Higher spin resonances in photoproduction processes

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JLAB Theory Seminar
Jan. 24, 2011
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- Basic science institute and KoRIA project

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BASIC SCIENCE INSTITUTE
& KoRIA
Motivation

Brain drain

- Foreign students in the US universities (as of 2008)
  - India: ~ 103,000 students
  - China: ~ 98,000 students
  - Korea: ~ 75,000 students (still increasing)

- Return to Korea after getting Ph.D degree
  - 2003: ~ 2,100
  - 2006: ~ 1,300 (decreasing)

- A survey of 100 young Korean scientists in the US
  (Are you willing to return to Korea?)

  - Yes (30%)
  - No (33%)
  - Don’t know (27%)
  - No answer (10%)
Crisis of science & technology in Korea

- High school student’s preference in S&T
  - Medical doctor (e.g. plastic surgeons)
  - Average score of entrance exam of Seoul National Univ.
    - Before 2000: physics ~ medical school >> math education
    - Now: medical school >> math education > physics

National labs in Korea

- Crucial role in the development of Korean economy
- Korea research council for fundamental sciences and technology
  - Supported by the Ministry of Education, Science and Technology
  - 13 institutions: atomic energy, standards & science, nuclear fusion, BT, NT, etc
- Korea research council for industrial sciences and technology
  - Supported by the Ministry of Knowledge-based Economics
  - 13 institutions: IT, materials, foods, electricity, train, NT, etc
- However, no national lab for pure sciences (cf. KIAS)
  - lack of large scale research facilities
Science Business Belt project

- Construction of a science city
  - (Inter-)national labs. (BSI): ~2,000 researchers
  - Large scale research facilities
  - Knowledge-based business center
  - Budget: ~3.2 billion US $ for the first 7 yrs (assumed US $1 = 1,100 KRW)

- New environment for R&D in pure sciences
  - Good payroll
  - Enough research fund
  - Freedom in research topics (but with responsibility for the output)

- Large scale research facilities
  - E.g.: Accelerators
    - At present, only one accelerator in Korea – a photon source at Pohang
  - Plan: Rare Isotope Accelerator
    KoRlA: Korea Rare Isotope Accelerator

- The law for SBB has passed the parliament at Dec. 2010.
  (But the details are yet to be discussed.)
Science Business Belt: construction of a new city
- The core should be Basic Science Institute.

Organization of BSI (tentative)
Scale
- Eventually ~50 research units
  - ~ 2,000 Ph.Ds and ~ 2,000 Graduate Students

Research Units
- Research fields: the WHOLE area in pure sciences
- Locate in the BSI campus or in other universities
- International manpower
  - open to ALL nationalities (researchers and Group Leaders)
  - Group leaders: selected by international committees
  - Non-Korean researchers: > 30% (required)
- The budget of each unit: 5M ~ 10M / year in the US $
  - (in principle, no overhead)
  - Execution of the budget (including the payroll): by the group leader
- Sunset system
  - E.g. 5 yrs + 5 yrs
  - Will be closed for the next research topics
Korea Rare Isotope Accelerator

Multipurpose, heavy-ion accelerator for basic science research
- Nuclear physics & nuclear astrophysics & atomic physics
- Material science
- Bio and medical science
- Nuclear data production
- Nuclear fusion

The first large scale research facility for pure science in Korea

Proposed construction budget: 460 B Won (~ $450M)
- ~$90M for experimental instruments + ~ $360M for accelerator
- Site purchasing & payroll from a separate budget

Schedule (plan)
- Design and R&D: 2009 ~ 2012
- Construction: 2012 ~ 2016
- No sunset system
Why Rare Isotopes (RIs)?

