Pseudo-Chern-Simons terms in the Standard Model (w/ applications)

Based on:

arXiv: 0704.1604 w/ S. Domokos

arXiv: 0708.1281 w/ C. Hill and R. Hill

Work in preparation w/ Hill and Hill

JLab
10/22/07

A toy model w/ PCS terms

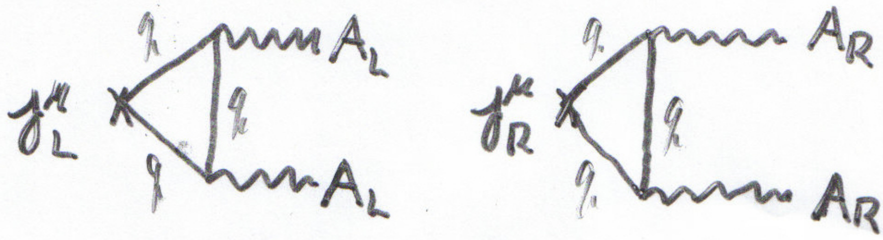
		B_L	B_R	L_L	L_R
"quarks"	q_L	1	0	0	0
	q_R	0	1	0	0
"leptons"	ℓ_L	0	0	1	0
	ℓ_R	0	0	0	1

Gauge $U(1)_L \times U(1)_R$ w/ generators

$$Q_L = B_L - L_L$$
$$Q_R = B_R - L_R$$

$$S = \int -\frac{1}{4}(F_L)^2 - \frac{1}{4}(F_R)^2 + \bar{q}_L(i\partial\!\!\!/ - A\!\!\!/_L)q_L + \bar{q}_R(i\partial\!\!\!/ - A\!\!\!/_R)q_R$$
$$+ \bar{\ell}_L(i\partial\!\!\!/ + A\!\!\!/_L)\ell_L + \bar{\ell}_R(i\partial\!\!\!/ + A\!\!\!/_R)\ell_R$$

The quark sector has anomalies



$$\partial_\mu j^\mu_{L,R} \propto \epsilon_{\mu\nu\lambda\rho} F^{\mu\nu}_{L,R} F^{\lambda\rho}_{L,R}$$

or in terms of

$$e^{iS_q^{eff}(A_L,A_R)} = \int \mathcal{D}q_L \mathcal{D}q_R \, e^{i\int \bar{q}_L (i\partial\!\!\!/ + A\!\!\!/_L) q_L + L\to R}$$

$$\delta_{L,R} S_q^{eff} = \frac{1}{24\pi^2}\int -\epsilon_L \, dA_L \, dA_L + \epsilon_R \, dA_R \, dA_R$$

but these are cancelled by leptons

$$\delta S_q^{eff} + \delta S_\ell^{eff} = 0$$

We now make this more like the Standard Model by adding a mass for the quarks

$$m_q e^{i\phi/f} \bar{q}_L q_R + h.c.$$

$$\phi/f \sim \pi^0/f_\pi \quad \text{in QCD}$$

$$q_L \to e^{i\epsilon_L} q_L \,, \quad \delta A_L = d\epsilon_L \,, \quad \delta_L \phi = f\epsilon_L$$

$$q_R \to e^{i\epsilon_R} q_R \,, \quad \delta A_R = d\epsilon_R \,, \quad \delta_R \phi = -f\epsilon_R$$

Consider the low-energy theory of ϕ, A_L, A_R after integrating out the massive quarks.

The anomalies must still cancel.

$$\Rightarrow \quad \Gamma_{WZW}(\phi, A_L, A_R)$$

$$\Gamma_{WZW}(\phi, A_L, A_R) = \frac{-1}{24\pi^2}\int \Big(A_L A_R dA_L$$

$$+ A_L A_R dA_R + \frac{\phi}{f}\left(dA_L dA_L + dA_R dA_R + dA_L dA_R\right)\Big)$$

w/ $\delta \Gamma_{WZW} = \delta S_q^{eff}$

$\Rightarrow$ anomalies still cancel

Now we add a new ingredient:

a background field coupling to the baryon current (the ω meson)

$$S_q \to \int d^4x \, \bar{q}_L (i\partial\!\!\!/ - A\!\!\!/_L - \omega\!\!\!/) q_L + \bar{q}_R (i\partial\!\!\!/ - A\!\!\!/_R - \omega\!\!\!/) q_R$$

