Lattice Studies of the Nearly Conformal Composite Higgs Mechanism

Julius Kuti

Jefferson Lab seminar
April 19, 2010

With Lattice Higgs Collaboration members:
Z. Fodor, K. Holland, D. Nogradi, C. Schroeder
Video Games in Technicolor

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Physicists at CERN in Geneva find the Higgs particle with unexpected characteristics

By Jane Ellis

The properties of the newly found Higgs particle shook the foundations of modern particle physics. Although its decay properties are very similar to what was expected, the mass at 507 GeV is far too heavy and the width far too narrow to accommodate what is known to be the Standard Model of modern particle physics. Physicists are turning now to lattice gauge theorists who are trying to explain with nearly conformal gauge theories the experiments at the Large Hadron Collider. Continued on page 11 ...

How to pull out the heavy Higgs particle to 507 GeV from the allowed oval region without violating EW precision data?

Nearly conformal gauge theories might work?

Requires unusual nonperturbative properties which can be studied on the lattice with extreme computing resources

Can the lattice be transformational when we do not know the answer?
1. Overview of three coordinated projects in our program
   - SU(3) color, fundamental rep, staggered Nf=4-20
   - sextet representation with SU(3) color
   - Running coupling (new ideas, first results)

2. Chiral symmetry breaking
   - Finite volume p-regime, delta-regime, epsilon-regime
   - Goldstone spectra and staggered CHPT
   - New results at Nf=4,8,9,12 will be presented

3. Inside and above the conformal window
   - Zero momentum dynamics at Nf=16,20

4. Conclusions and Outlook
   - Prospects towards model building ?
   - Can lattice studies be transformational ?
   - Is peta-scale to exa scale power needed for definitive phenomenology ?
Talk is based on published results last year:

1. **Topology and higher dimensional representations.**
   Published in *JHEP 0908:084,2009.*
   e-Print: [arXiv:0905.3586](http://arxiv.org/abs/0905.3586) [hep-lat]

2. **Nearly conformal gauge theories in finite volume.**
   e-Print: [arXiv:0907.4562](http://arxiv.org/abs/0907.4562) [hep-lat]

3. **Chiral properties of SU(3) sextet fermions**
   e-Print: [arXiv:0908.2466](http://arxiv.org/abs/0908.2466) [hep-lat]

4. **Chiral symmetry breaking in nearly conformal gauge theories**

5. **Calculating the running coupling in strong electroweak models**
   e-Print: [arXiv:0911.2934](http://arxiv.org/abs/0911.2934) [hep-lat]

and some unpublished new analysis
Standard Model: Charged currents in SU$_2^L \otimes U_1^Y$ sector

Phase diagram of TWO projects as nearly conformal gauge theories in flavor-color space?

- Project 1: Fundamental rep $N_f=4, 8-12, 14, 16, 20$ flavors and three colors with staggered fermions.
- Project 2: 2-index symmetric rep with $N_f=2$ flavors and three colors with overlap chiral fermions (will be briefly discussed here, but quenched results with interesting topology are published and full dynamical simulations are running)

Phenomenology goal: nearly conformal gauge theory with minimal realization of the composite Higgs mechanism
Consistent with ElectroWeak Precision Data?

Unified GPU/MPI code near the conformal window (walking):
Higgs phenomenology with nearly vanishing beta function

They are fun lattice field theories anyway!
Project 3: Important to complement the test of chirality with running coupling and beta function

Fundamental rep with $N_f=4,8,9$ should be similar
$N_f=10,11,12,14,16,20$ under continued study
$N_f=12$ controversial

How to reach walking scale which is wanted for several reasons in BSM?

Is 2-index symmetric rep nearly conformal?

DeGrand et al. (conformal?) our staggered simulations disagree with conformal phase important in model building

would be Banks-Zaks FP
1. Introduction

The LHC will probe the mechanism of electroweak symmetry breaking. A very attractive alternative to the standard Higgs mechanism, with fundamental scalars, involves new strongly-interacting gauge theories, known as technicolor [1, 2]. Such models avoid difficulties of theories with scalars, such as triviality and fine-tuning. Chiral symmetry must be spontaneously broken in a technicolor theory, to provide the technipions which generate the $W^\pm$ and $Z$ masses and break electroweak symmetry. Although this duplication of QCD is appealing, precise electroweak measurements have made it difficult to find a viable candidate theory. It is also necessary to enlarge the theory (extended technicolor) to generate quark masses, without generating large flavor-changing neutral currents, which is challenging.

