Nucleon-Nucleon Interactions from the Quark Model

Clark Downum (Oxford/Barcelona)

Jirina Stone (Oxford/UTK), Ted Barnes (ORNL/UTK), Eric Swanson (Pitt.), and Isaac Vidaña (Coimbra)
Talk Outline

- Review NN Models and the Force
- The 3P0 Model of NNm Coupling Constants and Form Factors
- One Gluon Exchange
- Oxford Model of NN Force (Initial Results)
  - Phase Shifts
  - Deuteron
  - Nuclear Matter
  - Looking at the Potential
- Conclusions, Thoughts, etc.
NN Models

- Phenomenological Density Dependent Models
  - Skyrme and Gogny.

- Nucleon-Nucleon Models
  - Operator Channel Models (Argonne v18, Reid)
  - N3LO Chiral Perturbation Theory
  - Meson Exchange Models (Bonn, Paris, Nijmegen)
Meson Exchange Theory of Nuclear Forces

- Choose the set of mesons.
- Choose a form-factor (dipole, exponential, etc.)
- Fit couplings and form-factor parameters
NN Models and Quarks/Gluons

- NN Models take little guidance from quark/gluon substructure of nucleons.
- These models work!
- QCD is non-perturbative $\Rightarrow$ a proper derivation of NN forces is impossible.
- Constituent Quark Model often estimates strong physics
  - 3P0 Decay Model
  - Quark Cluster Model studies of One Gluon Exchange

- Can the Quark Model Estimate NN Dynamics?
Schematic Picture of NN Forces
3P0 Model and Effective Strong 3-Point Vertices

\[ H_{3P0} = \gamma \sigma \cdot \vec{p} \left| q \bar{q} \right\rangle \langle 0 \]

Equating the Two \( \implies g_{NNm} \left( q^2 \right) = \cdots \)
3P0 NNm Coupling Constants

\[
g_{NN\pi} = \gamma 40 \sqrt{3} \pi^{3/4} m_N \sqrt{m_\pi} \frac{\beta^{3/2} (4\beta^2 + \alpha^2)}{(3\beta^2 + \alpha^2)^{5/2}}
\]

\[
g_{NN\sigma} = \gamma 108 \sqrt{2} \pi^{3/4} \frac{\alpha^4 \beta^{5/2} \sqrt{m_\sigma}}{(\alpha^2 + 3\beta^2)^{5/2}}
\]
3P0 Coupling Constants and Form Factors

\[ g_{\text{NN}\pi} = \gamma 40 \sqrt{3\pi^{3/4}} m_N \sqrt{m_\pi} \frac{\beta^{3/2} (4\beta^2 + \alpha^2)}{(3\beta^2 + \alpha^2)^{5/2}} \]

\[ \mathcal{F}_s = \exp \left\{ -\frac{(\vec{P}_i + \vec{P}_f)^2}{24(3\beta^2 + \alpha^2)} - \frac{(\vec{P}_i - \vec{P}_f)^2}{6\alpha^2} \right\} \]

\[ g_{\text{NN}\sigma} = \gamma 108 \sqrt{2\pi^{3/4}} \frac{\alpha^4 \beta^{5/2} \sqrt{m_\sigma}}{(\alpha^2 + 3\beta^2)^{5/2}} \]

\[ \mathcal{F}_p = \left( 1 + \frac{4\beta^2 + \alpha^2}{12\alpha^2(\alpha^2 + 3\beta^2)} (\vec{P}_f - \vec{P}_i)^2 \right) \exp \left\{ -\frac{(\vec{P}_i + \vec{P}_f)^2}{24(3\beta^2 + \alpha^2)} - \frac{(\vec{P}_i - \vec{P}_f)^2}{6\alpha^2} \right\} \]
Parameter Free Relations Among Coupling Constants

\[ g_{NN\eta'} = \frac{3}{10\sqrt{2}} \left( \frac{m_{\eta'}}{m_\pi} \right)^{1/2} g_{NN\pi} \]

\[ g_{NN\omega} = \frac{9}{10} \left( \frac{m_\omega}{m_\pi} \right)^{1/2} g_{NN\pi} \]

\[ g_{NN\rho} = \frac{1}{6} \left( \frac{m_\rho}{m_\omega} \right)^{1/2} g_{NN\omega} \]