We are not gauging $U(1)_B$ - it is anomalous - just adding a bkgnd field

$$\delta \Gamma_{WZW} = \frac{-1}{24\pi^2}\int \epsilon_L \left(2dA_L dw + (dw)^2\right)$$
$$- \epsilon_R \left(2dA_R dw + (dw)^2\right)$$

but the theory should still make sense
$\Rightarrow$ local counter term to cancel this

$$\Gamma_C = \frac{1}{24\pi^2}\int \left(-2wA_R dA_R - wA_R dw - (R \to L)\right)$$

Claim is

$$S^{eff} = \Gamma_{WZW}(\phi, A_L + w, A_R + w) + \Gamma_C$$
$$= \Gamma_{WZW}(\phi, A_L, A_R) + \Gamma_{PCS}$$

variation cancelled
by leptons.

gauge inv

$$\Gamma_{PCS} = \frac{1}{8\pi^2}\int \left(2w\,dA + w\,dZ + w\,dw\right)\left(Z - \frac{d\phi}{f}\right)$$

$A_L = Z + A, \quad A_R = A$

One can apply the same procedure to the SM:

$$\Gamma_{WZW}(U, A_L + B_L, A_R + B_R) \qquad U = e^{i\pi^a \lambda^a / f_\pi}$$

A - fundamental gauge fields - $SU(2)_L \times U(1)_Y$

B - background vector and axial-vector fields

Restricting to $U(2)_L \times U(2)_R$

$$B_L + B_R = \begin{pmatrix} \rho^0 + \omega & \sqrt{2}\rho^+ \\ \sqrt{2}\rho^- & -\rho^0 + \omega \end{pmatrix}$$

$$B_L - B_R = \begin{pmatrix} a_1^0 + f_1 & \sqrt{2}\, a_1^+ \\ \sqrt{2}\, a_1^- & -a_1^0 + f_1 \end{pmatrix}$$

and find Γ_C so that $\delta(\Gamma_{WZW} + \Gamma_C)$ is cancelled by leptons for $SU(2)_L \times U(1)_Y$ gauge variations in the presence of bkgnd B_L, B_R

Standard Model pCS terms:

MiniBoone excess

$$
\begin{aligned}
\Gamma_{AAB} &= \frac{N_c}{48\pi^2}\int dZZ\left[\frac{s_W^2}{c_W^2}\rho^0+\left(\frac{3}{2c_W^2}-3\right)\omega-\frac{1}{2c_W^2}f\right]+dAZ\left[-\frac{s_W}{c_W}\rho^0-\boxed{\frac{3s_W}{c_W}\omega}\right]+dZ\left[W^-\rho^++W^+\rho^-\right]\frac{s_W^2}{c_W} \\
&\quad +dA\left[W^-\rho^++W^+\rho^-\right](-s_W)+(dW^+W^-+dW^-W^+)\left[-\frac{3}{2}\omega-\frac{1}{2}f\right], \\
\Gamma_{ABB} &= \frac{N_c}{48\pi^2}\int Z\Big\{d\rho^0\Big[-\frac{3}{2c_W}\omega-\frac{s_W^2}{c_W}a^0+\left(-\frac{3}{2c_W}+3c_W\right)f\Big]+d\omega\Big[-\frac{3}{2c_W}\rho^0+\left(-\frac{3}{2c_W}+3c_W\right)a^0-\frac{s_W^2}{c_W}f\Big] \\
&\quad +da^0\left[\frac{s_W^2}{c_W}\rho^0+\left(\frac{3}{2c_W}-3c_W\right)\omega-\frac{1}{2c_W}f\right]+df\left[\left(\frac{3}{2c_W}-3c_W\right)\rho^0+\frac{s_W^2}{c_W}\omega-\frac{1}{2c_W}a^0\right]\Big\} \\
&\quad +dA\Big\{s_W\rho^0a^0+\boxed{3s_W\rho^0f}+3s_W\omega a^0+s_W\omega f\Big\}+dZ\Big\{-\frac{s_W^2}{c_W}\left(\rho^+a^-+\rho^-a^+\right)\Big\} \\
&\quad +dA\Big\{s_W\left(\rho^+a^-+\rho^-a^+\right)\Big\} \quad \text{Prediction for } f_1\to\rho+\gamma \\
&\quad +\frac{3}{2}\left[W^+d\rho^-+W^-d\rho^+)\right](-\omega+f)+\frac{3}{2}\left[W^+(-\rho^-+a^-)+W^-(-\rho^++a^+)\right]d\omega \\
&\quad +\frac{1}{2}\left[W^+da^-+W^-da^+\right](-3\omega-f)+\frac{1}{2}\left[W^+(-3\rho^--a^-)+W^-(-3\rho^+-a^+)\right]df, \\
\Gamma_{AAAB} &= \frac{N_c}{48\pi^2}\int i\Big\{W^+W^-\Big[3c_WZ+3s_WA\Big]\omega+W^+W^-\Big[\left(c_W+\frac{1}{2c_W}\right)Z+s_WA\Big]f\Big\}, \\
\Gamma_{AABB} &= \frac{N_c}{48\pi^2}\int i\Big\{W^+W^-\left[\frac{3}{2}(\rho^0+a^0)\omega-\frac{1}{2}(\rho^0-a^0)f\right] \\
&\quad +W^+Z\left[\frac{3c_W}{2}\rho^-f-\frac{3c_W}{2}\rho^-\omega-\frac{c_W}{2}a^-f+\frac{3c_W}{2}\omega a^--\frac{1}{c_W}\rho^-f\right] \\
&\quad +W^-Z\left[-\frac{3c_W}{2}\rho^+f+\frac{3c_W}{2}\rho^+\omega+\frac{c_W}{2}a^+f-\frac{3c_W}{2}\omega a^++\frac{1}{c_W}\rho^+f\right] \\
&\quad +W^+A\left[\frac{s_W}{2}\left(3\rho^-f-3\rho^-\omega-a^-f+3\omega a^-\right)\right]+W^-A\left[-\frac{s_W}{2}\left(3\rho^+f-3\rho^+\omega-a^+f+3\omega a^+\right)\right]\Big\}, \\
\Gamma_{ABBB} &= \frac{N_c}{48\pi^2}\int i\Big\{W^+\Big[\rho^-\rho^0(\omega-2f)-\rho^-\omega a^0+\rho^0\omega a^-+\omega a^-a^0\Big] \\
&\quad +W^-\Big[\rho^+\rho^0(-\omega+2f)+\rho^+\omega a^0-\rho^0\omega a^+-\omega a^+a^0\Big] \\
&\quad +Z\Big[\rho^+\rho^-\left(\frac{1}{c_W}\omega+\left(-4c_W+\frac{2}{c_W}\right)f\right)+\rho^+\omega a^-\left(-2c_W+\frac{1}{c_W}\right) \\
&\qquad +\rho^-\omega a^+\left(2c_W-\frac{1}{c_W}\right)+\omega a^+a^-\left(\frac{1}{c_W}\right)\Big]+A\Big[s_W\left(-4\rho^+\rho^-f-2\rho^+\omega a^-+2\rho^-\omega a^+\right)\Big]\Big\}. \qquad (2)
\end{aligned}
$$

$$S_{pcs} \sim \int A_1 \wedge A_2 \wedge dA_3 = \int \epsilon^{\mu\nu\lambda\rho} A_{1\mu} A_{2\nu} \partial_\lambda A_{3\rho}$$

with A_i = vector fields.

- If A_1 is a bkgnd vector field (e.g. baryon current) then in rest frame

$$S_{pcs} \sim \int A_1^0 \underbrace{\epsilon^{ijk} A_{2i} \partial_j A_{3k}}_{\text{CS term in } \mathbb{R}^3}$$

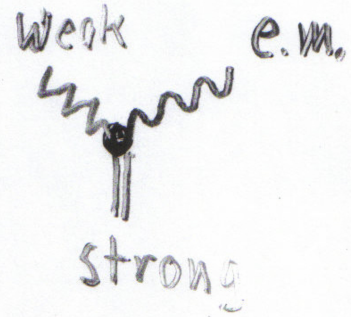
- S_{pcs} is not gauge inv. for fundamental gauge fields $\Rightarrow$ A_1, A_2 must be massive (Stuckelberg) gauge fields or bkgnd fields

- In a P inv. theory an odd number of the A_i must be axial-vector rather than vector fields.

We now make a leap in the spirit of vector meson dominance and regard these as couplings for dynamical vector mesons of QCD to fundamental gauge fields. I will focus on 2 terms.