“Nuclear science is entering a new era of discovery in understanding how nature works at the most basic level and in applying that knowledge in useful ways”. - National Academy 2007 RISAC Report -
You are welcome to join us!
Higher spin resonances in photoproduction processes

INTRODUCTION

With T.-S.H. Lee and K. Nakayama
Introduction

- Missing resonance puzzle
- At a mass region $> 1.8$ GeV $\rightarrow$ many high spin states $j \geq \frac{5}{2}$
  - N and $\Delta$ resonances as well as hyperons

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<td>$\Lambda(2950)$</td>
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$\Sigma$ states

| State | $J^P$ | $\Gamma$ (MeV) | Rating | $|g_{\Lambda\Sigma K}|$ |
|-------|-------|-----------------|--------|------------------|
| $\Sigma(1193)$ | $1/2^+$ | $\approx 150$ | **** | 4.2 |
| $\Sigma(1385)$ | $3/2^+$ | $\approx 60$ | **** | 4.0 |
| $\Sigma(1600)$ | $1/2^+$ | $\approx 300$ | *** | 1.0 |
| $\Sigma(1670)$ | $3/2^+$ | $\approx 80$ | **** | 2.8 |
| $\Sigma(1750)$ | $1/2^+$ | $\approx 90$ | *** | 0.5 |
| $\Sigma(1775)$ | $5/2^+$ | $\approx 120$ | **** | 2.8 |
| $\Sigma(1915)$ | $5/2^+$ | $\approx 120$ | **** | 2.8 |
| $\Sigma(1940)$ | $3/2^+$ | $\approx 220$ | *** | 2.8 |
| $\Sigma(2030)$ | $7/2^+$ | $\approx 180$ | **** | 2.8 |
| $\Sigma(2250)$ | ? | $\approx 100$ | *** | 2.8 |
**Fit:** $2 \text{ GeV} < \sqrt{s} < 2.4 \text{ GeV}

Free $5/2^+, 7/2^-$ with Locked Oh, Titov, Lee $t$-channel

Yields vs $\sqrt{s}$  
Phase Difference vs $\sqrt{s}$

Is this the **** $G_{17}(2190)$? Yes (most likely).  
Suggestive (but not conclusive) evidence for a *missing* $F_{15}(2000)$ state.
Testing hadron models (such as quark models)

- Data analyses: coupled-channels method
  - extract coupling constants of interaction Lagrangian

- Quark models give predictions on the decay (amplitudes)

- Decay widths cannot fix the sign of the coupling constant (sign ambiguity)
  - work with decay amplitudes
  - need for the relationship between coupling constants and the predicted decay amplitudes
FORMALISM FOR HIGHER SPIN RESONANCES
Propagators

- Propagators for bosons and fermions of arbitrary spin

\[ S(p) = \frac{i}{p^2 - M^2} \Delta \quad \text{for a boson} \]

\[ S(p) = \frac{i}{p^2 - M^2} (p \cdot \gamma + M) \Delta \quad \text{for a fermion} \]

- Rarita-Schwinger Formalism (wavefunctions)
  - **Boson of spin** \( s \): tensor of rank \( s \)
    
    \[ \left( p^2 + m^2 \right) \Phi_{\alpha_1 \alpha_2 \ldots \alpha_s} = 0 \]
    
    with subsidiary conditions
    
    \[ \Phi_{\ldots \alpha_i \ldots \alpha_j \ldots} = \Phi_{\ldots \alpha_j \ldots \alpha_i \ldots}, \quad p^{\alpha_i} \Phi_{\alpha_1 \ldots \alpha_s} = 0, \quad g^{\alpha_1 \alpha_2} \Phi_{\alpha_1 \alpha_2 \ldots \alpha_s} = 0 \]

