QCD Factorization and Transverse Single-Spin Asymmetry in ep collisions

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Based on work with many people

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Outline of my talk

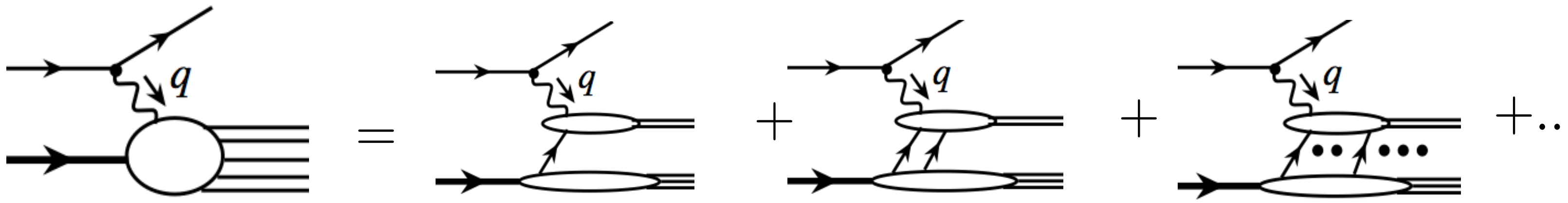
- Transverse single-spin asymmetry in ep collisions

  Ideal observable to go beyond the leading power collinear factorization

- Role of fundamental symmetries
- QCD TMD factorization approach
- QCD collinear factorization approach
- Connection between these two approaches
- Predictive power of QCD factorization approach
- Summary

Electron-proton collisions

❑ Cross sections:



✧ Every parton can participate the hard collision!

✧ Cross section depends on matrix elements of all possible fields

❑ Approximation – single large momentum transfer: Q >> 1/fm

$$\sigma(Q) = \sigma^{\mathrm{LP}}(Q) + \frac{Q_s}{Q}\sigma^{\mathrm{NLP}}(Q) + \frac{Q_s^2}{Q^2}\sigma^{\mathrm{NNLP}}(Q) + \ldots \approx \sigma^{\mathrm{LP}}(Q)$$

❑ Leading power QCD factorization - approximation:

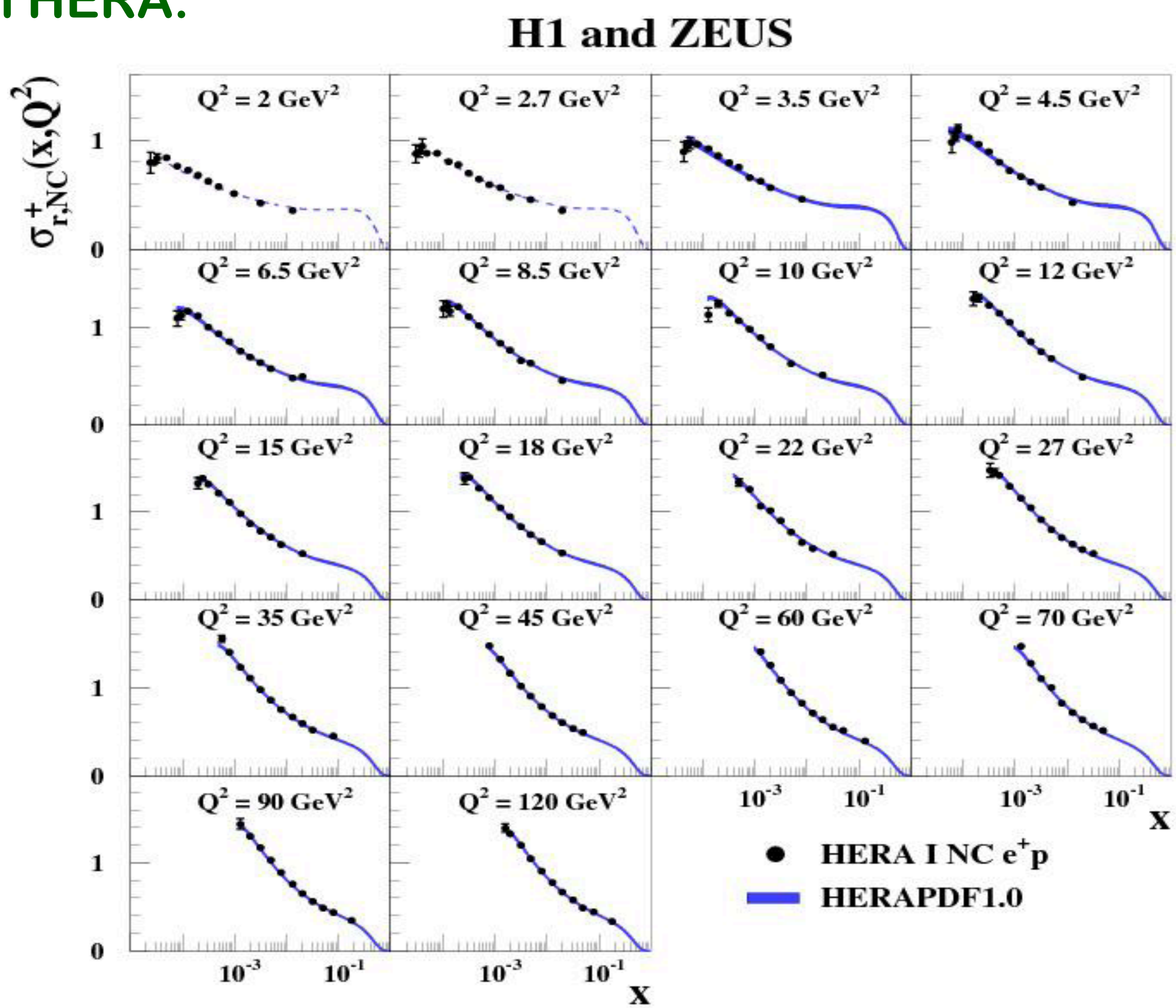
$$\sigma(Q) \approx \sigma^{\mathrm{LP}}(Q) \propto \hat{\sigma}(Q) \otimes \langle p,s|\tilde{\phi}^{\dagger}(k)\tilde{\phi}(k)|p,s\rangle + \ldots$$

Universal parton distributions
Hadron's partonic structure!

❑ How good the approximation is?

Inclusive DIS cross section

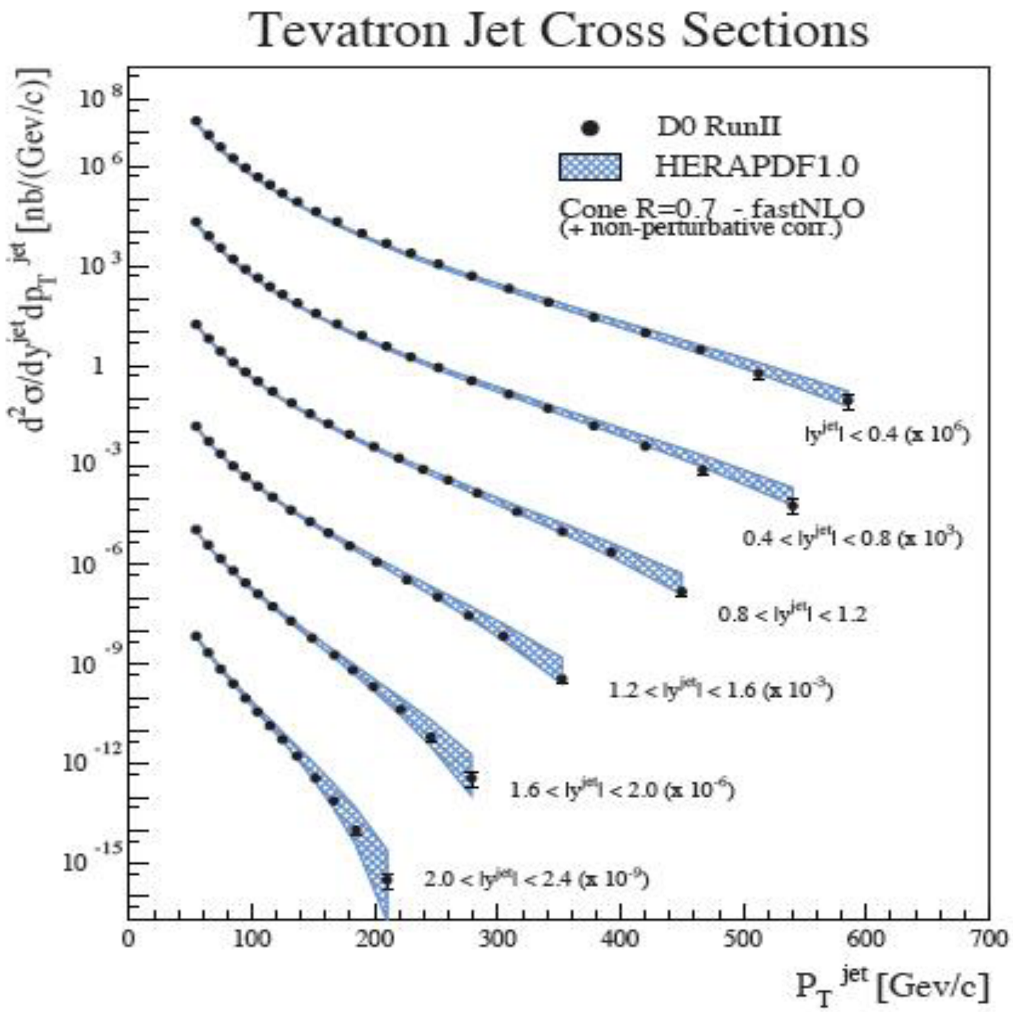
- ❑ **From HERA:**

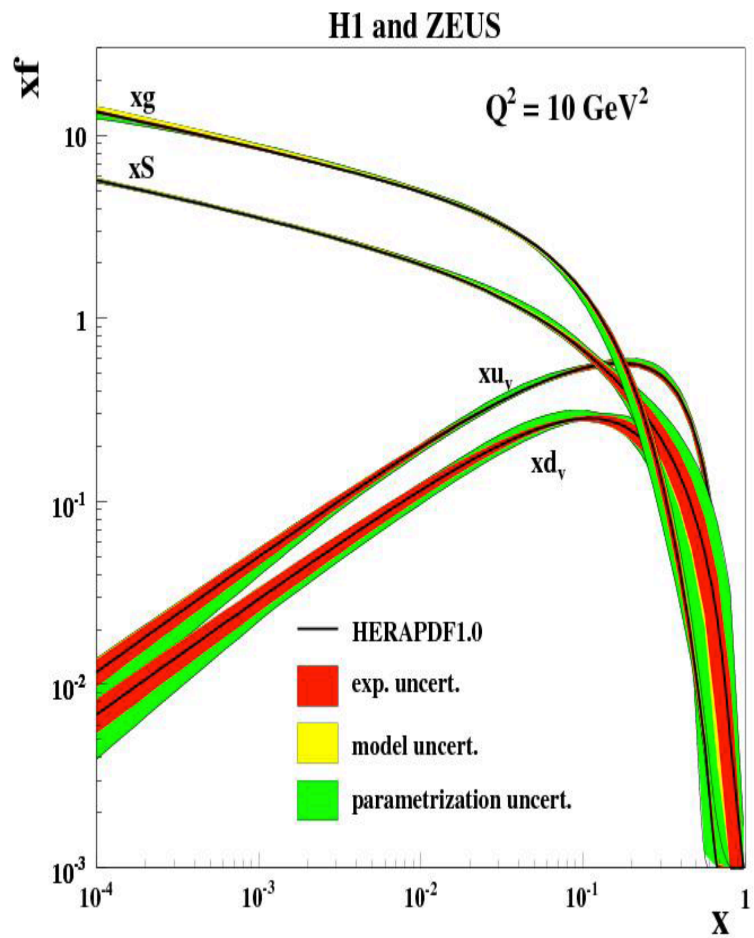


Inclusive single jet hadronic cross section

- To Tevatron:

With one set universal PDFs

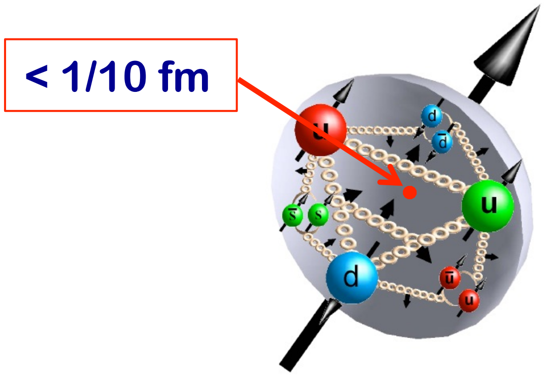




QCD is successful in last 30 years – we now believe it

Fact and questions

- **FACT:**
  - LP QCD collinear factorization/calculations have been very successful in interpreting HEP scattering data if Q > 2 GeV
  - QCD should be correct for the asymptotic regime: r < 1/10 fm!



- **QUESTIONS:**
  - How much have we learned about hadron's partonic structure?

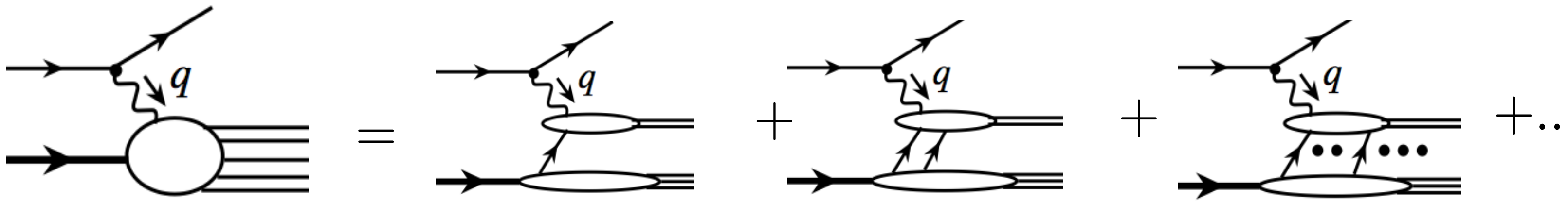
    Collinear PDFs, Helicity PDFs, …

    But, Not enough for the structure, …
  - How to test/explore QCD beyond the leading power formalism?

    Parton's transverse motion, and multiparton correlation beyond 1/10 fm?

Go beyond the LP collinear factorization

❑ **Recall:**



✧ **LP collinear term dominates the single scale cross section**

❑ **Need additional parameter – the LP term is not sensitive to:**

✧ **Nuclear A-dependence:**

– **result of multiple scattering and multiparton correlations**

$$R_A(Q) \equiv \sigma_A(Q)/\sigma_N(Q), \qquad \Delta\langle q_T^2\rangle_A \equiv \langle q_T^2\rangle_A - \langle q_T^2\rangle_N, \ldots$$

✧ **Transverse-spin:**

– **power of fundamental symmetries – cancels the LP collinear term**

$$A_N(Q, s_T) \propto \sigma(Q, s_T) - \sigma(Q, -s_T),$$

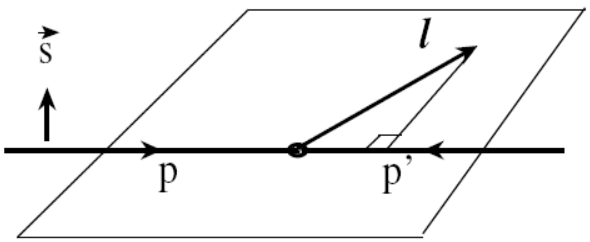
$$A_N(Q, q_T, s_T) \propto \sigma(Q, q_T, s_T) - \sigma(Q, q_T, -s_T), \ldots$$

Transverse SSA in collinear parton model

❑ **SSA corresponds to a naively T-odd triple product:**

$$A_N = [\sigma(p, s_T) - \sigma(p, -s_T)]/[\sigma(p, s_T) + \sigma(p, -s_T)]$$

$$A_N \propto i\, \vec{s}_p \cdot (\vec{p} \times \vec{\ell}) \;\Rightarrow\; i\, \epsilon^{\mu\nu\alpha\beta} p_\mu s_\nu \ell_\alpha p'_\beta$$

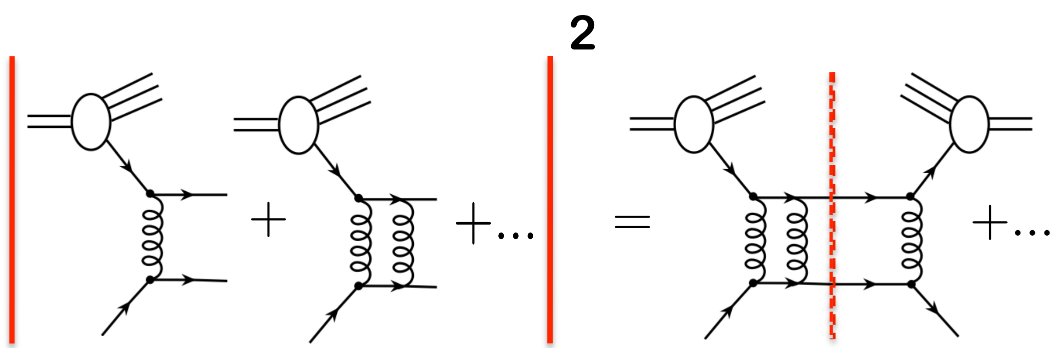


Novanish A_N requires a phase, enough vectors to fix a scattering plan, and a spin flip at the partonic scattering

❑ **Leading power in QCD:**

Kane, Pumplin, Repko, PRL, 1978

$\sigma_{AB}(p_T, \vec{s}) \propto$



$\propto \alpha_s \dfrac{m_q}{p_T}$

Need parton's transverse motion to generate the asymmetry!

Power of fundamental symmetries

- **Factorized cross sections – asymmetries:**

$$A \propto \sigma_{h(p)}(Q,\vec{s}) - \sigma_{h(p)}(Q,-\vec{s}) \propto \langle p,\vec{s}|\mathcal{O}(\psi_q, A^\mu)|p,\vec{s}\rangle - \langle p,-\vec{s}|\mathcal{O}(\psi_q, A^\mu)|p,-\vec{s}\rangle$$

$$e.g.\ \mathcal{O}(\psi, A^\mu) = \overline{\psi}(0)\,\hat{\Gamma}\,\psi(y^-) \qquad \text{with } \hat{\Gamma} = I, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}$$

- **Parity and Time-reversal invariance:**

$$\langle p,\vec{s}|\mathcal{O}(\psi, A^\mu)|p,\vec{s}\rangle = \langle p,-\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi, A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle$$

- **IF:** $\langle p,-\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi, A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi, A^\mu)|p,-\vec{s}\rangle$

  or $\langle p,\vec{s}|\mathcal{O}(\psi, A^\mu)|p,\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi, A^\mu)|p,-\vec{s}\rangle$

Operators lead to the "+" sign ⟶ **spin-averaged cross sections**

Operators lead to the "-" sign ⟶ **spin asymmetries**

- **Example:**

$$\mathcal{O}(\psi, A^\mu) = \overline{\psi}(0)\,\gamma^+\,\psi(y^-) \;\Rightarrow\; q(x)$$

$$\mathcal{O}(\psi, A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma_5\,\psi(y^-) \;\Rightarrow\; \Delta q(x)$$

$$\mathcal{O}(\psi, A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma^\perp\gamma_5\,\psi(y^-) \;\Rightarrow\; \delta q(x) \rightarrow h(x)$$

$$\mathcal{O}(\psi, A^\mu) = \frac{1}{xp^+}F^{+\alpha}(0)[-i\varepsilon_{\alpha\beta}]F^{+\beta}(y^-) \Rightarrow \Delta g(x)$$

$A_N = 0$ for inclusive DIS

- DIS cross section: $\sigma(Q, s_T) \propto L^{\mu\nu} W_{\mu\nu}(Q, s_T)$
- Leptionic tensor is symmetric:

$$L^{\mu\nu} = L^{\nu\mu}$$

- Hadronic tensor:

$$W_{\mu\nu}(Q, s_T) \propto \langle P, s_T | j^\dagger_\mu(0) j_\nu(y) | P, s_T \rangle$$

- Polarized cross section:

$$\Delta\sigma(Q, s_T) \propto L^{\mu\nu} \left[W_{\mu\nu}(Q, s_T) - W_{\mu\nu}(Q, -s_T) \right]$$

- P and T invariance:

$$\langle P, s_T | j^\dagger_\mu(0) j_\nu(y) | P, s_T \rangle = \langle P, -s_T | j^\dagger_\nu(0) j_\mu(y) | P, -s_T \rangle$$

$$\Rightarrow \; A_N(Q, s_T)^{\text{DIS}} = 0$$

Advantage of SIDIS

❑ Dominated by events with two different scales:

$$\ell(l, s_e) + A(P_A, s) \to \ell'(l') + h(p_h) + X$$

✧ A large momentum transfer: $Q = \sqrt{-(l-l')^2} \gg 1/\mathrm{fm}$

Localized probe, suppress contribution of complicate matrix elements

✧ A small momentum scale: $p_{hT} \sim 1/\mathrm{fm}$

Sensitive to parton's motion inside a hadron – TMD distributions

❑ Power of varying $p_{hT} \sim 1/\mathrm{fm} \to p_{hT} \sim Q$

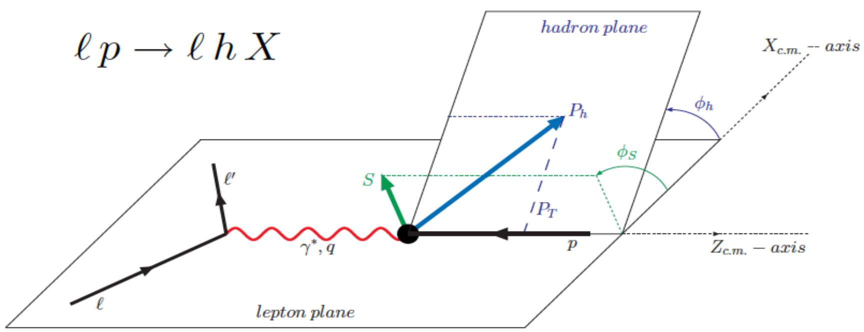
✧ Change from a two-scale problem to an one-scale problem