1. $$S_{pcs} = \frac{1}{16\pi^2}\frac{e\, g_w g_z}{\cos\theta_w}\int \epsilon^{\mu\nu\lambda\rho}\,\omega_\mu Z_\nu F_{\lambda\rho}$$

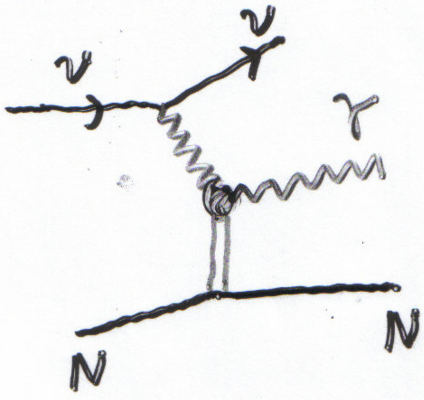
"321 widget"



2. $$S_{pcs} = \frac{3\, e\, g_\rho g_f}{16\pi^2}\int \epsilon^{\mu\nu\lambda\rho}\,\rho^a_\mu f^a_\nu\, \partial_\lambda A_\rho$$

leads to $f_1 \to \rho + \gamma$

The 1^{st} term leads to



which is a large potential bkgnd to charged current events

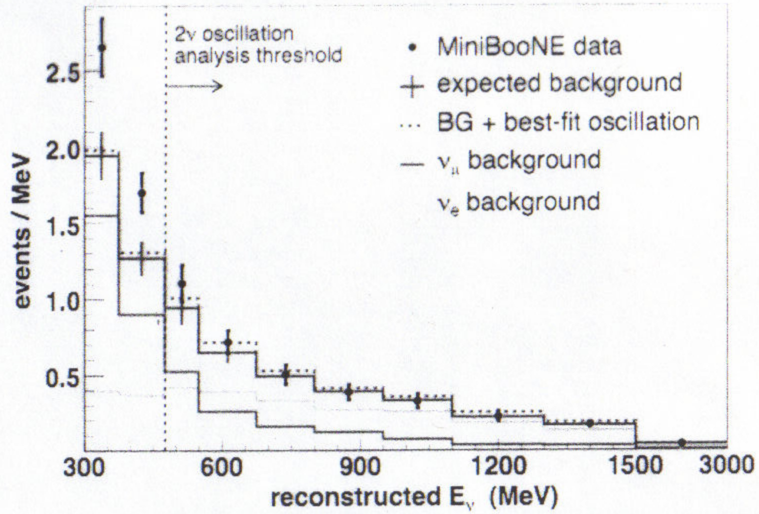
$$\nu_e + N \to e^- + N'$$

since γ, e^- are not distinguishable by Cerenkov radiation.

We find ~140 excess events using the peak ν_μ beam energy of ~700MeV for MiniBoone exp.

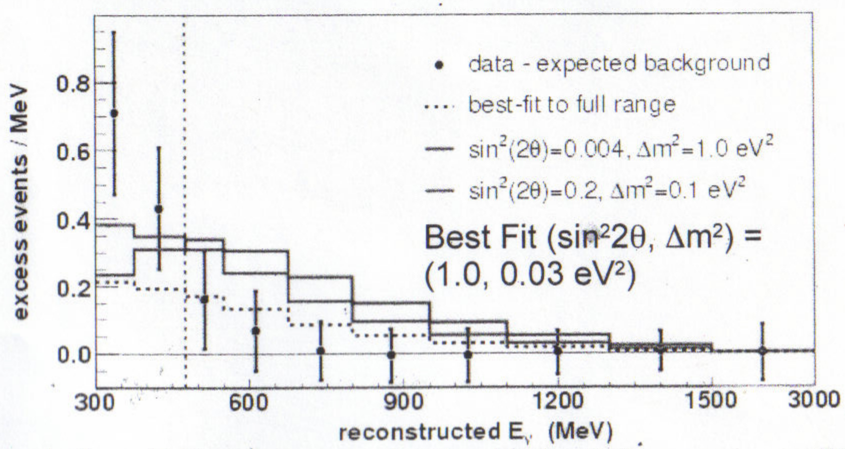
oscillation results: low-energy region

Track-based analysis
E_ν distributions:



For:
$300 < E_\nu < 475$ MeV
96 ± 17 ± 20 events
Excess: 3.7σ

background subtracted data:



The energy-dependence of excess is not consistent with $\nu_\mu \rightarrow \nu_e$ appearance assuming standard energy dependence