  - **Fermion of spin** \( s = n - \frac{1}{2} \): tensor of rank \( n - 1 \)
    
    \[ \left( \gamma \cdot \partial + m \right) \Phi_{\alpha_1 \alpha_2 \ldots \alpha_{n-1}} = 0 \]
    
    with subsidiary conditions
    
    \[ \Phi_{\ldots \alpha_i \ldots \alpha_j \ldots} = \Phi_{\ldots \alpha_j \ldots \alpha_i \ldots}, \quad \gamma^{\alpha_i} \Phi_{\alpha_1 \ldots \alpha_{n-1}} = 0, \quad g^{\alpha_1 \alpha_2} \Phi_{\alpha_1 \alpha_2 \ldots \alpha_{n-1}} = 0, \]
    
    \[ \partial^{\alpha_i} \Phi_{\alpha_1 \ldots \alpha_{n-1}} = 0 \]
The form of $\Delta$

$$\sum_{\text{spin}} \Phi_{\alpha_1 \alpha_2 \ldots} \Phi^{\beta_1 \beta_2 \ldots} = \Lambda_{\pm} \Delta_{\alpha_1 \ldots}$$

$\Lambda_{\pm}$: spin projection operator of spin -1/2 (for fermion)

$$= 1 \text{ (for boson)}$$

The general expressions

For integer spin $n$

$$\Delta_{\alpha_1 \ldots \alpha_n}^{\beta_1 \ldots \beta_n}(n,p) = \left(\frac{1}{n!}\right)^2 \sum_{P(\alpha), P(\beta)} \left[ \prod_{i=1}^{n} \theta_{\alpha_i}^{\beta_i} + a_1 \theta_{\alpha_1 \alpha_2} \theta_{\beta_1 \beta_2} \prod_{i=3}^{n} \theta_{\alpha_i}^{\beta_i} + \ldots \right]$$

for even $n$

with

$$a_r^{(n)} = \left(\frac{1}{2}\right)^r \frac{n!}{r!(n-2r)!} \frac{1}{(2n-1)(2n-3)\ldots(2n-2r+1)}$$

Similar expressions for odd $n$ and for half-integer spin

$$\theta_{\alpha}^{\beta} = -\left(g_{\alpha}^{\beta} - \frac{1}{M^2} p_{\alpha} p^{\beta}\right)$$

$$\Gamma^{\alpha} = i \left(\gamma^{\alpha} - \frac{1}{M^2} p \cdot \gamma p^{\alpha}\right)$$
Explicit expressions for several cases

- **Spin-1**
  \[ \Delta^\beta_\alpha (1) = \theta^\beta_\alpha \]

- **Spin-1/2**
  \[ \Delta \left( \frac{1}{2} \right) = \frac{1}{3} \gamma_\alpha \gamma_\beta \Delta^\beta_\alpha (1) = 1 \]

- **Spin-2**
  \[ \Delta^{\beta_1,\beta_2}_{\alpha_1,\alpha_2} (2) = \frac{1}{2} \left( \theta^{\beta_1}_{\alpha_1} \theta^{\beta_2}_{\alpha_2} + \theta^{\beta_2}_{\alpha_1} \theta^{\beta_1}_{\alpha_2} - \frac{2}{3} \theta_{\alpha_1,\alpha_2} \theta^{\beta_1,\beta_2} \right) \]

- **Spin-3/2**
  \[ \Delta \left( \frac{3}{2} \right) = -g^\beta_\alpha + \frac{1}{3} \gamma_\alpha \gamma^\beta + \frac{1}{3M} \left( \gamma_\alpha p^\beta - p_\alpha \gamma^\beta \right) + \frac{2}{3M^2} p_\alpha p^\beta \]

- Straightforward to get the expressions for higher spin propagators
III

EFFECTIVE LAGRANGIAN & COUPLING CONSTANTS
Interactions

- Number of independent couplings
  - Angular momentum conservation
  - $P$ and $T$ invariance

- Pion ($J^P = 0^-$) vertex: $J^\pm \to 0^- + \frac{1^+}{2}$
  - $R N \pi$ vertex where $N$: nucleon ($1/2^+$)
    - $R$: nucleon resonance of $J^P$
    - Only one interaction term

