Baryon chiral perturbation theory: one loop and beyond

Matthias R. Schindler

The George Washington University

JLab Theory Seminar

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Introduction: baryon chiral perturbation theory

Additional degrees of freedom

Nucleon mass at the two-loop level

Bayesian parameter estimation

Summary

Quantum chromodynamics (QCD)

- $SU(3)_c$ gauge theory describing the strong interactions
- Degrees of freedom: Quarks and gluons
- High energies: Perturbation theory in coupling constant
- Low energies:
  - Lattice QCD
  - Effective field theories

Effective field theory

Low-energy approximation to a more fundamental theory

- Description in terms of relevant degrees of freedom (e.g. pions, nucleons,...)
- Most general Lagrangian consistent with all symmetries of underlying theory $\rightarrow$ Ward identities satisfied
- Expansion in q/Λ, where
  - q: momenta and masses
  - Λ: energy scale
  - $q \ll \Lambda$

Examples:

- Heavy quark effective theory
- Chiral perturbation theory

Effective field theory II

Challenges:

- Infinite number of terms in Lagrangian
- Non-renormalizable in traditional sense

Solution:

- Finite order in q/Λ-expansion
  - $\Rightarrow$ Finite accuracy
  - $\Rightarrow$ Finite number of diagrams
- Most general Lagrangian
  - $\Rightarrow$ Renormalizable to given order

Chiral perturbation theory (ChPT)

- Effective field theory of the strong interaction
- Typical scale: $\Lambda \approx 1\text{GeV}$ (m_ρ, m_N, $4\pi F_\pi$)
- Relevant degrees of freedom: Pions and nucleons
- Symmetries: Lorentz invariance, C, P, T, chiral symmetry
- q: Pion mass, momenta $\ll \Lambda$
- Organization of Lagrangian in number of derivatives on Pion fields and powers of quark masses:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_\pi + \mathcal{L}_{\pi N} \\ &= \mathcal{L}_2 + \mathcal{L}_4 + \cdots + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \cdots\end{aligned}$$

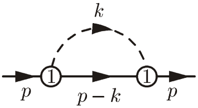
Power counting

Scheme to decide on relative importance of diagrams

- Each renormalized diagram is assigned chiral order D
- Counting rules
  - Loop integration in n dimensions $\sim \mathcal{O}(q^n)$
  - Vertex from $\mathcal{L}_{\pi N}^{(i)} \sim \mathcal{O}(q^i)$
  - Nucleon propagator $\sim \mathcal{O}(q^{-1})$
  - Pion propagator $\sim \mathcal{O}(q^{-2})$
- Diagrams with higher D are less important
- Relation between chiral and loop expansion

$D \leq 4 \rightarrow$ one-loop calculation

Example



- Power counting: $D = n + 2 \cdot 1 - 1 - 2 = n - 1 \xrightarrow{n \to 4} 3$
- $\widetilde{\mathrm{MS}}$ renormalization: $\Sigma_r \sim \mathcal{O}(q^2) \Leftrightarrow D = 2$

No consistent power counting?

- Terms violating power counting analytic in small parameters
- Suitable renormalization scheme preserves power counting
- Renormalization must also preserve all symmetries (Ward identities)

Examples: Heavy Baryon ChPT, Infrared regularization, ...

Solution 1: Heavy Baryon ChPT

- $1/m$ expansion of the Lagrangian
- $\widetilde{\mathrm{MS}}$ renormalization leads to consistent power counting as in mesonic sector
- Widely used
  - Nucleon mass up to order $\mathcal{O}(q^5) \to$ two loops
  - Leading contributions to axial coupling g_A up to order $\mathcal{O}(q^6)$
- For special cases with particular kinematics problems with analytic structure

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E. Jenkins and A. V. Manohar, Phys. Lett. B **255**, 558 (1991); V. Bernard, N. Kaiser, J. Kambor and U.-G. Meißner, Nucl. Phys. B **388**, 315 (1992)

J. A. McGovern and M. C. Birse, Phys. Lett. B **446**, 300 (1999), Phys. Rev. D **74**, 097501 (2006)

V. Bernard and U.-G. Meißner, Phys. Lett. B **639**, 278 (2006)

Example: Pion-nucleon scattering

Pole in s-channel due to nucleon propagator

- Covariant form:

$$\frac{1}{(p+q)^2 - m_N^2} = \frac{1}{2p\cdot q + M_\pi^2}$$

Pole at $2p\cdot q = -M_\pi^2 \leftrightarrow s = m_N^2$

- Heavy-baryon:

$$\begin{aligned}\frac{1}{2p\cdot q + M_\pi^2} &= \frac{1}{2m_N}\frac{1}{v\cdot q + \frac{M_\pi^2}{2m_N}} \\ &= \frac{1}{2m_N}\frac{1}{v\cdot q}\left(1 - \frac{M_\pi^2}{2m_N v\cdot q} + \cdots\right)\end{aligned}$$

Poles at $v\cdot q = 0$, not $v\cdot q = -M_\pi^2/(2m_N)$

Solution 2: Infrared renormalization

- Manifestly Lorentz-invariant formulation
- Split integral H into infrared singular part I and infrared regular part R:

$$\int_0^1 dz \cdots = \int_0^\infty dz \cdots - \int_1^\infty dz \cdots .$$

$$H = I + R$$

- Infrared singular part I:
  - Satisfies power counting
  - Satisfies Ward identities separately
- Infrared regular part R:
  - Contains terms that violate power counting
  - Satisfies Ward identities separately
  - Chiral expansion contains an infinite number of terms

T. Becher and H. Leutwyler, Eur. Phys. J. C **9**, 643 (1999)

Renormalization:

- Infrared regular part R absorbed by counterterms
- Renormalized integral defined as infrared singular part I
- Power counting and Ward identities satisfied

Applicability:

- In original formulation: Can be applied to one-loop integrals
- In reformulated version:
  - multi-loop integrals
  - Diagrams with additional heavy degrees of freedom
- IR renormalization similar to original formulation for spin-1 particles

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MRS, J. Gegelia and S. Scherer, Phys. Lett. B **586**, 258 (2004)

P. C. Bruns and U.-G. Meißner, arXiv:0808.3174 [hep-ph]

Solution 3: Extended on-mass-shell (EOMS) scheme

- Manifestly Lorentz-invariant formulation
- Preserves all symmetries up to the considered order
- Subtract only those terms which violate the power counting
  - Expand integrand in small quantities
  - Exchange summation and integration, i. e. integrate term by term
  - Subtract terms that violate power counting
- Reproduces chiral expansion of infrared regular part *R* up to any order
- Can be applied to multi-loop integrals and diagrams with additional heavy degrees of freedom

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T. Fuchs, J. Gegelia, G. Japaridze and S. Scherer, Phys. Rev. D **68**, 056005 (2003)
MRS, J. Gegelia and S. Scherer, Nucl. Phys. B **682**, 367 (2004)
T. Fuchs, MRS, J. Gegelia and S. Scherer, Phys. Lett. B **575**, 11 (2003)

Additional dynamical degrees of freedom

Heavy degrees of freedom in standard ChPT

- Do not appear explicitly
- Expand propagator in small momenta

$$\frac{1}{q^2 - M_V^2} = -\frac{1}{M_V^2}\left[1 + \frac{q^2}{M_V^2} + \left(\frac{q^2}{M_V^2}\right)^2 + \left(\frac{q^2}{M_V^2}\right)^3 + \mathcal{O}(q^8)\right]$$

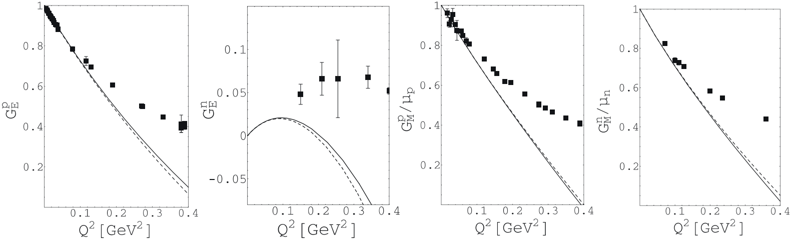
- Contributions absorbed in coupling constants at each order

Idea

- Treat as explicit degrees of freedom
- Resummation of some higher-order terms

Electromagnetic form factors in BChPT

Nucleon electromagnetic form factors in IR renormalization at order $\mathcal{O}(q^4)$:

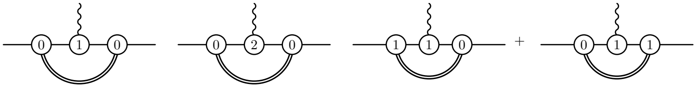


- Missing curvature → Higher-order terms

B. Kubis and U.-G. Meißner, Nucl. Phys. A **679**, 698 (2001)
MRS, J. Gegelia, and S. Scherer, Eur. Phys. J. A **26**, 1 (2005)

Inclusion of vector mesons

Diagrams to order $\mathcal{O}(q^4)$

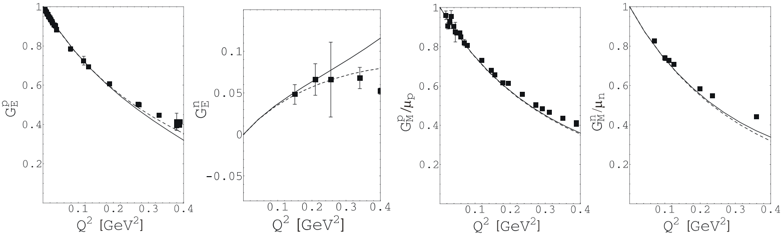


- Tree diagrams contribute to $F_1(q^2)$ and $F_2(q^2)$
- Loop diagrams: Only nucleon and vector meson propagators $\Rightarrow$ vanish in IR renormalization

Power counting + renormalization
$\Rightarrow$ Vector meson loops strongly suppressed

Electromagnetic form factors with vector mesons

Improved description by inclusion of vector mesons (ρ, ω, ϕ)



Calculations to order $\mathcal{O}(q^6)$

Nucleon mass

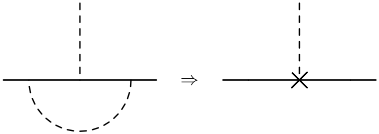
- Simplest quantity
- Of interest for chiral extrapolations

Contributions from

- Tree-level diagrams with vertices up to order $\mathcal{O}(q^6)$
- One-loop diagrams with vertices up to order $\mathcal{O}(q^4)$
- Two-loop integrals with vertices up to order $\mathcal{O}(q^2)$

IR-Renormalization of two-loop diagrams

- Determine infrared regular terms of one-loop subdiagrams



- Use as vertices in one-loop counterterm diagrams

- Perform additional renormalization to sum of two-loop diagram and counterterm diagrams

Complications

At one-loop level

- Splitting of integrals into infrared singular and infrared regular terms $\Rightarrow$ additional divergences

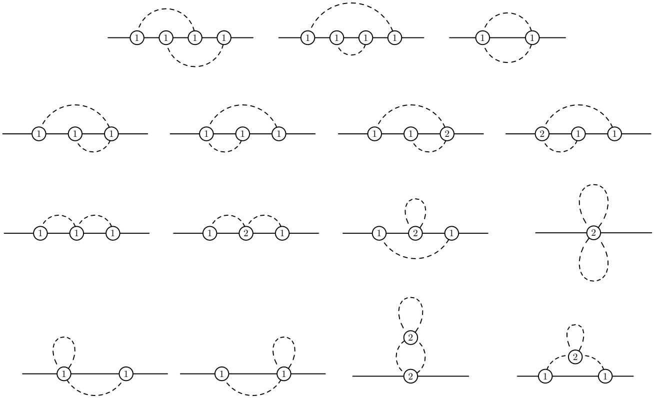
$$I = \widetilde{I} + \frac{I^{\mathrm{div}}}{\epsilon}, \quad R = \widetilde{R} + \frac{R^{\mathrm{UV}}}{\epsilon} + \frac{R^{\mathrm{div}}}{\epsilon}$$

- Cancel in the sum $I + R$
- IR renormalized integral defined as $\widetilde{I}$

At two-loop level

- Include terms $\sim \epsilon$ in subtraction terms
- Additional divergences in counterterm diagrams are multiplied by terms $\sim \epsilon$
  - Finite contributions
  - Necessary for chiral symmetry

Nucleon mass



Chiral expansion to $\mathcal{O}(q^6)$

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\frac{M}{\mu} + k_4 M^4$$
$$+ k_5 M^5 \ln\frac{M}{\mu} + k_6 M^5 + k_7 M^6 \ln^2\frac{M}{\mu} + k_8 M^6 \ln\frac{M}{\mu} + k_9 M^6$$

Examples:

$$k_5 = \frac{3g_A^2}{1024\pi^3 F^4}\left(16g_A^2 - 3\right)$$
$$k_6 = \frac{3g_A^2}{256\pi^3 F^4}\left[g_A^2 + \frac{\pi^2 F^2}{m^2} - 8\pi^2(3l_3 - 2l_4) - \frac{32\pi^2 F^2}{g_A}(2d_{16} - d_{18})\right]$$

MRS, D. Djukanovic, J. Gegelia and S. Scherer, Nucl. Phys. A **803**, 68 (2008)

Comparison with HBChPT

$$
\begin{aligned}
m_N &= m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln \frac{M}{\mu} + k_4 M^4 \\
&\quad + k_5 M^5 \ln \frac{M}{\mu} + k_6 M^5 + \cdots
\end{aligned}
$$

- Only leading non-analytic term ($k_5 M^5 \ln \frac{M}{\mu}$, $k_7 M^6 \ln^2 \frac{M}{\mu}$) renormalization scheme independent
- Coefficient k_5 agrees with HBChPT result
- $k_6 M^5$, $k_8 M^6 \ln \frac{M}{\mu}$ non-analytic, but renormalization scheme dependent $\rightarrow$ treatment of subdiagrams

J. A. McGovern and M. C. Birse, Phys. Lett. B **446**, 300 (1999)

Numerical estimates

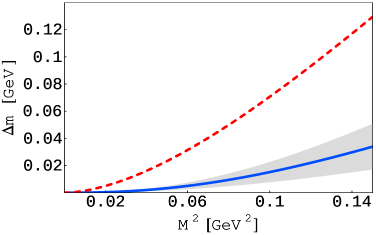
k_5

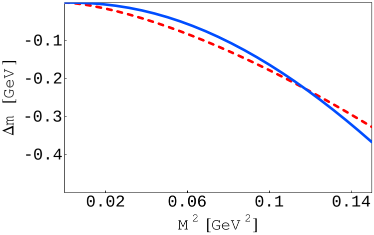
- Free of unknown low-energy constants
- $k_5 M^5 \ln(M/m_N) = -4.8\,\mathrm{MeV}$
- $\approx 31\%$ of leading nonanalytic contribution at one-loop order, $k_2 M^3$

k_6

- l_3, l_4, d_{18} known
- d_{16} not reliably determined: $\pi N \to \pi\pi N$ or lattice fit
- $k_6 M^5 = 3.7\,\mathrm{MeV}$ or $k_6 M^5 = -7.6\,\mathrm{MeV}$

Pion mass dependence





dashed: $k_3 M^4 \ln(M/m_N)$
solid: $k_7 M^6 \ln^2(M/m_N)$

dashed: $k_2 M^3$
solid: $k_5 M^5 \ln(M/m_N)$

Low-energy constants (LECs) in effective field theory

Generic physical quantity

$$A = A^{(0)} + A^{(1)} \frac{m}{\Lambda} + A^{(2)} \left(\frac{m}{\Lambda}\right)^2 + \cdots$$

LECs $A^{(i)}$

- Encode effects of high-energy physics
- Need to be known to make model-independent predictions
- Requirement: LECs are "natural" or "$\mathcal{O}(1)$"
- Determined from
  - Underlying theory $\rightarrow$ not always possible
  - Data

Bayes' theorem in data analysis

$$\mathrm{pr}(A|B) = \frac{\mathrm{pr}(B|A)\mathrm{pr}(A)}{\mathrm{pr}(B)}$$

$$\underbrace{\mathrm{pr}(\mathit{theory}|\mathit{data})}_{\mathit{posterior}} \propto \underbrace{\mathrm{pr}(\mathit{data}|\mathit{theory})}_{\mathit{likelihood}} \times \underbrace{\mathrm{pr}(\mathit{theory})}_{\mathit{prior}}$$

- Posterior: Given the data, what can we learn about the theory (parameter values)?
- Likelihood: Probability of observing the data given that the theory is true (particular parameter set)
  $\rightarrow$ Only part considered in "standard" statistics
- Prior: Incorporates any knowledge about the theory before analysis of the data
  Example: Measurement of mass $m > 0 \rightarrow \mathrm{pr}(m) = \Theta(m)$

Marginalization

- Theory can contain a large number of parameters
- Not necessarily interested in all parameters (e.g. higher-order constants, background signal,...)
- Can "integrate out" these nuisance parameters: Marginalization

$$\begin{aligned}
\mathrm{pr}(A, B|C) \mapsto \mathrm{pr}(A|C) &= \int dB\, \mathrm{pr}(A, B|C) \\
&= \int dB\, \mathrm{pr}(A|B, C) \times \mathrm{pr}(B|C)
\end{aligned}$$

Data analysis for EFTs

Which order in the m/Λ expansion is appropriate?

- There is no "right" order in the expansion
  $\Rightarrow$ Marginalize over order
  $\Rightarrow$ Uncertainty included in parameter estimates

How does one incorporate information on naturalness of LECs?

- Use Bayes' theorem to include naturalness via a prior pdf
- Notion of $\mathcal{O}(1)$ not precise
  $\Rightarrow$ Marginalize to take uncertainty into account

Higher-order parameters

- Not interested in actual values
  $\Rightarrow$ Marginalize over higher-order LECs
  $\Rightarrow$ Systematic treatment of related uncertainty

Final probability density function

$$\mathrm{pr}(\mathbf{a}_{res}|D) = \sum_{P=P_{min}}^{P_{max}} \int_{R_{min}}^{R_{max}} dR \int d\mathbf{a}_{marg} \frac{\mathrm{pr}(D|\mathbf{a},P)\mathrm{pr}(\mathbf{a}|P,R)\mathrm{pr}(P)\mathrm{pr}(R)}{\mathrm{pr}(D)}$$

- $\mathrm{pr}(\mathbf{a}_{res}|D)$: posterior pdf for parameter subset $\mathbf{a}_{res}$
- $\mathrm{pr}(D|\mathbf{a},P)$: likelihood, considered in “standard” approach
- $\mathrm{pr}(\mathbf{a}|P,R)$: prior pdf, incorporates naturalness
- $\sum_P \mathrm{pr}(P)$: marginalization over EFT order P with prior
- $\int dR\, \mathrm{pr}(R)$: marginalization over “naturalness parameter” R with prior
- $\int d\mathbf{a}_{marg}$: marginalization over higher-order parameters

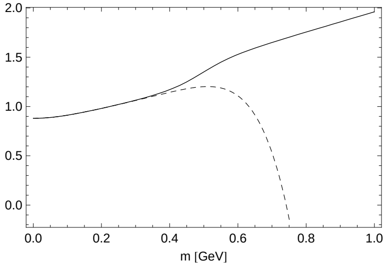
M. R. Schindler and D. R. Phillips, Annals Phys. **324**, 682 (2009)

Example: Data beyond domain of applicability

Generate data using:

$$M_N(m) = M_{\chi PT}(m)\left[1 - g\left(\frac{m}{\Lambda}\right)\right] + M_{\rm model}(m) g\left(\frac{m}{\Lambda}\right)$$

- $M_{\chi PT}(m) = M_0 + k_1 m^2 + \ldots$
  Chiral expansion of nucleon mass
- $M_{\rm model}(m) = \alpha + \beta m$
  Model deviation from chiral behavior for large pion mass
- $g(0) = 0$, $\lim_{x\to\infty} g(x) = 1$
  Transition between chiral and model region



Results: $m = 200 - m_{max}$ MeV, $P = 3 - 6$, $R = 0.1 - 10$

m_{max} (MeV)	M_0 (GeV)	k_1 (GeV^{-1})	$\log[\mathrm{pr}(\mathbf{a}_{res}\|D, M, \Lambda)]$
350	0.91 ± 0.04	1.61 ± 1.86	1.97
400	0.92 ± 0.04	1.54 ± 1.54	2.31
450	0.92 ± 0.03	1.35 ± 1.27	2.71
500	0.93 ± 0.03	1.11 ± 1.07	3.01
700	0.95 ± 0.04	0.50 ± 1.27	2.12
1000	0.88 ± 0.03	2.43 ± 0.94	-0.03

- Results for low m similar to those for data generated purely by $M_{\chi PT}(m)$
- Results for M_0 only mildly effected by m_{max}
- M_0 in good agreement with input value 0.88 MeV
- Data not sufficient to determine k_1
- Warning for high m_{max}: small probability

Summary

Chiral perturbation theory

- Effective field theory of QCD at low energies
- Requires consistent power counting
- Power counting linked to renormalization scheme
  - Mesonic ChPT: $\widetilde{\mathrm{MS}}$ renormalization
  - Baryonic ChPT: Heavy-baryon + $\widetilde{\mathrm{MS}}$, Infrared renormalization, ...

Reformulation of infrared renormalization

- Preserves power counting and all symmetries
- Inclusion of additional degrees of freedom
  - Resummation of some higher-order terms
  - Improved description of experimental data
  - Consistent theory for higher energies?
- Multi-loop diagrams
  - Nucleon mass at order $\mathcal{O}(q^6)$
  - Good convergence at physical pion mass
  - Lattice extrapolations?

Bayesian parameter estimation

- Bayes' theorem and marginalization $\rightarrow$ powerful alternative to standard approach
  - Incorporate prior information (naturalness)
  - Eliminate nuisance parameters
  - Avoid (arbitrary) choice of order of EFT calculation
  - Systematic treatment of uncertainties

- Generalize to non-linear dependence on parameters
- Theories with multiple scales
- Determination of higher-order parameters?
  - Refine priors
  - How well do low-order parameters have to be known?