Technicolor theories have lately enjoyed a resurgence, due to the exploration of various techniquark representations [3]. Feasible candidates have fewer new flavors, reducing tension with electroweak constraints. If a theory is almost conformal, it is possible this generates additional energy scales, which could help in building the extended technicolor sector. There are estimates of which theories are conformal for various representations, shown in Fig. 1. For $SU(N)$ gauge theory, if the number of techniquark flavors is less than some critical number, conformal and chiral symmetries are broken and the theory is QCD-like. For future model-building, it is crucial to go beyond these estimates and determine precisely where the conformal windows are. There have been a number of recent lattice simulations of technicolor theories, attempting to locate the conformal windows for various representations [4, 5, 6, 7, 8].

2. Dirac eigenvalues and chiral symmetry

The connection between the eigenvalues of the Dirac operator and chiral symmetry breaking is well understood. Predictions from Schwinger-Dyson approximations are not reliable.

Project 1: in fundamental rep with $N=3$ colors with $N_f=4,8,9,10,11,12,14,16,20$ flavors dynamical staggered

Project 2: 2-index symmetric rep (sextet) $N=3$ colors and $N_f=2$ flavors dynamical overlap

Running on CPU clusters and GPU clusters Very demanding Unified code
We are supported by the Wuppertal hardware/software infrastructure.

We built on Wuppertal software library:
Fodor, Szabo, Katz

Very limited use of Wuppertal hardware
We are USQCD!!

GPU HARDWARE

GTX 280
Flops: single 1 Tflop, double 80 Gflops
Memory 1GB, Bandwidth 141 GBs⁻¹
230 Watts, $350

For code development:
Small UCSD Tesla cluster
ARRA funded by DOE
waiting for Fermi cards

CUDA code:
Kalman Szabo
Sandor Katz
Chris Schroeder
Daniel Nogradi

Our new DYNQCD software
mostly by Szabo, Nogradi, Schroeder

Tesla 1060
Flops: single 1 Tflop, double 80 Gflops
Memory 4GB, Bandwidth 102 GBs⁻¹
230 Watts, $1200

Tesla 1070
Flops: single 4 Tflops, double 320 Gflops
Memory 16GB, Bandwidth 408 GBs⁻¹
900 Watts, $8000
Chiral regimes to identify in theory space:

Goldstone dynamics is different in each regime

We study $\delta$ and $\varepsilon$ -regimes (RMT) and p-regime (probing chiral loops) complement each other

interpretation of rotator levels in $m_q \to 0$ limit:

- $m_q = 0$
- $m_q \neq 0$

Veff: chiral condensate in flavor space arbitrary orientation of condensate

Not to misidentify rotator gaps as evidence of chirally symmetric phase!!
One-loop expansion in our analysis of p-regime:

\[ M^2 = M^2 \left[ 1 - \frac{M^2}{8\pi^2 N_f F^2} \ln \left( \frac{\Lambda_3}{M} \right) \right], \]

\[ F_\pi = F \left[ 1 + \frac{N_f M^2}{16\pi^2 F^2} \ln \left( \frac{\Lambda_4}{M} \right) \right], \]

Note Nf scaling of pion mass!

warning: 2-loop \sim Nf^2 (Bijnens)

\[ M_\pi(L_s, \eta) = M_\pi \left[ 1 + \frac{1}{2N_f} \frac{M^2}{16\pi^2 F^2} \cdot \bar{g}_1(\lambda, \eta) \right], \quad \lambda = ML_s \]

\[ F_\pi(L_s, \eta) = F_\pi \left[ 1 - \frac{N_f}{2} \frac{M^2}{16\pi^2 F^2} \cdot \bar{g}_1(\lambda, \eta) \right], \]

We use staggered action with stout smearing
Taste breaking included in staggered perturbation theory!
structure changing as Nf grows
\[ L^{(4)}_{\chi} = \frac{F^2}{4} \text{Tr}(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger) - \frac{1}{2} B m_q F^2 \text{Tr}(\Sigma + \Sigma^\dagger) + V^{(6)}_\chi \]

\[ M_\pi(T_a)^2 = 2Bm_q + a^2 \Delta(T_a) + O(a^2 m_q) + O(a^4) \]

\[ \Delta(\xi_5) = 0, \]
\[ \Delta(\xi_\mu) = \frac{8}{F^2} (C_1 + C_2 + C_3 + 3C_4 + C_5 + 3C_6), \]
\[ \Delta(\xi_{\mu5}) = \frac{8}{F^2} (C_1 + C_2 + 3C_3 + C_4 - C_5 + 3C_6), \]
\[ \Delta(\xi_{\mu\nu}) = \frac{8}{F^2} (2C_3 + 2C_4 + 4C_6). \]
\[ \left( \frac{m_{\pi_5}^{1-loop}}{2m} \right)^2 = \mu \left\{ 1 + \frac{1}{16\pi^2f^2} \left( \frac{2}{N_F} \left[ \ell(m_{\eta_1}^2) - \ell(m_{\pi_1}^2) \right] \right. \right. \\

