QCD Factorization
and
Transverse Single-Spin Asymmetry
in
ep collisions

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Based on work with many people

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Outline of my talk

- Transverse single-spin asymmetry in ep collisions
  Ideal observable to go beyond the leading power collinear factorization
- Role of fundamental symmetries
- QCD TMD factorization approach
- QCD collinear factorization approach
- Connection between these two approaches
- Predictive power of QCD factorization approach
- Summary
Electron-proton collisions

- Cross sections:

\[ \sigma(q) = \sigma_{\text{LP}}(Q) + \frac{Q_s}{Q} \sigma_{\text{NLP}}(Q) + \frac{Q_s^2}{Q^2} \sigma_{\text{NNLP}}(Q) + \ldots \approx \sigma_{\text{LP}}(Q) \]

- Every parton can participate the hard collision!
- Cross section depends on matrix elements of all possible fields

- Approximation – single large momentum transfer: \( Q >> 1/\text{fm} \)

\[ \sigma(Q) \approx \sigma_{\text{LP}}(Q) \propto \hat{\sigma}(Q) \otimes \langle p, s | \tilde{\phi}(k)^\dagger \tilde{\phi}(k) | p, s \rangle + \ldots \]

- Leading power QCD factorization - approximation:

How good the approximation is?

Universal parton distributions
Hadron’s partonic structure!
Inclusive DIS cross section

From HERA:

H1 and ZEUS

\[ \sigma^{+\text{NC}}_{\gamma p} (x, Q^2) \]

\( Q^2 = 2 \text{ GeV}^2 \)
\( Q^2 = 2.7 \text{ GeV}^2 \)
\( Q^2 = 3.5 \text{ GeV}^2 \)
\( Q^2 = 4.5 \text{ GeV}^2 \)
\( Q^2 = 6.5 \text{ GeV}^2 \)
\( Q^2 = 8.5 \text{ GeV}^2 \)
\( Q^2 = 10 \text{ GeV}^2 \)
\( Q^2 = 12 \text{ GeV}^2 \)
\( Q^2 = 15 \text{ GeV}^2 \)
\( Q^2 = 18 \text{ GeV}^2 \)
\( Q^2 = 22 \text{ GeV}^2 \)
\( Q^2 = 27 \text{ GeV}^2 \)
\( Q^2 = 35 \text{ GeV}^2 \)
\( Q^2 = 45 \text{ GeV}^2 \)
\( Q^2 = 60 \text{ GeV}^2 \)
\( Q^2 = 70 \text{ GeV}^2 \)
\( Q^2 = 90 \text{ GeV}^2 \)
\( Q^2 = 120 \text{ GeV}^2 \)

\( x \)

HERA INC e^+p
HERAPDF1.0
Inclusive single jet hadronic cross section

- To Tevatron:

QCD is successful in last 30 years – we now believe it
FACT:

✧ LP QCD collinear factorization/calculations have been very successful in interpreting HEP scattering data if $Q > 2$ GeV

✧ QCD should be correct for the asymptotic regime: $r < 1/10$ fm!

QUESTIONS:

✧ How much have we learned about hadron’s partonic structure?

  Collinear PDFs, Helicity PDFs, …

  But, Not enough for the structure, …

✧ How to test/explore QCD beyond the leading power formalism?

  Parton’s transverse motion, and multiparton correlation beyond 1/10 fm?
Go beyond the LP collinear factorization

- **Recall:**

\[ q \quad = \quad q + \ldots \]

- LP collinear term dominates the single scale cross section

- **Need additional parameter – the LP term is not sensitive to:**

  - **Nuclear A-dependence:**
    - result of multiple scattering and multiparton correlations
    \[ R_A(Q) \equiv \sigma_A(Q)/\sigma_N(Q), \quad \Delta \langle q_T^2 \rangle_A \equiv \langle q_T^2 \rangle_A - \langle q_T^2 \rangle_N, \ldots \]

  - **Transverse-spin:**
    - power of fundamental symmetries – cancels the LP collinear term
    \[ A_N(Q, s_T) \propto \sigma(Q, s_T) - \sigma(Q, -s_T), \]
    \[ A_N(Q, q_T, s_T) \propto \sigma(Q, q_T, s_T) - \sigma(Q, q_T, -s_T), \ldots \]
Transverse SSA in collinear parton model

- SSA corresponds to a naively T-odd triple product:

\[ A_N = \frac{\sigma(p, s_T) - \sigma(p, -s_T)}{\sigma(p, s_T) + \sigma(p, -s_T)} \]

\[ A_N \propto i \vec{s}_p \cdot (\vec{p} \times \vec{\ell}) \Rightarrow i \epsilon^{\mu\nu\alpha\beta} p_\mu s_\nu \ell_\alpha p'_\beta \]

Novanish \( A_N \) requires a phase, enough vectors to fix a scattering plan, and a spin flip at the partonic scattering