Transition from TMD factorization to Collinear factorization

$\ell\, p \to \ell\, h\, X$

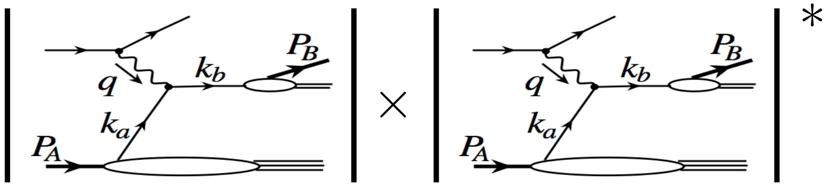
❑ Two natural scattering planes:

✧ Separation of various TMDs and spin states



TMD factorization – SIDIS

Collins' book



$$\propto \; L^{\mu\nu} \; W_{\mu\nu}(Q, p_{BT}, s_T)$$

$$W^{\mu\nu} = \sum_f |\mathcal{H}_f(Q;\mu)^2|^{\mu\nu} \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P^\uparrow}(x, \mathbf{k}_{1T}, S; \mu; \zeta_F)\, D_{h/f}(z, z\mathbf{k}_{2T}; \mu; \zeta_D)\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{q}_T - \mathbf{k}_{2T})$$

$$+ Y(Q, \mathbf{q}_T) + \mathcal{O}((\Lambda/Q)^a) \quad \Longrightarrow \quad \sigma_0\, \phi(x,\mu) \otimes D(z,\mu) \delta^2(p_{BT})$$

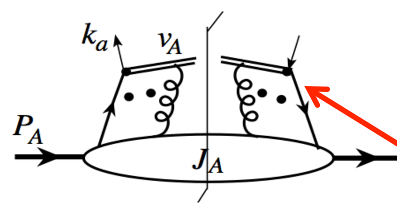
$$\zeta_F = M_P^2 x^2 e^{2(y_P - y_s)}$$

$$\zeta_D = (M_h^2/z^2) e^{2(y_s - y_h)}$$

$$\sqrt{\zeta_F \zeta_D} = Q^2$$

- ❑ **TMD parton distribution:**

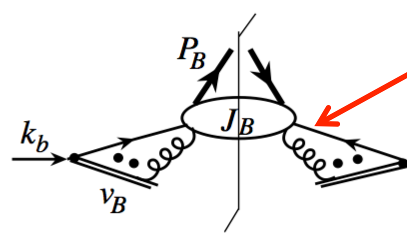
$$F_{f/P\uparrow}(x, k_{1T}, S, \mu, \zeta_F) = \mathrm{Tr}_{\rm color}\, \mathrm{Tr}_{\rm Dirac}\, \frac{\gamma^+}{2} \int \frac{k_1^-}{2\pi}$$



Gauge links

- ❑ **TMD fragmentation function:**

$$D_{h/f}(z, k_{2T}, \mu, \zeta_D) = \frac{\mathrm{Tr}_{\rm color}}{N_c}\, \frac{\mathrm{Tr}_{\rm Dirac}}{4}\, \frac{\gamma^+}{z} \int \frac{k_2^-}{2\pi}$$



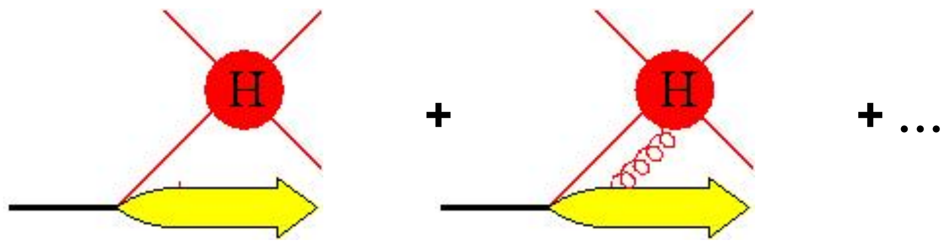
Phase for SSA

- ❑ **TMDs are more fundamental if we can measure them:**

Carry more information on hadron's partonic structure

Color flow – gauge links

❑ **Gauge link – QCD phase:**



Summation of leading power gluon field contribution produces the gauge link:

$$\Phi_n(\infty, y^-) = \mathcal{P} \exp\left(-ig \int_{y^-}^{\infty} d\lambda\, n \cdot A(\lambda n)\right)$$

Gauge invariant PDFs:

$$\phi(x,p,s) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \langle p,s|\overline{\psi}(0)_j \hat{\Gamma}_{ji} \Phi_n^\dagger(\infty,0)\Phi_n(\infty,y^-)\psi_i(y^-)|p,s\rangle$$

Collinear PDFs:

"Localized" operator with size ~ 1/xp ~ 1/Q

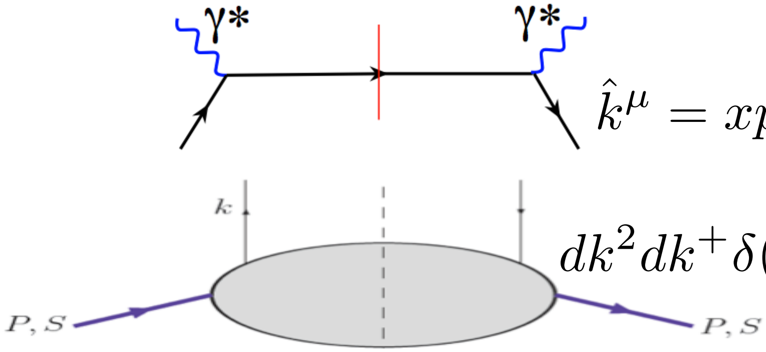
"localized" color flow

❑ **Universality of PDFs:**

Gauge link should be process independent!

TMD parton distributions

❑ Quark TMD distributions:



$$\hat{k}^\mu = xp^\mu + \frac{k_T^2}{2xp^+}n^\mu + k_T^\mu$$

$$dk^2 dk^+ \delta(x - k^+/P^+)$$

$$\begin{aligned}\Phi(x, \boldsymbol{k}_\perp) &= \frac{1}{2}\Big[f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \left(S_L g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^\perp\right)\gamma^5 \not{n}_+ \\ &+ h_{1T} i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu + \left(S_L h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} h_{1T}^\perp\right)\frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_\perp^\nu}{M} \\ &+ h_1^\perp \frac{\sigma_{\mu\nu}k_\perp^\mu n_+^\nu}{M}\Big]\end{aligned}$$

Total 8 TMD quark distributions

❑ Gluon TMD distributions, …

Production of quarkonium, two-photon, …

Most notable TMDs

- Sivers function – transverse polarized hadron:

Sivers function

$$f_{q/p,\boldsymbol{S}}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$
$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

- Boer-Mulder function – transverse polarized quark:

$$f_{q,\boldsymbol{s}_q/p}(x,\boldsymbol{k}_\perp) = \frac{1}{2} f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q^\uparrow/p}(x,k_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$
$$= \frac{1}{2} f_{q/p}(x,k_\perp) - \frac{1}{2}\frac{k_\perp}{M} h_1^{\perp q}(x,k_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

Boer-Mulder function

Affect angular distribution of Drell-Yan lepton pair production

Most notable TMDs – II

- Collins function – FF of a transversely polarized parton:

$$D_{h/q,s_q}(z,\boldsymbol{p}_\perp) = D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\, \boldsymbol{s}_q \cdot (\hat{\boldsymbol{p}}_q \times \hat{\boldsymbol{p}}_\perp)$$

$$= D_{h/q}(z,p_\perp) + \frac{p_\perp}{z\,M_h} H_1^{\perp q}(z,p_\perp)\, \boldsymbol{s}_q \cdot (\hat{\boldsymbol{p}}_q \times \hat{\boldsymbol{p}}_\perp)$$

Collins function

- Fragmentation function to a polarized hadron:

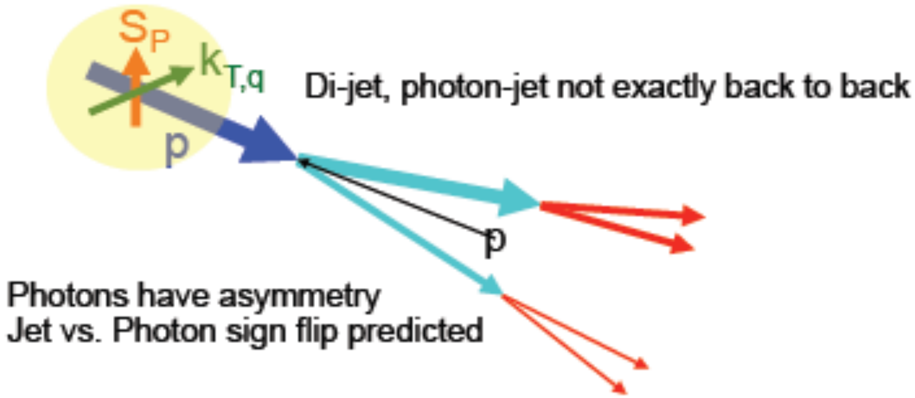
$$D_{\Lambda,S_\Lambda/q}(z,\boldsymbol{p}_\perp) = \frac{1}{2} D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{\Lambda^\uparrow/q}(z,p_\perp)\, \boldsymbol{S}_\Lambda \cdot (\hat{\boldsymbol{p}}_q \times \hat{\boldsymbol{p}}_\perp)$$

$$= \frac{1}{2} D_{h/q}(z,p_\perp) + \frac{p_\perp}{z\,M_\Lambda} D_{1T}^{\perp q}(z,p_\perp)\, \boldsymbol{S}_\Lambda \cdot (\hat{\boldsymbol{p}}_q \times \hat{\boldsymbol{p}}_\perp)$$

Unpolarized parton fragments into a polarized hadron - Λ

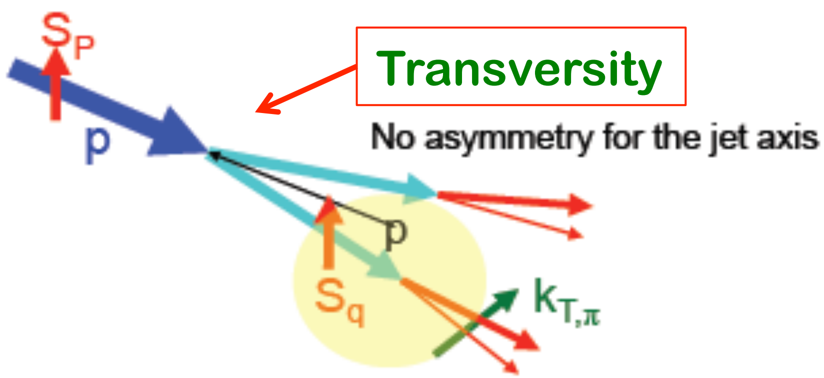
TMDs and spin asymmetries

- Sivers' effect – Sivers' function:



Hadron spin influences parton's transverse motion

- Collin's effect – Collin's function:



Parton's transverse spin affects its hadronization

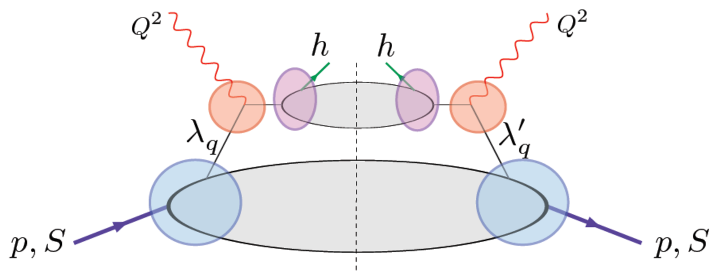
Separation of different effects?