+ \frac{2}{N_F} \left[ \ell(m_{\eta_1}^2) - \ell(m_{\pi_1}^2) \right] + \left. \frac{1}{2N_F} \ell(m_{\pi_1}^2) \right) \right. \right. \\

+ \frac{16\mu}{f^2} (2L_8 - L_5) (2m) + \frac{32\mu}{f^2} (2L_6 - L_4) (4N_Fm) + a^2C \right\} \]
Nf=9 NLO chiral analysis in p-regime:

Testing rooting (nothing unusual happens)
(useful for rooted sextet code, complete and running with Nf=2)
Provides additional independent info on chiral condensate trend
$N_f=12$ runs are far away from crossover region
Nf=12 NLO chiral analysis in p-regime:

Similar pattern to Nf=9 case!

All features exhibit chiral symmetry breaking

more work is needed
Some features of Nf=4,8,9,12 runs:

Nearly degenerate Goldstone spectra
stout action performs very well

Chiral condensate measured in F unit
is enhanced as Nf increases
\[ \frac{\langle \bar{\psi} \psi \rangle^{1/3}}{F} \]

Nf=4 \quad B/F = 53(6)
Nf=8 \quad B/F = 157(17)
Nf=9 \quad B/F = 125(19)
Nf=12 \quad B/F = 209(64)

large errors, preliminary, limited to Ls=32!

rho - A1 splitting

Better separation of rho and
Goldstones at Nf=12 would require
bigger runs at smaller fermion masses
Nf=12  mq=0.02  rho-A1 splitting  pulled out from single correlator with two parity partners

Nf=12  mq=0.015  rho-A1 splitting  pulled out from single correlator with two parity partners
Random Matrix Theory tests in epsilon regime:

Dirac spectrum
Integrated eigenvalue distributions of RMT
--> quartet degeneracy
--> RMT
First conclusions on our Nf sequence:

Nf=4,8,9,12 all appear to be in chirally broken phase according to several tests:

1. chiral Goldstone dynamics
2. nonvanishing condensate in chiral limit
3. rho-A1 parity doublet splitting close to chiral limit
4. epsilon regime and RMT
5. string tension and running coupling from potential/force?

Important warning is appropriate related to the size of F*L
Sextet representation with \( N_f=2 \)

Goldstone spectrum and \( F_{\pi} \rightarrow \) chirally broken
Will be easy 2 flavor analysis
Code reproduces Kogut-Sinclair results with the 2 stout steps disabled.
Preliminary indications on our \( N_f=2 \) sextet model:
It appears to be in chirally broken phase according to several tests like the ones discussed earlier:

1. chiral Goldstone dynamics
2. nonvanishing condensate in chiral limit
3. \( \rho - A_1 \) parity doublet splitting close to chiral limit ?
4. epsilon regime and RMT ?
5. string tension and running coupling from potential/force ?

More favorable to reach large enough \( F^*L \) values
When is $F*L$ large enough? This can be quantified (epsilon, delta and $p$ regimes are all connected)

$E_l = \frac{1}{2\theta} l(l+2)$ with $l = 0,1,2,...$ rotator spectrum for SU(2)

with $\theta = F^2 L_s^3 \left(1 + \frac{C(N_f = 2)}{F^2 L_s^2} + O(1 / F^4 L_s^4)\right)$ (P. Hasenfratz and F. Niedermayer)

there is overall factor $\frac{N_f^2 - 1}{N_f}$ for arbitrary $N_f$

$C(N_f = 2) = 0.45$ expected to grow with $N_f$

At $FL_s = 0.8$ the correction is 70% and grows with $N_f$

When expansion collapses in $\delta$ – regime, the $p$-regime analysis needs more scrutiny

Cross checks from several running coupling schemes is important
Our running coupling methods (important to have 3 methods)
SF, MCRG, Wilson loops (including $\alpha_W(R)$, $V(R)$, $F(R)$)
We use Wilson loop ratios, $V(r)$ potential, and $F(r)$ force to get the running coupling $g(R)$ in several schemes
Looks promising
We are running with $R$, or with $L$ when $R/L$ fixed (different schemes)