- Leading power in QCD: [Kane, Pumplin, Repko, PRL, 1978]

\[ \sigma_{AB}(p_T, \vec{s}) \propto \left| \begin{array}{c}
\text{diagram 1} \\
\text{diagram 2} \\
\text{...}
\end{array} \right| + \left| \begin{array}{c}
\text{diagram 3} \\
\text{diagram 4} \\
\text{...}
\end{array} \right| \]

\[ = \alpha_s \frac{m_q}{p_T} \]

Need parton’s transverse motion to generate the asymmetry!
Power of fundamental symmetries

- **Factorized cross sections – asymmetries:**
  \[ A \propto \sigma_{\uparrow}(Q, \bar{s}) - \sigma_{\downarrow}(Q, -\bar{s}) \propto \langle p, \bar{s}|O(\psi_q, A^\mu)|p, \bar{s}\rangle - \langle p, -\bar{s}|O(\psi_q, A^\mu)|p, -\bar{s}\rangle \]
  e.g. \( O(\psi, A^\mu) = \bar{\psi}(0) \hat{\Gamma} \psi(y^-) \) with \( \hat{\Gamma} = I, \gamma_5, \gamma^\mu, \gamma_5 \gamma^\mu, \sigma^{\mu\nu} \)

- **Parity and Time-reversal invariance:**
  \[ \langle p, \bar{s}|O(\psi, A^\mu)|p, \bar{s}\rangle = \langle p, -\bar{s}|\mathcal{P} \mathcal{T} O^\dagger(\psi, A^\mu) \mathcal{T}^{-1} \mathcal{P}^{-1}|p, -\bar{s}\rangle \]
  - **IF:** \( \langle p, -\bar{s}|\mathcal{P} \mathcal{T} O^\dagger(\psi, A^\mu) \mathcal{T}^{-1} \mathcal{P}^{-1}|p, -\bar{s}\rangle = \pm \langle p, -\bar{s}|O(\psi, A^\mu)|p, -\bar{s}\rangle \)
  - **or** \( \langle p, \bar{s}|O(\psi, A^\mu)|p, \bar{s}\rangle = \pm \langle p, -\bar{s}|O(\psi, A^\mu)|p, -\bar{s}\rangle \)

  Operators lead to the “+” sign \( \Rightarrow \) spin-averaged cross sections
  Operators lead to the “-” sign \( \Rightarrow \) spin asymmetries

- **Example:**
  \[ O(\psi, A^\mu) = \bar{\psi}(0) \gamma^+ \gamma_5 \psi(y^-) \Rightarrow q(x) \]
  \[ O(\psi, A^\mu) = \bar{\psi}(0) \gamma^+ \psi(y^-) \Rightarrow \Delta q(x) \]
  \[ O(\psi, A^\mu) = \bar{\psi}(0) \gamma^\perp \gamma_5 \psi(y^-) \Rightarrow \delta q(x) \rightarrow h(x) \]
  \[ O(\psi, A^\mu) = \frac{1}{xp^+} F^{+\alpha}(0)[-i\epsilon_{\alpha\beta}] F^{+\beta}(y^-) \Rightarrow \Delta g(x) \]
$A_N = 0$ for inclusive DIS

- **DIS cross section:**
  \[ \sigma(Q, s_T) \propto L^{\mu\nu} W_{\mu\nu}(Q, s_T) \]

- **Leptonic tensor is symmetric:**
  \[ L^{\mu\nu} = L^{\nu\mu} \]

- **Hadronic tensor:**
  \[ W_{\mu\nu}(Q, s_T) \propto \langle P, s_T | j_\mu(0) j_\nu(y) | P, s_T \rangle \]

- **Polarized cross section:**
  \[ \Delta \sigma(Q, s_T) \propto L^{\mu\nu} [W_{\mu\nu}(Q, s_T) - W_{\mu\nu}(Q, -s_T)] \]

- **P and T invariance:**
  \[ \langle P, s_T | j_\mu(0) j_\nu(y) | P, s_T \rangle = \langle P, -s_T | j_\nu(0) j_\mu(y) | P, -s_T \rangle \]
  \[ \Rightarrow A_N(Q, s_T)^{\text{DIS}} = 0 \]
Advantage of SIDIS

- Dominated by events with two different scales:
  \[ \ell(l, s_e) + A(P_A, s) \rightarrow \ell'(l') + h(p_h) + X \]
  - A large momentum transfer: \( Q = \sqrt{-(l - l')^2} \gg 1/\text{fm} \)
    Localized probe, suppress contribution of complicate matrix elements
  - A small momentum scale: \( p_{hT} \sim 1/\text{fm} \)
    Sensitive to parton’s motion inside a hadron – TMD distributions

- Power of varying \( p_{hT} \sim 1/\text{fm} \rightarrow p_{hT} \sim Q \)
  - Change from a two-scale problem to an one-scale problem
    Transition from TMD factorization to Collinear factorization