- TMD factorization is relevant for two-scale problems in QCD:

$$Q_1 \gg Q_2 \sim \Lambda_{\rm QCD}$$

SIDIS is ideal for studying TMDs

❑ SIDIS has the natural kinematics for TMD factorization:



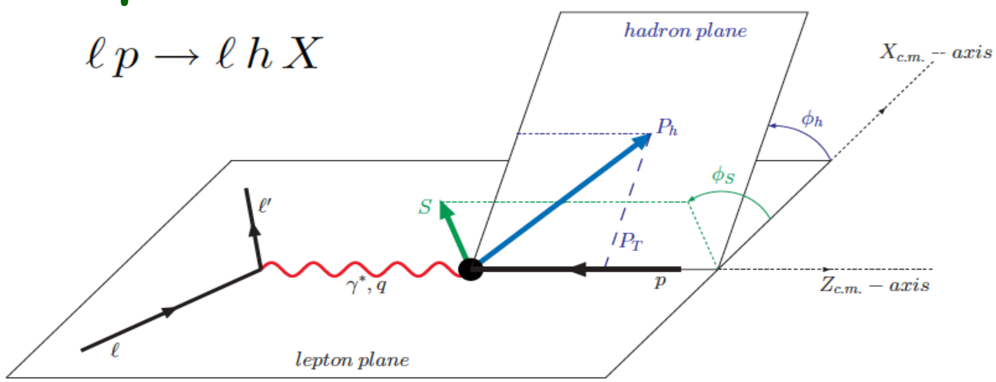
$$\ell(s_e) + p(s_p) \to \ell + h(s_h) + X$$

Natural event structure:

high Q and low p_T jet (or hadron)

❑ Separation of various TMD contribution by angular projection:

$$\ell\, p \to \ell\, h\, X$$



Lepton plane vs. hadron plane

$$A_{UT}(\varphi_h^l, \varphi_S^l) = \frac{1}{P}\frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow}$$

$$= A_{UT}^{Collins}\sin(\phi_h + \phi_S) + A_{UT}^{Sivers}\sin(\phi_h - \phi_S)$$

$$+ A_{UT}^{Pretzelosity}\sin(3\phi_h - \phi_S)$$

$$\longrightarrow$$

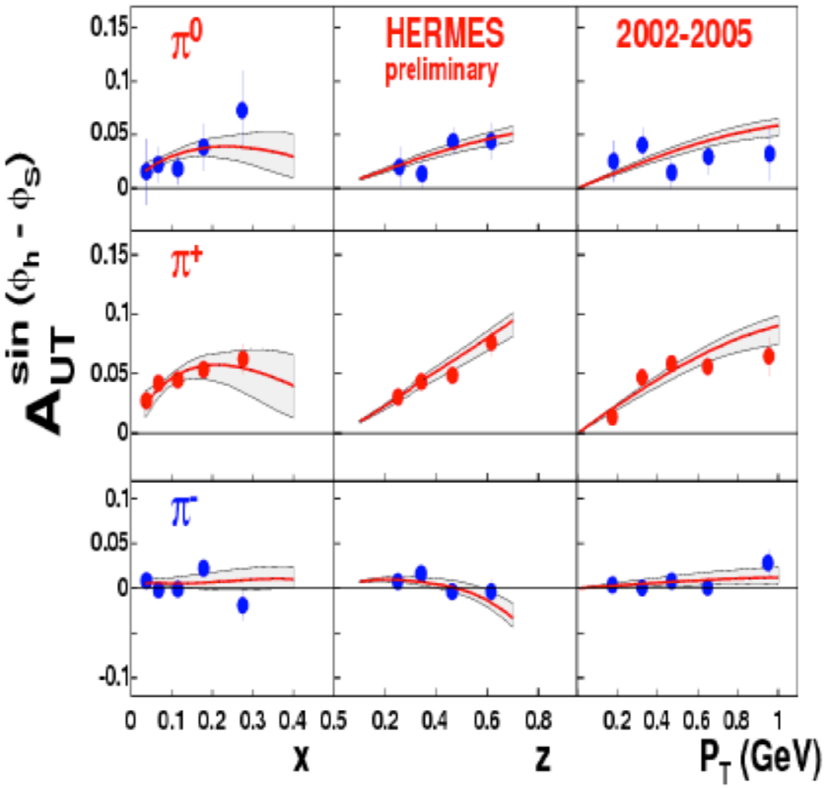
$$A_{UT}^{Collins} \propto \left\langle \sin(\phi_h + \phi_S) \right\rangle_{UT} \propto h_1 \otimes H_1^\perp$$

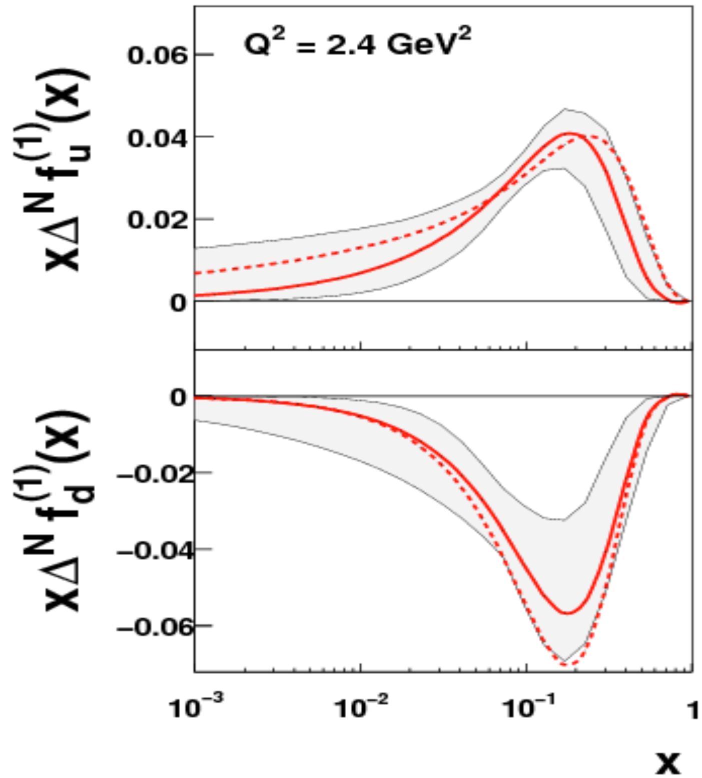
$$A_{UT}^{Sivers} \propto \left\langle \sin(\phi_h - \phi_S) \right\rangle_{UT} \propto f_{1T}^\perp \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \left\langle \sin(3\phi_h - \phi_S) \right\rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp$$

Our knowledge of TMDs

❑ Sivers function from low energy SIDIS:





EIC can do much better job in extracting TMDs

❑ NO TMD factorization for hadron production in p+p collisions!

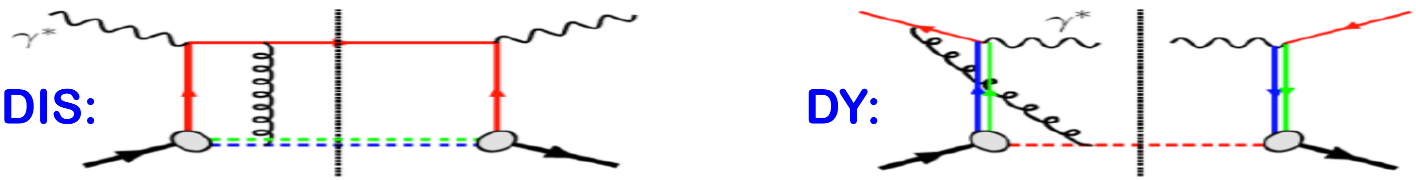
Collins and Qiu, 2007, Vogelsang and Yuan, 2007, Mulders and Rogers, 2010, …

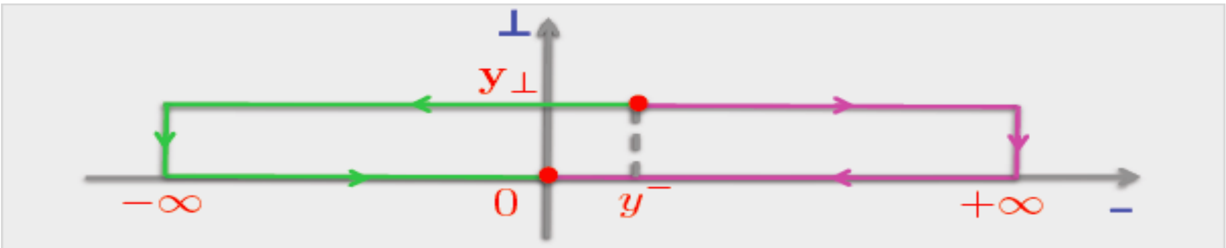
Critical test of TMD factorization

❑ **TMD distributions with non-local gauge links:**

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp) \boxed{\text{Gauge link}} \frac{\gamma^+}{2}\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

SIDIS: DY:





- For a fixed spin state:

$$f^{\text{SIDIS}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \neq f^{\text{DY}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S})$$

❑ **Parity + Time-reversal invariance:**

$$\Longrightarrow \quad f^{\text{Sivers}}_{q/h^\uparrow}(x,k_\perp)^{\text{SIDIS}} = -f^{\text{Sivers}}_{q/h^\uparrow}(x,k_\perp)^{\text{DY}}$$

The sign change is a critical test of TMD factorization approach

Another critical test of TMD factorization

- **Predictive power of QCD factorization:**
  - ✧ **Infrared safety of short-distance hard parts**
  - ✧ **Universality of the long-distance matrix elements**
  - ✧ **QCD evolution or scale dependence of the matrix elements**

