The $P\gamma$ transition form factor in QCD

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Outline:

• The $\pi\gamma$ trans. form factor in coll. factorization
• The new BaBar data
• Ways out
• The mod. pert. approach
• Generalization to $\eta, \eta', \eta_c$
• Summary
Measuring the $\pi\gamma$ form factor

$\gamma^*\gamma\pi$ vertex:

$$\Gamma_{\mu\nu} = -ie^2 F_{\pi\gamma^*}(Q^2) \epsilon^{\mu\nu\alpha\beta} q^{\alpha} q'^{\beta}$$

data from:

TPC/2$\gamma$(90), CELLO(91)
CLEO(95,98) $Q^2 \lesssim 8$ GeV$^2$
BaBar(09) $4 \lesssim Q^2 \lesssim 38$ GeV$^2$
also data on $\eta\gamma$, $\eta'\gamma$, $\eta_c\gamma$
L3(97), CLEO(95,98), BaBar(10)

BaBar(06)

$\eta\gamma$ and $\eta'\gamma$ at $s = 112$ GeV$^2$

two-photon decay width of the mesons:

normalization of FF at $Q^2 = 0$
Theory: collinear factorization

\[ F_{\pi\gamma}(Q^2) = \frac{\sqrt{2f_\pi}}{3} \int_0^1 dx \Phi_\pi(x, \mu_F) T_H(x, Q^2, \mu_R) \text{ at large } Q^2 \]

\[ T_H^{NLO} = \frac{1}{xQ^2} \left\{ 1 + C_F \frac{\alpha_s(\mu_R)}{2\pi} \left[ \frac{1}{2} \ln^2 x - \frac{x \ln x}{2(1-x)} - \frac{9}{2} + \left( \frac{3}{2} + \ln x \right) \ln \frac{Q^2}{\mu_R^2} \right] \right\} \]

\[ \Phi_\pi(x, \mu_F) = 6x(1-x) \left[ 1 + \sum_{n=2,4,\ldots} a_n(\mu_0) \left( \frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)} \right)^{\gamma_n/\beta_0} C_n^{3/2}(2x-1) \right] \]

- \( f_\pi \) pion decay constant;
- \( \mu_F, \mu_R, \mu_0 \) factorization, renormalization, initial scale
- \( a_n \) embody soft physics convenient choice: \( \mu_F = \mu_R = Q \) \( \overline{MS} \) scheme
- \( \gamma_n \) anomalous dimensions (pos. fractional numbers, growing with \( n \))

LO: Brodsky-Lepage (80)  NLO: del Aguila-Chase (81); Braaten (83)
Kadantseva et al (86)
LO: \[ Q^2 F_{\pi\gamma} = \frac{\sqrt{2} f_\pi}{3} \langle 1/x \rangle \quad \langle 1/x \rangle = 3 \left[ 1 + \sum a_n(\mu_F) \right] \]

due to evolution relative weights of the \( a_n \) vary with \( \ln Q^2 \)

for \( \ln Q^2 \to \infty \quad \Phi_\pi \to 6x(1-x) = \Phi_{\text{AS}} \quad Q^2 F_{\pi\gamma} \to \sqrt{2} f_\pi \)
Two virtual photons

\[ Q^2 = \frac{1}{2} (Q^2 + Q'^2) \quad ; \quad \omega = \frac{Q^2 - Q'^2}{Q^2 + Q'^2} \]

\[ F_{\pi \gamma^*}(Q^2, \omega) = \frac{\sqrt{2} f_\pi}{3 Q^2} \int_0^1 dx \frac{\Phi_\pi(x, \mu_F)}{1 - (2x - 1)^2 \omega^2} \left[ 1 + \frac{\alpha_s(\mu_R)}{\pi} K(\omega, x) \right] \]

for \( \omega \to 0 : \quad \overline{Q}^2 F_{\pi \gamma^*} = \frac{\sqrt{2} f_\pi}{3} \left[ 1 - \frac{\alpha_s}{\pi} \right] + \mathcal{O}(\omega^2) \]

\( \omega \to 0 \) limit \hspace{1cm} Cornwall(66)
\( a_n \) contribute to order \( \omega^n \) \hspace{1cm} Diehl-K-Vogt(01)
\( \alpha_s \) corrections \hspace{1cm} del Aguila-Chase (81)
\( \alpha_s^2 \) corrections \hspace{1cm} Melic-Müller-Passek (03)

parameter-free QCD prediction \hspace{1cm} (not end-point sens., power corr. small)
theor. status comparable with \( R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-) \)
Bjorken sum rule, Ellis-Jaffe sum rule, .. \hspace{1cm} but no data
Situation before the advent of the BaBar data

\[ Q^2 F_{\pi\gamma} \]

\[ Q'^2 = 0 \]

close to NLO result evaluated from asymptotic distribution amplitude

remaining \( \sim 10\% \) can be explained easily but differently:

non-asymptotic DA, low renormalization scale, twist-4 effects, quark transverse momenta, . . .

K-Raulfs (95), Ong (96), Musatov-Radyushkin (97), Brodsky-Pang-Robertson (98), Yakovlev-Schmedding (00), Diehl et al (01), Bakulev et al (03), . . .
The new situation

BaBar (09)

strong increase with $Q^2$

green: $\sqrt{2f_\pi} \left( Q^2 / 10 \text{ GeV} \right)^{0.25}$
(to guide the eyes)

AS: NLO corr. $< 0$

$a_2(1 \text{ GeV}) = 0.39$
$a_2(1 \text{ GeV}) = 0.39$, $a_4 = 0.24$

we have to worry:

a substantial increase of FF is difficult to accommodate in fixed order pQCD corr. due to $a_n (> 0)$ only shift NLO pred. upwards, don't change shape

(except unplausible solutions like $a_2 \approx 4$, $a_4 \approx -3.5$

in conflict with lattice QCD: $a_2(1 \text{ GeV}) = 0.252 \pm 0.143$  Braun et al (06))
Ways out

flat DA $\Phi \equiv 1$:

Polyakov(09) $Q^2 F_{\pi\gamma} \sim \int_0^1 dx \left[ x + M^2 / Q^2 \right]^{-1} = \ln \left[ Q^2 / M^2 + 1 \right]$

Radyushkin(09) with Gaussian w.f. $\sim k^2_\perp / x (1 - x)$

$$Q^2 F_{\pi\gamma} \sim \int_0^1 \frac{dx}{x} \left\{ 1 - \exp \left[ - \frac{x Q^2}{2 (1 - x) \sigma} \right] \right\} \rightarrow \ln \left[ Q^2 / 2 \sigma \right]$$

broad DA also found from AdS/QCD $\sim \sqrt{x (1 - x)}$ Brodsky-de Teramond(06)

but not by Mikhailov et al (10) QCD sum rules

Dorokhov(10) non-pert. effects (chiral quarks, instantons) $\rightarrow \ln \left[ Q^2 / M_q^2 \right]$

dispersion (LCSR) approach: Khodjamirian(09), Agaev et al (10)

corrections due to long-distance, hadron-like component of photon

quark-transverse momenta and resummation of Sudakov-like effects:
Li-Mishima(09), K(10)
The modified perturbative approach

LO \text{pQCD} + \text{quark transv. momenta} + \text{Sudakov suppr.} \, \text{Sterman et al (89,92)}

\implies \text{coll. fact. a. for } Q^2 \to \infty \quad (k_{\perp} \text{ fact. based on work by Collins-Soper})

Sudakov factor: higher order pQCD in NLL, resummed to all orders

\[ S \propto \ln \left( \ln \left( \frac{x_Q}{\sqrt{2} \Lambda_{QCD}} \right) \right) \ln \left( \frac{1}{b \Lambda_{QCD}} \right) + \text{NLL} + \text{RG}(\mu_F, \mu_R) \implies e^{-S} \quad \text{exponentiation in } b \text{ space} \]

\[ (q - \bar{q} \text{ separation}) \]

\[ \exp[-s(\xi_i, \bar{b}_i, Q)] \]

\[ \hat{\Psi}_\pi(x, b, \mu_F) = \frac{2\pi}{\sqrt{6}} f_\pi \Phi_\pi(x, \mu_F) \exp \left[ -\frac{x(1-x)b^2}{4\sigma^2_\pi} \right] \]

fact. scale \( \mu_F = 1/b \) \( (\mu_R = \max(\sqrt{xQ}, 1/b)) \)

\( b \) plays role of IR cut-off:

interface between soft gluons (in wave fct) and (semi-)hard gluons in Sudakov f. and \( T_H \)

\( \xi = x, 1 - x \)

\[ F_{\pi\gamma} = \int_0^1 dx \int_0^{1/\Lambda_{QCD}} db^2 \hat{\Psi}_\pi \left[ \frac{2}{\sqrt{3\pi}} K_0(\sqrt{xQ}b) \right] e^{-S} \]

