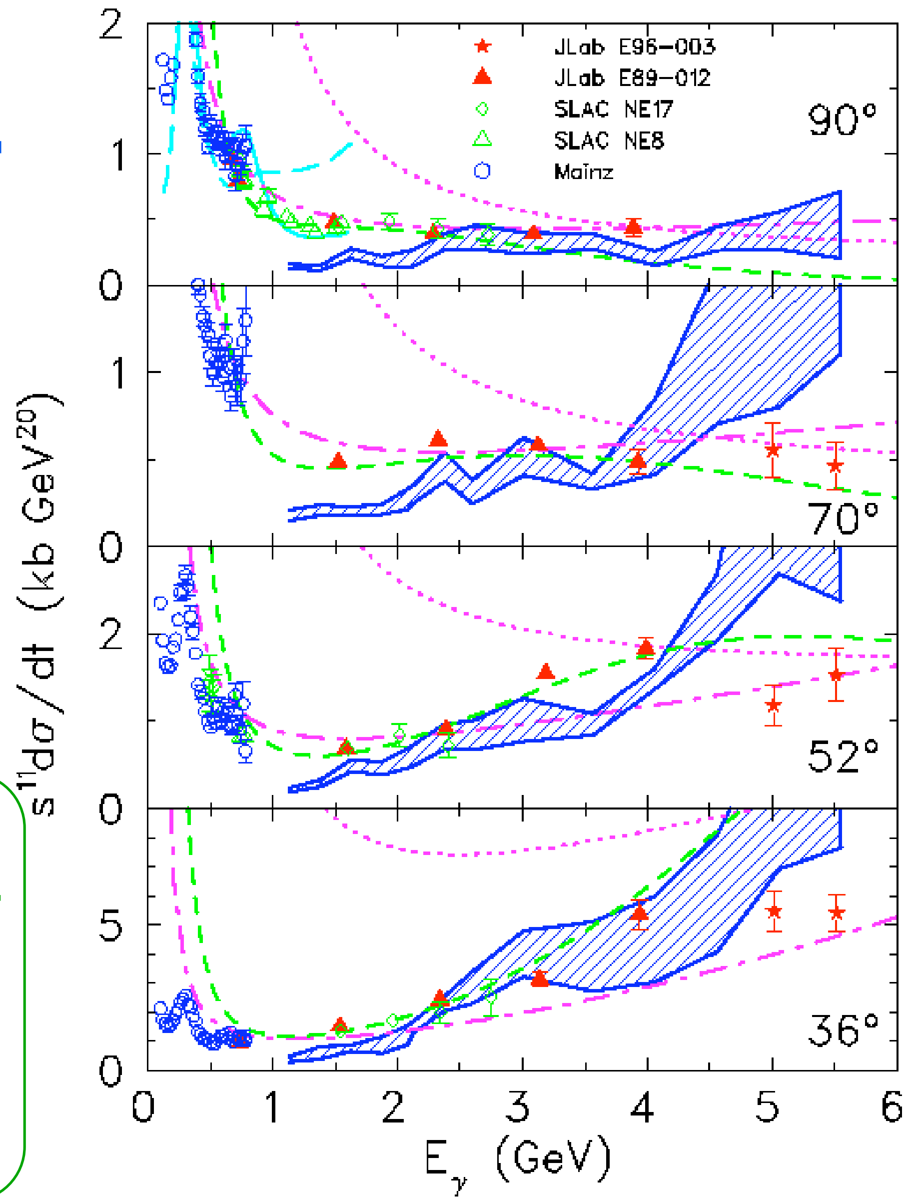
- Radyushkin
- Brodsky and Hiller (RNA)
- Kondratyuk, *et.al.* quark-gluon string model
- Frankfurt, Miller, Strikman, and Sargsian (final state *NN* scattering)

A quark-exchange diagram:



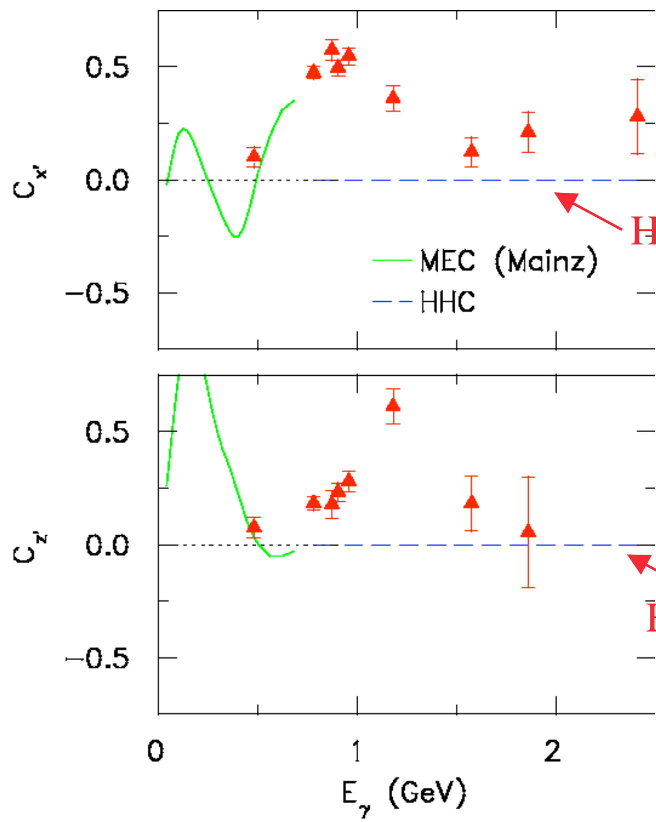
The QGS model





Polarization observables at high Q^2

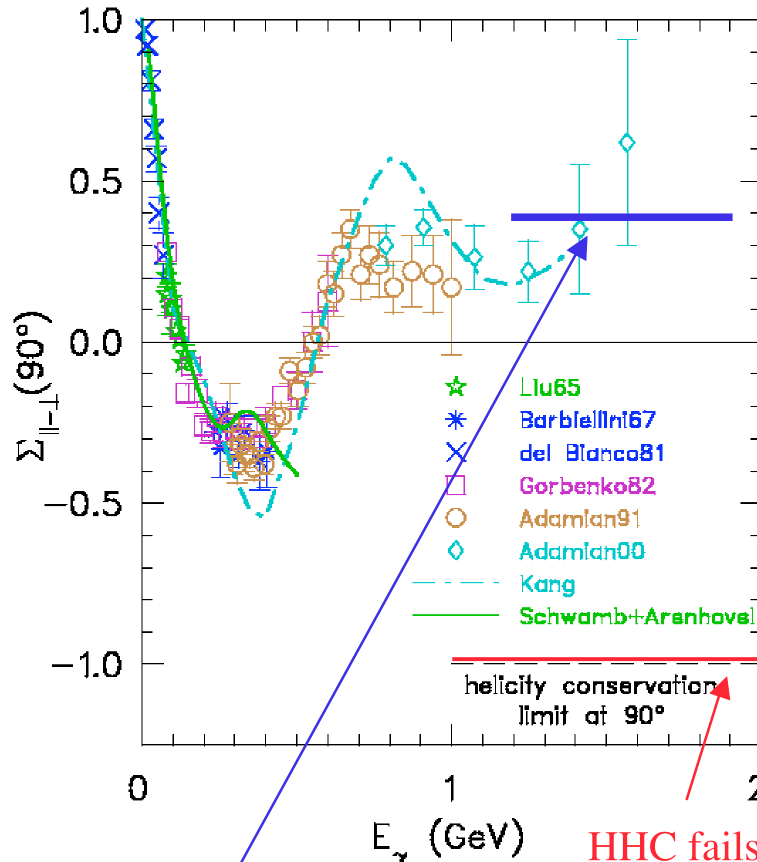
Are a sensitive test of pQCD
Hadron Helicity conservation (HHC)



HHC fails?

HHC OK

Schwamb and Arenhovel



HHC fails?

No! Isovector □s give □=1, so a combination gives □=constant

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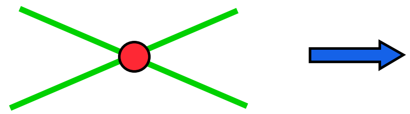
Conclusions from deuteron photodisintegration

- ★ The CHM will not work in this region unless explicitly supplemented by mechanisms that can describe *NN* scattering up to 8 GeV (and beyond)
- ★ This experiment could provide an ideal tool of studying the transition from *NN* to quark gluon degrees of freedom, but --
- ★ MORE COMPLETE, CONSISTENT CALCULATIONS ARE NEEDED: the bubble model teaches us that "energy dependence comes with a price"!
- ★ Electrodisintegration allows us to study the transition from x=2 (elastic form factors) to x=0 (photodisintegration)

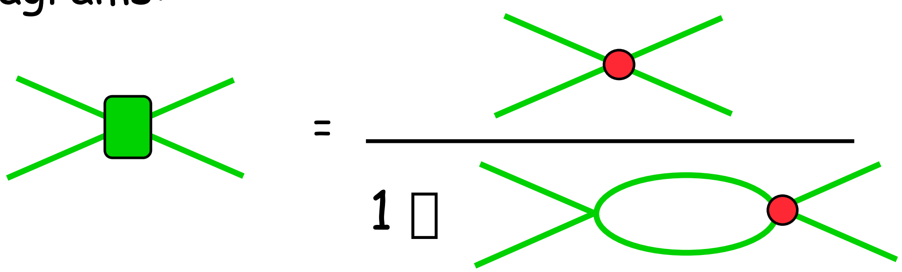
Lessons from the bubble sum (in 1+2 d for simplicity)

- suppose the *NN* interaction is an energy dependent four-point coupling:



$a + \Box(s - M_d^2)$

- then the scattering amplitude is a geometric sum of bubble diagrams:



$$M = \frac{a + \Box\left(s - M_d^2\right)}{1 - B(s)\left[a + \Box\left(s - M_d^2\right)\right]}$$

- the bound state condition fixes a, but the energy dependent parameter $\Box$ is undetermined