- **QCD evolution:**

  If there is a factorization/invariance, there is an evolution equation

- **Collinear factorization – DGLAP evolution:**

$$\sigma_{\rm phy}(Q,\Lambda_{\rm QCD}) \approx \sum_f \hat{\sigma}_f(Q,\mu) \otimes \phi_f(\mu,\Lambda_{\rm QCD}) \quad \rightarrow \quad \frac{d}{d\mu}\sigma_{\rm phy}(Q,\Lambda_{\rm QCD}) = 0$$

Scaling violation of nonperturbative functions

Evolution kernels are perturbative – a test of QCD

Evolution equations for TMDs

- **Collins-Soper equation:**
  - b-space quark TMD with γ^+

Boer, 2001, 2009, Idilbi, et al, 2004
Aybat, Rogers, 2010
Kang, Xiao, Yuan, 2011
Aybat, Collins, Qiu, Rogers, 2011

$$\frac{\partial \tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T;\mu)\tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; \zeta_F) \qquad \tilde{K}(b_T;\mu) = \frac{1}{2}\frac{\partial}{\partial y_s}\ln\left(\frac{\tilde{S}(b_T; y_s, -\infty)}{\tilde{S}(b_T; +\infty, y_s)}\right)$$

- **RG equations:**

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu)) \qquad \frac{d\tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; \zeta_F)}{d\ln\mu} = \gamma_F(g(\mu); \zeta_F/\mu^2)\tilde{F}_{f/P^\uparrow}(x, \mathrm{b_T}, S; \mu; \zeta_F).$$

- **Evolution equations for Sivers function:**

$$F_{f/P^\uparrow}(x, k_T, S; \mu, \zeta_F) = F_{f/P}(x, k_T; \mu, \zeta_F) - F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F)\frac{\epsilon_{ij}k_T^i S^j}{M_p}$$

CS: $$\frac{\partial \ln \tilde{F}_{1T}^{\prime\perp f}(x, b_T; \mu, \zeta_F)}{\partial \ln\sqrt{\zeta_F}} = \tilde{K}(b_T;\mu) \qquad \tilde{F}_{1T}^{\prime\perp f}(x, b_T; \mu, \zeta_F) \equiv \frac{\partial \tilde{F}_{1T}^{\perp f}(x, b_T; \mu, \zeta_F)}{\partial b_T}$$

RGs: $$\frac{d\tilde{F}_{1T}^{\prime\perp f}(x, b_T; \mu, \zeta_F)}{d\ln\mu} = \gamma_F(g(\mu); \zeta_F/\mu^2)\tilde{F}_{1T}^{\prime\perp f}(x, b_T; \mu, \zeta_F)$$

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu)) \quad \Longrightarrow \quad \frac{\partial \gamma_F(g(\mu); \zeta_F/\mu^2)}{\partial \ln\sqrt{\zeta_F}} = -\gamma_K(g(\mu)),$$

Scale dependence of Sivers function

Aybat, Collins, Qiu, Rogers, 2011

- Kernel is not perturbative for all b:

CSS prescription:
(not unique)

$$\mathrm{b}_* = \frac{\mathrm{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}} \qquad \mu_b = \frac{C_1}{b_*}$$

$$\tilde{K}(b_T;\mu) = \tilde{K}(b_*;\mu_b) - \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'}\gamma_K(g(\mu')) - g_K(b_T)$$

- Q^2-dependence of Sivers function:

$$\tilde{F}_{1T}'^{\perp f}(x,b_T;\mu,\zeta_F) = \tilde{F}_{1T}'^{\perp f}(x,b_T;\mu_0,Q_0^2)\exp\Bigg\{\ln\frac{\sqrt{\zeta_F}}{Q_0}\tilde{K}(b_*;\mu_b) + \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu');1) - \ln\frac{\sqrt{\zeta_F}}{\mu'}\gamma_K(g(\mu'))\right] + \int_{\mu_0}^{\mu_b}\frac{d\mu'}{\mu'}\ln\frac{\sqrt{\zeta_F}}{Q_0}\gamma_K(g(\mu')) - g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{Q_0}\Bigg\}$$

$$F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F) = \frac{-1}{2\pi k_T}\int_0^\infty db_T\, b_T J_1(k_T b_T)\tilde{F}_{1T}'^{\perp f}(x,b_T;\mu,\zeta_F)$$

– Evolved Sivers function

- Small-b perturbative contribution – match to twist-3:

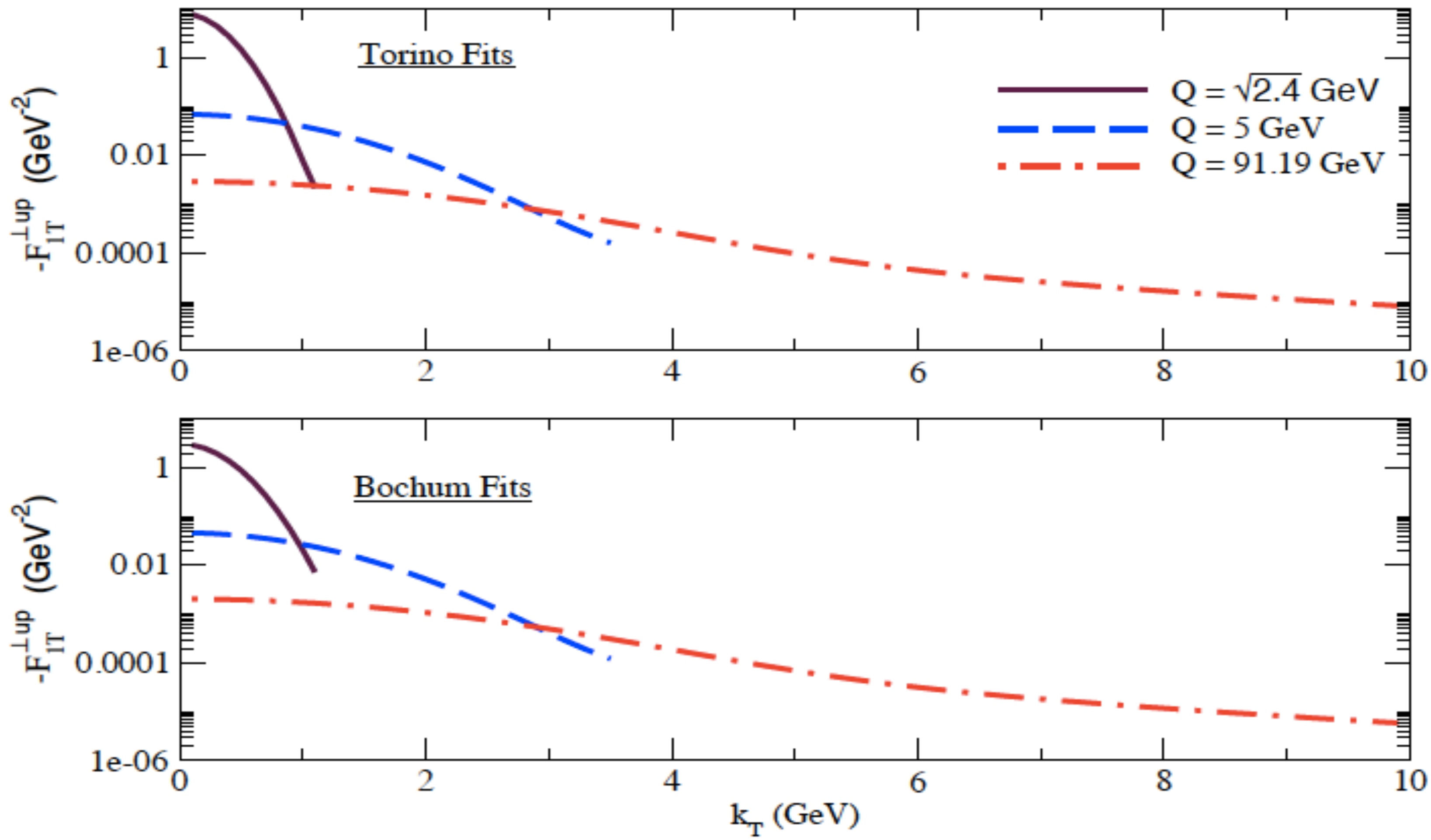
$$\tilde{F}_{1T}'^{\perp f}(x,b_T;\mu,\zeta_F) = \sum \frac{M_p b_T}{2}\int_x^1 \frac{d\hat{x}_1\, d\hat{x}_2}{\hat{x}_1\,\hat{x}_2}\tilde{C}_{f/j}^{\text{Sivers}}(\hat{x}_1,\hat{x}_2,b_*;\mu_b^2,\mu_b,g(\mu_b))\, T_{F\,j/P}(\hat{x}_1,\hat{x}_2,\mu_b)$$
$$\times\exp\left\{\ln\frac{\sqrt{\zeta_F}}{\mu_b}\tilde{K}(b_*;\mu_b) + \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu');1) - \ln\frac{\sqrt{\zeta_F}}{\mu'}\gamma_K(g(\mu'))\right]\right\}\times\exp\left\{-g_{j/P}^{\text{Sivers}}(x,b_T) - g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{Q_0}\right\}$$

Kang, Xiao, Yuan, 2011

Gaussian ansatz for input distributions

Aybat, Collins, Qiu, Rogers, 2011

- ❑ Up quark Sivers function:

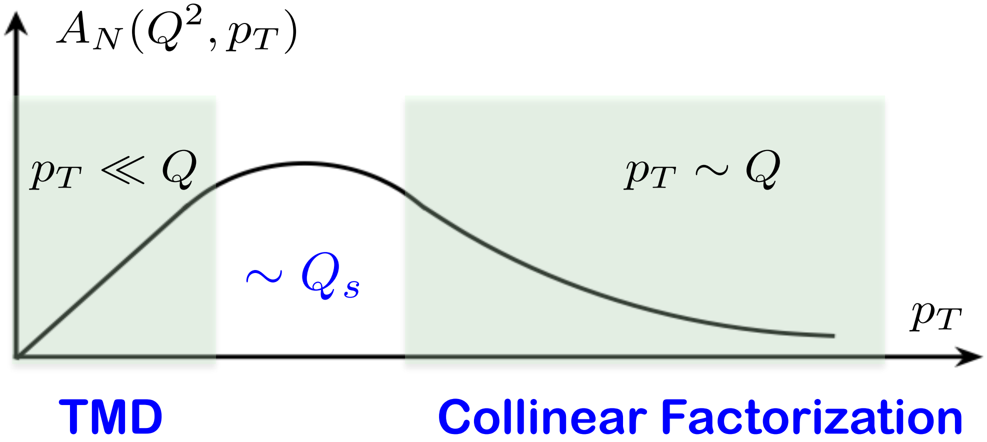


Very significant growth in the width of transverse momentum

From low p_T to high p_T

❑ TMD factorization to collinear factorization:

Ji, Qiu, Vogelsang, Yuan, Koike, Vogelsang, Yuan



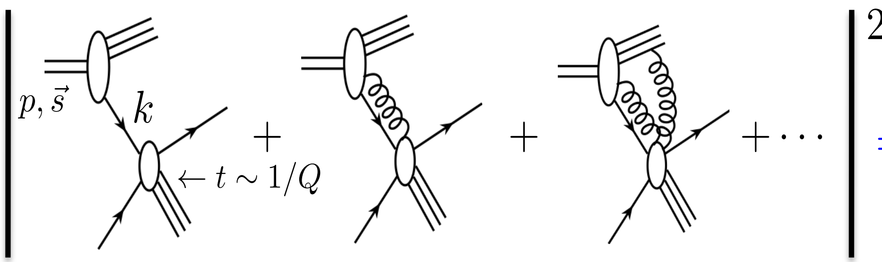
Two factorization are consistent in the overlap region where

$$\Lambda_{\rm QCD} \ll p_T \ll Q$$

❑ QCD collinear factorization:

Efremov, Teryaev, 82; Qiu, Sterman, 91, etc.

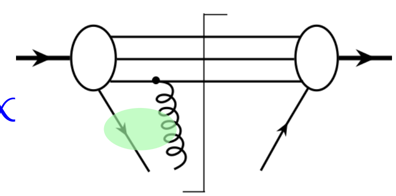
$\sigma(Q, \vec{s}) \propto$



$$= \sigma^{\rm LP}(Q, \vec{s}) + \frac{Q_s}{Q} \sigma^{\rm NLP}(Q, \vec{s}) + \ldots$$

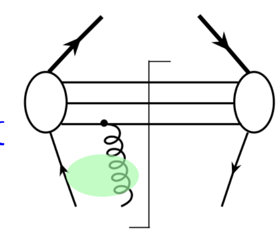
$$\Delta\sigma(s_T) \propto T^{(3)}(x, x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z, z) + \ldots$$

$T^{(3)}(x, x) \propto$



Qiu, Sterman, 1991, …

$D^{(3)}(z, z) \propto$



Kang, Yuan, Zhou, 2010

Twist-3 correlation functions

❑ Twist-2 parton distributions:

Kang, Qiu, PRD, 2009

✧ **Unpolarized PDFs:**

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$
$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

✧ **Polarized PDFs:**

$$\Delta q(x) \propto \langle P,S_{\parallel}|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P,S_{\parallel}\rangle$$
$$\Delta G(x) \propto \langle P,S_{\parallel}|F^{+\mu}(0)F^{+\nu}(y)|P,S_{\parallel}\rangle(i\epsilon_{\perp\mu\nu})$$

❑ Two-sets Twist-3 correlation functions:

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P,s_T|\overline{\psi}_q(0)\, \frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P,s_T|\overline{\psi}_q(0)\, \frac{\gamma^+\gamma^5}{2}\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

Role of color magnetic force!

Evolution equations and kernels

- Evolution equation is a consequence of factorization:

Factorization: $\Delta\sigma(Q, s_T) = (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

DGLAP for f_2: $\frac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

Evolution for f_3: $\frac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)}\right) \otimes f_3$

- Evolution kernel is process independent:

  - ✧ Calculate directly from the variation of process independent twist-3 distributions

    Kang, Qiu, 2009
    Yuan, Zhou, 2009

  - ✧ Extract from the scale dependence of the NLO hard part of any physical process

    Vogelsang, Yuan, 2009

  - ✧ Renormalization of the twist-3 operators

    Braun et al, 2009

Variation of twist-3 correlation functions

- **Closed set of evolution equations (spin-dependent):** Kang, Qiu, 2009

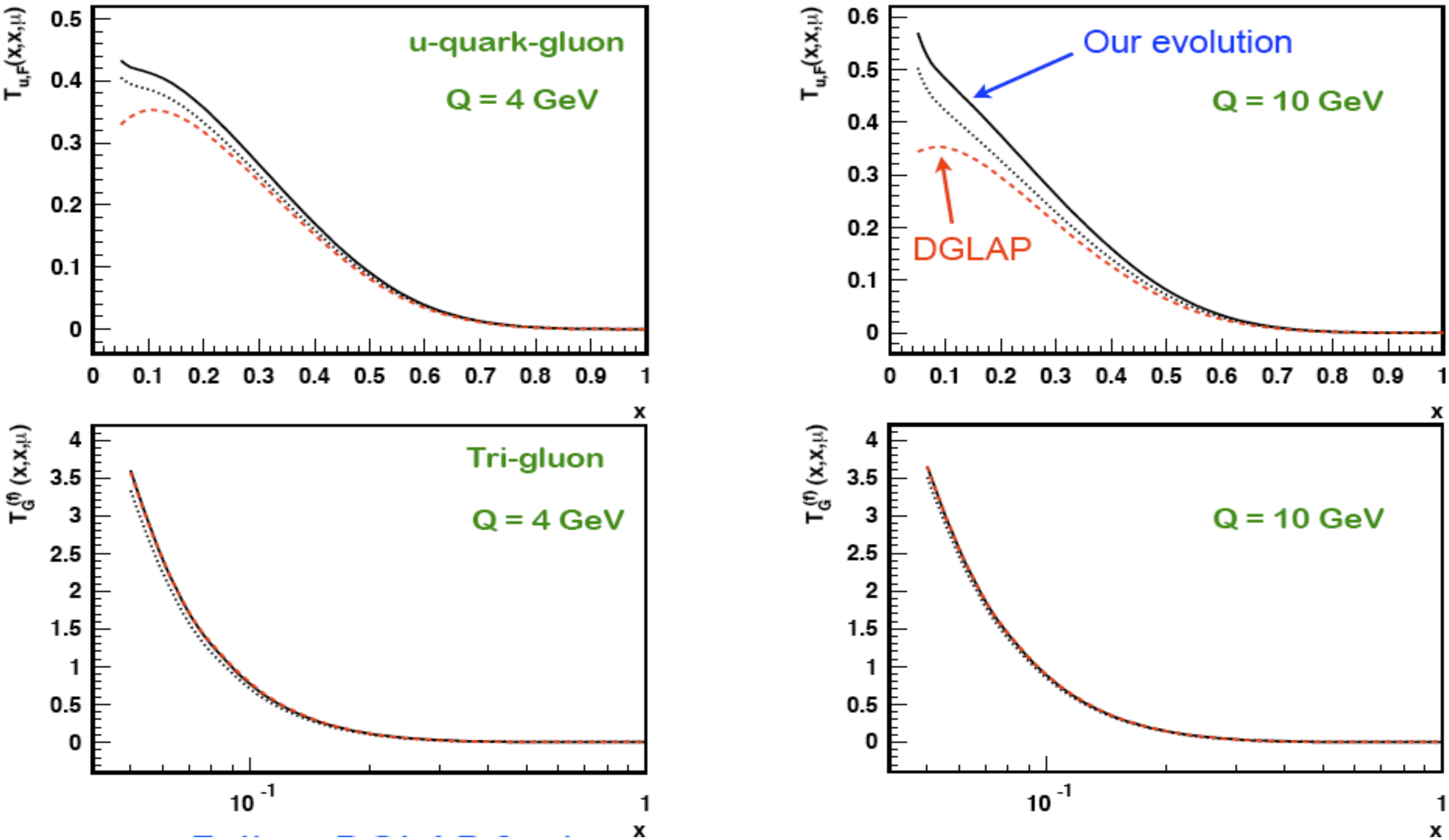
$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{q,F}(x, x+x_2, \mu_F, s_T) = \int d\xi d\xi_2 [\tilde{\mathcal{T}}_{q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K_{qq}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}_{\Delta q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K_{q\Delta q}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)]$$
$$+ \sum_{i=f,d} \int d\xi d\xi_2 [\tilde{\mathcal{T}}^{(i)}_{G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{qg}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{q\Delta g}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)].$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{G,F}(x, x+x_2, \mu_F, s_T) = \sum_{j=f,d} \int d\xi d\xi_2 [\tilde{\mathcal{T}}^{(j)}_{G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(ji)}_{gg}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}^{(j)}_{\Delta G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(ji)}_{g\Delta g}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)]$$
$$+ \sum_q \int d\xi d\xi_2 [\tilde{\mathcal{T}}_{q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{gq}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}_{\Delta q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{g\Delta q}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)],$$

Plus two more equations for:

$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{\Delta q,F}(x, x+x_2, \mu_F, s_T)$ **and** $\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(x, x+x_2, \mu_F, s_T)$

Scale dependence



- ✧ Follow DGLAP at large x
- ✧ Large deviation at low x (stronger correlation)

A sign "mismatch"

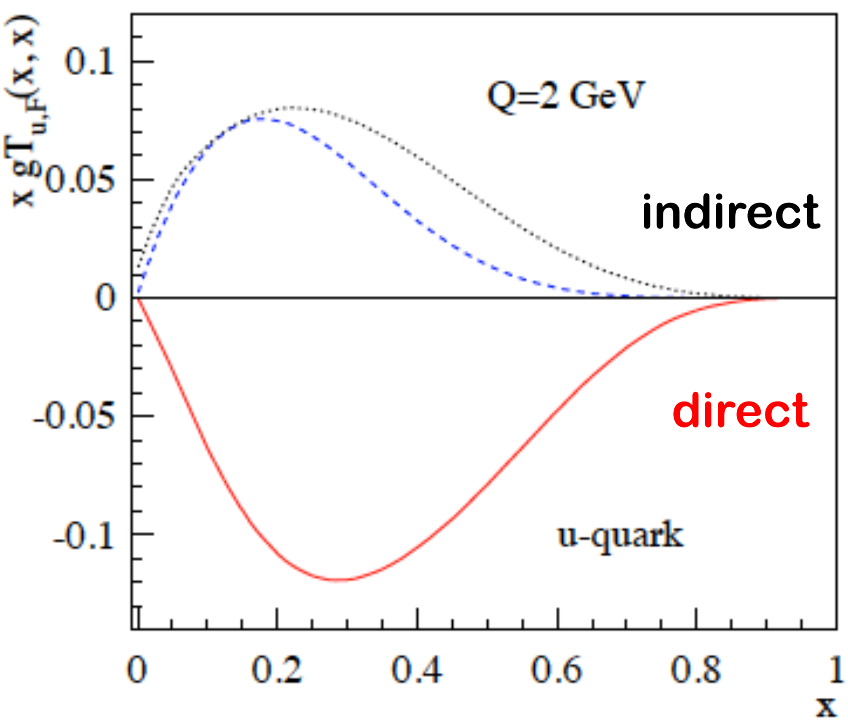
Kang, Qiu, Vogelsang, Yuan, 2011

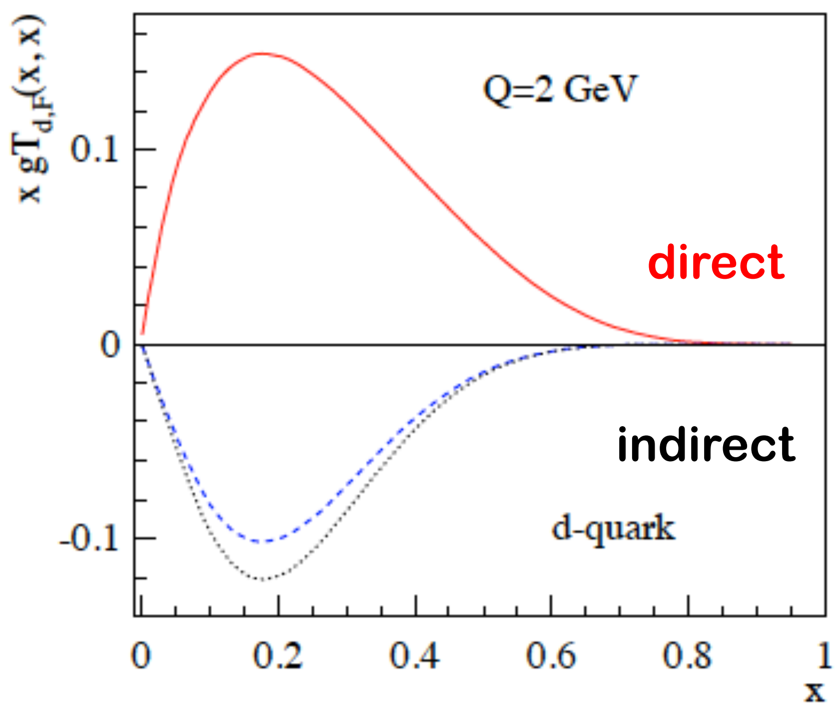
- Sivers function and twist-3 correlation:

$$gT_{q,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x,k_\perp^2)|_{\rm SIDIS} \quad \textbf{+ UVCT}$$