$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \, \sin^2(1.27 \Delta m^2 L/E)$

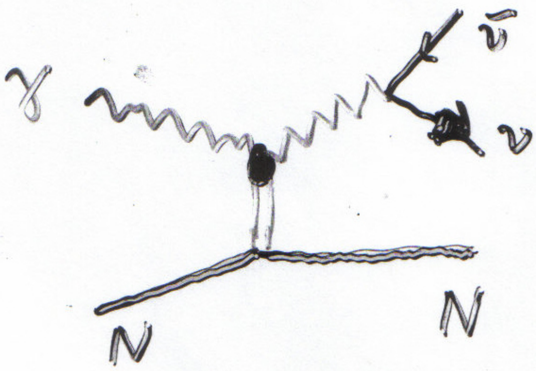
Including nuclear recoil and a simple form factor gives a reasonable fit to both energy and angular dependence of excess events.

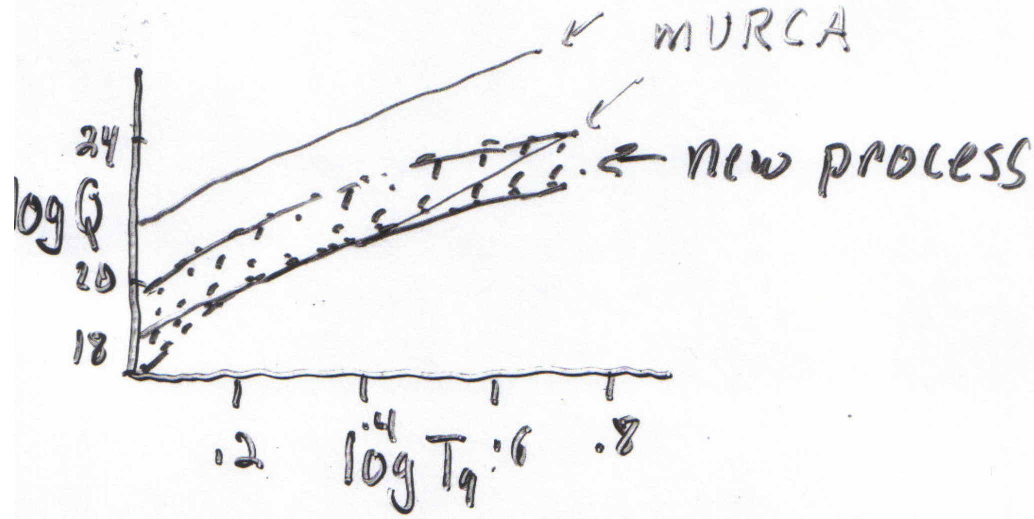
A detailed comparison requires

- coherence effects
- Fermi motion of nucleons in carbon nucleus
- Pauli blocking
- etc.

The acid test would be an experiment that can distinguish γ from e^- in the final state.

This effect also contributes to neutron star cooling:

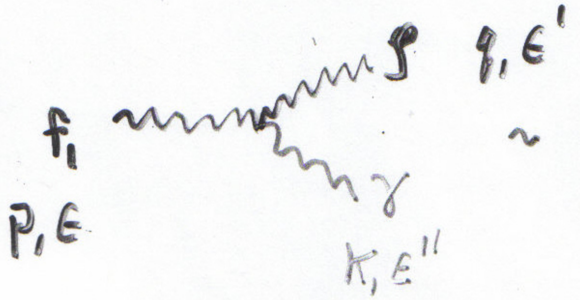




1. $S_{\rho f \gamma} = \lambda \int \epsilon^{\mu\nu\lambda\rho} \rho^0_\mu f^0_\nu \partial_\lambda A_\rho$

provides a sanity check. It leads to

$$f_1 \to \rho + \gamma$$



$$\sim \lambda \epsilon^{\mu\nu\lambda\rho} \epsilon_\mu \epsilon'^*_\nu k_\lambda \epsilon''^*_\rho$$

$$\Gamma(f_1 \to \rho + \gamma) = 2.9 \lambda^2 \text{ MeV}$$

$$\Gamma_{exp}(f_1 \to \rho + \gamma) = 1.32 \text{ MeV}$$

Agrees well if $\dfrac{g_\rho^2}{4\pi} \sim 2.7$

<u>and</u> one finds measured helicity structure. Unlike old calc. of Babcock + Rosner.

only QCD vector mesons! They are not fixed by these techniques, but they can be computed at large N_c using AdS/QCD ideas – at large 't Hooft coupling.

