Nucleon-Nucleon Interactions from the Quark Model

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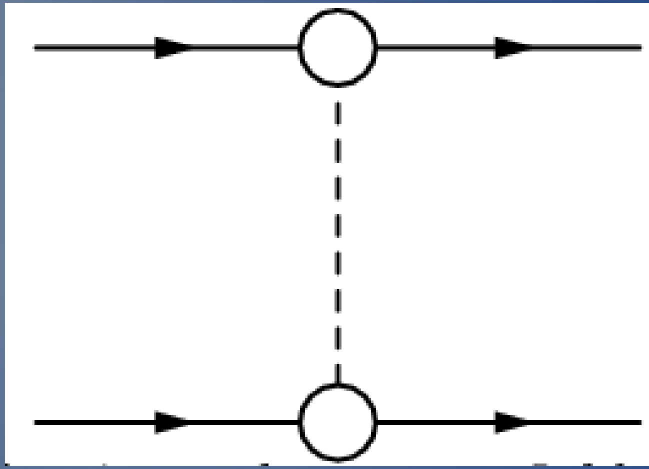
Talk Outline

- Review NN Models and the Force
- The 3P0 Model of NNm Coupling Constants and Form Factors
- One Gluon Exchange
- Oxford Model of NN Force (Initial Results)
  - Phase Shifts
  - Deuteron
  - Nuclear Matter
  - Looking at the Potential
- Conclusions, Thoughts, etc.

NN Models

- Phenomenological Density Dependent Models
  - Skyrme and Gogny.
- Nucleon-Nucleon Models
  - Operator Channel Models (Argonne v18, Reid)
  - N3LO Chiral Perturbation Theory
  - Meson Exchange Models (Bonn, Paris, Nijmegen)

Meson Exchange Theory of Nuclear Forces

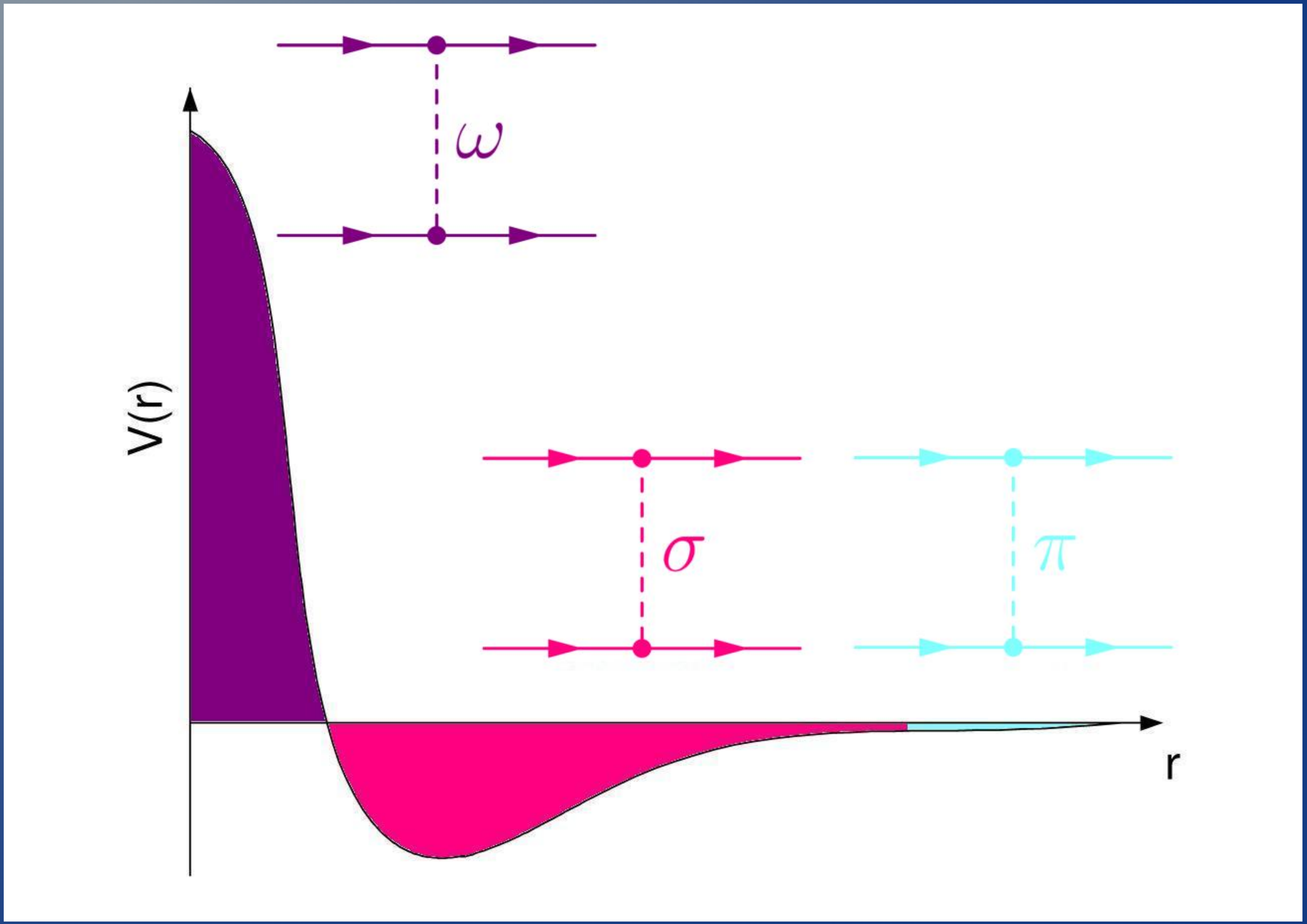


- Choose the set of mesons.
- Choose a form-factor (dipole, exponential, etc.)
- Fit couplings and form-factor parameters

NN Models and Quarks/Gluons

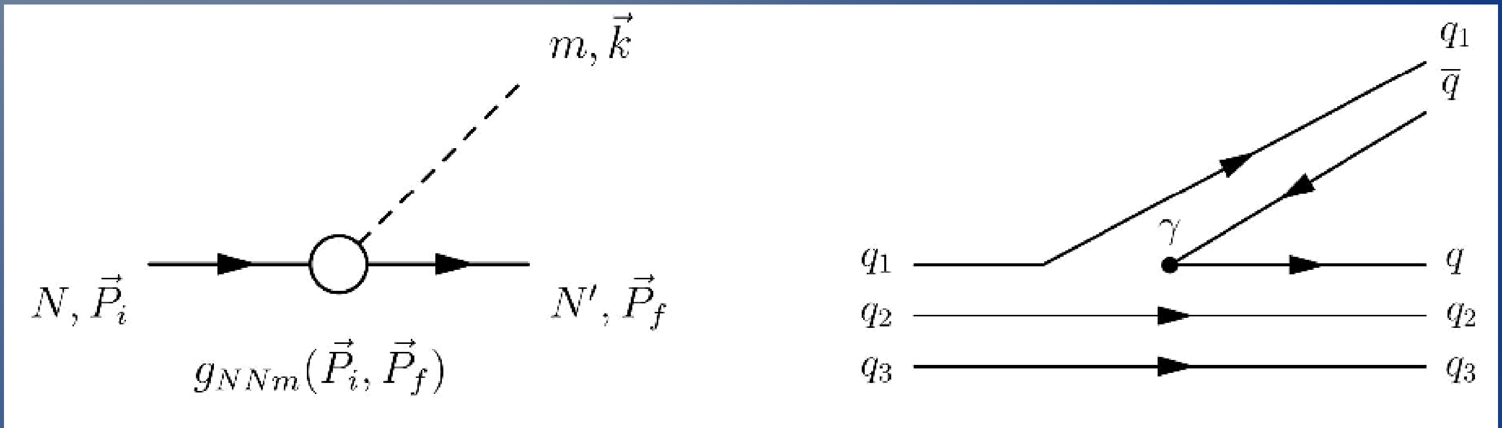
- NN Models take little guidance from quark/gluon substructure of nucleons.
- These models work!
- QCD is non-perturbative ⇒ a proper derivation of NN forces is impossible.
- Constituent Quark Model often estimates strong physics
  - 3P0 Decay Model
  - Quark Cluster Model studies of One Gluon Exchange
- Can the Quark Model Estimate NN Dynamics?

Schematic Picture of NN Forces



3P0 Model and Effective Strong 3-Point Vertices

$$H_{^3\mathrm{P}_0} = \gamma\boldsymbol{\sigma}.\vec{p}\,|q\bar{q}\rangle\,\langle 0|$$



$$\text{Equating the Two} \Longrightarrow g_{NNm}\left(q^2\right) = \cdots$$

3P0 NNm Coupling Constants

$$g_{\mathrm{NN}\pi} = \gamma 40\sqrt{3}\pi^{3/4} m_N \sqrt{m_\pi} \frac{\beta^{3/2}(4\beta^2 + \alpha^2)}{(3\beta^2 + \alpha^2)^{5/2}}$$

$$g_{\mathrm{NN}\sigma} = \gamma 108\sqrt{2}\pi^{3/4} \frac{\alpha^4 \beta^{5/2} \sqrt{m_\sigma}}{(\alpha^2 + 3\beta^2)^{5/2}}$$

3P0 Coupling Constants and Form Factors

$$g_{NN\pi} = \gamma 40\sqrt{3}\pi^{3/4} m_N \sqrt{m_\pi} \frac{\beta^{3/2}(4\beta^2+\alpha^2)}{(3\beta^2+\alpha^2)^{5/2}}$$

$$\mathcal{F}_S = \exp\left\{ -\frac{(\vec{P}_i+\vec{P}_f)^2}{24(3\beta^2+\alpha^2)} - \frac{(\vec{P}_i-\vec{P}_f)^2}{6\alpha^2} \right\}$$

$$g_{NN\sigma} = \gamma 108\sqrt{2}\pi^{3/4} \frac{\alpha^4\beta^{5/2}\sqrt{m_\sigma}}{(\alpha^2+3\beta^2)^{5/2}}$$

$$\mathcal{F}_P = \left(1 + \frac{4\beta^2+\alpha^2}{12\alpha^2(\alpha^2+3\beta^2)}(\vec{P}_f-\vec{P}_i)^2\right) \exp\left\{ -\frac{(\vec{P}_i+\vec{P}_f)^2}{24(3\beta^2+\alpha^2)} - \frac{(\vec{P}_i-\vec{P}_f)^2}{6\alpha^2} \right\}$$

Parameter Free Relations Among Coupling Constants

$$g_{NN\eta(\prime)} = \frac{3}{10\sqrt{2}} \left(\frac{m_{\eta(\prime)}}{m_\pi} \right)^{1/2} g_{NN\pi}$$

$$g_{NN\omega} = \frac{9}{10} \left(\frac{m_\omega}{m_\pi} \right)^{1/2} g_{NN\pi}$$

$$g_{NN\rho} = \frac{1}{6} \left(\frac{m_\rho}{m_\omega} \right)^{1/2} g_{NN\omega}$$

$$g_{NNa_0} = \frac{1}{3} \left(\frac{m_{a_0}}{m_\sigma} \right)^{1/2} g_{NN\sigma}$$

$$\kappa_\omega = -\frac{3}{2}$$

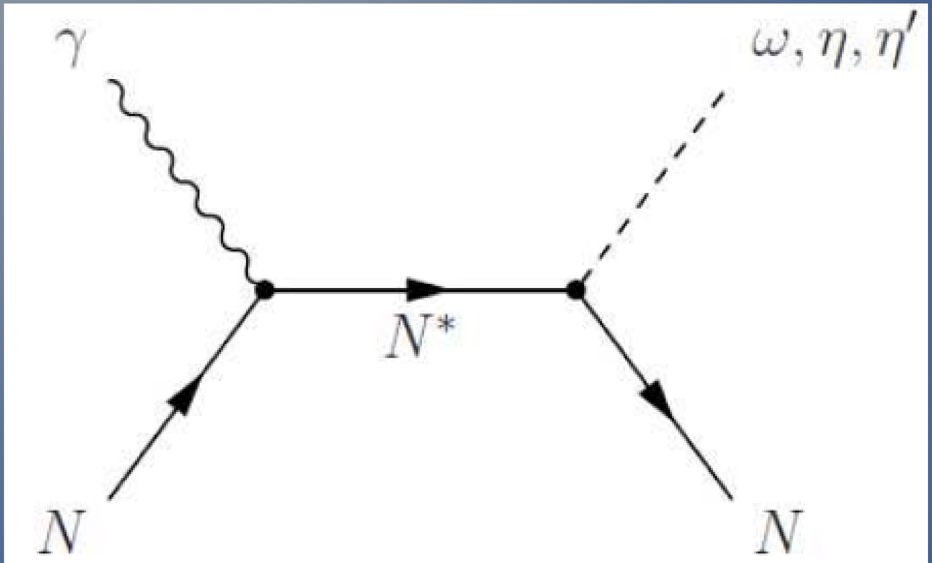
$$\kappa_\rho = +\frac{3}{2}$$

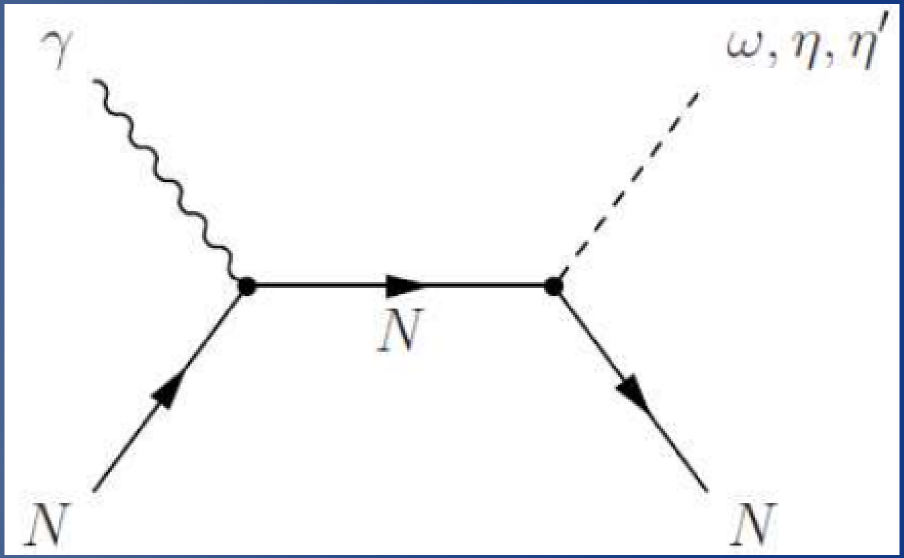
Numerical Values of Coupling Constants

Coupling	This Work	This Work	Paris	Nijmegen'93	CD-Bonn
$g_{NN\pi}$	14.2	[13.5]	[13.3]	13.3	[13.1]
$g_{NN\eta}$	6.0	5.7	–	9.8	–
$g_{NN\eta'}$	7.9	7.5	–	10.5	–
$g_{NN\sigma}$	5.0	N/A	–	17.9	(7.3; 14.9)
g_{NNa_0}	2.7	N/A	–	3.3	–
$g_{NN\omega}(\gamma_\mu)$	30.2	28.7	12.2	12.5	15.9
$g_{NN\rho}/g_{NN\omega}(\gamma_\mu)$	+.33	+.33	–	0.22	0.20
$\kappa_\omega(\sigma_{\mu\nu}/\gamma_\mu)$	–3/2	–3/2	–0.12	0.66	0
$\kappa_\rho(\sigma_{\mu\nu}/\gamma_\mu)$	+3/2	+3/2	–	6.6	6.1

Downnum et al. Phys. Lett. B (638) 455-460 (2006).

CLAS Measurement!





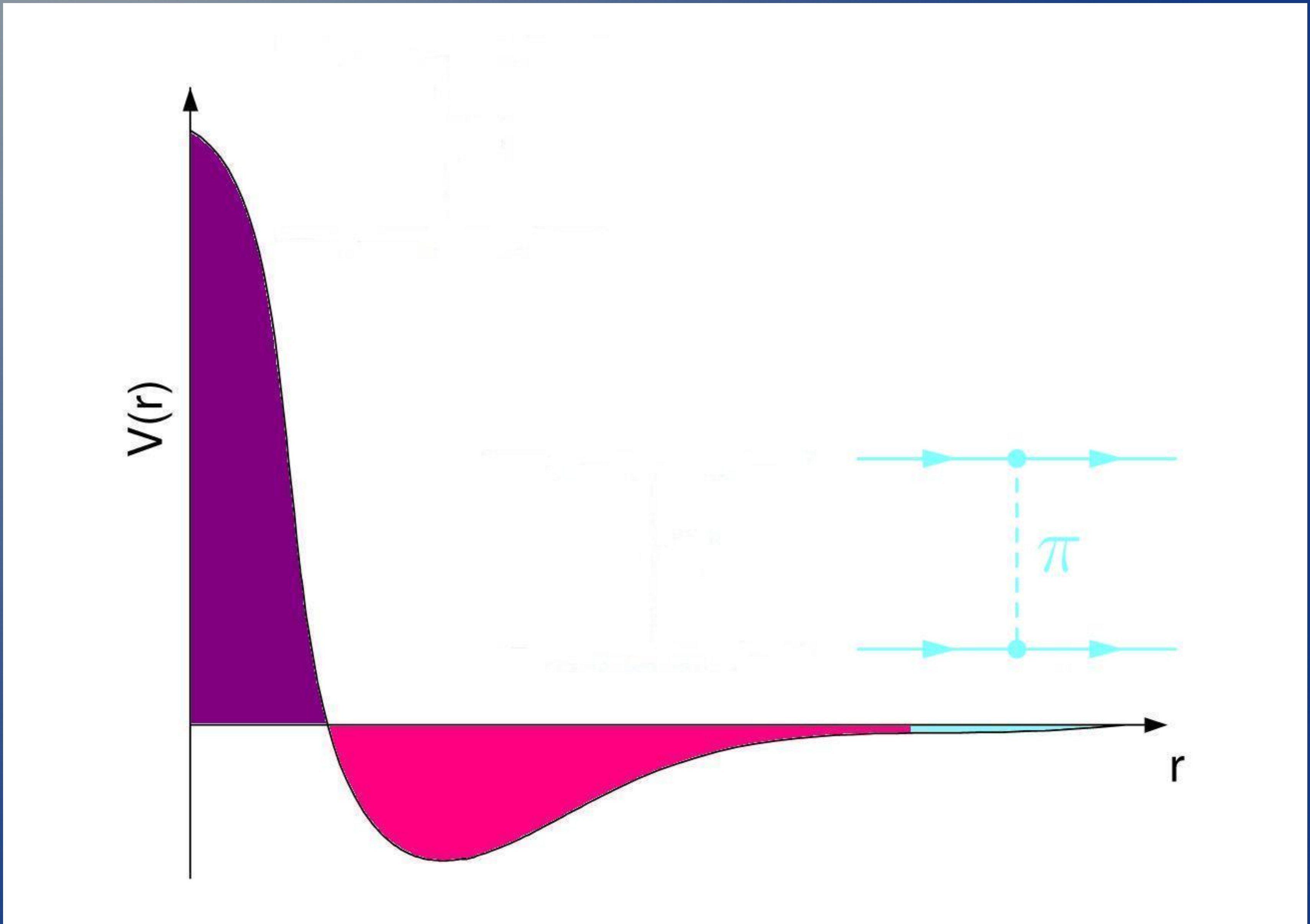
- Mike Williams reports (2007):

$$g_{\mathrm{NN}\omega} = 1.04$$

$$\kappa_\omega = -2.10$$

- Which is significantly different than our values for the coupling constant, but not the ratio.

Back to NN Force Schematic



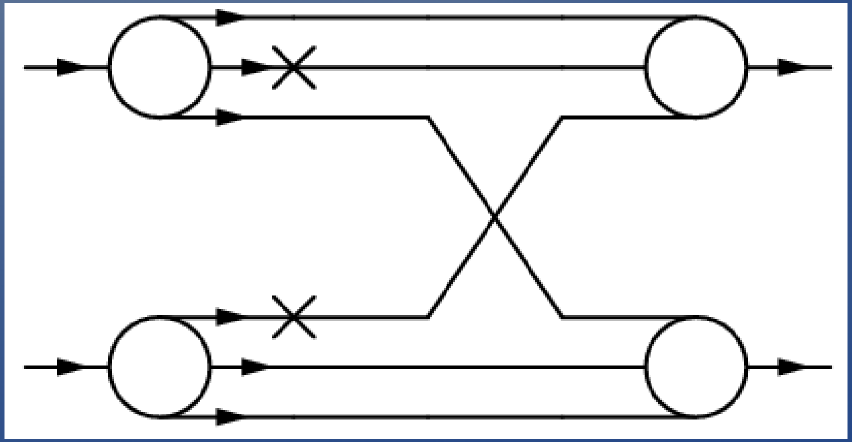
Quark Cluster Model Studies of One Gluon Exchange

- Date back to Liberman in 1977.
- Uses the Breit-Fermi Hamiltonian for perturbative QCD effects and a confining potential for non-perturbative QCD effects.
- Solve the Schrödinger Equation
- Extract a potential.
- They find: Tensor, Coulomb, Central and Spin-Orbit forces approximately cancel.
- Therefore, spin-spin contact interaction dominates.

Simple OGE Potential

Barnes et al. PRC 48 539(1993)

$$H_{\mathrm{OGE\ Hyp}} = \sum_{i<j;i,j=1}^{3} -\frac{8\pi\alpha_S}{3m_i m_j}\vec{s}_i.\vec{s}_j \sum_{a=1}^{8} \frac{\lambda_i^a}{2}\frac{\lambda_j^a}{2}$$



$$V_{\mathrm{OGE+CI}} = \frac{\alpha_S m}{3m_q^2}\sum_{n=1}^{8}\omega_n\eta_n \exp\left\{-A_n p_i{}^2 - C_n p_f{}^2 + B_n \vec{p}_i.\vec{p}_f\right\}$$

Oxford Model

$$\mathcal{L}_{\mathrm{Oxf}} = -ig_{\mathrm{NN}\pi}\mathcal{F}_S\bar{\psi}\gamma_5\vec{\tau}\psi\cdot\vec{\pi} - ig_{\mathrm{NN}\sigma}\mathcal{F}_S\bar{\Psi}\Psi\sigma + iV_{\mathrm{OGE+CI}}[\bar{\psi}\bar{\psi}\psi\psi].$$

- Wanted to try a simple model
- Introduced Charge Independence Breaking and Charge Symmetry Effects.
- Gave ourselves the freedom to vary m_σ and $\alpha=\beta$ by partial wave.
- Found a need for additional repulsion in the 1P1, 3P0, and 3P1 channels so we added an ω exchange to those channels.

NN Free Scattering Phase Shifts

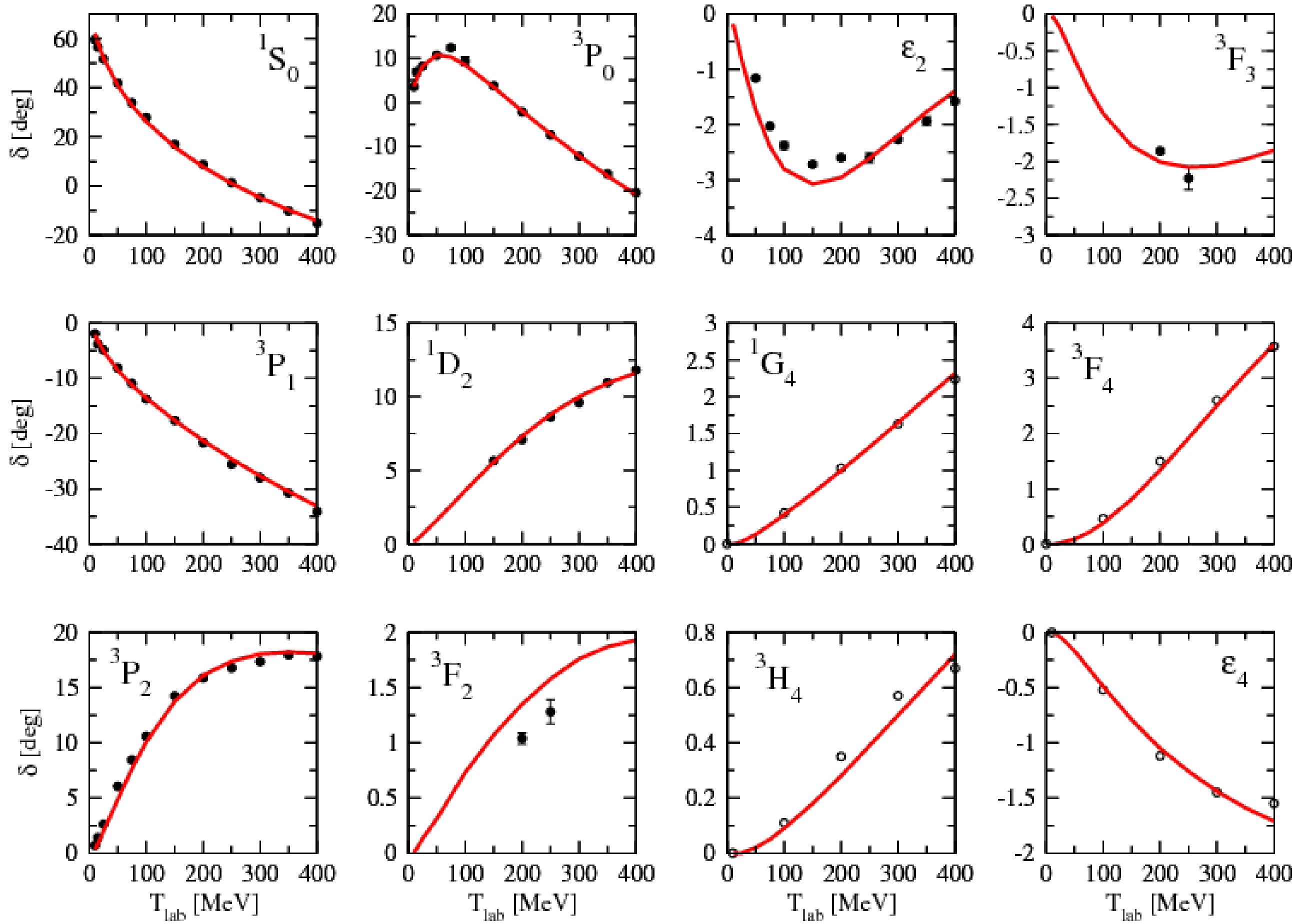
$$(T+V)\psi = H\psi = E\psi = \frac{p^2}{2m}\psi$$

$$r \to \infty \Rightarrow \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{l(l+1)}{2mr^2} + \frac{p^2}{2m}\right)\frac{u_l(r)}{r} \approx 0$$

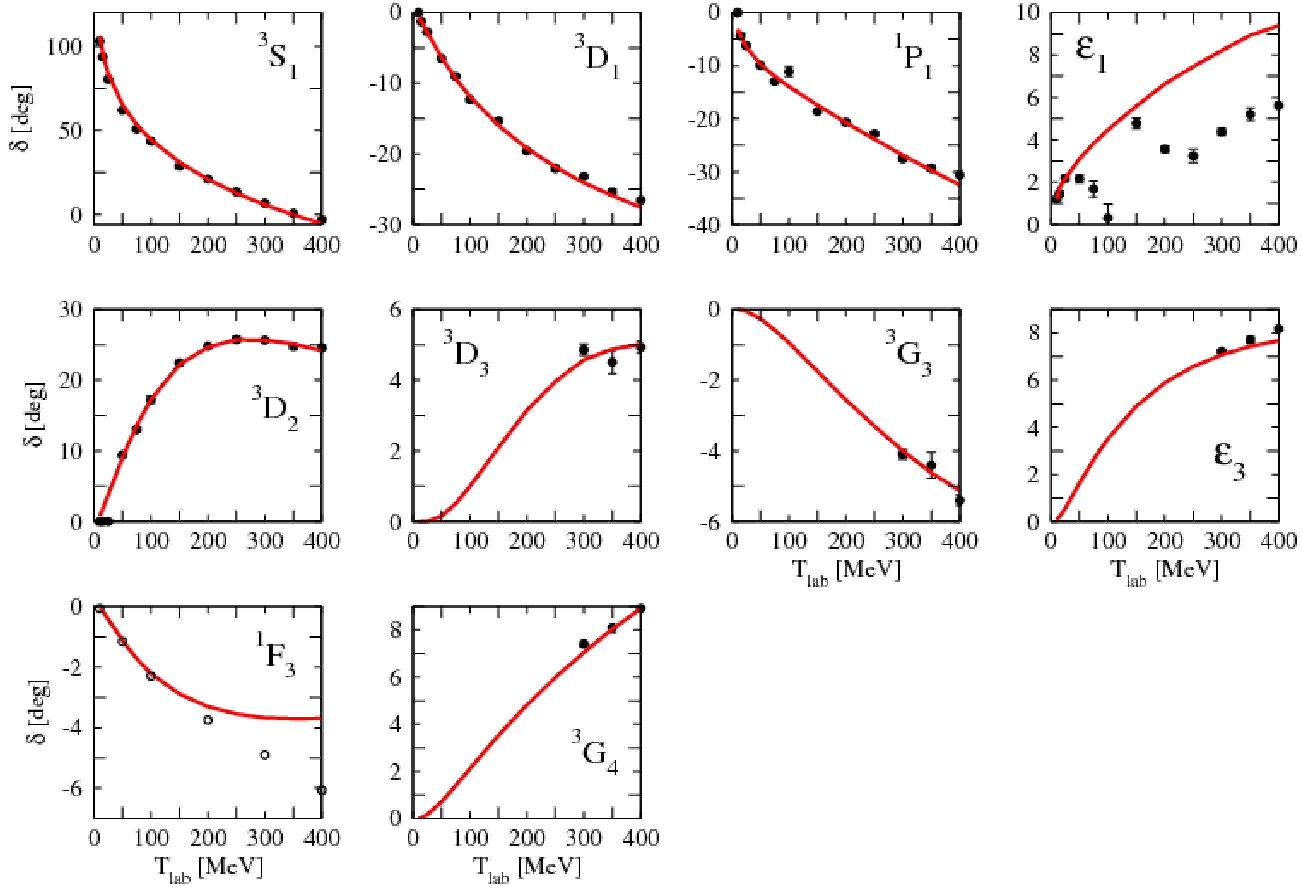
$$\text{Thus as } r \to \infty \Rightarrow u_l(r) \approx \sin\left(pr + \tfrac{1}{2}l\pi + \delta_l(p)\right)$$

$$\text{Spin 1 } \Rightarrow \delta \text{ for } {}^1\mathrm{S}_0, {}^3\mathrm{S}_1, {}^3\mathrm{P}_0, \ldots \text{ etc. and } \varepsilon_1, \varepsilon_2, \ldots \text{ etc.}$$

T=0 Phase Shifts



T=1 Phase Shifts



Scattering Lengths, Effective Ranges

$$(T+V)\psi = H\psi = E\psi = \frac{p^2}{2m}\psi$$

$$r \to \infty \Rightarrow \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{l(l+1)}{2mr^2} + \frac{p^2}{2m}\right)\frac{u_l(r)}{r} \approx 0$$

$$\text{Thus as } r \to \infty \Rightarrow u_l(r) \approx \sin\left(pr + \tfrac{1}{2}l\pi + \delta_l(p)\right)$$

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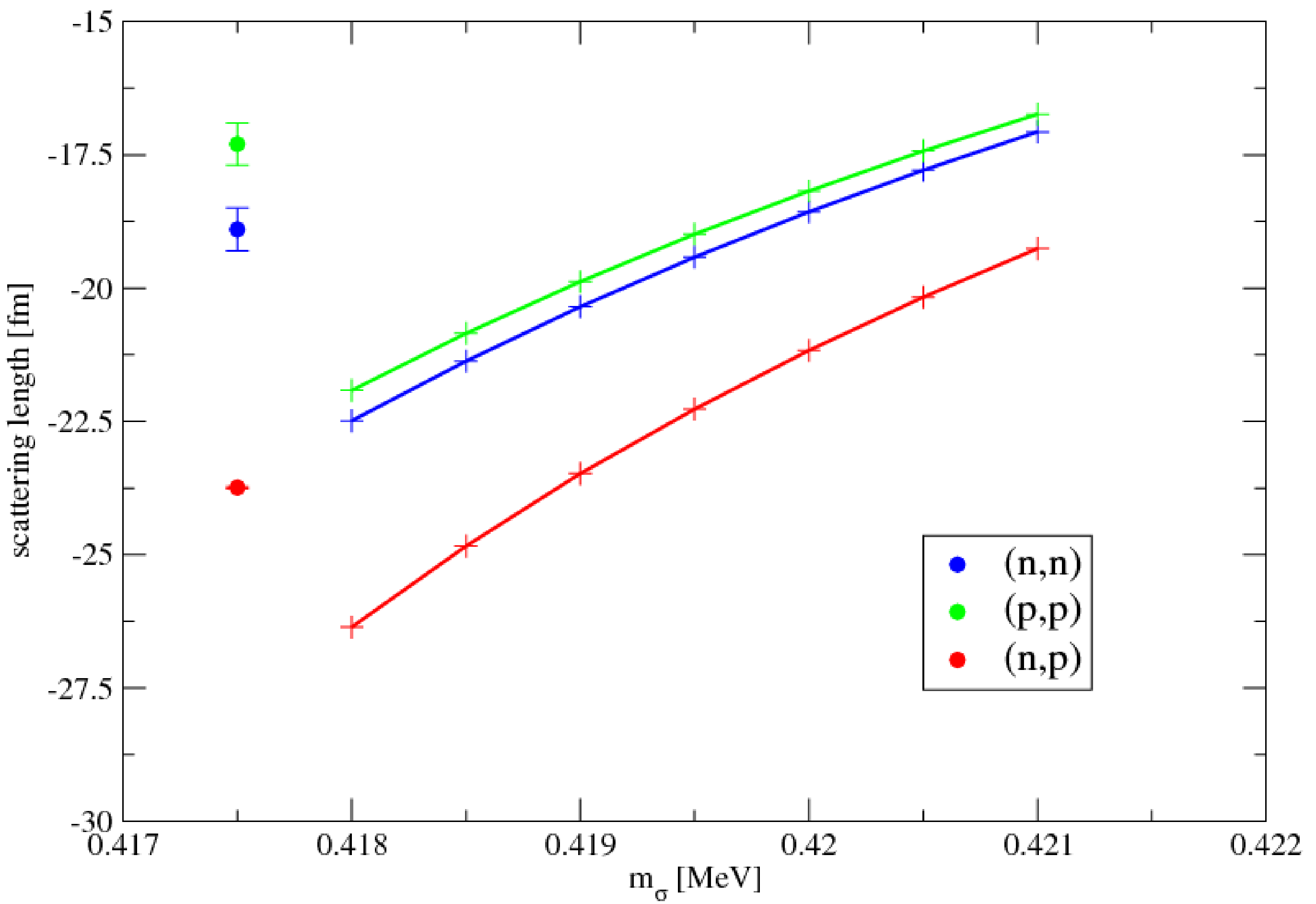
$$p\cot\left(\delta_S(p)\right) \approx -\frac{1}{a} + \frac{1}{2}r_{\mathrm{eff}}p^2 + O\left(p^4\right)$$

Scattering Lengths and Effective Ranges by Channel

	Oxford	CD-Bonn	Experiment
		$^1S_0$	
a^C_{pp}		-7.8154	-7.8149±0.0029
r^C_{pp}		2.773	2.769 ±0.014
a^N_{pp}	-17.33±0.03	-17.4602	
r^N_{pp}	2.806±0.004	2.845	
a^N_{nn}	-18.63±0.03	-18.9680	-18.9 ±0.4
r^N_{nn}	2.788±0.004	2.819	2.75 ±0.11
a^N_{np}	-23.6±0.1	-23.7380	-23.74 ±0.020
r^N_{np}	2.714±0.006	2.671	2.77±0.05
		$^3S_1$	
a^N_t	5.498±0.008	5.4196	5.419±0.007
r^N_t	1.763±0.007	1.751	1.753±0.008

Dependence of T=1 Scattering Length Against Mass of the Sigma



Deuteron Properties

$$H\Psi = (T+V)\Psi = E\Psi = \frac{(iq)^2}{2\mu}\Psi$$

$$\Rightarrow \left[\begin{pmatrix} T & 0 \\ 0 & T \end{pmatrix} + \begin{pmatrix} V_{\mathrm{SS}} & V_{\mathrm{SD}} \\ V_{\mathrm{DS}} & V_{\mathrm{DD}} \end{pmatrix} \right] \begin{pmatrix} \psi_{\mathrm{S}} \\ \psi_{\mathrm{D}} \end{pmatrix} = \frac{(iq)^2}{2\mu} \begin{pmatrix} \psi_{\mathrm{S}} \\ \psi_{\mathrm{D}} \end{pmatrix}$$

$$r_m = \frac{1}{2}\sqrt{\int (\psi_{\mathrm{S}}^2 + \psi_{\mathrm{D}}^2) r^2 dr}$$

$$P_D = \int \psi_{\mathrm{D}}^2 r^2 dr$$

$$Q_D = \frac{1}{20}\int \sqrt{8}\psi_{\mathrm{D}}^\dagger r^2 \psi_{\mathrm{S}} - \psi_{\mathrm{D}}^\dagger r^2 \psi_{\mathrm{D}} dr$$

Deuteron Properties (Cont'd)

$$H\psi = (T+V)\psi = E\psi = \frac{(iq)^2}{2\mu}\psi$$

$$\Rightarrow \left[\begin{pmatrix} T & 0 \\ 0 & T \end{pmatrix} + \begin{pmatrix} V_{\mathrm{SS}} & V_{\mathrm{SD}} \\ V_{\mathrm{DS}} & V_{\mathrm{DD}} \end{pmatrix} \right] \begin{pmatrix} \psi_{\mathrm{S}} \\ \psi_{\mathrm{D}} \end{pmatrix} = \frac{(iq)^2}{2\mu} \begin{pmatrix} \psi_{\mathrm{S}} \\ \psi_{\mathrm{D}} \end{pmatrix}$$

In the $\lim\limits_{r\to\infty} \Rightarrow$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{0}{2\mu r^2} + \frac{(iq)^2}{2\mu}\right)\psi_{\mathrm{S}} = 0$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{2(2+1)}{2\mu r^2} + \frac{(iq)^2}{2\mu}\right)\psi_{\mathrm{D}} = 0$$

$$\lim_{r\to\infty} \psi_{\mathrm{S}}(r) \approx A_{\mathrm{S}} h_0(iqr) = A_{\mathrm{S}}\frac{e^{-qr}}{qr}$$

$$\eta = \frac{A_{\mathrm{D}}}{A_{\mathrm{S}}} \qquad N^2 = A_{\mathrm{S}}^2 + A_{\mathrm{D}}^2$$

$$\lim_{r\to\infty} \psi_{\mathrm{D}}(r) \approx A_{\mathrm{D}} h_2(iqr) = A_{\mathrm{D}}\frac{e^{-qr}}{qr}\left(1 + \frac{3}{qr} + \frac{3}{(qr)^2}\right)$$

Deuteron Results

Property	Oxford	CD-Bonn	Nijmegen	Experiment
B_D[MeV]	2.226(2)	2.224575	2.224575	2.224575±0.000009
A_S[fm$^{1/2}$]	?	0.8846	0.8845	0.8848±0.0009
η	?	0.0256	0.0256	0.0256±0.004
r_m [fm]	?	1.966	1.9676	1.971±0.006
Q_d[fm^2]	?	0.270	0.271	0.28590±0.00030
P_D [%]	6.75(1)	4.85	5.67	

Obviously we still have some things to calculate here. The efforts are underway!

Nuclear Matter

Imagine a Volume V with A nucleons. We assume that the system has periodic boundary conditions and has translational invariance. The the limit as V and A $\rightarrow\infty$ but $\rho=A/V$ = constant of this matter is called Nuclear Matter.

Two variables:

- Density (ρ)
- Proton Fraction: x_p

  X_p=1/2 is Symmetric Nuclear Matter (SNM)

  X_p=0 is Pure Neutron Matter (PNM)

  X_p=else is Asymmetric Nuclear Matter (ANM)

We are interested in E/A, the Equation of State (EOS).

Properties of Nuclear Matter from Heavy Nuclei

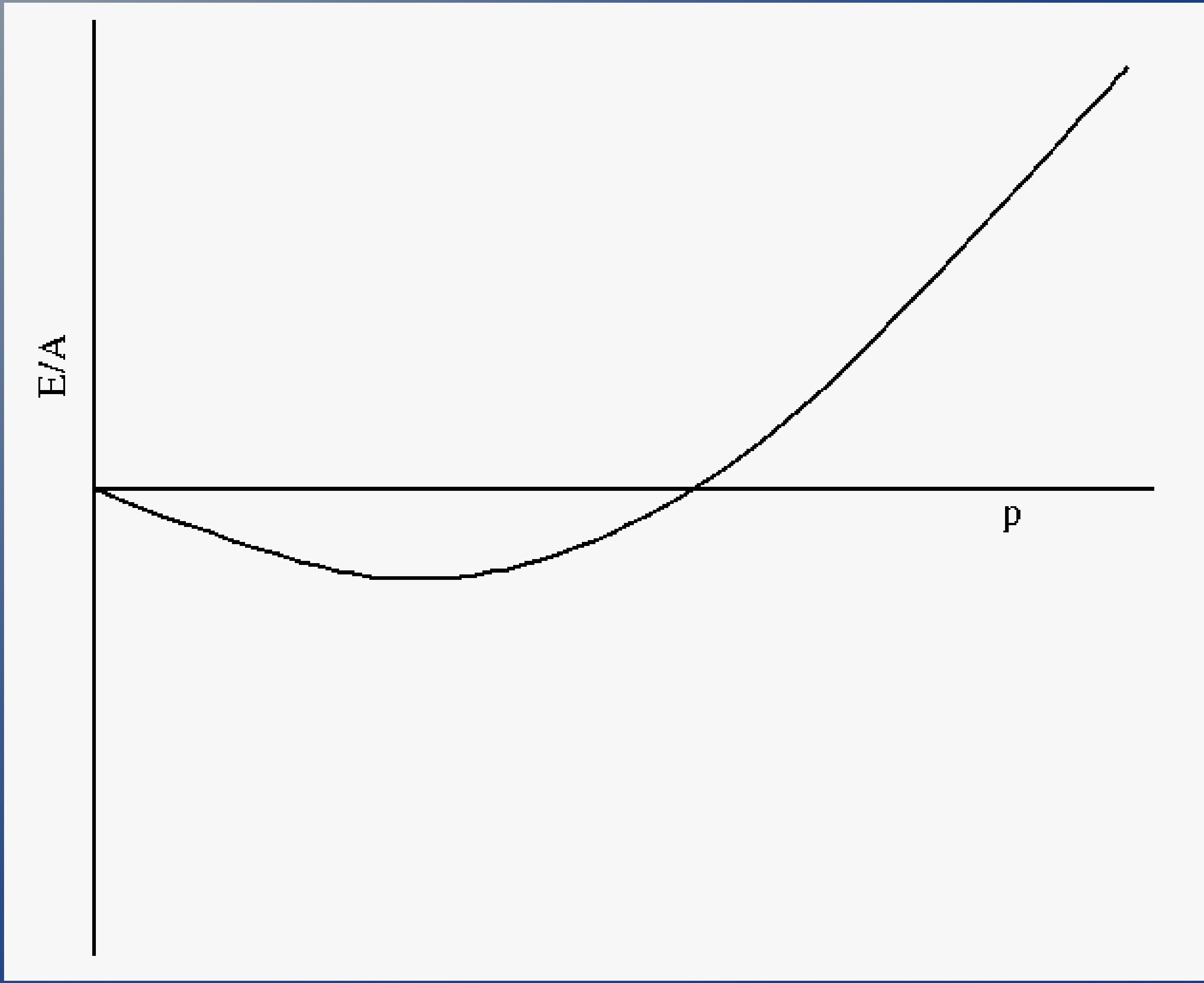
$$\frac{E}{A} = a_1 + a_2 A^{-\frac{1}{3}} + a_3 x_p^2 A^{-\frac{10}{3}} + a_4 (1 - 2x_p)^2 + a_5 A^{-\frac{7}{4}}$$

$$\lim_{A \to \infty} \left. \frac{E}{A} \right|_{x_p = \frac{1}{2}} = a_1 \approx -16.1\text{MeV}$$

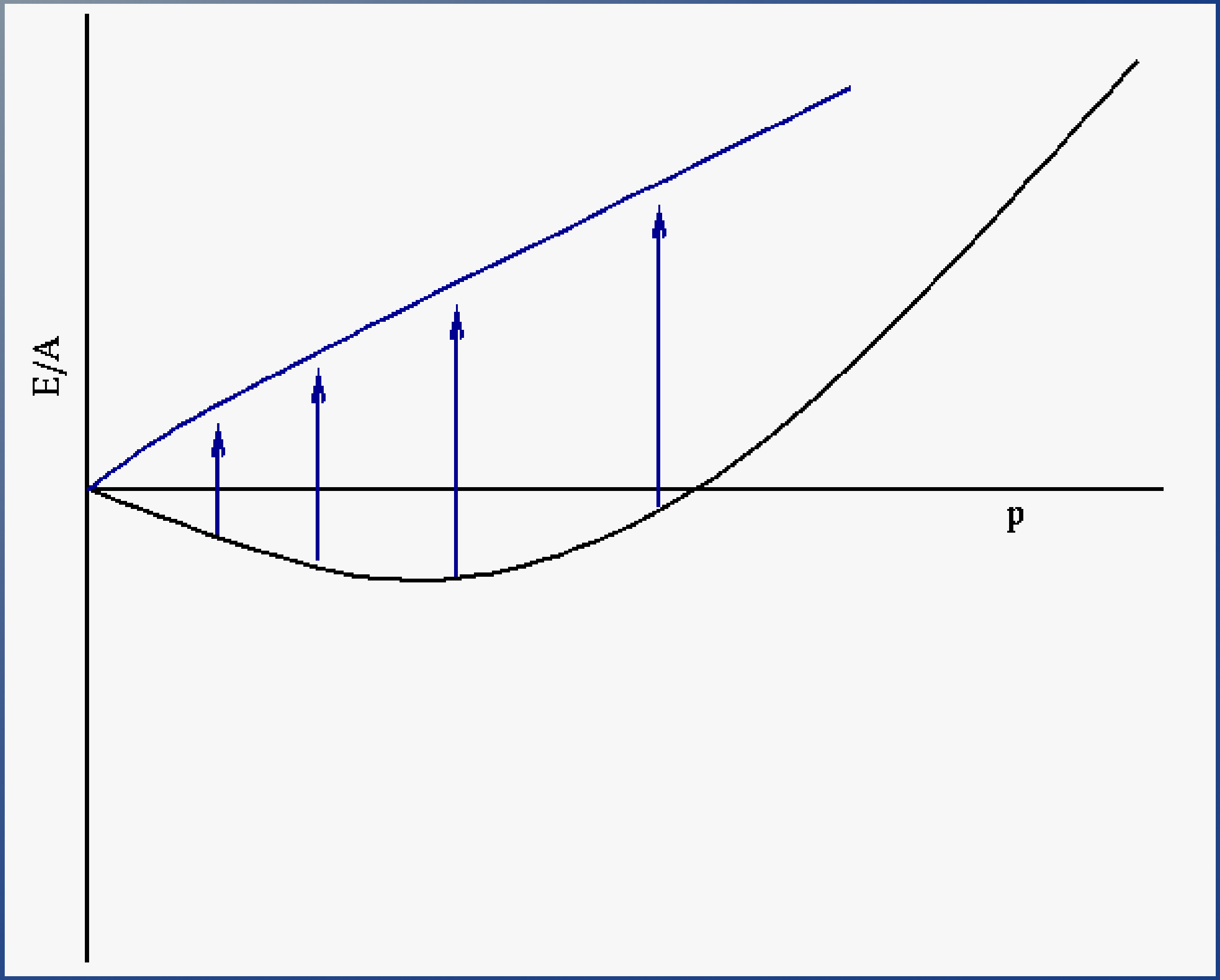
$$\lim_{A \to \infty} \frac{\partial^2}{\partial x_p^2} \frac{E}{A} = 8a_4 \Rightarrow \frac{1}{8} \lim_{A \to \infty} \frac{\partial^2}{\partial x_p^2} = a_4 \approx 23\text{MeV}$$

$$\lim_{A \to \infty} \frac{\partial^2}{\partial x_p^2} \frac{E}{A} \approx \lim_{A \to \infty} \left. \frac{E}{A} \right|_{x_p = \frac{1}{2}} - \lim_{A \to \infty} \left. \frac{E}{A} \right|_{x_p = 0}$$

Qualitative EOS for SNM



Change of EOS from SNM to PNM

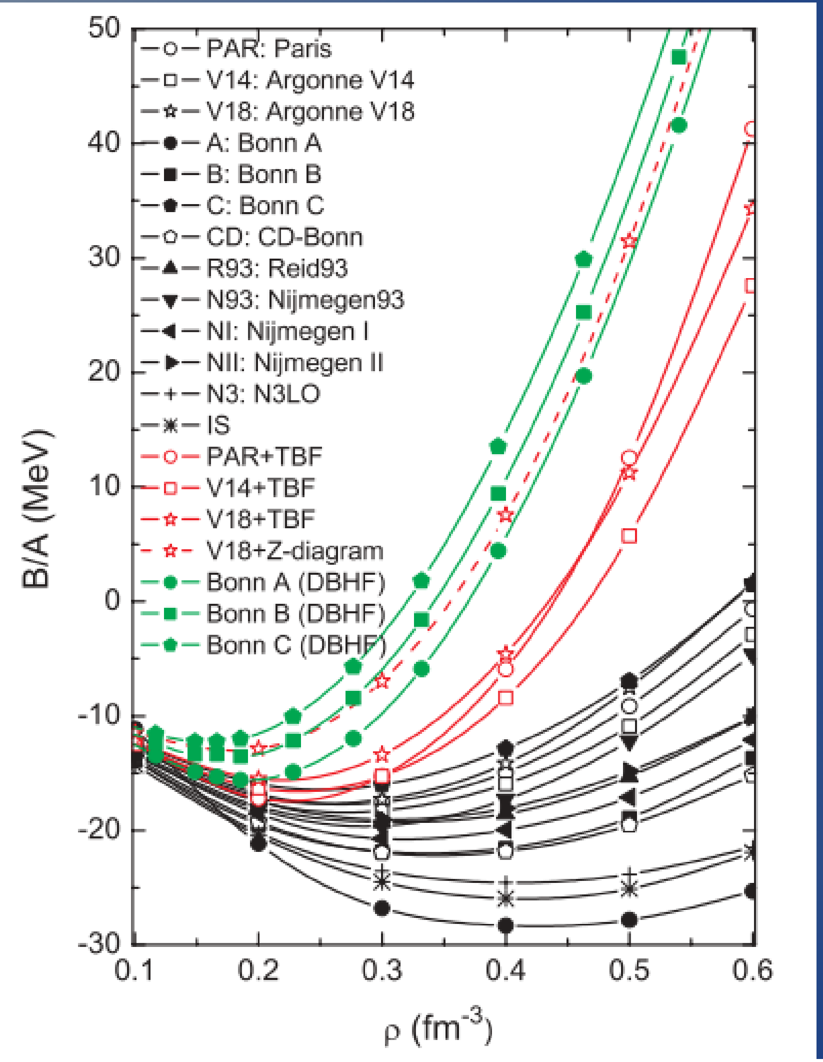


Calculating the EOS

$$H\psi = (T + V)\psi = E\psi$$

- Need to solve the many body Schrödinger equation.
- Non-trivial but technical.
- First person to try perturbation theory techniques to nuclear matter: Brueckner. Hence the Brueckner Hartree Fock formalism.

Formalism and Interaction Matter!

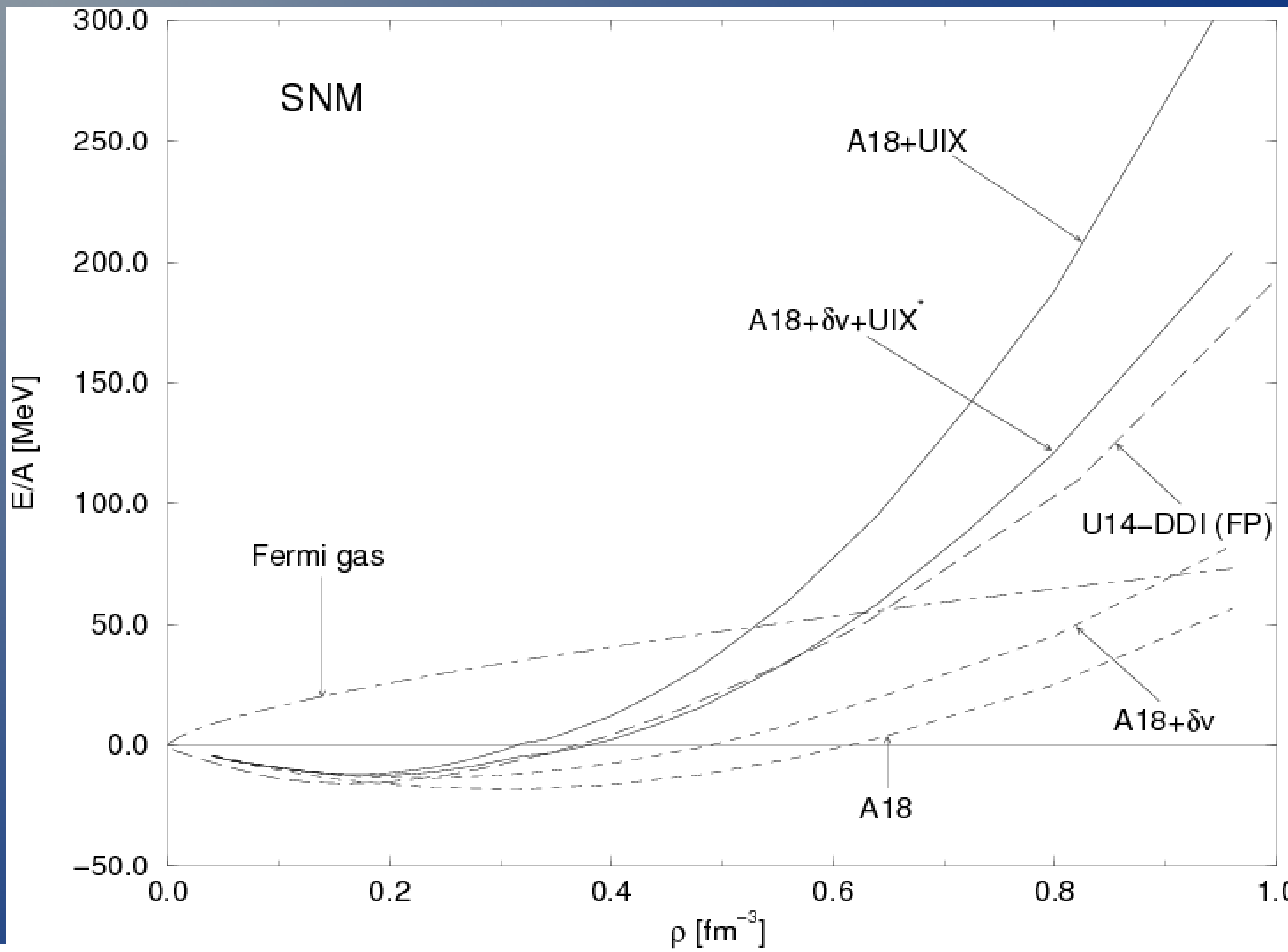


PHYSICAL REVIEW C **74**, 047304 (2006)

Nuclear matter saturation point and symmetry energy with modern nucleon-nucleon potentials

Z. H. Li,[1] U. Lombardo,[2] H.-J. Schulze,[3] W. Zuo,[4] L. W. Chen,[1] and H. R. Ma[1]

Making a 2-body Interaction Fit EOS Phenomenology

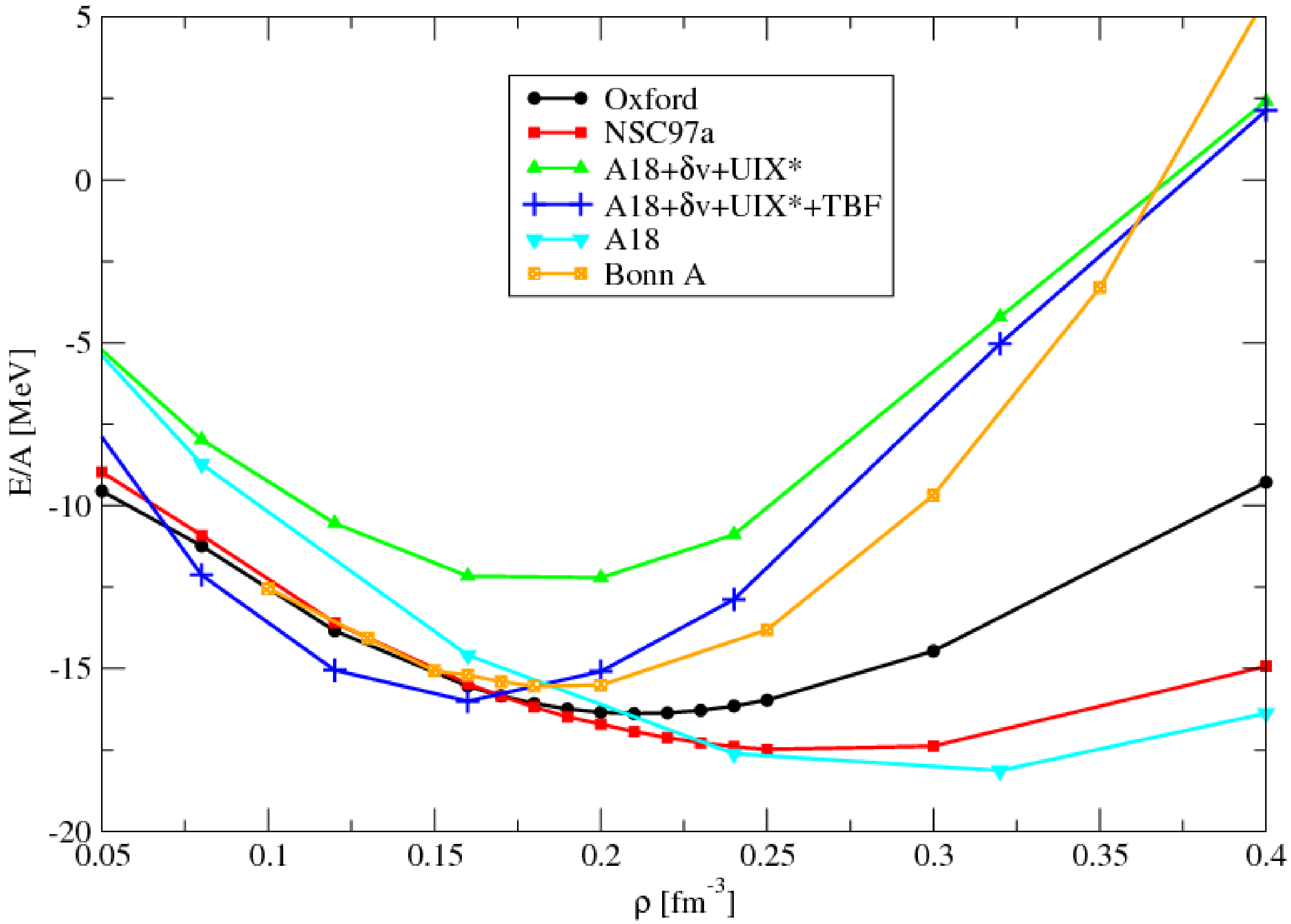


PHYSICAL REVIEW C VOLUME 58, NUMBER 3 SEPTEMBER 1998

Equation of state of nucleon matter and neutron star structure

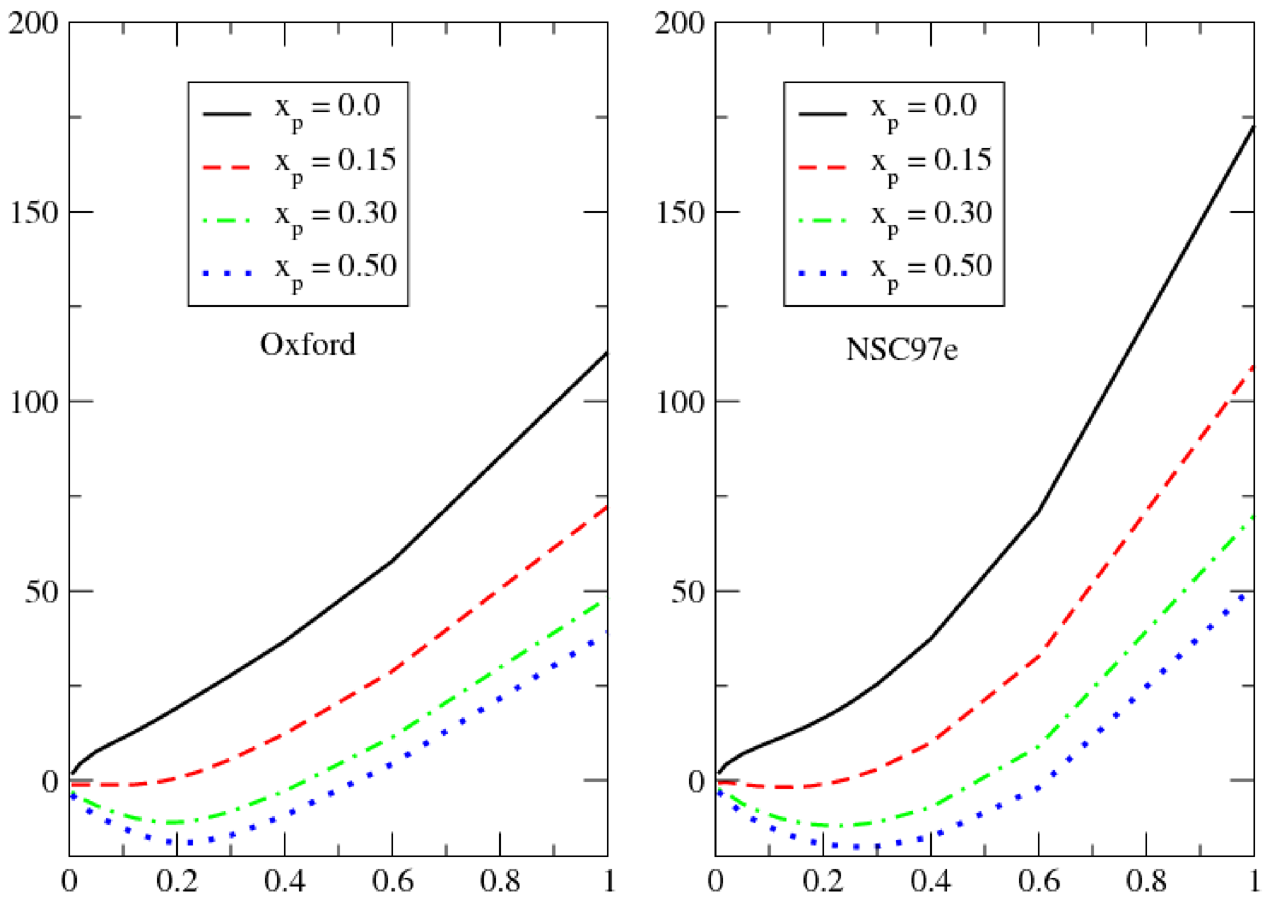
A. Akmal,* V. R. Pandharipande,† and D. G. Ravenhall‡

EOS Results

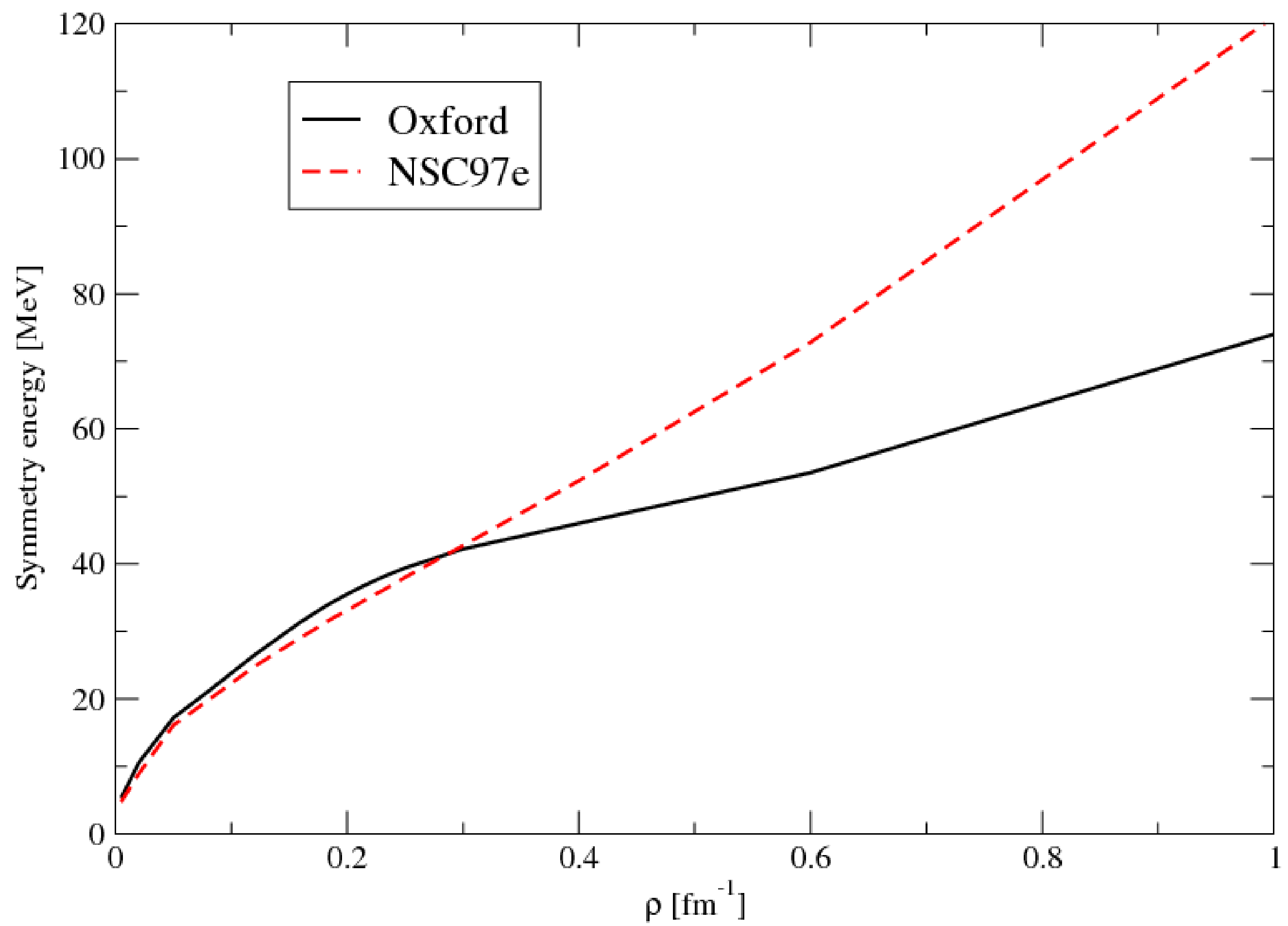


Oxford
NSC97a
A18+δv+UIX*
A18+δv+UIX*+TBF
A18
Bonn A
E/A [MeV]
ρ [fm^-3]
5
0
-5
-10
-15
-20
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4

EOS Results (Cont'd.)



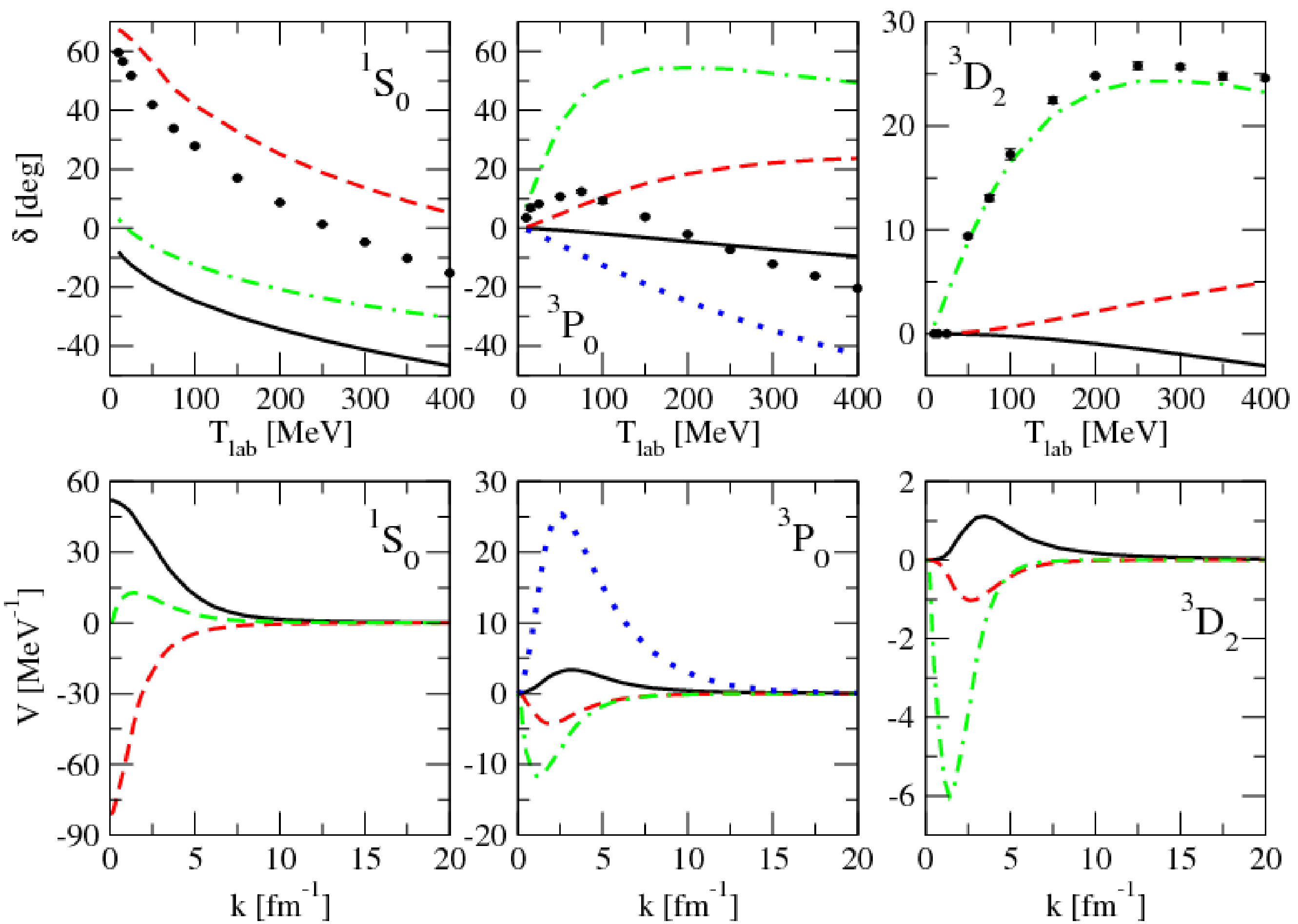
EOS Results (Cont'd)



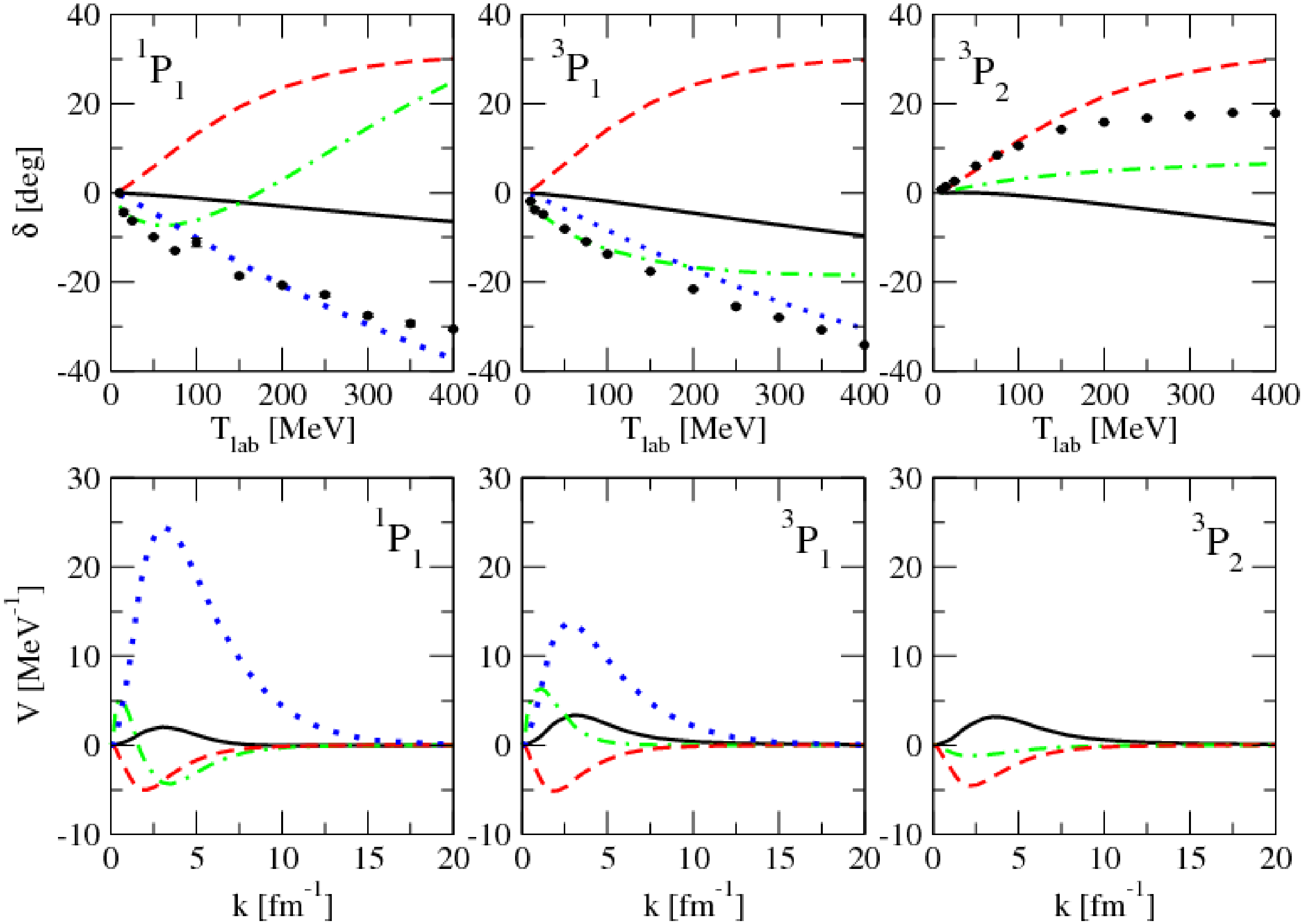
Comparison of EOS Results

Model	$\rho_0[\mathrm{fm}^{-3}]$	Symmetry Energy	B/A[MeV]
Paris	0.270	29.4	-17.6
Argonne 18	0.270	29.9	-17.3
Bonn A	0.259	32.1	-28.4
Bonn B	0.419	31.8	-22.0
Bonn C	0.341	28.5	-16.4
CD-Bonn	0.257	31.1	-21.9
Reid '93	0.328	30.0	-19.8
Nijmegen '93	0.285	30.4	-19.6
Nijmegen I	0.348	30.5	-20.7
Nijmegen II	0.326	29.5	-19.4
N^3LO	0.408	31.2	-24.5
NSC97e	0.25	38.0	-17.48
Oxford	0.21	36.4	-16.38

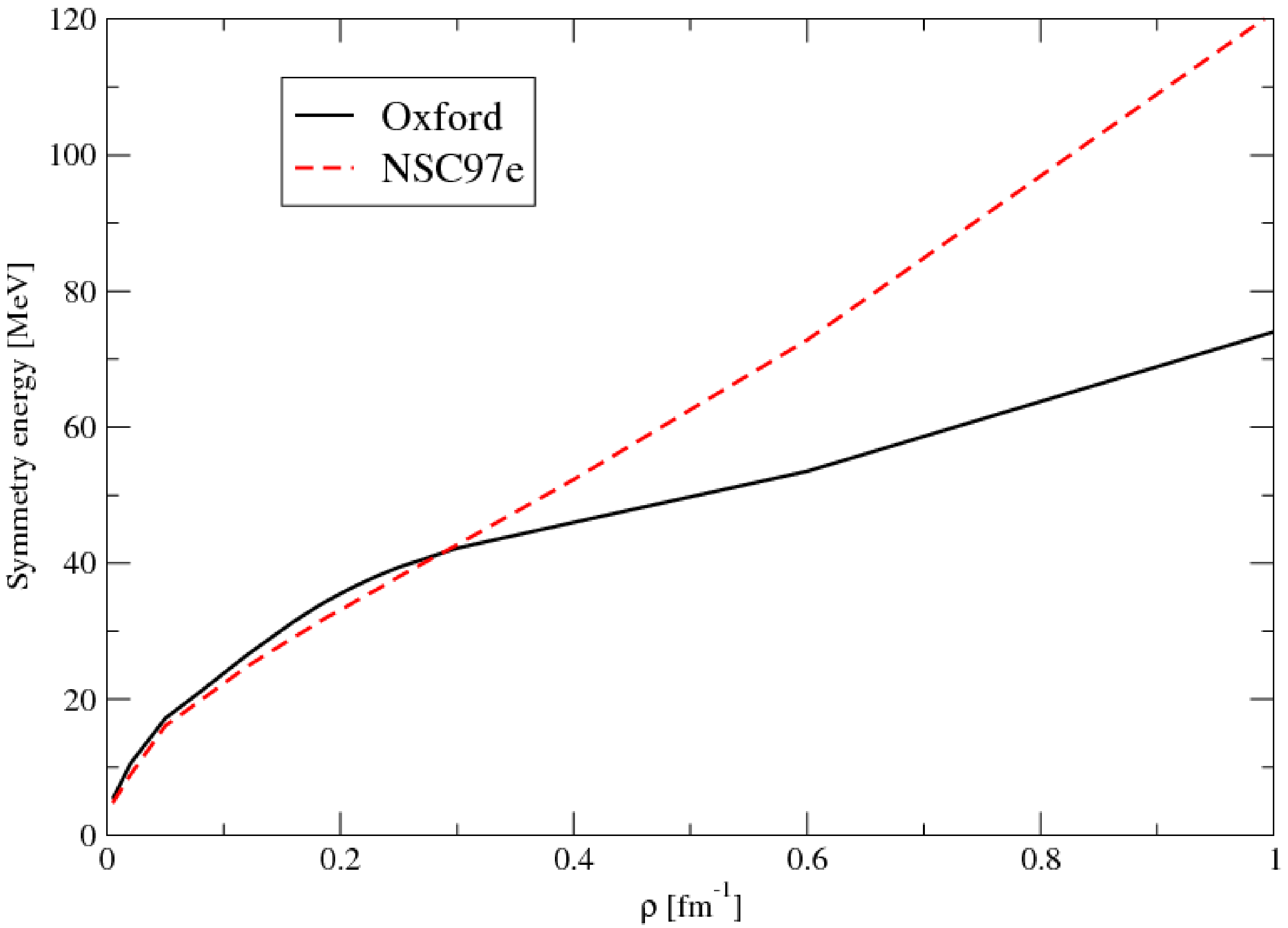
Phase Shifts and Potential Contributions



Phase Shifts and Potential Contributions (Cont'd.)

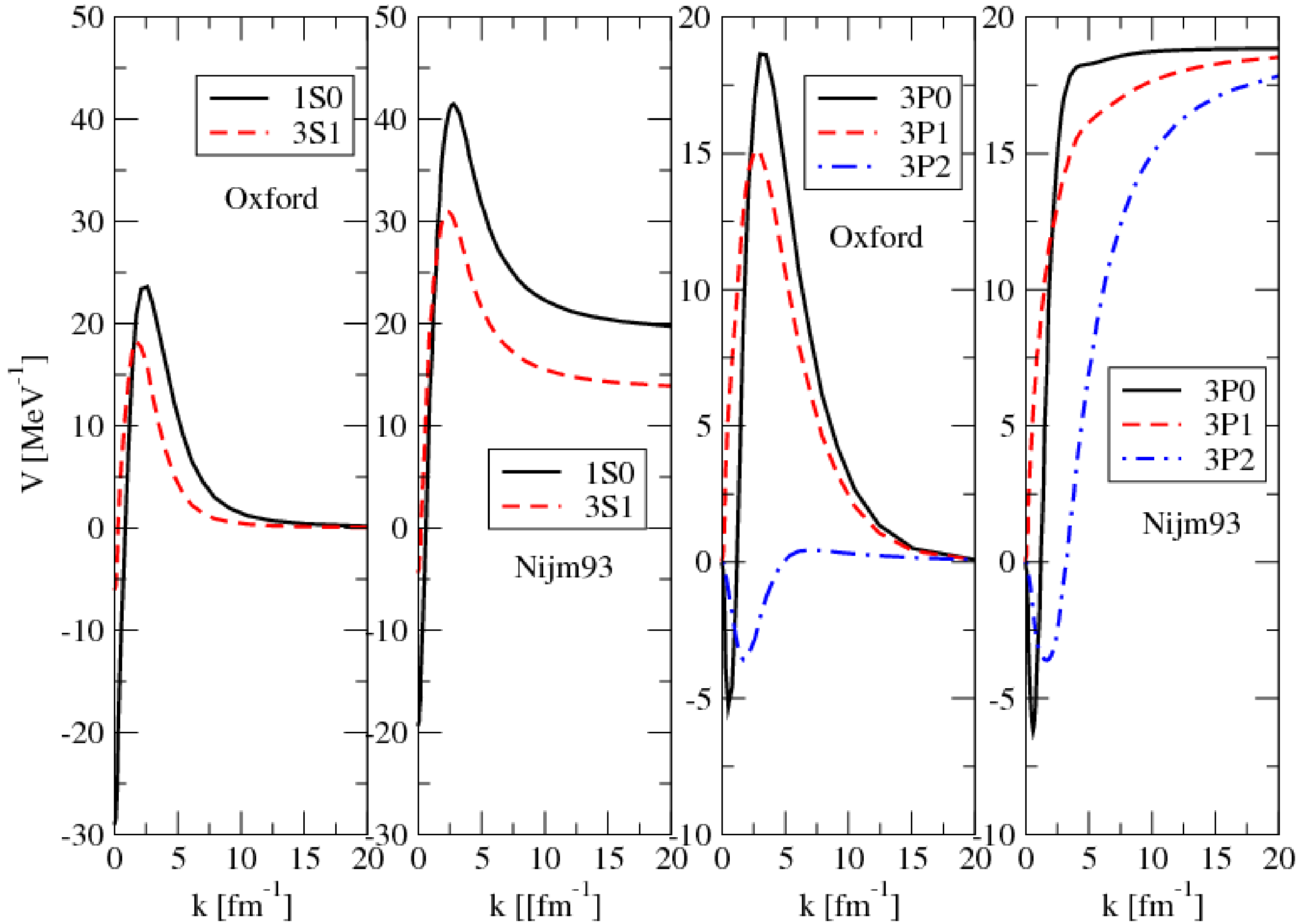


Mind the Gap.

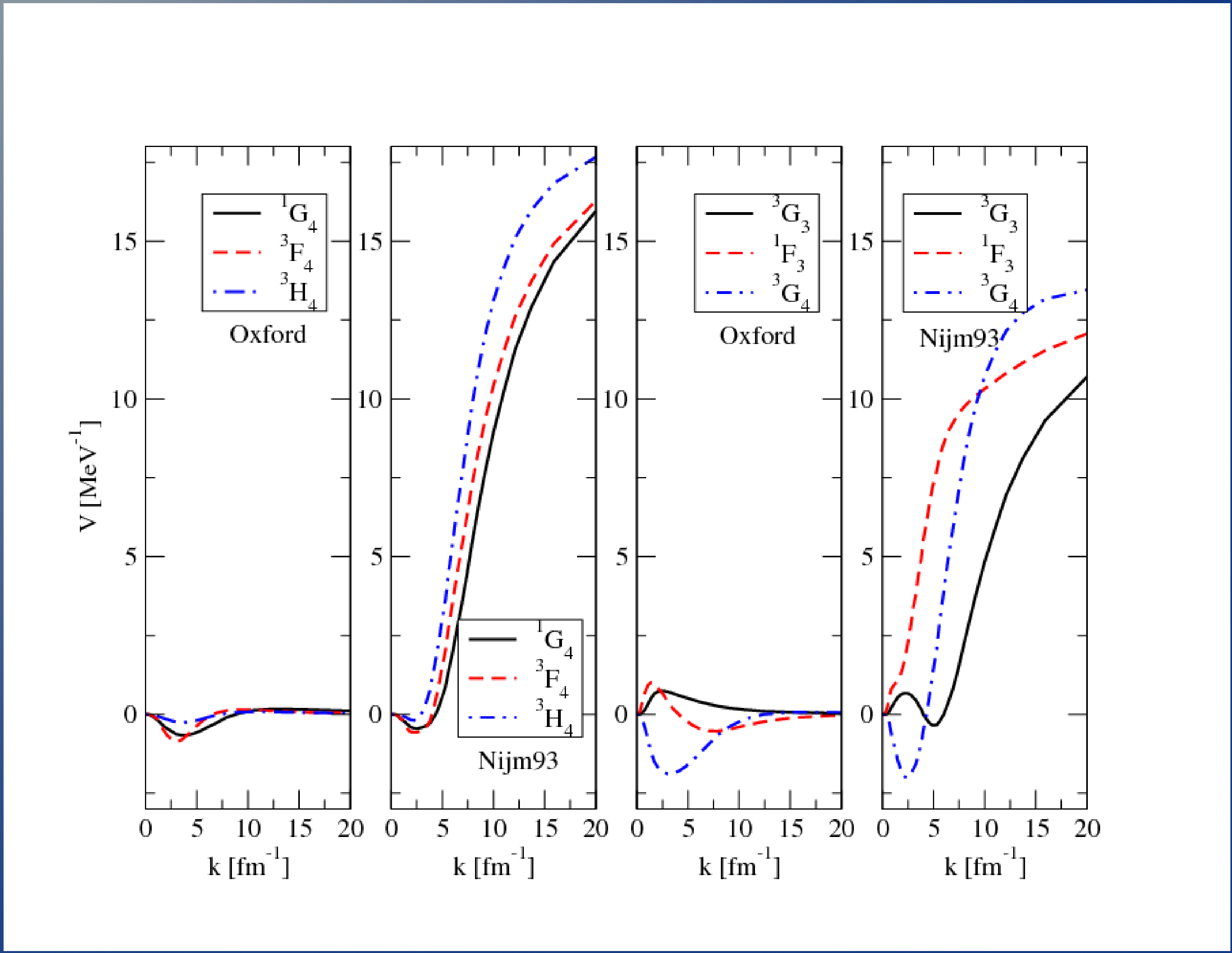


Oxford
NSC97e
Symmetry energy [MeV]
ρ [fm^-1]
0
20
40
60
80
100
120
0.2
0.4
0.6
0.8
1

Oxford and Nijmegen Potential by Partial Wave



Oxford and Nijmegen Potential by Partial Wave (Cont'd.)



Thoughts, Conclusions, etc.

- The most significant failure of the model is the necessity of including an ad-hoc ω exchange in some P-waves.
- The range of parameter variation by partial wave, while not extreme, is disappointing.
- We can alter the physics of the model self-consistently to explore other mechanisms (2π exchange, etc.)

Thoughts, Conclusions, etc. (Cont'd.)

- The ability of the simple and constrained model to well reproduce so many observables connected to the NN interaction is surprising and encouraging.
- The model raises important physics issues:
  - Non-locality
  - ρ exchange
  - High momentum behavior, etc.
- Work continues.

End of Talk

Thank you for your attention!

Questions?

Feedback?