- Two natural scattering planes:
  - Separation of various TMDs and spin states
TMD factorization – SIDIS

\[ W^{\mu \nu} = \sum_f |H_f(Q; \mu)\|^2 |^{\mu \nu} \int d^2 k_1T d^2 k_2T F_{f/P}^\uparrow (x, k_1T, S; \mu; \zeta_F) D_{h/f} (z, z k_2T; \mu; \zeta_D) \delta^{(2)}(k_1T + q_T - k_2T) 
\]

\[ + Y(Q, q_T) + \mathcal{O}((\Lambda/Q)^a) \]

\[ \sigma_0 \phi(x, \mu) \otimes D(z, \mu) \delta^2(p_{BT}) \]

- TMD parton distribution:

\[ F_{f/P}^\uparrow (x, k_1T, S; \mu, \zeta_F) = \text{Tr}_{\text{color}} \text{Tr}_{\text{Dirac}} \frac{\gamma^+}{2} \int \frac{k_1^-}{2 \pi} \]

- TMD fragmentation function:

\[ D_{h/f} (z, k_2T; \mu, \zeta_D) = \frac{\text{Tr}_{\text{color}}}{N_c} \frac{\text{Tr}_{\text{Dirac}}}{4} \frac{\gamma^+}{z} \int \frac{k_2^-}{2 \pi} \]

- TMDs are more fundamental if we can measure them:

Carry more information on hadron’s partonic structure
Color flow – gauge links

- **Gauge link – QCD phase:**

\[
\Phi_n(\infty, y^-) = \mathcal{P} \exp \left( -ig \int_{y^-}^{\infty} d\lambda n \cdot A(\lambda n) \right)
\]

Gauge invariant PDFs:

\[
\phi(x, p, s) = \int \frac{dy^-}{2\pi} e^{ip^+ y^-} \langle p, s | \bar{\psi}(0) j \hat{\Gamma}_{ji} \Phi_n^\dagger(\infty, 0) \Phi_n(\infty, y^-) \psi_i(y^-) | p, s \rangle
\]

Collinear PDFs: “Localized” operator with size \( \sim 1/xp \sim 1/Q \)

- **Universality of PDFs:**

Gauge link should be process independent!
TMD parton distributions

- Quark TMD distributions:

\[
\gamma^* \
\rightarrow \gamma^* \rightarrow \gamma^* \\
\hat{k}^\mu = x p^\mu + \frac{k_T^2}{2 x p^+} n^\mu + k_T^\mu
\]

\[
dk^2 dk^+ \delta(x - k^+/P^+)
\]

\[
\Phi(x, k_\perp) = \frac{1}{2} \left[ f_1 h_+ + f_{1T} \frac{\epsilon_{\mu\nu\rho\sigma} \gamma^\mu n_\perp^\rho k_\perp^\sigma}{M} + \left( S_L g_{1L} + \frac{k_\perp \cdot S_T}{M} g_{1T} \right) \gamma^5 h_+ \\
+ h_{1T}^\perp i \sigma_{\mu\nu} \gamma^5 n_\perp^\mu S_T^\nu + \left( S_L h_{1L}^\perp + \frac{k_\perp \cdot S_T}{M} h_{1T}^\perp \right) i \sigma_{\mu\nu} \gamma^5 n_\perp^\mu k_\perp^\nu \\
+ h_1^\perp \frac{\sigma_{\mu\nu} k_\perp^\mu n_\perp^\nu}{M} \right]
\]

Total 8 TMD quark distributions

- Gluon TMD distributions, ...

Production of quarkonium, two-photon, ...
Most notable TMDs

- **Sivers function** – transverse polarized hadron:

\[
f_{q/p, S}(x, k_{\perp}) = f_{q/p}(x, k_{\perp}) + \frac{1}{2} \Delta^N f_{q/p}^{\uparrow}(x, k_{\perp}) \mathbf{S} \cdot (\hat{p} \times \hat{k}_{\perp})
\]

\[
= f_{q/p}(x, k_{\perp}) - \frac{k_{\perp}}{M} f_{1T}^{\perp q}(x, k_{\perp}) \mathbf{S} \cdot (\hat{p} \times \hat{k}_{\perp})
\]

- **Boer-Mulder function** – transverse polarized quark:

\[
f_{q, s_{q/p}}(x, k_{\perp}) = \frac{1}{2} f_{q/p}(x, k_{\perp}) + \frac{1}{2} \Delta^N f_{q/p}^{\uparrow}(x, k_{\perp}) s_q \cdot (\hat{p} \times \hat{k}_{\perp})
\]

\[
= \frac{1}{2} f_{q/p}(x, k_{\perp}) - \frac{1}{2} \frac{k_{\perp}}{M} h_{1T}^{\perp q}(x, k_{\perp}) s_q \cdot (\hat{p} \times \hat{k}_{\perp})
\]

Affect angular distribution of Drell-Yan lepton pair production
Most notable TMDs – II