- Vector meson ($J^P = 1^-$) vertex: $J^\pm \to 1^- + \frac{1^+}{2}$
  - $R N V$ vertex
    - Two interaction terms for $R$ with $\frac{1^\pm}{2}$
    - Three interaction terms for $R$ with $J^\pm$ ($J \geq \frac{3}{2}$)
  - If $V$ is the photon, the numbers are reduced by one. ($\therefore q^2 = 0$)
\[ J^p = \frac{1}{2}^\pm \text{ case} \]
\[ \mathcal{L}_{1/2} = g_{\pi NR} \overline{N} \left[ i \lambda \Gamma^{(\pm)} \pi \mp \frac{1 - \lambda}{M_R \pm M_N} \Gamma^{(\pm)}_\mu \partial^\mu \pi \right] R + \text{H.c.} \]

\[ J^p = \frac{3}{2}^\pm \text{ case} \]
\[ \mathcal{L}_{3/2} = \frac{g_{\pi NR}}{M_\pi} \overline{N} \Gamma^{(\pm)} \partial^\mu \pi R_\mu + \text{H.c.} \]

\[ J^p = \frac{5}{2}^\pm \text{ case} \]
\[ \mathcal{L}_{5/2} = i \frac{g_{\pi NR}}{M_\pi^2} \overline{N} \Gamma^{(\pm)} \partial^\mu \partial^\nu \pi R_{\mu\nu} + \text{H.c.} \]

\[ J^p = \frac{7}{2}^\pm \text{ case} \]
\[ \mathcal{L}_{7/2} = \frac{g_{\pi NR}}{M_\pi^3} \overline{N} \Gamma^{(\pm)} \partial^\mu \partial^\nu \partial^\alpha \pi R_{\mu\nu\alpha} + \text{H.c.} \]
The general expression for the decay widths

\[ \Gamma(R \to N\pi) = \frac{3g_{\pi NR}^2}{4\pi} \frac{2^n (n!)^2}{n(2n)!} \frac{k_{\pi}^{2n-1}}{M_R M_{\pi}^{2(n-1)}} (E_N \pm M_N) \]

for \((-1)^n P_s = \pm 1\) with \(P_s\) being the parity of the spin - s resonance \(R\)

Examples

\[
\begin{align*}
\Gamma\left(\frac{1}{2}^\pm \to N\pi\right) &= \frac{3g_{\pi NR}^2}{4\pi} \frac{k_{\pi}^3}{M_R} (E_N \pm M_N) \\
\Gamma\left(\frac{3}{2}^\pm \to N\pi\right) &= \frac{3g_{\pi NR}^2}{4\pi} \frac{k_{\pi}^5}{5 M_R M_{\pi}^2} (E_N \pm M_N) \\
\Gamma\left(\frac{5}{2}^\pm \to N\pi\right) &= \frac{3g_{\pi NR}^2}{4\pi} \frac{6}{35} \frac{k_{\pi}^7}{M_R M_{\pi}^6} (E_N \pm M_N) \\
\end{align*}
\]

Isospin factor 3 is included.
$J^p = \frac{1}{2}$  case

$L_{1/2} = -\frac{1}{2M_N} \sqrt{N} \left[ g_3 \left( \pm \frac{\Gamma^{(\pm)}_\mu \partial^2}{M_R \mp M_N} - i\Gamma^{(\pm)}_\mu \right) V^\mu - g_1 \Gamma^{(\pm)}_{\mu \nu} \sigma^\nu V^\mu \right] R + \text{H.c.}$

$J^p = \frac{3}{2}$  case

$L_{3/2} = -i \frac{g_1}{2M_N} \sqrt{N} \Gamma^{(\pm)}_\nu V^{\mu \nu} R^\mu - \frac{g_2}{(2M_N)^2} \partial_\nu \sqrt{N} \Gamma^{(\pm)}_\nu V^{\mu \nu} R^\mu + \frac{g_3}{(2M_N)^2} \sqrt{N} \Gamma^{(\pm)}_\nu \partial_\nu V^{\mu \nu} R^\mu + \text{H.c.}$

$J^p = \frac{5}{2}$  case

$L_{5/2} = \frac{g_1}{(2M_N)^2} \sqrt{N} \Gamma^{(\pm)}_\nu \partial^\alpha V^{\mu \nu} R^\mu_{\alpha} - \frac{ig_2}{(2M_N)^3} \partial_\nu \sqrt{N} \Gamma^{(\pm)}_\nu \partial^\alpha V^{\mu \nu} R^\mu_{\alpha} + \frac{ig_3}{(2M_N)^3} \sqrt{N} \Gamma^{(\pm)}_\nu \partial^\alpha \partial_\nu V^{\mu \nu} R^\mu_{\alpha} + \text{H.c.}$

$J^p = \frac{7}{2}$  case

$L_{7/2} = \frac{ig_1}{(2M_N)^3} \sqrt{N} \Gamma^{(\pm)}_\nu \partial^\alpha \partial^\beta V^{\mu \nu} R^\mu_{\alpha \beta} + \frac{g_2}{(2M_N)^4} \partial_\nu \sqrt{N} \Gamma^{(\pm)}_\nu \partial^\alpha \partial^\beta V^{\mu \nu} R^\mu_{\alpha \beta} - \frac{g_3}{(2M_N)^4} \sqrt{N} \Gamma^{(\pm)}_\nu \partial^\alpha \partial^\beta \partial_\nu V^{\mu \nu} R^\mu_{\alpha \beta} + \text{H.c.}$
Similar to RNV interaction

But with $g_3 = 0$

\[
\mathcal{L}_{\text{RN}^\gamma} (1^\pm) = \frac{ef_1}{2M_N} \bar{N} \Gamma^{(\mp)} \sigma_{\mu\nu} \partial^\nu A^\mu R + \text{H.c.,}
\]

\[
\mathcal{L}_{\text{RN}^\gamma} (1^\pm) = -\frac{ief_1}{2M_N} \bar{N} \Gamma^{(\pm)} F^{\mu\nu} R_\mu - \frac{ef_2}{(2M_N)^2} \partial_\nu \bar{N} \Gamma^{(\pm)} F^{\mu\nu} R_\mu + \text{H.c.,}
\]

\[
\mathcal{L}_{\text{RN}^\gamma} (5^\pm) = \frac{ef_1}{(2M_N)^2} \bar{N} \Gamma^{(\pm)} \partial^\alpha F^{\mu\nu} R_{\mu\alpha} - \frac{ief_2}{(2M_N)^3} \partial_\nu \bar{N} \Gamma^{(\pm)} \partial^\alpha F^{\mu\nu} R_{\mu\alpha} + \text{H.c.,}
\]

Helicity amplitudes

\[
\Gamma(R \rightarrow N\gamma) = \frac{k_\gamma^2}{\pi} \frac{2M_N}{(2j + 1)M_R} [|A_{1/2}|^2 + |A_{3/2}|^2],
\]

\[
A_{1/2}(1^\pm) = \pm \frac{ef_1}{2M_N} \sqrt{\frac{k_\gamma M_R}{M_N}},
\]

\[
A_{1/2}(3^\pm) = \pm \frac{e\sqrt{6}}{12} \sqrt{\frac{k_\gamma}{M_N M_R}} \left[ f_1 \pm \frac{f_2}{4M_N^2} (M_R \mp M_N) \right],
\]

\[
A_{3/2}(3^\pm) = \pm \frac{e\sqrt{2}}{4M_N} \sqrt{\frac{k_\gamma M_R}{M_N M_R}} \left[ f_1 \mp \frac{f_2}{4M_N} (M_R \mp M_N) \right],
\]

\[
A_{1/2}(5^\pm) = \pm \frac{e}{4\sqrt{10} M_N} k_\gamma \sqrt{\frac{k_\gamma}{M_N M_R}} \left[ f_1 \pm \frac{f_2}{4M_N^2} (M_R \mp M_N) \right],
\]

\[
A_{3/2}(5^\pm) = \pm \frac{e}{4\sqrt{5} M_N^2} k_\gamma \sqrt{\frac{k_\gamma M_R}{M_N}} \left[ f_1 \pm \frac{f_2}{4M_N} (M_R \mp M_N) \right],
\]
Interactions of R with spin-3/2 baryons

- $J^* \to 0^- + \frac{3^+}{2}$

Lagrangian

\[
\mathcal{L}_{RK\Sigma^*}(\frac{1^\pm}{2}) = \frac{h_1}{M_K} \partial_\mu K \bar{\Sigma}^* \mu \Gamma^{(\mp)} R + \text{H.c.},
\]

\[
\mathcal{L}_{RK\Sigma^*}(\frac{3^\pm}{2}) = \frac{h_1}{M_K} \partial^\alpha K \bar{\Sigma}^* \mu \Gamma^{(\pm)} R_\mu + \frac{i h_2}{M_K^2} \partial^\mu \gamma^\alpha K \bar{\Sigma}^* \mu \Gamma^{(\mp)} R_\mu
\]

\[
+ \text{H.c.},
\]

\[
\mathcal{L}_{RK\Sigma^*}(\frac{5^\pm}{2}) = \frac{i h_1}{M_K^2} \partial^\mu \partial^\beta K \bar{\Sigma}^* \mu \Gamma^{(\mp)} R_{\alpha \beta}
\]

\[
- \frac{h_2}{M_K^3} \partial^\mu \gamma^\alpha \partial^\beta K \bar{\Sigma}^* \mu \Gamma^{(\mp)} R_{\alpha \beta} + \text{H.c.}
\]

Decay widths

\[
\Gamma(\frac{1^\pm}{2} \to K \Sigma^*) = \frac{h_1^2 q^2 M_R}{2 \pi M_K^2 M_{\Sigma^*}^2} (E_{\Sigma^*} \pm M_{\Sigma^*})
\]

\[
\times \left\{ \frac{h_1^2}{M_K^2} (M_R \mp M_{\Sigma^*})^2
\right\}
\]

\[
+ \frac{i h_1^2}{M_K^3} M_R q^2 (M_R \mp M_{\Sigma^*})(2E_{\Sigma^*} \mp M_{\Sigma^*})
\]

\[
+ \frac{h_2^2}{M_K^5} M_R^2 q^4 \right\},
\]

\[
\Gamma(\frac{3^\pm}{2} \to K \Sigma^*) = \frac{1}{24 \pi M_R M_{\Sigma^*}^2} q (E_{\Sigma^*} \pm M_{\Sigma^*})
\]

\[
\times \left\{ \frac{h_1^2}{M_K^2} (M_R \mp M_{\Sigma^*})^2
\right\}
\]

\[
+ \frac{i h_1^2}{M_K^3} M_R q^2 (M_R \mp M_{\Sigma^*})(2E_{\Sigma^*} \mp M_{\Sigma^*})
\]

\[
+ \frac{h_2^2}{M_K^5} M_R^2 q^4 \right\}
\]

\[
\Gamma(\frac{5^\pm}{2} \to K \Sigma^*) = \frac{1}{60 \pi M_R M_{\Sigma^*}^2} q^3 (E_{\Sigma^*} \pm M_{\Sigma^*})
\]

\[
\times \left\{ \frac{h_1^2}{M_K^2} (M_R \mp M_{\Sigma^*})^2
\right\}
\]

\[
+ \frac{i h_1^2}{M_K^3} M_R q^2 (M_R \mp M_{\Sigma^*})(2E_{\Sigma^*} \mp M_{\Sigma^*})
\]

\[
+ \frac{h_2^2}{M_K^5} M_R^2 q^4 \right\}
\]
Matching with quark model predictions

