Chiral extrapolation of nucleon form factors from lattice data

P. Wang, D. B. Leinweber, A. W. Thomas, and R. Young

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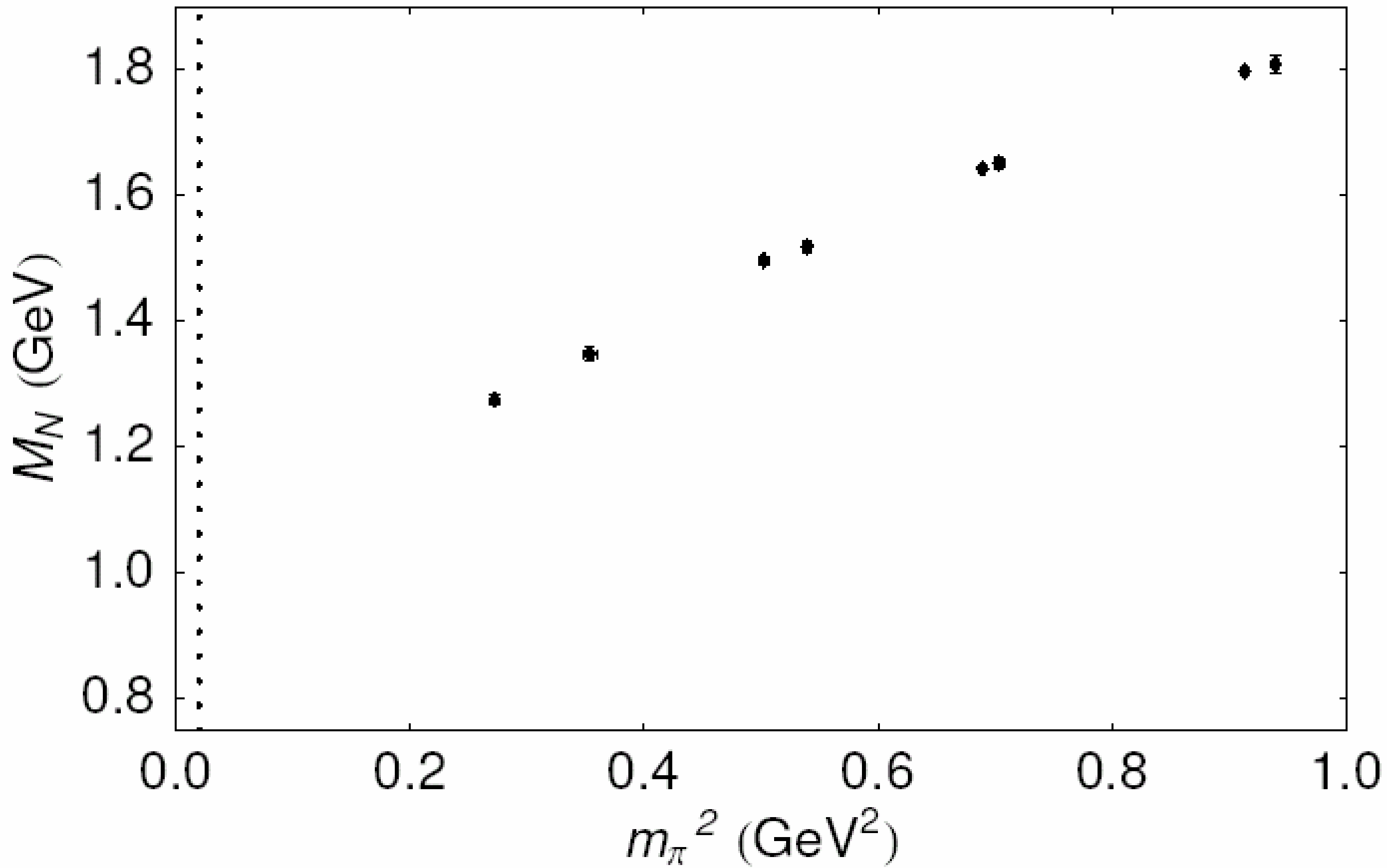
Introduction



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$$M_N = c_0 + c_2\, m_\pi^2 + c_4\, m_\pi^4 + \ldots$$

$$M_N = \{\text{Terms Analytic in } m_q\} + \{\text{Chiral loop corrections}\}$$

Chiral perturbation theory:

Chiral perturbation theory provides a systematic method for discussing the physics at low energy by means of an effective field theory.

The general Lagrangian can be written with an increasing number of derivatives and quark(meson) mass terms.

$$£_m = £_2 + £_4 + \ \ldots\ , \ £_{MB} = £_{MB}^{(1)} + £_{MB}^{(2)} + \ldots$$

$$\mathcal{L}_2 = \frac{F_0^2}{4}\mathrm{Tr}[D_\mu U (D^\mu U)^\dagger] + \frac{F_0^2}{4}\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)$$

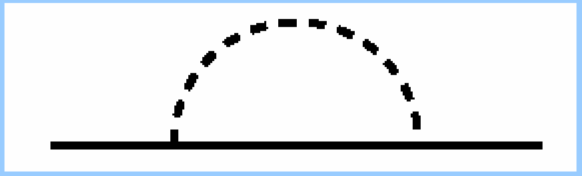
The lowest order Lagrangian including baryons:

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\not{D} - m_N + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu\right)\Psi$$

$$u_\mu = i\left[u^\dagger \partial_\mu u - u \partial_\mu u^\dagger\right]$$

The Feynman diagrams can be arranged as power counting scheme (q^D or m_π^D).

$$D = 4N_L - 2I_M - I_B + \Sigma(2n \times N_{2n}{}^M) + \Sigma(n \times N_n{}^B)$$



$$D = 4 - 2 - 1 + 2 = 3$$

Treating the nucleons as relativistic Dirac fields does not satisfy the power counting.

$$\psi_{\vec{p}}^{(+)(\alpha)}(\vec{x},t) = u^{(\alpha)}(\vec{p})e^{-ip\cdot x}$$

$$u^{(\alpha)}(\vec{p}) = \sqrt{E(\vec{p})+m}\begin{pmatrix} \chi^{(\alpha)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E(\vec{p})+m}\chi^{(\alpha)} \end{pmatrix}$$

Heavy baryon chiral perturbation theory:

Separate the original baryon field into light and heavy components.
Consider baryons as extremely heavy static source.
The light component satisfies the massless Dirac equation.
The heavy component is suppressed by powers of *1/m*.

$$\Psi(x) = e^{-imv\cdot x}\left[\mathcal{N}_v(x) + \mathcal{H}_v(x)\right]$$

$\mathcal{N}_v$ is the massless field which has the following properties:

$$\bar{\mathcal{N}}_v\gamma_5\mathcal{N}_v = 0$$

$$\bar{\mathcal{N}}_v\gamma^\mu\mathcal{N}_v = v^\mu\bar{\mathcal{N}}_v\mathcal{N}_v$$

$$\bar{\mathcal{N}}_v\gamma^\mu\gamma_5\mathcal{N}_v = 2\bar{\mathcal{N}}_v S_v^\mu\mathcal{N}_v$$

$$\bar{\mathcal{N}}_v\sigma^{\mu\nu}\mathcal{N}_v = 2\epsilon^{\mu\nu\rho\sigma}v_\rho\bar{\mathcal{N}}_v S_\sigma^v\mathcal{N}_v$$

S_v^μ is the spin operator:

$$S_v^\mu = \frac{i}{2}\gamma_5\sigma^{\mu\nu}v_\nu$$

The Lagrangian in chiral perturbation theory

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\not{D} - m_N + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right)\Psi$$

is changed to be:

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\mathcal{N}}_v\left[iv\cdot D + g_A S_v\cdot u\right]\mathcal{N}_v$$

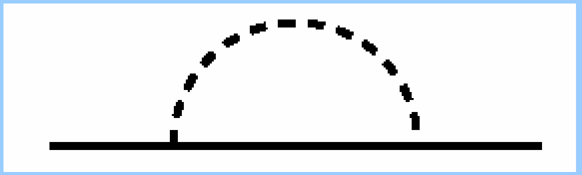
The propagators for octet and decuplet baryons:

$$\frac{i}{v\cdot k - \delta^{jN} + i\varepsilon}, \quad \delta^{ab} = m_b - m_a$$

$$\frac{iP^{\mu\nu}}{v\cdot k - \delta^{jN} + i\varepsilon}, \quad P^{\mu\nu} \text{ is } v^\mu v^\nu - g^{\mu\nu} - (4/3)S_v^\mu S_v^\nu$$

$$M_N = a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \ldots + \chi_\pi I_\pi (m_\pi)$$

Leading order one loop Feynman diagram:



$$\chi_\pi I_\pi(m_\pi) = -\frac{3\, g_A^2}{32\,\pi\, f_\pi^2} \frac{2}{\pi} \int_0^\infty dk\, \frac{k^4}{k^2 + m^2}$$

With the dimensional regularization:

$$I_\pi \to \infty + \infty m_\pi^2 + m_\pi^3$$

$$M_N = c_0 + c_2 m_\pi^2 + \chi_\pi m_\pi^3 + c_4 m_\pi^4$$

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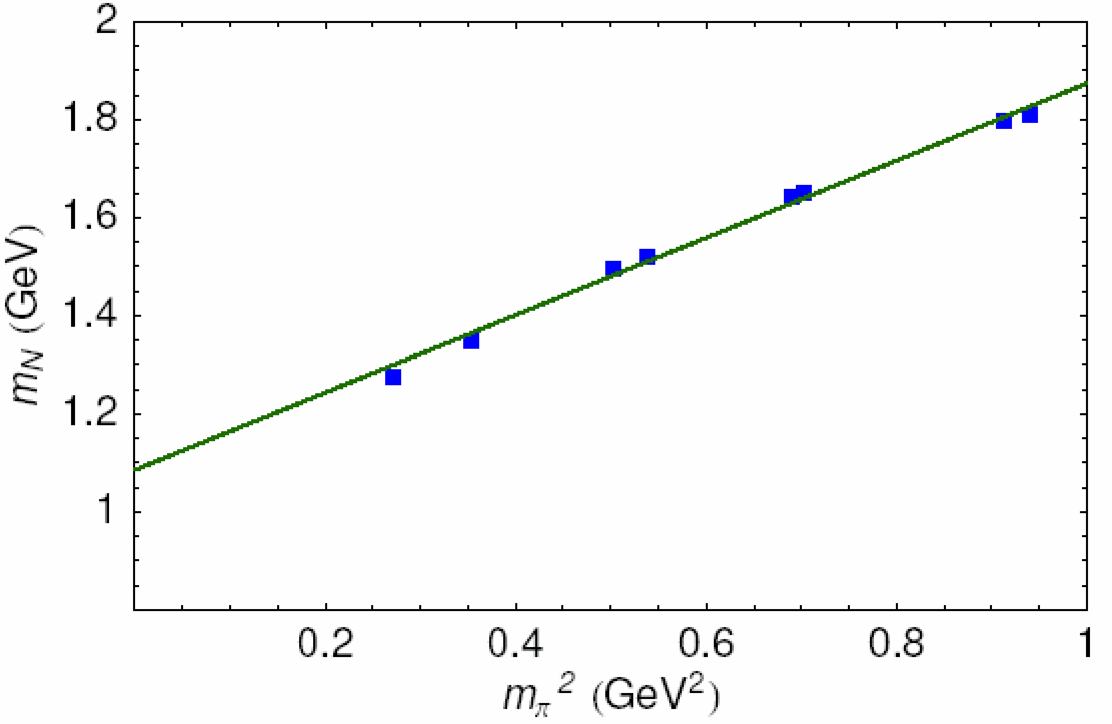

- A: $a_0 + a_2 m_\pi^2$

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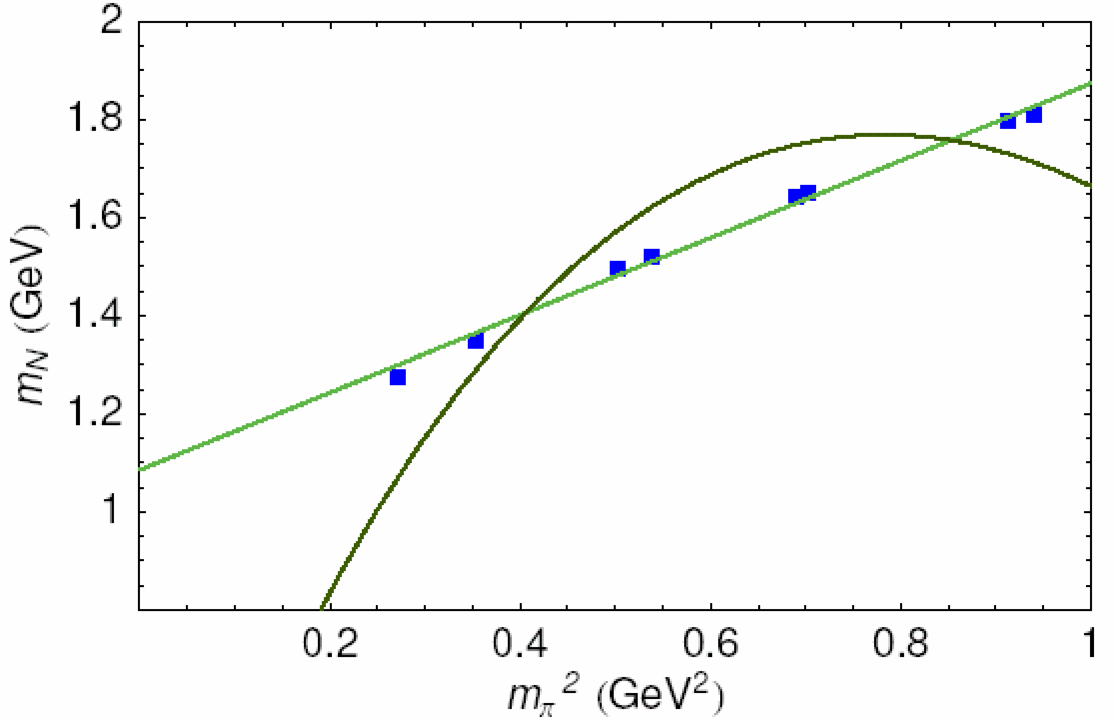

- B: $a_0 + a_2 m_\pi^2 + \chi_\pi I_\pi$

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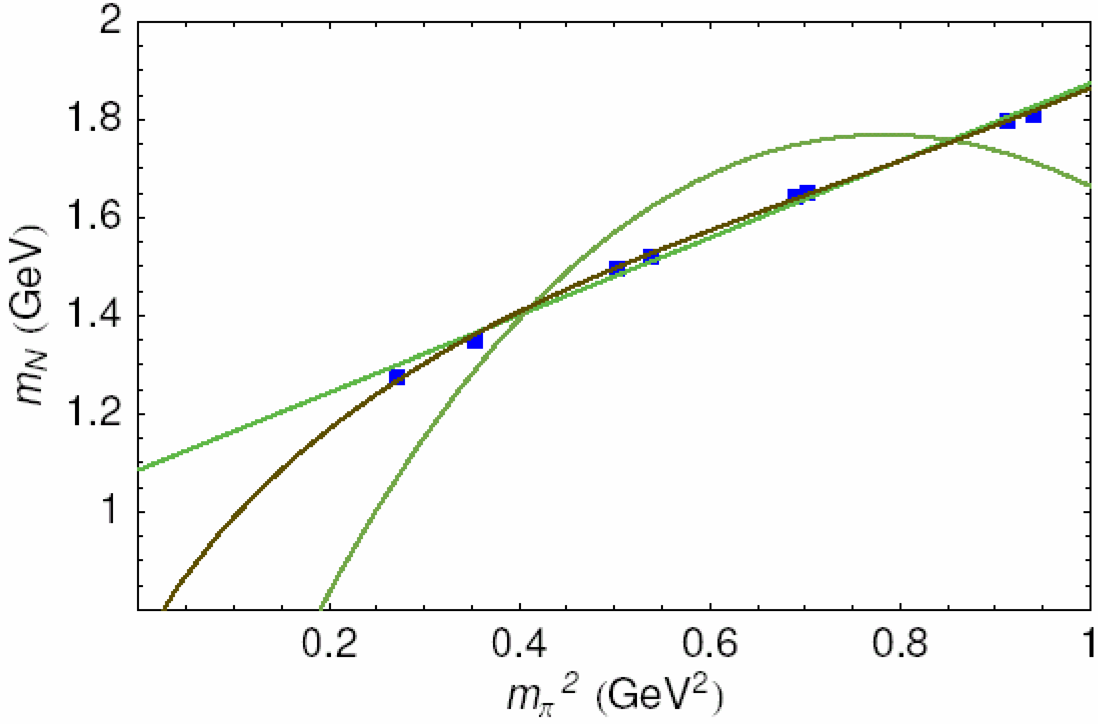


- C: $a_0 + a_2 m_\pi^2 + \chi_\pi I_\pi + a_4 m_\pi^4$

Chiral perturbation theory with dimensional regularization fails to fit the lattice data.

The chiral expansion is not convergent.

High order terms are important at large pion mass.

How to solve this problem? How to build the high order terms into the one loop contribution and make the chiral expansion be convergent quickly.

$\rightarrow$ **Finite Range Regularization**

DR:

Large contributions to the integral from $k \to \infty$ portion of integral.

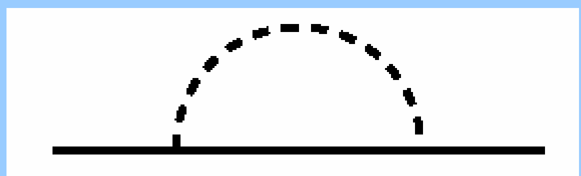
Baryons are hard point particle.

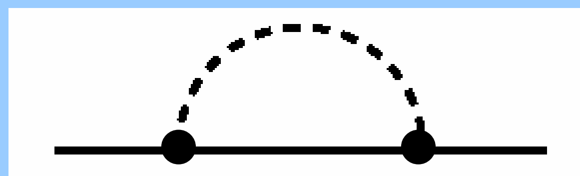
Short distance physics is highly overestimated.

FRR:

Remove the incorrect short distance contribution associated with the suppression of loop integral at ultraviolet region.

Baryons are soft particle with structure which results in a vertex function in the loop integral.





