Analyticity of the Scattering Amplitude and Resonance Parameters in the Meson Exchange Picture

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The Jülich model of pion-nucleon interaction

- Analytic coupled channel model based on meson exchange (chiral Lagrangian of Wess and Zumino).
- Channels $\pi N, \eta N$; effective $\pi \pi N$ channels $\sigma N, \rho N, \pi \Delta$.
- Baryonic resonances up to $J = 3/2$ with derivative couplings as required by chiral symmetry.

Scattering equation in the \( JLS \) basis

Scattering equation guarantees two-body unitarity: \( T = V + VGT \)

\[
\langle L'S' k' | T_{IJ}^{\mu\nu} | LSk \rangle = \langle L'S' k' | V_{IJ}^{\mu\nu} | LSk \rangle \\
+ \sum_{\gamma,L''S''} \int_{0}^{\infty} dk'' \langle L'S' k' | V_{IJ}^{\mu\gamma} | L''S'' k'' \rangle \frac{1}{Z - E_{\gamma}(k'') + i\epsilon} \langle L''S'' k'' | T_{IJ}^{\gamma\nu} | LSk \rangle
\]

\( T \): Amplitude \hspace{1cm} \( V \): Pseudopotential \hspace{1cm} \( G \): Propagator

\( J \): total angular momentum
\( L \): orbital angular momentum
\( S \): total Spin of MB system
\( I \): isospin
\( k(k', k'') \): incoming(outgoing, intermediate) momentum, on- or off-shell
\( \mu(\nu, \gamma) \): incoming(outgoing, intermediate) channel \([\pi N, \eta N, \pi \Delta, \rho N, \sigma N]\)
Partial waves in $\pi N \rightarrow \pi N$

Data: GWU/SAID analysis, single energy solution

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Partial waves in $\pi N \rightarrow \pi N$

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Coupled channels and gauge invariance
Haberzettl, PRC56 (1997), Haberzettl, Nakayama, Krewald, PRC74 (2006)

Hadronic scattering:

\[
\pi N \rightarrow \pi N\]

\[
\gamma N \rightarrow \pi N
\]

Photo-production:

\[
\text{Gauge invariance: Generalized Ward-Takahashi identity (WTI)}
\]

(Note the condition of current conservation \( k_{\mu}M^{\mu} = 0 \) is necessary but not sufficient!)

\[
k_{\mu}M^{\mu} = - |F_{sT}\rangle S_{p+k}Q_{i}S^{-1}_{p} + S_{p+k}^{-1}Q_{j}S_{p'} - k |F_{uT}\rangle + \Delta_{p-p'}^{-1} + \sum_{Q} \Delta_{p-p'}^{\pi} |F_{rT}\rangle
\]

Strategy: Replace \( \text{by phenomenological contact term such that the generalized WTI is satisfied} \)
Results

Analytic continuation

$d\sigma/d\Omega$ and $\Sigma_\gamma$ for $\gamma p \rightarrow \pi^+ n$

preliminary (F. Huang, K. Nakayama)

**Differential cross section for $\gamma p \rightarrow \pi^+ n$**

**Photon spin asymmetry for $\gamma p \rightarrow \pi^+ n$**
Results

Analytic continuation

\[ \frac{d\sigma}{d\Omega} \text{ and } \Sigma_{\gamma} \text{ for } \gamma n \rightarrow \pi^- p \]

preliminary (F. Huang, K. Nakayama)

Differential cross section for \( \gamma n \rightarrow \pi^- p \)

Photon spin asymmetry for \( \gamma n \rightarrow \pi^- p \)
$d\sigma/d\Omega$ and $\Sigma_\gamma$ for $\gamma p \rightarrow \pi^0 p$

preliminary (F. Huang, K. Nakayama)

Differential cross section for $\gamma p \rightarrow \pi^0 p$

Photon spin asymmetry for $\gamma p \rightarrow \pi^0 p$
Coupled channel effects in photo- & electro-production

Dynamically generated resonances

- Next task: Evaluate the Roper transition form factor!
- Example: the $N^*(1535)$ as a dynamically generated resonance:
  [D. Jido, E. Oset, M.D., PRC77 (2008)]

Data: JLab

- Predictions possible.

Coupled channel effects in $\gamma N \rightarrow \eta N$

[M.D., K. Nakayama, EPJA43 (2010), PLB683 (2010)]
Coupled channel effects in photo- & electro-production

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Coupled channel effects in $\gamma N \rightarrow \eta N$

[M.D., K. Nakayama, EPJA43 (2010), PLB683 (2010)]
Propagator of stable particles $\pi\pi$, $\pi N$, $\eta N$ (c.m. system)

$$\Pi_\sigma(z) = \int_0^\infty q^2 dq \frac{(\nu_{\sigma\pi\pi}(q, z))^2}{z - E_1 - E_2 + i\epsilon}$$

$$E_{1,2} = \sqrt{m_{1,2}^2 + q^2}$$

→ Righthand cut, two sheets, and one branch point!
Contour deformation
...enables access to both Riemann sheets

\[ \Pi_\sigma(z) = \int_0^\infty q^2 dq \frac{(v^{\sigma\pi\pi}(q))^2}{z - E_1 - E_2 + i\epsilon} \]

\[ z - E_1 - E_2 = 0 \iff q = q_{c.m.} \]

\[ q_{c.m.} = \frac{1}{2z} \sqrt{[z^2 - (m_1 - m_2)^2][z^2 - (m_1 + m_2)^2]} \]

- Plot \( q_{c.m.}(z) \) in the \( q \) plane of integration (X: Pole positions).
- case (a), Im \( z > 0 \): straight integration from \( q = 0 \) to \( q = \infty \).
- case (b), Im \( z = 0 \): Pole is on real \( q \) axis.
- case (c), Im \( z < 0 \): Deformation gives analytic continuation.

Special case: Pole at \( q = 0 \)
\[ \iff \text{branch point at } z = m_1 + m_2 (= \text{threshold}). \]
Effective $\pi\pi N$ channels: Analytic structure

- The cut along $\text{Im} \, z = 0$ is induced by the cut of the self energy of the unstable particle.
- The poles of the unstable particle ($\sigma$) induce branch points in the $\sigma N$ propagator at

$$z_{b_2} = m_N + z_0, \quad z_{b'_2} = m_N + z_0^*$$

Three branch points and four sheets for each of the $\sigma N$, $\rho N$, and $\pi \Delta$ propagators.
The cut along $\text{Im } z = 0$ is induced by the cut of the self energy of the unstable particle.

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$$z_{b_2} = m_N + z_0, \quad z_{b_2'} = m_N + z_0^*$$

Three branch points and four sheets for each of the $\sigma N$, $\rho N$, and $\pi \Delta$ propagators.
Pole search

on the unphysical sheets, given by the analytic continuation.

<table>
<thead>
<tr>
<th>Pole position and residues</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure of the P11 partial wave</td>
</tr>
</tbody>
</table>

| Pole | Re $z_0$ [MeV] | -2 Im $z_0$ [MeV] | $|R|$ [MeV] | $\theta$ [deg] |
|------|----------------|------------------|-----------|-------------|
| $N^*$ (1535) $S_{11}$ | 1519 | 129 | 31 | -3 |
| ARN | 1502 | 95 | 16 | -16 |
| HOE | 1487 | | | |
| CUT | 1510±50 | 260±80 | 120±40 | +15±45 |
| $N^*$ (1650) $S_{11}$ | 1669 | 136 | 54 | -44 |
| ARN | 1648 | 80 | 14 | -69 |
| HOE | 1670 | 163 | 39 | -37 |
| CUT | 1640±20 | 150±30 | 60±10 | -75±25 |
| $N^*$ (1720) $P_{13}$ | 1663 | 212 | 14 | -82 |
| ARN | 1666 | 355 | 25 | -94 |
| HOE | 1686 | 187 | 15 | |
| CUT | 1680±30 | 120±40 | 8±12 | -160±30 |
| $\Delta$ (1232) $P_{33}$ | 1218 | 90 | 47 | -37 |
| ARN | 1211 | 99 | 52 | -47 |
| HOE | 1209 | 100 | 50 | -48 |
| CUT | 1210±1 | 100±2 | 53±2 | -47±1 |
| $\Delta^*$ (1620) $S_{31}$ | 1593 | 72 | 12 | -108 |
| ARN | 1595 | 135 | 15 | -92 |
| HOE | 1608 | 116 | 19 | -95 |
| CUT | 1600±15 | 120±20 | 15±2 | -110±20 |
| $\Delta^*$ (1700) $D_{33}$ | 1637 | 236 | 16 | -38 |
| ARN | 1632 | 253 | 18 | -40 |
| HOE | 1651 | 159 | 10 | |
| CUT | 1675±25 | 220±40 | 13±3 | -20±25 |
| $\Delta^*$ (1910) $P_{31}$ | 1840 | 221 | 12 | -153 |
| ARN | 1771 | 479 | 45 | +172 |
| HOE | 1874 | 283 | 38 | |
| CUT | 1880±30 | 200±40 | 20±4 | -90±30 |

[ARN]: Arndt et al., PRC 74 (2006), [HOE]: Höhler, $\pi N$ Newsl. 9 (1993), [CUT]: Cutkowski et al., PRD 20 (1979).

Residues to $\eta N$, $\sigma N$, $\rho N$, $\pi \Delta$. Zeros. Branching ratios to $\pi N$, $\eta N$.
Hidden poles in the $S_{11}$ partial wave

[Data: Arndt et al., FA08, EPJA 35 (2008)]

- Laurent series,

$$T^{(2)}_{ij} = \frac{a_{ij}^{0}}{z - z_0} + a_{ij}^{0} + \ldots$$

- Resonance interference of $N^*(1535)$ and $N^*(1650)$. 

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Hidden poles in the $S_{11}$ partial wave

- Laurent series,
  \[ T^{(2)}_{ij} = \frac{a^{ij}_{-1}}{z - z_0} + a^{ij}_0 + \ldots \]

- Resonance interference of $N^*(1535)$ and $N^*(1650)$.

[Data: Arndt et al., FA08, EPJA 35 (2008)]
Hidden poles in the $S_{11}$ partial wave

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- Different poles on different sheets produce the cusp.
- Two-pole structure! (x2 resonances)
Hidden poles in the $S_{11}$ partial wave

[Data: Arndt et al., FA08, EPJA 35 (2008)]

- Different poles on different sheets produce the cusp.
- Two-pole structure! (x2 resonances)
The two poles of the Roper resonance

  - two poles of the Roper, in all cases on two different $\pi\Delta$ sheets.

- Poles on hidden sheets can be only seen through the corresponding branch point \(\Rightarrow\) Structures from hidden poles appear as...
  - ... cusps if branch point on physical axis
    - \(N(1535)\) case close to $\eta N$ threshold
  - ... washed out cusps (non-standard resonance shape) if branch point in the complex plane (Roper case close to $\pi\Delta$ threshold).

- Proposed two-pole structure of the $\Lambda(1405)$ [Jido et al., NPA 725 (2003); M.D. et al. EPJA 32 (2007)]: both poles on same, non-hidden, sheet \(\rightarrow\) experimental tests may be possible.
An additional state in P11

* New pole in P11 found at $z=1620 + 297$ i MeV.

* Very weak branching to $\pi N$.
* Very large branching to $\pi \Delta$.
* Resonance transition amplitude $\pi N \rightarrow \pi \Delta$:
  Manley ($\pi N \rightarrow \pi \pi N$): -0.21
  here: -0.25
* Roper: generated from $\sigma N$.
* N(1620): generated from $\pi \Delta$.
* Dynamics of the nucleon pole: Pole repulsion N $\leftrightarrow$ Roper
Pole repulsion in $P_{33}$

- Poles in $T^{NP}$ may occur $\Rightarrow$ pole repulsion in $T = T^{NP} + T^P$.

- Back to P11:
  - $T^P$ given by renormalized/dressed nucleon.
  - Roper: pole in $T^{NP}$.
  - $T = T^{NP} + T^P$: Roper is moved to physical position; N $\leftrightarrow$ Roper repulsion.
Summary: The analytic structure of the P11 partial wave
Summary: The analytic structure of the P11 partial wave

EBAC
arXiv:0909.1356

N (1820)
Outlook: Matching with lattice results

- Pole position is quark mass dependent, Re $\Sigma(m_\pi)$.
- Quark mass dependence of effective $2\pi N$ channels (to be done).
- Analytic model allows for matching with lattice results ($m_\pi >> m_\pi^{\text{phys}}$).
Conclusions

- First results for the photoproduction have been obtained.
- Gauge invariant amplitude.
- The analytic continuation allows to access the sheets of the amplitude in the complex plane.
- Poles are found (in particular the dynamically generated Roper).
  - Positions.
  - Residue strengths and angles.
- Structure of the P11 amplitude.
  - Nucleon renormalized to its physical properties (is the only genuine pole term in $P_{11}$).
  - Roper dynamically generated mainly from $\sigma N$ channel.
  - Two poles of the Roper resonance, but on different sheets.
  - An additional state at $W = 1620$ MeV far in the complex plane, generated mainly from $\pi \Delta$; shares properties of the $N(1710) P_{11}$.
  - Resonance interference ("pole repulsion") between nucleon and Roper through intermediate states is important for the dynamics.
- Caution when identifying $T^{NP}$ as "background". $T^{NP}$ is model dependent; $T = T^P + T^{NP}$ is physically meaningful.
Current and future projects

- $\pi$ and $\eta$ photo- and Electroproduction.
- $\pi\pi N$ production.
- Inclusion of higher spin resonances ($J > 3/2$).
- Inclusion of $KY$ channels.
- Conceptual improvements: Consistent treatment of three-body cuts; three-body unitarity.
- Conceptual improvements: Matching to chiral perturbation theory.
### Couplings “ $g = \sqrt{a_{-1}}$ ” to other channels

<table>
<thead>
<tr>
<th>Channels</th>
<th>$N\pi$</th>
<th>$N\rho^{(1)} (S = 1/2)$</th>
<th>$N\rho^{(2)} (S = 3/2)$</th>
<th>$N\rho^{(3)} (S = 3/2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^*,(1535),S_{11}$</td>
<td>$S_{11}$ 8.1 + 0.5i</td>
<td>$S_{11}$ 2.2 − 5.4i</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>$N^*,(1650),S_{11}$</td>
<td>$S_{11}$ 8.6 − 2.8i</td>
<td>$S_{11}$ 0.9 − 9.1i</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>$N^*,(1440),P_{11}$</td>
<td>$P_{11}$ 11.2 − 5.0i</td>
<td>$P_{11}$ 1.3 − 3.2i</td>
<td>$P_{11}$ 3.6 − 2.6i</td>
<td>−</td>
</tr>
<tr>
<td>$\Delta^*,(1620),S_{31}$</td>
<td>$S_{31}$ 2.9 − 3.7i</td>
<td>$S_{31}$ 0.0 − 0.0i</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>$N^*,(1910),P_{31}$</td>
<td>$P_{31}$ 1.2 − 3.5i</td>
<td>$P_{31}$ 0.2 − 0.4i</td>
<td>$P_{31}$ 0.2 − 0.4i</td>
<td>−</td>
</tr>
<tr>
<td>$N^*,(1720),P_{13}$</td>
<td>$P_{13}$ 5.7 − 2.6i</td>
<td>$P_{13}$ 0.1 + 0.8i</td>
<td>$P_{13}$ 0.1 + 0.8i</td>
<td>$F_{13}$ 0.1 + 0.4i</td>
</tr>
<tr>
<td>$N^*,(1520),D_{13}$</td>
<td>$D_{13}$ 8.4 − 0.8i</td>
<td>$D_{13}$ 0.9 − 2.0i</td>
<td>$D_{13}$ 0.9 − 2.0i</td>
<td>$S_{13}$ −2.5 − 22.8i</td>
</tr>
<tr>
<td>$\Delta,(1232),P_{33}$</td>
<td>$P_{33}$ 17.9 − 3.2i</td>
<td>$P_{33}$ 0.9 − 3.0i</td>
<td>$P_{33}$ 0.9 − 3.0i</td>
<td>$F_{33}$ 0.0 + 0.0i</td>
</tr>
<tr>
<td>$\Delta^*,(1700),D_{33}$</td>
<td>$D_{33}$ 4.9 − 1.0i</td>
<td>$D_{33}$ 0.4 − 0.4i</td>
<td>$D_{33}$ 0.4 − 0.4i</td>
<td>$S_{33}$ −0.1 − 0.9i</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Channels</th>
<th>$\Delta\pi^{(1)}$</th>
<th>$\Delta\pi^{(2)}$</th>
<th>$N\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^*,(1535),S_{11}$</td>
<td>−</td>
<td>−</td>
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<tr>
<td>$N^*,(1650),S_{11}$</td>
<td>−</td>
<td>−</td>
<td>$D_{11}$ 4.3 + 0.4i</td>
</tr>
<tr>
<td>$N^*,(1440),P_{11}$</td>
<td>$P_{11}$ −0.1 + 0.0i</td>
<td>$P_{11}$ −4.6 − 1.7i</td>
<td>$D_{11}$ −2.1 + 1.0i</td>
</tr>
<tr>
<td>$\Delta^*,(1620),S_{31}$</td>
<td>−</td>
<td>−</td>
<td>$S_{11}$ −8.3 + 27.7i</td>
</tr>
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<td>$\Delta^*,(1910),P_{31}$</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>$N^*,(1720),P_{13}$</td>
<td>$P_{13}$ −7.7 + 5.5i</td>
<td>$P_{13}$ −14.1 + 3.0i</td>
<td>$F_{13}$ 0.0 + 0.3i</td>
</tr>
<tr>
<td>$N^*,(1520),D_{13}$</td>
<td>$D_{13}$ 0.16 − 0.60i</td>
<td>$D_{13}$ 0.0 + 0.4i</td>
<td>$D_{13}$ −0.8 + 0.4i</td>
</tr>
<tr>
<td>$\Delta,(1232),P_{33}$</td>
<td>−</td>
<td>−</td>
<td>$S_{13}$ −12.9 − 0.7i</td>
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</tr>
</tbody>
</table>

Resonance couplings $g_i$ [$10^{-3}$ MeV$^{-1/2}$] to the coupled channels $i$. Also, the $LJS$ type of each coupling is indicated. For the $\rho N$ channels, the total spin $S$ is also indicated.
Zeros and branching ratio to $\pi N$, $\eta N$

<table>
<thead>
<tr>
<th>first sheet</th>
<th>second sheet</th>
<th>[FA02]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{11}$</td>
<td>$S_{11}$</td>
<td>$1587 - 45,i$</td>
</tr>
<tr>
<td>$D_{33}$</td>
<td>$S_{31}$</td>
<td>$1585 - 17,i$</td>
</tr>
<tr>
<td>$P_{31}$</td>
<td>$1848 - 83,i$</td>
<td></td>
</tr>
<tr>
<td>$P_{13}$</td>
<td>$1607 - 38,i$</td>
<td></td>
</tr>
<tr>
<td>$P_{33}$</td>
<td>$1702 - 64,i$</td>
<td></td>
</tr>
<tr>
<td>$D_{13}$</td>
<td>$1702 - 64,i$</td>
<td></td>
</tr>
</tbody>
</table>


| $N^*(1535)$ $S_{11}$ | $48\, [33\text{ to } 55]$ | $38\, [45\text{ to } 60]$ |
| $N^*(1650)$ $S_{11}$ | $79\, [60\text{ to } 95]$ | $6\, [3\text{ to } 10]$ |
| $N^*(1440)$ $P_{11}$ | $64\, [55\text{ to } 75]$ | $0\, [0 \pm 1]$ |
| $\Delta^*(1620)$ $S_{31}$ | $34\, [20\text{ to } 30]$ | $-\, 0$ |
| $\Delta^*(1910)$ $P_{31}$ | $11\, [15\text{ to } 30]$ | $-\, 0$ |
| $N^*(1720)$ $P_{13}$ | $13\, [10\text{ to } 20]$ | $38\, [4 \pm 1]$ |
| $N^*(1520)$ $D_{13}$ | $67\, [55\text{ to } 65]$ | $0.10\, [0.23 \pm 0.04]$ |
| $\Delta(1232)$ $P_{33}$ | $100\, [100]$ | $-\, 0$ |
| $\Delta^*(1700)$ $D_{33}$ | $13\, [10\text{ to } 20]$ | $-\, 0$ |

Branching ratios into $\pi N$ and $\eta N$. The values in brackets are from the PDG, [Amsler et al., PLB 667 (2008)].
Couplings and dressed vertices

Residue $a_{-1}$ vs. dressed vertex $\Gamma$ vs. bare vertex $\gamma$. 

$\Gamma_D^{(\dagger)} = \gamma_B^{(\dagger)} + \gamma_B^{(\dagger)} G T^{NP}$

$\Sigma = \gamma_B^{(\dagger)} G \Gamma_D$

$S_D = S_B + S_B \Sigma S_D$

$T^P = \Gamma_D S_D \Gamma_D^{(\dagger)}$

$a_{-1} = \frac{\Gamma_d \Gamma_d^{(\dagger)}}{1 - \frac{\partial}{\partial Z} \Sigma}$

$g = \sqrt{a_{-1}}$

$r = \left| \frac{\Gamma_D - \gamma_B}{\Gamma_D} \right|$

$r' = \left| 1 - \sqrt{1 - \Sigma'} \right|$

- Dressed $\Gamma$ depends on $T^{NP}$.

- $\sqrt{a_{-1}} \neq \Gamma \neq \gamma$

<table>
<thead>
<tr>
<th>$N^*(1520)$ $D_{13}$</th>
<th>$N^*(1720)$ $P_{13}$</th>
<th>$\Delta(1232)$ $P_{33}$</th>
<th>$\Delta^*(1620)$ $S_{31}$</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$\gamma^C$</td>
<td>$\Gamma^C$</td>
<td>$r$ [%]</td>
<td>$r'$ [%]</td>
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<tr>
<td>$6.4 - 0.6i$</td>
<td>$13.2 + 1.2i$</td>
<td>53</td>
<td>61</td>
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<tr>
<td>$-0.1 + 5.4i$</td>
<td>$0.9 + 4.8i$</td>
<td>24</td>
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<td>$0.1 + 14.3i$</td>
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<td></td>
</tr>
<tr>
<td>$9.4 + 0.3i$</td>
<td>$1.9 - 3.2i$</td>
<td>222</td>
<td>22</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The $D_{13}$ partial wave

The $N^*(1520)$ and a dynamically generated pole in $T^{\text{NP}}$.

- $T^{\text{NP}}$: no $s$-channel $N^*(1520)$.
- Pole in $T^{\text{NP}}$ on 3rd $\rho N$ sheet.
- On physical axis visible through branch point $b_2$.
- Pole invisible in full solution.
  → We do not identify it with a dynamically generated $N^*(1700)$.