\[ g_{NNa_0} = \frac{1}{3} \left( \frac{m_{a_0}}{m_\sigma} \right)^{1/2} g_{NN\sigma} \]

\[ \kappa_\omega = -\frac{3}{2} \]

\[ \kappa_\rho = +\frac{3}{2} \]
## Numerical Values of Coupling Constants

<table>
<thead>
<tr>
<th>Coupling</th>
<th>This Work</th>
<th>This Work</th>
<th>Paris</th>
<th>Nijmegen’93</th>
<th>CD-Bonn</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{NN\pi}$</td>
<td>14.2</td>
<td>[13.5]</td>
<td>[13.3]</td>
<td>13.3</td>
<td>[13.1]</td>
</tr>
<tr>
<td>$g_{NN\eta}$</td>
<td>6.0</td>
<td>5.7</td>
<td>–</td>
<td>9.8</td>
<td>–</td>
</tr>
<tr>
<td>$g_{NN\eta'}$</td>
<td>7.9</td>
<td>7.5</td>
<td>–</td>
<td>10.5</td>
<td>–</td>
</tr>
<tr>
<td>$g_{NN\sigma}$</td>
<td>5.0</td>
<td>N/A</td>
<td>–</td>
<td>17.9</td>
<td>(7.3; 14.9)</td>
</tr>
<tr>
<td>$g_{NNa_0}$</td>
<td>2.7</td>
<td>N/A</td>
<td>–</td>
<td>3.3</td>
<td>–</td>
</tr>
<tr>
<td>$g_{NN\omega}(\gamma_\mu)$</td>
<td>30.2</td>
<td>28.7</td>
<td>12.2</td>
<td>12.5</td>
<td>15.9</td>
</tr>
<tr>
<td>$g_{NN\rho}/g_{NN\omega}(\gamma_\mu)$</td>
<td>+.33</td>
<td>+.33</td>
<td>–</td>
<td>0.22</td>
<td>0.20</td>
</tr>
<tr>
<td>$\kappa_\omega(\sigma_{\mu\nu}/\gamma_\mu)$</td>
<td>−3/2</td>
<td>−3/2</td>
<td>−0.12</td>
<td>0.66</td>
<td>0</td>
</tr>
<tr>
<td>$\kappa_\rho(\sigma_{\mu\nu}/\gamma_\mu)$</td>
<td>+3/2</td>
<td>+3/2</td>
<td>–</td>
<td>6.6</td>
<td>6.1</td>
</tr>
</tbody>
</table>

Mike Williams reports (2007):

\[ g_{NN\omega} = 1.04 \]
\[ \kappa_\omega = -2.10 \]

Which is significantly different than our values for the coupling constant, but not the ratio.
Back to NN Force Schematic
Quark Cluster Model Studies of One Gluon Exchange

- Date back to Liberman in 1977.
- Uses the Breit-Fermi Hamiltonian for perturbative QCD effects and a confining potential for non-perturbative QCD effects.
- Solve the Schrödinger Equation
- Extract a potential.
- They find: Tensor, Coulomb, Central and Spin-Orbit forces approximately cancel.
- Therefore, spin-spin contact interaction dominates.
Simple OGE Potential
Barnes et al. PRC 48 539(1993)

\[ H_{\text{OGE Hyp}} = \sum_{i<j; i, j=1}^{3} -\frac{8\pi\alpha_s}{3m_i m_j} \vec{s}_i \cdot \vec{s}_j \sum_{a=1}^{8} \frac{\lambda_i^a \lambda_j^a}{2} \]

\[ \mathcal{V}_{\text{OGE+CI}} = \frac{\alpha_s m}{3m_q^2} \sum_{n=1}^{8} \omega_n \eta_n \exp \left\{ -A_n p_i^2 - C_n p_f^2 + B_n \vec{p}_i \cdot \vec{p}_f \right\} \]
Oxford Model

\[ L_{Oxf} = -ig_{\text{NN}\pi} F_S \bar{\psi} \gamma_5 \psi \cdot \vec{p} - ig_{\text{NN}\sigma} F_S \bar{\psi} \psi \sigma + i V_{\text{OGE+CI}} [\bar{\psi} \psi \psi]. \]