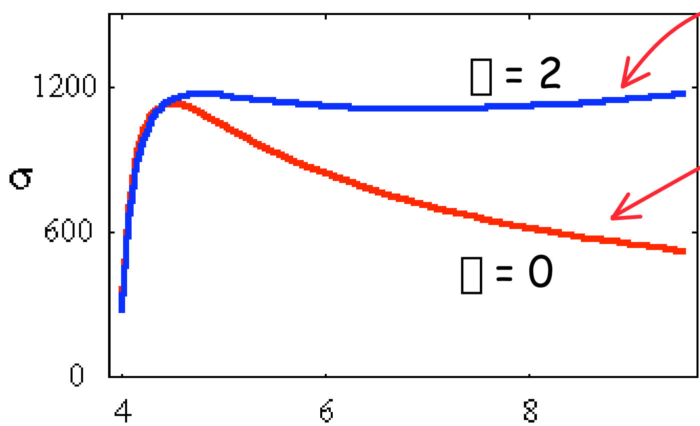
$$a\,B\left(M_d^2\right) = 1$$

Lessons from the bubble sum (2)

★ the deuteron wave function is independent of □,

$$\square(p,M_d) = \frac{N}{\left(m^2 - \left(\tfrac{1}{2}P + p\right)^2\right)\left(m^2 - \left(\tfrac{1}{2}P - p\right)^2\right)}; \quad P^2 = M_d^2$$

★ but the NN cross section is not:



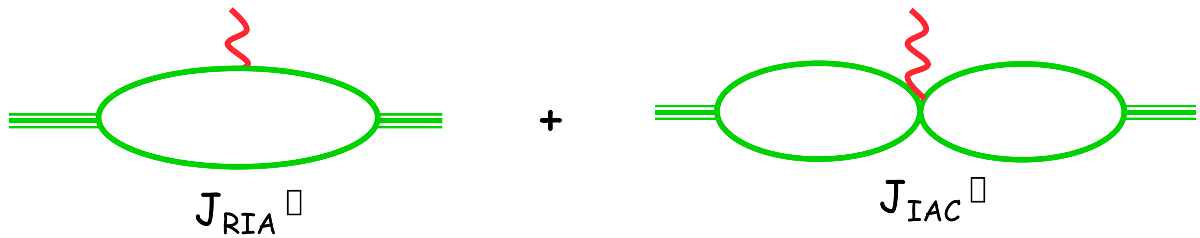
s (in units of m^2)

$$\square(s) = \frac{1}{\sqrt{s}}\left|M(s)\right|^2; \quad M = \frac{a + \square\left(s - M_d^2\right)}{1 - B(s)\left[a + \square\left(s - M_d^2\right)\right]}$$

Lessons from the bubble sum

"energy dependence comes with a price"

- ★ the deuteron form factor is the sum of two terms:

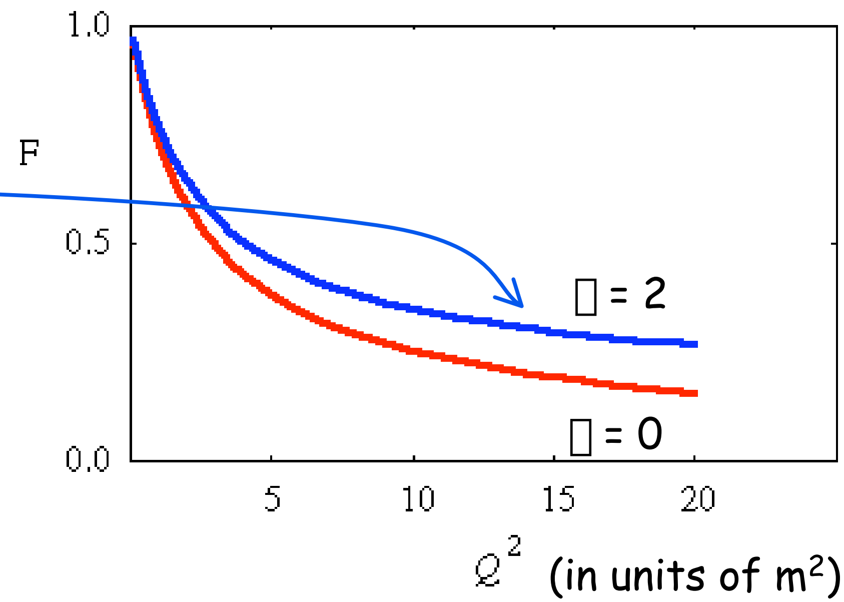


- ★ the energy dependence of the interaction generates an interaction current (IAC) which depends on □

- ★ the IAC required by the interaction is unique and separately gauge invariant

$$J^{□}_{IAC}(Q^2) = □\, N^2 (P + P')^{□} B^2(Q^2)$$

- ★ FSI and IAC must be consistent with the dynamics! Calculations must be consistent.