2. AdS/QCD

- **Review**

AdS_5 w/ IR cutoff: $ds^2 = \frac{1}{z^2}\left(-dz^2 + dx^\mu dx_\mu\right)$

$$\underset{\substack{\uparrow \\ \text{UV}}}{0} < z \leq \underset{\substack{\uparrow \\ \text{IR - scale}}}{z_m}$$

Fields: $A_L^{a\mu} \sim J_L^{a\mu} = \bar{q}_L t^a \gamma^\mu q_L$

$A_R^{a\mu} \sim J_R^{a\mu} = \bar{q}_R t^a \gamma^\mu q_R$

$X^{\alpha\beta} \sim \bar{q}^\alpha q^\beta$

We will gauge $U(N_f)_L \times U(N_f)_R$ and focus on $N_f = 2$.

$$S = \int d^4x\, dz \sqrt{g}\, \mathrm{Tr}\left[|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\right]$$

$$g_5^2 = 12\pi^2/N_c$$

One finds π, ρ, a_1, f_1 ••• by expandin[g]
around

$$X_0(z) = \frac{1}{2} m_q \mathbb{I} z + \frac{1}{2} \sigma \mathbb{I} z^3$$

coeff of $\bar{q}q$
in S_{QCD}

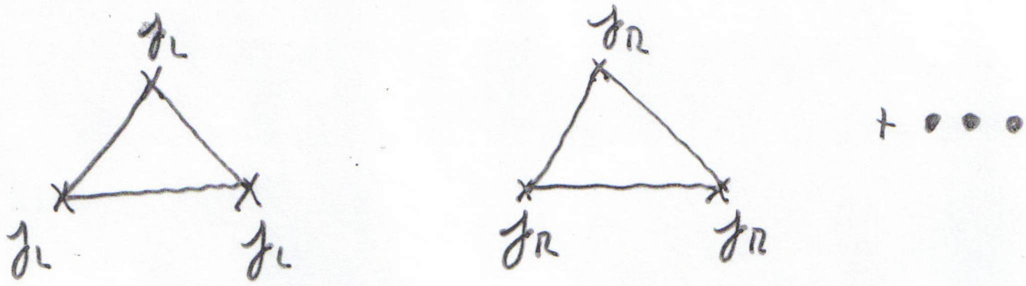
$\langle \bar{q}q \rangle$ exp. value

Parameters: z_m, m_q, σ

Fits spectrum, decay constants
to 10 - 15%

• Add Chern-Simons terms

In QCD there are anomalies



The AdS dual involves terms which are gauge inv. in bulk, but vary on the bndy in the same way that QCD would if coupled to ficticious $U(N_f)_L \times U(N_f)_R$ gauge fields.

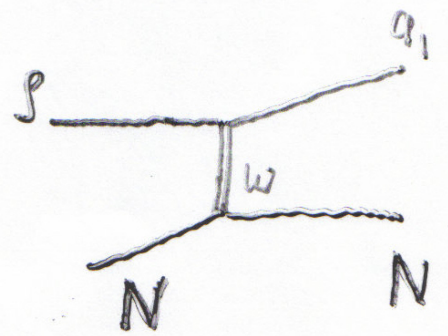
$$S_{CS} = \frac{N_c}{24\pi^2} \int \omega_5 (A_L) - \omega_5 (A_R)$$

w/ $$d\omega_5 = \mathrm{Tr}\, F^3$$
$$\delta \omega_5 = d\omega_4^1$$

$W_5 \sim \mathrm{Tr}\,(AF^2 + \cdots)$ and this leads to a variety of pcs coupling including

$$S_{\omega-\rho-a_1} \approx g_{\omega a\rho} \int d^4x\, \epsilon^{\mu\nu\lambda\rho}\, \omega_\mu\, a_\nu\, \partial_\lambda\, a_\rho$$

Can one study this by looking for a_1 decays from polarized ρs?



1. The SM has pCS terms mixing A, Z w/ QCD vector mesons.

2. These have observeable consequence

   Miniboone excess

   $f_1 \to \rho + \gamma$

3 pCS terms in pure QCD are also possible, and it measurable provide new tests of AdS/QCD framework.