The loop integral in FRR:

$$I_\pi(m_\pi) = \frac{2}{\pi}\int_0^\infty dk\, \frac{k^4\, u^2(k)}{k^2+m^2}$$

$u\,(k)$ is the regulator, for example for dipole:

$$u(k) = \left(\frac{\Lambda^2}{\Lambda^2+k^2}\right)^2$$

$$I_\pi = \frac{1}{16}\frac{\Lambda^5(m_\pi^2+4m_\pi\Lambda+\Lambda^2)}{(m_\pi+\Lambda)^4}$$

Expand in m_π,

$$I_\pi \to \frac{\Lambda^3}{16} - \frac{5\Lambda}{16}m_\pi^2 + m_\pi^3 - \frac{35}{16\Lambda}m_\pi^4 + \frac{4}{\Lambda^2}m_\pi^5 + \ldots$$

In FRR, the nucleon mass is expressed as:

$$M_N = a_0^\Lambda + a_2^\Lambda m_\pi^2 + \chi_\pi I_\pi(m_\pi, \Lambda) + a_4^\Lambda m_\pi^4$$

$$\begin{aligned} M_N &= \left(a_0 + \chi_\pi \frac{\Lambda^3}{16}\right) + \left(a_2 - \chi_\pi \frac{5\Lambda}{16}\right) m_\pi^2 + \chi_\pi m_\pi^3 \\ &\quad + \left(a_4 - \chi_\pi \frac{35}{16\Lambda}\right) m_\pi^4 + \cdots \\ &= c_0 + c_2 m_\pi^2 + \chi_\pi m_\pi^3 + c_4 m_\pi^4 \end{aligned}$$

To any finite order, FRR is mathematically equivalent to dimensional regularization.

For small pion mass, FRR and DR give almost same results.

For large pion mass, can FRR fit lattice data?

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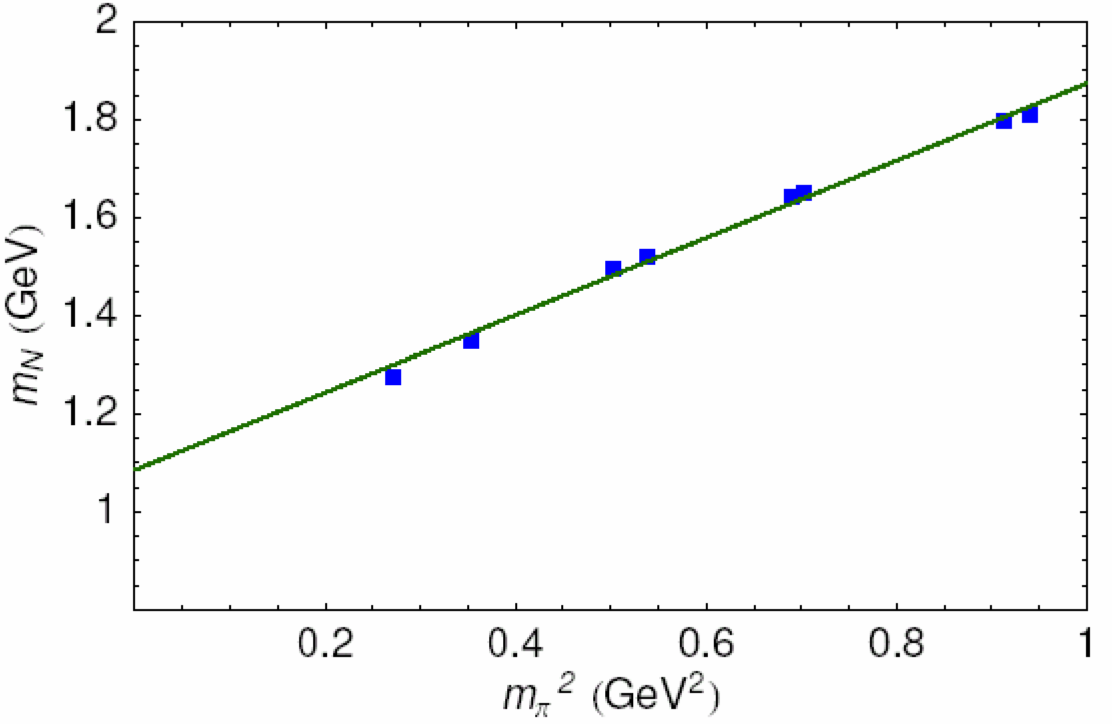
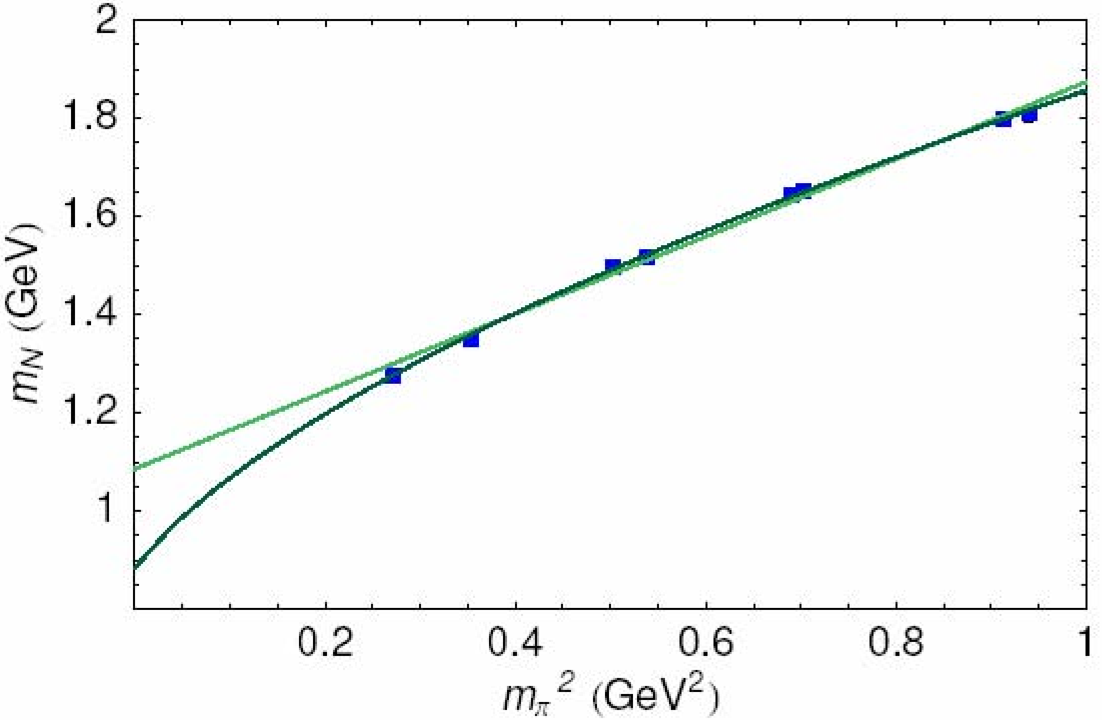


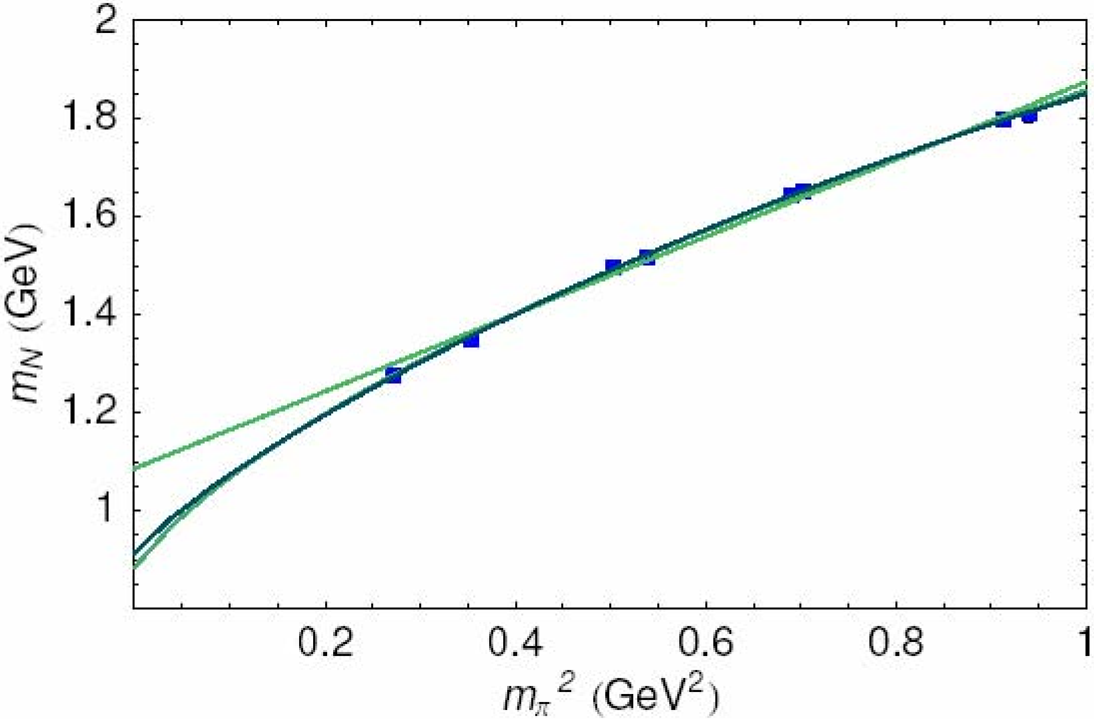
- A: $a_0 + a_2 m_\pi^2$

- Dipole Regularisation



- B: $a_0 + a_2 m_\pi^2 + \chi_\pi I_\pi$

- Dipole Regularisation



- C: $a_0 + a_2 m_\pi^2 + \chi_\pi I_\pi + a_4 m_\pi^4$

D. Leinweber, A. Thomas and R. Young, PRL 92 (2004) 242002;
R. Young, D. Leinweber, A. Thomas, S. Wright, PRD 66 (2002) 094797.

For other regulators:

Sharp cut-off

$$\theta(\Lambda - k)$$

Monopole

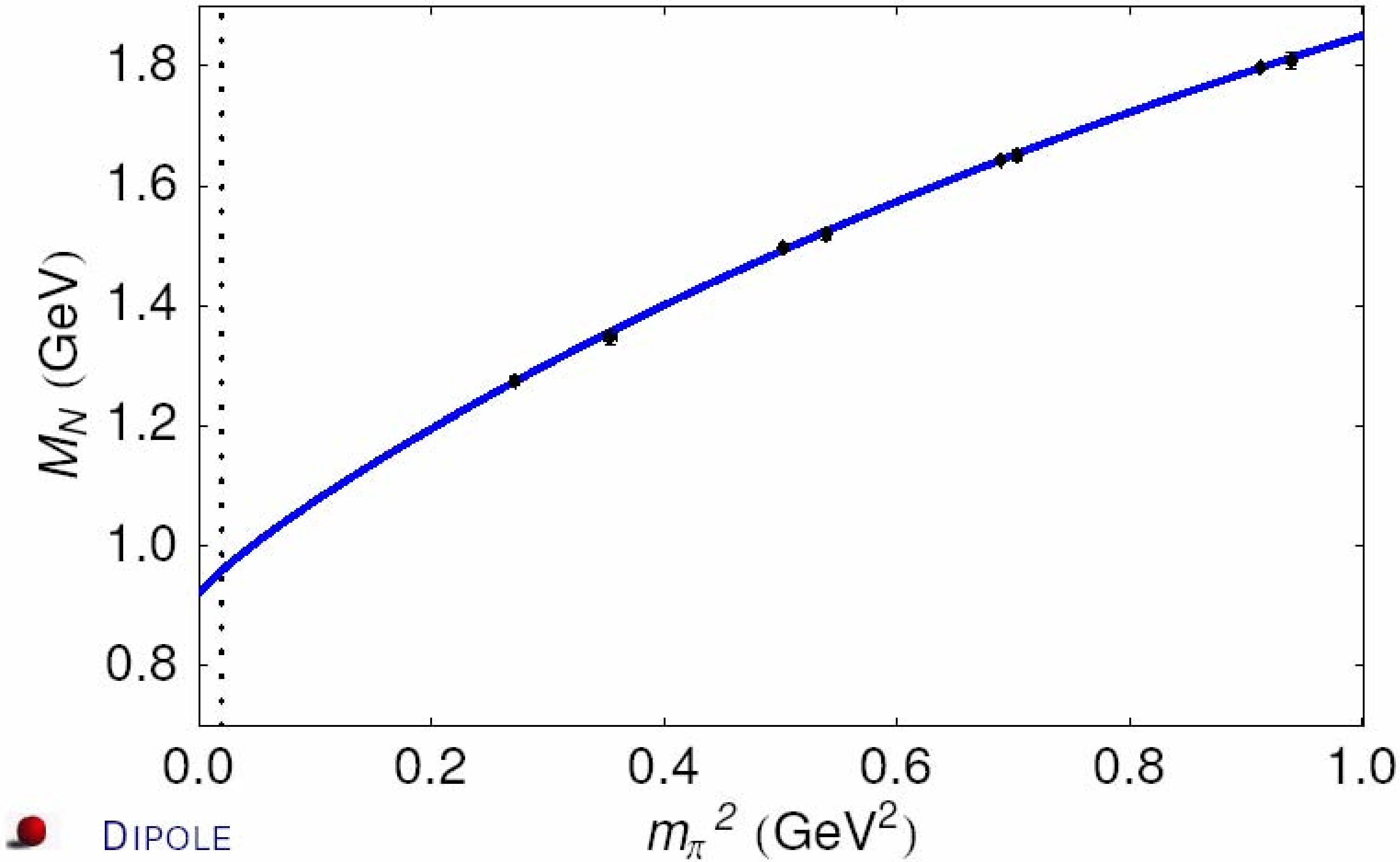
$$\left(\frac{\Lambda^2}{\Lambda^2 + k^2}\right)$$

Gaussian

$$\exp(-\frac{k^2}{\Lambda^2})$$

$$M_N = a_0^\Lambda + a_2^\Lambda m_\pi^2 + \chi_\pi I_\pi(m_\pi, \Lambda) + a_4^\Lambda m_\pi^4$$

Regulator	a_4 (GeV^{-3})
Dipole	-0.25
Sharp cutoff	-0.29
Monopole	-0.24
Gaussian	-0.27
Dim. reg.	2.4



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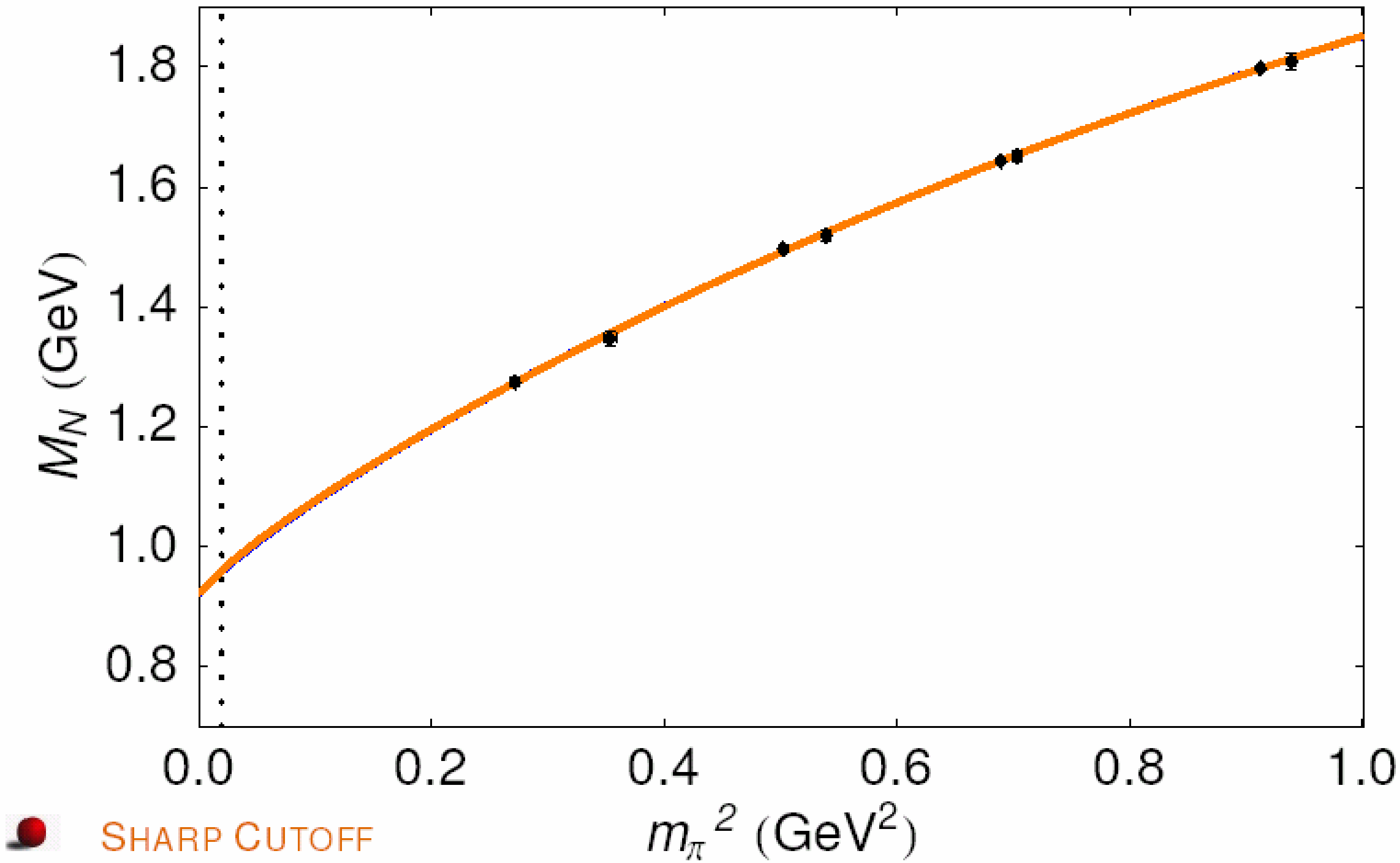

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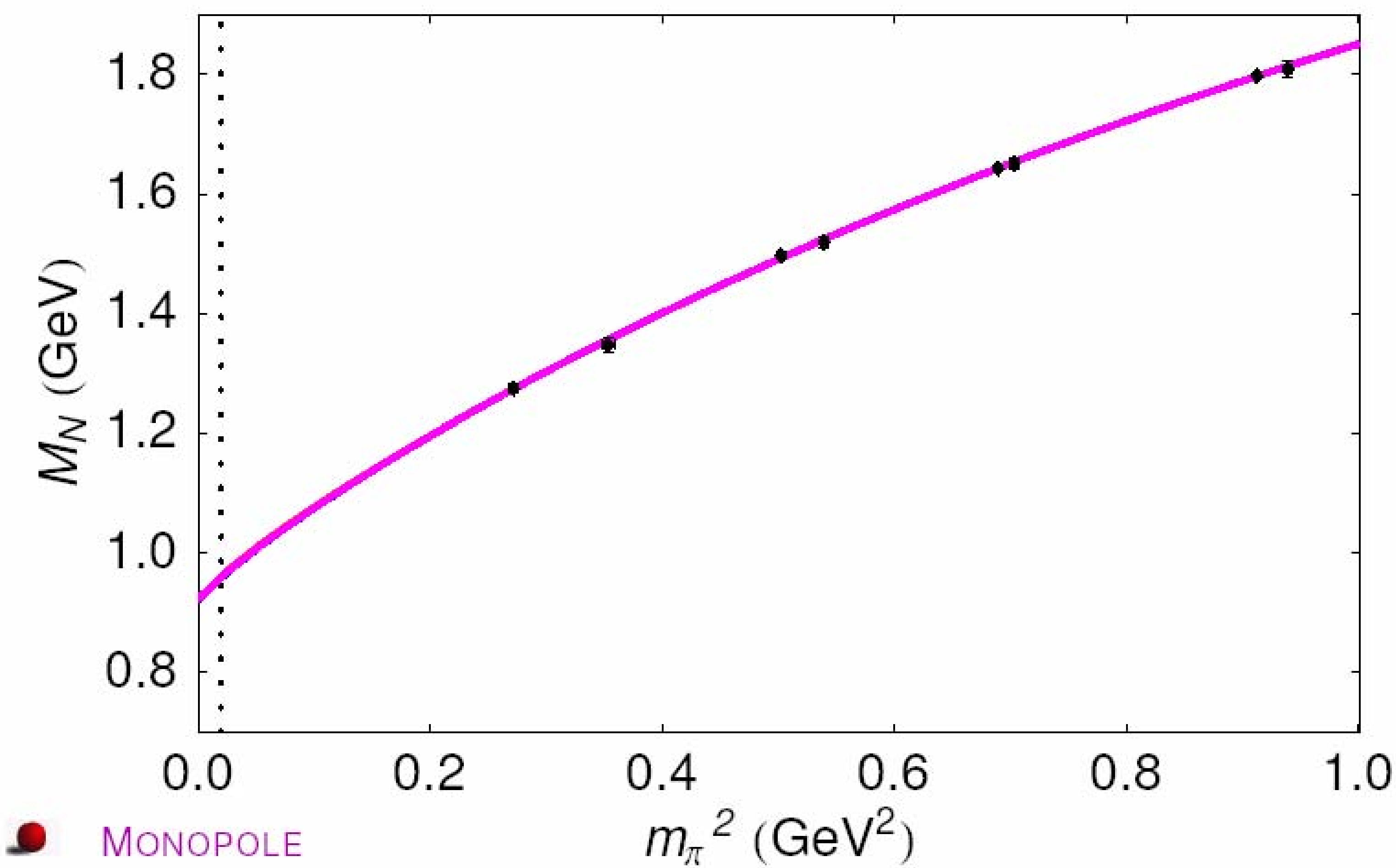

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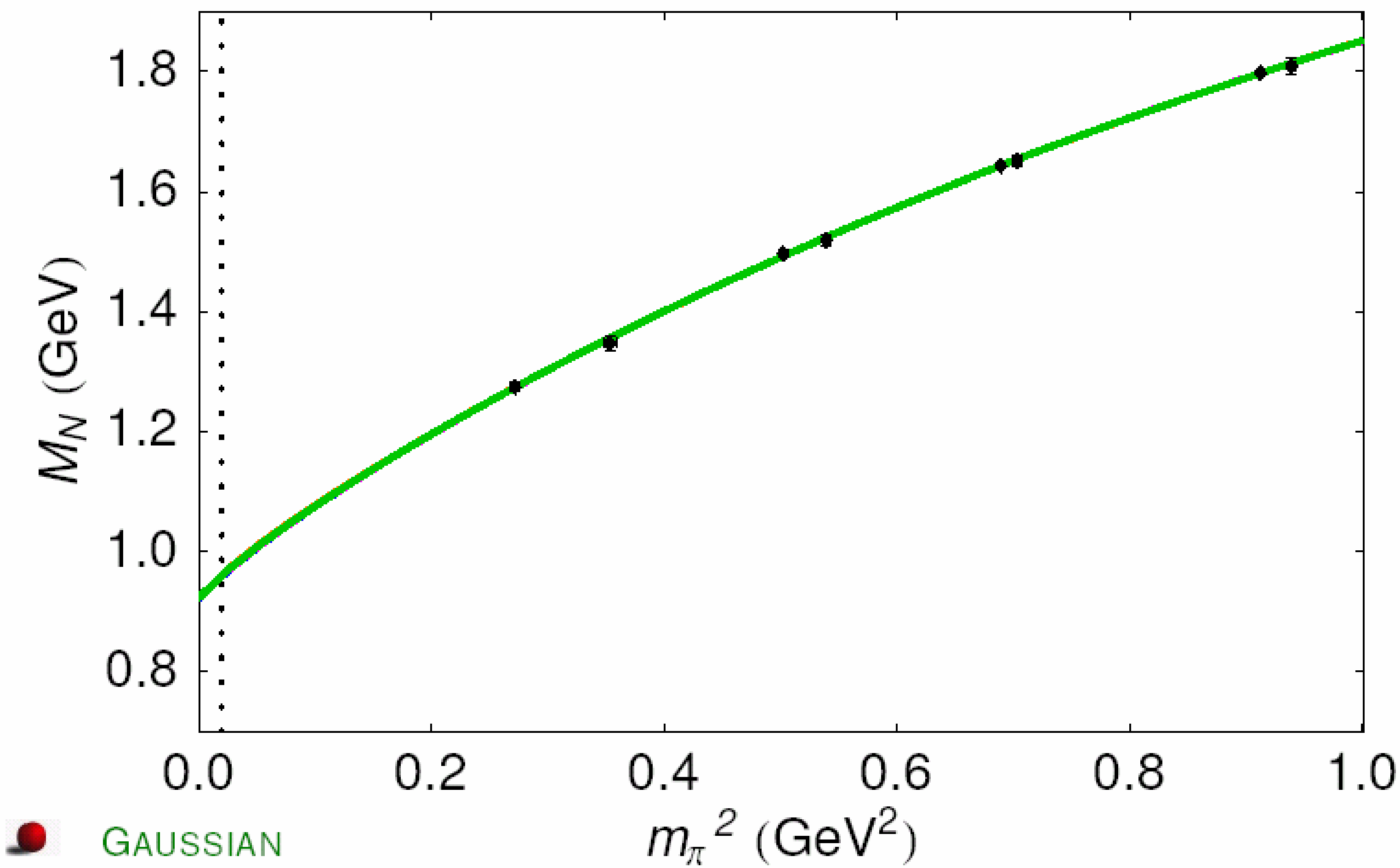


D. Leinweber, A. Thomas and R. Young, PRL 92 (2004) 242002

The FRR reproduces the lattice results very well.

The high order terms are really automatically included in the one loop contribution in FRR.

The residual analytic terms have a good convergent behavior.

At any finite order, FRR and DR are equivalent. In this sense, DR is an approximation of FRR at low pion mass.

Magnetic Form factors

The lowest order interaction:

$$\mathcal{L}_{MB}^{(1)} = \mathrm{Tr}\left[\bar{B}\left(i\not{D} - M_0\right)B\right] - \frac{D}{2}\mathrm{Tr}\left(\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}\right) - \frac{F}{2}\mathrm{Tr}\left(\bar{B}\gamma^\mu\gamma_5[u_\mu, B]\right)$$

In the heavy baryon formalism:

$$\begin{aligned}\mathcal{L}_v = & \, i\mathrm{Tr}\bar{B}_v(v\cdot\mathcal{D})B_v + 2D\mathrm{Tr}\bar{B}_v S_v^\mu\{A_\mu, B_v\} + 2F\mathrm{Tr}\bar{B}_v S_v^\mu[A_\mu, B_v] \\ & -i\bar{T}_v^\mu(v\cdot\mathcal{D})T_{v\mu} + \mathcal{C}(\bar{T}_v^\mu A_\mu B_v + \bar{B}_v A_\mu T_v^\mu),\end{aligned}$$

The compact notation:

$$(\bar{\mathcal{T}}^\mu \Gamma \mathcal{T}_\mu) \equiv \bar{\mathcal{T}}^\mu_{kji,\alpha}\Gamma^\alpha{}_\beta \mathcal{T}^\beta_{\mu,ijk}$$

$$(\bar{\mathcal{B}}\Gamma A^\mu \mathcal{T}_\mu) \equiv \bar{\mathcal{B}}^\alpha_{kji}\Gamma^\alpha{}_\beta A^\mu_{ii'}\mathcal{T}^\beta_{\mu,i'jk}$$

Meson matrix:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

Octet baryons:

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

Decuplet baryons:

$$T_{111} = \Delta^{++}, \quad T_{112} = \frac{1}{\sqrt{3}}\Delta^+, \quad T_{122} = \frac{1}{\sqrt{3}}\Delta^0$$

$$T_{222} = \Delta^-, \quad T_{113} = \frac{1}{\sqrt{3}}\Sigma^{*,+}, \quad T_{123} = \frac{1}{\sqrt{6}}\Sigma^{*,0}$$

$$T_{223} = \frac{1}{\sqrt{3}}\Sigma^{*,-}, \quad T_{133} = \frac{1}{\sqrt{3}}\Xi^{*,0}, \quad T_{233} = \frac{1}{\sqrt{3}}\Xi^{*,-}, \quad T_{333} = \Omega^-$$

Baryon octet magnetic moment Lagrangian:

$$\mathcal{L} = \frac{e}{4m_N}\left(\mu_D \mathrm{Tr}\bar{B}_v \sigma^{\mu\nu}\left\{F^{+}_{\mu\nu}, B_v\right\} + \mu_F \mathrm{Tr}\bar{B}_v \sigma^{\mu\nu}\left[F^{+}_{\mu\nu}, B_v\right]\right)$$

At the lowest order:

$$\mu_p = \frac{1}{3}\mu_D + \mu_F, \qquad \mu_n = -\frac{2}{3}\mu_D$$

Baryon decuplet magnetic moment Lagrangian:

$$\mathcal{L} = -i\frac{e}{m_N}\mu_C q_{ijk}\bar{T}^{\mu}_{v,ikl}T^{\nu}_{v,jkl}F_{\mu\nu}$$

The transition magnetic operator:

$$\mathcal{L} = i\frac{e}{2m_N}\mu_T F_{\mu\nu}\left(\epsilon_{ijk}Q^i_l \bar{B}^j_{vm} S^{\mu}_v T^{\nu,klm}_v + \epsilon^{ijk}Q^l_i \bar{T}^{\mu}_{v,klm} S^{\nu}_v B^m_{vj}\right)$$

The baryon magnetic moments can also be expressed as quark magnetic moments.

For particular choice: $\mu_s = \mu_d = -\frac{1}{2}\mu_u$

$$\mu_D = \frac{3}{2}\mu_u, \quad \mu_F = \frac{2}{3}\mu_D, \quad \mu_C = \mu_D, \quad \mu_T = -4\mu_D$$

Nucleon magnetic moments in one loop level:

$$\mu_p(m_\pi^2) = a_0^p + a_2^p m_\pi^2 + a_4^p m_\pi^4 + \sum_{k=a}^{g} G_M^{p(1k)}(Q^2 = 0, m_\pi^2)$$

$$\mu_n(m_\pi^2) = a_0^n + a_2^n m_\pi^2 + a_4^n m_\pi^4 + \sum_{k=a}^{g} G_M^{n(1k)}(Q^2 = 0, m_\pi^2)$$

The Pauli and Dirac form factors:

$$\langle B(p') \,|\, \mathcal{J}_\mu \,|\, B(p) \rangle = \quad \bar{u}(p') \left\{ \gamma_\mu F_1^B(t) + \frac{i\sigma_{\mu\nu} q^\nu}{2m_B} F_2^B(t) \right\} u(p)$$

In terms of charge and magnetic form factors:

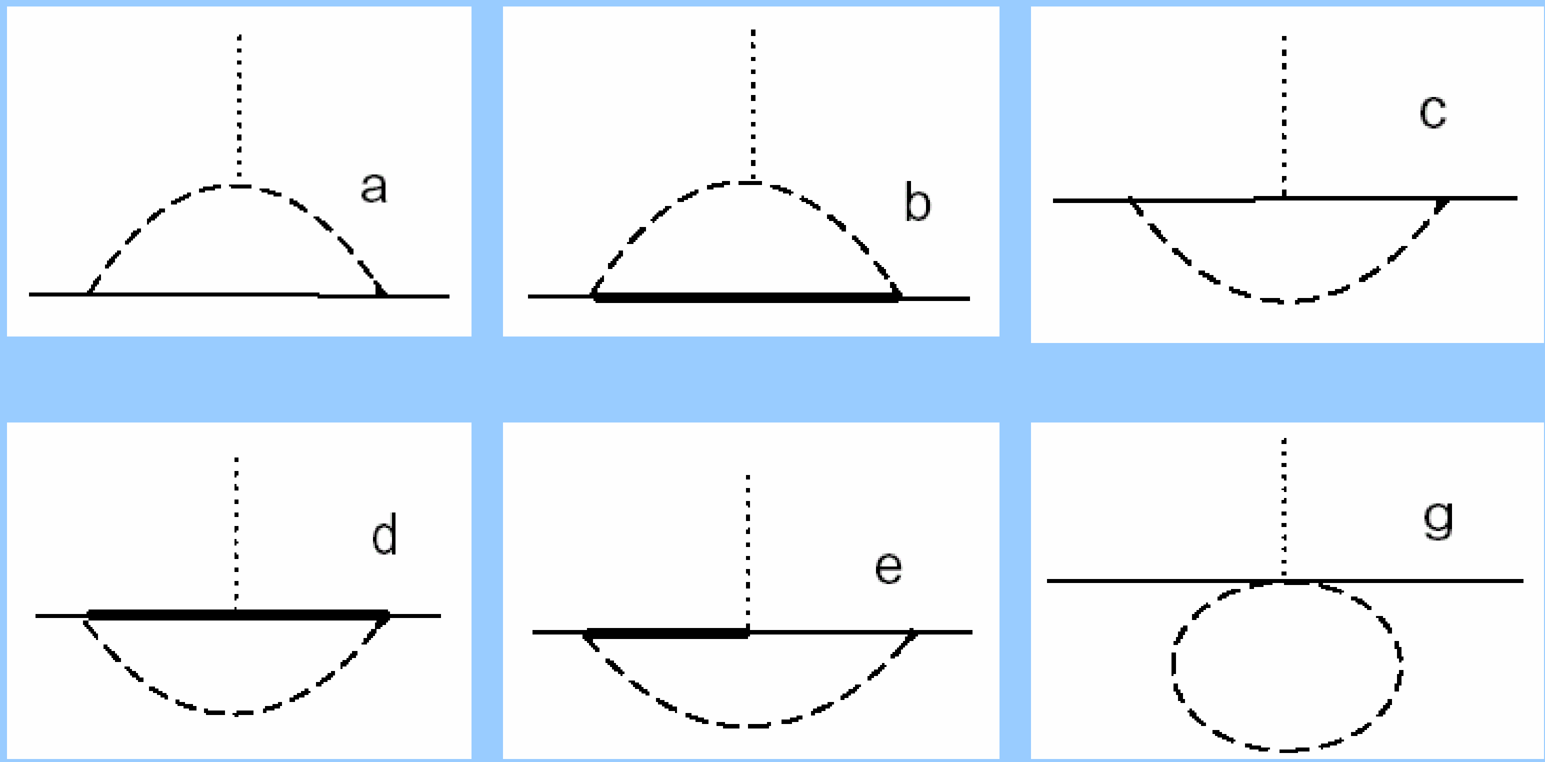
$$< B(p') | J_\mu | B(p) > = \bar{u}(p') \left\{ v_\mu G_E(Q^2) + \frac{i\epsilon_{\mu\nu\alpha\beta} v^\alpha S_v^\beta q^\nu}{m_N} G_M(Q^2) \right\} u(p)$$

Equivalent definition of charge and magnetic form factors:

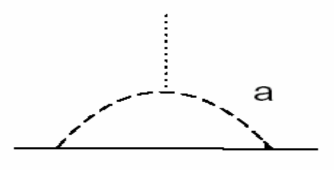
$$\left\langle N_{s'}\left(\frac{\vec{q}}{2}\right) | J^0(0) | N_s\left(-\frac{\vec{q}}{2}\right) \right\rangle = \chi^\dagger_{N_{s'}} \chi_{N_s} G_E^N(Q^2)$$

$$\left\langle N_{s'}\left(\frac{\vec{q}}{2}\right) | \vec{J}(0) | N_s\left(-\frac{\vec{q}}{2}\right) \right\rangle = \chi^\dagger_{N_{s'}} \frac{i\vec{\sigma}_N \times \vec{q}}{2m_N} \chi_{N_s} G_M^N(Q^2)$$

The one loop diagrams for magnetic form factors:



The contribution of diagram a:



$$G_M^{p(1a)} = \frac{m_N(D+F)^2}{8\pi^3 f_\pi^2} I_{1\pi}^{NN} + \frac{m_N(D+3F)^2 I_{1K}^{N\Lambda} + 3m_N(D-F)^2 I_{1K}^{N\Sigma}}{48\pi^3 f_\pi^2}$$

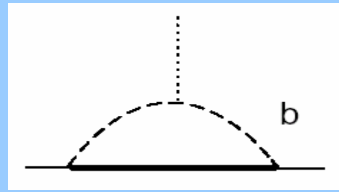
$$G_M^{n(1a)} = -\frac{m_N(D+F)^2}{8\pi^3 f_\pi^2} I_{1\pi}^{NN} + \frac{m_N(D-F)^2}{8\pi^3 f_\pi^2} I_{1K}^{N\Sigma}.$$

$$I_{1j}^{\alpha\beta} = \int d\vec{k}\, \frac{k_y^2 u(\vec{k}+\vec{q}/2)u(\vec{k}-\vec{q}/2)(\omega_j(\vec{k}+\vec{q}/2)+\omega_j(\vec{k}-\vec{q}/2)+\delta^{\alpha\beta})}{A_j^{\alpha\beta}}$$

$$A_j^{\alpha\beta} = \omega_j(\vec{k}+\vec{q}/2)\omega_j(\vec{k}-\vec{q}/2)(\omega_j(\vec{k}+\vec{q}/2)+\delta^{\alpha\beta})$$
$$(\omega_j(\vec{k}-\vec{q}/2)+\delta^{\alpha\beta})(\omega_j(\vec{k}+\vec{q}/2)+\omega_j(\vec{k}-\vec{q}/2))$$

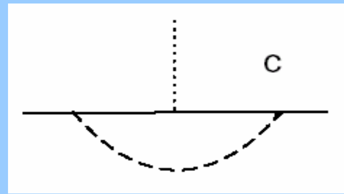
For dimensional regularization, $I_{1\pi}^{NN} \propto m_\pi$.

The contribution of diagram b: $O(m_\pi \ln m_\pi)$



$$G_M^{p(1b)} = \frac{m_N \mathcal{C}^2}{36\pi^3 f_\pi^2} I_{1\pi}^{N\Delta} - \frac{m_N \mathcal{C}^2}{144\pi^3 f_\pi^2} I_{1K}^{N\Sigma^*},$$

$$G_M^{n(1b)} = -\frac{m_N \mathcal{C}^2}{36\pi^3 f_\pi^2} I_{1\pi}^{N\Delta} - \frac{m_N \mathcal{C}^2}{72\pi^3 f_\pi^2} I_{1K}^{N\Sigma^*}$$

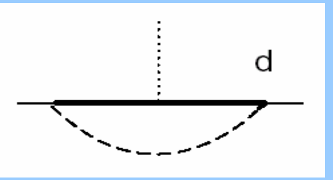


The contribution of diagram c: $O(m_\pi^2 \ln m_\pi)$

$$G_M^{p(1c)} = \frac{(D+F)^2(\mu_D - \mu_F)}{192\pi^3 f_\pi^2} I_{2\pi}^{NN} - \frac{1}{192\pi^3 f_\pi^2}\Big[(D-F)^2(2\mu_F + \mu_D) I_{2K}^{N\Sigma} - (\frac{D}{3}+F)^2 \mu_D I_{2K}^{N\Lambda} - (D-F)(\frac{2D}{3}+2F)\mu_D I_{5K}^{N\Lambda\Sigma}\Big] - \frac{(\frac{D}{3}-F)^2(\mu_D + 3\mu_F)}{192\pi^3 f_\pi^2} I_{2\eta}^{NN},$$

$$G_M^{n(1c)} = -\frac{(D+F)^2 \mu_F}{96\pi^3 f_\pi^2} I_{2\pi}^{NN} - \frac{1}{192\pi^3 f_\pi^2}\Big[(D-F)^2(\mu_D - 2\mu_F) I_{2K}^{N\Sigma} - (\frac{D}{3}+F)^2 \mu_D I_{2K}^{N\Lambda} + (\frac{2D}{3}+2F)(D-F)\mu_D I_{5K}^{N\Lambda\Sigma}\Big] + \frac{(\frac{D}{3}-F)^2 \mu_D}{96\pi^3 f_\pi^2} I_{2\eta}^{NN},$$

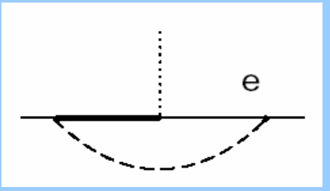
The contribution of diagram d: $O(m_\pi^2 \ln m_\pi)$



$$G_M^{p(1d)} = \frac{5\mathcal{C}^2\mu_C}{162\pi^3 f_\pi^2} I_{2\pi}^{N\Delta} + \frac{5\mathcal{C}^2\mu_C}{1296\pi^3 f_\pi^2} I_{2K}^{N\Sigma}$$

$$G_M^{n(1d)} = -\frac{5\mathcal{C}^2\mu_C}{648\pi^3 f_\pi^2} I_{2\pi}^{N\Delta} - \frac{5\mathcal{C}^2\mu_C}{1296\pi^3 f_\pi^2} I_{2K}^{N\Sigma^*}$$

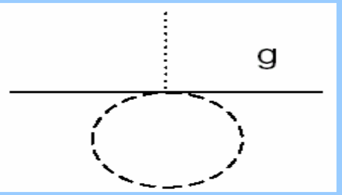
The contribution of diagram e: $O(m_\pi^2 \ln m_\pi)$



$$G_M^{p(1e+1f)} = \frac{(D+F)\mathcal{C}\mu_T}{108\pi^3 f_\pi^2} I_{3\pi}^{N\Delta} + \frac{5(D-F)\mathcal{C}\mu_T}{864\pi^3 f_\pi^2} I_{5K}^{N\Sigma\Sigma^*} + \frac{(D+3F)\mathcal{C}\mu_T}{864\pi^3 f_\pi^2} I_{5K}^{N\Lambda\Sigma^*}$$

$$G_M^{n(1e+1f)} = -\frac{(D+F)\mathcal{C}\mu_T}{108\pi^3 f_\pi^2} I_{3\pi}^{N\Delta} + \frac{(D-F)\mathcal{C}\mu_T}{864\pi^3 f_\pi^2} I_{5K}^{N\Sigma\Sigma^*} - \frac{(D+3F)\mathcal{C}\mu_T}{864\pi^3 f_\pi^2} I_{5K}^{N\Lambda\Sigma^*}$$

The contribution of diagram g: $O(m_\pi^2 \ln m_\pi)$



$$G_M^{p(1g)} = -\frac{(\mu_D + \mu_F)}{32\pi^3 f_\pi^2} I_{4\pi} - \frac{\mu_F}{16\pi^3 f_\pi^2} I_{4K}$$

$$G_M^{n(1g)} = \frac{(\mu_D + \mu_F)}{32\pi^3 f_\pi^2} I_{4\pi} + \frac{(\mu_D - \mu_F)}{32\pi^3 f_\pi^2} I_{4K}$$

The integrals are defined as:

$$I_{2j}^{\alpha\beta} = \int d\overrightarrow{k} \frac{k^2 u(\overrightarrow{k})^2}{\omega_j(\overrightarrow{k})(\omega_j(\overrightarrow{k}) + \delta^{\alpha\beta})^2}$$

$$I_{3j}^{\alpha\beta} = \int d\overrightarrow{k} \frac{k^2 u(\overrightarrow{k})^2}{\omega_j(\overrightarrow{k})^2(\omega_j(\overrightarrow{k}) + \delta^{\alpha\beta})}$$

$$I_{4j} = \int d\overrightarrow{k} \frac{u(\overrightarrow{k})^2}{\omega_j(\overrightarrow{k})}$$

$$I_{5j}^{\alpha\beta\gamma} = \int d\overrightarrow{k} \frac{k^2 u(\overrightarrow{k})^2}{\omega_j(\overrightarrow{k})(\omega_j(\overrightarrow{k}) + \delta^{\alpha\beta})(\omega_j(\overrightarrow{k}) + \delta^{\alpha\gamma}))}$$

Extrapolation results

The extrapolation of magnetic moments:

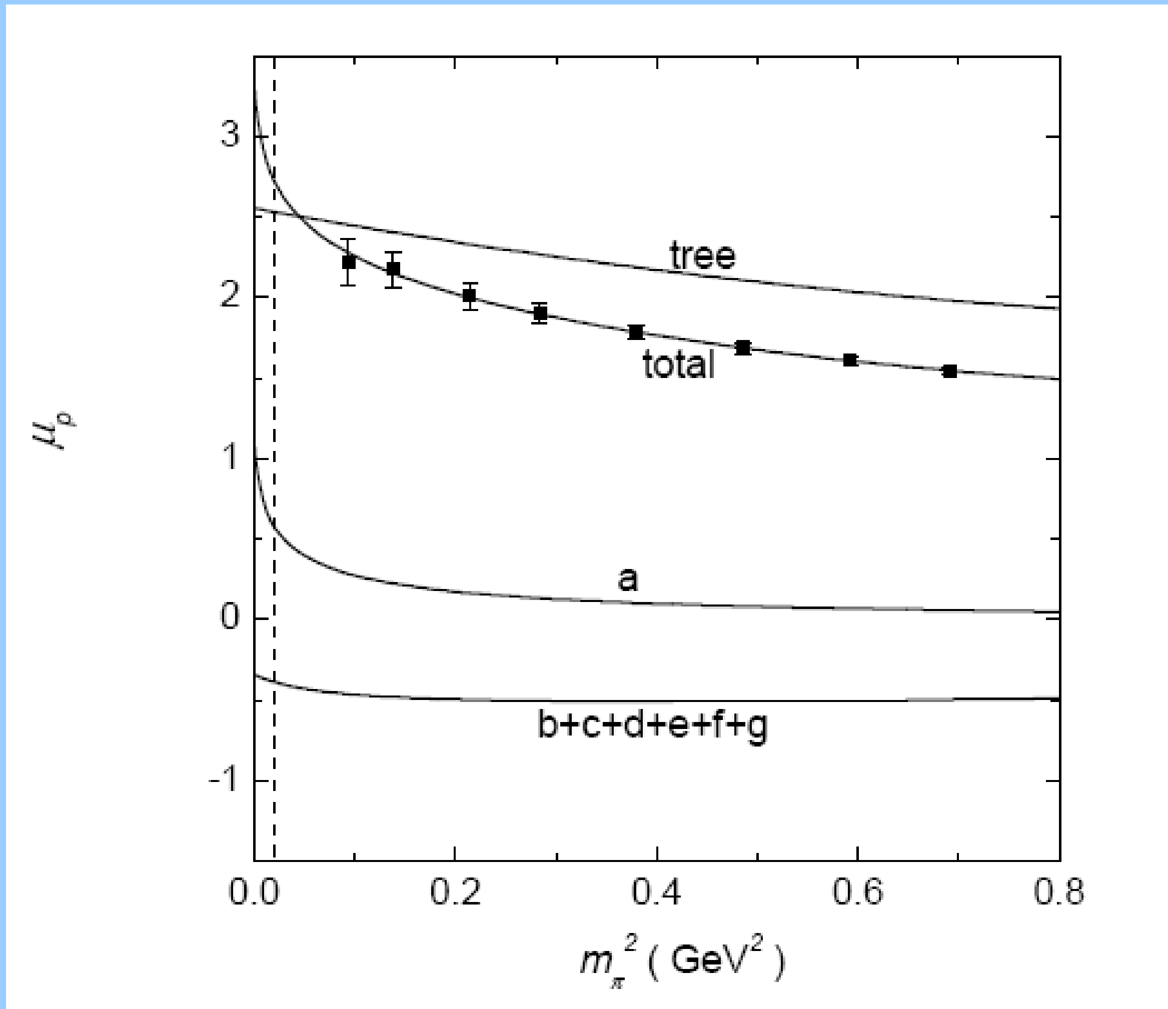
$$\mu_p(m_\pi^2) = a_0^p + a_2^p m_\pi^2 + a_4^p m_\pi^4 + \sum_{k=a}^{g} G_M^{p(1k)}(Q^2=0, m_\pi^2)$$

$$\mu_n(m_\pi^2) = a_0^n + a_2^n m_\pi^2 + a_4^n m_\pi^4 + \sum_{k=a}^{g} G_M^{n(1k)}(Q^2=0, m_\pi^2)$$

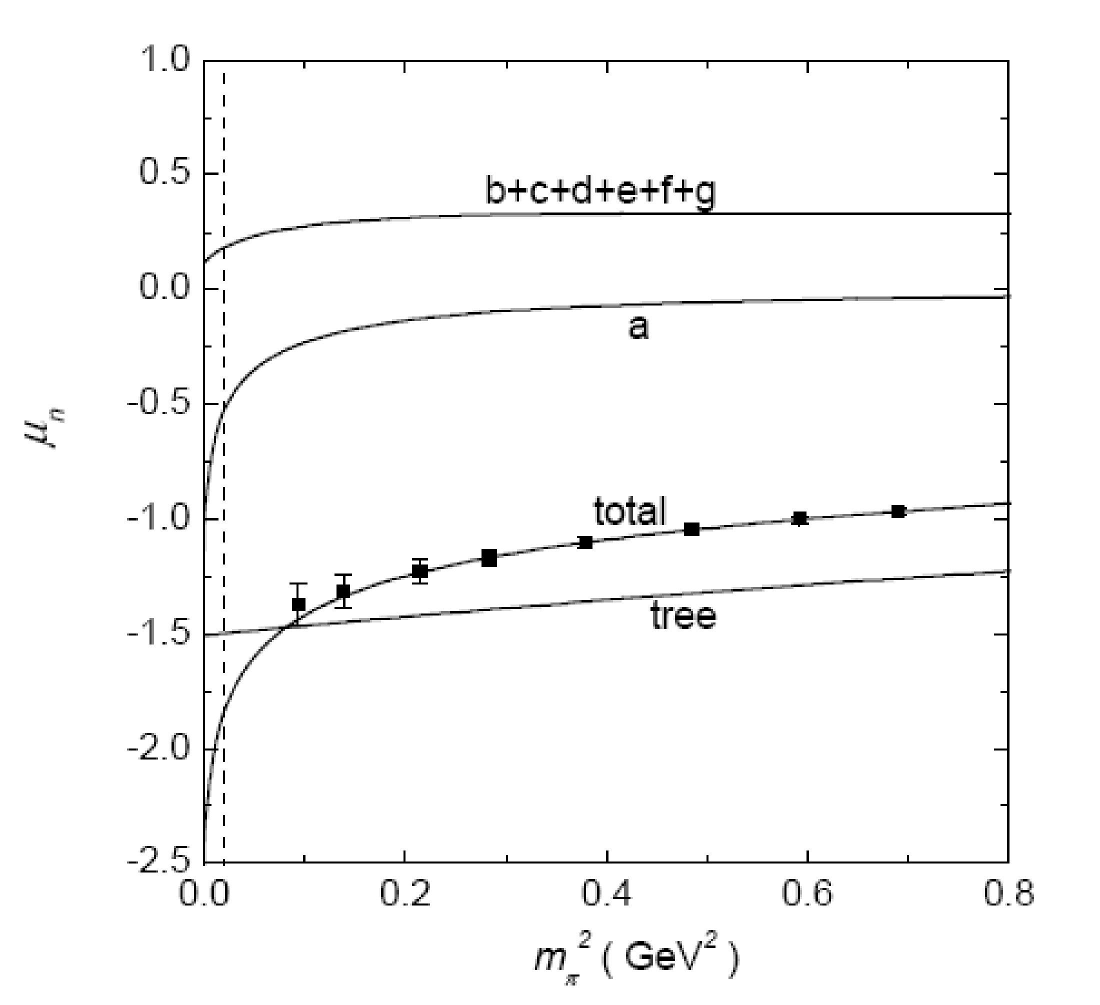
The mass relationships between mesons:

$$m_K^2 = \frac{1}{2} m_\pi^2 + m_K^2|_{\mathrm{phy}} - \frac{1}{2} m_\pi^2|_{\mathrm{phy}}$$

$$m_\eta^2 = \frac{1}{3} m_\pi^2 + m_\eta^2|_{\mathrm{phy}} - \frac{1}{3} m_\pi^2|_{\mathrm{phy}}$$



P. Wang, D. Leinweber, A. Thomas and R. Yong, PRD 75 (2007) 073012

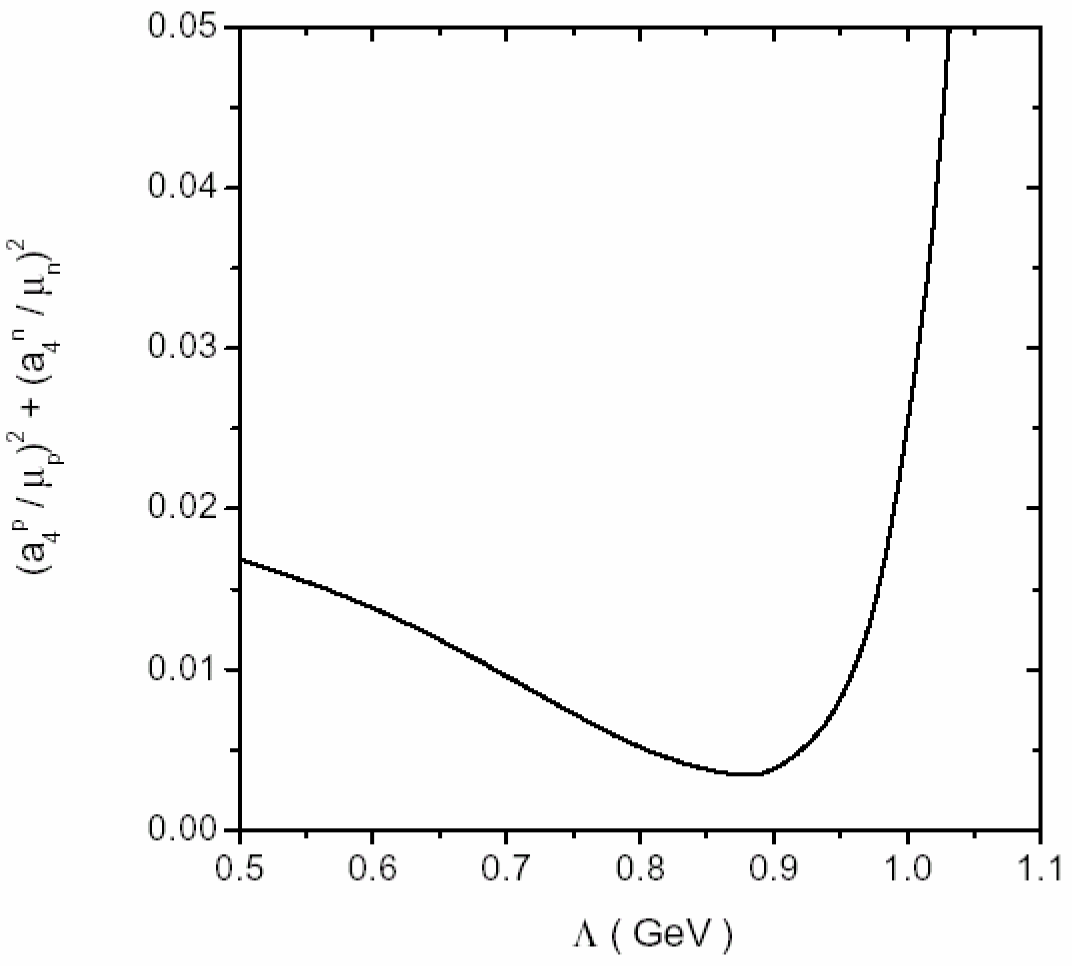


1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0.0
0.2
0.4
0.6
0.8
μ_n
m_π^2 (GeV^2)
b+c+d+e+f+g
a
total
tree

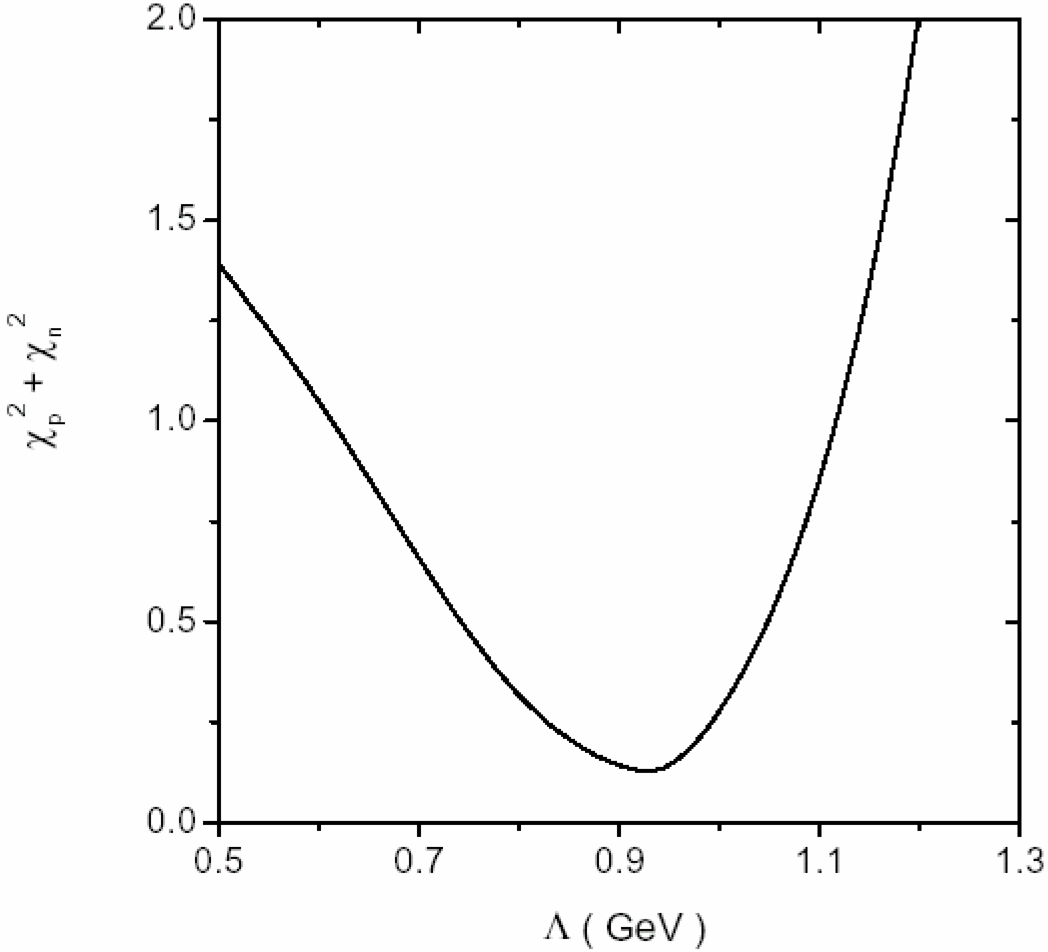
Q^2	a_0^p	a_0^n	a_2^p (GeV^{-2})	a_2^n (GeV^{-2})	a_4^p (GeV^{-4})	a_4^n (GeV^{-4})	G_M^p	G_M^n
0	2.554	−1.506	−1.135	0.420	0.446	−0.090	2.73 ± 0.20	-1.84 ± 0.19

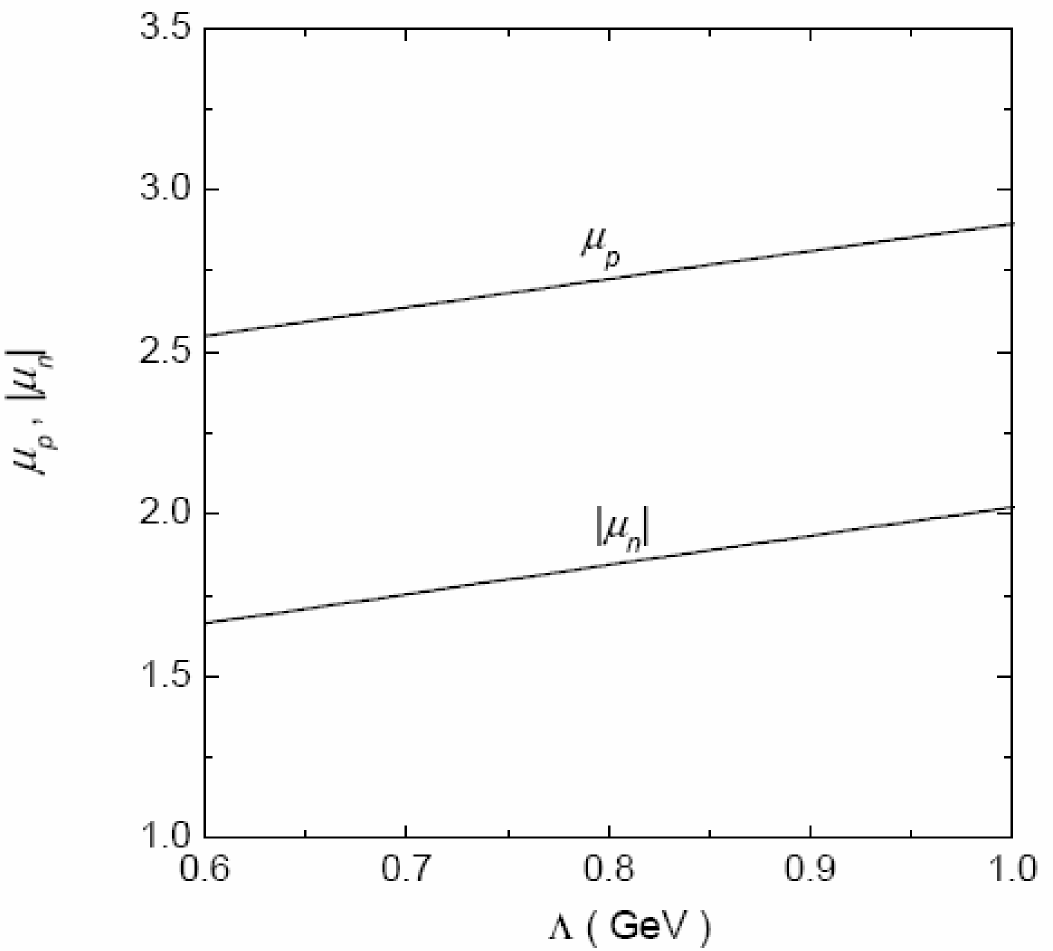
Determination of Optimal Λ:

1. Have the best convergence, $a_4 \to 0$.
2. Have the best fit of lattice data, $\chi^2 = \sum (\mu_{fit} - \mu_{lat})^2$ is small.
3. Produce reasonable nucleon magnetic moments.



0.05
0.04
0.03
0.02
0.01
0.00
0.5
0.6
0.7
0.8
0.9
1.0
1.1
Λ (GeV)
$(a_4^p / \mu_p)^2 + (a_4^n / \mu_n)^2$


2.0
1.5
1.0
0.5
0.0
0.5
0.7
0.9
1.1
1.3
$\chi_p^2 + \chi_n^2$
Λ (GeV)


3.5
3.0
2.5
2.0
1.5
1.0
0.6
0.7
0.8
0.9
1.0
μ_p
$|\mu_n|$
μ_p , $|\mu_n|$
Λ (GeV)

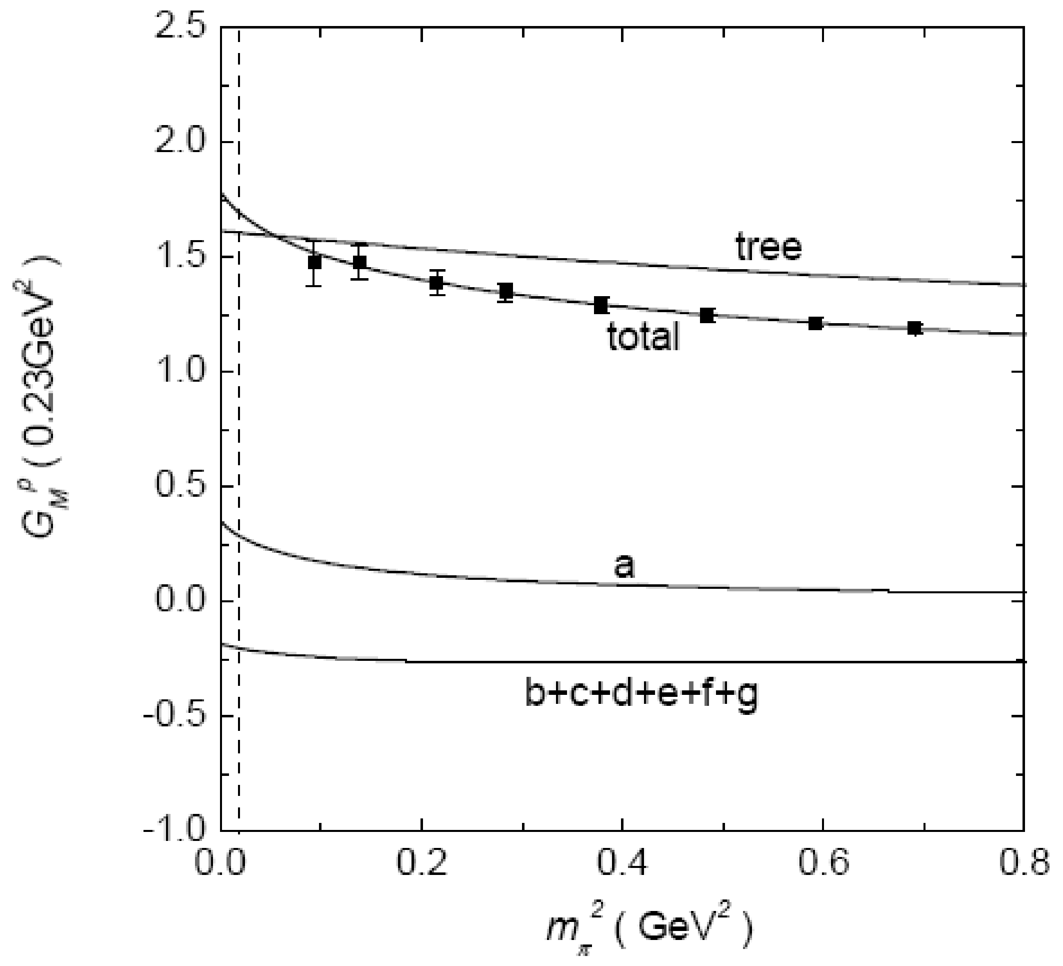
The extrapolation of magnetic form factors:

At finite momentum, we do not expand the form factors in terms of Q^2 and Q^4 as we did for the m_π dependence.

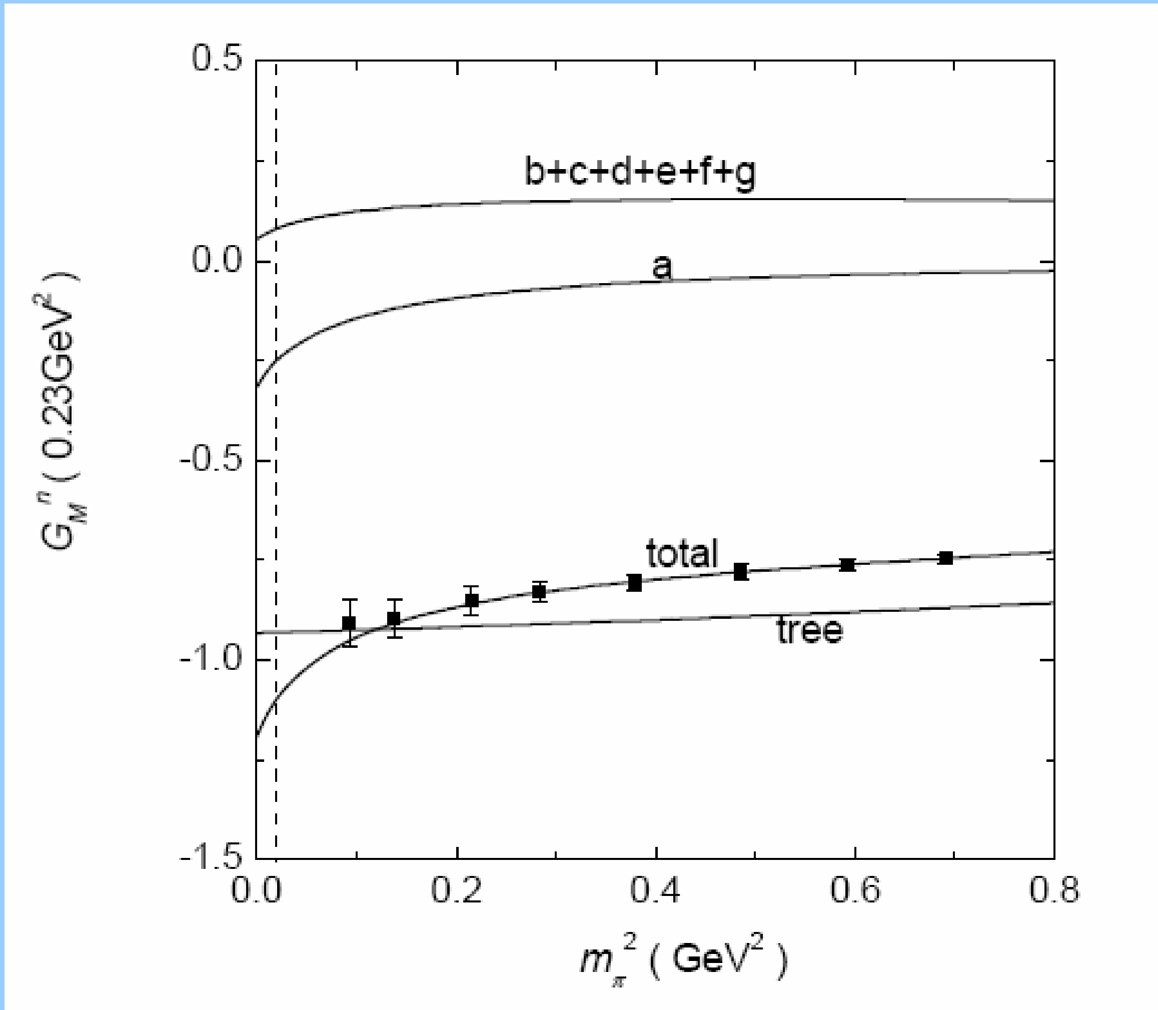
$$G_M^N(m_\pi^2) = a_0^N + a_2^N m_\pi^2 + a_4^N m_\pi^4 + \sum G_M^N(\text{Loop})$$

$$a_0^N = b_1^N + b_2^N Q^2 + b_3^N Q^4 + \ldots$$
$$a_2^N = b_4^N + b_5^N Q^2 + b_6^N Q^4 + \ldots$$
$$a_4^N = b_7^N + b_8^N Q^2 + b_9^N Q^4 + \ldots$$

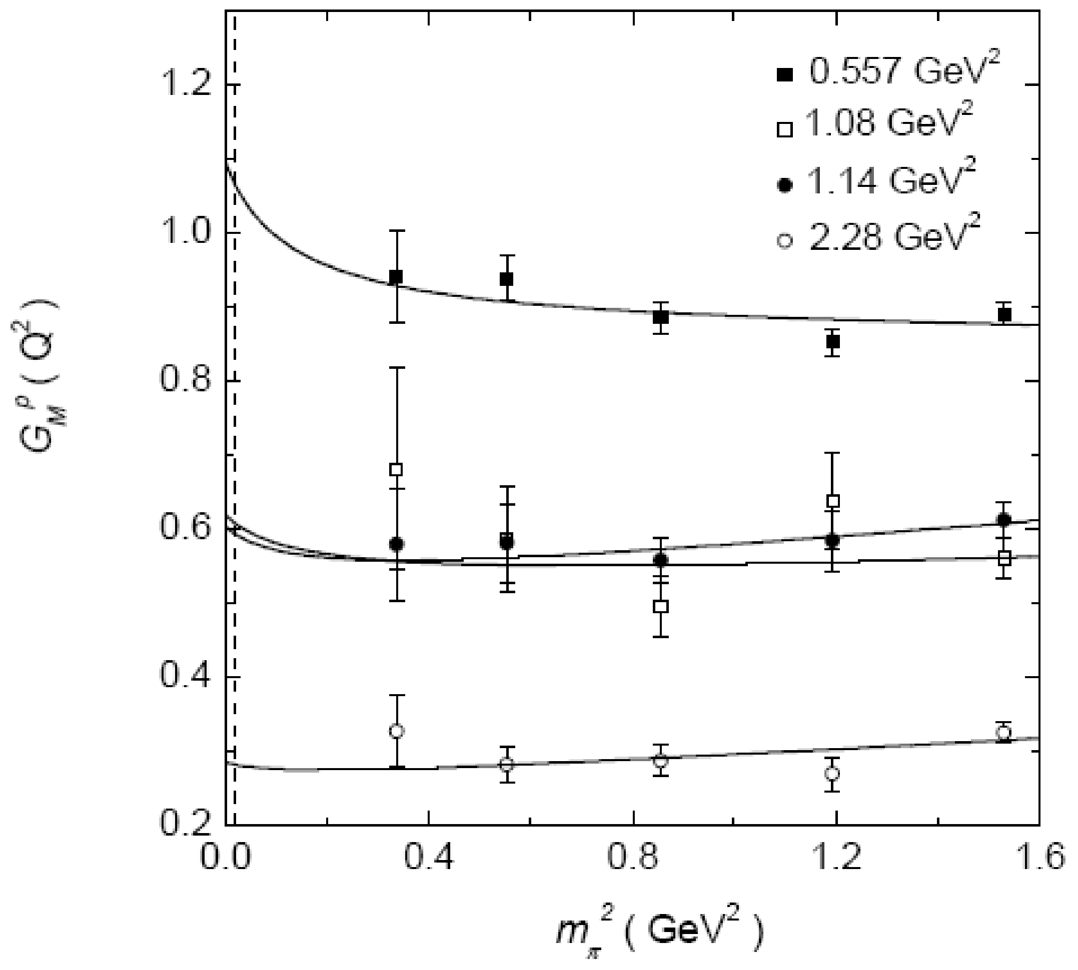
a_0 , a_2 and a_4 are determined from the lattice data at finite momentum and Q^2 dependence is included in the parameters.



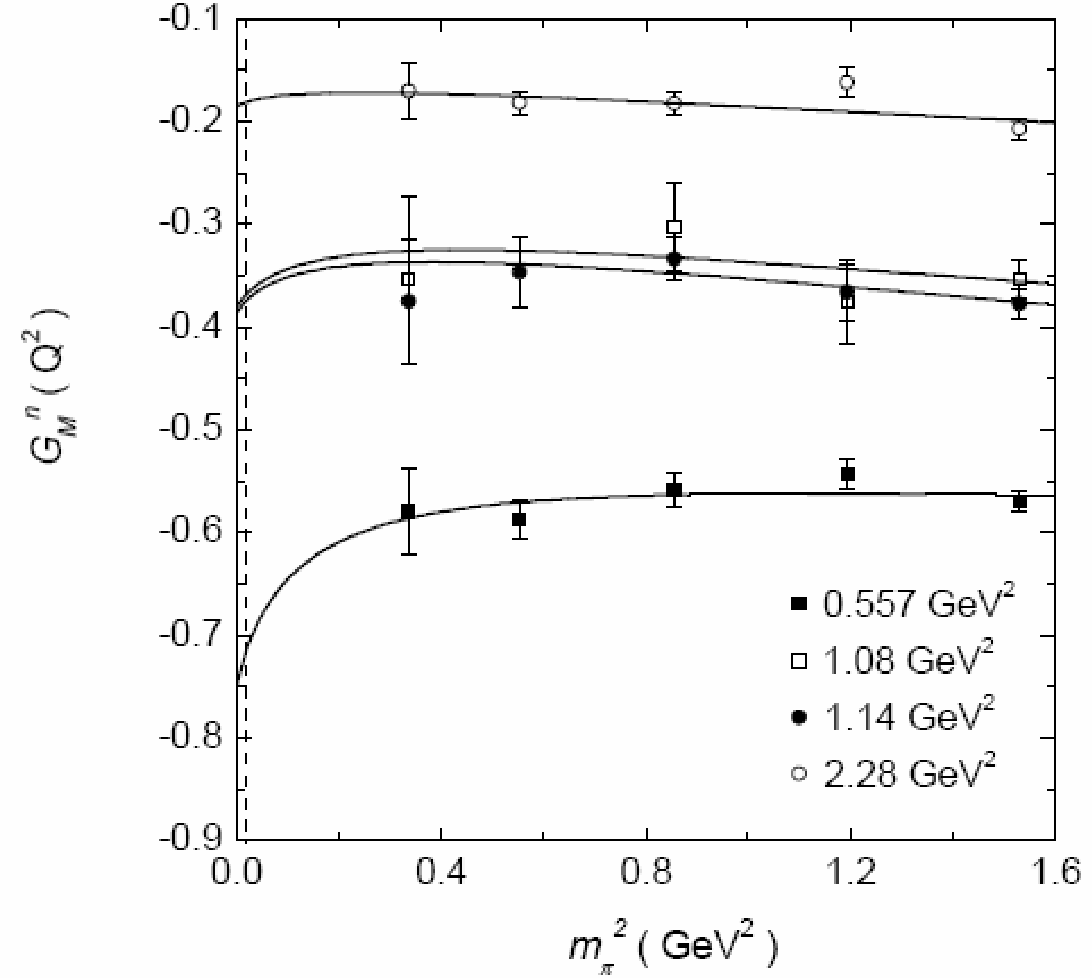
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
0.0
0.2
0.4
0.6
0.8
tree
total
a
b+c+d+e+f+g
G_M^p (0.23GeV^2)
m_π^2 (GeV^2)



0.5
0.0
-0.5
-1.0
-1.5
0.0
0.2
0.4
0.6
0.8
b+c+d+e+f+g
a
total
tree
G_M^n (0.23GeV^2)
m_π^2 (GeV^2)

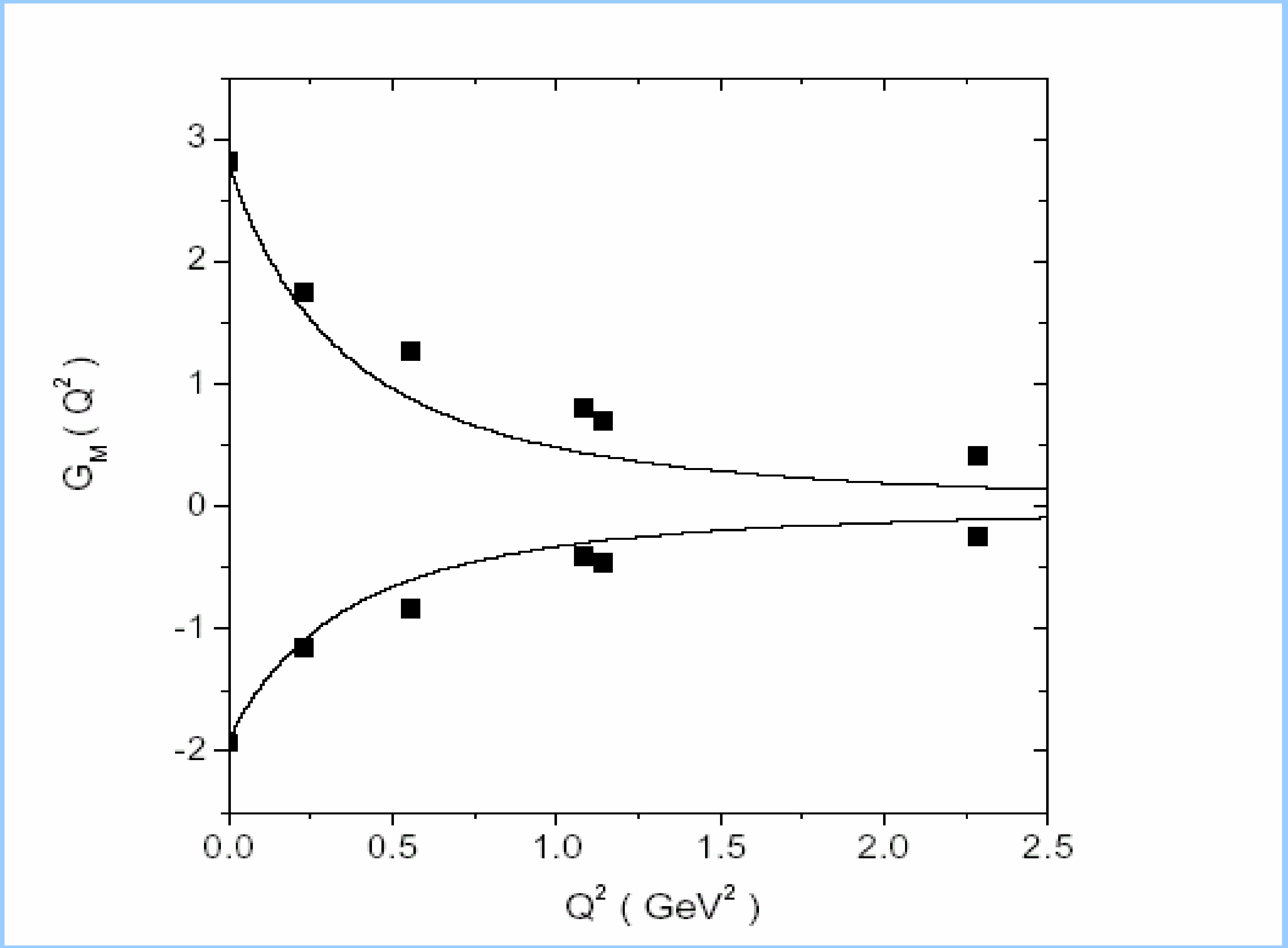


0.557 GeV2
1.08 GeV2
1.14 GeV2
2.28 GeV2
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.4
0.8
1.2
1.6
$G_M^p(Q^2)$
m_π^2 (GeV^2)

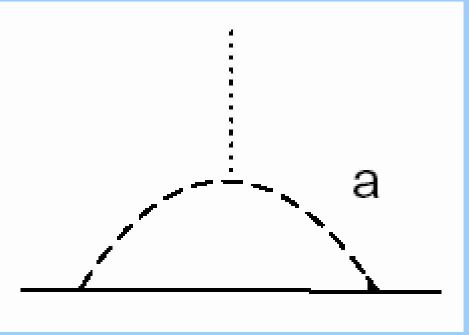

$G_M^n(Q^2)$
m_π^2 (GeV^2)
0.557 GeV^2
1.08 GeV^2
1.14 GeV^2
2.28 GeV^2
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
0.0
0.4
0.8
1.2
1.6

Q^2	a_0^p	a_0^n	a_2^p (GeV^{-2})	a_2^n (GeV^{-2})	a_4^p (GeV^{-4})	a_4^n (GeV^{-4})	G_M^p	G_M^n
0.23	1.617	-0.932	-0.411	0.070	0.144	0.031	1.70 ± 0.12	-1.10 ± 0.11

Q^2 (GeV^2)	a_0^p	a_0^n	a_2^p (GeV^{-2})	a_2^n (GeV^{-2})	G_M^p	G_M^n
0.557	1.042	-0.638	-0.024	-0.00	1.07 ± 0.17	-0.71 ± 0.14
1.08	0.609	-0.337	0.015	-0.04	0.61 ± 0.13	-0.37 ± 0.13
1.14	0.598	-0.348	0.052	-0.04	0.59 ± 0.11	-0.37 ± 0.09
2.28	0.293	-0.178	0.035	-0.03	0.28 ± 0.09	-0.18 ± 0.05



Big error bars on lattice data.
Meson form factors.



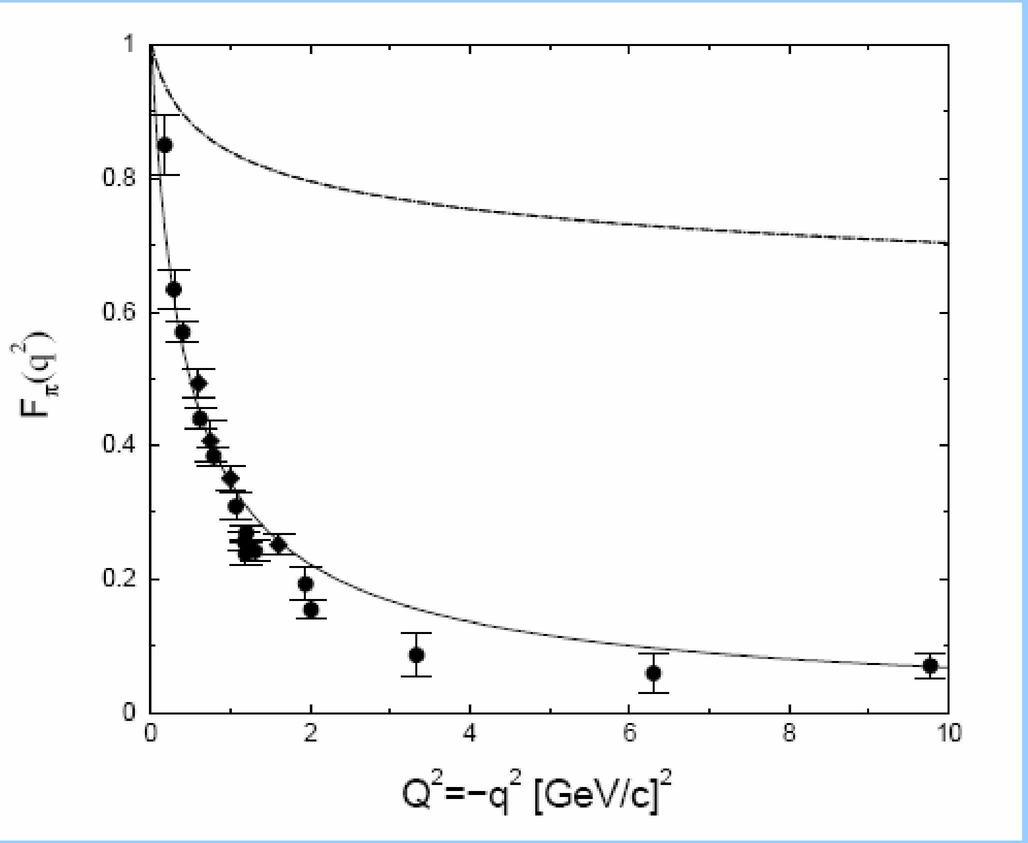
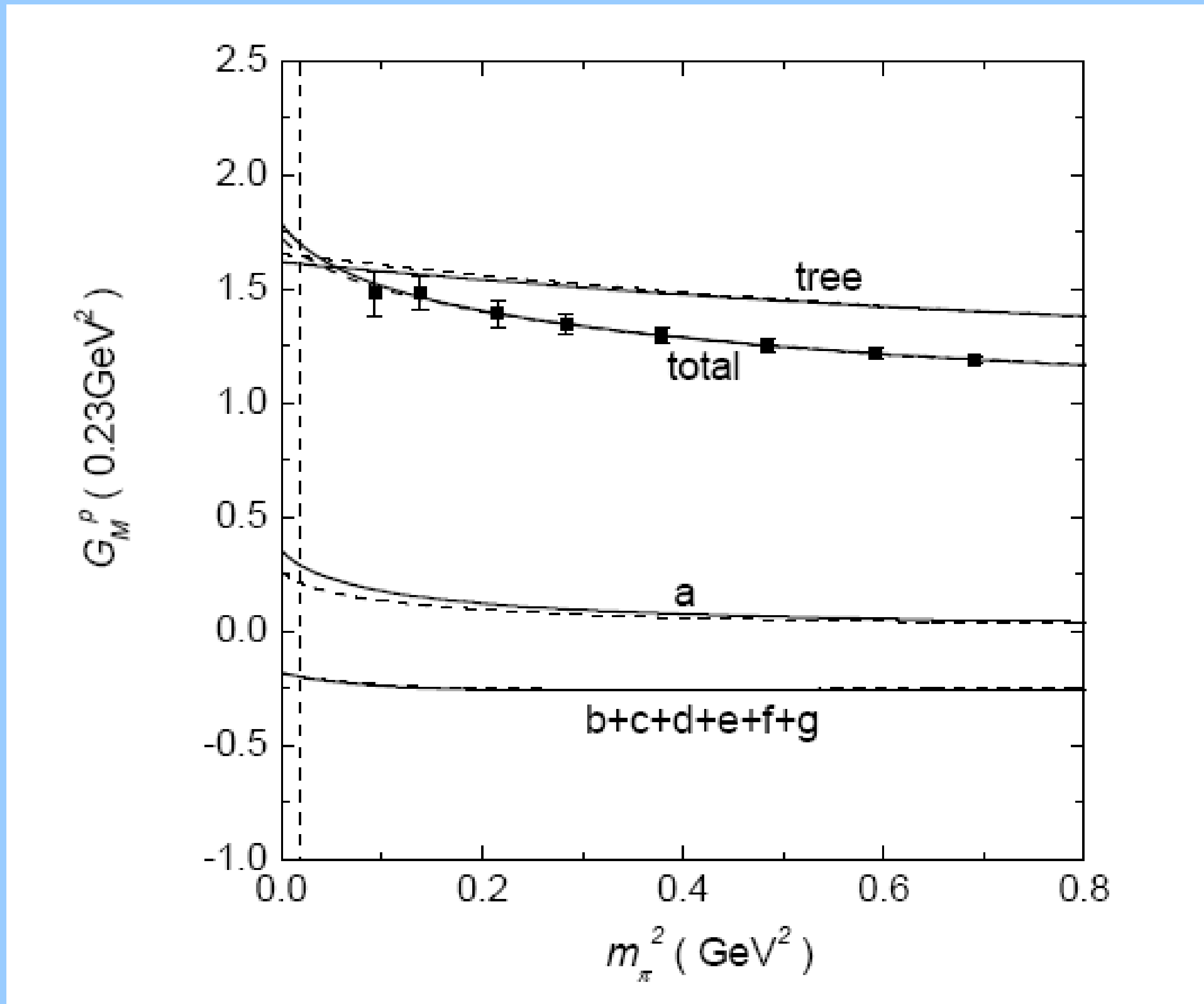


TABLE III: Pion electromagnetic form factor, F_π, at various Q^2.

Q^2 (GeV^2)	0.23	0.557	1.08	1.14	2.28
F_π	0.70	0.50	0.31	0.29	0.18

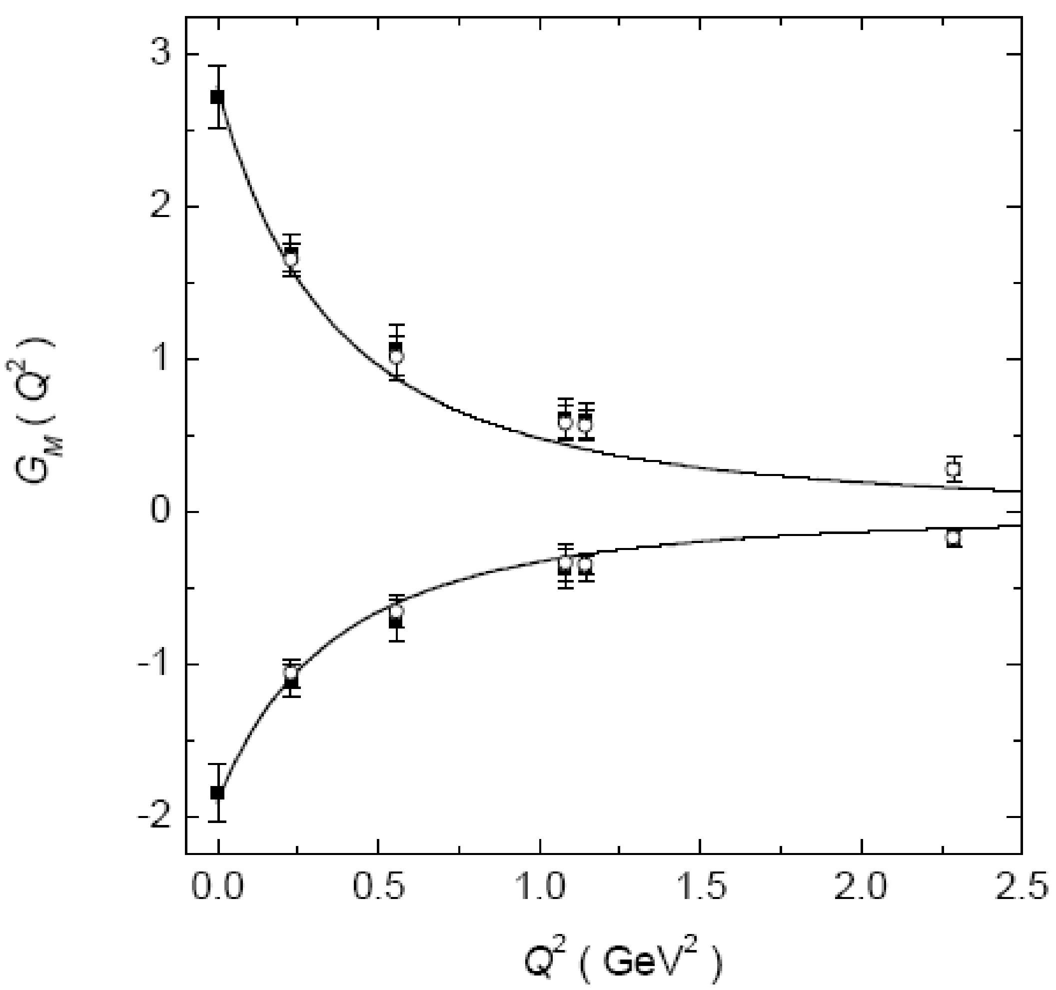
R. Baldini et al., EPJC 11 (1999) 709; NPA 666 (2000) 3.



2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
0.0
0.2
0.4
0.6
0.8
tree
total
a
b+c+d+e+f+g
G_M^p (0.23GeV^2)
m_π^2 (GeV^2)

Q^2	a_0^p	a_0^n	a_2^p (GeV^{-2})	a_2^n (GeV^{-2})	a_4^p (GeV^{-4})	a_4^n (GeV^{-4})	G_M^p	G_M^n
0.23	1.617	-0.932	-0.411	0.070	0.144	0.031	1.70 ± 0.12	-1.10 ± 0.11
0.23	1.652	-0.968	-0.499	0.159	0.201	-0.027	1.65 ± 0.10	-1.06 ± 0.09

Q^2 (GeV^2)	a_0^p	a_0^n	a_2^p (GeV^{-2})	a_2^n (GeV^{-2})	G_M^p	G_M^n
0.557	1.042	-0.638	-0.024	-0.00	1.07 ± 0.17	-0.71 ± 0.14
1.08	0.609	-0.337	0.015	-0.04	0.61 ± 0.13	-0.37 ± 0.13
1.14	0.598	-0.348	0.052	-0.04	0.59 ± 0.11	-0.37 ± 0.09
2.28	0.293	-0.178	0.035	-0.03	0.28 ± 0.09	-0.18 ± 0.05
0.557	1.051	-0.650	-0.033	0.01	1.01 ± 0.15	-0.66 ± 0.10
1.08	0.620	-0.349	0.008	-0.03	0.58 ± 0.12	-0.34 ± 0.12
1.14	0.610	-0.360	0.044	-0.04	0.57 ± 0.10	-0.35 ± 0.07
2.28	0.300	-0.185	0.032	-0.02	0.27 ± 0.09	-0.17 ± 0.05



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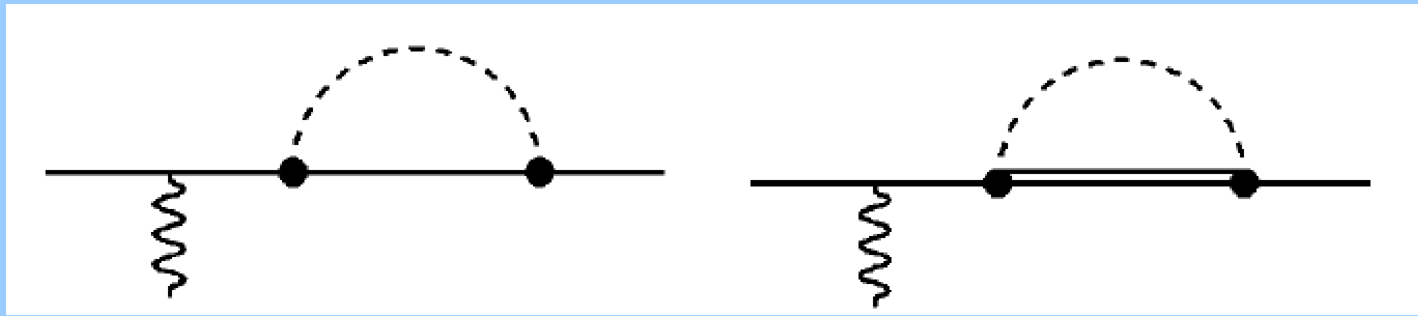
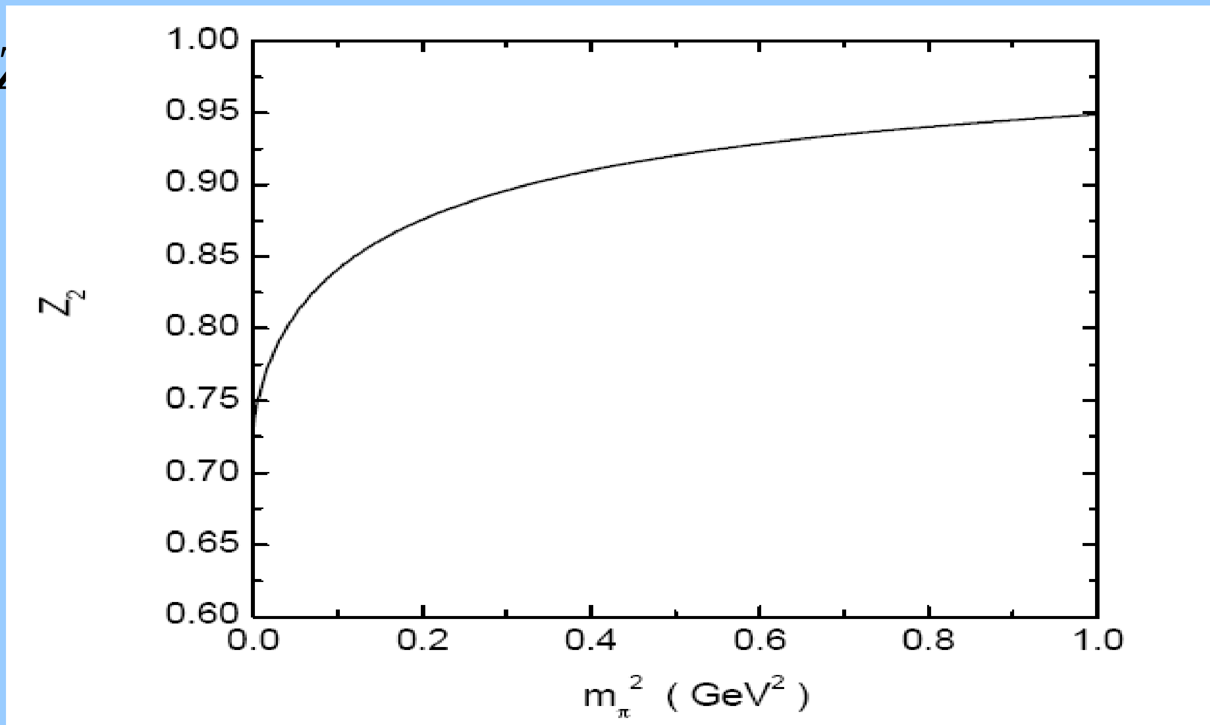
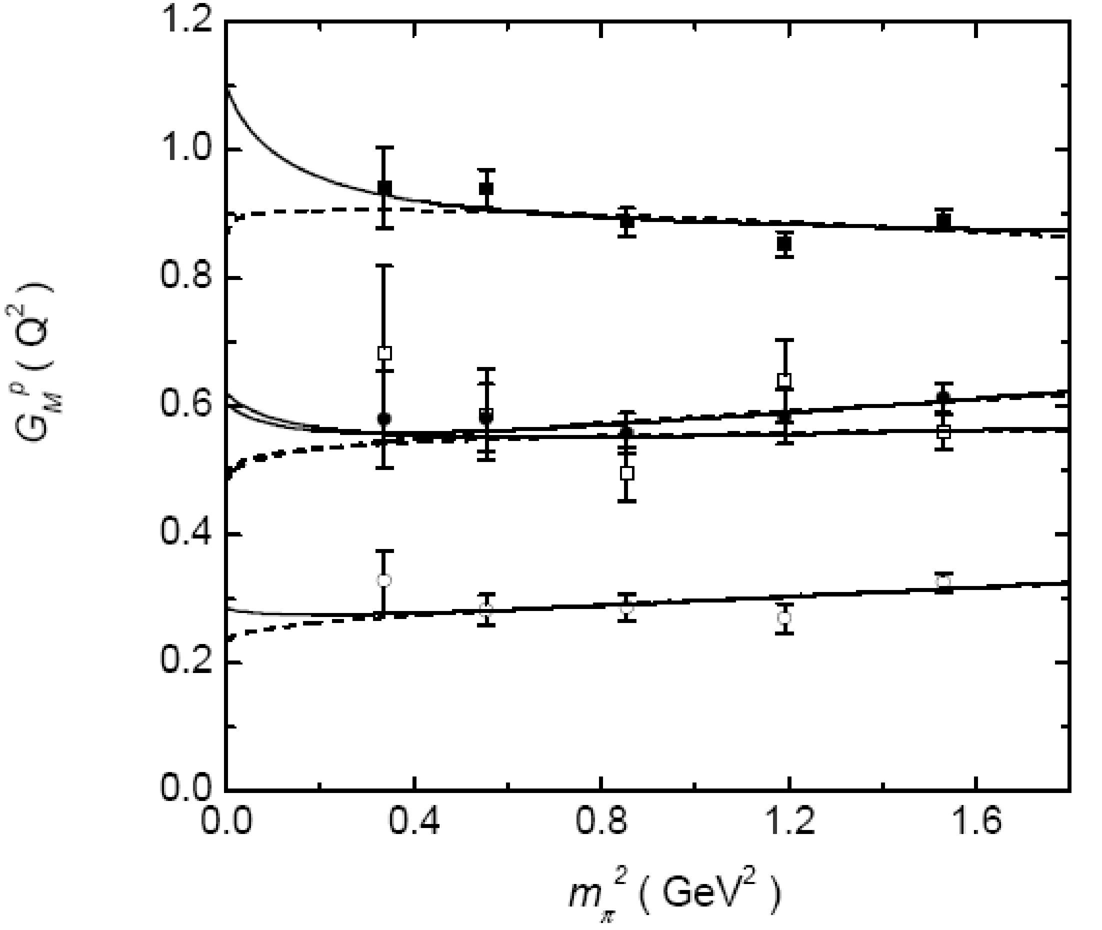
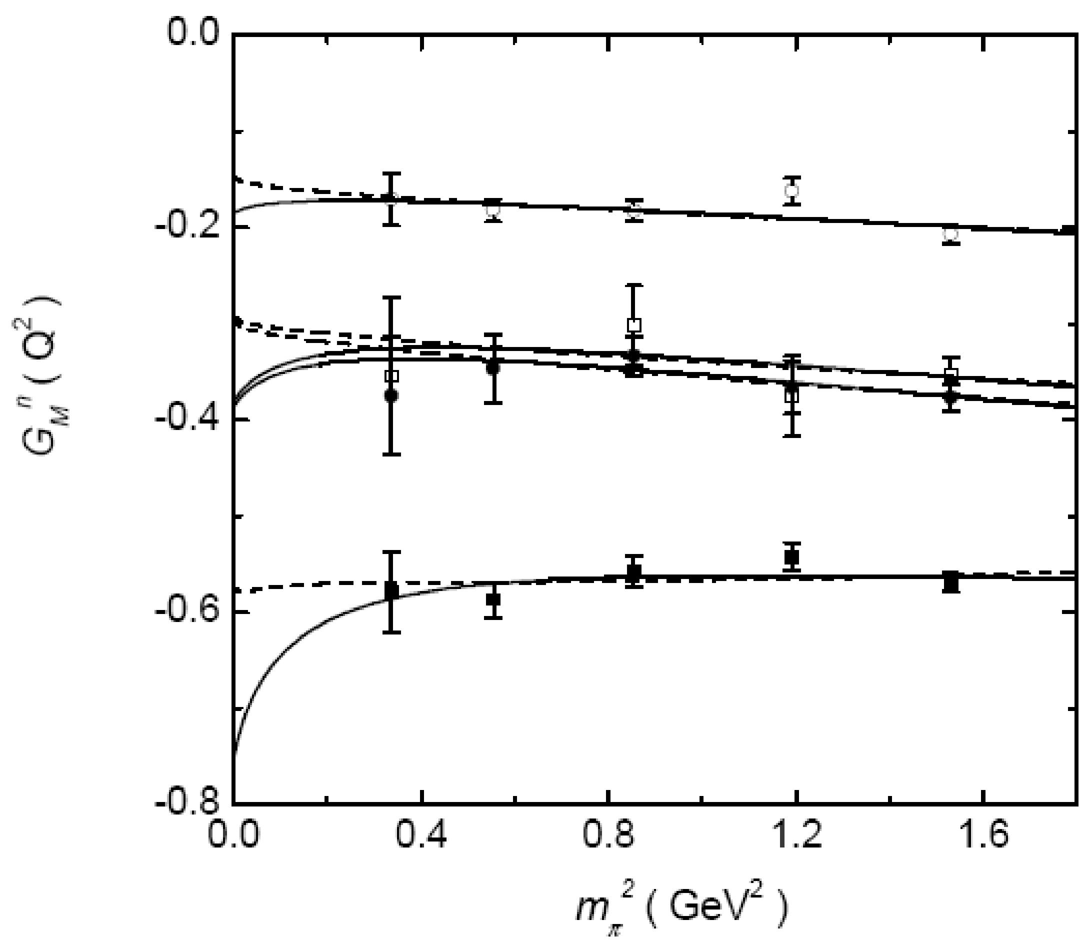


$$G_M^N(m_\pi^2) = Z(a_0^N + a_2^N m_\pi^2 + a_4^N m_\pi^4) + \sum G_M^N(\text{Loop})$$

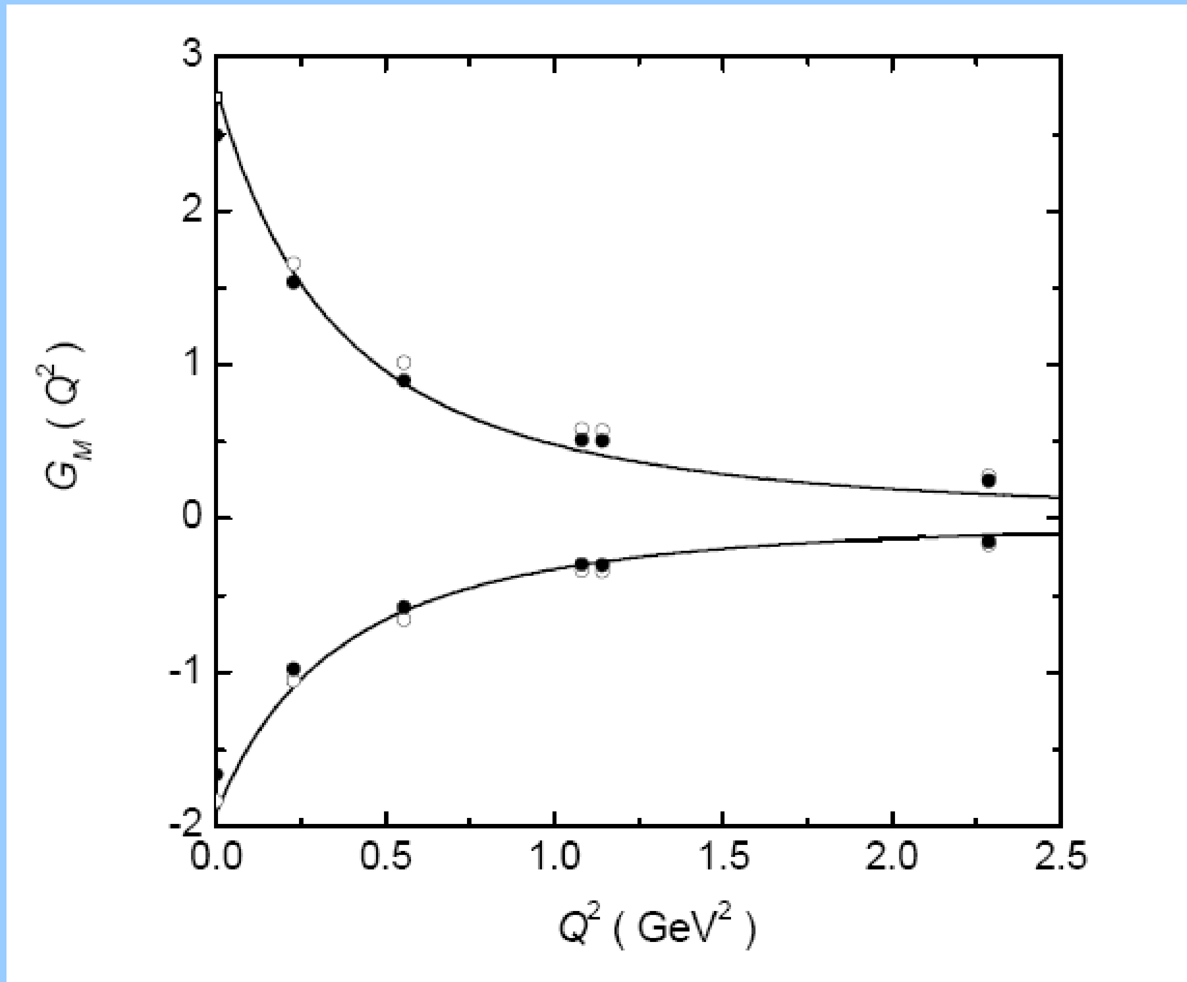




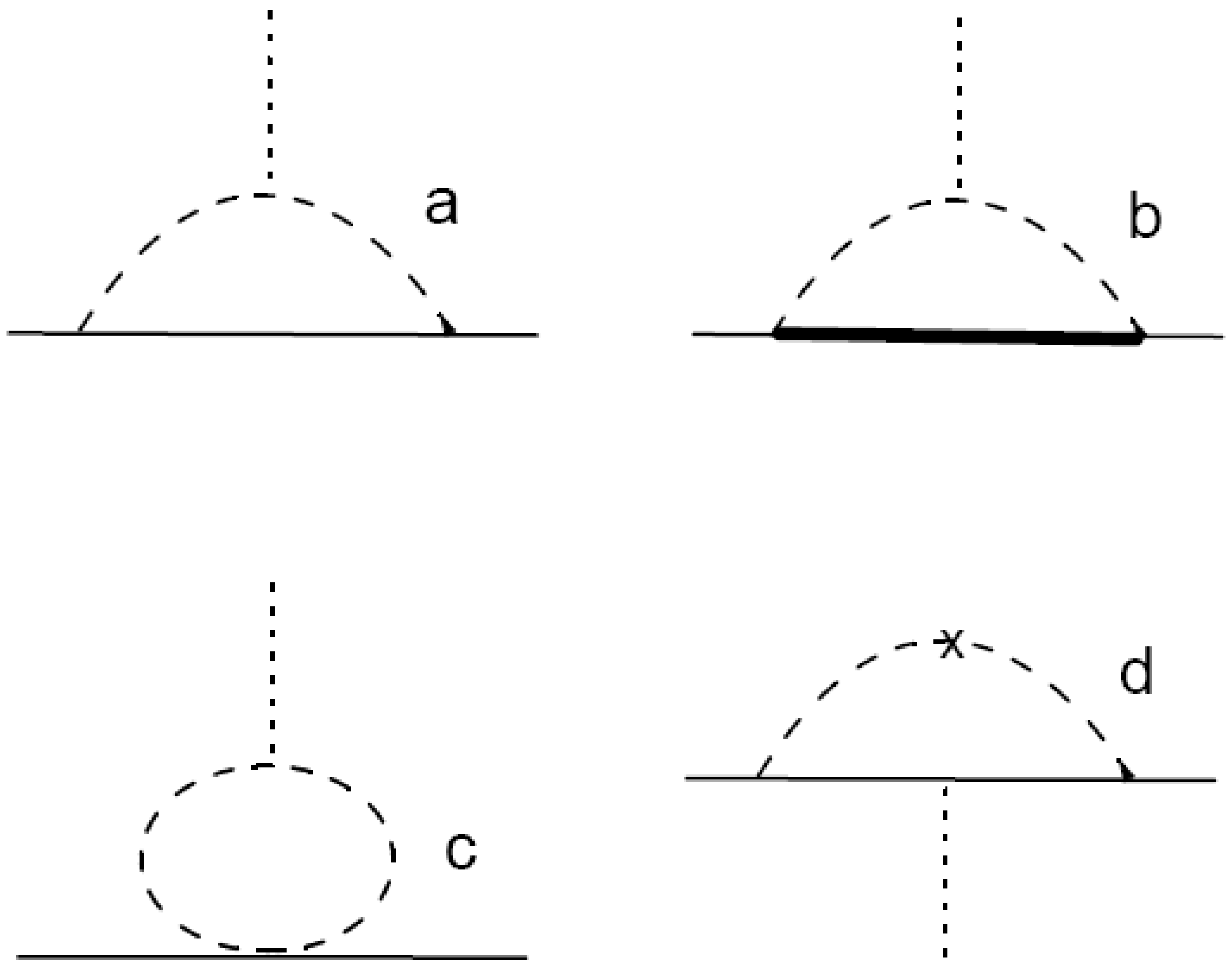
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.4
0.8
1.2
1.6
$G_M^p(Q^2)$
m_π^2 (GeV^2)

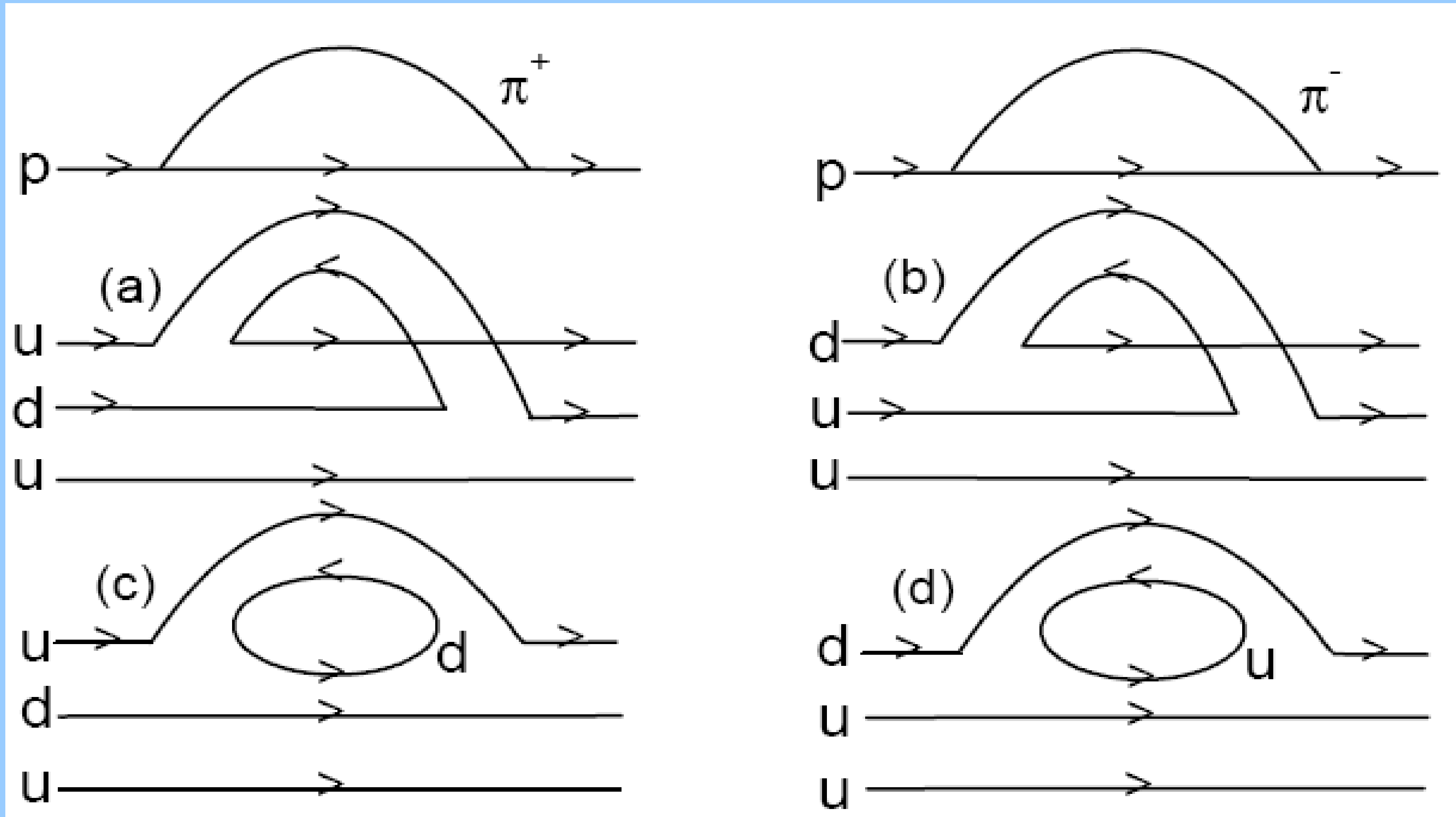


0.0
-0.2
-0.4
-0.6
-0.8
0.0
0.4
0.8
1.2
1.6
$G_M^n(Q^2)$
m_π^2 (GeV^2)

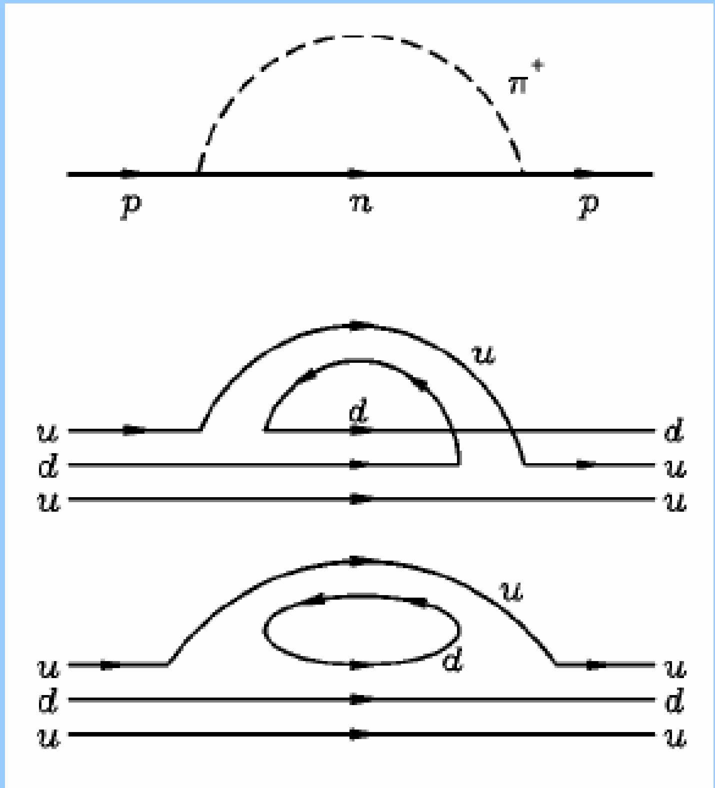


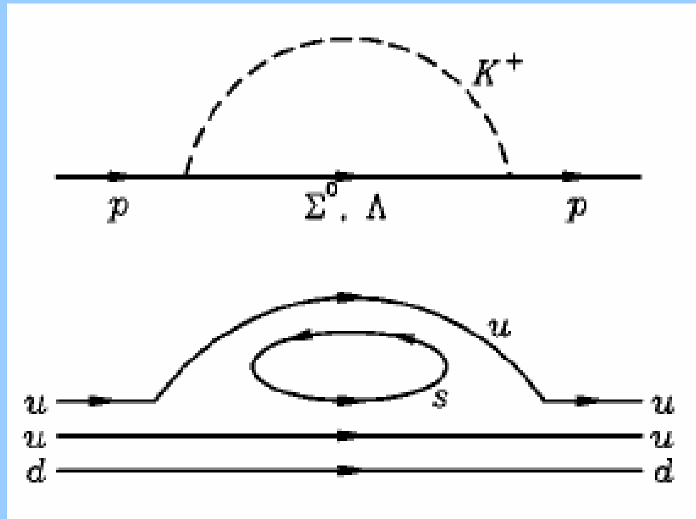
3
2
1
0
-1
-2
0.0
0.5
1.0
1.5
2.0
2.5
$G_M(Q^2)$
Q^2 (GeV^2)


a
b
c
d



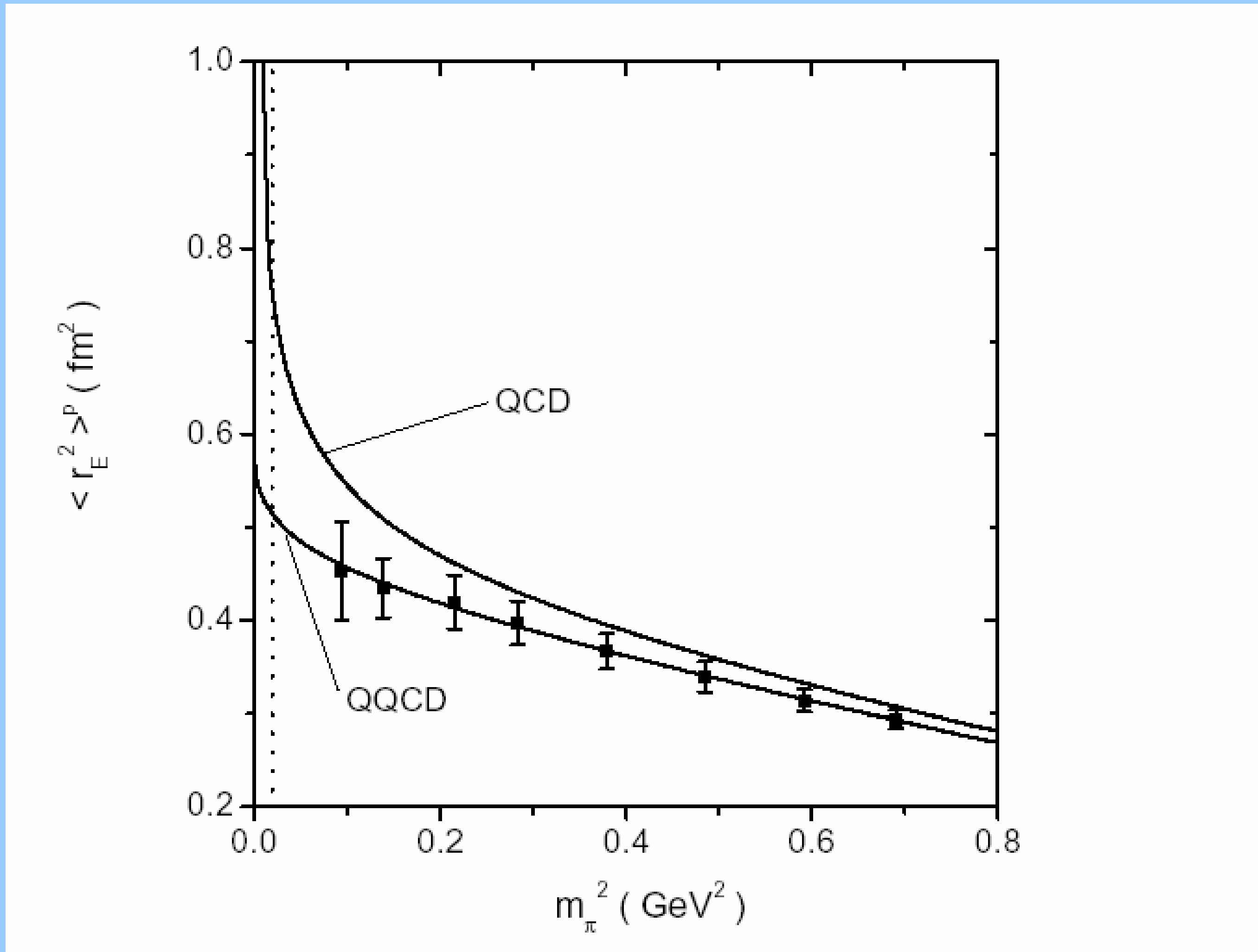
π+
p
(a)
u
d
u
π-
p
(b)
d
u
u
(c)
u
d
d
u
(d)
d
u
u
u



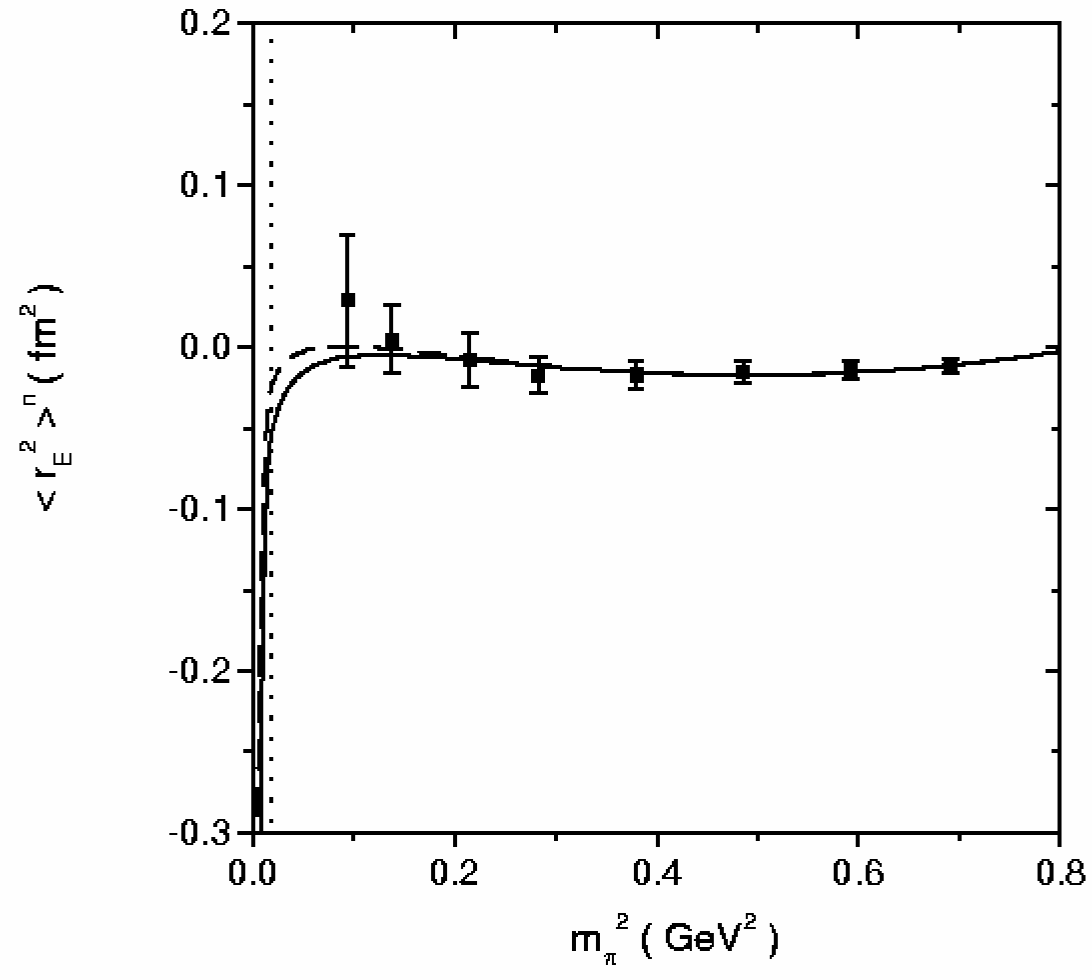


$$(D+F)^2 - \frac{2}{3}D^2 - 2F^2$$

D. B. Leinweber, PRD 69 (2004) 014005.



$$< r_E^2 > = - 6\, dG_E / dQ^2$$



0.2
0.1
0.0
-0.1
-0.2
-0.3
0.0
0.2
0.4
0.6
0.8
$< r_E^2 >^n$ (fm^2)
m_π^2 (GeV^2)

Summary

The nucleon magnetic moments and form factors are extrapolated from the lattice data with FRR in chiral perturbation theory. The optimal Lambda is discussed in different methods.

To any finite order, FRR is mathematically equivalent to dimensional regularization. At low pion mass, both FRR and DR give reasonable results. At large pion mass, DR fails while FRR works well.

High order terms in the chiral expansion are important which can be built in the one loop contribution in FRR. The residual analytic terms have a good convergent behavior.

The extrapolated magnetic moments, form factors and charge radii at physical pion mass are reasonable compared with the experimental values. The difference could due to the quenched approximation, finite volume effect of lattice, etc.

The same method can be applied to the extrapolation of many other lattice data, such as the charge radii, magnetic form factors of baryon octet and decuplet, strange from factors, transition form factors etc, in full QCD or quenched QCD.