- Collins function – FF of a transversely polarized parton:

\[
D_{h/q,s_q}(z, p_\perp) = D_{h/q}(z, p_\perp) + \frac{1}{2} \Delta^N D_{h/q,s_q}(z, p_\perp) s_q \cdot (\hat{p}_q \times \hat{p}_\perp) \\
= D_{h/q}(z, p_\perp) + \frac{p_\perp}{z M_h} H_{1q}^{+q}(z, p_\perp) s_q \cdot (\hat{p}_q \times \hat{p}_\perp)
\]

- Fragmentation function to a polarized hadron:

\[
D_{\Lambda,S_{\Lambda},q}(z, p_\perp) = \frac{1}{2} D_{h/q}(z, p_\perp) + \frac{1}{2} \Delta^N D_{\Lambda,q}(z, p_\perp) S_\Lambda \cdot (\hat{p}_q \times \hat{p}_\perp) \\
= \frac{1}{2} D_{h/q}(z, p_\perp) + \frac{p_\perp}{z M_\Lambda} D_{1T}^{\perp q}(z, p_\perp) S_\Lambda \cdot (\hat{p}_q \times \hat{p}_\perp)
\]

Unpolarized parton fragments into a polarized hadron - \( \Lambda \)
**TMDs and spin asymmetries**

- **Sivers’ effect – Sivers’ function:**
  - Hadron spin influences parton’s transverse motion
  - Sivers' function: $q^\perp$

- **Collin’s effect – Collin’s function:**
  - Transversity
  - Parton’s transverse spin affects its hadronization
  - Separation of different effects?

- **TMD factorization is relevant for two-scale problems in QCD:**
  - $Q_1 \gg Q_2 \sim \Lambda_{\text{QCD}}$
SIDIS is ideal for studying TMDs

- SIDIS has the natural kinematics for TMD factorization:

$$\ell(s_e) + p(s_p) \rightarrow \ell + h(s_h) + X$$

**Natural event structure:** high Q and low $p_T$ jet (or hadron)

- Separation of various TMD contribution by angular projection:

$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^\perp$$

$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^\perp \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp$$
Our knowledge of TMDs

- Sivers function from low energy SIDIS:

EIC can do much better job in extracting TMDs

- NO TMD factorization for hadron production in p+p collisions!

Collins and Qiu, 2007, Vogelsang and Yuan, 2007, Mulders and Rogers, 2010, …
Critical test of TMD factorization

- **TMD distributions with non-local gauge links:**

\[
f_{q/h^{+}}(x, k_{\perp}, \vec{S}) = \int \frac{dy^{-} d^{2}y_{\perp}}{(2\pi)^{3}} e^{i x p^{+} y^{-} - i k_{\perp} \cdot y_{\perp}} \langle p, \vec{S} | \bar{\psi}(0^{-}, 0_{\perp}) | y^{-}, y_{\perp} \rangle \frac{\gamma^{+}}{2} \psi(y^{-}, y_{\perp}) | p, \vec{S} \rangle
\]

- **Parity + Time-reversal invariance:**

  - For a fixed spin state:

  \[
f_{q/h^{+}}^{\text{SIDIS}}(x, k_{\perp}, \vec{S}) \neq f_{q/h^{+}}^{\text{DY}}(x, k_{\perp}, \vec{S})
  \]

- **Parity + Time-reversal invariance:**

  \[
f_{q/h^{+}}^{\text{Sivers}}(x, k_{\perp})^{\text{SIDIS}} = -f_{q/h^{+}}^{\text{Sivers}}(x, k_{\perp})^{\text{DY}}
  \]

The sign change is a critical test of TMD factorization approach.
Another critical test of TMD factorization

- **Predictive power of QCD factorization:**
  - Infrared safety of short-distance hard parts
  - Universality of the long-distance matrix elements
  - QCD evolution or scale dependence of the matrix elements

- **QCD evolution:**
  If there is a factorization/invariance, there is an evolution equation

- **Collinear factorization – DGLAP evolution:**
  \[
  \sigma_{\text{phy}}(Q, \Lambda_{\text{QCD}}) \approx \sum_f \hat{\sigma}_f(Q, \mu) \otimes \phi_f(\mu, \Lambda_{\text{QCD}}) \rightarrow \frac{d}{d\mu} \sigma_{\text{phy}}(Q, \Lambda_{\text{QCD}}) = 0
  \]
  Scaling violation of nonperturbative functions

  Evolution kernels are perturbative – a test of QCD
Evolution equations for TMDs

- **Collins-Soper equation**: b-space quark TMD with $\gamma^+$

  \[
  \frac{\partial \tilde{F}_{f/P\uparrow}(x, b_T, S; \mu; \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T; \mu) \tilde{F}_{f/P\uparrow}(x, b_T, S; \mu; \zeta_F) \\
  \tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_s} \ln \left( \frac{\tilde{S}(b_T; y_s, -\infty)}{\tilde{S}(b_T; +\infty, y_s)} \right)
  \]

