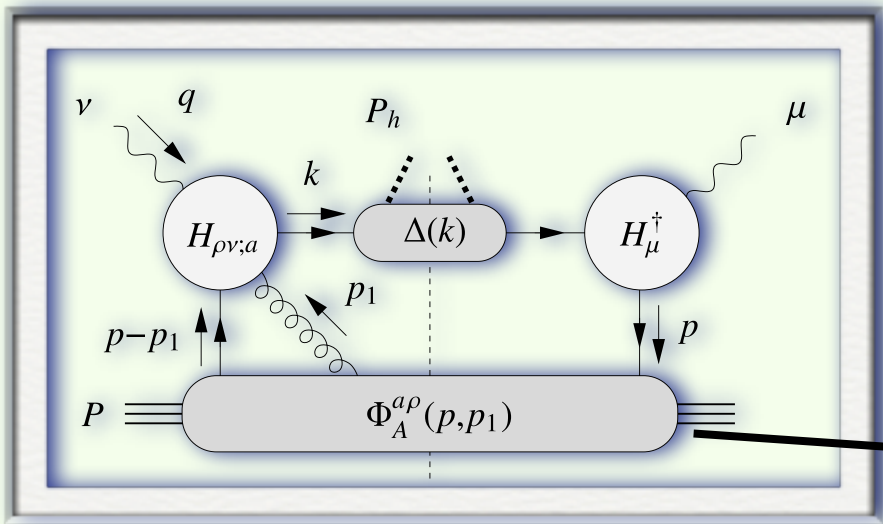
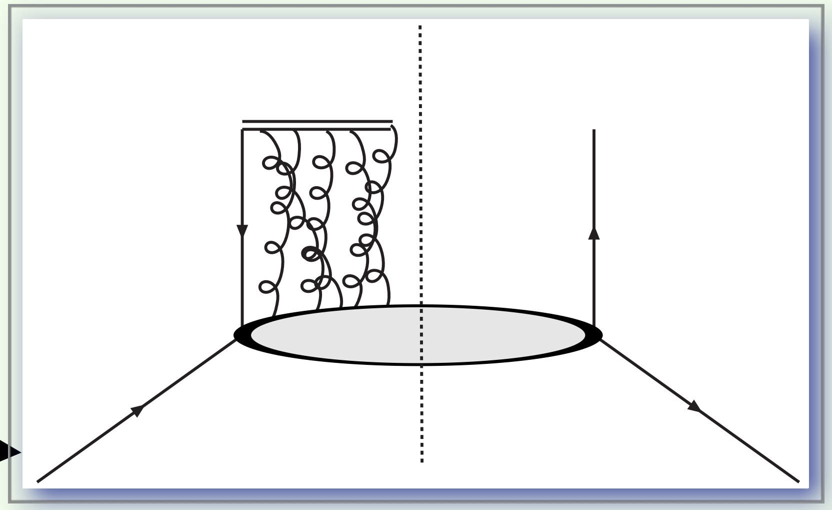
T-Odd Effects From Color Gauge Inv. via Wilson Line

Gauge link determined re-summing gluon interactions btwn soft and hard

Efremov,Radyushkin *Theor. Math. Phys. 1981*

Belitsky, Ji, Yuan NPB 2003, *Boer, Bomhof, Mulders Pijlman, et al. 2003 - 2008- NPB, PLB, PRD*

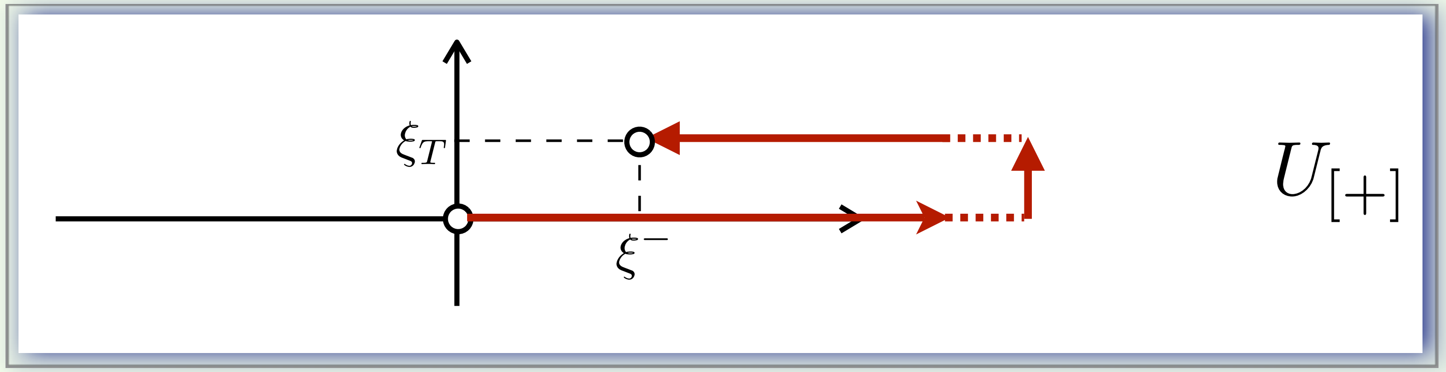




$$\Phi_{ij}(x,\vec{p}_T) = \int \frac{dz^- d^2 z_T}{(8\pi^3)} e^{ip\cdot z} \langle P|\bar{q}_j(0)\mathcal{U}^{[+]}_{[0,\xi]} q_i(z)|P\rangle|_{z^+=0}$$

- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

$$\int d^4p d^4k \delta^4(p+q-k) \mathrm{Tr}\left[\Phi^{[U^C_{[\infty;\xi]}}(p) H^\dagger_\mu(p,k)\Delta(k)H_\nu(p,k)\right]$$



Factorization Sensitivity to $P_T \sim k_\perp \longrightarrow$ TMDs

John Collins

Nuclear Physics B396 (1993) 161–182

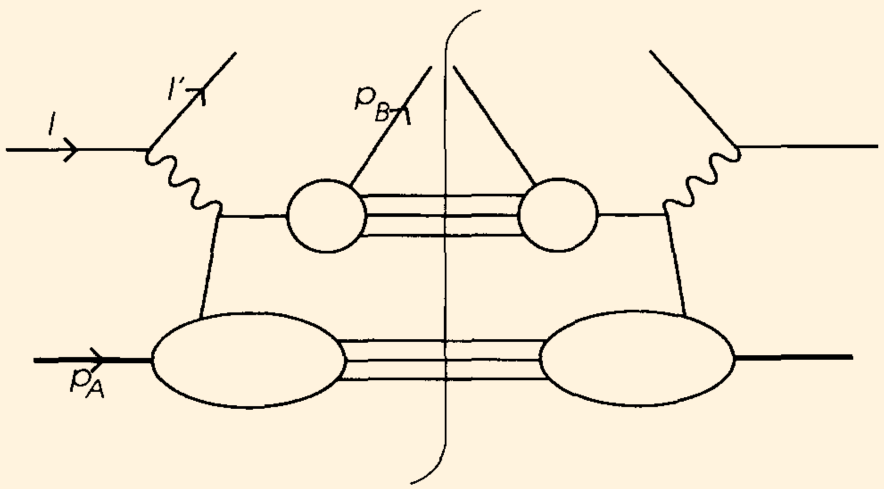


Fig. 2. Parton model for semi-inclusive deeply inelastic scattering.

3.4. FACTORIZATION WITH INTRINSIC TRANSVERSE MOMENTUM AND POLARIZATION

We now explain factorization for the semi-inclusive deep inelastic cross section when the incoming hadron A is transversely polarized but the lepton remains unpolarized. (It is left as an exercise to treat the most general case.) The factorization theorems, eq. (12) and eq. (14), continue to apply when we include polarization for the incoming hadron, but with the insertion of helicity density matrices for in and out quarks; this is a simple generalization of the results in refs. [10,23].

....

$$E'E_B \frac{\mathrm{d}\sigma}{\mathrm{d}^3 l'\, \mathrm{d}^3 p_B} = \sum_a \int \mathrm{d}\xi \int \frac{\mathrm{d}\zeta}{\zeta} \int \mathrm{d}^2 k_{a\perp} \int \mathrm{d}^2 k_{b\perp}\, \hat{f}_{a/A}(\xi, k_{a\perp})$$

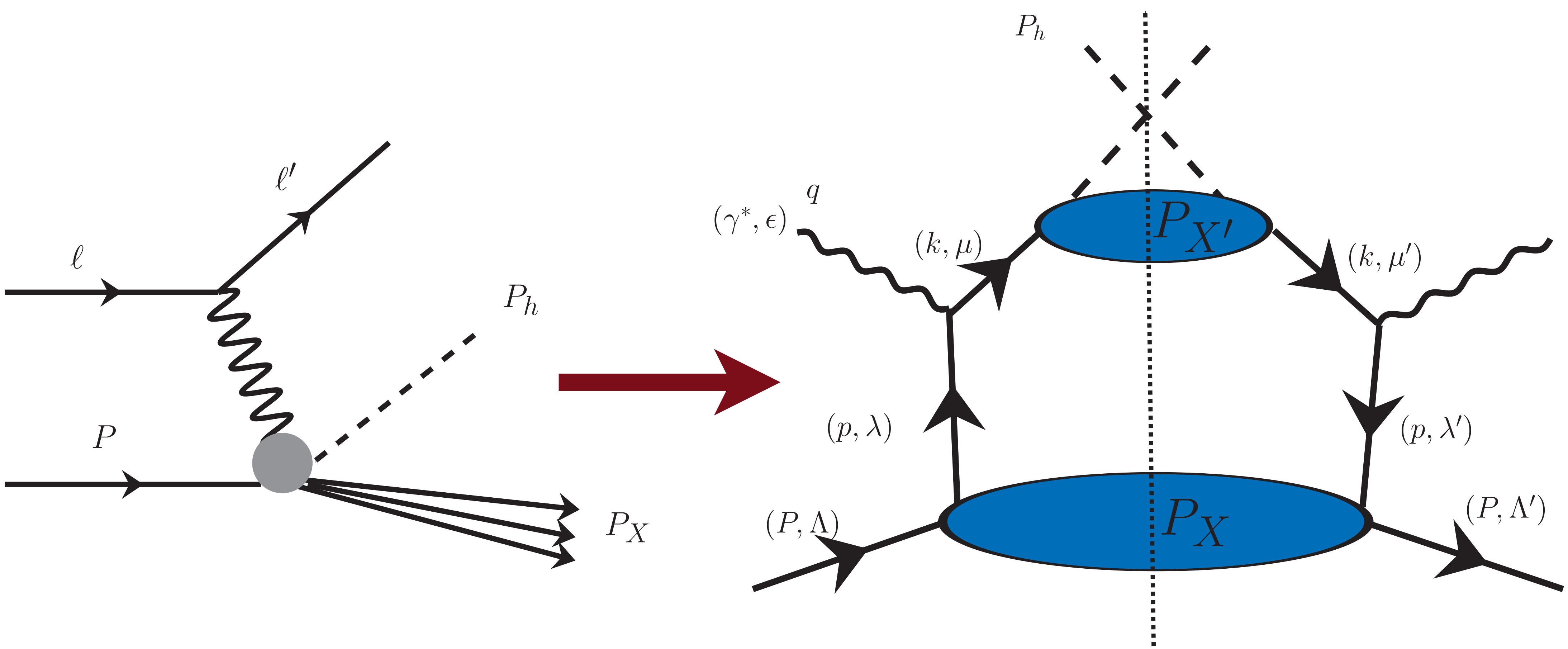
$$\times E'E_{k_b} \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}^3 l'\, \mathrm{d}^3 k_b} \hat{D}_{B/a}(\zeta, k_{b\perp}) + Y(x_{\mathrm{Bj}}, Q, z, q_\perp/Q)$$

The function $\hat{f}_{a/A}$ defined earlier gives the intrinsic transverse-momentum dependence of partons in the initial-state hadron. Similarly, $\hat{D}_{B/a}$ gives the distribution of hadrons in a parton, with $k_{b\perp}$ being the transverse momentum of the parton relative to the hadron.

How to motivate the PDFs and FFs from SIDIS at moderate P_T

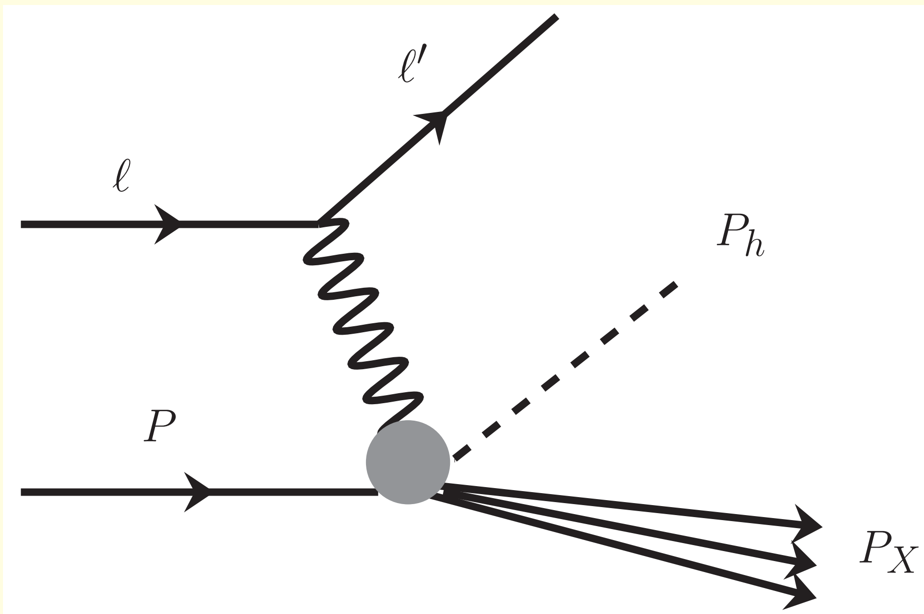
Consider the limit where Q^2 is large and

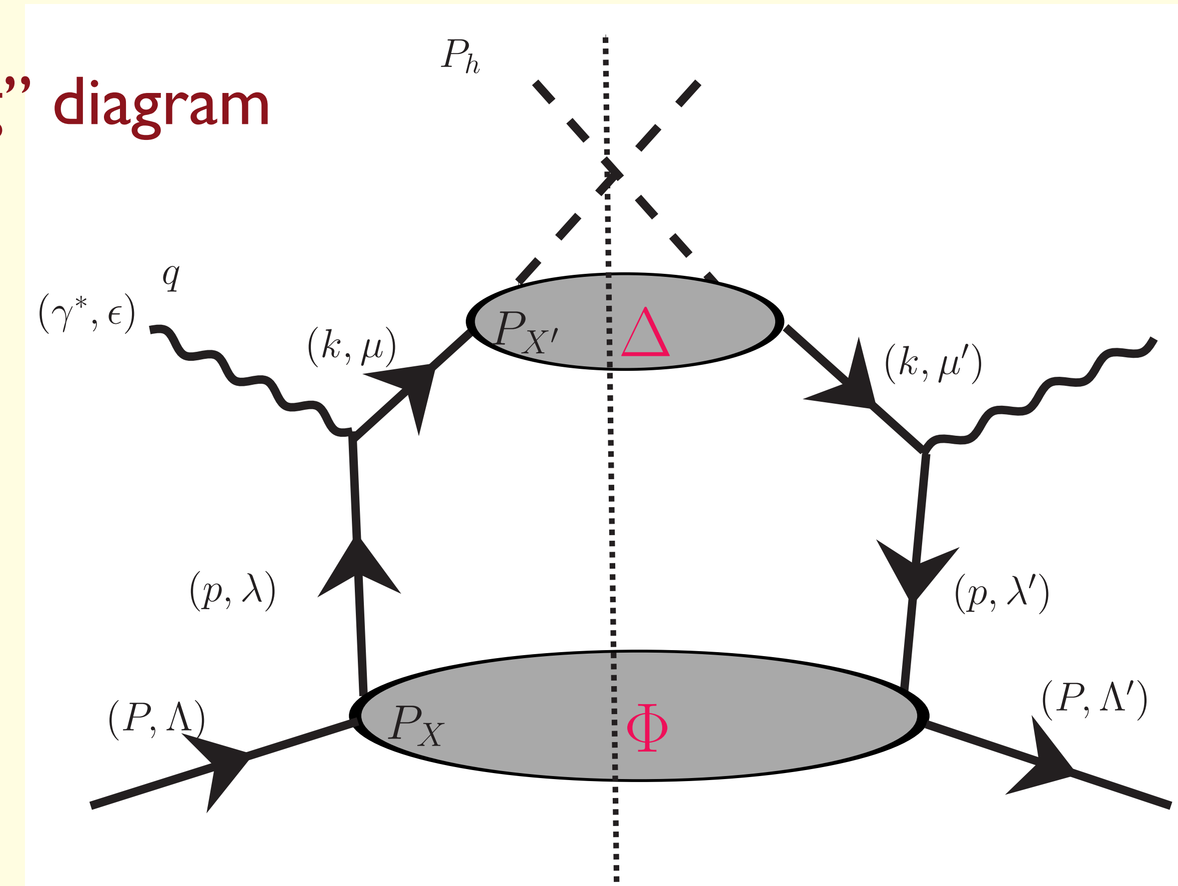
γ^* assumed to scatter *incoherently* off constituents. Currents treated as in free field theory. Interactions btwn. struck quark and target remnant are neglected at least most of the time---color gauge links will modify this statement



SIDIS and the parton model

Hand Bag" diagram





1) A "DIS" reaction where hadron in current region is detected in the final: state-rapidity sep.

2) Parton model assumption: virtual photon strikes quark inside nucleon.

3) In case of SIDIS the tagged final state hadron comes from fragmentation of struck quark.

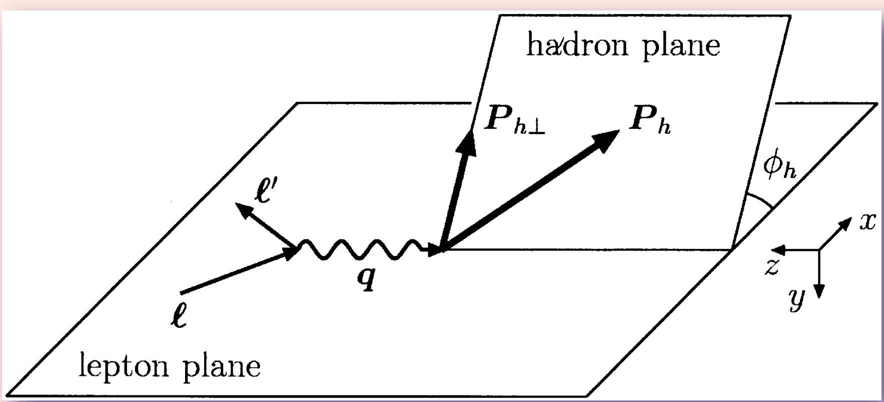
Hand Bag" diagram

4)The scattering process can be "factorized" into two soft hadronic parts connected by a hard scattering piece

Kinematics and “Geometry” of SIDIS

$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X,$$

$$x_B = \frac{Q^2}{2\,P\cdot q}, \qquad y = \frac{P\cdot q}{P\cdot l}, \qquad z_h = \frac{P\cdot P_h}{P\cdot q}$$



Lepton and hadron planes in semi-inclusive leptoproduction.

1) Interested in limit where Q^2 $p.q$, $P_h.q$ and $P.P_h$ become large while x_B and z_h remain finite

2)In case of SIDIS the tagged final state hadron comes from the fragmentation of the struck quark

Comments

- Interactions btwn. struck quark and target remnant are neglected at least most of the time---color gauge links will modify this statement
- Conserved quantities Helicity and Transversity (will come back to this & “transversity”)
- Tree level factorization-”diagramatic”
- Agrees with Landshoff-Polkinghorne-hints at non-perturbative dynamics

”*Covariant parton model” where non-perturbative dynamics are treated in terms of cuts and poles of soft factors. Will come back to this in seminar in May*

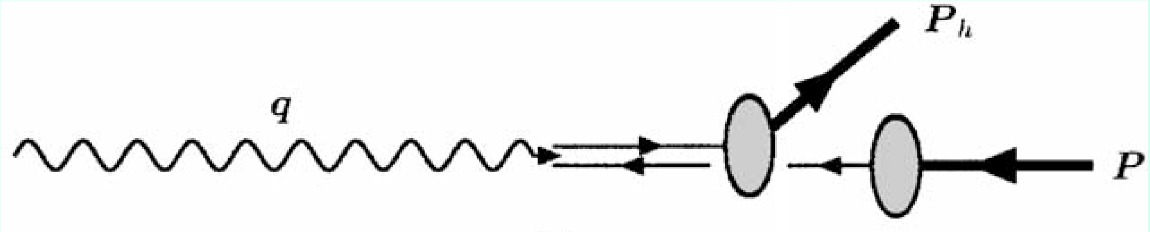
Light-cone kinematics Photon-Hadron Frame

Light Cone Vectors

$$V^+ = \frac{V^0 + V^3}{\sqrt{2}}, \quad V^- = \frac{V^0 - V^3}{\sqrt{2}}, \quad \mathbf{V}_T$$

$$V^\mu = (V^+, V^-, \mathbf{V}_T)$$

Photon – Hadron Frame

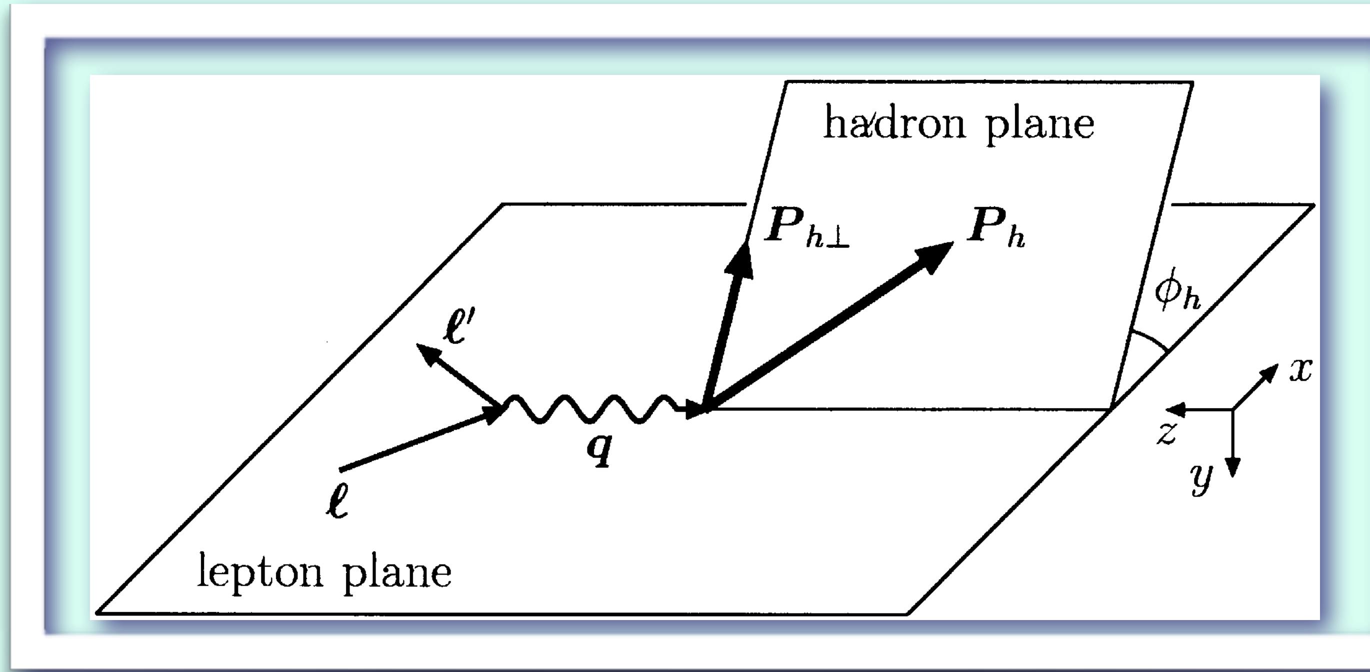


$$P^\mu = \left(P^+, \frac{M^2}{2P^+}, 0\right), \quad q^\mu = \left(-x_B P^+, \frac{Q^2}{2x_B P^+}, 0\right) \quad P_h = \left(\frac{m_h^2 + P_{h\perp}^2}{2P_h^-}, P_h^-, P_{h\perp}\right)$$

$$P^\mu = P^+ n_+^\mu + \frac{M^2}{2P^+} n_-^\mu$$

note from $z_h = \frac{P \cdot P_h}{P \cdot q} \approx \frac{P_h^-}{q^-}$

$$q^\mu = \left(-x_B P^+, \frac{P_h^-}{z_h}, 0\right) \Longrightarrow \frac{Q^2}{2x_B P^+} = \frac{P_h^-}{z_h}$$



Diagramatic Factorization

Cross Section for SIDIS

- Sum over spin of out going electron
- Differential in phase space of final detected electron and hadron
- Sum over all unobserved final states "X"

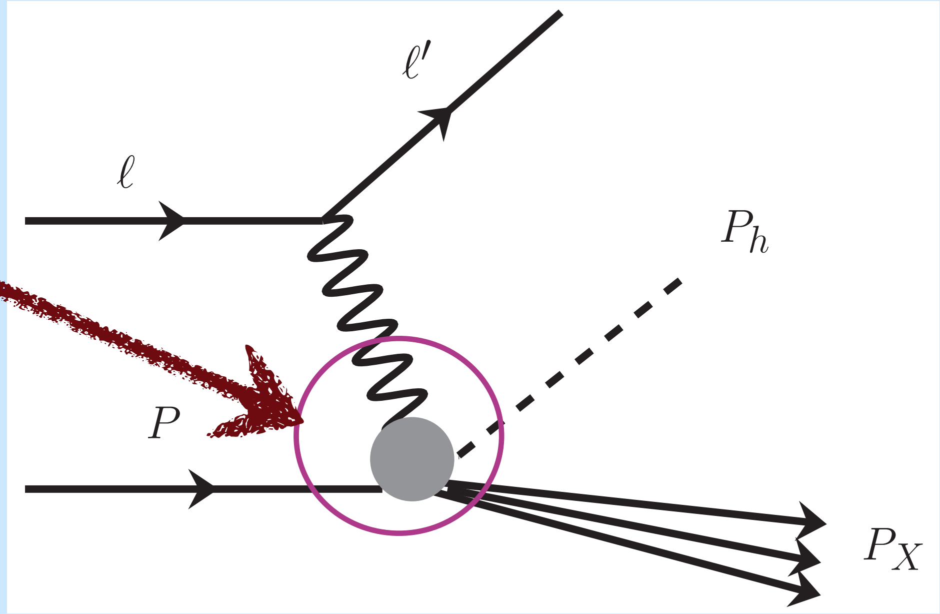
$$d\sigma = \frac{1}{4\cdot P}\sum_{s_{\ell'}}\sum_X \left\{\int \frac{d^3\mathbf{P}_X}{(2\pi)^3 2E_X}(2\pi)^4\delta^4(P+\ell-P_X-P_h-\ell')|M|^2\right\}\frac{d^3\ell'}{(2\pi)^3 2E'}\frac{d^3\mathbf{P}_h}{(2\pi)^3 2E_h}$$

$$|M|^2 = \frac{e^4}{q^4}L_{\mu\nu}W^{\mu\nu}$$

Leptonic & Hadronic Tensor

$$W^{\mu\nu} = \frac{1}{(2\pi)^4}\sum_X\int \frac{d^3\mathbf{P}_X}{(2\pi)^3 2E_X}(2\pi)^4\delta^4(P+\ell-P_X-P_h-\ell')$$
$$\times\langle PS|J^\mu(0)|X,P_hS_h\rangle\langle X,P_hS_h|J^\nu(0)|PS\rangle$$

$$d\sigma = \frac{1}{4\cdot P}\frac{e^4}{q^4}L_{\mu\nu}W^{\mu\nu}(2\pi)^4\frac{d^3\ell'}{(2\pi)^3 2E'}\frac{d^3\mathbf{P}_h}{(2\pi)^3 2E_h}$$



$$2E_h\frac{d\sigma}{d^3\mathbf{P}_h dE' d\Omega} = \frac{\alpha_{\rm em}^2}{2MQ^4}\frac{E'}{E}L_{\mu\nu}W^{\mu\nu}$$

Comments

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Cross Section for SIDIS in terms of invariants

$$x_B = \frac{Q^2}{2\,P\cdot q}, \qquad y = \frac{P\cdot q}{P\cdot l}, \qquad z_h = \frac{P\cdot P_h}{P\cdot q}$$

In terms of invariants

$$2E_h \frac{d\sigma}{dx_B dy d^3\mathbf{P}_h} = \frac{\pi\alpha_{\rm em}^2 y}{Q^4} L_{\mu\nu} W^{\mu\nu}$$

When $P_{h\perp} \ll P_h \sim E_h \Longrightarrow \quad \dfrac{d^3\mathbf{P}_h}{2E_h} = \dfrac{dz_h d^2\mathbf{P}_{h\perp}}{2z_h}$

Compact expression

$$\frac{d\sigma}{dx_B dy dz_h d^2\mathbf{P}_{h\perp}} = \frac{\pi\alpha_{\rm em}^2}{2Q^4}\frac{y}{z_h} L_{\mu\nu} W^{\mu\nu}$$

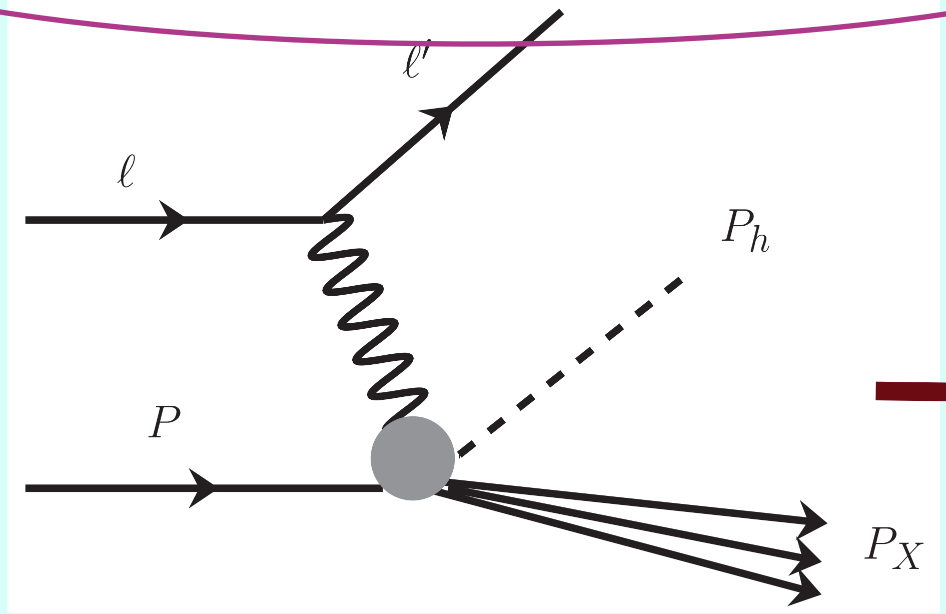
Comment on DIS from SIDIS

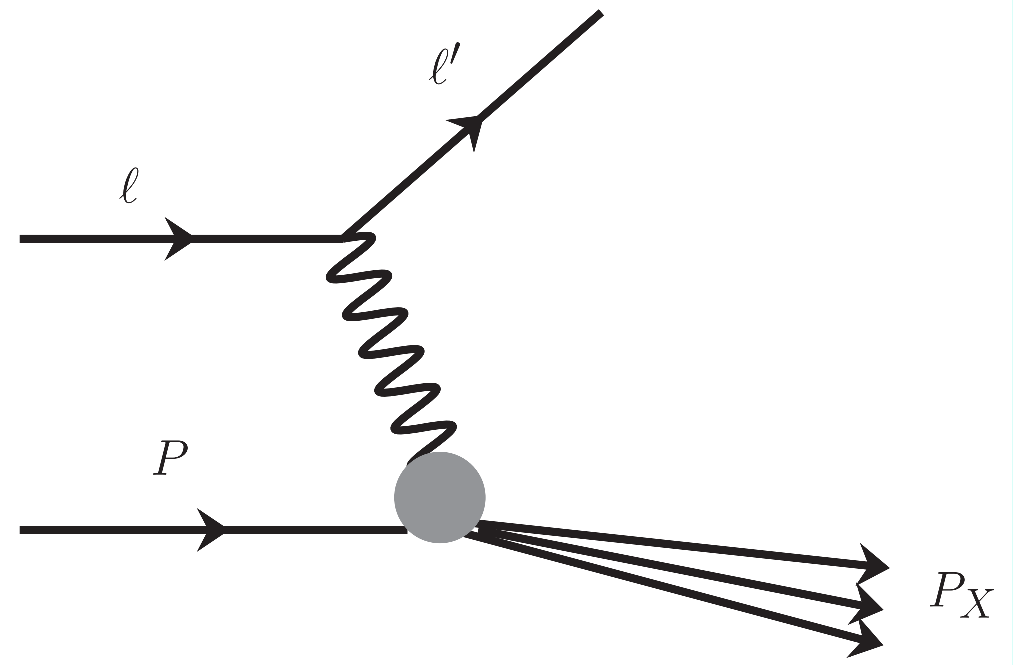
$$W^{\mu\nu}(q,P,S,P_h) = \frac{1}{(2\pi)^4}\sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3 2E_X}(2\pi)^4\delta^4(P+\ell-P_X-P_h-\ell')$$

$$\times\langle PS|J^\mu(0)|X,P_hS_h\rangle\langle X,P_hS_h|J^\nu(0)|PS\rangle$$

Summing over all possible hadrons and integrating over the final state hadron recover inclusive DIS result

$$\sum_h \int \frac{d^3\mathbf{P}_h}{(2\pi)^3 2E_h} W^{\mu\nu}(q,P,S,P_h) = W^{\mu\nu}(q,P,S)$$





Parton Model Factorization of SIDIS

$$W^{\mu\nu} = \frac{1}{(2\pi)^4}\sum_a e^2 \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3 2E_X} \sum_{X'} \int \frac{d^3\mathbf{P}_{X'}}{(2\pi)^3 2E_{X'}} \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4}$$

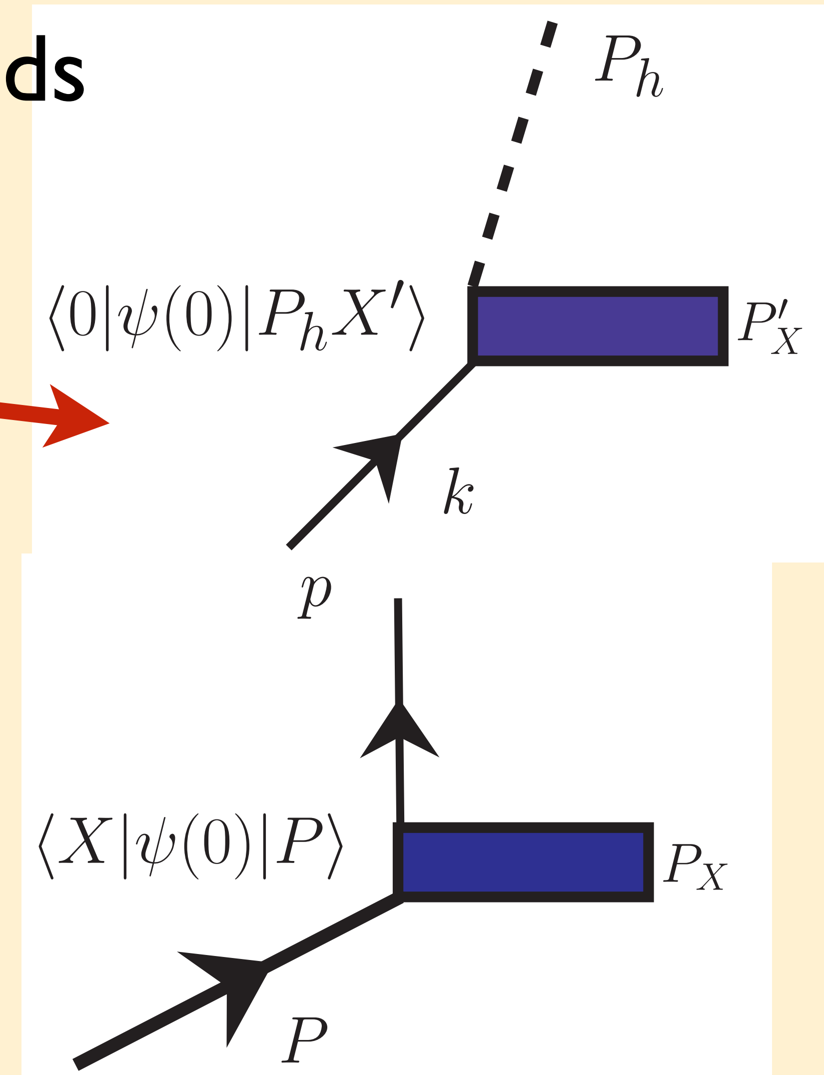
$$\times \int \frac{d^4k}{(2\pi)^4} (2\pi)^4 \delta^4(P - p - P_X)(2\pi)^4\delta^4(p + q - k)(2\pi)^4\delta^4(k - P_h - P_{X'})$$

$$\times [\bar{\chi}(k; P_h, S_h)\gamma^\mu \phi(p; P, S)]^\dagger \, [\bar{\chi}(k; P_h, S_h)\gamma^\nu \phi(p; P, S)]$$

Hadron Matrix element of quark fields

$$\chi(k; P_h, S_h) = \langle 0|\psi(0)|P_h S_h; X\rangle$$

$$\phi(p; P, S) = \langle X|\psi(0)|PS\rangle$$



Non-perturbative quark-quark Correlators from Hadron matrix elements of quark fields

$$\Phi_{ji}(p;P,S) = \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3 2E_X}(2\pi)^4\delta^4(P-p-P_X)\phi_i(p;P,S)\bar{\phi}_j(p;P,S)$$

exponentiate delta function

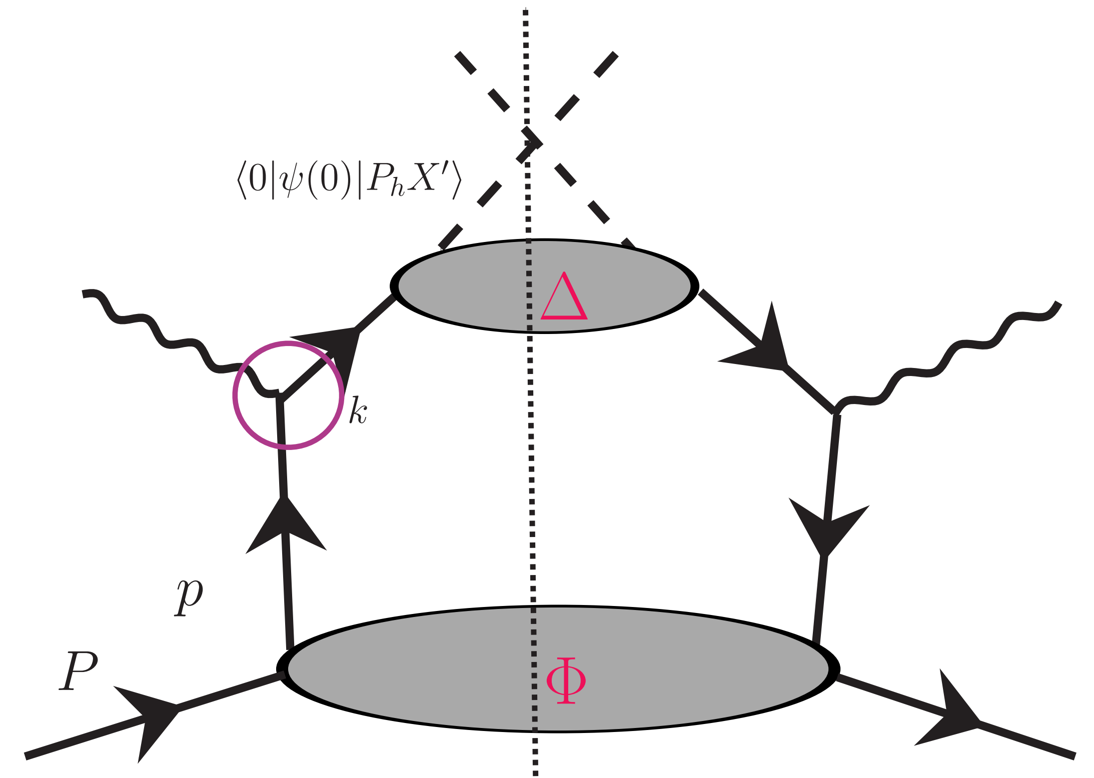
$$= \int d^4\xi\, e^{ip\cdot\xi}\langle PS|\bar{\psi}_j(0)\psi_i(\xi)|PS\rangle$$

and

$$\Delta_{ij}(k;P_hS_h) = \sum_{X'} \int \frac{d^3\mathbf{P}_{X'}}{(2\pi)^3 2E_{X'}}(2\pi)^4\delta^4(P_h+P_X-k)\chi_i(k;P_hS_h)\bar{\chi}(k;P_hS_h)$$

$$= \sum_{X'} \int \frac{d^3\mathbf{P}_{X'}}{(2\pi)^3 2E_{X'}} \int d^4\xi e^{ik\cdot\xi}\langle 0|\psi_i(\xi)|P_hS_h,X\rangle\langle P_hS_h,X|\bar{\psi}_j(0)|0\rangle$$

$$W^{\mu\nu}(q,P,S,P_h) = \sum_a e^2 \int \frac{d^4p}{(2\pi)^4}\int \frac{d^4k}{(2\pi)^4}\delta^4(p+q-k)\mathrm{Tr}\,[\Phi\gamma^\mu\Delta\gamma^\nu]$$



Light-cone kinematics quark momenta

$$\mathrm{QUARK} \quad \mathrm{Momenta}$$

$$p^\mu = (p^+, \frac{p^2 + p_\perp^2}{2p^+}, \mathbf{p}_\perp)$$ initial quark

$$k^\mu = (\frac{k^2 + k_\perp^2}{2k^-}, k^-, \mathbf{k}_\perp)$$ fragmenting quark

$$\mathrm{Parton} \quad \mathrm{assumptions}$$

$$p^+ \propto P^+, \quad k^- \propto P_h^- \qquad \Longleftrightarrow \mathrm{LARGE-Momenta} \sim Q$$

$$\mathrm{therefore} \quad p^- \propto 1/P^+, \quad k^+ \propto 1/P_h^- \qquad \Longleftrightarrow \mathrm{SMALL-Momenta} \sim 1/Q$$

Factorization of Long. Momenta

$$W^{\mu\nu}(q,P,S,P_h) = \sum_a e^2 \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \delta^4(p+q-k) \mathrm{Tr}\left[\Phi\gamma^\mu \Delta\gamma^\nu\right]$$

$$\delta^4(p+q-k) = \delta(p^+ + q^+ - k^+)\delta(p^- + q^- - k^-)\delta^2(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T)$$

Parton model assumptions

$$p^+ \propto P^+, \quad k^- \propto P_h^- \quad \Longleftrightarrow \mathrm{LARGE-Momenta} \sim Q$$

therefore

$$p^- \propto 1/P^+, \quad k^+ \propto 1/P_h^- \quad \Longleftrightarrow \mathrm{SMALL-Momenta} \sim 1/Q$$

$$\Longrightarrow \delta^4(p+q-k) \approx \delta(p^+ - x_B P^+)\delta(k^- - \frac{P_h}{z_h})\delta^2(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T)$$

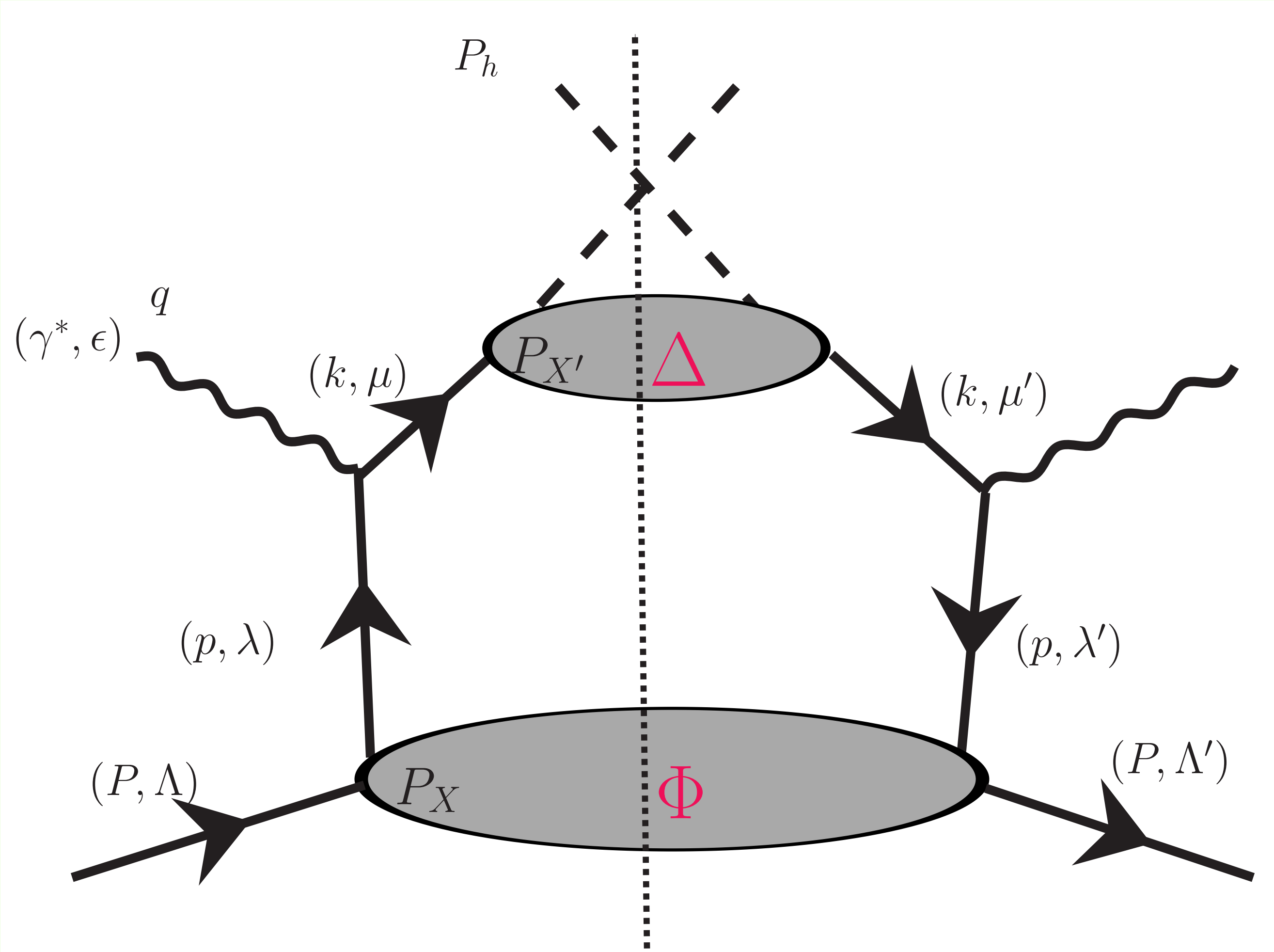
Message: the P_T of the hadron is small!

$$W^{\mu\nu}(q,P,S,P_h) \approx \sum_a e^2 \int \frac{d^2\mathbf{p}_T dp^- dp^+}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T dk^- dk^+}{(2\pi)^4} \delta(p^+ - x_B P^+)\delta(k^- - \frac{P_h^-}{z_h})\delta^2(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T)$$

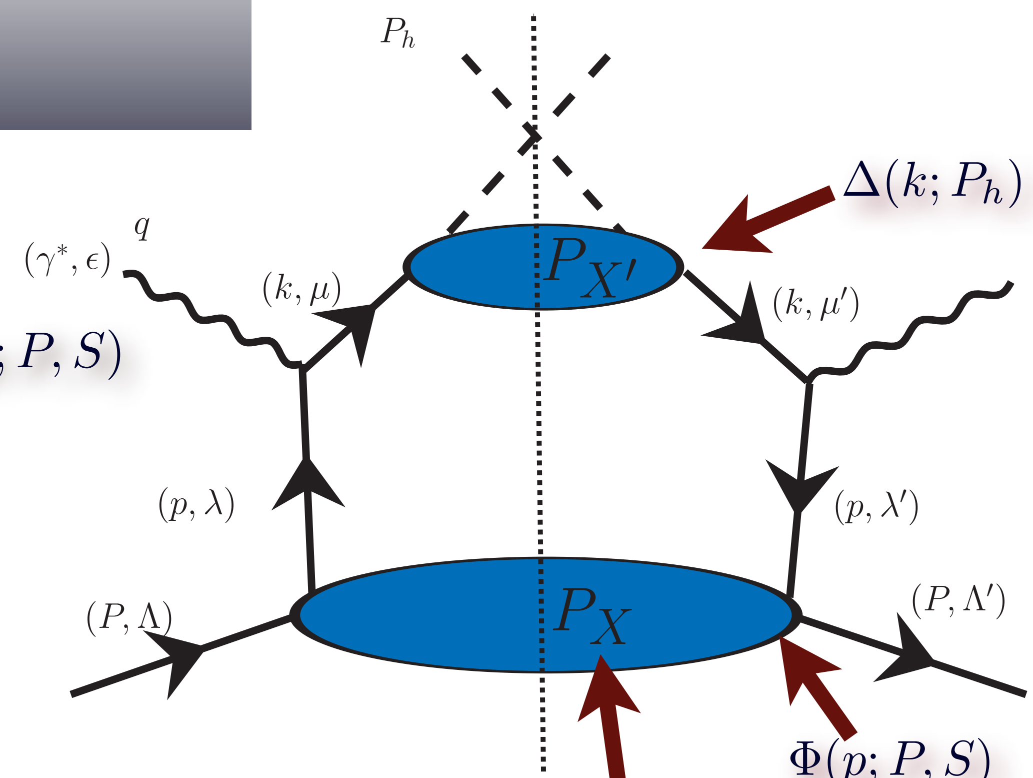
$$\times \mathrm{Tr}\left[\Phi(p,P,S)\gamma^\mu \Delta(k,P_h)\gamma^\nu\right]$$

$$W^{\mu\nu}(q,P,S,P_h) = \int \frac{d^2\mathbf{p}_T}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T}{(2\pi)^4} \delta^2(\mathbf{p}_T - \frac{\mathbf{P}_{h\perp}}{z_h} - \mathbf{k}_T)\mathrm{Tr}\left[\Phi(x,\mathbf{p}_T)\gamma^\mu \Delta(z,\mathbf{k}_T)\gamma^\nu\right]$$

$$\Phi(x,\mathbf{p}_T) = \int \frac{dp^-}{2}\Phi(p,P,S)|_{p^+=x_B P^+}\,, \qquad \Delta(z,\mathbf{k}_T) = \int \frac{dk^-}{2}\Delta(k,P_h)|_{k^-=\frac{P^-}{z_h}}$$



From soft quark-quark Correlators to TMDs



$$\Phi_{ji}(p; P, S) = \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3 2E_X}(2\pi)^4\delta^4(P - p - P_X)\phi_i(p; P, S)\bar{\phi}_j(p; P, S)$$

exponentiate delta function

$$= \int d^4\xi\, e^{ip\cdot\xi}\langle PS|\bar{\psi}_j(0)\psi_i(\xi)|PS\rangle$$

and

$$\Delta_{ij}(k; P_h S_h) = \sum_{X'} \int \frac{d^3\mathbf{P}_{X'}}{(2\pi)^3 2E_{X'}}(2\pi)^4\delta^4(P_h + P_X - k)\chi_i(k; P_h S_h)\bar{\chi}(k; P_h S_h)$$

$$= \sum_{X'} \int \frac{d^3\mathbf{P}_{X'}}{(2\pi)^3 2E_{X'}} \int d^4\xi e^{ik\cdot\xi}\langle 0|\psi_i(\xi)|P_h S_h, X\rangle\langle P_h S_h, X|\bar{\psi}_j(0)|0\rangle$$

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

$$W^{\mu\nu}(q, P, S, P_h) = \sum_a e^2 \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4}\delta^4(p + q - k)\mathrm{Tr}\left[\Phi\gamma^\mu\Delta\gamma^\nu\right]$$

Factorization proceeds by Sudakov/L.C. decomp of quark momenta ... next time

Ji, Ma, Yuan: PLB, PRD 2004, 2005 Extend factorization of CS-NPB: 81

PHYSICAL REVIEW D **71,** 034005 (2005)

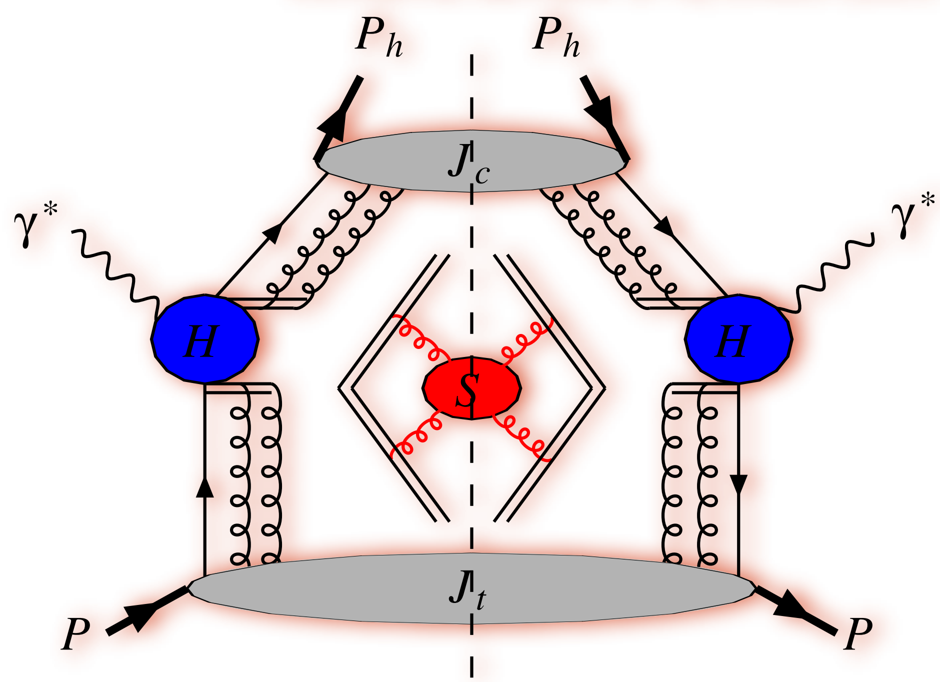


FIG. 8 (color online). The leading region for SIDIS after soft and collinear factorizations.

And Bacchetta Boer Diehl Mulders JHEP 08

$$F_{UU,T}(x,z,P_{h\perp}^2,Q^2) = \mathcal{C}\left[f_1 D_1\right]$$
$$= \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T - \boldsymbol{P}_{h\perp}/z\right)$$
$$x \sum_a e_a^2\, f_1^a(x, p_T^2, \mu^2)\, D_1^a(z, k_T^2, \mu^2)\, U(l_T^2, \mu^2) H(Q^2, \mu^2)$$

TMD PDF TMD FF Soft factor Hard part

Collins, Soper, NPB 193 (81)
Ji, Ma, Yuan, PRD 71 (05)

Few extra Slides on Transversity
should come back to this

Comment: why Transversity is a good quantity

- **Conserved quantities, mass & Polarization**

$$
\begin{aligned}
W^\mu &\equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J_{\nu\rho} P_\sigma - \mathrm{Pauli-Lubanski} \\
& P^\mu - \mathrm{Momentum}, \\
& J_{\nu\rho} - \mathrm{AngularMomentum}
\end{aligned}
$$

where,

$$
\begin{aligned}
W^2 &= -m^2 s(s+1) - \mathrm{Casimir} \\
P^2 &= m^2 - \mathrm{Casimir}
\end{aligned}
$$

- **Eigen States of polarization**

$$
\begin{aligned}
P^\mu |p,s> &= p^\mu |p,s> \\
-\frac{W\cdot n}{m} |p,s> &= \mp \frac{1}{2} |p,s> \\
\text{where} \quad [H, P^\mu] &= [H, W\cdot n] = 0, \quad \text{and} \quad n\cdot p = 0,\, n^2 = -1
\end{aligned}
$$

- Polarization Longitudinal and Transverse helicity and transversity states

When $\vec{n}||\vec{p}$, $\quad -\dfrac{W\cdot n}{m} = \dfrac{1}{2m}\gamma_5 \not{n}\not{p} = \dfrac{\vec{\Sigma}\cdot\vec{p}}{2p}$

helicity spinors

$$\frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}\, u_\pm(p) = \pm\, u_\pm(p)\,,$$
$$\frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}\, v_\pm(p) = \mp\, v_\pm(p)\,.$$

When $\vec{n}\cdot\vec{p} = 0$, $\quad -\dfrac{W\cdot n}{m} = \dfrac{\gamma_5\gamma_\perp\cdot\vec{n}_\perp\not{p}}{2m} = \dfrac{\gamma_0\vec{\Sigma}_\perp\cdot\vec{n}_\perp\not{p}}{2m}$

$$\frac{\gamma_0\vec{\Sigma}_\perp\cdot\vec{n}_\perp}{2}\, u_{\uparrow\downarrow}(p) = \pm\, u_{\uparrow\downarrow}(p)\,,$$

transversity spinors

$$\frac{\gamma_0\vec{\Sigma}_\perp\cdot\vec{n}_\perp}{2}\, v_{\uparrow\downarrow}(p) = \mp\, v_{\uparrow\downarrow}(p)\,.$$

- Classification of quark spin states depends on Dirac Matricies that
    - 1) commute with each other and
    - 2) Hamiltonian
- Helicity is a good quantum number
- Transversity is a good quantum number in the parton model

- Parton model-nucleon at leading twist yields 3 leading quark distribution functions. Introducing projection ops for good component of Dirac spinor reveals meaning.

$$
\begin{aligned}
P_{[+/-]} &= \frac{1}{2}(1 \mp \gamma_5) \\
& f_1(x) \sim \int d^2k_\perp \langle P|b_+^\dagger(xp,k_\perp)b_+(xp,k_\perp) + b_-^\dagger(xp,k_\perp)b_-(xp,k_\perp)|P\rangle \\
& g_1(x) \sim \int d^2k_\perp \langle PS_z|b_+^\dagger(xp,k_\perp)b_+(xp,k_\perp) - b_-^\dagger(xp,k_\perp)b_-(xp,k_\perp)|PS_z\rangle \\
& h_1(x) \sim \int d^2k_\perp \mathrm{Re}\langle PS_\perp|b_-^\dagger(xp,k_\perp)b_+(xp,k_\perp)|PS_\perp\rangle \\
Q_{[\perp/\top]} &= \frac{1}{2}(1 \pm \gamma^5\gamma^\perp) \to \gamma^\perp = \gamma^y \\
& f_1(x) \sim \int d^2k_\perp \langle P|b_\perp^\dagger(xp,k_\perp)b_\perp(xp,k_\perp) + b_\top^\dagger(xp,k_\perp)b_\top(xp,k_\perp)|P\rangle \\
& g_1(x) \sim \int d^2k_\perp \mathrm{Re}\langle PS_z|b_\perp^\dagger(xp,k_\perp)b_\top(xp,k_\perp)|PS_z\rangle \\
& h_1(x) \sim \int d^2k_\perp \langle PS_\perp|b_\perp^\dagger(xp,k_\perp)b_\perp(xp,k_\perp) - b_\top^\dagger(xp,k_\perp)b_\top(xp,k_\perp)|PS_\perp\rangle
\end{aligned}
$$

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Need another chiral odd soft factor ie fragmentation function to flip the helicity of the initial quark e.g. SIDIS