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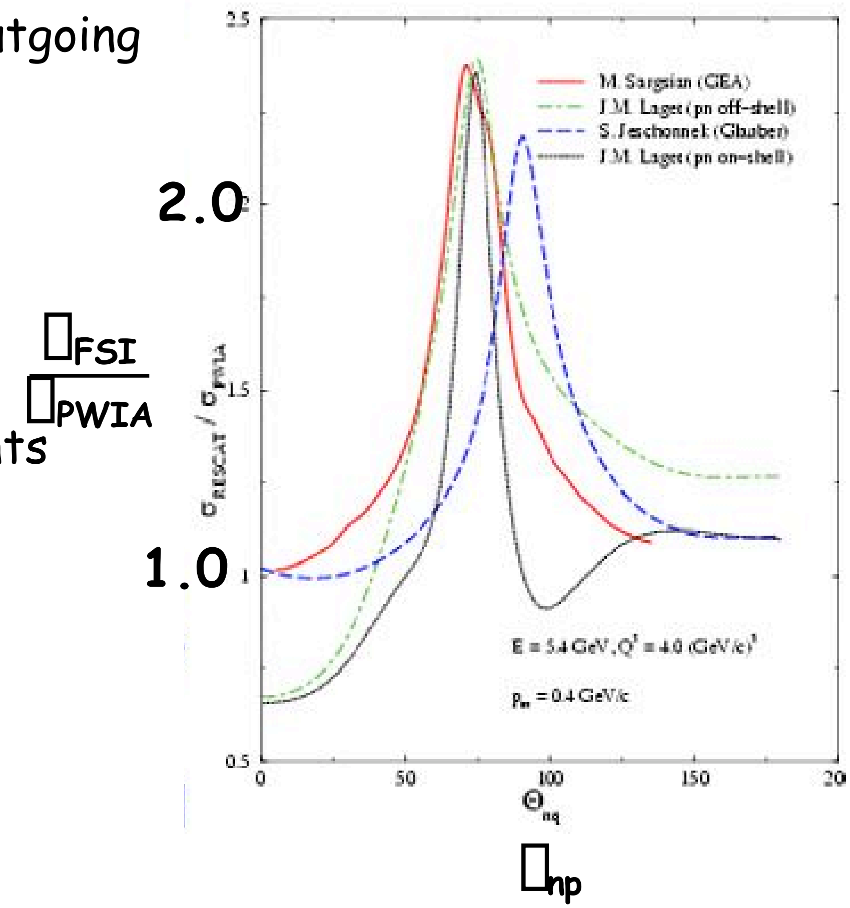
Study of deuteron electrodisintegration

Study of FSI in d(e,e'p)n (Boeglin, Ulmer, *et. al.*)

- Test predictions of FSI as a function of the scattering angle of the outgoing np pair at various Q^2
- predictions of Sargsian's GEA, Laget, and Jeschonnek
- also, study of longitudinal currents and complete separations



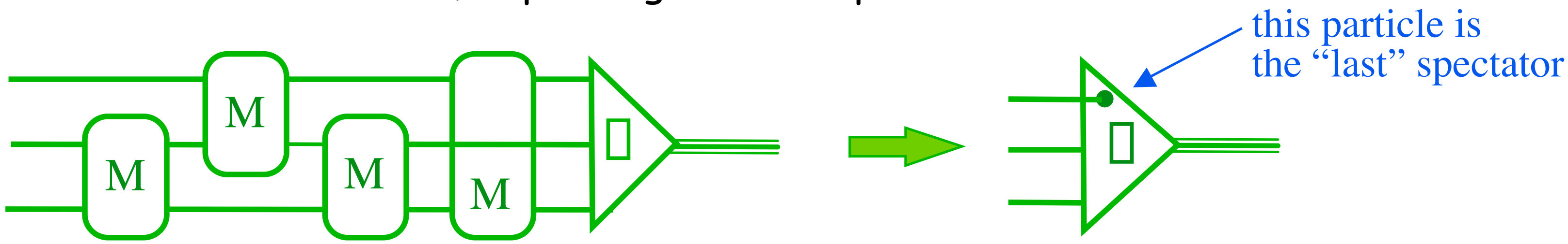
Franz Gross - JLab/W&M

II. The *NNN* interaction and correlations

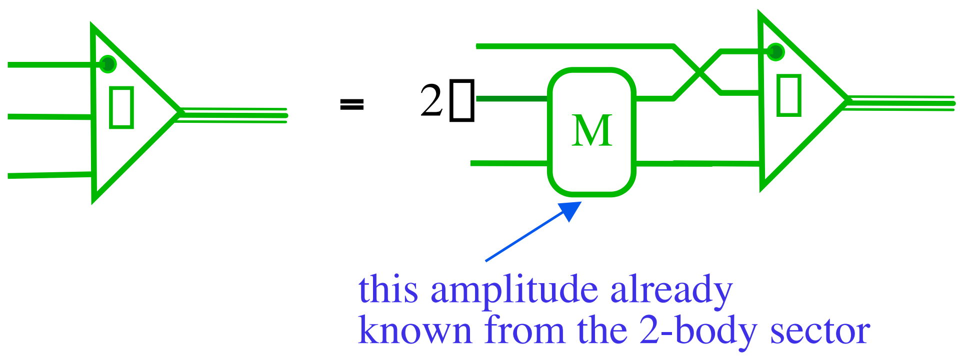
Electrodisintegration of ^3He

Theory overview (3 body bound state)

- ★ three-body scattering amplitudes and vertex functions are constructed from the two-body solutions. If there no three body forces, there are three kinds of vertex function, depending on which pair was the last to interact:



- ★ for identical nucleons, this gives the (relativistic) three body Faddeev (or AGS) equations for the relativistic vertex



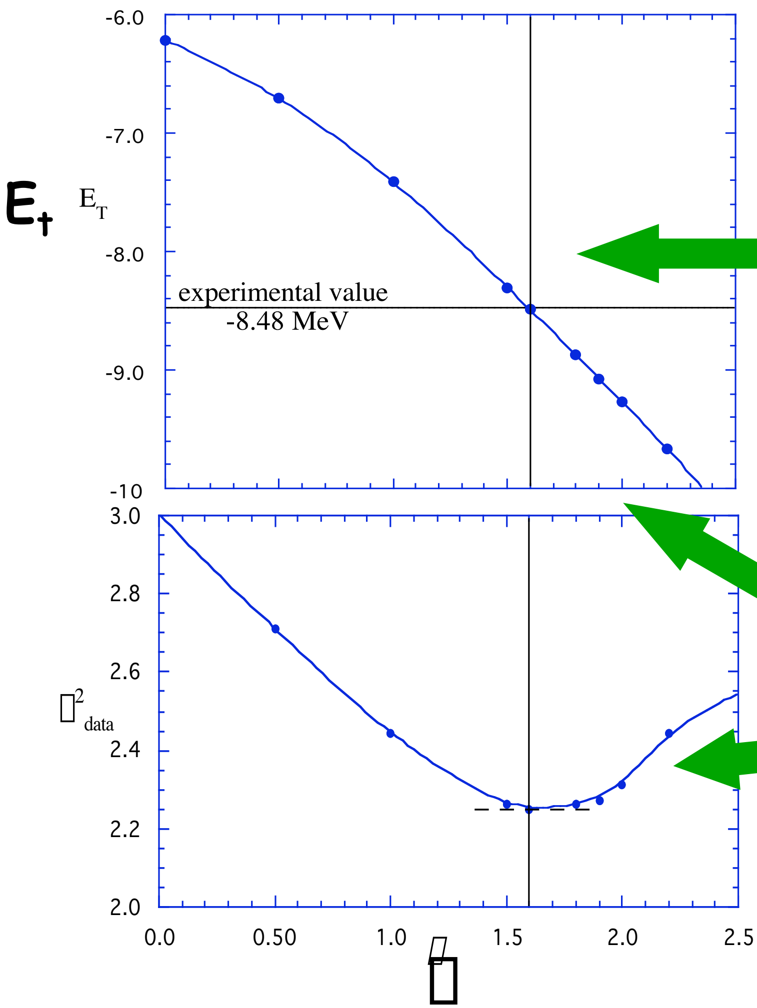
These equations in the covariant spectator theory* were solved exactly by Alfred Stadler**

(32 □ 148 channels!)

*Alfred Stadler, FG, and Michael Frank, Phys. Rev. C 56, 2396 (1997)

**Alfred Stadler and FG, Phys. Rev. Letters 78, 26 (1997)

Relativistic effects in ^3H binding*



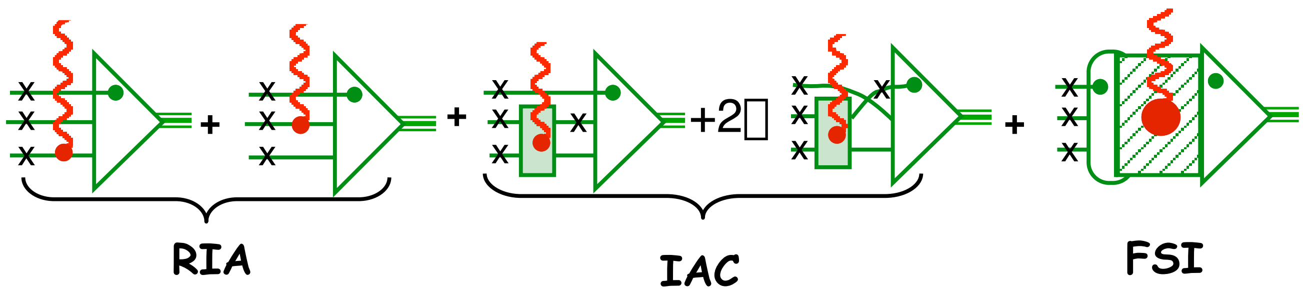
It turns out that the relativistic calculation of the three body binding energy is sensitive to a new, relativistic off-shell coupling (described by the parameter □). Non-zero □ is equivalent to effective three-body (and n-body forces).

The value of □ that gives the correct binding energy is close to the value that gives the best fit to the two-body data!

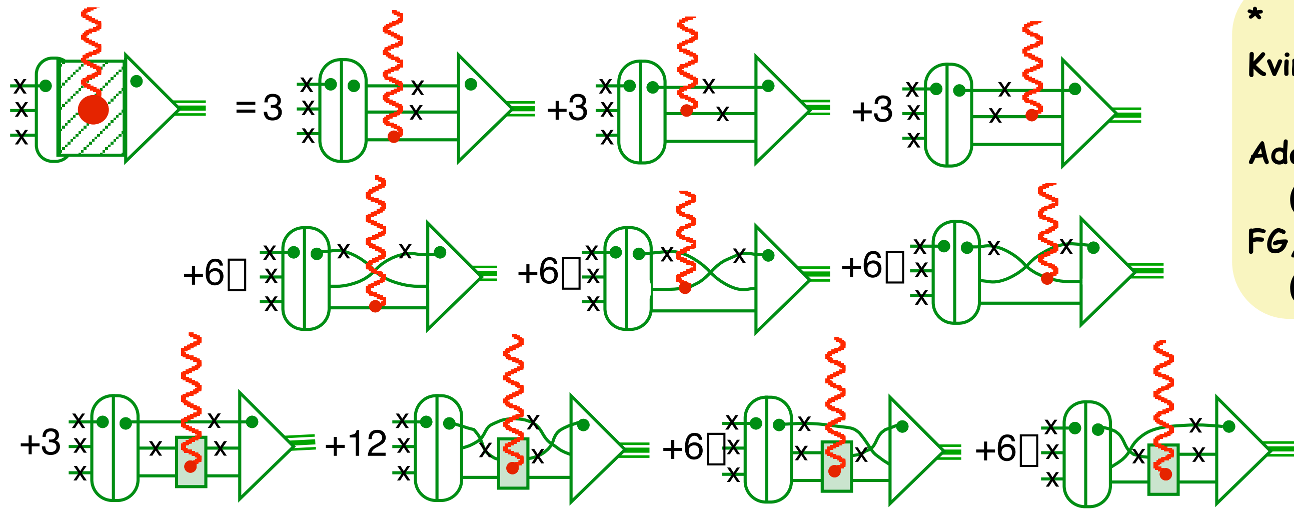
*three body calculations done with Alfred Stadler, Phys. Rev. Letters **78**, 26 (1997)

Theory overview (3 body currents - in the spectator theory)*

★ The gauge invariant three-body breakup current in the spectator theory (with on-shell particles labeled by an x) requires many diagrams



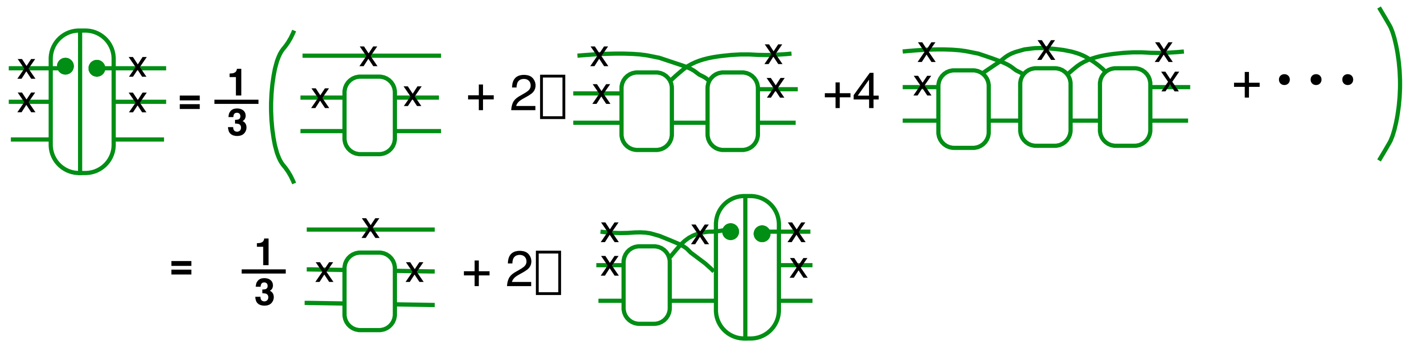
where the FSI term is



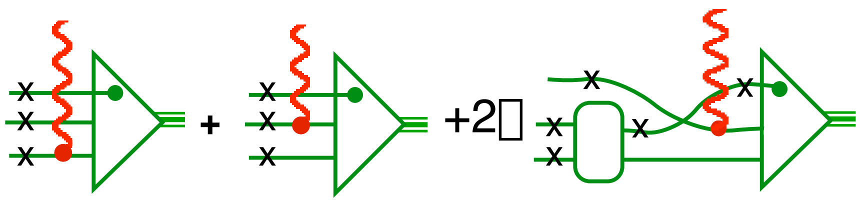
*
Kvinikhidze & Blankleider, PRC 56, 2973 (1997)
Adam & Van Orden (in preparation)
FG, A. Stadler, & T. Pena (in preparation)

Theory overview (scattering in the final state)

