Recent progress on two-boson exchange effects in electron scattering

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Theory seminar at JLAB

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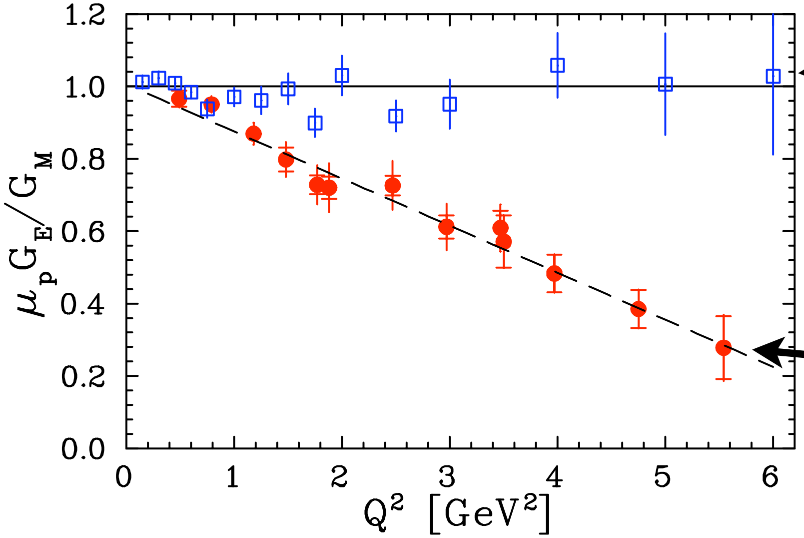
Outline

- Review of two-photon exchange (TPE): Rosenbluth vs polarization measurements of G_E and G_M of nucleon
- Hadronic model of two-photon exchange (TPE)
- pQCD results at high Q^2
- Parity violating asymmetry A_{PV} ($\gamma\gamma$ and γZ)
  - utility of generalized form factors
  - relation to atomic PV, MS calculation
- TPE effect on pion form factor

Proton G_E/G_M Ratio



Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

→ G_E from slope in ε plot

→ suppressed at large Q^2

PT method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

→ $P_{T,L}$ recoil proton polarization in $\vec{e}\,p \to e\,\vec{p}$

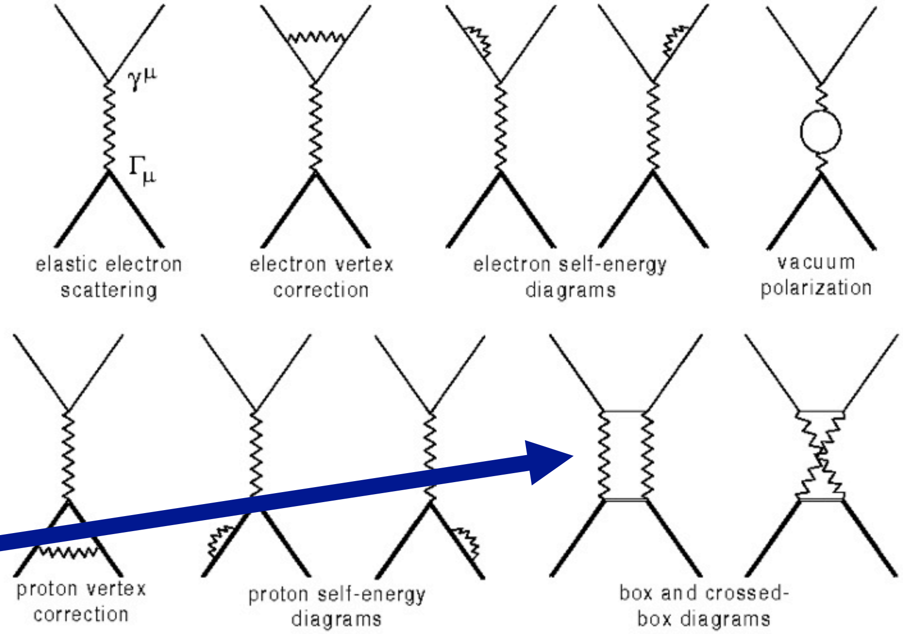
Radiative corrections

$d\sigma_0 \rightarrow d\sigma = d\sigma_0\,(1+\delta_{RC})$

Missing effect is

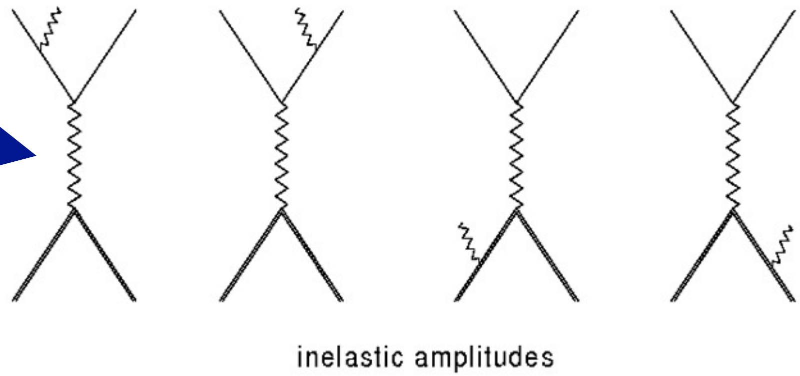
- approximately linear in ε
- not strongly Q^2 dependent

Two-photon exchange



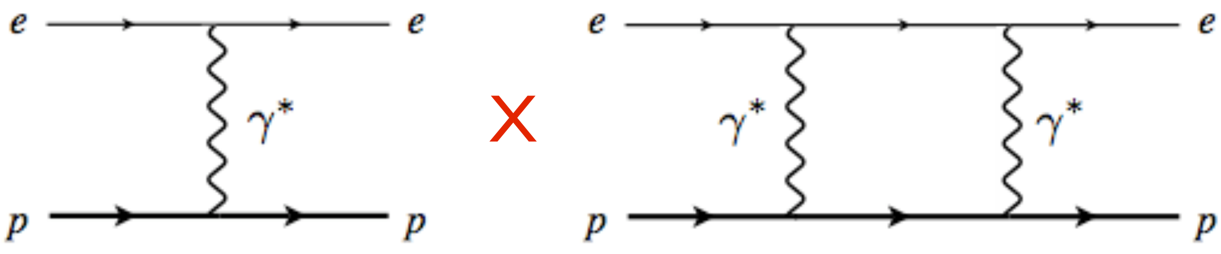
Bremsstrahlung

- SuperRosenbluth (detect proton)



Two-photon exchange

- interference between Born and two-photon exchange amplitudes



$\mathcal{M}_0$ $\qquad\qquad$ $\mathcal{M}_{\gamma\gamma}$

- contribution to cross section:

$$\delta^{(2\gamma)} = \frac{2\mathcal{R}e\left\{\mathcal{M}_0^\dagger\, \mathcal{M}_{\gamma\gamma}\right\}}{|\mathcal{M}_0|^2}$$

- standard "soft photon approximation" (used in most data analyses)

$\longrightarrow$ approximate integrand in $\mathcal{M}_{\gamma\gamma}$ by values at γ^* poles

$\longrightarrow$ neglect nucleon structure (no form factors) *Mo, Tsai (1969)*

Various Approaches

Rely on Models

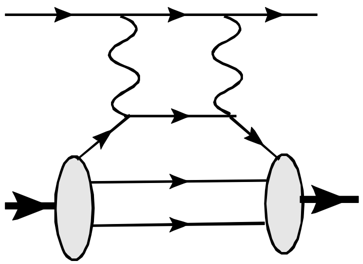
- Low to moderate Q^2: hadronic: N + Δ + N* etc.
  - more and more parameters, less and less reliable
- Moderate to high Q^2:
  - GPD approach: assumption of 1 active quark
    - Valid only in certain kinematic range
  - pQCD: recent work indicates 2 active quarks dominate

Rely on data

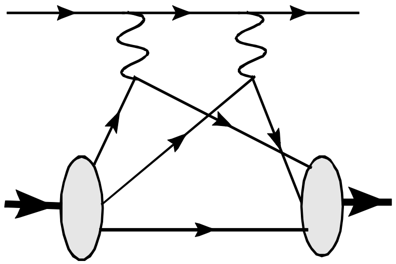
- Use dispersion integrals to relate Real and Imaginary parts. Imaginary parts fixed by cross section data
  - Valid at forward angles: must use models to extrapolate
  - Incomplete: not all data is available (e.g. axial hadron coupling and isospin dependence in γZ diagrams

Partonic (GPD) calculation of two-photon exchange contribution

(Chen et al.)



"handbag"



"cat's ears"

valid at large Q^2 : δ^{hard}

handbag diagrams (one active quark)

to reproduce the IR divergent contribution at nucleon correctly (Low Energy Theorem): δ^{soft}

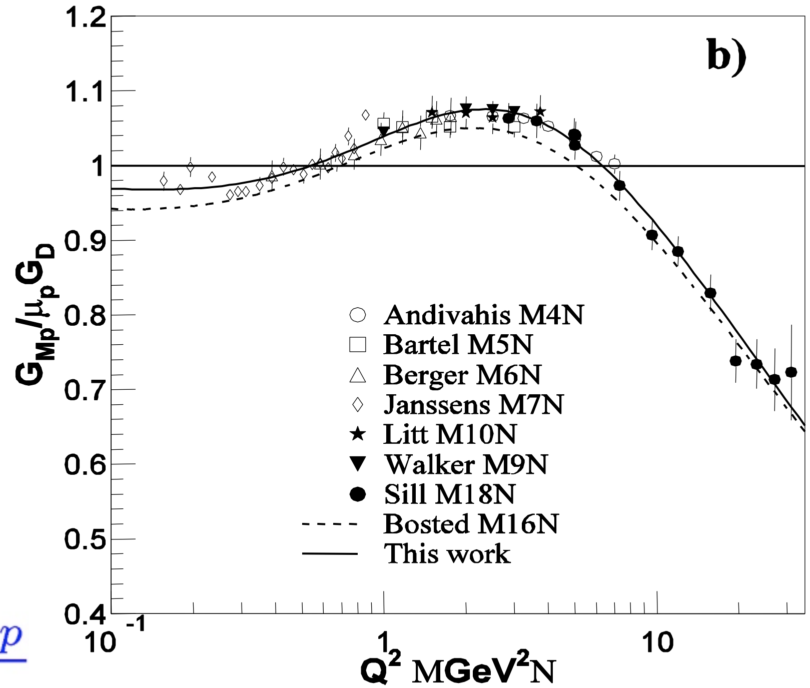
need cat's ears diagrams (two active quarks)

Nucleon elastic contribution (BMT)

Model form factors used as input in calculation

magnetic proton form factor

Brash et al. (2002)



electric proton form factor :

G_E/G_M of proton fixed from polarization data

Gayou et al. (2002)

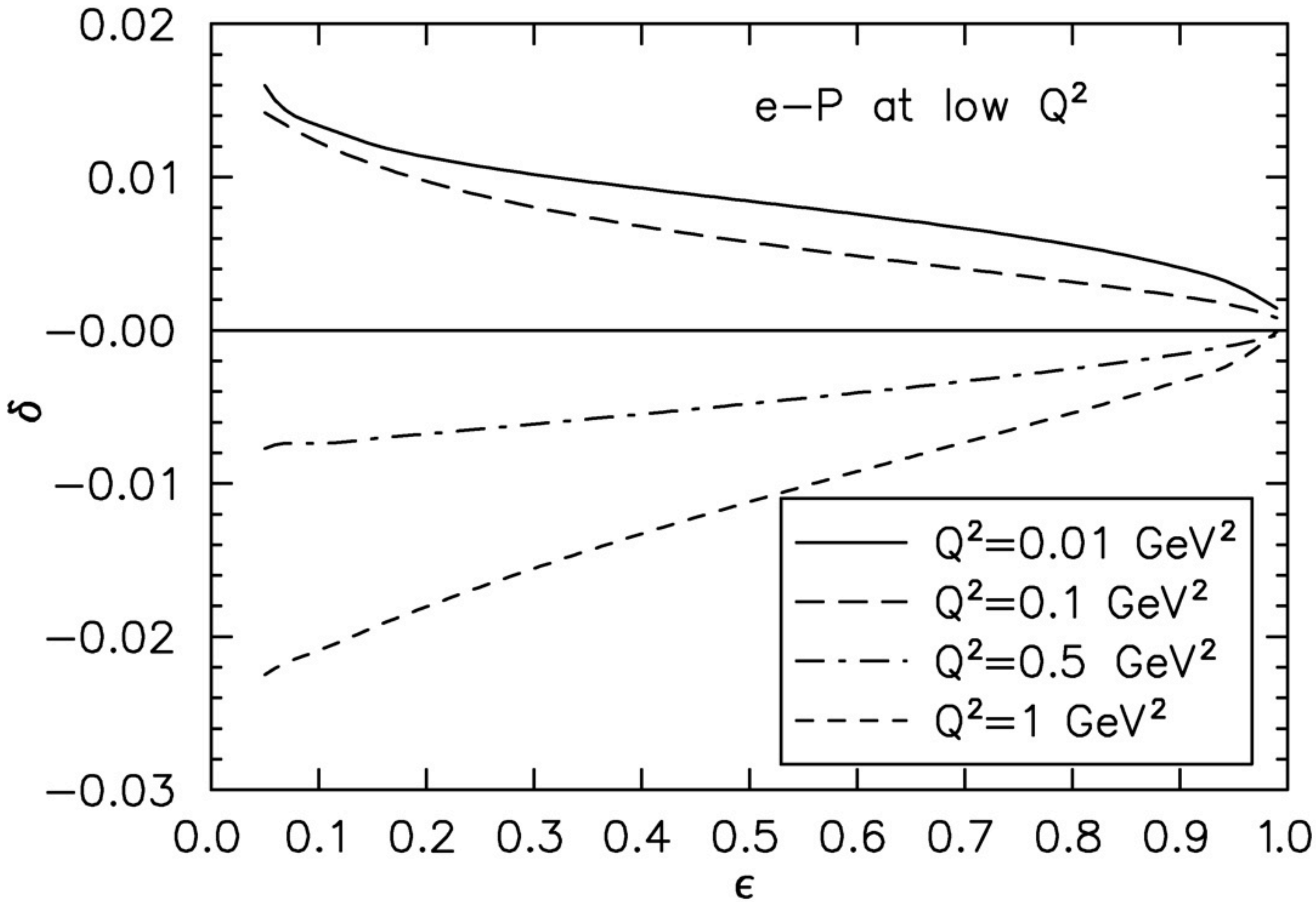
$$G_{E\,p} = \left(1 - 0.13(Q^2 - 0.04)\right)\frac{G_{M\,p}}{\mu_p}$$

Parametrize as sum of monopoles

maintains analytic form of result (Passarino-Veltman functions)

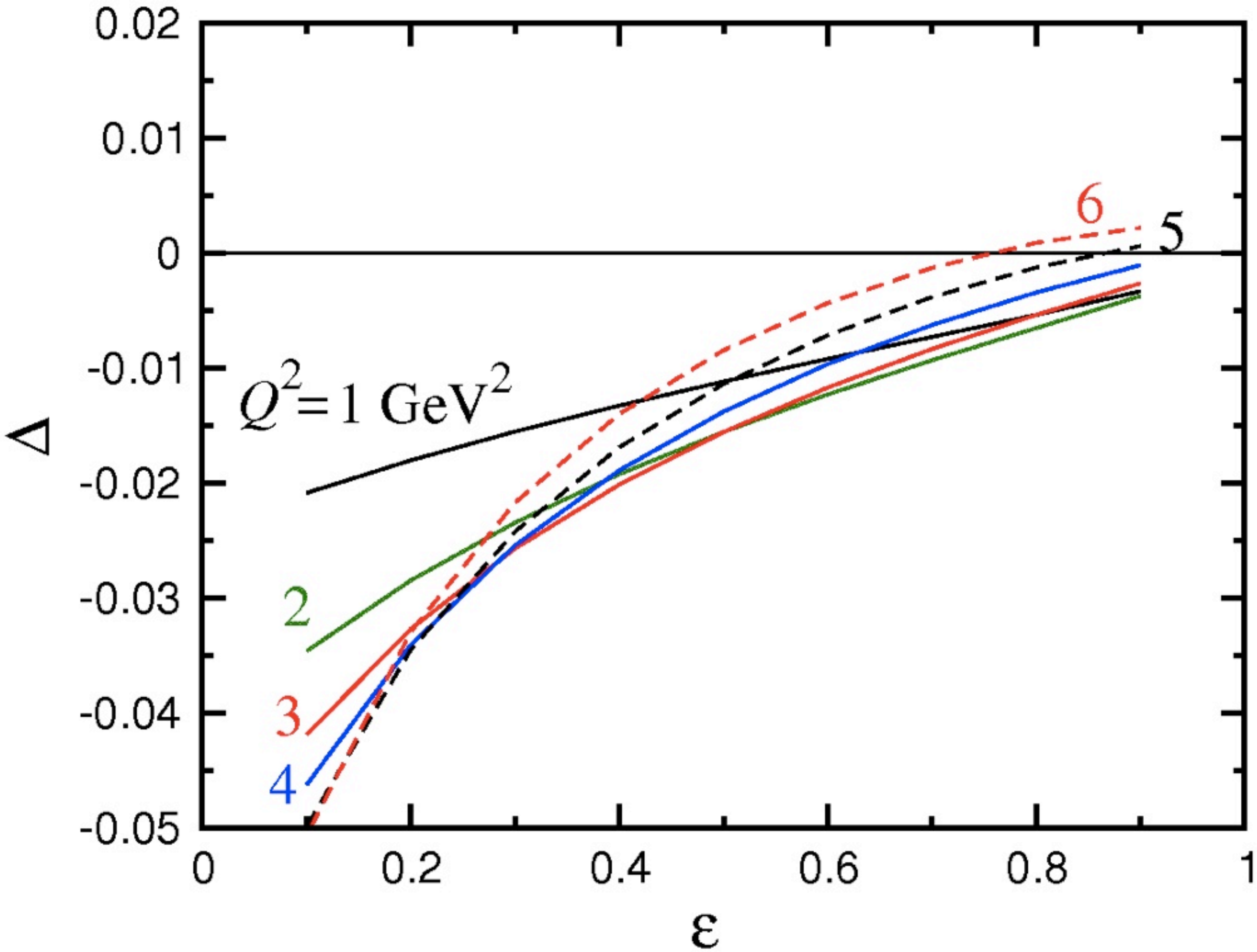
Numerical results not terribly sensitive to model for G_E, or to details of G_M; dipole form factors work well too

proton correction at low Q^2



- Essentially independent of mass (same for muon, free quarks)
- At high Q^2, G_M dominates the loop integral
- At low Q^2, G_E dominates
- neutron correction vanishes at low Q^2 (pointlike neutron)

Corrections to unpolarized cross sections for Q^2=1 to 6 GeV^2



Effect largest at small ε (backward angles)

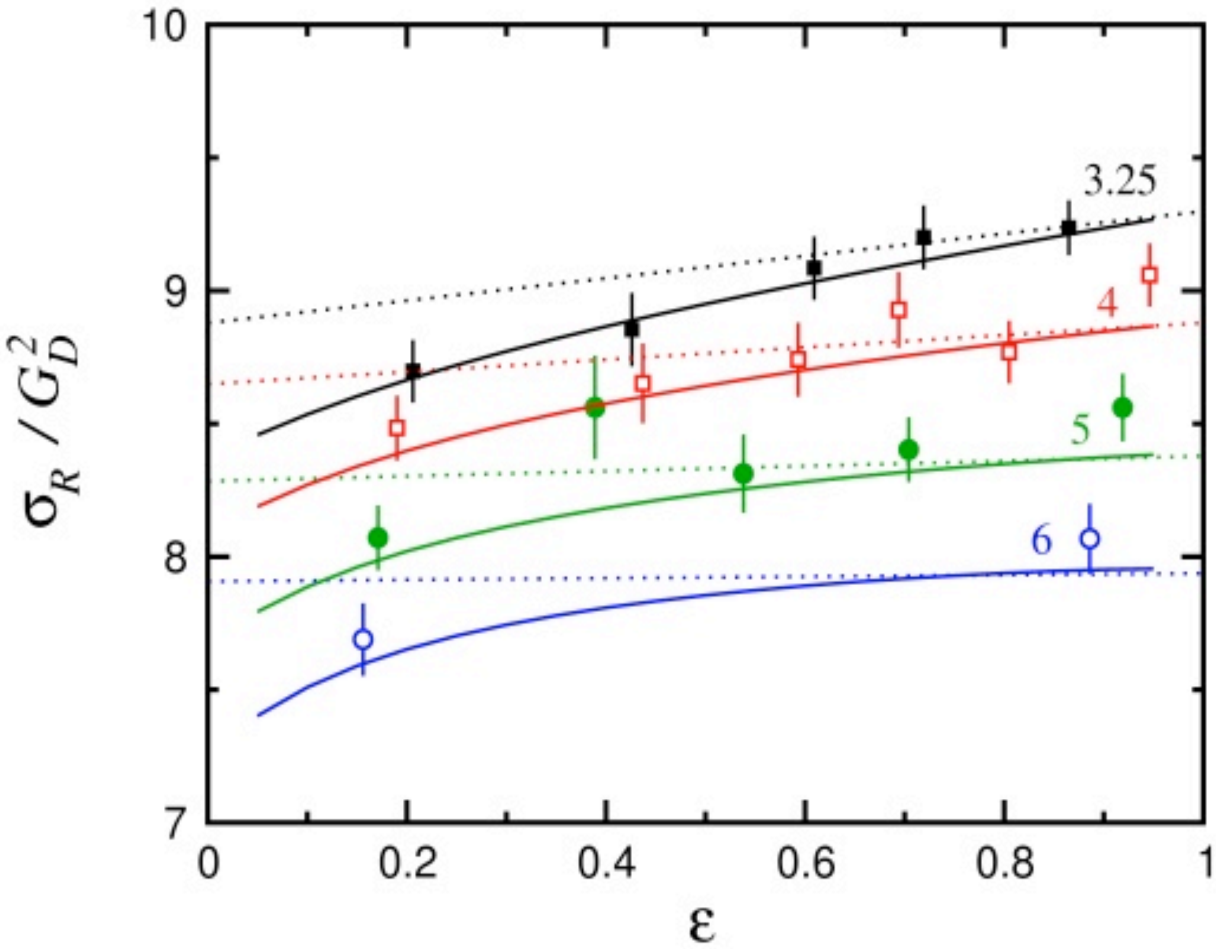
Vanishes as ε ->1

Nonlinearity grows with Q^2

JLAB E05-017 (Arrington) will set limits on nonlinearity

Effect on SLAC reduced cross sections at different Q^2

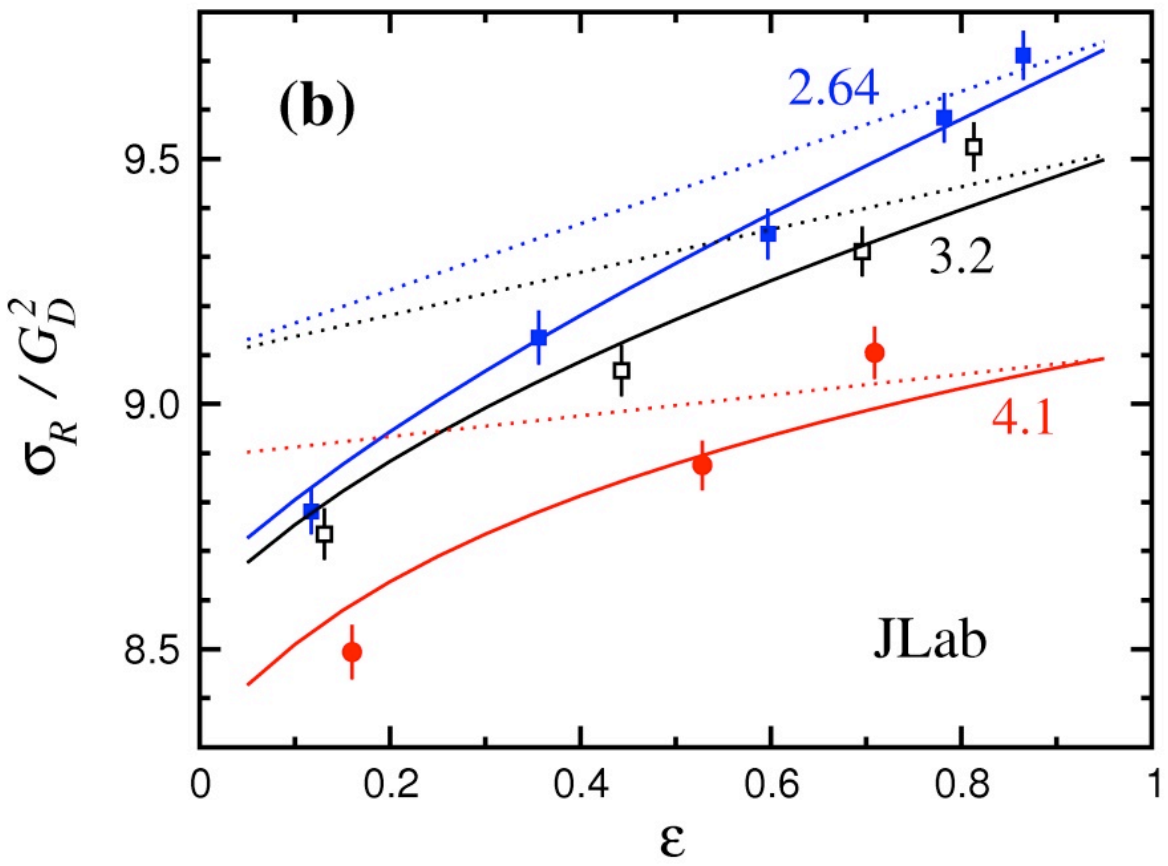
(normalized to dipole $G_D{}^2$)



Nonlinearity in ε is displayed here

JLAB proposals to measure nonlinearity

SuperRosenbluth (JLAB) data



Curves shifted by

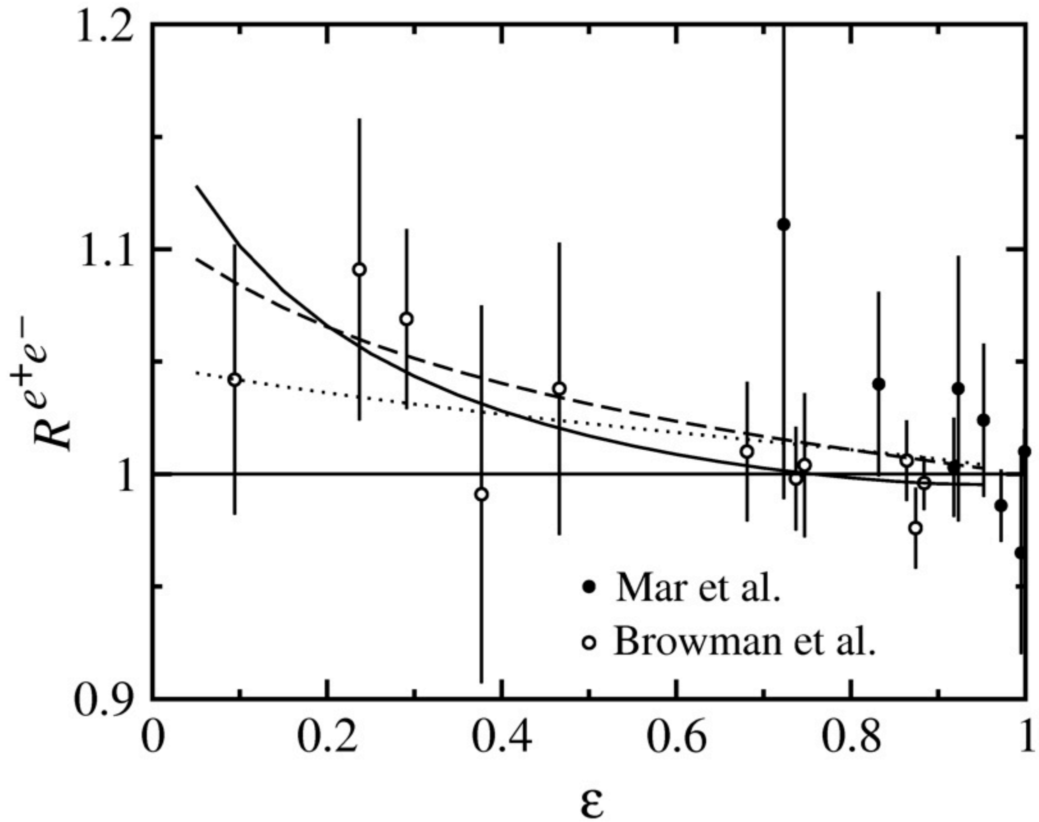
+1.0% 2.64

+2.1% 3.20

+3.0% 4.10

(Effect on determination of G_M)

Effect on ratio of e^+p to e^-p cross sections (SLAC, Q^2 from 0.01 to 5 GeV^2)



M_{Born} opposite sign for e^+p vs. e^-p, so enhancement instead of suppression as $\varepsilon \rightarrow 0$

$R(e^+p/e^-p) = (1-\Delta)/(1+\Delta)$

$= 1-2\Delta$

Curves are elastic results for Q^2=1, 3, 6 GeV^2

Expts.

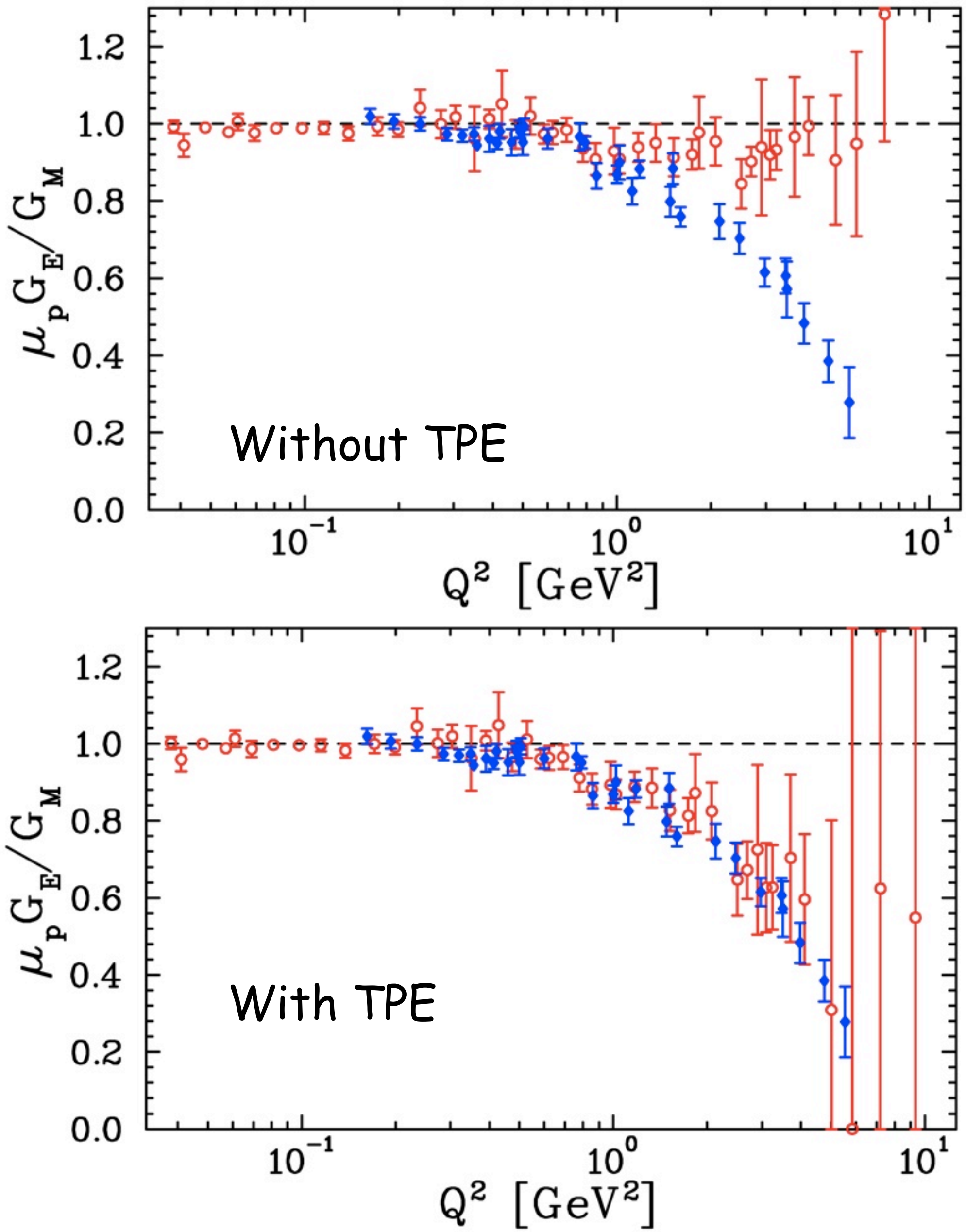
E04-116 $Q^2 < 2$ GeV^2

VEPP-3 Q^2=1.6 GeV^2, ε~0.4

Effect on ratio R

Global Analysis:

(Arrington, Melnitchouk & Tjon, PRC, 2007)



Resonance (Δ) contribution:

$\gamma(q^\alpha) + \Delta(p^\mu) \rightarrow N$

- Lorentz covariant form
- Spin $\frac{1}{2}$ decoupled
- Obeys gauge symmetries

$$
\begin{aligned}
p_\mu \Gamma^{\alpha\mu}(p,q) &= 0 \\
q_\alpha \Gamma^{\alpha\mu}(p,q) &= 0
\end{aligned}
$$

$$
\begin{aligned}
\Gamma^{\alpha\mu}_{\gamma\Delta\rightarrow N}(p,q) = \frac{ieF_\Delta(q^2)}{2M_\Delta^2}\{ & g_1(g^{\alpha\mu}\not{p}\not{q} - p^\alpha\gamma^\mu\not{q} - \gamma^\alpha\gamma^\mu p\cdot q + \gamma^\alpha\not{p}q^\mu) \\
& + g_2\,(p^\alpha q^\mu - g^{\alpha\mu}p\cdot q) \\
+ \; & (g_3/M_\Delta)\left(q^2(p^\alpha\gamma^\mu - g^{\alpha\mu}\not{p}\right) + q^\alpha\,(q^\mu\not{p} - \gamma^\mu p\cdot q)\}\gamma_5 T_3
\end{aligned}
$$

3 coupling constants g_1, g_2, and g_3

At Δ pole:

	g_1	magnetic
	$(g_2\text{-}g_1)$	electric
	g_3	Coulomb

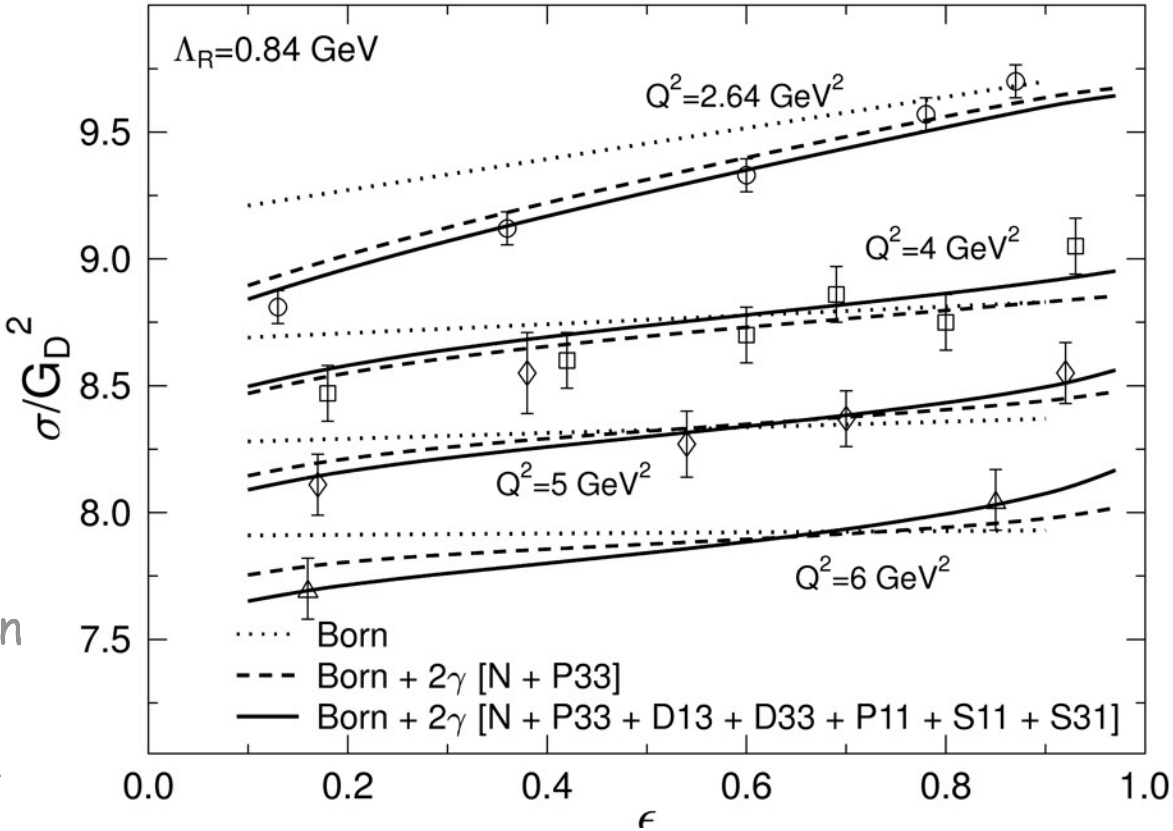
Take dipole FF $F_\Delta(q^2) = 1/(1-q^2/\Lambda_\Delta^{\;2})^2$ with $\Lambda_\Delta = 0.84$ GeV

Other resonances

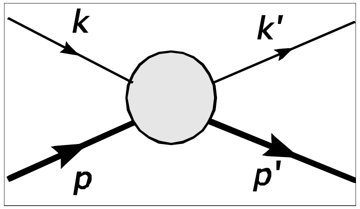
- N (P11), Δ (P33) + D13, D33, P11, S11, S31
- Parameters from dressed K-matrix model

Results

• contribution of heavier resonances much smaller than N and Δ
• D13 next most important (consistent with second resonance shape of Compton scattering cross section)
• partial cancellation between spin 1/2 and spin 3/2
• leads to better agreement, especially at high Q^2



Phenomenology: Generalized form factors



$$P \equiv \frac{p+p'}{2}, \qquad K \equiv \frac{k+k'}{2}$$

Kinematical invariants :

$$\begin{aligned} q^2 &= (p'-p)^2 \equiv -Q^2 \\ \nu &= K \cdot P = p \cdot k + q^2/4 \end{aligned}$$

In limit $m_e \to 0$ (helicity conservation) general amplitude can be put in form

$$T = (\gamma_\mu)^{(e)} \otimes \left(\tilde{F}_1 \gamma^\mu + i \frac{\tilde{F}_2}{2M} \sigma^{\mu\nu} q_\nu + \frac{F_3}{M^2} \gamma \cdot K P^\mu \right)^{(p)}$$

In general, 16 independent amplitudes:

parity 16 → 8; time reversal 8 → 6; helicity conservation (m_e=0) 6 → 3

Generalized (complex) form factors

$$\begin{aligned} \tilde{F}_1(\nu, Q^2) &= F_1(Q^2) + \delta F_1 \\ \tilde{F}_2(\nu, Q^2) &= F_2(Q^2) + \delta F_2 \end{aligned}$$

$$\begin{aligned} \tilde{G}_M &= \tilde{F}_1 + \tilde{F}_2 \\ \tilde{G}_E &= \tilde{F}_1 - \tau \tilde{F}_2 \\ Y_2 &= \frac{\nu}{M^2} \frac{F_3}{G_M} \end{aligned}$$

Observables including two-photon exchange

$$\frac{\delta\sigma}{\sigma_0} = 2\frac{\left\{\epsilon\left(\frac{\delta G_E}{G_E}\right)G_E^2 + \tau\left(\frac{\delta G_M}{G_M}\right)G_M^2 + \epsilon Y_2(\tau G_M^2 + G_M G_E)\right\}}{\epsilon G_E^2 + \tau G_M^2}$$

$$\frac{\delta P_L}{P_L} = 2\left(\frac{\delta G_M}{G_M}\right) + 2\frac{\epsilon}{1+\epsilon}Y_2 - \frac{\delta\sigma}{\sigma_0}$$

$$\frac{\delta P_T}{P_T} = \left(\frac{\delta G_M}{G_M}\right) + \left(\frac{\delta G_E}{G_E}\right) + \frac{G_M}{G_E}Y_2 - \frac{\delta\sigma}{\sigma_0}$$

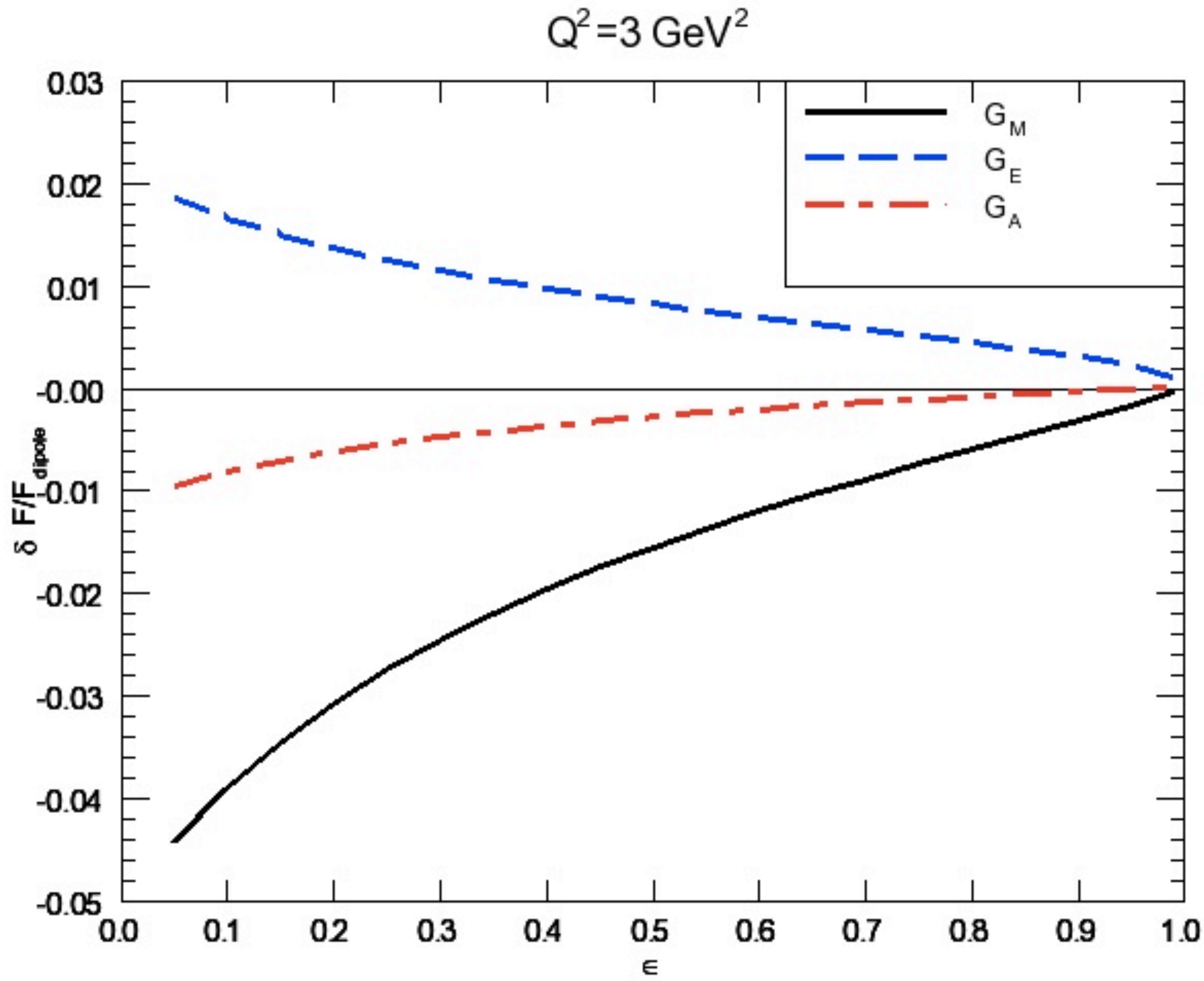
Caution needed about assumptions (generalized FF's are not observables)

- Parametrization of amplitude NOT unique

  Axial parametrization: G_A' $(\gamma_\mu\gamma_5)^{(e)}$ $(\gamma^\mu\gamma_5)^{(p)}$ instead of F_3 (or Y_2) term

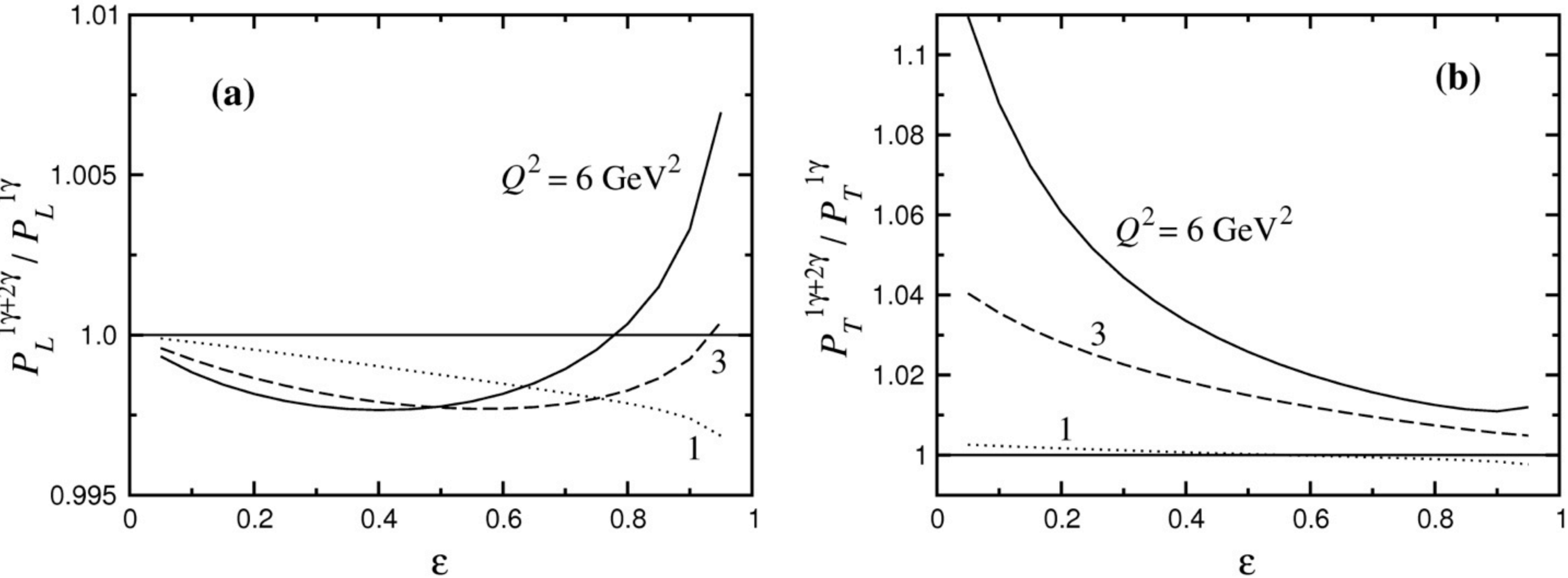
  shifts some F_3 into δF_1 (and hence into δG_E and δG_M)

Real part of elastic results



$$\vec{e} + p \to e + \vec{p}$$

Corrections to P_L and P_T at Q^2=1, 3, and 6 GeV^2



P_T/P_L will show some variation with ε, esp. at low ε

JLab data taken at ε~0.7

JLAB expt (Gilman) measures P_T/P_L at low ε

GPD calculation predicts suppression of P_T/P_L

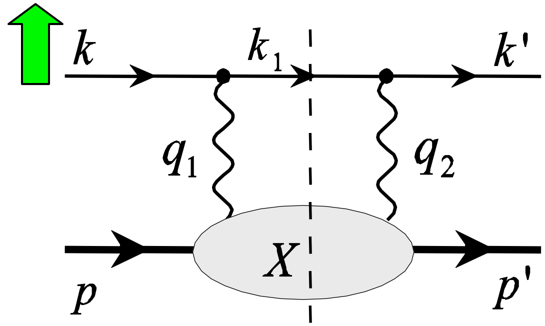
SSA in elastic eN scattering

spin of beam OR target

OR recoil proton

NORMAL to scattering

plane



$s = (k+p)^2$

on-shell intermediate state (M_X = W)

involves the imaginary part of two-photon exchange amplitudes

Target: general formula of order e^2

- GPD model allows connection of real and imaginary amplitudes
- Hadronic models sensitive to intermediate state contributions, no reliable theoretical calculations at present

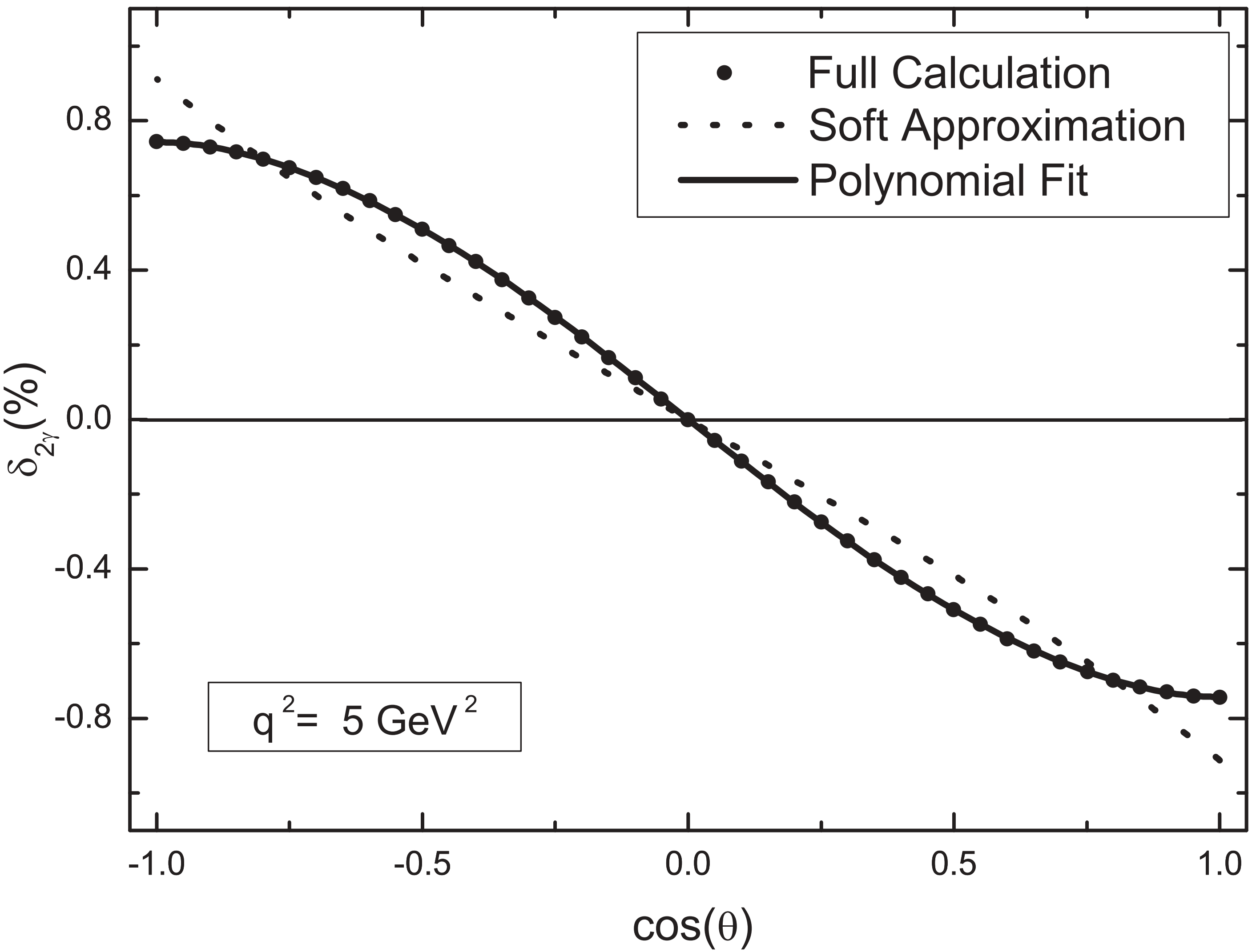
Beam: general formula of order $m_e\ e^2$ (few ppm)

- Measured in PV experiments (longitudinally polarized electrons) at SAMPLE and A4 (Mainz)
- Only non-zero result so far for TPEX

TPE contribution to proton FF's in time-like region:

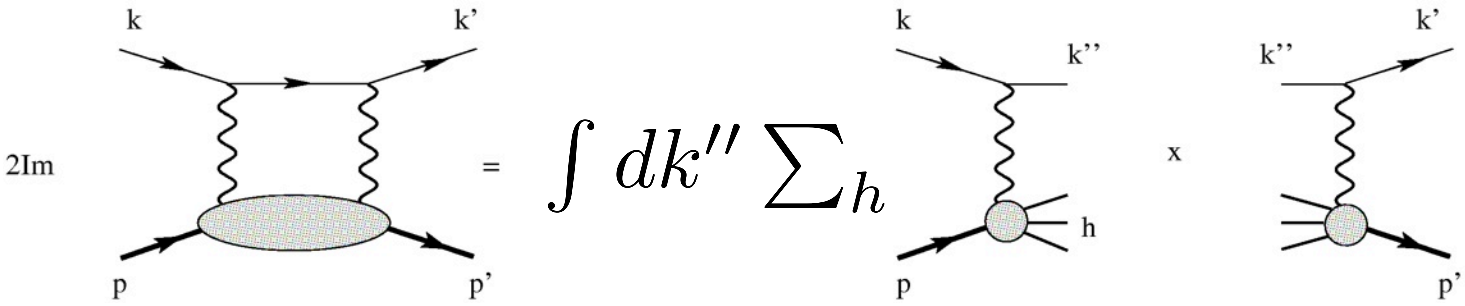
$$e^+ + e^- \rightarrow p + \bar{p}$$

Chen, Zhou & Dong, PRC **78** (2008)



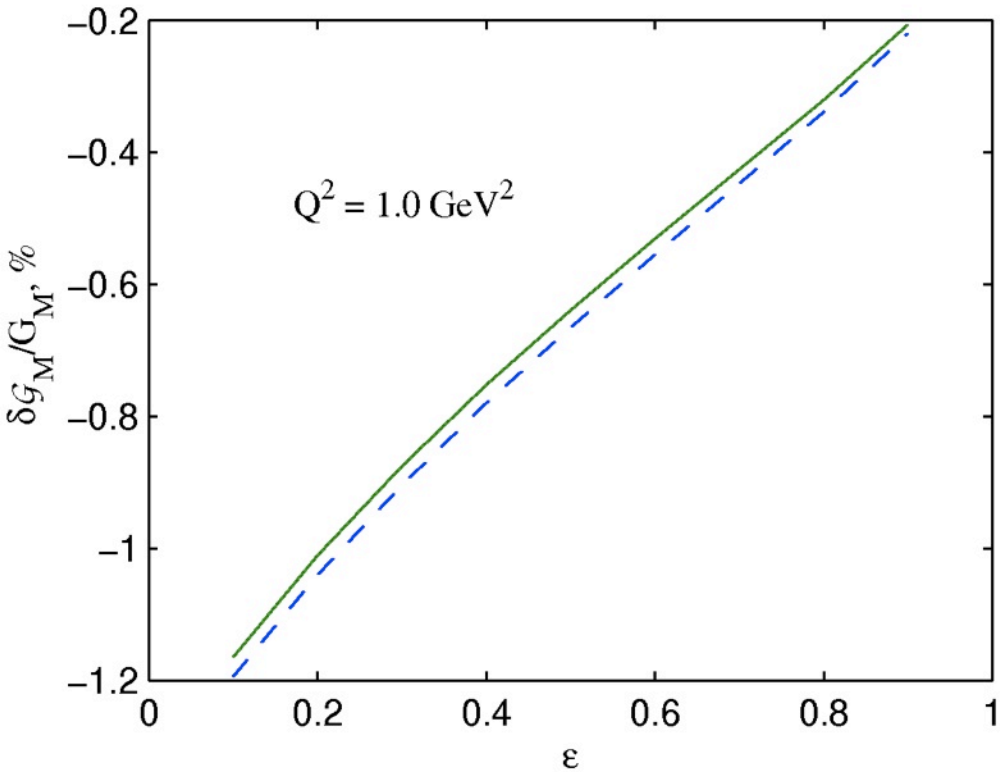
TPEX using dispersion relations

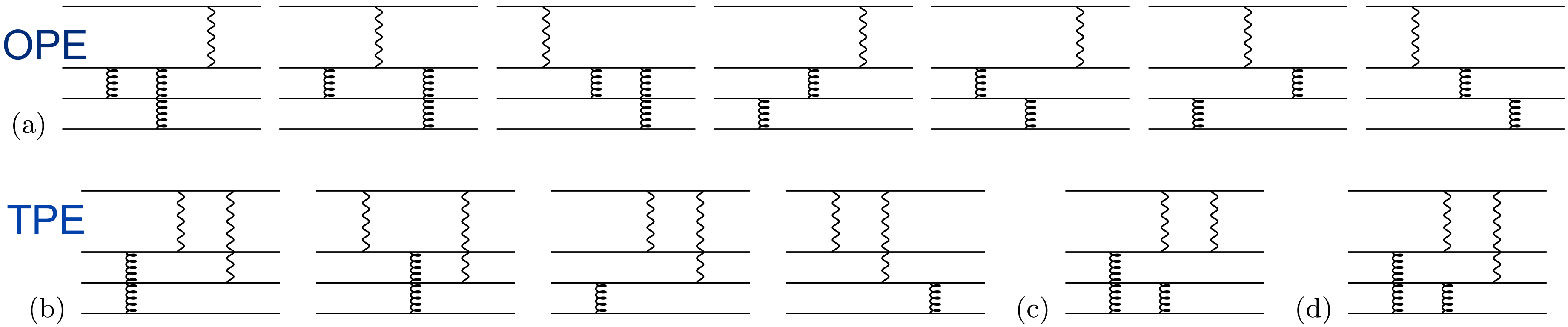
(Borisyuk & Kobushkin, PRC **78**, 2008)



- Imaginary part determined by unitarity
- Only on-shell form factors
- Real part determined from dispersion relations
- Numerical differences between naive (solid) and dispersion (dashed) analyses are small

- Similar insensitivity seen for Δ (Tjon, Blunden, Melnitchouk)





Recent pQCD calculation: Borisyuk & Kobushkin, PRD **79**, 2009

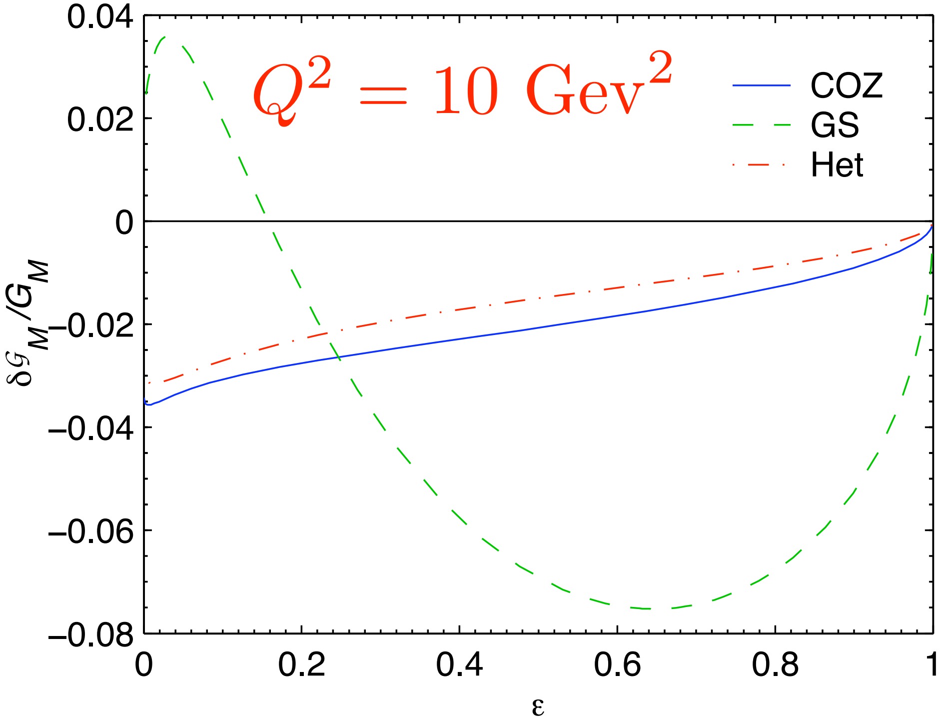
(a) one-photon exchange: need 2 hard gluons to turn momentum of all 3 quarks

$$\alpha\alpha_s^2/Q^6$$

(b) two-photon exchange:
leading order needs 1 hard gluon

$$\alpha^2\alpha_s/Q^6 \quad \text{TPE/OPE} \sim \alpha/\alpha^s$$

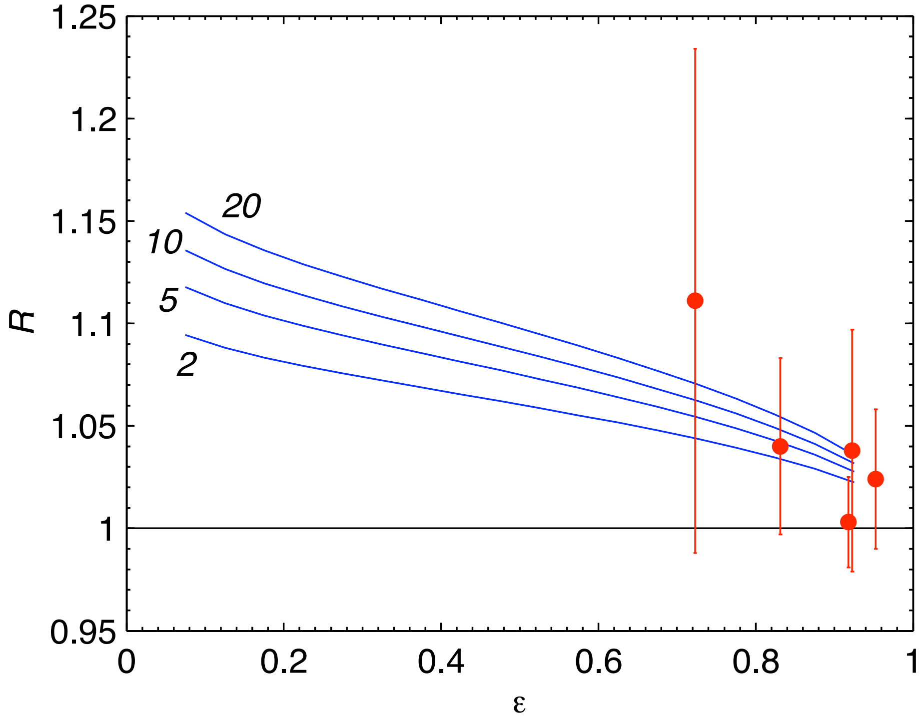
subleading order (both photons on one quark) requires 2 hard gluons

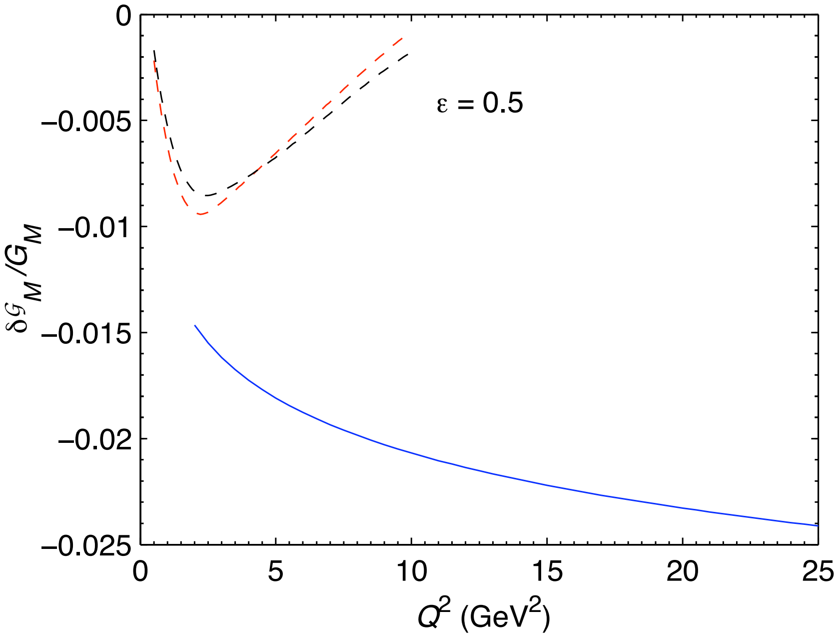


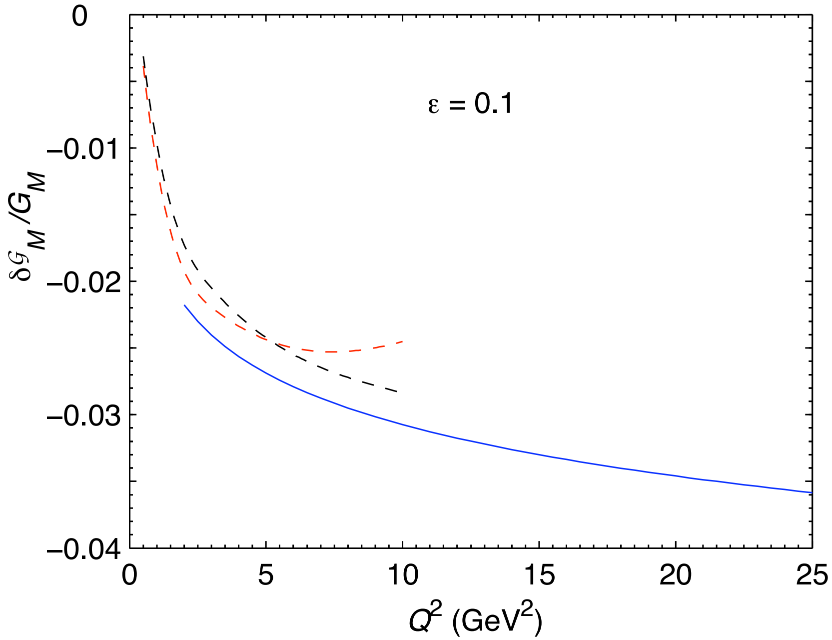
Contribution to *ep* for different quark wavefunctions

Approx linear in ε

$$e^+e^-\text{ratio}$$





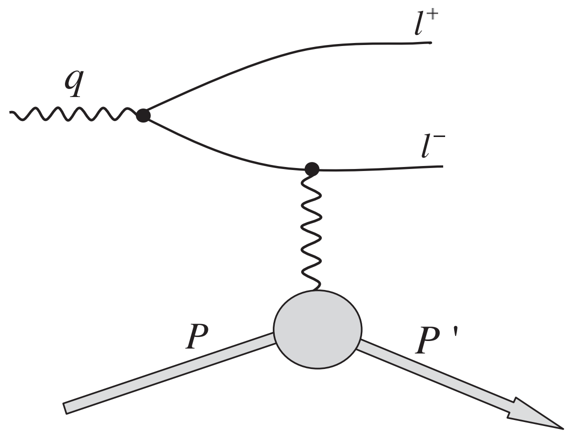


Comparison of hadronic and pQCD results

Connect smoothly around $Q^2 = 3\ \text{GeV}^2$

Lepton-antilepton photoproduction using real photons

(Pervez Hoodbhoy, PRD 2006)



TWO-PHOTON EFFECTS IN LEPTON-ANTILEPTON . . .

PHYSICAL REVIEW D **73,** 054027 (2006)

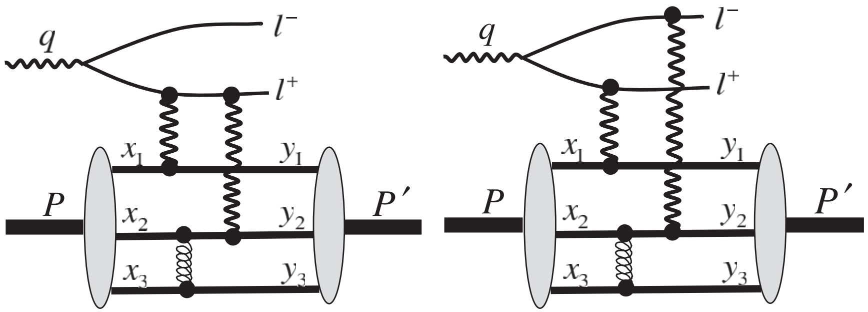


FIG. 6. Typical diagrams for lepton pair production from a 3-quark proton.

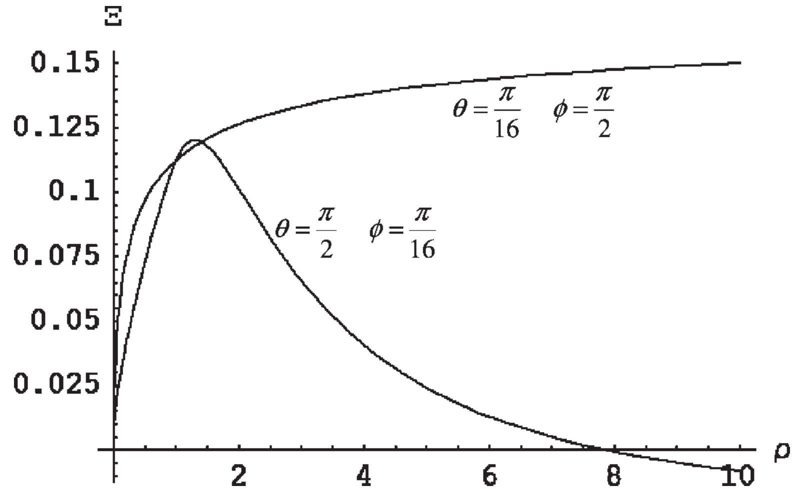


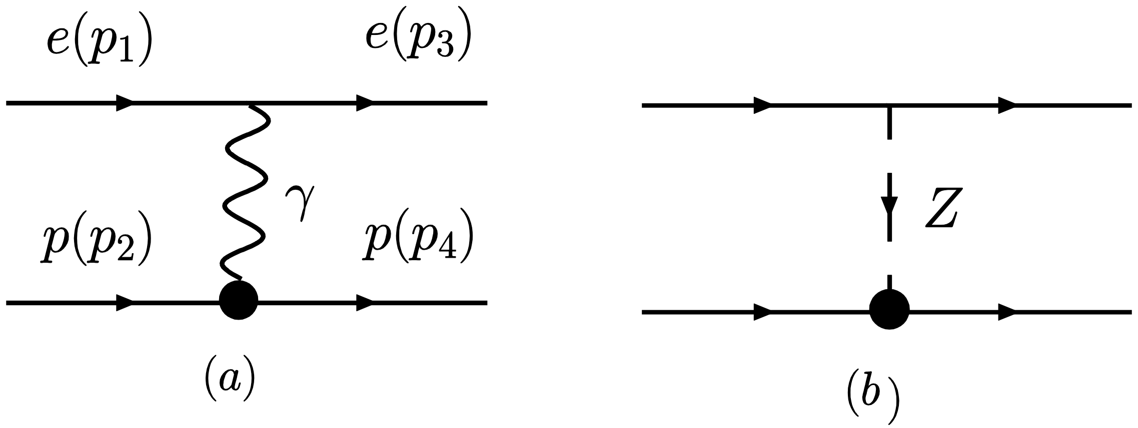
FIG. 7. Lepton pair asymmetry from a proton target.

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\, p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

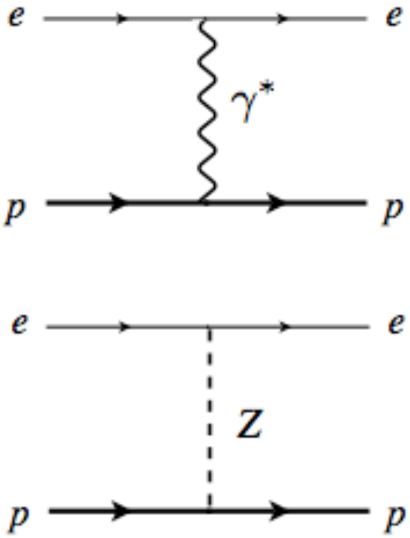
$\longrightarrow$ measure interference between e.m. and weak currents



$$\begin{aligned} j^Z_\mu &= \bar{u}\,(g^e_V \gamma_\mu + g^e_A \gamma_\mu \gamma_5)\, u \\ J^\mu_Z(q) &= \bar{U}\left(F^Z_1 \gamma^\mu + F^Z_2 i \frac{\sigma^{\mu\nu} q_\nu}{2M} + G^Z_A \gamma^\mu \gamma_5\right) U \end{aligned}$$

$$\begin{aligned} g^e_V &= -(1 - 4 s^2_W) \\ g^e_A &= +1 \\ F^Z_i &= (1 - 4 s^2_W) F^p_i - F^n_i - F^s_i \\ G^Z_A &= -G_A \tau_3 + G^s_A \end{aligned}$$

Two-boson exchange corrections



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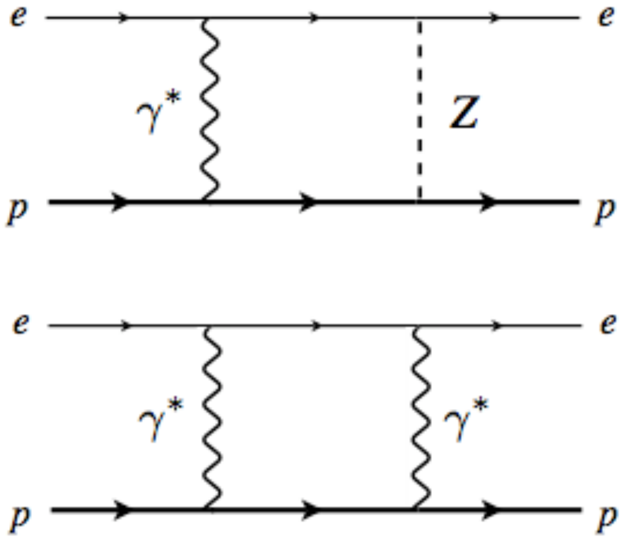


"$\gamma(Z\gamma)$"

"$Z(\gamma\gamma)$"

- current PDG estimates (of "$\gamma(Z\gamma)$") computed at $Q^2 = 0$

Marciano, Sirlin (1980)

Erler, Ramsey-Musolf (2003)

Zhou, Kao & Yang, PRL 2007; Tjon & Melnitchouk, PRL 2008; Tjon, Melnitchouk & Blunden, PRC 2009

Marciano-Sirlin (PV in atoms)

$$H = \frac{G_F}{2\sqrt{2}}\left(C_1^p \bar{u}_e \gamma_\mu \gamma_5 u_e \bar{U}_p \gamma^\mu U \quad + \quad C_2^p \bar{u}_e \gamma_\mu u_e \bar{U}_p \gamma^\mu \gamma_5 U_p\right)$$

$$C_{1p} = \frac{1}{2}\rho(1 - 4\kappa s_w^2) + \frac{5}{2}\Delta$$

$$= \frac{1}{2}\rho'(1 - 4\kappa' s_w^2)$$

Perturbative (free quark result) $\Delta_{\mathrm{quark}} = \frac{\alpha}{2\pi}(1 - 4s^2)\left(\ln\frac{M_Z^2}{\mu^2} + \frac{3}{2}\right)$

Nonperturbative $\Delta = \frac{\alpha}{2\pi}(1 - 4s^2)\left(K + \frac{4}{5}(\xi_1)_B^p\right)$

$$K = M_Z^2 \int_{\mu^2}^{\infty} \frac{du}{u(u + M_Z^2)}\left[1 - \frac{\alpha_s(u)}{\pi}\right]$$

$K = 8.58$ for $\mu = 1$ GeV, and $(\xi_1)_B^p = 2.55$ using dipole proton form factors, showing that the quark contribution dominates.

Effect on Parity-violating asymmetry in elastic e+p

$$\mathcal{A}_{PV} = \frac{2\Re\left\{M_\gamma^\dagger M_Z\right\}}{|M_\gamma|^2}$$

Electromagnetic radiative corrections interfere with M_Z ($M_\gamma \to M_\gamma + M_{\gamma\gamma}$)

plus weak radiative corrections interfere with M_γ ($M_Z \to M_Z + M_{\gamma Z}$)

Afanasev and Carlson (PRL 2005) used generalized form factors to analyze effect of $\gamma\gamma$ on A (GPD model)

$$\begin{aligned}\mathcal{A}_{PV} &= -\frac{G_F Q^2}{e^2\sqrt{2}} \times \\ &\frac{g_A^e\left(\epsilon G_E G_E^Z + \tau G_M G_M^Z + (\epsilon G_E^Z \delta G'_E + \tau G_M^Z \delta G'_M + \epsilon' G_M^Z G'_A)\right) + g_V^e G_A^Z\left(\epsilon' G_M + (1+\tau) G'_A\right)}{\epsilon G_E^2 + \tau G_M^2 + 2(\epsilon G_E \delta G'_E + \tau G_M \delta G'_M + \epsilon' G_M G'_A)} \\ &= -\frac{G_F Q^2}{e^2\sqrt{2}} (A_V + A_A)\end{aligned}$$

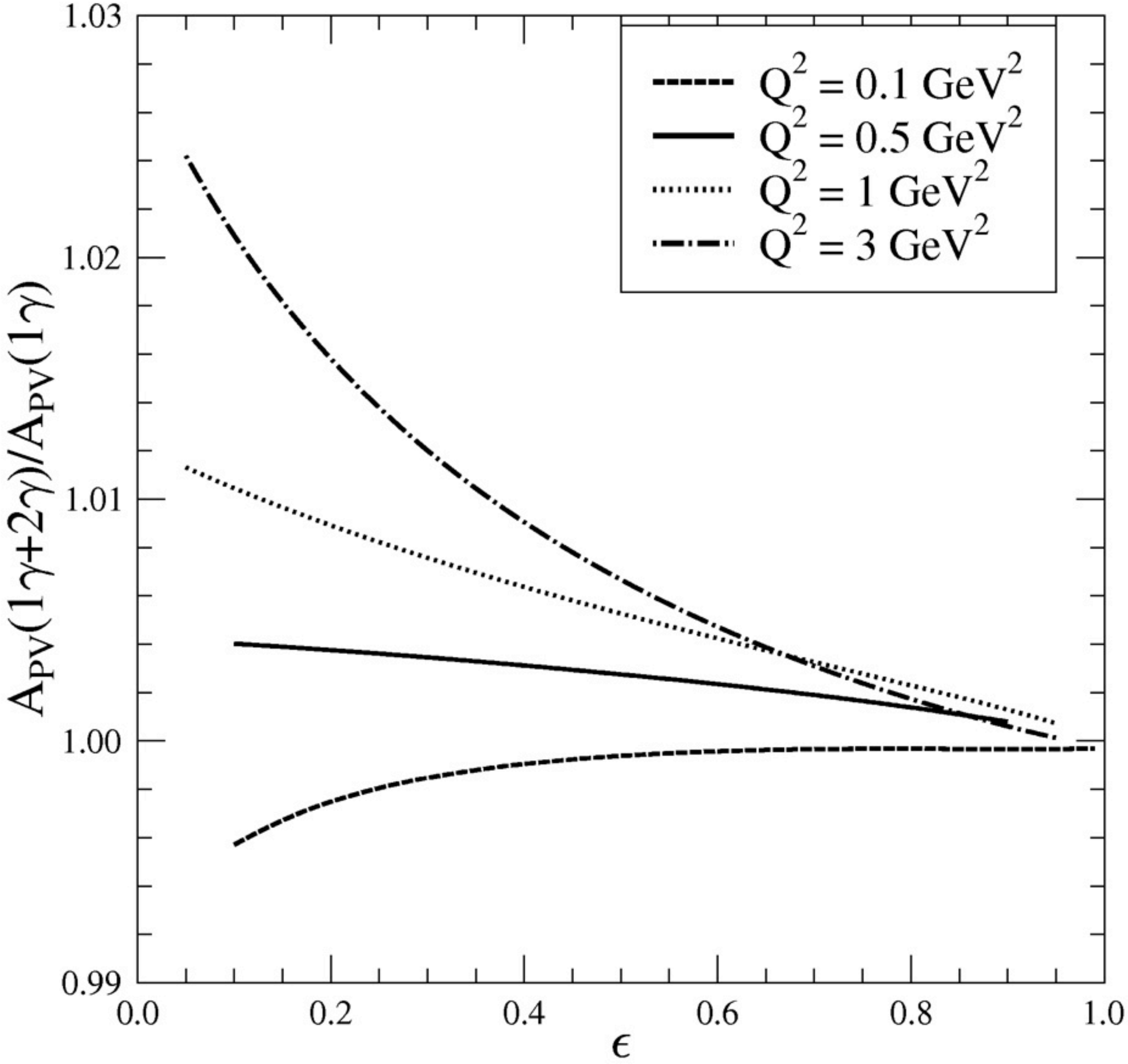
$$G_{E,M}^Z = (1 - 4s_W^2) G_{E,M}^p - G_{E,M}^n$$

Equivalently,

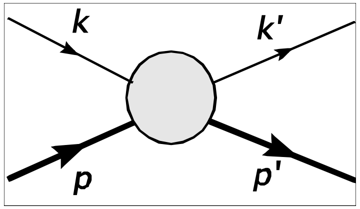
$$G_{E,M} \to G'_{E,M} = G_{E,M} + \delta G'_{E,M}$$

Therefore, $A_V = (1-4s_W^2) + \ldots$

A_{PV} vs. ε for Q^2 = 0.1, 0.5, 1.0, 3.0 GeV^2 (TPE only)



Phenomenology: Generalized form factors



$$T = (\gamma_\mu \gamma_5)^{(e)} \otimes \left(\tilde{F}_1 \gamma^\mu + i \tilde{F}_2 \frac{\sigma^{\mu\nu} q_\nu}{2M} \right)^{(p)}$$
$$+ \quad (\gamma_\mu)^{(e)} \otimes \left(\tilde{G}_A \gamma^\mu \gamma_5 \right)^{(p)}$$

In general, 16 independent amplitudes:

parity NC 16 $\rightarrow$ 8; time reversal 8 $\rightarrow$ 6; helicity conservation (m_e=0) 6 $\rightarrow$ 3

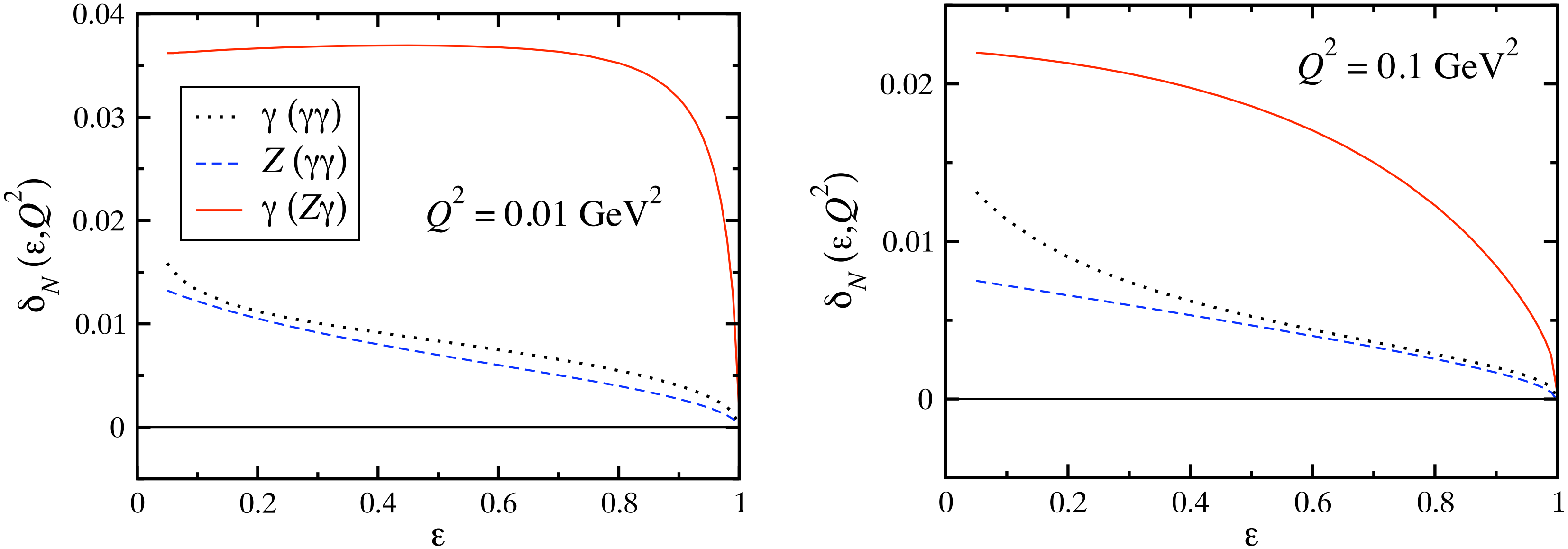
Generalized (complex) form factors

$$\begin{aligned}
\tilde{F}_1(\nu, Q^2) &= F_1^Z + \delta F_1 \\
\tilde{F}_2(\nu, Q^2) &= F_2^Z + \delta F_2 \\
\tilde{G}_A(\nu, Q^2) &= G_A^Z + \delta G_A
\end{aligned}$$

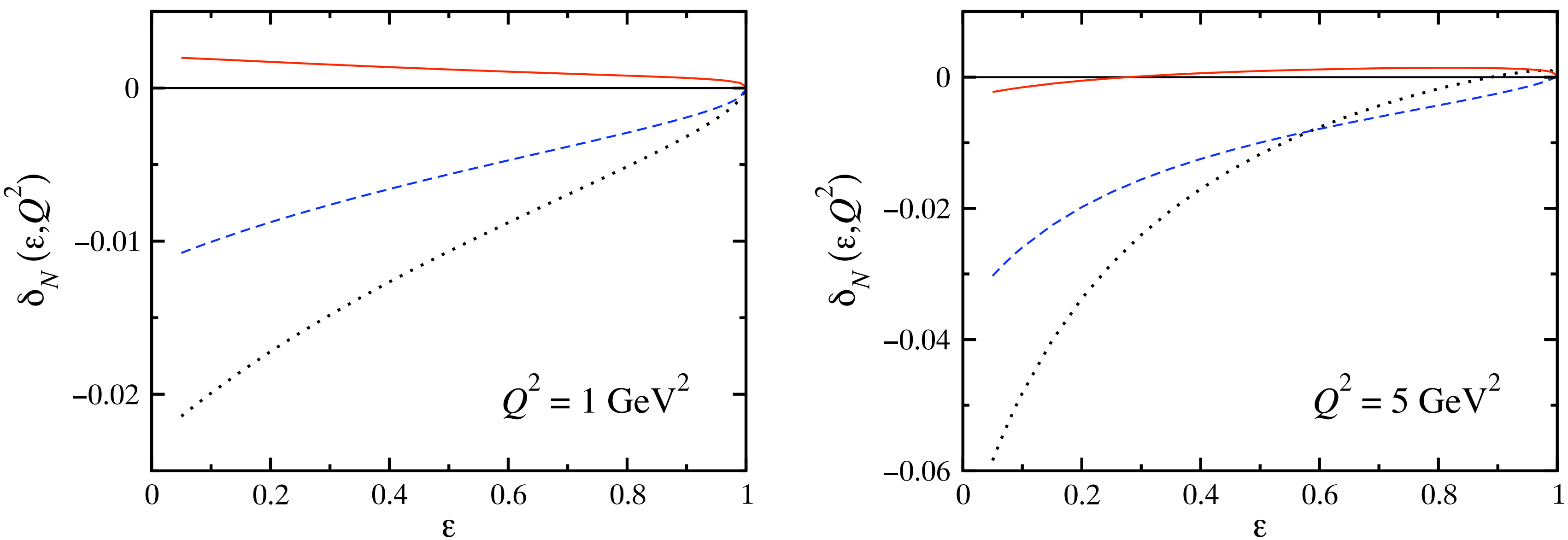
At $Q^2 = 0$ only 2 needed: related to C_1^p and C_2^p of Marciano-Sirlin

No new terms arise in Afanasev-Carlson expression

Tjon, Blunden & Melnitchouk, PRC (2009)



$$\delta \approx \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}$$



Delta resonance contribution

Vector coupling

CVC and isospin symmetry relate $\gamma N \Delta$ to $Z N \Delta$ form factors

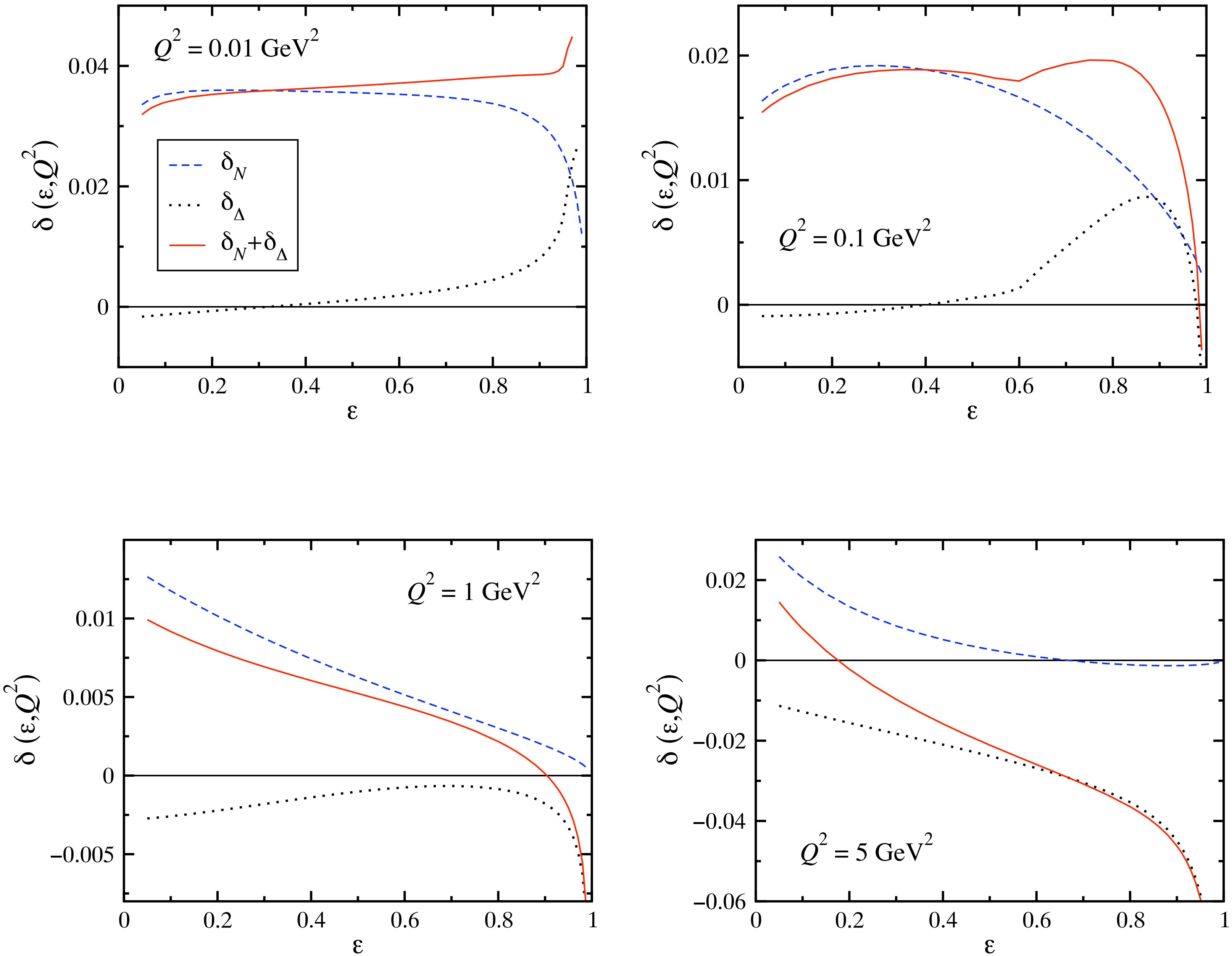
$$g_i^V = 2(1 - 2s_w^2)g_i$$

For N: $g_i^Z = 2(1 - 2s_w^2)g_i^{(1)} - 2s_w^2 g_i^{(0)} = (1 - 4s_w^2)g_i^p - g_i^n$

Axial vector coupling

Take from neutrino scattering parametrization of Lalakulich & Paschos

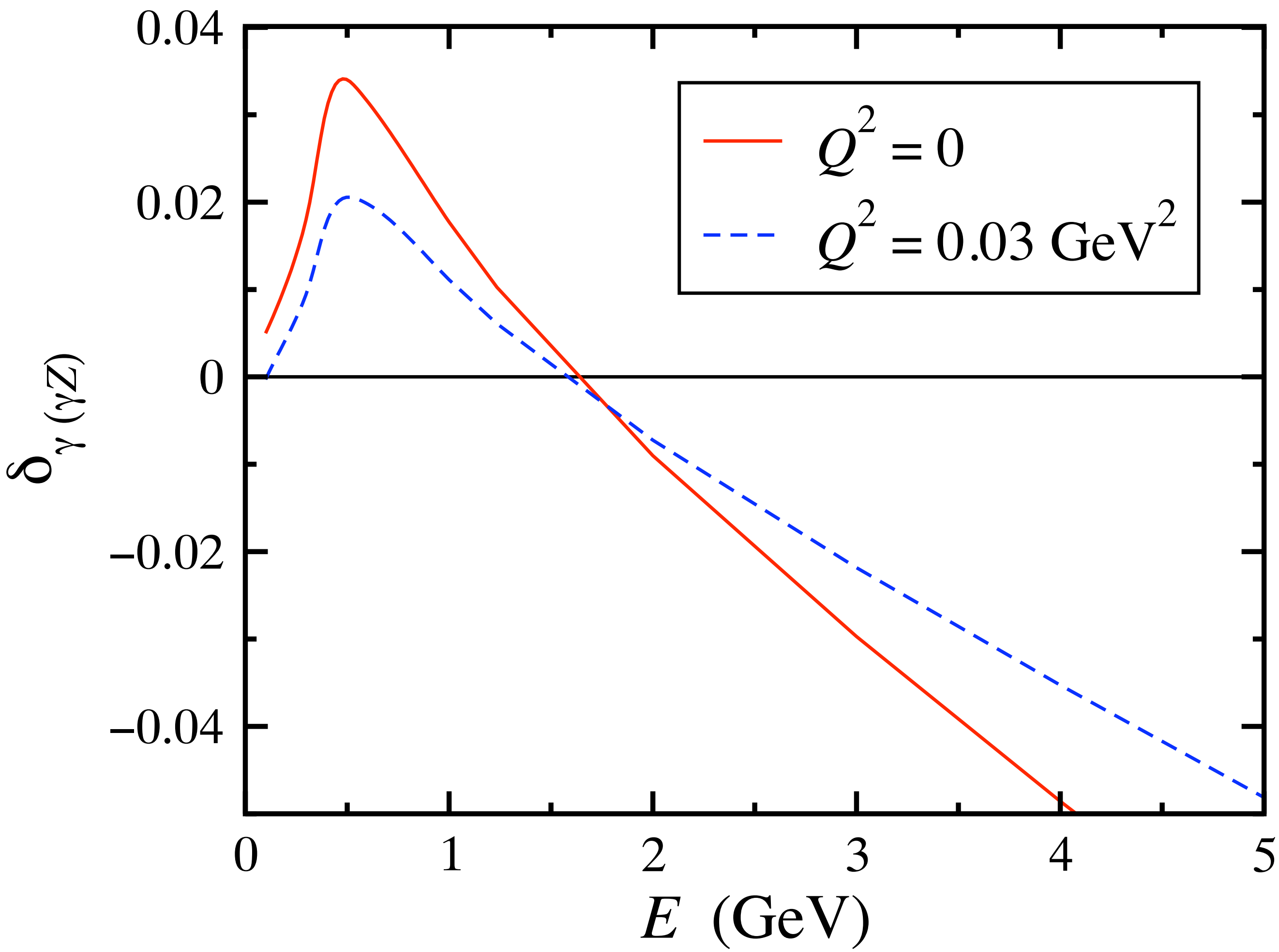
Nucleon and Delta contribution



$\delta_{\gamma(\gamma Z)}$ Δ contribution enhanced at forward angles and low Q^2

enhancement: $g_A^e 2(1-2s_w^2)/(1-4s_w^2) = (1+Q_w^p)/Q_w^p \approx 14$

+ energy dependence (this correction vanishes at E=0, not in Marciano-Sirlin)



γZ contribution to Q_{weak} using dispersion relations

(Gorchtein & Horowitz, PRL 2009)

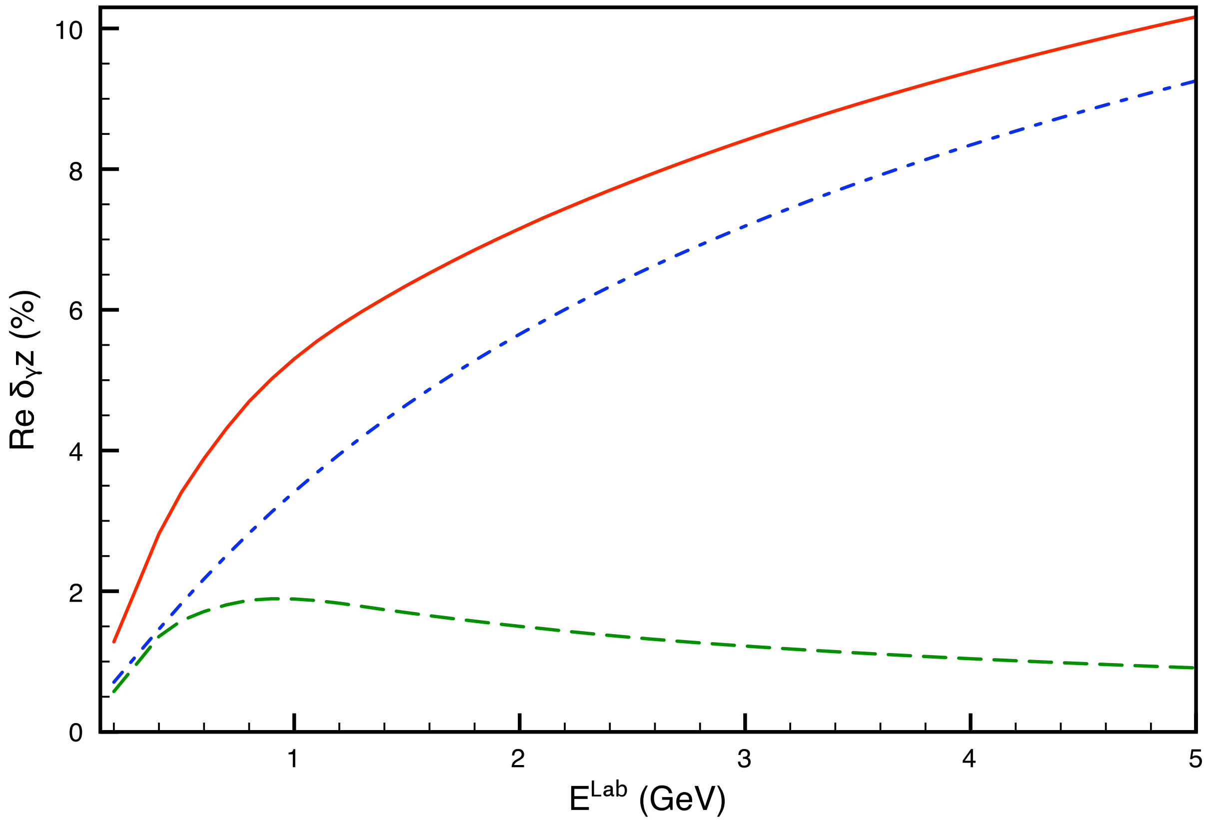


FIG. 3: Results for $\mathrm{Re}\delta_{\gamma Z_A}$ as function of energy. The contributions of nucleon resonances (dashed line), Regge (dash-dotted line) and the sum of the two (solid line) are shown.

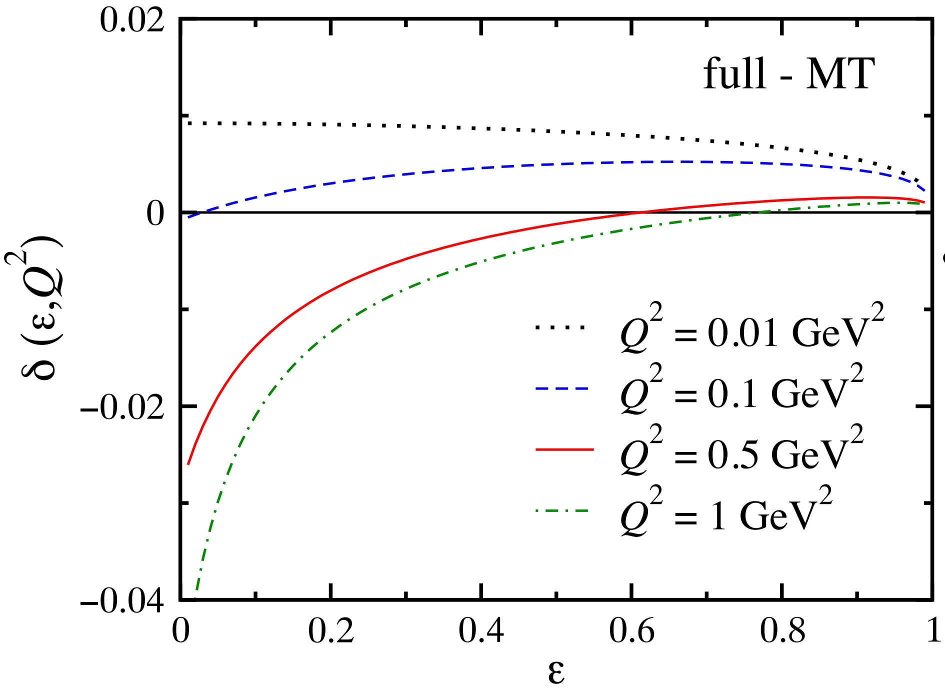
TPE effect on Pion Form Factor: BMT 2009

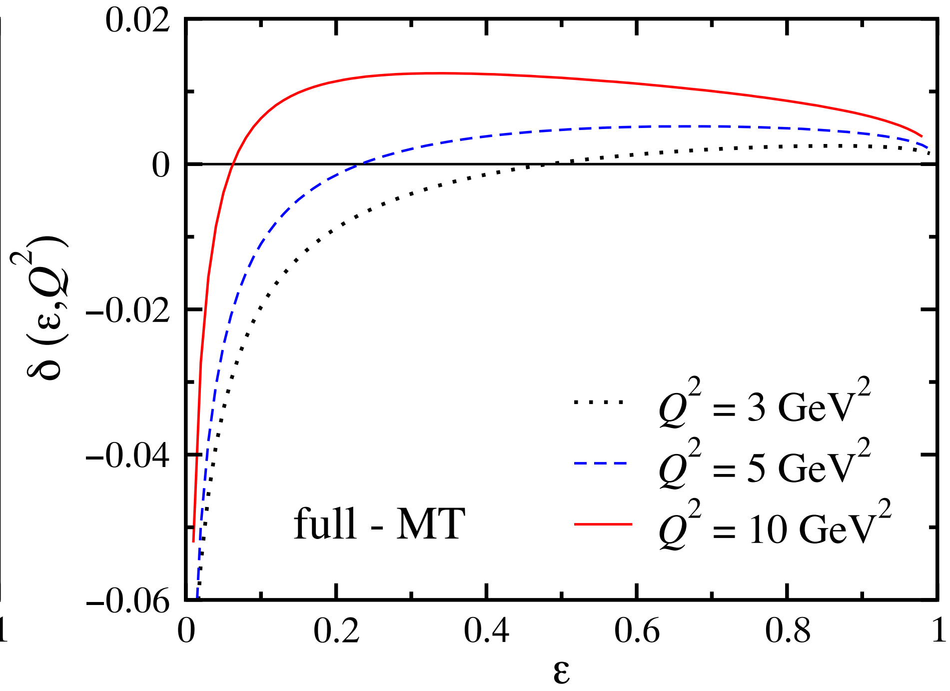
$$\frac{d\sigma}{d\Omega} = \sigma_{\rm Mott} F_\pi^2(q^2)$$

TPE: $F_\pi(q^2) \to F_\pi(q^2)(1+\delta)$

Current is: $\langle \pi(p')|J^\mu|\pi(p)\rangle = (p'+p)^\mu F_\pi(q^2)$

Form factor in loop: used both VMD (ρ meson), and VMD + pQCD





Outlook

- Use phenomenological form factors in analyzing data, extracting strange form factors, etc.
- Merge hadronic models with GPD or pQCD calculations for $\gamma\gamma$ and γZ?
- Recent work on TPE seems to indicate insensitivity to off-shell form factors
- Dispersion relations that use cross section data are useful at forward angles, however still need for models to extrapolate (not all data is available, e.g. γZ interference, axial part)

Collaborators: Melnitchouk, Tjon + Kondratyuk