- Wanted to try a simple model
- Introduced Charge Independence Breaking and Charge Symmetry Effects.
- Gave ourselves the freedom to vary \( m_\sigma \) and \( \alpha = \beta \) by partial wave.
- Found a need for additional repulsion in the 1P1, 3P0, and 3P1 channels so we added an \( \omega \) exchange to those channels.
NN Free Scattering Phase Shifts

\[(T + V)\psi = H\psi = E\psi = \frac{p^2}{2m}\psi\]

\[r \to \infty \Rightarrow \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{l(l + 1)}{2mr^2} + \frac{p^2}{2m} \right) \frac{u_l(r)}{r} \approx 0\]

Thus as \(r \to \infty \Rightarrow u_l(r) \approx \sin(pr + \frac{1}{2}l\pi + \delta_l(p))\)

Spin 1 \(\Rightarrow \delta\) for \(^1S_0, ^3S_1, ^3P_0, \ldots\) etc. and \(\varepsilon_1, \varepsilon_2, \ldots\) etc.
T=0 Phase Shifts
T=1 Phase Shifts
Scattering Lengths, Effective Ranges

\[(T + V)\psi = H\psi = E\psi = \frac{p^2}{2m}\psi\]

\[r \to \infty \Rightarrow \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{l(l + 1)}{2mr^2} + \frac{p^2}{2m} \right) \frac{u_l(r)}{r} \approx 0\]

Thus as \(r \to \infty \Rightarrow u_l(r) \approx \sin(pr + \frac{1}{2}l\pi + \delta_l(p))\)

Spin 1 \(\Rightarrow \delta \) for \(^1S_0, ^3S_1, ^3P_0, \ldots \) etc. and \(\varepsilon_1, \varepsilon_2, \ldots \) etc.

\[pcot(\delta_S(p)) \approx -\frac{1}{a} + \frac{1}{2}r_{\text{eff}}p^2 + O(p^4)\]
## Scattering Lengths and Effective Ranges by Channel

<table>
<thead>
<tr>
<th>Channel</th>
<th>Oxford</th>
<th>CD-Bonn</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1S_0$</td>
<td>$-7.8154$</td>
<td>$-7.8149 \pm 0.0029$</td>
<td></td>
</tr>
<tr>
<td>$r_{pp}^C$</td>
<td>2.773</td>
<td>2.769 ± 0.014</td>
<td></td>
</tr>
<tr>
<td>$a_{pp}^C$</td>
<td>-17.33 ± 0.03</td>
<td>-17.4602</td>
<td></td>
</tr>
<tr>
<td>$r_{pp}^N$</td>
<td>2.806 ± 0.004</td>
<td>2.845</td>
<td></td>
</tr>
<tr>
<td>$a_{pp}^N$</td>
<td>-18.63 ± 0.03</td>
<td>-18.9680</td>
<td>-18.9 ± 0.4</td>
</tr>
<tr>
<td>$r_{nn}^N$</td>
<td>2.788 ± 0.004</td>
<td>2.819</td>
<td>2.75 ± 0.11</td>
</tr>
<tr>
<td>$a_{np}^N$</td>
<td>-23.6 ± 0.1</td>
<td>-23.7380</td>
<td>-23.74 ± 0.020</td>
</tr>
<tr>
<td>$r_{np}^N$</td>
<td>2.714 ± 0.006</td>
<td>2.671</td>
<td>2.77 ± 0.05</td>
</tr>
<tr>
<td>$^3S_1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_t^N$</td>
<td>5.498 ± 0.008</td>
<td>5.4196</td>
<td>5.419 ± 0.007</td>
</tr>
<tr>
<td>$r_t^N$</td>
<td>1.763 ± 0.007</td>
<td>1.751</td>
<td>1.753 ± 0.008</td>
</tr>
</tbody>
</table>
Dependence of $T=1$ Scattering Length Against Mass of the Sigma
Deuteron Properties

\[ H\psi = (T + V)\psi = E\psi = \frac{(iq)^2}{2\mu}\psi \]

\[ \Rightarrow \left[ \begin{pmatrix} T & 0 \\ 0 & T \end{pmatrix} + \begin{pmatrix} V_{SS} & V_{SD} \\ V_{DS} & V_{DD} \end{pmatrix} \right] \begin{pmatrix} \psi_S \\ \psi_D \end{pmatrix} = \frac{(iq)^2}{2\mu} \begin{pmatrix} \psi_S \\ \psi_D \end{pmatrix} \]

\[ r_m = \frac{1}{2} \sqrt{\int (\psi_S^2 + \psi_D^2) r^2 \, dr} \]

\[ P_D = \int \psi_D^2 r^2 \, dr \]

\[ Q_D = \frac{1}{20} \int \sqrt{8} \psi_D^\dagger r^2 \psi_S - \psi_D^\dagger r^2 \psi_D \, dr \]
Deuteron Properties (Cont'd)

\[ H\psi = (T + V)\psi = E\psi = \frac{(iq)^2}{2\mu}\psi \]

\[ \Rightarrow \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} + \begin{bmatrix} V_{SS} & V_{SD} \\ V_{DS} & V_{DD} \end{bmatrix} \begin{bmatrix} \psi_S \\ \psi_D \end{bmatrix} = \frac{(iq)^2}{2\mu} \begin{bmatrix} \psi_S \\ \psi_D \end{bmatrix} \]

In the limit \( r \to \infty \) \( \Rightarrow \)

\[ \begin{bmatrix} \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{0}{2\mu r^2} + \frac{(iq)^2}{2\mu} \\ \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{2(2+1)}{2\mu r^2} + \frac{(iq)^2}{2\mu} \end{bmatrix} \begin{bmatrix} \psi_S \\ \psi_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

\[ \lim_{r \to \infty} \psi_S(r) \approx A_S h_0(iqr) = A_S \frac{e^{-qr}}{qr} \]

\[ \lim_{r \to \infty} \psi_D(r) \approx A_D h_2(iqr) = A_D \frac{e^{-qr}}{qr} \left(1 + \frac{3}{qr} + \frac{3}{(qr)^2}\right) \]

\[ \eta = \frac{A_D}{A_S} \quad N^2 = A_S^2 + A_D^2 \]
Deuteron Results

<table>
<thead>
<tr>
<th>Property</th>
<th>Oxford</th>
<th>CD-Bonn</th>
<th>Nijmegen</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_D[\text{MeV}]$</td>
<td>2.226(2)</td>
<td>2.224575</td>
<td>2.224575</td>
<td>2.224575 ± 0.000009</td>
</tr>
<tr>
<td>$A_S[\text{fm}^{1/2}]$</td>
<td>?</td>
<td>0.8846</td>
<td>0.8845</td>
<td>0.8848 ± 0.0009</td>
</tr>
<tr>
<td>$\eta$</td>
<td>?</td>
<td>0.0256</td>
<td>0.0256</td>
<td>0.0256 ± 0.004</td>
</tr>
<tr>
<td>$r_m[\text{fm}]$</td>
<td>?</td>
<td>1.966</td>
<td>1.9676</td>
<td>1.971 ± 0.006</td>
</tr>
<tr>
<td>$Q_d[\text{fm}^2]$</td>
<td>?</td>
<td>0.270</td>
<td>0.271</td>
<td>0.28590 ± 0.00030</td>
</tr>
<tr>
<td>$P_D[%]$</td>
<td>6.75(1)</td>
<td>4.85</td>
<td>5.67</td>
<td></td>
</tr>
</tbody>
</table>

Obviously we still have some things to calculate here. The efforts are underway!
Imagine a Volume $V$ with $A$ nucleons. We assume that the system has periodic boundary conditions and has translational invariance. The limit as $V$ and $A \to \infty$ but $\rho = A/V = \text{constant}$ of this matter is called Nuclear Matter.

Two variables:
- Density $(\rho)$
- Proton Fraction: $x_p$
  - $x_p = 1/2$ is Symmetric Nuclear Matter (SNM)
  - $x_p = 0$ is Pure Neutron Matter (PNM)
  - $x_p = \text{else}$ is Asymmetric Nuclear Matter (ANM)

We are interested in $E/A$, the Equation of State (EOS).
Properties of Nuclear Matter from Heavy Nuclei

\[
\frac{E}{A} = a_1 + a_2 A^{-\frac{1}{3}} + a_3 x_p^2 A^{-\frac{10}{3}} + a_4 (1 - 2x_p)^2 + a_5 A^{-\frac{7}{4}}
\]

\[
\lim_{A \to \infty} \left. \frac{E}{A} \right|_{x_p = \frac{1}{2}} = a_1 \approx -16.1 \text{MeV}
\]

\[
\lim_{A \to \infty} \frac{\partial^2 E}{\partial x_p^2} = 8a_4 \Rightarrow \frac{1}{8} \lim_{A \to \infty} \frac{\partial^2 E}{\partial x_p^2} = a_4 \approx 23 \text{MeV}
\]

\[
\lim_{A \to \infty} \frac{\partial^2 E}{\partial x_p^2} \approx \lim_{A \to \infty} \left. \frac{E}{A} \right|_{x_p = \frac{1}{2}} - \lim_{A \to \infty} \left. \frac{E}{A} \right|_{x_p = 0}
\]
Qualitative EOS for SNM
Change of EOS from SNM to PNM
Calculating the EOS

\[ H\psi = (T + V)\psi = E\psi \]

• Need to solve the many body Schrödinger equation.
• Non-trivial but technical.
• First person to try perturbation theory techniques to nuclear matter: Brueckner. Hence the Brueckner Hartree Fock formalism.
Formalism and Interaction Matter!

Nuclear matter saturation point and symmetry energy with modern nucleon-nucleon potentials


PHYSICAL REVIEW C 74, 047304 (2006)
Making a 2-body Interaction Fit
EOS Phenomenology

Equation of state of nucleon matter and neutron star structure

A. Akmal,* V. R. Pandharipande,† and D. G. Ravenhall†
EOS Results
EOS Results (Cont'd.)
EOS Results (Cont'd)

Symmetry energy [MeV] vs. $\rho [\text{fm}^{-1}]$

- Oxford
- NSC97e
## Comparison of EOS Results

<table>
<thead>
<tr>
<th>Model</th>
<th>$\rho_0$ [fm$^{-3}$]</th>
<th>Symmetry Energy</th>
<th>B/A [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paris</td>
<td>0.270</td>
<td>29.4</td>
<td>-17.6</td>
</tr>
<tr>
<td>Argonne 18</td>
<td>0.270</td>
<td>29.9</td>
<td>-17.3</td>
</tr>
<tr>
<td>Bonn A</td>
<td>0.259</td>
<td>32.1</td>
<td>-28.4</td>
</tr>
<tr>
<td>Bonn B</td>
<td>0.419</td>
<td>31.8</td>
<td>-22.0</td>
</tr>
<tr>
<td>Bonn C</td>
<td>0.341</td>
<td>28.5</td>
<td>-16.4</td>
</tr>
<tr>
<td>CD-Bonn</td>
<td>0.257</td>
<td>31.1</td>
<td>-21.9</td>
</tr>
<tr>
<td>Reid ’93</td>
<td>0.328</td>
<td>30.0</td>
<td>-19.8</td>
</tr>
<tr>
<td>Nijmegen ’93</td>
<td>0.285</td>
<td>30.4</td>
<td>-19.6</td>
</tr>
<tr>
<td>Nijmegen I</td>
<td>0.348</td>
<td>30.5</td>
<td>-20.7</td>
</tr>
<tr>
<td>Nijmegen II</td>
<td>0.326</td>
<td>29.5</td>
<td>-19.4</td>
</tr>
<tr>
<td>N$^3$LO</td>
<td>0.408</td>
<td>31.2</td>
<td>-24.5</td>
</tr>
<tr>
<td>NSC97e</td>
<td>0.25</td>
<td>38.0</td>
<td>-17.48</td>
</tr>
<tr>
<td>Oxford</td>
<td>0.21</td>
<td>36.4</td>
<td>-16.38</td>
</tr>
</tbody>
</table>
Phase Shifts and Potential Contributions
Phase Shifts and Potential Contributions (Cont'd.)
Mind the Gap.
Oxford and Nijmegen Potential by Partial Wave
Oxford and Nijmegen Potential by Partial Wave (Cont'd.)
Thoughts, Conclusions, etc.

- The most significant failure of the model is the necessity of including an ad-hoc $\omega$ exchange in some P-waves.
- The range of parameter variation by partial wave, while not extreme, is disappointing.
- We can alter the physics of the model self-consistently to explore other mechanisms ($2\pi$ exchange, etc.)
Thoughts, Conclusions, etc. (Cont'd.)

- The ability of the simple and constrained model to well reproduce so many observables connected to the NN interaction is surprising and encouraging.

- The model raises important physics issues:
  - Non-locality
  - $\rho$ exchange
  - High momentum behavior, etc.

- Work continues.
End of Talk

Thank you for your attention!

Questions?

Feedback?