- and the three body scattering amplitude is

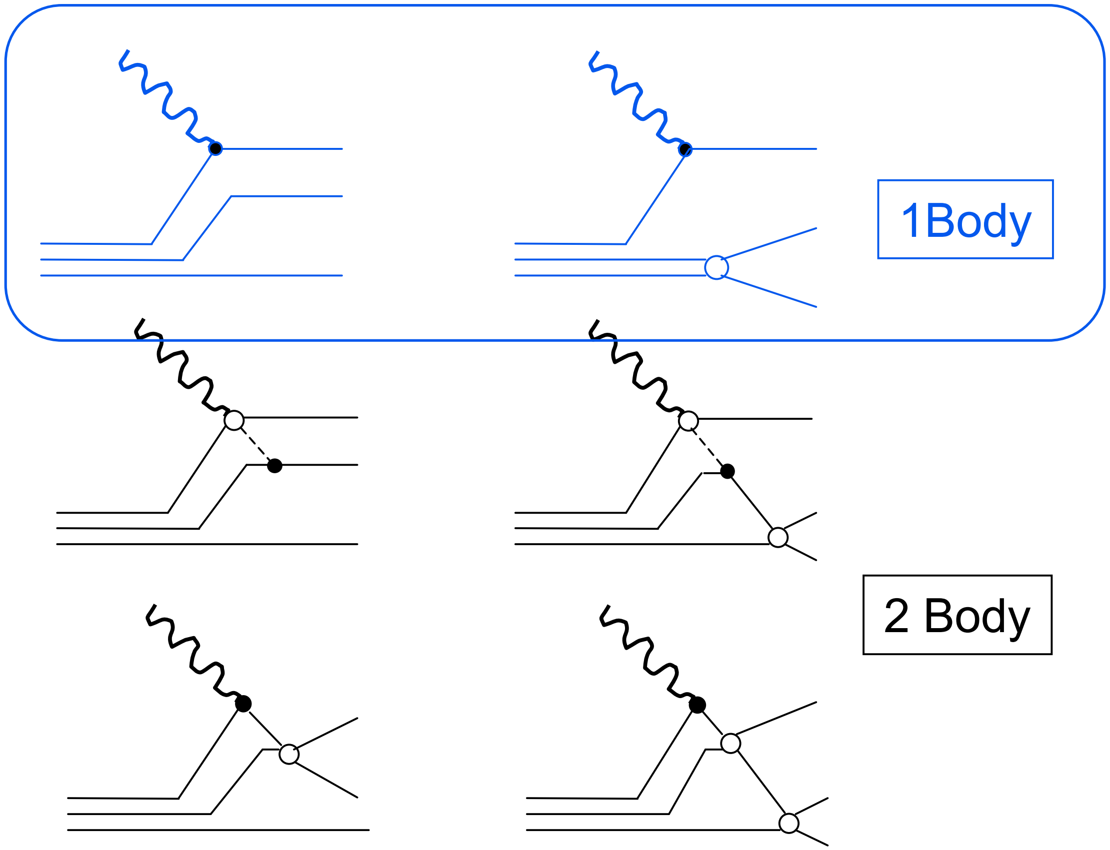


- If we neglect IAC, then the RIA with first FSI correction is



- these are to be compared to the Glockle and Laget calculations; we know the first FSI term will suppress the RIA by about a factor of 6

Laget's one and two body terms



to be compared
to the relativistic
calculation

Ulmer showed that
the Laget and Sargsian
calculations (based on
the 1 body diagrams)
give the major contributions

much more work to be done!

III: What have we learned? [Conclusions to Parts I & II]

- ★ Relativistic calculations are essential at JLab energies -- and JLab data has stimulated the development of the *relativistic theory of composite few body systems*;
- ★ excitations to low mass final states (e.g. the deuteron form factors, where $W^2 = M_d^2$) can be efficiently and correctly described by an effective theory based only on composite nucleon degrees of freedom ("left hides right");
- ★ when W^2 is large (e.g. high energy photodisintegration) additional physics, *perhaps involving the explicit appearance of quark degrees of freedom*, is needed (but: "energy dependence comes with a price");
- ★ pQCD has been very successful in motivating experiments, and is remarkably robust. It is unlikely to be correct because:
  - B has a minimum (?)
  - normalization is off by orders of magnitude
  - soft processes can easily explain the results