- **Decay amplitude**

\[
\langle K(q)\Sigma^+(-q,m_f) | -i\mathcal{H}_{\text{int}} | R(0,m_j) \rangle
\]

\[= 2\pi M_R \sqrt{\frac{2}{q}} \sum_{\ell,m_\ell} \langle \ell m_\ell | m_f | j m_j \rangle Y_{\ell m_\ell}(q) G(\ell), \]

\[\Gamma(R \rightarrow K \Sigma^+) = \sum_\ell |G(\ell)|^2,\]

For \(\frac{1}{2}^+\)

\[G(1) = -\frac{1}{\sqrt{2\pi}} \frac{q}{M_{\Sigma^*}} \sqrt{q M_R} \sqrt{E_{\Sigma^*}} + M_{\Sigma^*} \frac{h_1}{M_K},\]

For \(\frac{1}{2}^-\)

\[G(2) = -\frac{1}{\sqrt{2\pi}} \frac{q}{M_{\Sigma^*}} \sqrt{q M_R} \sqrt{E_{\Sigma^*}} - M_{\Sigma^*} \frac{h_1}{M_K},\]

For \(\frac{3}{2}^+\)

\[G(1) = G_{11}^{(3/2)} \frac{h_1}{M_K} + G_{12}^{(3/2)} \frac{h_2}{M_K},\]

\[G(3) = G_{31}^{(3/2)} \frac{h_1}{M_K} + G_{32}^{(3/2)} \frac{h_2}{M_K},\]

and so on …

\[G_{11}^{(3/2)} = \frac{\sqrt{30}}{60\sqrt{\pi}} \frac{1}{M_{\Sigma^*}} \frac{q}{M_R} \sqrt{E_{\Sigma^*}} - M_{\Sigma^*} (M_R + M_{\Sigma^*}),\]

\[G_{12}^{(3/2)} = -\frac{\sqrt{30}}{60\sqrt{\pi}} \frac{q^2}{M_{\Sigma^*}} \sqrt{E_{\Sigma^*}} - M_{\Sigma^*},\]

\[G_{31}^{(3/2)} = -\frac{\sqrt{30}}{20\sqrt{\pi}} \frac{1}{M_{\Sigma^*}} \frac{q}{M_R} \sqrt{E_{\Sigma^*}} - M_{\Sigma^*},\]

\[G_{32}^{(3/2)} = \frac{\sqrt{30}}{20\sqrt{\pi}} \frac{q^2}{M_{\Sigma^*}} \sqrt{E_{\Sigma^*}} - M_{\Sigma^*},\]

Quark model predictions on \(G(l)\)

G’s can be related to cc
IV

APPLICATION
The reaction of $\gamma N \rightarrow K \Sigma^*$

Considered resonances

<table>
<thead>
<tr>
<th>Resonance</th>
<th>PDG [29]</th>
<th>Amplitudes of $R \rightarrow K \Sigma(1385)$$^a$</th>
<th>$h_1$</th>
<th>$h_2$</th>
<th>Amplitudes of $R \rightarrow N \gamma$$^b$</th>
<th>$f_1$</th>
<th>$f_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_\frac{1}{2}^-$ (1945)</td>
<td>$S_{11}^*$ (2000)</td>
<td>$G(2) = +1.7$</td>
<td>$-$</td>
<td>$-$</td>
<td>$G(\ell_1)$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>$N_{\frac{3}{2}}^-$ (1960)</td>
<td>$D_{13}^*$ (2005)</td>
<td>$G(0) = +1.3$</td>
<td>$G(2) = +1.4$</td>
<td>$0.24$</td>
<td>$-0.54$</td>
<td>$A^p_{\frac{1}{2}}$</td>
<td>$+$</td>
</tr>
<tr>
<td>$N_{\frac{3}{2}}^+$ (2005)</td>
<td>$D_{13}^*$ (2005)</td>
<td>$G(2) = -2.0$</td>
<td>$G(4) = 0.0$</td>
<td>$0.29$</td>
<td>$-0.08$</td>
<td>$A^p_{\frac{3}{2}}$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\Delta_\frac{1}{2}^-$ (2080)</td>
<td>$D_{33}^*$ (1940)</td>
<td>$G(0) = -4.1$</td>
<td>$G(3) = -0.5$</td>
<td>$-0.68$</td>
<td>$1.00$</td>
<td>$A^p_{\frac{1}{2}}$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\Delta_\frac{3}{2}^+$ (1990)</td>
<td>$F_{33}^{**} (2000)$</td>
<td>$G(1) = +4.0$</td>
<td>$G(3) = -0.1$</td>
<td>$-0.87$</td>
<td>$0.11$</td>
<td>$A^p_{\frac{3}{2}}$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