Nf=16 Creutz coupling running

$g^2(L)$

$g^\star^2 \sim 0.5$

$N_f=12$, 2-stout $32^3 \times 64$, $\beta=2.2$, $m=0.015$

Not Coulomb-like

Not Coulomb-like
Our new calculation:
use $V(r)$ potential and $F(r)$ force to get traditional running $g(r)$ in several schemes just like in QCD
Looks very promising

$N_f=12$, 2-stout $32^3\times64$, $\beta=2.2$, $m=0.015$
At short distances the force may be obtained by an integration of the perturbative infinite volume, continuum extrapolated which is in excellent agreement with our results for theory [20]. This suggests that the agreement with Eq. (3.5) is rather accidental. In any of 

The continuum force is plotted in Fig. 4 using Eq. (2.5) to combine the two regimes of

For large values of which is in excellent agreement with our results for theory [20]. This suggests that the agreement with Eq. (3.5) is rather accidental. In any

Quenched test works Necco-Sommer

\[ F(r) = \frac{C_F g^2_{qq}(r)}{4\pi r^2}, \quad C_F = \frac{4}{3}, \]
\[ -r \frac{d}{dr} g_{qq} = \beta(g_{qq}) = -\sum_{\nu=0}^{2} b_{\nu} g_{qq}^{2\nu+3}, \quad b_0 = \frac{11}{16\pi^2}, \quad b_1 = \frac{102}{(16\pi^2)^2}, \]
\[ b_2 = \frac{1}{(4\pi)^6} \left( -3470 + 2519 \frac{\pi^2}{3} - 99 \frac{\pi^4}{4} + 726 \zeta(3) \right). \]

\[ F(r) = \sigma + \frac{\pi}{12r^2}, \quad \sigma r_0^2 = 1.65 - \frac{\pi}{12} \]

Infinite volume, continuum extrapolated limited \( r/r_0 \) range between 0.15 and 0.3 we try to run with the volume!
Running coupling from force and SF running nicely match for Nf=0 and Nf=2

We have difficulties to match data from the Force/Creutz running coupling and the Yale SF running at Nf=12 in the relevant coupling range

Will requires further careful studies

Very difficult to resolve fixed point from walking

Interesting collection of renormalized running couplings
Running gauge coupling from RG on large Wilson loops

two groups: our group and Bilgici et al. (generalization from earlier work)

\[
W(R, T; L, T_0; a; g) = \frac{a}{g^2} + \frac{a^2}{g^4} + \frac{a^3}{g^6} + \cdots
\]

\[
k \cdot g^2_R(L_0, R/L_0) = -R^2 \left( \frac{\partial^2}{\partial R \partial T} \ln \langle W(R, T; L_0, T_0) \rangle \right)_{T=R}
\]

k is geometric factor (cutoff dependent on lattice) defined from tree level relation with the bare coupling \( g_0 \)

\[
-R^2 \left( \frac{\partial^2}{\partial R \partial T} \ln \langle W(R, T; L_0) \rangle \right)_{T=R} = kg_0^2
\]

Lattice implementation requires the study of the step function together with its cutoff dependence

Useful alternative to Schrodinger functional?

Wilson loops could be replaced by Polyakov loop correlators
We first tested the method at weak coupling for large Wilson loops. Rather than simulating Wilson loops with Monte Carlo, we calculated (simulated) them analytically using the boosted coupling procedure of Lepage and McKenzie which reproduces even large Wilson loops accurately. The finite volume dependence was obtained from Urs Heller’s code who calculated the Wilson loops in bare perturbation theory. (thanks to Urs Heller and Paul Mackenzie for the help they provided)

Onto Monte Carlo now -->
Quenched SU(3) simulation extrapolating the step function renormalized coupling to zero cutoff at fixed finite physical box size:
The running coupling of our Wilson-Creutz scheme

In quenched SU(3) simulation renormalized coupling is running with physical box size $L$ without lattice cutoff effects

Onto dynamical fermions: fairly strong cutoff effects but no sign of $N_f=12$ conformal fixed point
Conclusions and Outlook

- Our focus shifted to Nf=10-16 range (and beyond?)
  Nf=12 chiral symmetry breaking and running coupling from V(r) and F(r) and Wilson loops

- Nf=12 might be close enough to realize walking technicolor but otherwise (like the S-parameter) we do not know how it will perform

- What is the fate of the Nf=2 sextet model? Next controversy is brewing? Walking Nf=2 sextet would be a favorite candidate for composite Higgs (mass generation?)

- Zero mode dynamics important at weak coupling inside conformal window

- Reliable EW precision quantities (S/T/U) will be important to get accurately once we settle on the candidate model(s)