- "direct" and "indirect" twist-3 correlation functions:

Calculate $T_{q,F}(x,x)$ by using the measured Sivers functions



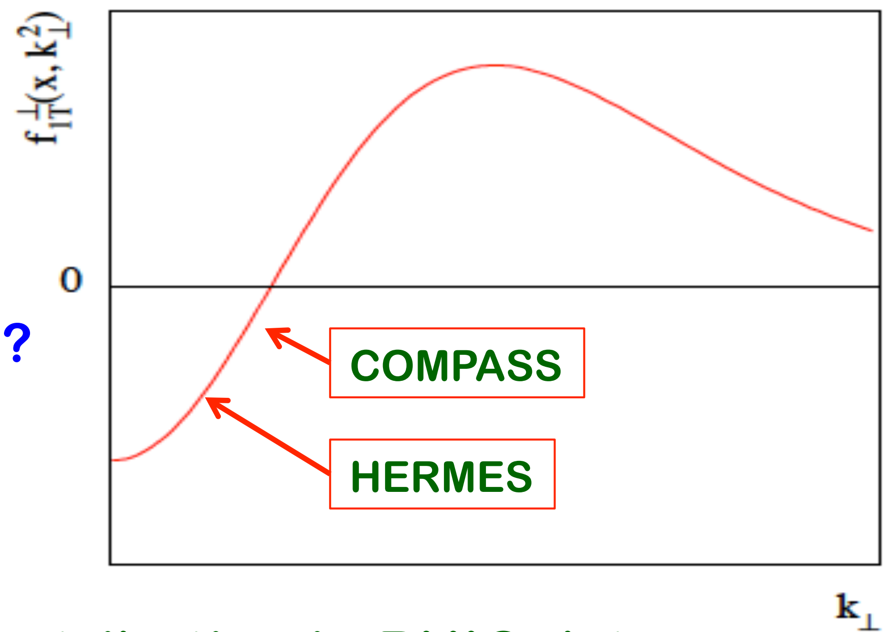


Possible interpretations

Kang, Qiu, Vogelsang, Yuan, 2011

- ❑ A node in k_T-distribution:
  - ✧ Like the DSSV's $\Delta G(x)$
  - ✧ HERMES vs COMPASS
  - ✧ Physics behind the sign change?

  EIC can measure TMDs
  for a wide range of k_T



- ❑ Large twist-3 fragmentation contribution in RHIC data:

  If Sivers-type initial-state effect is much smaller than fragmentation effect and two effects have an opposite sign

  Can be tested by A_N of single jet or direct photon at RHIC

- ❑ A node in x-dependence of Sivers or twist-3 distributions

  Physics behind the node if there is any

Boer, …

Propose new observables for ep collisions

Kang, Metz, Qiu, Zhou, 2011

- **Process:** $e(\ell) + h(p) \to \mathrm{jet}(p_j)(\mathrm{or}\ \pi, ...) + X$

  Lepton-hadron scattering without measuring the scattered lepton

  Single hard scale: p_{jT} in lepton-hadron frame

- **Complement to SIDIS:** $e(\ell) + h(p) \to e'(\ell') + \mathrm{jet}(p_j)(\mathrm{or}\ \pi, ...) + X$

  Two scales: $Q,\ p_{jT}$ in virtual-photon-hadron frame

- **Key difference in theory treatment:**

  Collinear factorization for $e(\ell) + h(p) \to \mathrm{jet}(p_j)(\mathrm{or}\ \pi, ...) + X$

  TMD factorization for $e(\ell) + h(p) \to e'(\ell') + \mathrm{jet}(p_j)(\mathrm{or}\ \pi, ...) + X$

Test the consistency between TMD and Twist-3 to SSA
in the same experimental setting
Jlab, Compass, Future EIC, …

Analytical formulae

Kang, Metz, Qiu, Zhou, 2011

❑ Factorization is valid:

Same as hadron-hadron collision to jet + X

$$\frac{d\sigma^{lh\to \text{jet}(P_J)X}}{dP_{JT}dy} \approx \sum_{ab}\int dx\, f_1^{a/l}(x,\mu)\int dx'\, f_1^{b/h}(x',\mu)\frac{d\hat{\sigma}^{ab\to \text{Jet}(P_J)X}}{dP_{JT}dy}(x,x',P_{JT},y,\mu)$$

$$a = l,\gamma,q,\bar{q},g$$
$$b = q,\bar{q},g$$

❑ Leading order results:

$$\begin{aligned} P_J^0\frac{d^3\sigma}{d^3P_J} \quad = \quad & \frac{\alpha_{em}^2}{s}\sum_a\frac{e_a^2}{(s+t)\,x}\Big\{f_1^a(x)\,H_{UU}+\lambda_l\lambda_p\,g_1^a(x)\,H_{LL} \\ & + 2\pi M\,\varepsilon_T^{ij}S_T^iP_{JT}^j\left[T_F^a(x,x)-x\,\frac{d}{dx}T_F^a(x,x)\right]\frac{\hat{s}}{\hat{t}\hat{u}}\,H_{UU} \\ & + \lambda_l\,2M\,\vec{S}_T\cdot\vec{P}_{JT}\left[\left(\tilde{g}^a(x)-x\frac{d}{dx}\tilde{g}^a(x)\right)\frac{\hat{s}}{\hat{t}\hat{u}}\,H_{LL}+x\,g_T^a(x)\,\frac{2}{\hat{t}}\right]\Big\}\end{aligned}$$

$\lambda_l,\ \lambda_p$: Lepton, hadron helicity, respectively

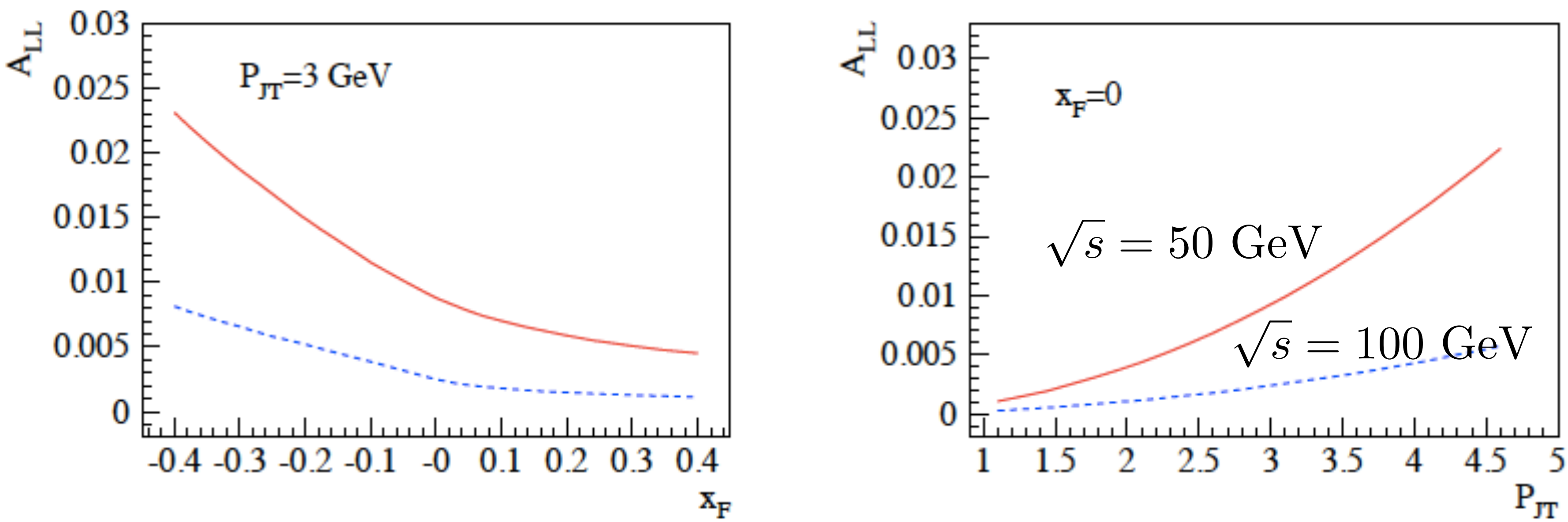
$\vec{S}_T$: Hadron's transverse spin vector

Numerical results

- Asymmetries:

$$A_{LL} = \frac{\sigma_{LL}}{\sigma_{UU}}, \qquad A_{UT} = \frac{\sigma_{UT}}{\sigma_{UU}}, \qquad A_{LT} = \frac{\sigma_{LT}}{\sigma_{UU}}$$

- Double spin asymmetries – very small:

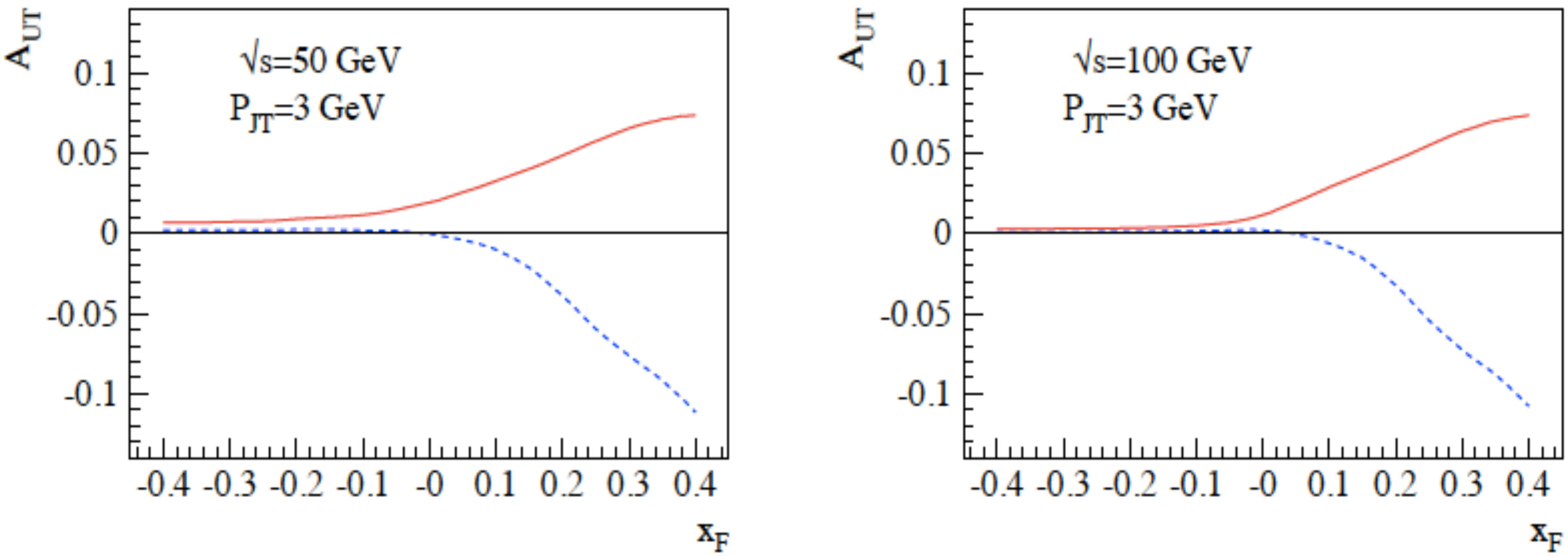


Wandzura-Wilczek approximation:

$$g_T(x) \approx \int_x^1 \frac{dy}{y} g_1(y) \qquad \tilde{g}(x) \approx x \int_x^1 \frac{dy}{y} g_1(y) \qquad \to \; A_{LT} \sim 0.001$$

Good probe of Sivers function

- Independent check of the "sign mismatch":



Red line: $T_F(x,\mu)$ extracted from fitting SSA in hadronic collisions

Blue line: $\pi T_F(x,x) = -\int d^2k_T \frac{\vec{k}_T^2}{2M^2} f_{1T}^{\perp}(x,\vec{k}_T^2)\Big|_{DIS}$ Sivers function

Excellent test for the mechanism of SSA
possibly at Jlab, surely at future EIC

More on future directions

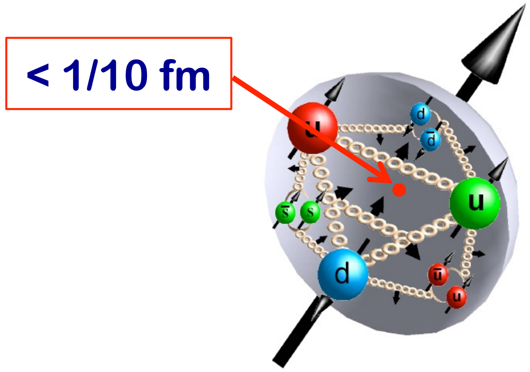
- RHIC spin, JLab at 12 GeV, possibly at Compass, …
- Future EIC:
  – a dedicated QCD machine for the visible matter

  Yellow book on EIC physics from INT workshop is available:

  arXiv: submit/0295324 [nucl-th]
- A white paper on EIC physics:
  – a writing group appointed by BNL and Jlab is working hard
- Physics opportunities at EIC:
  - Inclusive DIS – Spin, F_L, …
  - SIDIS – TMDs, spin-orbital correlations,
  - One jet or particle inclusive – multiparton quantum correlation, …
  - GPDs – parton spatial distributions
  - …

Summary

- QCD factorization/calculation have been very successful in interpreting HEP scattering data



- What about the hadron structure?

  Not much!

- RHIC spin, Jlab12, a future EIC with a polarized hadron beams opens up many new ways to test QCD and to study hadron structure: TMDs, GPDs, …

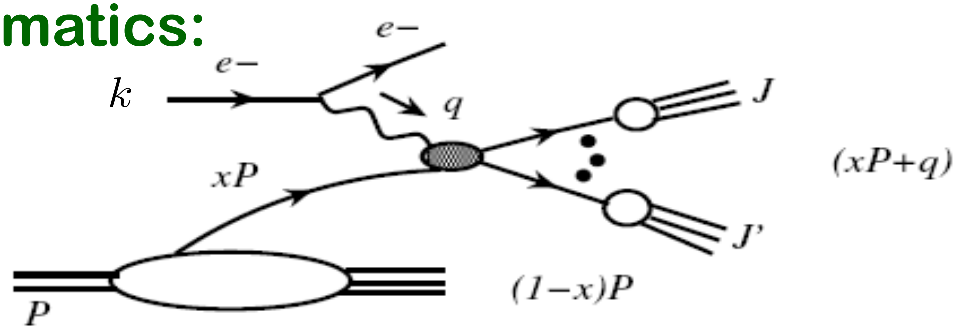
- The challenge for theorists:
  – to indentify new and calculable observables that carry rich information on hadron's partonic structure
  – to make measureable predictions

Thank you!

Backup slices

EIC Kinematics

- **DIS kinematics:**



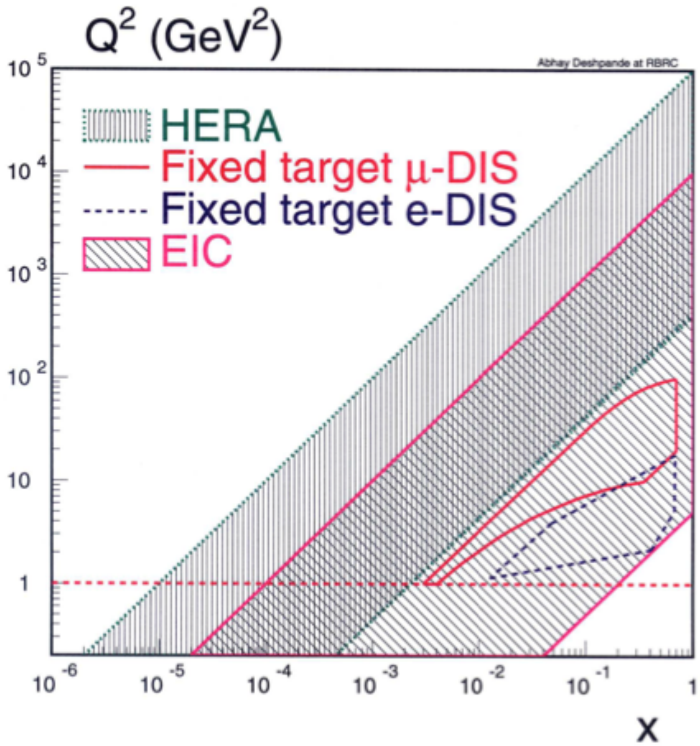
$$Q^2 = -q^2 = x_B\, y\, S$$

$$x_B = \frac{Q^2}{2p \cdot q}$$

$$y = \frac{p \cdot q}{p \cdot k}$$

$$S = (p+k)^2$$

- **EIC (eRHIC – ELIC) basic parameters:**



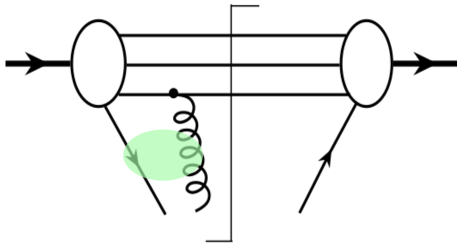
- $E_e = 10$ GeV (5-30 GeV available)
- $E_p = 250$ GeV (50-325 GeV available)
- $\sqrt{S} = 100$ GeV (30-200 GeV available)
- **"localized" probe:** $Q^2 \gtrsim 1$ GeV
- $x_{\rm min} \sim 10^{-4}$
- **Luminosity ~ 100 x HERA**
- **Polarization, heavy ion beam, …**

"Interpretation" of twist-3 correlation functions

❑ **Measurement of direct QCD quantum interference:**

Qiu, Sterman, 1991, …

$$T^{(3)}(x, x, S_\perp) \;\propto$$



Interference between a single active parton state and an active two-parton composite state

❑ **"Expectation value" of QCD operators:**

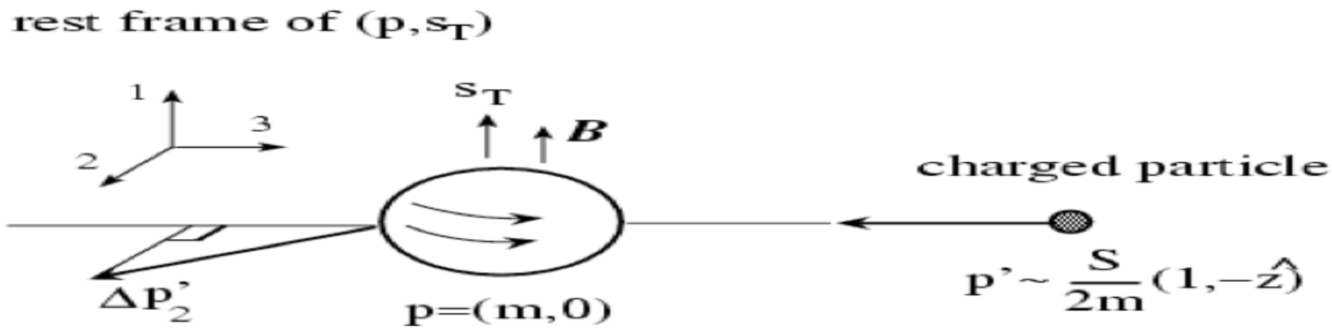
$$\langle P, s|\overline{\psi}(0)\gamma^+\psi(y^-)|P, s\rangle \;\longrightarrow\; \langle P, s|\overline{\psi}(0)\gamma^+ \left[\epsilon_\perp^{\alpha\beta} s_{T\alpha} \int dy_2^- F_\beta{}^+(y_2^-)\right] \psi(y^-)|P, s\rangle$$

$$\langle P, s|\overline{\psi}(0)\gamma^+\gamma_5\psi(y^-)|P, s\rangle \;\longrightarrow\; \langle P, s|\overline{\psi}(0)\gamma^+ \left[i\, g_\perp^{\alpha\beta} s_{T\alpha} \int dy_2^- F_\beta{}^+(y_2^-)\right] \psi(y^-)|P, s\rangle$$

How to interpret the "expectation value" of the operators in RED?

A simple example

- The operator in Red – a classical Abelian case:



- Change of transverse momentum:

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

- In the c.m. frame:

$$(m, \vec{0}) \to \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \to n = (0, 1, 0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\, \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+$$

- The total change: $\boxed{\Delta p_2' = e \int dy^- \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+(y^-)}$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton