QCD structure of hard processes in nuclear interactions

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Motivation
Motivation

- understanding the structure and the dynamics of matter at the smallest scales or at its most fundamentals.
- of nucleons and of nucleon-nucleon interaction in the light of quantum chromodynamics (QCD) degrees of freedom.
- identifying the observables and the kinematics where this description explicitly breaks from the domain of phenomenological hadron theories.
Introduction

Hard Processes

Hard Processes Involving Nuclei

Hard Rescatrering Model of $\gamma + NN \rightarrow NN$

$^3$He Photodisintegration within the HRM

Scattering Amplitude

pp and pn breakup

Polarization Transfer

Summary

Δ-Isobar Production in Deuteron Breakup

Angular Distributions

Further Applications
GOAL: Describe the Nucleon Nucleon Force at Small Distances

Long range attraction

![Graph showing internucleon potential vs. separation with key features including repulsive core and long range attraction.]
Hard Processes

To probe short distance structure, introduce the study of hard processes.

FEATURES

▷ Large kinematic variables, $|t|, |u| > m^2_N$,
▷ Hadron wavefunctions dominated by minimal Fock components, $|N⟩ \sim |qqq⟩$
▷ Facilitate factorization schemes.

$ep \rightarrow eX$ at large $Q^2$ revealed partons in nucleons

Explore the role of quarks and gluons through Hard NN scattering
\[ \langle \psi_c \psi_d | T | \psi_a \psi_b \rangle = \sum_{\alpha, \beta, \gamma} \langle \psi_c | \alpha'_2, \beta'_1, \gamma'_1 \rangle \langle \psi_d | \alpha'_1, \beta'_2, \gamma'_2 \rangle \]
\[ \times \langle \alpha'_2, \beta'_2, \gamma'_2, \alpha'_1 \beta'_1 \gamma'_1 | H | \alpha_1, \beta_1, \gamma_1, \alpha_2 \beta_2 \gamma_2 \rangle \]
\[ \times \langle \alpha_1, \beta_1, \gamma_1 | \psi_a \rangle \langle \alpha_2, \beta_2, \gamma_2 | \psi_b \rangle, \]
Factorization

- **Segment analysis in subprocesses according to scale.**
- **Hard subprocesses insensitive to large distance scale effects.** Potentially workable through perturbative methods.
- **Soft factors group long-distance scale effects.** They’re mostly approached phenomenologically

\[
\langle \psi_c \psi_d \mid T \mid \psi_a \psi_b \rangle = \sum_{\alpha, \beta, \gamma} \langle \psi_c \mid \alpha_2', \beta_1', \gamma_1' \rangle \langle \psi_d \mid \alpha_1', \beta_2', \gamma_2' \rangle \\
\times \langle \alpha_2', \beta_2', \gamma_2', \alpha_1' \beta_1' \gamma_1' \mid H \mid \alpha_1, \beta_1, \gamma_1, \alpha_2 \beta_2 \gamma_2 \rangle \\
\times \langle \alpha_1, \beta_1, \gamma_1 \mid \psi_a \rangle \langle \alpha_2, \beta_2, \gamma_2 \mid \psi_b \rangle,
\]
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\times \langle \alpha_1, \beta_1, \gamma_1 | \psi_a \rangle \langle \alpha_2, \beta_2, \gamma_2 | \psi_b \rangle,
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\times \langle \alpha_1, \beta_1, \gamma_1 \mid \psi_a \rangle \langle \alpha_2, \beta_2, \gamma_2 \mid \psi_b \rangle,
\]
Dimensional Counting Rules
Dimensional Counting Rules

In exclusive reactions \( a + b \to c + d \), for hard subprocesses in the asymptotic limit \((-t, s \to \infty \) and fixed \( \frac{t}{s} \)),

\[
\sim f(t/s)
\]

\[
\sim s^{-1}
\]

\[
\sim s^{1/2}
\]

leading to,

\[
\mathcal{M}_{ab \to cd} \sim s^{-\frac{n_a+n_b+n_c+n_d-4}{2}}
\]

and consequently,

\[
\frac{d\sigma}{dt}_{ab \to cd} \to s^{-(n_a+n_b+n_c+n_d-2)}
\]

with \( n_i \) being the number of minimal constituents of particle \( i \) taking part in the hard subprocess.


Dimensional Counting Rules

\[
\frac{d\sigma}{dt}_{ab \rightarrow cd} \rightarrow s^{-\left(n_a + n_b + n_c + n_d - 2\right)}
\]

\[
NN \rightarrow NN
\]

\[
\frac{d\sigma}{dt} \sim s^{-10}
\]
Dimensional Counting Rules

$pp$ elastic scattering

$$\frac{d\sigma}{dt} \sim s^{-10}$$
Dimensional Counting Rules

**pp elastic scattering**

\[ \frac{d\sigma}{dt} \sim s^{-10} \]

Oscillations point to a more complex picture of the \( NN \) interaction for the studied region.
Dimensional Counting Rules

\textit{pp elastic scattering}

- \textbf{Landshoff/Sudakof interference} \textit{(Pire, Ralston, 1983)}

- \textbf{Heavy flavor resonances interfering with a pQCD background} \textit{(Brodsky, de Teramond, 1988)}

\textbf{Oscillations point to a more complex picture of the NN interaction for the studied region.}
Understanding of the strong force can be improved by studies of $\gamma + NN \rightarrow N + N$ reactions.

Hadronic and QCD descriptions of the $NN$ force can be studied through such reactions.

**Hadron picture**

**QCD picture**
Kinematic Advantages of Hard $\gamma + NN \rightarrow N + N$

$N + N$ emerge at large transverse momentum from a short range $NN$ interaction.

$$-t \sim s \rightarrow \infty \left( \frac{s}{2} \frac{1 - \cos(\theta_{c.m.})}{1} \right)$$

Large center of mass energy of the emerging nucleons $\sqrt{s}/2$ more efficiently reached in $\gamma + NN \rightarrow N + N$,

$$s_{\gamma NN} \approx 4m_N^2 + 2E_\gamma \cdot 2m_N,$$

compared to

$$s_{NN} = 2m_N^2 + 2E \cdot m_N$$

in $N_{beam} + N_{target} \rightarrow N + N$ processes (with $E$ being the energy of the nucleon beam).

Then, at moderate $E_\gamma \sim GeV$ and at large $\theta_{c.m.}$,

$\gamma + NN \rightarrow N + N$ reaches the hard kinematic regime.
Producing High $p_T$ $N + N$ in $\gamma + NN \rightarrow N + N$

There are two basic approaches,

- **Photon probes a preexisting compact $NN$ system (large $p_T$ component in initial state) in nucleus.** This is assumed for instance in approaches such as the reduced nuclear amplitude (RNA) (Brodsky, Hiller, 1983).

Or,

- **the energetic photon is absorbed by a low $p_T$ $NN$ system.** The energy transfer triggers a final state interaction from which the nucleons emerge at large relative transverse.

The latter is the scenario for the hard rescattering model (HRM) (Frankfurt et al. 2000) for which we develop applications in what follows.
Hard rescattering model, HRM

Hard rescattering model, HRM

$\gamma + d \rightarrow p + n$ HRM amplitude results in a convolution of a hard process amplitude and a nuclear wave function,

$$
\langle \lambda_{1f}, \lambda_{2f} | M | \lambda_{\gamma}, \lambda_{d} \rangle = -\frac{i[\lambda_{\gamma}]e_Q f \sqrt{2} (2\pi)^3}{\sqrt{2S_{NN}^{QIM}}} \times \sum_{\lambda_{2i}} \int \langle \lambda_{2f}; \lambda_{1f} | T_{NN}^{QIM} (s_{NN}, t_N) | \lambda_{\gamma}; \lambda_{2i} \rangle \\
\times \psi_{d, NR}^{\lambda_{2i}, \lambda_{\gamma} \lambda_{\gamma} \lambda_{2i}} (\vec{p}_1, \vec{p}_2) m_{N} \frac{d^2 p_\perp}{(2\pi)^2},
$$

Hard rescattering model, HRM

A parameter free cross section calculable through the input of experimental data,

\[
\frac{d\sigma^{\gamma d \rightarrow pn}}{dt}(s, \theta_{c.m.}) = \frac{8\alpha}{9} \pi^4 \frac{1}{s'} C(\frac{\tilde{t}}{s}) \times \frac{d\sigma^{pn \rightarrow pn}}{dt}(s, \theta_{c.m.}) \frac{1}{s'} \int \psi_d^{NR}(p_z = 0, p_t) \sqrt{m_n} \frac{d^2 p_t}{(2\pi)^2} \left| \psi_d^{NR}(p_z = 0, p_t) \right|^2
\]

Energy and Angular distributions in pn elastic scattering data should be reflected by \( \gamma + d \rightarrow p + n \) data.
Energy Distribution

\[
\frac{d\sigma}{dt}(s, \theta_{c.m.}) = \frac{8\alpha}{9} \pi^4 \frac{1}{s'} C\left(\frac{t}{s}\right) \times \frac{d\sigma}{dt}(s, \theta^N_{c.m.})
\]

&

\times \left| \int \psi^{NR}_d(p_z = 0, p_t) \sqrt{m_n} \frac{d^2 p_t}{(2\pi)^2} \right|^2

\gamma + d \rightarrow p + n

\[d(\gamma, p)n\]

\[s (\text{GeV}^2)\]

\[\text{d}(\gamma, p)n\]

\[90^\circ \text{c.m.}\]

\[s (\text{GeV}^2)\]

\[d(\gamma, p)n\]

\[\text{d}(\gamma, p)n\]

\[90^\circ \text{c.m.}\]

\[s (\text{GeV}^2)\]

\[\theta_{c.m.} = 60^\circ\]

\[\text{d}(\gamma, p)n\]

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Bochna et al. 1998

\[\theta_{c.m.} = 60^\circ\]

\[\text{d}(\gamma, p)n\]

\[\text{d}(\gamma, p)n\]
Energy Distribution

\[ \gamma + d \rightarrow p + n \]

\[ p + n \rightarrow p + n \]

\[ \gamma + d \rightarrow p + n \sim s^{-11} \quad \text{for} \quad E_{\gamma} > 2 \text{GeV}, \quad s > 10 \text{GeV}^2 \]

However, pn elastic scattering data is not good enough to clearly confirm HRM prediction
Angular Distributions

\( \gamma + d \rightarrow p + n \)

\( p + n \rightarrow p + n \)

Figure: pn elastic scattering for \( p_{\text{Lab}} = 8\text{GeV/c} \)

$NN$ breakup studies can be extended to $\gamma + pp \rightarrow p + p$

- Because of much better $pp$ elastic scattering data, the HRM approach applied to $pp$ photodisintegration can be tested experimentally better than the $pn$ case.

- The energy behavior of $\gamma^3He \rightarrow (p + p) + n_s$ potentially probes the transition from a Hadronic to a QCD description of the $NN$ force.

At low beam energies $\gamma + pp \rightarrow p + p$ is suppressed because of the neutral charge of the mesons mediating the force. $\gamma^3He \rightarrow (p + p) + n_s$ proceeds largely through a three body (two steps) interaction:

In hard $pp$ breakup however, the photon couples to quark currents that now mediate the $pp$ interaction making possible for
Hard Photodisintegration of a $NN$ System in $^3\text{He}$

$\gamma^3\text{He} \rightarrow (p + p) + n_s$ to proceed through $\gamma + pp \rightarrow p + p$ which becomes dominant.

- Features of $pp$ elastic scattering such as oscillations in energy distributions are expected to appear as well in $\gamma^3\text{He} \rightarrow (p + p) + n_s$.

- Predictions on spectator momentum distributions of $\gamma^3\text{He} \rightarrow (p + p) + n_s$ cross sections and on polarization observables provide yet more experimental checks on the validity of the HRM approach to $NN$ breakup.
Main assumptions

- Photon interacts with a $NN$ system with relative momentum $p_\perp << m_N$
- Third nucleon ($N_s$) is spectator to $\gamma NN$ reaction.
- The photon is absorbed by a valence quark of a nucleon from the NN system.
- Struck quark rescatters off valence quark of 2\textsuperscript{nd} nucleon.
- This rescattering produces 2 nucleons emerging at large transverse momentum.

Calculation of Scattering Amplitude

\[ \langle \lambda_{f1}, \lambda_{f2}, \lambda_s \mid \mathcal{M} \mid \lambda_\gamma, \lambda_A \rangle = \]

\[(N1) : \int \frac{-i\Gamma_{N1f}^\dagger i[p_{1f} - k_1 + m_q]}{(p_{1f} - k_1)^2 - m_q^2 + i\epsilon} \cdot \cdot \cdot [-igT_c^F \gamma_\mu] \cdot \cdot \cdot \]

\[iS(k_1) \frac{i[p_{1i} - k_1 + m_q](-i)\Gamma_{N1i}}{(p_{1i} - k_1 + q)^2 - m_q^2 + i\epsilon} \cdot \cdot \cdot [-iQ_i \epsilon \gamma^\perp] \]

\[(\gamma q) : \int \frac{-i\Gamma_{N2f}^\dagger i[p_{2f} - k_2 + m_q]}{(p_{2f} - k_2)^2 - m_q^2 + i\epsilon} \cdot \cdot \cdot [-igT_c^F \gamma_\nu] \]

\[iS(k_2) \frac{i[p_{2i} - k_2 + m_q](-i)\Gamma_{N2i}}{(p_{2i} - k_2)^2 - m_q^2 + \epsilon} \cdot \cdot \cdot \frac{d^4k_1}{(2\pi)^4} \]

\[\frac{d^4k_2}{(2\pi)^4} \]

\[d^4p_{2i} \]

\[g : \frac{i\delta_{\mu\nu} \delta_{ab}}{[(p_{2i} - k_2) - (p_{1i} - k_1) - (q - l)]^2 + i\epsilon}, \]
Light cone variables

\[ p = (p^+, p^-, p_\perp) \]

with

\[ p^+ = p_0 + p_z; \quad p^- = p_0 - p_z; \quad p_\perp = (p_x, p_y). \]

Light cone momentum fractions

\[ \alpha = \frac{p^+_{2i}}{p^+_\text{NN}}, \]

and

\[ x_l = \frac{k^+_l}{p^+_l}; \quad x'_l = \frac{k^+_l}{p^+_{lf}}. \]

Then,

\[ d^4 p_{2i} = \frac{1}{2} p^+_\text{NN} d\alpha dp^-_{2i} d^2 p_\perp, \]

and

\[ d^4 k_l = \frac{1}{2} p^+_{li} dx dk^-_l d^2 k_\perp. \]
\( \gamma - NN \) Reference Frame

We consider a reference frame where \( q^+ = 0 \). Having that \( s_{NN} = (p_{NN} + q)^2 \) and \( s'_{NN} = s_{NN} - M_{NN}^2 \), the 4-momenta of the photon and of the \( NN \) system are respectively:

\[
q \equiv (q^+, q^-, q^\perp) = (0, \sqrt{s'_{NN}}, 0),
\]

\[
p \equiv (p_{NN}^+, p_{NN}^-, q_{NN\perp}) = (\sqrt{s'_{NN}}, \frac{M_{NN}^2}{\sqrt{s'_{NN}}}, 0).
\]

For \( p_{NN}^+ \gg M_{NN}^2 \), the \( \gamma - NN \) system approaches its center of mass reference frame.
HRM Scattering Amplitude for $\gamma + ^3He \rightarrow N + N + N_s$

After performing loop integrations on $p_{21}^-$ and $k^-$ variables, $N_2$ and spectator systems are put on mass shell. Using that,

$$\not{p} + m \approx \sum_s u_s(p)\bar{u}_s(p),$$

for internal lines off mass-shell in the $p_{NN}^+ >> M_{NN}^2$ approximation and by defining nuclear and nucleonic wave functions,

$$\Psi_{^3He}^{\lambda_A, \lambda_1, \lambda_2, \lambda_s}(\alpha, p_\perp) = \frac{\bar{u}_{\lambda_1}(p_{NN} - p)\bar{u}_{\lambda_2}(p)\bar{u}_{\lambda_s}(p_s)\Gamma_{^3He}^{\lambda_A}}{M_{NN}^2 - \frac{m_N^2 + p_\perp^2}{\alpha(1-\alpha)}}$$

(2)

$$\Psi_N^{\lambda, \eta}(p, x, k_\perp) = \frac{\bar{u}_\eta(p - k)\psi_s^\dagger(k)\Gamma_N u_N^\lambda(p)}{m_N^2 - \frac{m_s^2(1-x) + m_q^2x + (k_\perp - xp_\perp)^2}{x(1-x)}}$$

(3)
HRM Scattering Amplitude for $\gamma + ^3 He \rightarrow N + N + N_s$

After performing loop integrations on $p_{21}^-$ and $k^-$ variables, $N_2$ and spectator systems are put on mass shell. Using that,

$$\phi + m \approx \sum_s u_s(p)\bar{u}_s(p),$$

for internal lines off mass-shell in the $p_{NN}^+ >> M_{NN}^2$ approximation and by defining nuclear and nucleonic wave functions, the invariant amplitude is simplified to:

$$\langle \lambda_1f, \lambda_2f, \lambda_s \mid M \mid \lambda_\gamma, \lambda_A \rangle = \sum_{(\eta')s,(\lambda'_i)s,(\zeta)} \int \left\{ \frac{\psi^\lambda_2f,\eta_2f(p_{2f},x',k_{2\perp})}{1 - x'_2} \bar{u}_{\eta_2f}(p_{2f} - k_2)[-igT^F_{c\gamma} \gamma^\nu] \right. $$

$$\left. \psi_{N}^{\lambda_1i,\eta_1i}(p_{1i},x_1,k_{1\perp}) \bar{u}_{\eta_1f}(p_{1f} - k_1)[-iQie \gamma_{\perp}]u_{\eta_1i}(p_{1i} - k_1) \frac{\psi_{N}^{\lambda_1i,\eta_1i}(p_{1i},x_1,k_{1\perp})}{(1 - x_1)} \right\} \times$$

$$\left\{ \frac{\psi^\lambda_1f,\eta_1f(p_{1f},x'_1,k_{1\perp})}{1 - x'_1} \bar{u}_{\eta_1f}(p_{1f} - k_1)[-igT^F_{c\gamma} \gamma^\mu]u_{\eta_2i}(p_{2i} - k_2) \psi_{N}^{\lambda_2i,\eta_2i}(p_{2i},x_2,k_{2\perp}) \frac{\psi_{N}^{\lambda_2i,\eta_2i}(p_{2i},x_2,k_{2\perp})}{(1 - x_2)} \right\} \times$$

$$G^{\mu,\nu}(r) \frac{dx_1}{x_1} \frac{d^2k_{1\perp}}{2(2\pi)^3} \frac{dx_2}{x_2} \frac{d^2k_{2\perp}}{2(2\pi)^3} \psi_{3He}^{\lambda_A,\lambda_1i,\lambda_2i,\lambda_s}(\alpha, p_{\perp}, p_s) \frac{d\alpha}{\alpha} \frac{d^2p_{\perp}}{2(2\pi)^3} - (p_{1f} \leftrightarrow p_{2f}),$$

(2)
\[ \bar{u}_\zeta(p_{1i} - k_1 + q)[-iQ_i e^{\lambda \gamma} \gamma^\perp] u_{\eta_1i}(p_{1i} - k_1) \]

\[ (1 - x_1)s'(\alpha - (\alpha_c + i\epsilon)) \]

can be calculated in the approximation \( p_{1i}^+ - k_1^+ >> k_\perp, m_q \), in which the spinor of a quark of a given helicity \( \eta_q = \pm 1 \) and a given energy \( E_q \) can be written

\[ u_{\eta_q} \approx \sqrt{E_q} \left( \begin{array} {c} \chi_{\eta_q} \\ \eta_q \chi_{\eta_q} \end{array} \right). \]

in which \( \chi \) are Pauli spinors.

Then using also that \( \frac{1}{\alpha - \alpha_c + i\epsilon} \approx -i\pi \delta(\alpha - \alpha_c) + P(\frac{1}{\alpha - \alpha_c}) \) and neglecting the principal value term we obtain,

\[ \frac{\bar{u}_\zeta[-iQ_i e^{\lambda \gamma} \gamma^\perp] u_{\eta_1i}}{(1 - x_1)s'(\alpha - (\alpha_c + i\epsilon))} \approx i\pi \delta(\alpha - \alpha_c) Q_i e^{\lambda \gamma} \sqrt{\frac{2(1 - (1 - x_1)(1 - \alpha))}{s'(1 - x_1)}} \delta_{\zeta \lambda \gamma} \delta_{\eta_1i \lambda \gamma} \]

then \( \gamma \) only interacts with a quark having its same helicity, i.e., if \( \eta_{1i} = \lambda \gamma \) and consequently we have that \( \zeta = \eta_{1i} \)
Selection in $\gamma + ^3\text{He} \rightarrow (N + N) + N_s$

\[
\frac{\bar{u}_\zeta [ - i Q_i e^\gamma \gamma^\perp ] u_{\eta_1 i}}{(1 - x_1)s' (\alpha - (\alpha_c + i \epsilon))} \approx i \pi \delta (\alpha - \alpha_c) Q_i e^\lambda \gamma \sqrt{\frac{2(1 - (1 - x_1)(1 - \alpha))}{s'(1 - x_1)}} \delta_\zeta \lambda \gamma \delta_{\eta_1 i \lambda \gamma}
\]

allows to perform the $\alpha$ integration in calculating $\mathcal{M}$. $\alpha_c$ is then approximated to $\frac{1}{2}$ which corresponds to the situation in which both nucleons in $NN$ contribute equally to $p_{NN}^\perp$. This is justified by the strong peak of $\Psi_{3\text{He}}$'s magnitude at $\alpha = \frac{1}{2}$.

From

\[
\alpha_c = 1 + \frac{1}{s'_{\text{NN}}} \left[ \tilde{m}_N^2 - \frac{m_s^2(1 - x_1) + m_q^2x_1 + (k_1 - x_1 p_1)^2_\perp}{x_1 (1 - x_1)} \right].
\]

in the large $s'$ limit, $\alpha_c = 1/2$ leads to $x_1 \sim k_1^2 / s'$. Because $k_1^2 << s'$, this corresponds to having the valence quark contributing to most of the momentum of its parent nucleon $\Rightarrow p_q \approx p_N$ and $\eta_q \approx \lambda_N$.

These approximations are also extended to the valence quark of the second nucleon in $NN$. 

\[
\begin{array}{c}
\text{\psi}^2
\end{array}
\begin{array}{c}
\text{vs. } \alpha
\end{array}
\]
HRM scattering amplitude of $\gamma + He \rightarrow N + N + N_s$

After the considered approximations are applied to $\mathcal{M}$, the scattering amplitude for $\gamma + He \rightarrow N + N + N_s$ takes the form,

$$\langle \lambda_1, \lambda_2, \lambda_s | \mathcal{M}_i | \lambda_\gamma, \lambda_A \rangle = i[\lambda_\gamma] e^{i \sum_{(\eta_1f, \eta_2f), (\eta_2i), \lambda_1i, \lambda_2i} \int \frac{Q_i}{\sqrt{2s^f}}}$$

$$\left\{ \begin{array}{c}
\int \frac{\psi_{\lambda_f, \eta_2f}^{\lambda_2f, \eta_2f}}{1-x_2} \bar{u}_{\eta_2f} [-i g T^F_{\gamma, \mu}] u_{\eta_1i} \psi_{\lambda_1i, \eta_1i}^{\lambda_1i, \eta_1i} (1-x_1) \\
\int \frac{\psi_{\lambda_1f, \eta_1f}^{\lambda_1f, \eta_1f}}{1-x_1} \bar{u}_{\eta_1f} [-i g T^F_{\gamma, \mu}] u_{\eta_2i} \psi_{\lambda_2i, \eta_2i}^{\lambda_2i, \eta_2i} (1-x_2) \end{array} \right\} \times$$

$$G^{\mu, \nu} \frac{dx_1}{x_1} \frac{d^2 k_1}{2(2\pi)^3} \frac{dx_2}{x_2} \frac{d^2 k_2}{2(2\pi)^3} \frac{d^2 p_2}{(2\pi)^2} \frac{d^2 p_2}{(2\pi)^2} .$$

which is written as a convolution of a $NN$ elastic scattering amplitude with a component of a nuclear wave function.
HRM scattering amplitude of $\gamma + \text{He} \rightarrow N + N + N_s$

Accounting for all possible quark interchange diagrams we obtain,

$$\langle \lambda_{1f}, \lambda_{2f}, \lambda_s \mid M \mid \lambda_\gamma, \lambda_A \rangle = \frac{i[\lambda_\gamma]e\sqrt{2}(2\pi)^3}{\sqrt{2S'_{NN}}} \times \left\{ \begin{array}{c} Q_F^{N_1} \sum_{\lambda_{2i}} \int \langle \lambda_{2f}; \lambda_{1f} \mid T_{NN}^{\text{QIM}}(s_{NN}, t_N) \mid \lambda_\gamma; \lambda_{2i} \rangle \psi^{\lambda_A}_{3\text{He}, \text{NR}}(\vec{p}_1, \lambda_\gamma; \vec{p}_2, \lambda_{2i}; \vec{p}_s, \lambda_s) m_N \frac{d^2 \vec{p}_\perp}{(2\pi)^2} + \\
Q_F^{N_2} \sum_{\lambda_{1i}} \int \langle \lambda_{2f}; \lambda_{1f} \mid T_{NN}^{\text{QIM}}(s_{NN}, t_N) \mid \lambda_{1i}; \lambda_\lambda \rangle \psi^{\lambda_A}_{3\text{He}, \text{NR}}(\vec{p}_1, \lambda_{1i}; \vec{p}_2, \lambda_\gamma; \vec{p}_s, \lambda_s) m_N \frac{d^2 \vec{p}_\perp}{(2\pi)^2} \end{array} \right\}$$

(3)

where charge factors $Q_F$ are introduced such that,

$$\sum_{i \in N} Q_i^N \langle a' b' \mid T_{NN,i}^{\text{QIM}} \mid ab \rangle = Q_F^N \cdot \langle a' b' \mid T_{NN}^{\text{QIM}} \mid ab \rangle$$
$\gamma + ^3 He \rightarrow N + N + N_s$ Scattering Amplitudes

$NN$ scattering amplitudes

**Dominance of the quark interchange (QI) mechanism (White et al. 1994)**

For the quark interchange $NN$ amplitudes, we choose $NN$ helicity amplitudes labeled as follows:

\begin{align*}
< +, + | T_{NN}^{QIM} | +, + > &= \phi_1 \\
< +, + | T_{NN}^{QIM} | +, - > &= \phi_5 \\
< +, + | T_{NN}^{QIM} | -, - > &= \phi_2 \\
< +, - | T_{NN}^{QIM} | +, - > &= \phi_3 \\
< +, - | T_{NN}^{QIM} | - + > &= -\phi_4.
\end{align*}

(4)

All other helicity combinations can be related to the above amplitudes through the parity and time-reversal symmetry. The cross section of a $NN$ scattering is defined as

\[ \frac{d\sigma^{NN\rightarrow NN}}{dt} = \frac{1}{16\pi} \frac{1}{s(s - 4m_N^2)} \frac{1}{2} (|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 4|\phi_5|^2). \]

(5)
\( \gamma + ^3 \text{He} \rightarrow N + N + N_s \) Scattering Amplitudes

\( NN \) scattering amplitudes II

Then for \( NN \) breakup in \( ^3 \text{He} \), using the antisymmetry of the \( ^3 \text{He} \) ground state wave function we have,

\[
\langle +, +, \lambda_s | M | +, \lambda_A \rangle = B \int \left[ Q_F \phi_5 \psi_{^3 \text{He}}^\lambda (+, -, \lambda_s) + Q_F \phi_1 \psi_{^3 \text{He}}^\lambda (+, +, \lambda_s) \right] m_N \frac{d^2 p_\perp}{(2\pi)^2}
\]

\[
\langle +, -, \lambda_s | M | +, \lambda_A \rangle = B \int \left[ (Q_F^N \phi_3 + Q_F^N \phi_4) \psi_{^3 \text{He}}^\lambda (+, -, \lambda_s) - Q_F \phi_5 \psi_{^3 \text{He}}^\lambda (+, +, \lambda_s) \right] \times m_N \frac{d^2 p_\perp}{(2\pi)^2}
\]

\[
\langle -, +, \lambda_s | M | +, \lambda_A \rangle = B \int \left[ -(Q_F^N \phi_4 + Q_F^N \phi_3) \psi_{^3 \text{He}}^\lambda (+, -, \lambda_s) + Q_F \phi_5 \psi_{^3 \text{He}}^\lambda (+, +, \lambda_s) \right] \times m_N \frac{d^2 p_\perp}{(2\pi)^2}
\]

\[
\langle -, -, \lambda_s | M | +, \lambda_A \rangle = B \int \left[ Q_F \phi_5 \psi_{^3 \text{He}}^\lambda (+, -, \lambda_s) + Q_F \phi_2 \psi_{^3 \text{He}}^\lambda (+, +, \lambda_s) \right] m_N \frac{d^2 p_\perp}{(2\pi)^2}
\]

where \( B = \frac{ie\sqrt{2}(2\pi)^3}{\sqrt{2s'_{NN}}} \).
**NN Breakup cross sections**

Then for the averaged squared amplitude one obtains,

\[
|\bar{M}|^2 = \frac{e^2 2 (2\pi)^6}{2s'_NN} \frac{1}{2} \left\{ 2Q_F^2 |\phi_5|^2 S_0 + Q_F^2 (|\phi_1|^2 + |\phi_2|^2) S_{12} + \right. \\
\left. \left[ (Q_{F1}^N \phi_3 + Q_{F2}^N \phi_4)^2 + (Q_{F1}^N \phi_4 + Q_{F2}^N \phi_3)^2 \right] S_{34} \right\} 
\]

(6)

where \( Q_F = Q_{F1}^N + Q_{F2}^N \) and \( S_{12}, S_{34}, \) and \( S_0 \) are partially integrated nuclear spectral functions:

\[
S_{12}(t_1, t_2, \alpha, \bar{p}_s) = N_{NN} \sum_{\lambda_1=\lambda_2=-\frac{1}{2}}^{\frac{1}{2}} \sum_{\lambda_3=-\frac{1}{2}}^{\frac{1}{2}} \left| \int \psi_\frac{1}{2}^{3He,NR}(\bar{p}_1, \lambda_1, t_1; \bar{p}_2, \lambda_2, t_2; \bar{p}_s, \lambda_3) m_N \frac{d^2p_\perp}{(2\pi)^2} \right|^2 ,
\]

\[
S_{34}(t_1, t_2, \alpha, \bar{p}_s) = N_{NN} \sum_{\lambda_1=-\lambda_2=-\frac{1}{2}}^{\frac{1}{2}} \sum_{\lambda_3=-\frac{1}{2}}^{\frac{1}{2}} \left| \int \psi_\frac{1}{2}^{3He,NR}(\bar{p}_1, \lambda_1, t_1; \bar{p}_2, \lambda_2, t_2; \bar{p}_s, \lambda_3) m_N \frac{d^2p_\perp}{(2\pi)^2} \right|^2 ,
\]

and

\[
S_0 = S_{12} + S_{34} .
\]

(9)
NN Breakup cross sections

Assuming the massless approximation for the interchanging quarks, their corresponding helicities are conserved during the hard subprocess, leading to the vanishing of NN amplitudes that don’t conserve helicity, i.e.,

\[ \phi_2 = 0 \]
\[ \phi_5 = 0 \]

then for NN scattering,

\[
\frac{d\sigma_{NN\rightarrow NN}}{dt} = \frac{1}{16\pi} \frac{1}{s(s - 4m_N^2)} \frac{1}{2} \left( |\phi_1|^2 + |\phi_3|^2 + |\phi_4|^2 \right),
\] (10)

while for NN breakup

\[
|\tilde{M}|^2 = \frac{e^22(2\pi)^6}{2s'_{NN}} \frac{1}{2} \left\{ Q_F^2 |\phi_1|^2 S_{12} + \left[ (Q_{F_1}^N \phi_3 + Q_{F_2}^N \phi_4)^2 + (Q_{F_1}^N \phi_4 + Q_{F_2}^N \phi_3)^2 \right] S_{34} \right\}
\] (11)
We now consider the $\gamma + ^3\text{He} \rightarrow p + n + p_s$ reaction. Using that,

- for large angle pn scattering $\phi_3 \approx \phi_4$,
- in the QI model it is found that $Q_{F}^{pn} = \frac{1}{3}$,
- and that for $^3\text{He}$ in its ground state, $S_{12}^{pn} \approx S_{34}^{pn} \approx \frac{S_0^{pn}}{2}$,

we obtain that,

$$|\tilde{M}|^2 = \frac{(eQ_{F, pn})^2(2\pi)^6}{s_N'} 16\pi s_{NN}(s_{NN} - 4m_N^2) \frac{d\sigma^{pn\rightarrow pn}(s_{NN}, t_N)}{dt_N} S_0^{pn},$$

and for the differential cross section,

$$\frac{d\sigma^{^3\text{He}\rightarrow(pn)p}}{dt \frac{d^3p_s}{E_s}} = \alpha Q_{F, pn}^2 16\pi^4 S_0^{pn}(\alpha = \frac{1}{2}, \vec{p}_s) \frac{s_{NN}(s_{NN} - 4m^2)}{(s_{NN} - p_{NN}^2)^2(s - M_{^3\text{He}}^2)} \frac{d\sigma^{pn\rightarrow pn}(s_{NN}, t_N)}{dt_N},$$
Hard pp Breakup

To calculate $\gamma + ^3\text{He} \rightarrow p + p + n_s$ we use that

- for pp scattering, $\phi_4 \sim -\phi_3$ for large $\theta_{c.m}$ (Ramsey, Sivers, 1992),
- from the exclusion principle and $S$ state dominance of the nuclear wave function, $S_{12}^{pp} \ll S_{34}^{pp}$,
- and obtaining from QI diagrams that $Q_F^{pp} = \frac{5}{3}$.

Then, for hard pp breakup in $^3\text{He}$ photodisintegration,

$$|\vec{M}|^2 = \frac{(e^2 2(2\pi)^6}{2 s_{NN}'} \frac{1}{2} \left\{ 2 Q_F^{pp} (|\phi_3| - |\phi_4|)^2 S_{34} \right\},$$

and for the corresponding differential cross section,

$$\frac{d\sigma}{dt} \frac{d^3 p_s}{E_s} = \alpha Q_{F,pp}^2 16\pi^4 S_{34}^{pp} (\alpha = \frac{1}{2} \cdot \vec{p}_s) \frac{2 \beta^2}{1 + 2 C^2 (s_{NN} - p_{NN}^2)^2 (s - M_{^3\text{He}}^2)} \times$$

$$\frac{d\sigma_{pp \rightarrow pp}}{dt}(s_{NN}, t_N),$$

where,

$$C^2 = \frac{\phi_3^2}{\phi_1^2} \approx \frac{\phi_4^2}{\phi_1^2}, \quad \text{and,} \quad \beta = \frac{|\phi_3| - |\phi_4|}{|\phi_1|},$$
$\gamma NN$ and $N + N$ Kinematics I

Calculating $\frac{d\sigma}{dt} \frac{d^3p_s}{E_s} (s_{NN}, \theta_{c.m.})$ within HRM, using $\frac{d\sigma}{dt} \frac{d^3p_s}{E_s} (s_{NN}, \theta_{c.m.})$ requires using the right correspondence between $\theta_{c.m}$ and $\theta_{c.m.}^N$.

In the center of mass reference frame for,

$\gamma + NN \to N + N$

\[ t = (p_{NN} - p_{2f})^2 \]

$N + N \to N + N$

\[ t^N = (p_{2i} - p_{2f})^2 \approx \left( \frac{p_{NN}}{2} - p_{2f} \right)^2 \]
\( \gamma NN \) and \( N + N \) Kinematics II

which leads to,

\[
t^N = \frac{t}{2} + \frac{m_N^2}{2} - \frac{M_{NN}^2}{4}
\]

that together with

\[
t^N = -\frac{s_{NN} - 4m_N^2}{2} (1 - \cos(\theta_{c.m.}^N))
\]

and

\[
t = -\frac{(s_{NN} - M_{NN}^2)}{2} (1 - \sqrt{1 - 4 \frac{m_N^2}{s_{NN}} \cos(\theta_{c.m.})}) + m_N^2
\]

is used to obtain that,

\[
\cos(\theta_{c.m.}^N) = 1 - \frac{(s_{NN} - M_{NN}^2)}{2(s_{NN} - 4m_N^2)} \left( \sqrt{s_{NN}} - \sqrt{s_{NN} - 4m_N^2} \cos(\theta_{c.m.}) \right) + \frac{4m_N^2 - M_{NN}^2}{2(s_{NN} - 4m_N^2)}.
\]

Then for instance, to calculate a HRM \(^3\)He photodisintegration cross section for \( \theta_{c.m.} = 90^\circ \), from the above equation we’ll need to have a numerical input for the corresponding \( NN \) elastic scattering cross section at \( \theta_{c.m.}^N = 60^\circ \).
pn and pp breakup at $\theta_{c.m.} = 90^\circ$

\[
\frac{d\sigma}{dt} \frac{d^3p_s}{E_s} = \alpha Q_{F,pn}^2 16\pi^4 S_{0}^{pn}(\alpha = \frac{1}{2}, \bar{\rho_s}) 2
\]

\[
\frac{s_{NN}(s_{NN} - 4m^2)}{(s_{NN} - p_{NN}^2)^2(s - M_{3He}^2)} d\sigma_{pn \rightarrow pn}(s_{NN}, t_N) dt_N
\]

\[
\frac{s_{NN}(s_{NN} - 4m^2)}{(s_{NN} - p_{NN}^2)^2(s - M_{3He}^2)} \times
\frac{d\sigma_{pp \rightarrow pp}(s_{NN}, t_N)}{dt}
\]

\[
\frac{d\sigma}{dt} \frac{d^3p_s}{E_s} = \alpha Q_{F,pp}^2 16\pi^4 S_{34}^{pp}(\alpha = \frac{1}{2}, \bar{\rho_s}) 2\beta^2 1 + 2C^2
\]

\[
\frac{s_{NN}(s_{NN} - 4m^2)}{(s_{NN} - p_{NN}^2)^2(s - M_{3He}^2)} \times
\frac{d\sigma_{pp \rightarrow pp}(s_{NN}, t_N)}{dt}
\]

**Figure:** Energy dependence of $s^{11}$ weighted differential cross sections at $90^\circ$ c.m. angle scattering in "$\gamma$-NN" system. In these calculations one integrated over the spectator nucleon momenta in the range of 0-100 MeV/c.

$pn$ and $pp$ breakup at $\theta_{c.m.} = 90^\circ$

HRM calculated $^3\text{He}(\gamma pp)n$ energy distribution in agreement with experimental data.

Note

$$s\sigma ^3\text{He}(\gamma pp)n(s) \sim \sigma^{pp}(s)$$
Polarization Transfer

For a circularly polarize photon $C'_z$ measures the asymmetry of the hard breakup reaction with respect to the helicity of the outgoing proton.

$$C'_z = \frac{\sum_{\lambda_1,\lambda_2,\lambda_s,\lambda_a} \left\{ |\langle +, \lambda_2f, \lambda_s | M | +, \lambda_A \rangle|^2 - |\langle -, \lambda_2f, \lambda_s | M | +, \lambda_A \rangle|^2 \right\}}{\sum_{\lambda_1,\lambda_2,\lambda_s,\lambda_a} |\langle \lambda_1f, \lambda_2f, \lambda_s | M | +, \lambda_A \rangle|^2}.$$  \hfill (12)

Then

$$C'_z = \frac{(|\phi_1|^2 - |\phi_2|^2)S^{++} + (|\phi_3|^2 - |\phi_4|^2)S^{+-}}{2|\phi_5|^2 S^+ + (|\phi_1|^2 + |\phi_2|^2)S^{++} + (|\phi_3|^2 + |\phi_4|^2)S^{+-}},$$  \hfill (13)

with,

$$S^\pm,\pm(t_1, t_2, \alpha, \bar{p}_s) = \sum_{\lambda_A = -\frac{1}{2}}^{\frac{1}{2}} \sum_{\lambda_3 = -\frac{1}{2}}^{\frac{1}{2}} \left| \int \psi_{\lambda A}^{\lambda A} (\bar{p}_1, \lambda_1 = \pm \frac{1}{2}, t_1; \bar{p}_2, \lambda_2 = \pm \frac{1}{2}, t_2; \bar{p}_s, \lambda_3)mN \frac{d^2p_2, \perp}{(2\pi)^2} \right|^2 \hfill (14)$$

and $S^+ = S^{++} + S^{+-}$. 
Polarization Transfer

\[ C_{z'} = \frac{(|\phi_1|^2 - |\phi_2|^2)S^{++} + (|\phi_3|^2 - |\phi_4|^2)S^{+-}}{2|\phi_5|^2 S^+ + (|\phi_1|^2 + |\phi_2|^2)S^{++} + (|\phi_3|^2 + |\phi_4|^2)S^{+-}}, \tag{15} \]

Through previous assumptions for \textit{pp} and \textit{pn} breakup,

\[ C_{z'}^{pp} \approx \frac{|\phi_3|^2 - |\phi_4|^2}{|\phi_3|^2 + |\phi_4|^2} \sim 0, \tag{16} \]

which results from the suppression of \( S^{++} \) and from \( \phi_3 \sim \phi_4 \).

And,

\[ C_{z'}^{pn} \approx \frac{|\phi_1|^2 + |\phi_3|^2 - |\phi_4|^2}{|\phi_1|^2 + |\phi_3|^2 + |\phi_4|^2} \sim \frac{2}{3}, \tag{17} \]
Summary

Large angle high energy breakup of a NN system in $^3$He has been studied in $pp$ and $pn$ breakup channels within the framework of the QCD hard rescattering model, HRM.

HRM predicted energy dependencies, in accordance with counting rules, agree with recent experimental observations which supports the explicit role of QCD degrees of freedom in the reaction.

- Calculated HRM $\gamma + ^3$He $\rightarrow p + p + n_s$ differential cross section at $\theta_{c.m.}^{NN} = 90^\circ$ agrees well with experimental data without the introduction of adjustable parameters. (Note suppression from $|\phi_3| - |\phi_4|$ factor with respect to $pn$ breakup).

- The $s^{-11}$ behavior of the observed cross section for $\gamma + ^3$He $\rightarrow p + p + n_s$ indicates the dominance of the two body process in the reaction.

Additional observables are calculated from the HRM applied to $\gamma + ^3$He $\rightarrow N + N + N_s$, which predicts

- a broader spectator’s momentum distribution for $pp$ breakup in relations to $pn$ breakup.

- $C_{z'}^{pp}$ suppression in relation to $C_{z'}^{pn}$

Experimental confirmation of these predictions will further reinforce the HRM picture of breakup processes.

The encouraging experimental results motivate the study of more processes that can be described through the hard rescattering model.
The study of the deuteron photodisintegration into \( \Delta \Delta \)-isobars channels was proposed as a venue for investigating the evolution of a nucleon-nucleon system into a six quark system.

The onset of a six quark picture of the deuteron could then be marked by a large increase of the \( \gamma d \rightarrow \Delta \Delta \) cross section. This prediction assumes that such cross section is small for a nucleon dominated deuteron wave function because of its suppressed \( \Delta \Delta \) components.

In contrast, for a six quark deuteron, \( NN \) and \( \Delta \Delta \) components contribute with comparable strength to the deuteron wave function (roughly 10\% and 8\% respectively) while more than 80\% is contributed by \( CC \) (hidden color) components for which unlike \( N \) or \( \Delta \), \( C \) has a color charge (Brodsky, Ji, Lepage, 1983).

\[
\psi_d = \sqrt{\frac{1}{9}} \psi_{NN} + \sqrt{\frac{4}{45}} \psi_{\Delta \Delta} + \sqrt{\frac{4}{5}} \psi_{cc}
\]
High energy $\gamma d \rightarrow \Delta \Delta$ with $\Delta \Delta$ emerging at large transverse momentum is thought to probe the onset of hidden color components in the deuteron. Assuming that the high $p_T$ $\Delta \Delta$ system was created in the initial state of the interaction, in the asymptotic limit we have that

$$\frac{d\sigma}{dt}^{\gamma d \rightarrow \Delta \Delta} \sim \frac{d\sigma}{dt}^{\gamma d \rightarrow pn}.$$ 

Under this same assumption we also have that,

$$\frac{d\sigma}{dt}^{\gamma d \rightarrow \Delta^+ \Delta^-} = \frac{d\sigma}{dt}^{\gamma d \rightarrow \Delta^+ \Delta^0},$$

since both $\Delta \Delta$ channels in general contribute with the same strength to the spin-isospin wave function of the deuteron. In the QCD hard rescattering model (HRM), the high $p_T$ $\Delta \Delta$ system is created in a final state interaction mainly through a $pn$ rescattering reaction. As it’ll be shown in what follows under this scenario, the hard rescattering model predicts a dominance of

$$\frac{d\sigma}{dt}^{\gamma d \rightarrow \Delta^+ \Delta^-} \text{ over } \frac{d\sigma}{dt}^{\gamma d \rightarrow \Delta^+ \Delta^0}$$

which contrasts the picture described above.
For comparison, we also estimate within the HRM the strength of $\Delta\Delta$ channels in deuteron breakup relative to that of $\gamma d \rightarrow pn$.
As it was the case for $\gamma d \rightarrow pn$, through the HRM for $\gamma d \rightarrow \Delta\Delta$ we find that,

$$
\frac{d\sigma_{\gamma d\rightarrow\Delta\Delta}(s, \theta_{c.m.})}{dt} = \frac{\alpha Q^2_F,\Delta\Delta}{s'} 8\pi^4 \frac{d\sigma_{pn\rightarrow\Delta\Delta}(s, \theta_{c.m.}^N)}{dt} \tilde{S}_{0,NR},
$$

But $\frac{d\sigma_{pn\rightarrow\Delta\Delta}(s, \theta_{c.m.}^N)}{dt}$ has not been well measured in the required kinematic regime.

As it was the case for \( \gamma d \rightarrow pn \), through the HRM for \( \gamma d \rightarrow \Delta\Delta \) we find that,

\[
\frac{d\sigma_{\gamma d \rightarrow \Delta\Delta}(s, \theta_{c.m.})}{dt} = \frac{\alpha Q_F^2,\Delta\Delta}{s'} 8\pi^4 \frac{d\sigma_{pn \rightarrow \Delta\Delta}(s, \theta_{c.m.}^N)}{dt} S_{0, NR},
\]

But \( \frac{d\sigma_{pn \rightarrow \Delta\Delta}(s, \theta_{c.m.}^N)}{dt} \) has not been well measured in the required kinematic regime. Then model dependent \( pn \rightarrow \Delta\Delta \) amplitudes \( \phi \) are input in

\[
|\tilde{M}|^2_{\gamma d \rightarrow \Delta\Delta} = \frac{11}{23} e^2 \frac{1}{2s'}
\times \left[ S_{12} \left\{ |(\hat{Q}^N_1 + \hat{Q}^N_2)\phi_1|^2 + |(\hat{Q}^N_1 + \hat{Q}^N_2)\phi_6|^2 + |(\hat{Q}^N_1 + \hat{Q}^N_2)\phi_7|^2 \right\} 
+ S_{34} \left\{ |\hat{Q}^N_1 \phi_3 + \hat{Q}^N_2 \phi_4|^2 + |\hat{Q}^N_1 \phi_4 + \hat{Q}^N_2 \phi_3|^2 
+ |\hat{Q}^N_1 \phi_8 + \hat{Q}^N_2 \phi_9|^2 + |\hat{Q}^N_1 \phi_9 + \hat{Q}^N_2 \phi_8|^2 \right\} \right],
\]

$\gamma d \rightarrow \Delta \Delta \text{ in the HRM}$

$pn \rightarrow \Delta \Delta \text{ scattering amplitudes}$

*Helicity conserving $pn \rightarrow \Delta \Delta \text{ amplitudes}$*

\[
\begin{align*}
\langle +\frac{1}{2}, +\frac{1}{2} | T | +\frac{1}{2}, +\frac{1}{2} \rangle &= \phi_1 \\
\langle +\frac{1}{2}, -\frac{1}{2} | T | +\frac{1}{2}, -\frac{1}{2} \rangle &= \phi_3 \\
\langle -\frac{1}{2}, +\frac{1}{2} | T | +\frac{1}{2}, -\frac{1}{2} \rangle &= \phi_4 \\
\langle +\frac{3}{2}, -\frac{1}{2} | T | +\frac{1}{2}, +\frac{1}{2} \rangle &= \phi_6 \\
\langle -\frac{1}{2}, +\frac{3}{2} | T | +\frac{1}{2}, +\frac{1}{2} \rangle &= \phi_7 \\
\langle +\frac{3}{2}, -\frac{3}{2} | T | +\frac{1}{2}, -\frac{1}{2} \rangle &= \phi_8 \\
\langle -\frac{3}{2}, +\frac{3}{2} | T | +\frac{1}{2}, -\frac{1}{2} \rangle &= \phi_9,
\end{align*}
\]
\[ \gamma d \rightarrow \Delta \Delta \text{ in the HRM} \]
\[ pn \rightarrow \Delta \Delta \text{ scattering amplitudes} \]

Helicity conserving \( pn \rightarrow \Delta \Delta \) amplitudes \( \text{ Obtained in the quark exchange model } \)

\[ pn \rightarrow \Delta^+ \Delta^0 \]

\[
\begin{align*}
\phi_1 &= \frac{2}{9} N_{\Delta \Delta} (2f(\theta_{c.m.}^N) - f(\pi - \theta_{c.m.}^N)) \\
\phi_3 &= \frac{1}{9} N_{\Delta \Delta} (4f(\theta_{c.m.}^N) + f(\pi - \theta_{c.m.}^N)) \\
\phi_4 &= \frac{2}{9} N_{\Delta \Delta} (f(\theta_{c.m.}^N)) + f(\pi - \theta_{c.m.}^N) \\
\phi_6 &= \frac{N_{\Delta \Delta}}{3 \sqrt{3}} (2f(\theta_{c.m.}^N) - f(\pi - \theta_{c.m.}^N)) \\
\phi_7 &= \frac{N_{\Delta \Delta}}{3 \sqrt{3}} (2f(\theta_{c.m.}^N) - f(\pi - \theta_{c.m.}^N)) \\
\phi_8 &= \frac{2}{9} N_{\Delta \Delta} (f(\theta_{c.m.}^N)) \\
\phi_9 &= \frac{1}{3} N_{\Delta \Delta} f(\pi - \theta_{c.m.}^N),
\end{align*} \]

\[ pn \rightarrow \Delta^+ \Delta^- \]

\[
\begin{align*}
\phi_1 &= -\frac{2}{3} N_{\Delta \Delta} f(\theta_{c.m.}^N) \\
\phi_3 &= -\frac{2}{3} N_{\Delta \Delta} f(\theta_{c.m.}^N) \\
\phi_4 &= -\frac{1}{3} N_{\Delta \Delta} f(\theta_{c.m.}^N) \\
\phi_6 &= -\frac{N_{\Delta \Delta}}{\sqrt{3}} f(\theta_{c.m.}^N) \\
\phi_7 &= -\frac{N_{\Delta \Delta}}{\sqrt{3}} f(\theta_{c.m.}^N) \\
\phi_8 &= -N_{\Delta \Delta} f(\theta_{c.m.}^N) \\
\phi_9 &= 0,
\end{align*} \]
\[ \gamma d \rightarrow \Delta \Delta \text{ in the HRM} \]

\[
|\mathcal{M}|^2_{\gamma d \rightarrow \Delta^+\Delta^-} = \frac{1}{6} \frac{e^2}{2 s'} Q^2_{F, \Delta \Delta} \left\{ S_{12} \left[ |\phi_1|^2 + |\phi_6|^2 + |\phi_7|^2 \right] + S_{34} \left[ \left( \frac{\phi_3 + \phi_4}{2} + 2\phi_u - \phi_3 \right)^2 + \left( \frac{\phi_4 + \phi_3}{2} + 2\phi_u - \phi_4 \right)^2 \right. \\
+ \left. \left( \frac{\phi_8 + \phi_9}{2} + 2\phi_u - \phi_8 \right)^2 + \left( \frac{\phi_9 + \phi_8}{2} + 2\phi_u - \phi_9 \right)^2 \right] \right\},
\]

\[
|\mathcal{M}|^2_{\gamma d \rightarrow \Delta^+\Delta^-} = \frac{1}{6} \frac{e^2}{2 s'} Q^2_{F, \Delta \Delta} \left\{ S_{12} \left[ |\phi_1|^2 + |\phi_6|^2 + |\phi_7|^2 \right] + S_{34} \left[ (2\phi_3 - \phi_4)^2 + (2\phi_4 - \phi_3)^2 + 5|\phi_8|^2 \right] \right\}.
\]
\( \gamma d \rightarrow \Delta \Delta \) in the HRM

Angular Distributions

\[ R \text{ in Fig. (a)} \]

- is sensitive to model of baryon wavefunction
- shows a strong dependence on \( \theta_{c.m.} \)
  contrasting \( R \sim 1 \) expected from the onset of hidden color components.

\[ R \text{ in Fig. (b)} \]

- \( R \) doesn’t depend on the choice of wave function
- \( \Delta^{++}\Delta^- \) channel is consistently larger than that of the \( \Delta^+\Delta^0 \) channel. If the isobars were produced from a \( \Delta\Delta \) component of the initial deuteron wave function, both channels should have the same strength and \( R=1 \).

Further Applications

- Energy scaling in $\gamma^3\text{He} \rightarrow pd$
Further Applications

- Energy scaling in $\gamma^3\text{He} \rightarrow \text{pd}$
- Energy scaling in Compton Scattering off a Proton
Further Applications

- Energy scaling in $\gamma^3\text{He} \rightarrow pd$
- Energy scaling in Compton Scattering off a Proton
- Heavy Quark Meson Production
Thank you
Three-body/two-step reaction

- Dominant at low to intermediate beam energies \( (E_\gamma \sim 200\text{MeV}) \).
- HRM amplitude does not interfere with two-body/one-step amplitude.
- HRM cross section scales like \( s^{-12} \) at large energies from second rescattering.
Introducing the light-cone wave function of \(^3\)He \([?, ?, ?]\)

\[
\psi_{^3\text{He}}^{\lambda_4, \lambda_1, \lambda_2, \lambda_s}(\alpha, p_\perp) = \frac{\Gamma_{^3\text{He}}^{\lambda}}{M_{NN}^2 - \frac{m_N^2 + p_\perp^2}{\alpha(1-\alpha)}} \bar{u}_{\lambda_1}(p_{NN} - p)\bar{u}_{\lambda_2}(p)\bar{u}_{\lambda_s}(p_s)
\] (18)

defining quark wave function of the nucleon as

\[
\psi_N^{\lambda, \eta}(p, x, k_\perp) = \frac{u_N^{\lambda}(p)\Gamma_N \bar{u}_\eta(p - k)\psi^\dagger_s(k)}{m_N^2 - \frac{m_s^2(1-x)+m_q^2x+(k_\perp - xp_\perp)^2}{x(1-x)}}
\] (19)
\begin{equation}
\psi_{3\text{He}}(\alpha, p_\perp, \alpha_s, p_s, \perp) = \sqrt{2}(2\pi)^3 m_N \psi_{3\text{He},\text{NR}}(\alpha, p_\perp, \alpha_s, p_s, \perp) \tag{20}
\end{equation}


SU(6) Helicity Amplitudes

For $pp$ scattering:

$$\phi_1(\theta_{CM}) = 144f(\theta_{CM}) + 144f(\pi - \theta_{CM}) \quad (21)$$
$$\phi_2(\theta_{CM}) = 0$$
$$\phi_3(\theta_{CM}) = 56f(\theta_{CM}) + 68f(\pi - \theta_{CM})$$
$$\phi_4(\theta_{CM}) = -68f(\theta_{CM}) - 56f(\pi - \theta_{CM})$$
$$\phi_5(\theta_{CM}) = 0$$

Spectator Momentum Distributions

\[ \frac{\sigma^{pp}(\text{solid})}{\sigma^{pn}(\text{dotted})} \]

(a) Ratio \[ \frac{\sigma^{pp}}{\sigma^{pn}} \] with

\[ \alpha_s \equiv \frac{E_s - p_{s,z}}{M_A/A} = \alpha_A - \alpha_{NN} \]

with

\[ s_{NN} = M_{NN}^2 + E_\gamma m_n \alpha_{NN} \].

- Asymmetry around \( \alpha = 1 \) due to \( s^{-11} \) dependence.
- \( pp \) distribution broader than \( pn \).
- \( R \) drops around \( \alpha = 1 \) from the suppression of same helicity two proton components of the nuclear wave function at small momenta.