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<tr>
<th>Resonance</th>
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<th>$f_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\frac{3}{2}}^-$ (2095)</td>
<td>$G(0) = +7.7$</td>
<td>$G(2) = -0.8$</td>
<td>$0.99$</td>
<td>$0.27$</td>
<td>$A^p_{\frac{1}{2}}$</td>
<td>$-$</td>
</tr>
<tr>
<td>$N_{\frac{3}{2}}^+$ (1980)</td>
<td>$G(1) = -3.6$</td>
<td>$G(3) = -0.1$</td>
<td>$0.59$</td>
<td>$0.24$</td>
<td>$A^p_{\frac{3}{2}}$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\Delta_\frac{1}{2}^-$ (2145)</td>
<td>$G(0) = +5.2$</td>
<td>$G(2) = -1.9$</td>
<td>$0.25$</td>
<td>$0.46$</td>
<td>$A^p_{\frac{1}{2}}$</td>
<td>$-$</td>
</tr>
</tbody>
</table>
Results (cross section)
\( \gamma N \rightarrow KK\Xi \)

### Table: \( \Lambda \) states

| State | \( j^P \) | \( \Gamma \) (MeV) | Rating | \( |g_{\Lambda\Lambda K}| \) |
|-------|--------|-----------------|--------|-----------------|
| \( \Lambda^{(1116)} \) | 1/2\(^+\) | \( \approx 50 \) | **** | \( |g_{\Lambda\Lambda K}| \) |
| \( \Lambda^{(1405)} \) | 1/2\(^-\) | \( \approx 50 \) | **** | \( |g_{\Lambda\Lambda K}| \) |
| \( \Lambda^{(1520)} \) | 3/2\(^-\) | \( \approx 16 \) | **** | \( |g_{\Lambda\Lambda K}| \) |

### Table: \( \Sigma \) states

| State | \( j^P \) | \( \Gamma \) (MeV) | Rating | \( |g_{\Sigma\Sigma K}| \) |
|-------|--------|-----------------|--------|-----------------|
| \( \Sigma^{(1193)} \) | 1/2\(^+\) | \( \approx 100 \) | **** | 2.5 |
| \( \Sigma^{(1385)} \) | 3/2\(^+\) | \( \approx 90 \) | **** | \( |g_{\Sigma\Sigma K}| \) |

\[ |M_{1/2^+}|^2, |M_{5/2^+}|^2 \propto \left( E_N \pm M_N \right) \left( E_{\Xi} \pm M_{\Xi} \right) \]

\[ |M_{3/2^+}|^2, |M_{7/2^+}|^2 \propto \left( E_N \mp M_N \right) \left( E_{\Xi} \pm M_{\Xi} \right) \]

Nakayama, YO, Haberzettl, PRC 74

This work
$\gamma p \rightarrow K^+ K^+ \Xi^-$ (new result)

Nakayama, YO, Haberzettl, PRC 76 (2007)

This work
SUMMARY
Summary

- Inclusion of higher spin resonances
  - To understand the mechanism of photoproduction processes
  - To search for the missing resonances
  - To test various hadron models in the market

- To-do
  - Matching the coupling constants to models for hadron structure
    - A complete list for the relationship between the CC and the decay amplitudes (of hadron models)
  - Off-shell parameters
  - Etc

THANK YOU!