The Dipole Model &
the DAF-BFKL Pomeron
@ HERA and LHeC & EIC

Henri Kowalski

BNL- JLAB
24, 29 of September 2008
Evidence for the discrete asymptotically-free BFKL Pomeron from HERA data

J. Ellis\textsuperscript{a}, H. Kowalski\textsuperscript{b}, D.A. Ross\textsuperscript{a,c,*}

Physics Letters B  

in print
Outline of the talk:

Short review of low $x$ HERA data, Dipole Picture, Saturation, oomph factor and all that

Why Pomeron at HERA?,

What is DAF-BFKL Pomeron
Evidence for DAF-Pomeron from HERA data

Relation with DGLAP
MRST $\leftrightarrow$ EKR

Pomeron-Graviton Correspondence

Consequences for RHIC, LHC, EIC, LHeC
Low-x Physics @ HERA

- At low $x$ and high $Q^2$, steep rise in structure function

Behavior of $F_2$ is dominated by gluon density at small-$x$
Hard Diffraction – the HERA surprise

Non-Diffractive Event

Diffractive Event

expected before HERA
<0.01%, seen over 10% at $Q^2 = 10 \text{ GeV}^2$

Diffraction at HERA is so large because it is a shadow of DIS (i.e. inelastic processes) $\rightarrow$ dipole picture

$$\sigma_{tot}^{\gamma^* p} = \frac{I}{W^2} \text{Im} A_{el}(W^2, t = 0)$$

$\tau_{qq} \approx \frac{1}{\Delta E} \approx \frac{1}{m_p x} \approx 10 - 1000 \text{ fm}$
Dipole description of DIS
equivalent to Parton Picture in the perturbative region

\[ \sigma_{qg} \sim r^2 x g(x, \mu) \text{ for small } r \]

Mueller, Nikolaev, Zakharov

\[ \sigma^\gamma_p = \int d^2 \vec{r} \int_0^1 dz \Psi^\star_q(x, r^2) \Psi_q \]

\[ \frac{d\sigma^\gamma_{VM}}{dt} \bigg|_{t=0} = \frac{1}{16\pi} \int d^2 \vec{r} \int_0^1 dz \Psi^*_{VM}(Q^2, z, \vec{r}) \sigma_{qg}(x, r^2) \Psi(Q^2, z, \vec{r}) \]

\[ \frac{d\sigma^\gamma_{diff}}{dt} \bigg|_{t=0} = \frac{1}{16\pi} \int d^2 \vec{r} \int_0^1 dz \Psi^* \sigma_{q\bar{q}}^2(x, r^2) \Psi \]
Total $\gamma^p$ cross-section

$\sigma_{tot}(\text{mb})$ (scaled)

$Q^2 (\text{GeV}^2) (\text{scale})$

$0.25 (2.1)$
$0.30 (2.4)$
$0.40 (1.8)$
$0.50 (1.4)$
$0.65 (1.1)$
$2.5 (1.3)$
$2.7 (1.1)$
$3.0 (1.2)$
$3.5 (1)$
$4.5 (1)$
$5.5 (1)$
$6.0 (1)$
$7.0 (1)$
$8.0 (1)$
$12.0 (1)$
$15.0 (1)$
$20.0 (1)$
$25.0 (1)$
$30.0 (1)$
$35.0 (1)$
$40.0 (1)$
$45.0 (1)$
$50.0 (1)$
$60.0 (1)$
$70.0 (1)$
$90.0 (1)$
$120.0 (1)$
$150.0 (1)$

$W^2 (\text{GeV}^2)$

$x < 10^{-2}$

$\lambda_{tot} = \frac{\sigma_{tot}(\gamma^p)}{(W^2)^2} \sim (1/x)^{\lambda_{tot}}$

$b$-Sat

$\mu^2 = \frac{c}{r^2} + \mu_0^2$

$b$-CGC

$IIM+KMW$

$\lambda_{tot} = \frac{\sigma_{tot}(\gamma^p)}{(W^2)^2} \sim (1/x)^{\lambda_{tot}}$

$\sigma_{tot}(\gamma^p) \sim (W^2)^{\lambda_{tot}} \sim (1/x)^{\lambda_{tot}}$

$\sigma_{tot}(\gamma^p) = A_g \left( \frac{1}{x} \right)^{\lambda_g} (1-x)^{a_b}$

$\frac{d\sigma_{q\bar{q}}}{d^2b} = 2 \left[ 1 - \exp \left( -\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b) \right) \right] x g(x, \mu_0^2) = A_g \left( \frac{1}{x} \right)^{\lambda_g} (1-x)^{a_b}$

$\frac{d\sigma_{q\bar{q}}}{d^2b} = 2 N(x, r, b) = 2 \times \begin{cases} \mathcal{N}_0 \left( \frac{\gamma Q_s}{2} \right)^2 \left( \gamma + \frac{1}{\gamma} \ln \frac{2}{r Q_s} \right) & : \ r Q_s \leq 2 \\ 1 - e^{-A \ln^2(B r Q_s)} & : \ r Q_s > 2 \end{cases}$

$b$-Sat

$b$-CGC

$IIM+KMW$
Low-x Physics @ HERA

Diffraction at HERA is a shadow of DIS
→
dipole picture, equivalent to LO p-QCD for small dipoles, $Q \sim 1/r$

$$\sigma_{tot}^{p} = \frac{1}{W^2} \text{Im} A_{el}(W^2)$$

$x \approx \frac{Q^2}{W^2} < 0.01$

$$\sigma_{tot}^{p}(W,Q^2) = \frac{4 \pi^2 \alpha_{em}}{Q^2} \cdot F_2(x,Q^2)$$

NNPZ, AM, GLM, FKS, GBW, DGKP, BGBK, IIM, FSS......
KT - Kowalski, Teaney
KMW - K, Motyka, Watt
KLMV - K, Lappi, Marquet, Venugopalan
$F_L(x,Q^2)$


K+Watt
Dipole Picture—gluon density convoluted with the dipole wave functions \( \Rightarrow \) simultaneous prediction/description of many reactions

Diffractive Di-jets

\( Q^2 > 5 \text{ GeV}^2 \)
Diffractive production of a $q\bar{q}$ pair

$\tilde{f}(x_{IP}, l_t^2) \sim$ probability to find a Pomeron(2g) in $p$

$= \text{Fourier transform of } \sigma_{qq}$

$\tilde{f}(x_{IP}, l_t^2) = \frac{3\sigma_0}{4\pi^2} R_0^2(x_{IP}) l_t^2 \exp(-R_0^2(x_{IP}) l_t^2)$

$$\frac{d\sigma^T}{dt \, dM^2} \bigg|_{t=0} = \sum_f e_f^2 \frac{\pi^2 \alpha_{em}}{12 \, Q^4} \frac{\beta^2}{(1-\beta)^2} \int dk_t^2 \frac{k_t^2 + m_q^2}{k_t^2 \sqrt{1 - 4\beta k_t^2 / Q^2}} \left\{ \left[ 1 - \frac{2\beta k_t^2}{Q^2} \right] P^T_{IPq} + 4 \frac{k_t^2 m_q^2}{k_t^4} P^{L}_{IPq} \right\}$$

$$k_t^2 = \frac{(k_t^2 + m_q^2)}{(1-\beta)}$$

$$\frac{d\sigma^L}{dt \, dM^2} \bigg|_{t=0} = \sum_f e_f^2 \frac{4\pi^2 \alpha_{em}}{3 \, Q^6} \frac{\beta^3}{(1-\beta)} \int dk_t^2 \frac{k_t^2}{\sqrt{1 - 4\beta k_t^2 / Q^2}} P^{L}_{IPq}$$

$$P^T_{IPq}(\beta, k^2) = \int \frac{dl_t^2}{l_t^2} \tilde{f}(l_t^2) \left[ 1 - 2\beta - 2 \frac{m_q^2}{k_t^2} + \frac{l_t^2 - (1-2\beta)k_t^2 + 2m_q^2}{\sqrt{(l_t^2 + k_t^2)^2 - 4l_t^2 k_t^2}} \right]^2$$

$$P^{L}_{IPq}(\beta, k^2) = \int \frac{dl_t^2}{l_t^2} \tilde{f}(l_t^2) \left[ 1 - \frac{k_t^2}{\sqrt{(l_t^2 + k_t^2)^2 - 4l_t^2 k_t^2}} \right]^2$$

$\sim$ probability for a Pomeron(2g) to couple to a quark
Diffractive production of a qqg state

\[
\frac{d\sigma}{dt\,dM^2}\bigg|_{t=0} = 2 \sum_f e_f^2 \frac{4\pi^2\alpha_{em}}{Q^2} \frac{\beta^2}{Q^2} \int dk_t^2 \int d\phi \int dq_t^2
\]

\[
\frac{\alpha_s}{2\pi} \int \frac{dz}{z} P_{qg} \left( \frac{\beta}{z} \right) P_{PP_g}(z, \frac{k_t^2}{1-z})
\]

D.C.: \(Q^2 \gg q_t^2 \gg k_t^2\)

\[P_{PP_g}(z, k_t^2) = \frac{9}{64} \frac{1}{z^2} \left( \frac{1}{1-z} \right)^3 \int \frac{dP_t^2}{l_t^2} \tilde{f}(x_P, l_t^2) \]

\[
\left. z^2 + (1-z)^2 + \frac{l_t^2}{k_t^2} - \frac{[(1-2z)k_t^2 - l_t^2]^2 + 2z(1-z)k_f^4}{k^2 \sqrt{(l_t^2 + k_t^2)^2 - 4(1-z)l_t^2k_f^2}} \right)^2
\]

\[
z = \frac{(Q^2 + m^2)}{(Q^2 + M^2)}
\]

\[
m^2 = (q + \bar{q})^2 + k_t^2
\]

\[P_{qg} \left( \frac{\beta}{z} \right) = \frac{1}{2} \frac{1 - 2q_t^2 + m_q^2}{q_t^2 + m_q^2} \left[ \frac{(\beta/z)^2}{z} + \left(1 - \frac{\beta}{z}\right)^2 \right]
\]

\[+ \frac{4}{q_t^2 + m_q^2} \frac{\beta}{z} \left(1 - 2\frac{\beta}{z}\right) m_q^2 Q^2
\]

\[+ \frac{2m^2}{(q_t^2 + m_q^2)^2} \left( \frac{\beta}{z} \right)^2 \left(1 - 2\frac{\beta}{z}\right) m_q^2 Q^2
\]
\[
\frac{\alpha_s f(x, l^2)}{l^4} = \frac{3}{4\pi} \int \frac{d^2r}{(2\pi)^2} \exp\{il \cdot r\} \{\hat{\sigma}_\infty(x) - \hat{\sigma}(x, r)\} = \\
= \frac{3}{8\pi^2} \int_0^\infty dr \ J_0(lr) \{\hat{\sigma}_\infty(x) - \hat{\sigma}(x, r)\}.
\]

In the original dipole model we find [2]:

\[
\alpha_s f(x, l^2) = \frac{3\sigma_0}{4\pi^2} \ R_0^2(x) \ l^4 \ \exp\{-R_0^2(x) \ l^2\}.
\]
\[ \sigma \sim s^{\alpha-1} \Rightarrow \frac{dN}{d\log M_X^2} \sim \text{const} \]
Diffractive Signature

\[ \ln(M_X^2) \sim \Delta Y = \log(W^2/M_X^2) \]

Non-Diffraction

- Uniform, uncorrelated particle emission along the rapidity axis
- Probability to see a gap \( \Delta Y \) is \( \sim \exp(-\langle n \rangle \Delta Y) \)
- \( \langle n \rangle \) - average multipl. per unit of \( Y \)

Diffraction

- \( dN/dM_X^2 \sim 1/M_X^2 \) => \( dN/d\log M_X^2 \sim \text{const} \)
ZEUS

- \( \text{Fit}(c \exp(b \ln M_X^2)) \)
- \( \text{Fit}(D + c \exp(b \ln M_X^2)) \)
- DJANGO
- SATRAP + ZEUSVM
- SANG\((M_N < 2.3 \text{ GeV})\)
- (ZEUS 98-99) - PYTHIA - SANG\((M_N > 2.3 \text{ GeV})\)

\[ W = 37 - 55 \text{ GeV} \quad \text{and} \quad W = 200 - 245 \text{ GeV} \]
Inclusive Diffraction
LPS

ZEUS – BGBK Dipole

\[ Q^2 (\text{GeV}^2) \]

\[ \beta = 0.007 \quad \beta = 0.03 \quad \beta = 0.13 \quad \beta = 0.48 \]

\[ x_{IP} F_2 D(3) \]

\[ x_{IP} \]

ZEUS 97
BGK

\[ x_{IP} < 0.01 \]

\[ x_{IP} > 0.01 \]
Observation of diffraction indicates that single ladder may not be sufficient (partons produced from a single chain produce exponentially suppressed rap. gaps)

Diffractive Structure Function

Dipole Model

Initial Diff. SF
$Q^2_0 \sim 4 \text{ GeV}^2$

Study of exclusive diffractive states may clarify which pattern is right

Only few final states present in DiMo: $qq, qqq$ (aligned and as jets)

VM
Diffractive Di-jets
$Q^2 > 5 \text{ GeV}^2$
Diffractive Di-jets

$Q^2 > 5 GeV^2$

ZEUS

$E_{T, \text{jets}}$ (GeV)

$M_X$ (GeV)

$Z_{IP}$
Diffractive Di-jets

$Q^2 > 5 \text{ GeV}^2$

ZEUS

- $\frac{d\sigma}{dz_{TP}} \text{ (pb)}$
- $\frac{d\sigma}{dx_{\gamma'}} \text{ (pb)}$

- ZEUS (prel.) 99-00
- Correlated syst. uncertainty
- RAPGAP(di+res.) $\times 0.92$
- direct only $\times 1.03$
- resolved only

- SATRAP $\times 1.12$
Dipole Picture—gluon density convoluted with the dipole wave functions \( \rightarrow \) simultaneous prediction/description of many reactions

Vector Mesons

Note: educated guesses for VM wf are working very well
Note: educated guesses for $J/\Psi$ and $\phi$ wave functions are working very well.

More work to do for $\rho$ meson wave function.
Can use vector meson production to extract proton profile:

\[
\frac{d\sigma_{VM}}{dt} = \frac{1}{16\pi} \left| \int d^2 \vec{r} \int d^2 b e^{-i\vec{b} \cdot \vec{A}} \int_0^1 dz \Psi_{VM}^* \right|^2 \left\{ 1 - \exp\left(-\frac{\Omega}{2}\right) \right\}
\]

\[
\Omega = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2) xg(x, \mu^2) T(b)
\]

**T(b)-proton shape**
Description of the size of interaction region $B_D$

Modification by Bartels, Golec-Biernat, Peters

$e^{i\vec{b} \cdot \Delta} \rightarrow e^{i(\vec{b} + (1-z)\vec{r}) \cdot \Delta}$

\[
\frac{d\sigma^{\text{diff}}}{dt} \sim \exp(B_D \cdot t) \quad \Rightarrow T(b) \sim \exp(-\vec{b}^2 / 2B_G)
\]

\[\sqrt{R_p^2 = 3B_G} = 0.68 \text{ fm}\]

$R_p = 0.870 \pm 0.008 \text{ fm}$

$\Rightarrow B_G = 6.48 \text{ GeV}^2$

the gluonic proton radius smaller than the quark radius
measurement of $\alpha'$

$\gamma p \to J/\psi p$ (photoproduction)

$J_n(t) = 1 + \frac{c}{n + \delta(t)}$

Significant slope is expected for a leading pomeron trajectory
Lipatov (1986)

$\alpha'_p = 0.115 \pm 0.018 \text{(stat.)}^{+0.008}_{-0.015} \text{(syst.)} \text{GeV}^{-2}$

$\alpha' = \frac{R^2}{g_{YM} \sqrt{N}}$
Universal, “Pomeron like” QCD object soft and hard Pomeron join together
Pomerons at work
Rise of the DVCS cross-sections

\[ \gamma^* p \rightarrow \gamma p \]

At EIC (LHeC) it should be possible to reduce the errors by a large factor, \( O(100) \)

\( \Rightarrow \) detailed study of the Pomeron possible
Gluon density test?
Teubner

$\gamma^* p \rightarrow J/\psi p$

IP-Dipole Model

VM wave function dependence?
Saturation

Estimate of $Q_s$

- Apparently little saturation at $Q_s^2 = 4 \text{GeV}^2$

- $\Rightarrow$ Oomph factor
- Increase saturation by going to nuclei
Nuclear enhancement of universal dynamics of high parton densities

Kowalski, Lappi, Venugopalan
K, L, Marquet, V

\[
\frac{Q_{s,\Lambda}^2}{Q_{s,B}^2} = \frac{A^1}{B^1} \frac{T_A(b_\perp)}{T_B(b_\perp)} \frac{F(x, Q_{s,\Lambda}^2)}{F(x, Q_{s,B}^2)} \sim A^{1/3} \frac{F(x, Q_{s,\Lambda}^2)}{B^{1/3} F(x, Q_{s,B}^2)}
\]

large enhancement of saturation scale in nuclei
200^{1/3} \sim 6
Equivalent center of mass energy \sim 14 time larger than in ep
$t$-distributions for coherent (i.e., exclusive) diffractive meson production on proton and nuclei at EIC

KLMV

First estimate of the expected measurement precision:

$\Delta p_T < 30$ MeV, $t \sim p_T^2$

$\Delta t < 0.01$ GeV$^2$

for proton and light nuclei
Basics of BFKL

BFKL equation at $q=0$

Eigenfunctions

Conformal invariance

solved by finding a complete set of eigenfunctions

Characteristic function

$\omega = \bar{\alpha}_s \chi(\nu)$

$\chi(\nu) = -2\gamma_E - \psi(1/2 + iv) - \psi(1/2 - iv)$

$\psi$ is the Digamma function

Green function
NLO BFKL with running $\alpha_s$

NLO

\[ \omega \equiv \chi(\alpha_s, \nu) = \bar{\alpha}_s (1 - A\bar{\alpha}_s) \chi_0 \left( \frac{1}{2} + \bar{\alpha}_s B + i\nu + \frac{\omega}{2} \right) + \bar{\alpha}_s^2 \chi_1(\nu). \]

running coupling

\[ \omega = \chi(\alpha_s(k), \nu_\omega(k)). \]

\[ \omega = \chi(\alpha_s(k_{\text{crit}}), 0). \]

Airy functions are solving BFKL eq. around $k \sim k_{\text{crit}}$

\[ \left[ \frac{d^2}{d \ln(k^2/k_0^2)} + \frac{\beta_0}{2\pi} \frac{\chi''(\alpha_s(k_{\text{crit}}), 0)}{\chi''(\alpha_s(k_{\text{crit}}), 0)} \ln \left( \frac{k^2}{k_0^2} \right) \right] f_\omega(k) = 0, \]

\[ f_\omega(k^2) = \frac{f_\omega(k)}{\sqrt{k^2}}, \]

Fadin, Lipatov
G. Salam
resummation

property of $\chi$:
largest $\omega$ at $\nu=0$
Matching the solutions at $k=k_{\text{crit}}$ determines the phase of oscillation $= \pi/4$. Lipatov 86 encode the infrared behaviour of QCD by assuming a fixed phase $\eta$ at $k_0$.

\[
\phi_\omega(k) = 2 \int_{k}^{k_{\text{crit}}} \frac{dk'}{k'} |\nu_\omega(k)|.
\]

\[
\eta = -0.16\pi
\]
Structure functions in DIS

\[ F_2(x, Q^2) = \int \frac{1}{x} dz \int \frac{dk}{k} \Phi_{\text{DIS}}(z, Q, k) x g \left( \frac{x}{z}, k \right). \]

unintegrated gluon density

\[ x g(x, k) = \sum_n \int \frac{dk'}{k'} \Phi_p(k') \left( \frac{k' x}{k} \right)^{-\omega_n} k^2 f_{\omega_n}^*(k') f_{\omega_n}(k), \]

enhancement of leading eigenfun.by \((1/x)^\omega\)

\[ x g(x, k) = \sum_n a_n x^{-\omega_n} k^{(2+\omega_n)} f_{\omega_n}(k). \]

no enhancement of leading eigenfun.

\[ \Phi_p(k) = \sum_n a_n k^{(2-\omega_n)} f_{\omega_n}(k), \]
\[
S_q(x, Q^2) = \frac{Q^2}{4\pi^2} \int \frac{dk^2}{k^4} \int_0^1 d\beta \int d^2\kappa' \alpha_s \left\{ \left[ \beta^2 + (1 - \beta)^2 \right] \left( \frac{\kappa}{D_{1q}} - \frac{\kappa - k}{D_{2q}} \right)^2 \right. \\
+ \left. \left[ m_q^2 + 4Q^2\beta^2(1 - \beta)^2 \right] \left( \frac{1}{D_{1q}} - \frac{1}{D_{2q}} \right)^2 \right\} f\left( \frac{x}{z}, k^2 \right) \Theta \left( 1 - \frac{x}{z} \right)
\]

\[\kappa' = \kappa - (1 - \beta)k\] and

\[D_{1q} = \kappa^2 + \beta(1 - \beta)Q^2 + m_q^2\]

\[D_{2q} = (\kappa - k)^2 + \beta(1 - \beta)Q^2 + m_q^2\]

\[z = \left[ 1 + \frac{\kappa'^2 + m_q^2}{\beta(1 - \beta)Q^2} + \frac{k^2}{Q^2} \right]^{-1} \]

\[F_2 = \sum_q e_q^2 (S_q + V_q)\]
**Fit with charm**

Correct qualitative behaviour from leading singularity

Excellent fit to data for $x < 10^{-2}$ with 4 poles

---

The qualities of fits using up to 4 poles, and the corresponding pole residues, assuming $\eta = -0.16 \pi$ at $k_0 = 0.3 \text{ GeV}$

<table>
<thead>
<tr>
<th>Number of poles</th>
<th>$\chi^2/N_{\text{df}}$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.894/101</td>
<td>0.478</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>1157/100</td>
<td>0.566</td>
<td>-0.08</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>167/99</td>
<td>0.707</td>
<td>0.87</td>
<td>3.70</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>83.3/98</td>
<td>0.483</td>
<td>-6.32</td>
<td>-26.0</td>
<td>-26.9</td>
</tr>
</tbody>
</table>
Contributions to $F_2$ of the individual eigenfunctions

good data description due to interferences

$\Rightarrow$ phase $\eta$ precisely determined
Sum of contributions with small eigenvalues can give a larger rate of rise than the leading eigenvalue !!!
\[ F_2(x,Q^2) = x \sum_q e_q^2 q(x,Q^2) \]

\[ q(x,Q) = \int_0^Q \frac{dk}{k} \Phi_{DIS}(Q,k)xg(x,k) \]

\[ q(x,\mu^2) = q_0(x) + \frac{\alpha_s}{\pi} \int_x^{\mu^2} \frac{d\xi}{\xi} q_0(\xi) \left\{ P_{qg} \left( \frac{x}{\xi} \right) \ln \frac{\mu}{\xi} \right\} + \ldots \]

**Graph:**

- **Legend:**
  - EKR
  - MRST

- **Lines:**
  - \( x = 10^{-4} \)
  - \( x = 10^{-3} \)
  - \( x = 10^{-2} \)
Where Do BFKL and DGLAP Meet

Lipatov, private communication

Unintegrated BFKL gluon density (LO, no running $\alpha_s$)

$$xg(x, k^2) = \int d\gamma \Phi_p(\gamma) \left( \frac{k^2}{\mu^2} \right)^\gamma x^{-\alpha_s \chi(\gamma)} = \int d\gamma \Phi_p(\gamma) \exp(F(\gamma))$$

$\gamma = \frac{1}{2} + iv$

Saddle point

$$(F(\gamma))' = (\gamma \ln(k^2/\mu^2) + \bar{\alpha}_s \ln(1/x)\chi(\gamma))' = 0$$

$$\chi(\gamma) = \frac{1}{\gamma} - 2\zeta(3)\gamma^2 + \cdots$$

$$\gamma^2 = \frac{\bar{\alpha} \ln(1/x)}{\ln(k^2/\mu^2)}$$

valid if

$$\bar{\alpha}(k^2) \ln(1/x) \ll 1,$$

not fulfilled for HERA or even Higgs at LHC!
Eigenfunctions

1-7

8-14

15-21

DAF Pomeron fit with
\[ \Phi_p = k^2 \exp(-bk^2) \]
DAF Pomeron fit with
\[ \Phi_p = k^2 \exp(-bk^2) \]

\[ b \sim 6 \text{ GeV}^{-2} \]

for \( x < 0.001 \)
\[ \chi^2/\text{ndf} = 25/30 \]

for \( x < 0.01 \)
\[ \chi^2/\text{ndf} = 260/100 \]
Pomeron and Gauge/String Duality
Brower, Polchinski, Strassler, and Tan, hep-th/0603115

Pomeron is a coherent color-singlet object, built from gluons, with universal properties; it is a closed string propagating in ADS space, when the conformal symmetry is broken at some infrared point in the fifth dimension

\[ 1 + \omega = 2 - \frac{2}{\sqrt{4\pi \alpha_s N}} \]  
in ADS/CFT

\[ \omega = 0.26 \]  
in N=4 YM SuSy QCD


\[ \omega = 0.26 \]  

BFKL at fixed weak coupling
bare graviton at fixed strong coupling
Stasto, hep-ph/0702195
QCD resummation
Consequences for LHC

Good knowledge of gluon density around $x \sim 10^{-2}$ and $Q^2 \sim 10000$ GeV$^2$ is essential for LHC physics (Higgs region).

Large effort is going into precise measurement of $W$ and $Z$ inclusive $X$-sections $\Rightarrow$ precise determination of sea-quark distributions $\Rightarrow$ precise gluon density.

Is the sea-quark $\leftrightarrow$ gluon density relation the same in the DGLAP-like picture (MRST/CTEQ) and DAF-Pomeron?

![Diagram showing the comparison between EKR and MRST models for sea-quark and gluon densities.](image)

The sea-quark $\leftrightarrow$ gluon relation can be checked by the jets with $p_T$ around 50 GeV.
**LHeC will substantially improve the knowledge of gluon density**

*Consequences*

**HERA finding:**
Proton consists mainly of gluons, gluons determine its main properties, like the mass.

\[ xg(x, k) = \sum_n \int \frac{dk'}{k'} \, \Phi_p(k') \left( \frac{k' x}{k} \right)^{-\omega_n} k^2 f_{\omega_n}^{*}(k') f_{\omega_n}(k), \]

\[ \Phi_{p,N} \] Direct access to the gluonic structure of Protons and Nuclei
Diffraction measurements at EIC and LHeC will substantially improve the knowledge of \( \Phi_{p,N} \) - gluonic structure of matter.

**Consequences**

- Spatial distribution of gluonic field
- \( t \)-dependence of \( \omega \): is DAF-Pomeron moving towards or away from the Graviton?

**Advantage of EIC:**

- Precision measurements of the \( t \)-dependence

Build detectors which are able to measure diffractive reaction with high precision.
Instead of Conclusions

Study of Gluon Density are very important because it is the analog of Black Body Radiation in QED

It seemed hopeless to study pure Gluon Radiation since it is never free. However, it is becoming free for a short moment in HEP reactions

HERA has shown that physics processes at low-x are completely dominated by pure Gluon Density,

Investigation of Gluon Density has a chance to become as fundamental as Black Body radiation
The QCD improved parton model

MRST/CTEQ approach

sea quark densities (input: $q_0(x)$ for every quark species...)

$$q(x, \mu^2) = q_0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} q_0(\xi) \left( P_{qq} \left( \frac{x}{\xi} \right) \ln \frac{\mu^2}{\kappa^2} + C_q \left( \frac{x}{\xi} \right) \right) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} g_0(\xi) \left( P_{qg} \left( \frac{x}{\xi} \right) \ln \frac{\mu^2}{\kappa^2} + C_g \left( \frac{x}{\xi} \right) \right)$$

MS-bar scheme

$$F_2(x,Q^2) = x \sum_q e_q^2 \int_x^1 \frac{d\xi}{\xi} q(\xi, Q^2) \left\{ \delta(1-\frac{x}{\xi}) + \frac{\alpha_s}{2\pi} C_q^{\overline{MS}} \left( \frac{x}{\xi} \right) + ... \right\} + x \sum_q e_q^2 \int_x^1 \frac{d\xi}{\xi} g(\xi, Q^2) \left\{ \frac{\alpha_s}{2\pi} C_g^{\overline{MS}} \left( \frac{x}{\xi} \right) + ... \right\}$$

second term gives a very small contribution
How Important is Saturation?

- Eikonal exponentiation (Glauber-Mueller, MV):

\[
\frac{d\sigma_{qq}}{d^2b} = 2 \left[ 1 - \exp \left( -\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) xg(x, \mu^2) T(b) \right) \right]
\]

- Depends on impact parameter, momentum scale
- Define saturation scale \( Q_s \) by

\[
\frac{d\sigma_{qq}(x, r^2 = 2/Q_s^2(x, b))}{d^2b} = 2 \cdot \left\{ 1 - \exp(-1/2) \right\}
\]

- Estimate \( Q_s \) using indicative models for proton impact-parameter profile and gluon distribution:

\[
xg(x, \mu_0^2) = A_g \left( \frac{1}{x} \right)^{2g} (1-x)^{5.6} + \text{DGLAP evol.}
\]
Low-x Physics @ HERA

- Increasing rate of growth with $Q^2$, well described by DGLAP evolution
Basics of BFKL

Conformal invariance

BFKL equation at $q=Q/\mu$

solved by finding a complete set of eigenfunctions

\[
\omega \tilde{f}(\omega, k_1, k_2) = \delta^2(k_1 - k_2) + \frac{\alpha_s}{\pi} \int \frac{d^2k'}{(k_1 - k')^2} [\tilde{f}(\omega, k_1, k_2) + \frac{k_1^2}{k^2 + (k' - k_1)^2} \tilde{f}(\omega, k_1, k_2)]
\]

Eigenfunctions

\[
f_\omega(k^2) = \frac{(k^2)^{i\nu}}{\sqrt{k^2}}
\]

Characteristic function

\[
\omega = \bar{\alpha}_s \chi(\nu)
\]

\[
\chi(\nu) = -2 \gamma_E - \psi(1/2 + i\nu) - \psi(1/2 - i\nu)
\]

Green function

\[
f_{BFKL}(\omega, k_1, k_2) = \int_{-\infty}^{\infty} d\nu \left( \frac{k_1^2}{k_2^2} \right)^{i\nu} \frac{1}{2\pi^2 k_1 k_2} \frac{1}{(\omega - \bar{\alpha}_s \chi(\nu))}
\]

\(\psi\) is the Digamma function.
String theory emerged out of phenomenology of hadron-hadron scattering

Dolan-Horn-Schmid duality between s-channel and t-channel Regge-pole description of hadronic X-sections

\[ \sum \frac{g^2(t)}{s - (M_r - i\Gamma_r)^2} \approx \beta(t)(-\alpha's^{\alpha(t)}) . \]

\[ \Rightarrow \ \text{Veneziano amplitude} \]

\[ A_{\pi^+\pi^-\pi^+\pi^-}(s,t) = g^2 \frac{\Gamma[1-\alpha_p(t)]\Gamma[1-\alpha_p(s)]}{\Gamma[1-\alpha_p(s)-\alpha_p(t)]} \]

generalization by Virasoro \Rightarrow dual models

\[ \Rightarrow \ \text{mesons are open strings, closed strings necessary for unitarity} \]

\[ L - \text{string length, } E = c L, \quad J = \alpha' E^2 \]

Virasoro-amplitude for \( \alpha(0)=1 \) has a pole at \( s=t=0 \)

with \( J = 2, \) a graviton \Rightarrow starting point for theory of quantum gravity

\[ \alpha(t) = \alpha_0 + \alpha't \]

Superstring Theory, Green, Schwarz, Witten (1987)

R. Brower hep-th/0508036
**Maldacena Conjecture**
from the talk by J. Maldacena

**Particle theory** = **gravity theory**

```
Most supersymmetry QCD theory = String theory on AdS$_5 \times S^5$
```

(N colors)

Radius of curvature

\[ R_{S^5} = R_{AdS_5} = \left( \frac{g_{YM}^2 N}{l_s^4} \right)^{1/4} l_s \]

**Duality:**

- \( g^2 N \) is small \( \rightarrow \) perturbation theory is easy – gravity is bad
- \( g^2 N \) is large \( \rightarrow \) gravity is good – perturbation theory is hard

Strings made with gluons become fundamental strings.
Consequences for LHC

Proton dissociation

In high pp (ep) reactions particles are emitted approximately uniformly along the rapidity axis →
rapidity range in ND = \ln(s/m_p^2)

rapidity range in SD = \ln(M_X^2/m_p^2) → rapidity gap \Delta Y = \ln(s/M_X^2)

M_X^2/s = \xi - fraction of the proton momentum carried by the pomeron
\xi is called \xi_{IP} in ep,
minimum \xi at LHC = 1.4/s = 7 \times 10^{-9} \ (deep \ in \ the \ saturation \ region)
probability to find \xi, \ P(\xi) \sim 1/\xi \ d\xi

Can be measured with help of forward detectors down to x~\times 10^{-7}?
Consequences for LHC

- Consider diffractive production of a 'small' object
- Single or double diffraction?
  - $y = \ln(s/m_X^2)$ or $y_1 + y_2 = \ln(s/m_X^2)$?

- Examples:
  - $pp \rightarrow p$ (jet pair), $pp \rightarrow p$ ($D \Lambda_c$)
  - $pp \rightarrow p \eta_c p$, (RHIC) $pp \rightarrow p \ H \ p$ (LHC)
  - $pp \rightarrow p$ (jet pair) $p$

Measure the $t$-dependence of $\omega$:
Is DAF-Pomeron moving towards the Graviton or away from it? (EIC)
Consequences for EIC

Measure precisely the dependence of inclusive and exclusive diffractive processes (DVCS, J/Psi, rho, phi...)

$\Rightarrow$ Investigate QCD evolution with diffractive processes
  - pure evolution of $\sim$(gluon density)$^2$
  - $t$-dependence of effective exponents

$\Rightarrow$!!! Investigate Structure of Matter as $x$ increases !!!

Note: QCD evolution at HERA studied only with $F_2$
H1 and ZEUS measurement for $F_2$ agree to $\sim$3%
for diffractive processes systematic differences are factor 10 larger

Main reason: HERA experiments control only $\sim$2/3 of the rapidity range
NLO BFKL with running $\alpha_S$

solution away from $k_{\text{crit}}$

$$\bar{f}_{\omega}(k) = e^{\pm i\varphi_\omega(k)},$$

$$\varphi_\omega(k) = 2 \int_k^{k_{\text{crit}}} \frac{dk'}{k'} |\nu_\omega|$$

for all regions:

$$\bar{f}_{\omega}(k) = \sqrt[3]{\varphi_\omega(k)} \left[ J_{\frac{2}{3}}(\varphi_\omega(k)) + J_{-\frac{1}{3}}(\varphi_\omega(k)) \right], \quad (k < k_{\text{crit}}),$$

$$= \sqrt{3} \sqrt[3]{\varphi_\omega(k)} K_{\frac{1}{3}}(\varphi_\omega(k)), \quad (k > k_{\text{crit}}),$$

Matching the solutions at $k=k_{\text{crit}}$ determines the phase of oscillations $= \pi/4$

near $k\sim k_0$

$$\bar{f}_{\omega}(k) \sim \sin \left( \frac{\nu_\omega(k_0)}{k_0^2} (k^2 - k_0^2) - \eta \right).$$

Lipatov 86 $\Rightarrow$ encode the infrared behaviour of QCD by assuming a fixed phase $\eta$ at $k_0$

Quantization condition

$$\varphi_\omega(k_0) \equiv 2 \int_{k_0}^{k_{\text{crit}}} \frac{dk'}{k'} |\nu_\omega(k)| = \left( n - \frac{1}{4} \right) \pi + \eta,$$
The QCD improved parton model
MRST/CTEQ approach

sea quark densities

\[ q(x,\mu^2) = q_0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\zeta}{\zeta} q_0(\zeta) \left\{ P_{gg} \left( \frac{x}{\zeta} \right) \ln \frac{\mu^2}{\zeta^2} + C_q \left( \frac{x}{\zeta} \right) \right\} + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\zeta}{\zeta} g_0(\zeta) \left\{ P_{gg} \left( \frac{x}{\zeta} \right) \ln \frac{\mu^2}{\zeta^2} + C_g \left( \frac{x}{\zeta} \right) \right\} \]

MS-bar scheme

\[ F_2(x,Q^2) = x \sum_q e_q^2 \int_x^1 \frac{d\zeta}{\zeta} q(\zeta, Q^2) \left\{ \delta(1-\frac{x}{\zeta}) + \frac{\alpha_s}{2\pi} C_{\overline{MS}} \left( \frac{x}{\zeta} \right) + \ldots \right\} + x \sum_q e_q^2 \int_x^1 \frac{d\zeta}{\zeta} g(\zeta, Q^2) \left\{ \frac{\alpha_s}{2\pi} C_{\overline{MS}} \left( \frac{x}{\zeta} \right) + \ldots \right\} \]

second term gives a very small contribution
Consequences for EIC

Measure precisely the dependence of inclusive and exclusive diffractive processes (DVCS, J/Psi, rho, phi...)

Investigate QCD evolution with diffractive processes

pure evolution of $(\text{gluon density})^2$

t-dependence of effective exponents!

Note: QCD evolution at HERA studied only with $F_2$

H1 and ZEUS measurement for F agree to $\sim 3\%$

for diffractive processes systematic differences are factor 10 larger

Main reason: HERA experiments control only $\sim 2/3$

of the rapidity range
Diffraction at HERA is a shadow of DIS in the dipole picture, equivalent to LO pQCD for small dipoles, $Q \sim 1/r$. 

- ZEUS data (FPC)
- H1 data (LRG)
- IPsat
- bCGC
universal rate of rise
of all hadronic cross-sections

$$\lambda_{\text{tot}} = 1 + \lambda$$

$$\sigma_{zep}^{p} \sim (W^2)^{\lambda_{\text{tot}}} \sim (1/x)^{\lambda_{\text{tot}}}$$

**ZEUSS**

**ZEUS 98-99**

$$\alpha_{\text{TP}}^{\text{tot}}(0)$$ ($\pi < 0.01$)

$$\alpha_{\text{TP}}^{\text{eff}}(0)$$ ($2 < M < 15$ GeV)

$$\alpha_{\text{tot}} = 1 + \lambda$$

**soft Pomeron**