- **RG equations**:

  \[
  \frac{d\tilde{K}(b_T; \mu)}{d\ln \mu} = -\gamma_K(g(\mu)) \\
  \frac{d\tilde{F}_{f/P\uparrow}(x, b_T, S; \mu; \zeta_F)}{d\ln \mu} = \gamma_F(g(\mu); \zeta_F/\mu^2) \tilde{F}_{f/P\uparrow}(x, b_T, S; \mu; \zeta_F).
  \]

- **Evolution equations for Sivers function**:

  \[
  F_{f/P\uparrow}(x, k_T, S; \mu, \zeta_F) = F_{f/P}(x, k_T; \mu, \zeta_F) - F_{1T}^{\uparrow f}(x, k_T; \mu, \zeta_F) \frac{\epsilon_{ij} k_i^S S_j}{M_p}
  \]

  **CS**:

  \[
  \frac{\partial \ln \tilde{F}_{1T}^{\uparrow f}(x, b_T; \mu, \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T; \mu) \\
  \tilde{F}_{1T}^{\uparrow f}(x, b_T; \mu, \zeta_F) \equiv \frac{\partial \tilde{F}_{1T}^{\uparrow f}(x, b_T; \mu, \zeta_F)}{\partial b_T}
  \]

  **RGs**:

  \[
  \frac{d\tilde{F}_{1T}^{\uparrow f}(x, b_T; \mu, \zeta_F)}{d\ln \mu} = \gamma_F(g(\mu); \zeta_F/\mu^2) \tilde{F}_{1T}^{\uparrow f}(x, b_T; \mu, \zeta_F)
  \]

  \[
  \frac{d\tilde{K}(b_T; \mu)}{d\ln \mu} = -\gamma_K(g(\mu)) \\
  \frac{\partial \gamma_F(g(\mu); \zeta_F/\mu^2)}{\partial \ln \sqrt{\zeta_F}} = -\gamma_K(g(\mu)),
  \]
Kernel is not perturbative for all $b$:

$$b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{\text{max}}^2}}$$

$$\mu_b = \frac{C_1}{b_*}$$

CSS prescription: (not unique)

$$\tilde{K}(b_T; \mu) = \tilde{K}(b_*; \mu_b) - \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \gamma_K(g(\mu')) - g_K(b_T)$$

$Q^2$-dependence of Sivers function:

$$\tilde{F}_{1T}^{\perp f}(x, b_T; \mu, \zeta_F) = \tilde{F}_{1T}^{\perp f}(x, b_T; \mu_0, Q_0^2) \exp \left\{ \ln \frac{\sqrt{\zeta_F}}{Q_0} \tilde{K}(b_*; \mu_b) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\}$$

$$+ \int_{\mu_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{\sqrt{\zeta_F}}{Q_0} \gamma_K(g(\mu')) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0}$$

$\tilde{F}_{1T}^{f}(x, k_T; \mu, \zeta_F) = -\frac{1}{2\pi k_T} \int_0^\infty \! \! db_T \, b_T \, J_1(k_T b_T) \tilde{F}_{1T}^{\perp f}(x, b_T; \mu, \zeta_F)$ – Evolved Sivers function

Small-$b$ perturbative contribution – match to twist-3:

$$\tilde{F}_{1T}^{\perp f}(x, b_T; \mu, \zeta_F) = \sum \frac{M_P b_T}{2} \int_R d\hat{x}_1 \, d\hat{x}_2 \, C_{j/j}^{\text{Sivers}}(\hat{x}_1, \hat{x}_2, b_*; \mu_b, \mu_b, g(\mu_b)) \, T_{F_{j/P}}(\hat{x}_1, \hat{x}_2, \mu_b)$$

$$\times \exp \left\{ \ln \frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \exp \left\{ -g_{j/P}^{\text{Sivers}}(x, b_T) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}$$

Kang, Xiao, Yuan, 2011
Gaussian ansatz for input distributions

- Up quark Sivers function:

Aybat, Collins, Qiu, Rogers, 2011

Very significant growth in the width of transverse momentum
From low $p_T$ to high $p_T$

**TMD factorization to collinear factorization:**

Two factorization are consistent in the overlap region where

$$\Lambda_{QCD} \ll p_T \ll Q$$

**QCD collinear factorization:**

Efremov, Teryaev, 82; Qiu, Sterman, 91, etc.

$$\sigma(Q, \bar{s}) \propto \sigma^{LP}(Q, \bar{s}) + \frac{Q_s}{Q} \sigma^{NLP}(Q, \bar{s}) + ...$$

$$\Delta \sigma(s_T) \propto T^{(3)}(x, x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z, z) + ...$$

$$T^{(3)}(x, x) \propto$$

Qiu, Sterman, 1991, ...

$$D^{(3)}(z, \bar{z}) \propto$$

Kang, Yuan, Zhou, 2010
Twist-3 correlation functions

- **Twist-2 parton distributions:**

  - **Unpolarized PDFs:**
    \[
    q(x) \propto \langle P | \overline{\psi}_q(0) \frac{\gamma^+}{2} \psi_q(y) | P \rangle \\
    G(x) \propto \langle P | F^{+\mu}(0) F^{+\nu}(y) | P \rangle (-g_{\mu\nu})
    \]

  - **Polarized PDFs:**
    \[
    \Delta q(x) \propto \langle P, S_{\parallel} | \overline{\psi}_q(0) \frac{\gamma^+ \gamma^5}{2} \psi_q(y) | P, S_{\parallel} \rangle \\
    \Delta G(x) \propto \langle P, S_{\parallel} | F^{+\mu}(0) F^{+\nu}(y) | P, S_{\parallel} \rangle (i \epsilon_{\perp \mu\nu})
    \]

- **Two-sets Twist-3 correlation functions:**

  \[
  \widetilde{\mathcal{T}}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ix_1^+ y_1^-} e^{ix_2^+ y_2^-} \langle P, s_T | \overline{\psi}_q(0) \frac{\gamma^+}{2} [e^{s_T \sigma n n} F_\sigma^{+}(y_2^-)] \psi_q(y_1^-) | P, s_T \rangle
  \]

  \[
  \widetilde{\mathcal{T}}_{(f,d)}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ix_1^+ y_1^-} e^{ix_2^+ y_2^-} \frac{1}{P^+} \langle P, s_T | F^{+\rho}(0) [e^{s_T \sigma n n} F_\sigma^{+}(y_2^-)] F^{+\lambda}(y_1^-) | P, s_T \rangle (-g_{\rho\lambda})
  \]

  \[
  \widetilde{\mathcal{T}}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ix_1^+ y_1^-} e^{ix_2^+ y_2^-} \langle P, s_T | \overline{\psi}_q(0) \frac{\gamma^+ \gamma^5}{2} [i s_T^\sigma F_\sigma^{+}(y_2^-)] \psi_q(y_1^-) | P, s_T \rangle
  \]

  \[
  \widetilde{\mathcal{T}}_{\Delta G,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ix_1^+ y_1^-} e^{ix_2^+ y_2^-} \frac{1}{P^+} \langle P, s_T | F^{+\rho}(0) [i s_T^\sigma F_\sigma^{+}(y_2^-)] F^{+\lambda}(y_1^-) | P, s_T \rangle (i \epsilon_{\perp \rho\lambda})
  \]

**Role of color magnetic force!**
Evolution equations and kernels

- Evolution equation is a consequence of factorization:
  
  \[ \Delta \sigma(Q, s_T) = \frac{1}{Q} H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F) \]

  - DGLAP for \( f_2 \):
    \[ \frac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F) \]
  
  - Evolution for \( f_3 \):
    \[ \frac{\partial}{\partial \ln(\mu_F)} f_3 = \left( \frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)} \right) \otimes f_3 \]

- Evolution kernel is process independent:
  
  - Calculate directly from the variation of process independent twist-3 distributions
    
    Kang, Qiu, 2009
    Yuan, Zhou, 2009
  
  - Extract from the scale dependence of the NLO hard part of any physical process
    
    Vogelsang, Yuan, 2009
  
  - Renormalization of the twist-3 operators
    
    Braun et al, 2009
Variation of twist-3 correlation functions

Closed set of evolution equations (spin-dependent):

\[ \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{T}_{q,F}(x, x + x_2, \mu_F, s_T) = \int d\xi d\xi_2 [\tilde{T}_{q,F}(\xi, \xi + \xi_2, \mu_F, s_T)K_{qq}(\xi, \xi + \xi_2, x, x + x_2, \alpha_s) \\
+ \tilde{T}_{\Delta q,F}(\xi, \xi + \xi_2, \mu_F, s_T)K_{q\Delta q}(\xi, \xi + \xi_2, x, x + x_2, \alpha_s)] \\
+ \sum_{i=f,d} \int d\xi d\xi_2 [\tilde{T}^{(i)}_{G,F}(\xi, \xi + \xi_2, \mu_F, s_T)K_{qg}^{(i)}(\xi, \xi + \xi_2, x, x + x_2, \alpha_s) \\
+ \tilde{T}^{(i)}_{\Delta G,F}(\xi, \xi + \xi_2, \mu_F, s_T)K_{q\Delta g}^{(i)}(\xi, \xi + \xi_2, x, x + x_2, \alpha_s)]. \]

\[ \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{T}^{(i)}_{G,F}(x, x + x_2, \mu_F, s_T) = \sum_{j=f,d} \int d\xi d\xi_2 [\tilde{T}^{(j)}_{G,F}(\xi, \xi + \xi_2, \mu_F, s_T)K_{gg}^{(j)}(\xi, \xi + \xi_2, x, x + x_2, \alpha_s) \\
+ \tilde{T}^{(j)}_{\Delta G,F}(\xi, \xi + \xi_2, \mu_F, s_T)K_{g\Delta g}^{(j)}(\xi, \xi + \xi_2, x, x + x_2, \alpha_s)] \\
+ \sum_{q} \int d\xi d\xi_2 [\tilde{T}_{q,F}(\xi, \xi + \xi_2, \mu_F, s_T)K_{\Delta q}^{(i)}(\xi, \xi + \xi_2, x, x + x_2, \alpha_s) \\
+ \tilde{T}_{\Delta q,F}(\xi, \xi + \xi_2, \mu_F, s_T)K_{gq}^{(i)}(\xi, \xi + \xi_2, x, x + x_2, \alpha_s)]. \]

Plus two more equations for:

\[ \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{T}_{\Delta q,F}(x, x + x_2, \mu_F, s_T) \quad \text{and} \quad \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{T}^{(i)}_{\Delta G,F}(x, x + x_2, \mu_F, s_T) \]

Kang, Qiu, 2009
Scale dependence

✧ Follow DGLAP at large $x$
✧ Large deviation at low $x$ (stronger correlation)
A sign “mismatch”

- Sivers function and twist-3 correlation:
  \[ gT_{q,F}(x,x) = - \int d^2 k_\perp \frac{|k_\perp|^2}{M} f_{1T}^q(x, k_\perp^2) \text{SIDIS} + \text{UVCT} \]

- “direct” and “indirect” twist-3 correlation functions:

Calculate \( T_{q,F}(x,x) \) by using the measured Sivers functions
Possible interpretations

- **A node in $k_T$-distribution:**
  - Like the DSSV’s $\Delta G(x)$
  - HERMES vs COMPASS
  - Physics behind the sign change?

  EIC can measure TMDs for a wide range of $k_T$

- **Large twist-3 fragmentation contribution in RHIC data:**
  - If Sivers-type initial-state effect is much smaller than fragmentation effect and two effects have an opposite sign
  - Can be tested by $A_N$ of single jet or direct photon at RHIC

- **A node in $x$-dependence of Sivers or twist-3 distributions**
  - Physics behind the node if there is any

Kang, Qiu, Vogelsang, Yuan, 2011
Propose new observables for ep collisions

- **Process:** \( e(\ell) + h(p) \rightarrow \text{jet}(p_j)(\text{or } \pi, \ldots) + X \)
  
  Lepton-hadron scattering without measuring the scattered lepton
  
  Single hard scale: \( p_{jT} \) in lepton-hadron frame

- **Complement to SIDIS:** \( e(\ell) + h(p) \rightarrow e'(\ell') + \text{jet}(p_j)(\text{or } \pi, \ldots) + X \)
  
  Two scales: \( Q, p_{jT} \) in virtual-photon-hadron frame

- **Key difference in theory treatment:**
  
  Collinear factorization for \( e(\ell) + h(p) \rightarrow \text{jet}(p_j)(\text{or } \pi, \ldots) + X \)
  
  TMD factorization for \( e(\ell) + h(p) \rightarrow e'(\ell') + \text{jet}(p_j)(\text{or } \pi, \ldots) + X \)

  Test the consistency between TMD and Twist-3 to SSA in the same experimental setting
  
  Jlab, Compass, Future EIC, …
Analytical formulae

- **Factorization is valid:**

  Same as hadron-hadron collision to jet + X

  \[
  \frac{d\sigma_{lh\rightarrow\text{jet}(P_J)X}}{dP_{JT}dy} \approx \sum_{ab} \int dx f_1^{a/l}(x, \mu) \int dx' f_1^{b/h}(x', \mu) \frac{d\sigma^{ab\rightarrow\text{Jet}(P_J)X}}{dP_{JT}dy}(x, x', P_{JT}, y, \mu)
  \]

  \[a = l, \gamma, q, \bar{q}, g\]

  \[b = q, \bar{q}, g\]

- **Leading order results:**

  \[
  P_j^0 \frac{d^3\sigma}{d^3P_j} = \frac{\alpha_{em}^2}{s} \sum_a \frac{e_a^2}{(s + t) x} \left\{ f_1^a(x) H_{UU} + \lambda_l \lambda_p g_1^a(x) H_{LL} \right. \]

  \[
  + 2\pi M \varepsilon_T^j S_T^i P_{JT}^j \left[ T^a_F(x, x) - x \frac{d}{dx} T^a_F(x, x) \right] \frac{\hat{s}}{\hat{t}u} H_{UU} \]

  \[
  + \lambda_l 2M \vec{S}_T \cdot \vec{P}_{JT} \left[ \left( \bar{g}^a(x) - x \frac{d}{dx} \bar{g}^a(x) \right) \frac{\hat{s}}{\hat{t}u} H_{LL} + x g_1^a(x) \frac{2}{t} \right] \}
  \]

  \[\lambda_l, \lambda_p: \text{ Lepton, hadron helicity, respectively}\]

  \[\vec{S}_T: \text{ Hadron’s transverse spin vector}\]
Numerical results

- Asymmetries:

  \[ A_{LL} = \frac{\sigma_{LL}}{\sigma_{UU}}, \quad A_{UT} = \frac{\sigma_{UT}}{\sigma_{UU}}, \quad A_{LT} = \frac{\sigma_{LT}}{\sigma_{UU}} \]

- Double spin asymmetries – very small:

  \[ \sqrt{s} = 50 \text{ GeV}, \quad \sqrt{s} = 100 \text{ GeV} \]

  Wandzura-Wilczek approximation:

  \[ g_T(x) \approx \int_x^1 \frac{dy}{y} g_1(y) \quad \tilde{g}(x) \approx x \int_x^1 \frac{dy}{y} g_1(y) \]

  \[ \rightarrow A_{LT} \sim 0.001 \]
Good probe of Sivers function

- Independent check of the “sign mismatch”:

Red line: \( T_F(x, \mu) \) extracted from fitting SSA in hadronic collisions

Blue line: \( \pi T_F(x, x) = -\int d^2k_T \frac{k_T^2}{2M^2} f_{1T}^T(x, \vec{k}_T^2)\bigg|_{DIS} \) Sivers function

Excellent test for the mechanism of SSA possibly at Jlab, surely at future EIC
More on future directions

- RHIC spin, JLab at 12 GeV, possibly at Compass, ...

- Future EIC:
  - a dedicated QCD machine for the visible matter

  Yellow book on EIC physics from INT workshop is available:

  arXiv: submit/0295324 [nucl-th]

- A white paper on EIC physics:
  - a writing group appointed by BNL and Jlab is working hard

- Physics opportunities at EIC:
  - Inclusive DIS – Spin, $F_L$, ...
  - SIDIS – TMDs, spin-orbital correlations,
  - One jet or particle inclusive – multiparton quantum correlation, ...
  - GPDs – parton spatial distributions
  - ...

...
Summary

- QCD factorization/calculation have been very successful in interpreting HEP scattering data

- What about the hadron structure?
  Not much!

- RHIC spin, Jlab12, a future EIC with a polarized hadron beams opens up many new ways to test QCD and to study hadron structure: TMDs, GPDs, ...

- The challenge for theorists:
  - to indentify new and calculable observables that carry rich information on hadron’s partonic structure
  - to make measureable predictions

Thank you!
Backup slices
**EIC Kinematics**

- **DIS kinematics:**
  \[ Q^2 = -q^2 = x_B y S \]
  \[ x_B = \frac{Q^2}{2p \cdot q} \]
  \[ y = \frac{p \cdot q}{p \cdot k} \]
  \[ S = (p + k)^2 \]

- **EIC (eRHIC – ELIC) basic parameters:**
  - \( E_e = 10 \) GeV (5-30 GeV available)
  - \( E_p = 250 \) GeV (50-325 GeV available)
  - \( \sqrt{S} = 100 \) GeV (30-200 GeV available)
  - "localized" probe: \( Q^2 \gtrsim 1 \) GeV
  - \( x_{\text{min}} \sim 10^{-4} \)
  - Luminosity \( \sim 100 \times \) HERA
  - Polarization, heavy ion beam, …
“Interpretation” of twist-3 correlation functions

- Measurement of direct QCD quantum interference:
  \[ T^{(3)}(x, x, S_{\perp}) \propto \]
  \[ \text{Interference between a single active parton state and an active two-parton composite state} \]

- “Expectation value” of QCD operators:
  \[
  \langle P, s | \bar{\psi}(0) \gamma^+ \psi(y^-) | P, s \rangle \quad \rightarrow \quad \langle P, s | \bar{\psi}(0) \gamma^+ \left[ \epsilon_\perp^{\alpha\beta} s T_\alpha \int dy_2^- F_\beta^+(y_2^-) \right] \psi(y^-) | P, s \rangle
  \]
  \[
  \langle P, s | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(y^-) | P, s \rangle \quad \rightarrow \quad \langle P, s | \bar{\psi}(0) \gamma^+ \left[ i g_\perp^{\alpha\beta} s T_\alpha \int dy_2^- F_\beta^+(y_2^-) \right] \psi(y^-) | P, s \rangle
  \]

How to interpret the “expectation value” of the operators in RED?
A simple example

- The operator in Red – a classical Abelian case:

  \[ \Delta p_2' \]

  In the c.m. frame:

  \[ (m, \vec{0}) \rightarrow \vec{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \rightarrow n = (0, 1, 0_T) \]

  \[ \implies \frac{d}{dt} p_2' = e \epsilon^{s_T \sigma n \bar{n}} F_\sigma^+ \]

- Change of transverse momentum:

  \[ \frac{d}{dt} p_2' = e (\vec{\nu}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23} \]

- The total change:

  \[ \Delta p_2' = e \int d\gamma^- \epsilon^{s_T \sigma n \bar{n}} F_\sigma^+(\gamma^-) \]

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton