Extracting the resonance information
with a dynamical coupled-channels model

Hiroyuki Kamano

Cake Seminar, April 14th, 2010
1. Motivation and research program for the N* study at EBAC

2. Extraction of resonances and their dynamical origins
Motivation and research program for the N* study at EBAC (1 of 2)
N* states - \( \Delta(1232) \) and others -

- The Delta \( (1232) \) resonance stands as a clear peak.
- The region \( s^{1/2} = 1.4 - 2 \) GeV hosts ~ 20 resonances.
# $N^*$ states and PDG $^*$s

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<th>Particle</th>
<th>$L_{2I,2J}$ status</th>
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**Diagram:**

Isospin = $I$, Spin = $J$
Parity = $(-)^{L+1}$
**N* states and PDG *s**

All of these studies essentially agree on the existence and (most) properties of the 4-star states. For the 3-star and lower states, however, even a statement of existence is problematic.


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Parity = $(-)^{L+1}$

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Thomas Jefferson National Accelerator Facility
N* states and PDG *s

All of these studies essentially agree on the existence and (most) properties of the 4-star states. For the 3-star and lower states, however, even a statement of existence is problematic.

--- Arndt, Briscoe, Strakovsky, Workman PRC 74 045205 (2006)

Most of the N*\s were extracted from

\[ \pi N \rightarrow \pi N, \ \gamma N \rightarrow \pi N \]

Need combined analysis of

\[ \pi N, \eta N, \pi\pi N, KY, \omega N, ... \] channels!
Excited Baryon Analysis Center (EBAC) of Jefferson Lab

Founded in January 2006

http://ebac-theory.jlab.org/

Objectives and goals:

Through the comprehensive analysis of world data of $\pi N$, $\gamma N$, $N(e,e')$ reactions,

✓ Determine $N^*$ spectrum (masses, widths)

✓ Extract $N^*$ form factors

✓ Provide information about reaction mechanism necessary to interpret the $N^*$ properties

Dynamical Coupled-Channels Analysis @ EBAC

Reaction Data

$\pi N \rightarrow \pi N, \eta N, \pi\pi N, KY, \omega N...$

$\gamma^{(\nu)} N \rightarrow \pi N, \eta N, \pi\pi N, KY, \omega N...$

Hadron Models

Lattice QCD

QCD
Basic reaction model

Dynamical coupled-channels model of meson production reactions


- Maintain coupled-channels unitarity of $\pi N$, $\eta N$, $\pi\pi N$, $K\Lambda$, $K\Sigma$, $\omega N$...
- Can treat 3-body $\pi\pi N$ cut

Singular!
Partial wave (LSJ) amplitude of an $a \rightarrow b$ reaction:

$$T^{(LSJ)}_{a,b}(p_a, p_b; E) = V^{(LSJ)}_{a,b}(p_a, p_b) + \sum_c \int_0^\infty q^2 dq V^{(LSJ)}_{a,c}(p_a, q) G_c(q; E) T^{(LSJ)}_{c,b}(q, p_b; E)$$

Reaction channels:

$$a, b, c = (\gamma^{(*)} N, \pi N, \eta N, \pi \Delta, \sigma N, \rho N, K \Lambda, K \Sigma, \omega N)$$

Potential:

$$V_{a,b} = v_{a,b} + \sum_{N^*} \frac{\Gamma^\dagger_{N^*,a} \Gamma_{N^*,b}}{E - M_{N^*}}$$

- Exchange potential of ground state mesons and baryons
- Bare $N^*$ state
Dynamical coupled-channels model \( @ \) EBAC


\[
\begin{align*}
7. \pi(k, i) + N(p) & \rightarrow \rho(k', j) + N(p'):
\tilde{V}(7) = \tilde{V}_a^7 + \tilde{V}_b^7 + \tilde{V}_c^7 + \tilde{V}_d^7 + \tilde{V}_e^7
\end{align*}
\]

with
\[
\begin{align*}
\tilde{V}_a^7 &= i \frac{f_{\pi NN}}{m_\pi} g_{\rho NN} \Gamma_\rho' S_N(p + k) \gamma_5 \tau^i,

\tilde{V}_b^7 &= i \frac{f_{\pi NN}}{m_\pi} g_{\rho NN} k \gamma_5 \tau^i S_N(p - k') \Gamma_\rho',

\tilde{V}_c^7 &= \frac{f_{\pi NN}}{m_\pi} g_{\rho NN} \epsilon_{i j l} \tau^l \frac{(q - k) \cdot \epsilon_{\rho'}^{*} \cdot \gamma_5}{q^2 - m_\pi^2},

\tilde{V}_d^7 &= -\frac{f_{\pi NN}}{m_\pi} g_{\rho NN} \epsilon_{i j l} \epsilon_{\rho'}^{*} \gamma_5 \epsilon_{j l} \tau^l,

\tilde{V}_e^7 &= \frac{g_{\omega NN} g_{\omega NN}}{m_\omega} \delta_{i j} \frac{\epsilon_{\rho'}^{*} \epsilon_{\rho'}^{*} K_{\rho'} K_{\rho'}}{q^2 - m_\omega^2} \left[ \gamma^\delta + \frac{\kappa_\omega}{4m_N} (\gamma^\delta q - \gamma^\delta \gamma_5) \right],
\end{align*}
\]

where
\[
\Gamma_\rho' = \frac{\tau^j}{2} \left[ \epsilon_{\rho'}^{*} + \frac{\kappa_\rho}{4m_N} \left( \epsilon_{\rho'}^{*} k' - k' \epsilon_{\rho'}^{*} \right) \right].
\]
Dynamical coupled-channels model @ EBAC


- Partial wave (LSJ) amplitude of a $\rightarrow$ b reaction:

$$T_{a,b}^{(LSJ)}(p_a, p_b; E) = V_{a,b}^{(LSJ)}(p_a, p_b) + \sum_c \int_0^\infty q^2 dq V_{a,c}^{(LSJ)}(p_a, q) G_c(q; E) T_{c,b}^{(LSJ)}(q, p_b; E)$$

- Reaction:

$$\Gamma_{N^*,a(LS)}(p) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{m_N}} \left( p \over m_\pi \right)^L C_{N^*,a} \left( \frac{\Lambda_{N^*,a(LS)}^2}{\Lambda_{N^*,a(LS)}^2 + p^2} \right)^{(2+L)}$$

- Potential:

$$V_{a,b} = v_{a,b} + \sum_{N^*} \frac{\Gamma_{N^*,a}^{\dagger} \Gamma_{N^*,b}}{E - M_{N^*}}$$

exchange potential of ground state mesons and baryons

bare N* state
Strategy for the N* study @ EBAC

Stage 1
Construct a reaction model through the comprehensive analysis of meson production reactions

Stage 2
Extract resonance information from the constructed reaction model
- N* pole positions; N* → γN, MB transition form factors
- Confirm/reject N* with low-star status; Search for new N*

Stage 3
Make a connection to hadron structure calculations; Explore the structure of the N* states.
- CQM, DSE, Large Nc, Soliton models,…
- Connection to the Lattice QCD data
Current status of the EBAC-DCC analysis

**Hadronic part**

- $\pi N \rightarrow \pi N$: fitted to the data up to $W = 2$ GeV.
  
  Julia-Diaz, Lee, Matsuyama, Sato, PRC76 065201 (2007)

- $\pi N \rightarrow \pi \pi N$: cross sections calculated with the $\pi N$ model; fit is ongoing.
  
  Kamano, Julia-Diaz, Lee, Matsuyama, Sato, PRC79 025206 (2009)

- $\pi N \rightarrow \eta N$: fitted to the data up to $W = 2$ GeV
  

**Electromagnetic part**

- $\gamma(\ast) N \rightarrow \pi N$: fitted to the data up to $W = 1.6$ GeV (and up to $Q^2 = 1.5$ GeV$^2$)

- $\gamma N \rightarrow \pi \pi N$: cross sections calculated with the $\gamma N$ & $\pi N$ model; fit is ongoing.
  
  Kamano, Julia-Diaz, Lee, Matsuyama, Sato, PRC80 065203 (2009)

- $\gamma(\ast) N \rightarrow \eta N$: in progress

- $\gamma N \rightarrow K \Lambda$: in progress (Sandorfi, Hoblit, Kamano, Lee, arXiv:0912.3505)
$MB = \pi N, \eta N, \pi\pi N (\varpi \pi\Delta, \sigma N, \rho N)$ coupled channels are considered.

Angular distribution

Target polarization

EBAC

SAID (SP06)
$MB = \pi N, \eta N, \pi\pi N (\equiv \pi\Delta, \sigma N, \rho N)$ coupled channels are considered.

\[ \pi^- p \text{ total} = \sum_{MB} \sigma(\pi^- p \rightarrow MB) \]

\[ \pi^- p \rightarrow \pi^- p + \pi^0 n \]

\[ \pi^+ p \text{ total} = \sum_{MB} \sigma(\pi^+ p \rightarrow MB) \]

\[ \pi^+ p \rightarrow \pi^+ p \]
pi N → pi pi N reaction

Parameters used in the calculation are from πN → πN analysis.

π− p → π+ π− n

\[ W = 1.44 \text{ (GeV)} \]

\[ W = 1.6 \text{ (GeV)} \]

\[ W = 1.79 \text{ (GeV)} \]

\[ \text{Full result C.C. effect off} \]

\[ \text{PID analysis} \]

\[ \text{Parameters used in the calculation are from πN → πN analysis.} \]
Single pion photoproduction

✓ Fitted up to $W = 1.6$ GeV.
✓ Only $\Gamma_{\gamma N \rightarrow N^*}^{\text{bare}}$ is varied.

- Comparison to data
  - Total cross section

Single pion photoproduction

- Fitted up to $W = 1.6$ GeV.
- Only $\Gamma_{\gamma N \rightarrow N^*}^{\text{bare}}$ is varied.

Comparison to data
- Total cross section
- Differential cross section


\[ \gamma p \rightarrow \pi^0 p \]
\[ \gamma p \rightarrow \pi^+ n \]
Single pion photoproduction


- Fitted up to $W = 1.6$ GeV.
- Only $\Gamma_{\gamma N \rightarrow N^*}^{\text{bare}}$ is varied.

- Comparison to data
  - Total cross section
  - Differential cross section
  - Photon asymmetry

$\gamma p \rightarrow \pi^+ n$

$W = 1154$ MeV  $W = 1162$ MeV  $W = 1178$ MeV

$W = 1209$ MeV  $W = 1217$ MeV  $W = 1232$ MeV

$W = 1285$ MeV  $W = 1299$ MeV  $W = 1417$ MeV

$W = 1496$ MeV  $W = 1513$ MeV  $W = 1544$ MeV

$\theta$ (deg)
Double pion photoproduction

Parameters used in the calculation are from $\pi N \rightarrow \pi N$ & $\gamma N \rightarrow \pi N$ analyses.

\[ \gamma p \rightarrow \pi^+ \pi^- p \quad \gamma p \rightarrow \pi^0 \pi^0 p \quad \gamma p \rightarrow \pi^+ \pi^0 n \]

- Good description near threshold
- Reasonable shape of invariant mass distributions
- Above 1.5 GeV, the total cross sections of $p\pi^0\pi^0$ and $p\pi^+\pi^-$ overestimate the data.
## Plan for EBAC-DCC analysis in 2010

### EBAC second generation model

### Full combined analysis (global fit) of:

| ~ End of 2010 | 
|---|---|
| $\pi N \rightarrow \pi N$ | $(W < 2 \text{ GeV})$ |
| $\pi N \rightarrow \eta N$ | $(W < 2 \text{ GeV})$ |
| $\gamma N \rightarrow \pi N$ | $(W < 1.6 \text{ GeV} \rightarrow 2 \text{ GeV})$ |
| $\gamma N \rightarrow \eta N$ | $(W < 2 \text{ GeV})$ |
| $\gamma N \rightarrow \text{KY}$ | $(W < 2 \text{ GeV})$ |

| 2010 ~ 2011 | 
|---|---|
| $\pi N \rightarrow \pi \pi N$ | $(W < 2 \text{ GeV})$ |
| $\gamma N \rightarrow \pi \pi N$ | $(W < 1.5 \text{ GeV} \rightarrow 2 \text{ GeV})$ |

New $N^*$ states may be found!!
Extraction of resonances and their dynamical origins
(2 of 2)
How can we extract N* information?

PROPER definition of

- N* mass and width

- N* → MB, γN decay vertices

Pole position of the amplitudes

Residue of the pole

\[
\left< p_a | \hat{T}(E) | p_b \right> \bigg|_{E \rightarrow E_0} \rightarrow \frac{\Gamma(E_0, p_a) \Gamma(E_0, p_b)}{E - E_0} + \text{(regular terms)}
\]

N* → b decay vertex

N* pole position (Im(E_0) < 0)
How can we extract N* information?

PROPER definition of

✓ N* mass and width ➔ Pole position of the amplitudes
✓ N* → MB, γN decay vertices ➔ Residue of the pole

Need analytic continuation of the amplitudes !!

Multi-layered structure of the scattering amplitudes

e.g.) single-channel meson-baryon scattering

\[ T(p, p'; E) = V(p, p') + \int q^2 dq V(p, q)G(q; E)T(q, p'; E) \]

Scattering amplitude is a double-valued function of E!!
Multi-layered structure of the scattering amplitudes

e.g.) single-channel meson-baryon scattering

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Multi-layered structure of the scattering amplitudes

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Scattering amplitude is a double-valued function of \( E \)!!

\[ \text{physical sheet} \]

\[ \text{unphysical sheet} \]

\[ E + i\varepsilon \text{ ("physical world")} \]

\[ E_{th} \text{ (branch point)} \]

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Multi-layered structure of the scattering amplitudes

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\[ \text{E + i\epsilon ("physical world")} \]

E\text{\textsubscript{th}} (branch point)

\[ \text{physical sheet} \]

\[ \text{unphysical sheet} \]
Multi-layered structure of the scattering amplitudes

e.g.) single-channel meson-baryon scattering

\[ T(p, p'; E) = V(p, p') + \int q^2 dq V(p, q) G(q; E) T(q, p'; E) \]

Scattering amplitude is a double-valued function of \( E \)!!
Multi-layer structure of the scattering amplitudes

e.g.) single-channel meson-baryon scattering

\[ T(p, p'; E) = V(p, p') + \int q^2 dq \]

Scattering amplitude is a double-valued function of \( E \) !!

2-channel case (4 sheets):

\( (\text{channel 1, channel 2}) = (p, p), (u, p), (p, u), (u, u) \)

\( p = \) physical sheet
\( u = \) unphysical sheet

\( N \)-channels \( \rightarrow \) Need \( 2^N \) Riemann sheets

\( \text{Re (E)} \) \( \rightarrow \) \( \text{Im (E)} \)
How to choose Riemann sheet of complex E-plane

\[ T(p, p'; E) = V(p, p') + \int_C q^2 dq V(p, q) G_{MB}(q; E) T(q, p'; E) \]

Meson-Baryon Green function
\[ G_{MB}(q, E) = \frac{1}{E - E_M(q) - E_B(q) + i\epsilon} \]

For real E
\[ E_M(q) = \sqrt{m_M^2 + q^2}, \quad E_B(q) = \sqrt{m_B^2 + q^2} \]

Solution of
\[ E - E_M(q) - E_B(q) + i\epsilon = 0 \]
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Solution of

E_{th}
How to choose Riemann sheet of complex E-plane

For complex E (Im E < 0)

\[ T(p, p'; E) = V(p, p') + \int_C q^2 dq V(p, q) G_{MB}(q; E) T(q, p'; E) \]

Meson-Baryon Green function

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How to choose Riemann sheet of complex E-plane


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$$G_{MB}(q, E) = \frac{1}{E - E_M(q) - E_B(q) + i\epsilon}$$

For complex E (Im E < 0)

$$E_M(q) = \sqrt{m_M^2 + q^2}, \quad E_B(q) = \sqrt{m_B^2 + q^2}$$

[Diagram showing the unphysical sheet and the real and imaginary parts of E]
How to choose Riemann sheet of complex E-plane

\[ T(p, p'; E) = V(p, p') + \int_C q^2dqV(p, q)G_{MB}(q; E)T(q, p'; E) \]

Meson-Baryon Green function

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For complex E (Im E < 0)

\[ E_M(q) = \sqrt{m_M^2 + q^2}, \quad E_B(q) = \sqrt{m_B^2 + q^2} \]

Momentum-integral path to avoid singularities

In addition, momentum-integral path must be taken not to cross any other singularities.

Discontinuity in $\pi\Delta$, $\rho N$, $\sigma N$ Green functions (coming from $\pi\pi N$ cut)

Momentum plane

$\rightarrow$ Path to look at unphysical sheet of complex energy plane.

Singularity from t-ch. exch. pot.'s.

$\rightarrow$ Path to look at physical sheet of complex energy plane.
# N* poles from EBAC-DCC analysis


<table>
<thead>
<tr>
<th>$L_{2I , 2J}$</th>
<th>EBAC (MeV)</th>
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**Two resonance poles in the Roper resonance region!!**
## N* poles from EBAC-DCC analysis


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**Two resonance poles in the Roper resonance region!!**

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<td>CMB (1990)</td>
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<td>GWU (2006)</td>
<td>(1359 - 82i), (1388 - 83i)</td>
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<td>Jülich (2009)</td>
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Thomas Jefferson National Accelerator Facility

**Jefferson Lab**
Delta(1232) : The 1st P33 resonance


Complex E-plane

Real energy axis “physical world”

Im (E)  Re (E)

P33

πN physical & πΔ physical sheet

πN unphysical & πΔ physical sheet

Riemann-sheet for other channels: (ηN,ρN,σN) = (-, p, -)
Delta(1232) : The 1st P33 resonance


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Real energy axis “physical world”

P33

Im (E)

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Riemann-sheet for other channels: (ηN, ρN, σN) = (-, p, -)

πN unphysical & πΔ physical sheet

Complex E-plane

**Delta(1232): The 1st P33 resonance**


In this case, BW mass & width can be a good approximation of the pole position.

- Pole: 1211, 50
- BW: 1232, 118/2=59

Riemann-sheet for other channels: $(\eta N, \rho N, \sigma N) = (-, p, -)$

- Small background
- Isolated pole
- Simple analytic structure of the complex E-plane

Real energy axis "physical world"

Complex E-plane

P33

$\pi N$ unphysical

$\pi \Delta$ unphysical

1211-50i
Two-pole structure of the Roper P11(1440)

Two-pole structure of the Roper P11(1440)


Pole A cannot generate a resonance shape on “physical” real E axis.

πN unphysical & πΔ physical sheet

πΔ branch point prevents pole B from generating a resonance shape on “physical” real E axis.

Riemann-sheet for other channels: (ηN,ρN,σN) = (p,p,p)
Two-pole structure of the Roper P11(1440)


Pole A cannot generate a resonance shape on “physical” real E axis.

In this case, BW mass & width has NO clear relation with the resonance poles:

- **Two poles**
  - 1356, 78
  - 1364, 105

- **BW**
  - 1440, 300/2 = 150
Dynamical origin of P11 resonances

All three P11 poles below 2 GeV are generated from a *same, single* bare state!

Multi-channel reactions can generate *many* resonance poles from a *single* bare state

- Eden, Taylor, Phys. Rev. 133 B1575 (1964)

---

P11 N* resonances in the EBAC-DCC model

(1357, -76)
(1364, -105)
(1820, -248)
Dynamical origin of P11 resonances


\[
\frac{1}{E - m_{N^*}^0 - \sigma(E)} \rightarrow \frac{1}{E - m_{N^*}^0 - \sum_{MB} x_{MB} \sigma_{MB}(E)}
\]

\[x_{MB} : 0 \rightarrow 1\]

\[
\eta N \text{ threshold}
\]

\[
(E = m_{N^*}^0, \text{ all } x_{MB} = 0)
\]

\[
\pi \Delta \text{ threshold}
\]

\[
A: 1357-76i
\]

\[
B: 1364-105i
\]

\[
\rho N \text{ threshold}
\]

\[
C: 1820-248i
\]

\[
(\pi N, \sigma N) = (u, p)
\]

for three P11 poles

Re E (MeV)

Im E (MeV)
Dynamical origin of P11 resonances


Pole trajectory of $N^*$ propagator

\[
\frac{1}{E - m_{N^*}^0 - \sigma(E)} \rightarrow \frac{1}{E - m_{N^*}^0 - \sum_{MB} x_{MB} \sigma_{MB}(E)} \quad x_{MB} : 0 \to 1
\]

For three P11 poles:

\[
\sigma(E) = \sum_{MB} \sigma_{MB}(E) = \sum_{MB} \frac{1}{E - m_{N^*}^0 - \sum_{MB} x_{MB} \sigma_{MB}(E)}
\]

\[MB = (\pi N, \eta N, \pi \Delta, \sigma N, \rho N)\]
Dynamical origin of P11 resonances


\[
\frac{1}{E - m_{N^*}^0 - \sigma(E)} \rightarrow \frac{1}{E - m_{N^*}^0 - \sum_{MB} x_{MB}\sigma_{MB}(E)}
\]

\(x_{MB} : 0 \rightarrow 1\)

Pole trajectory of \(N^*\) propagator

\((\eta N, \rho N, \pi\Delta) = (u, u, u)\)

\((\pi N, \sigma N) = (u, p)\) for three P11 poles

\(\eta N\) threshold

\(\pi\Delta\) threshold

\(\rho N\) threshold

Bare state

\((E = m_{N^*}^0, \text{ all } x_{MB} = 0)\)

\(\eta N\) threshold

A: 1357–76i

B: 1364–105i

C: 1820–248i
Dynamical origin of P11 resonances


\[
\frac{1}{E - m_{N^*}^0 - \sigma(E)} \rightarrow \frac{1}{E - m_{N^*}^0 - \sum_{MB} x_{MB}\sigma_{MB}(E)}
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\[x_{MB} : 0 \rightarrow 1\]

\[\begin{align*}
\eta N &\text{ threshold} \\
\pi\Delta &\text{ threshold} \\
\rho N &\text{ threshold}
\end{align*}\]

\[\begin{align*}
\eta N, \rho N, \pi\Delta &= (p, u, -) \\
\pi N, \sigma N &= (u, p)
\end{align*}\]

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A: 1357–76i \\
B: 1364–105i \\
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Dynamical origin of P11 resonances


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\frac{1}{E - m_{N^*}^0 - \sigma(E)} \rightarrow \frac{1}{E - m_{N^*}^0 - \sum_{MB} x_{MB} \sigma_{MB}(E)} \quad x_{MB} : 0 \rightarrow 1
\]

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\text{Pole trajectory of } N^* \text{ propagator} \\
\frac{1}{E - m_{N^*}^0 - \sigma(E)} & \rightarrow \frac{1}{E - m_{N^*}^0 - \sum_{MB} x_{MB} \sigma_{MB}(E)} \\
& \quad x_{MB} : 0 \rightarrow 1
\end{align*}

\( (\eta N, \rho N, \pi\Delta) = (p, u, u) \)

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\( (\eta N, \rho N, \pi\Delta) = (u, u, u) \)

\( (\pi N, \sigma N) = (u, p) \) for three P11 poles

\( x_{\pi\Delta} : 0 \rightarrow 1 \)

\( x_{\text{otherMBs}} = 1 \)
Continuous effort for exploring the N* states is being made at EBAC of Jefferson Lab.

Resonance poles have been successfully extracted from the EBAC-DCC analysis.

Dynamical origin of the P11 nucleon resonances:

- The Roper resonance is associated with two resonance poles.
- (Two) Roper and N(1710) originate from a same, single bare state.

N* → γN transition form factors have also been extracted.

Treatment of multi-reaction channels is key to understanding the N* spectrum!!