III. What have we learned (cont'd)?

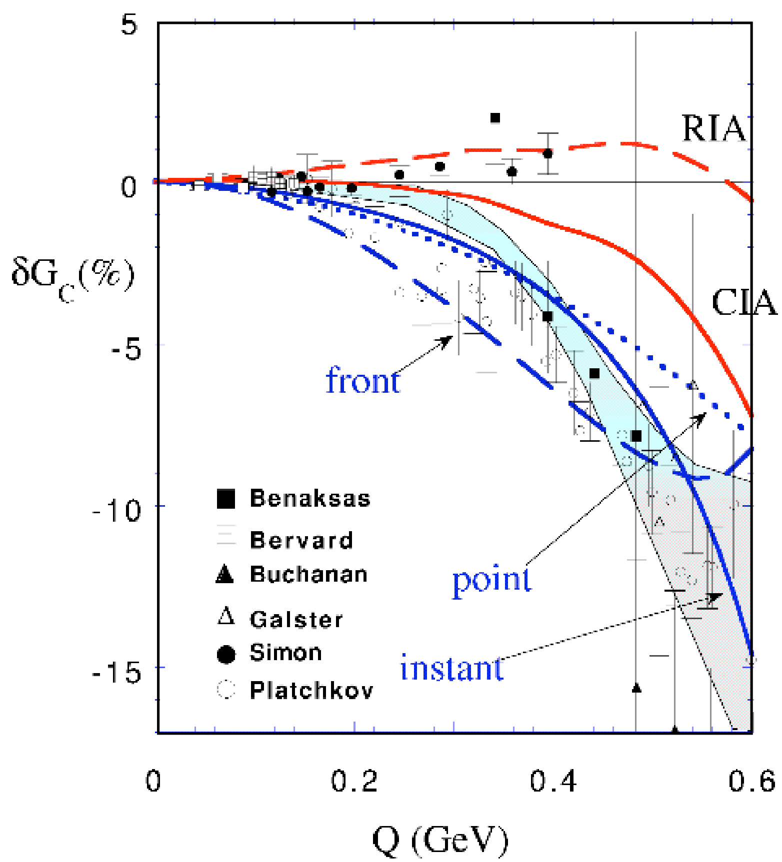
- ★ predictions will not be reliable unless the currents are constrained by the strong interaction dynamics (*i.e. calculations must be consistent*);
  - only the VODG and SP models work for the deuteron form factors
- ★ electromagnetic currents cannot be completely determined by an effective theory with composite degrees of freedom
  - recall that the new off-shell nucleon form factor, F_3, must be constrained by data

IV: What is left to be done?

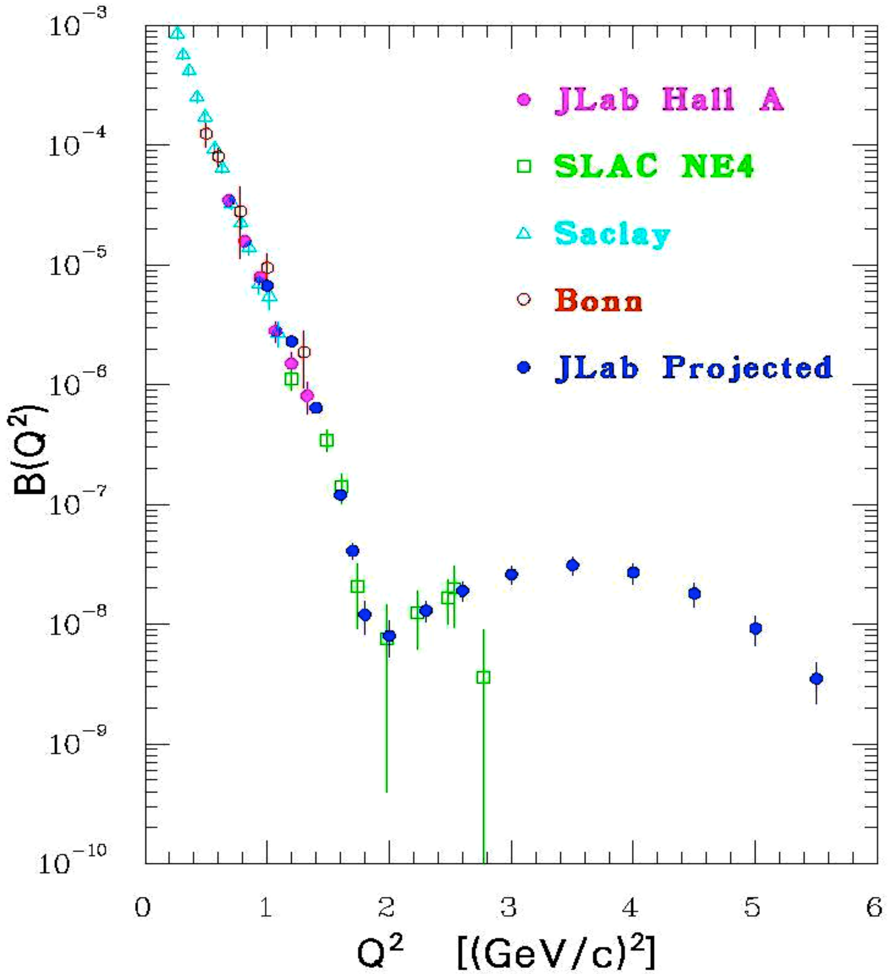
- we need a theory that puts both nuclEON and nuclEAR structure on the same footing (structure of the nucleon cannot be factored out)
- we must extend CHM to the description of high energy scattering
- important near term measurements:
  - presion measurement of A at low Q
  - measure B near the minimum and to very high Q2
  - push □d to as high an energy as possible
  - "fill in" the x dependence from x=0 to x=2 using electrodisintegration
- apply relativistic few body techniques to the study of 2 and 3 quark systems

Precision measurement of *A* at low Q^2

- Discrepancy(?) between Platchkov and Simon at low Q^2
- different relativistic models give different results -- yet all can calculate to order $(v/c)^2$
- should be able to use data to advance out understanding of relativistic corrections



New JLab Proposal



Precise measurement near minimum.

Extend to higher Q^2.

From Paul Ulmer

New Proposal: Petratos, Gomez, Beise *et al.*

END