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A remarkable property

with the Gegenbauer expansion

\[ Q^2 F_{\pi\gamma} = \sqrt{2} f_\pi C_0(Q^2, \mu_0, \sigma_\pi) \left[ 1 + \sum_{n=2,4,\ldots} a_n(\mu_0) \frac{C_n}{C_0} \right] \]

\( Q^2 \to \infty: \) \( C_0 \to 1 \) and \( C_n \to 0 \)
due to evolution

low \( Q^2: \) strong suppr. of higher terms

increasing \( Q^2: \) higher \( n \) terms become gradually more important

intrinsic \( k_\perp \) omitted

only lowest few Gegenbauer terms influence results on \( F_{\pi\gamma} \)

\( \Phi_{AS} \) suffices for low \( Q^2 \) (see fit to CLEO data)
Nature of corrections in m.p.a.

can be understood by replacing $e^{-S}$ by $\Theta(1/\Lambda_{QCD} - b)$ and wave fct $\propto \delta(k_{\perp}^2)$:

$$\int_{0}^{\Lambda_{QCD}^{-1}} bdbK_0(\sqrt{x}Qb) = \frac{1}{xQ^2} \left[ 1 - \frac{\sqrt{x}Q}{\Lambda_{QCD}} K_1\left( \frac{\sqrt{x}Q}{\Lambda_{QCD}} \right) \right]$$

(coll. fact: suppression of large $b$ only by pert. prop.)

$\sqrt{x}Q \gg \Lambda_{QCD}$: $K_1$ term exponentially suppressed

$\sqrt{x}Q \sim \Lambda_{QCD}$: $K_1$ term of $O(1)$

multiplication with distr. ampl. and integration over $x$

$$F_{\pi\gamma} \sim 1 + a_2 + a_4 + \ldots - 8\frac{\Lambda_{QCD}^2}{Q^2}(1 + 6a_2 + 15a_4 + \ldots) + O\left(\frac{\Lambda_{QCD}^4}{Q^4} \right)$$

Sudakov factor provides series of power suppressed terms which come from region of soft quark momenta $(x, 1-x \rightarrow 0)$ and grow with Gegenbauer index $n$

intrinsic transverse momentum: power suppressed terms from all $x$

which do not grow with $n$

Agaev et al (10): similar observation in the framework of LCSR

large non-pert. corrections suppress higher order Gegenbauer terms
Suppression of soft regions

$F_{\pi\gamma}(\mu_c)/F_{\pi\gamma}(0)$

accumulation profile at $Q^2 = 10 \text{GeV}^2$

$\mu_c$ cut-off parameter - any contribution to form factor is set to zero if $\mu_R < \mu_c$
present data allow to fix only one Gegenbauer coefficient
fit to CLEO and Babar data (initial scale \( \mu_0 = 1 \text{ GeV} \)):

\[
a_2 = 0.25 \quad \text{(fixed from lattice Braun(06))}
\]

\[
a_4 = 0.01 \pm 0.06 \quad \sigma_\pi = 0.40 \pm 0.06 \text{ GeV}^{-1} \quad \text{(trans. size parameter)}
\]

dashed line: \( \Phi_{AS} \) K.-Raulfs(95);
dotted line: coll. NLO result

fit to BaBar data
Extension to $\eta \gamma$ and $\eta' \gamma$

\[ P = \eta, \eta' : \quad F_{P \gamma} = F_{P \gamma}^\eta + F_{P \gamma}^{\eta'} \]

$F_{P \gamma}^{\eta'}$ as $F_{\pi \gamma}$ except of diff. wave fct. and charge factors

octet-singlet basis favored because of evolution behavior:

flavor-octet part as for pion

flavor-singlet part: due to mixing with the two-gluon Fock component

if intrinsic glue is small $a_n^g(\mu_0) \simeq 0$: evolution with $\gamma_n^{(+)} \simeq \gamma_n$

to NLO: also direct contribution from gg Fock state (K-Passek(03))

quark-flavor mixing scheme (Feldmann-K-Stech (98))

\[
F_{\eta \gamma} = \cos \theta_8 F_8^\eta - \sin \theta_1 F_1^\eta \\
F_{\eta' \gamma} = \sin \theta_8 F_8^\eta + \cos \theta_1 F_1^\eta \\
Q^2 F_8^\eta \rightarrow \sqrt{\frac{2}{3}} f_8 \\
Q^2 F_1^\eta \rightarrow \frac{4}{\sqrt{3}} f_1 \\
f_8 = 1.26 f_\pi \quad f_1 = 1.17 f_\pi \quad \theta_8 = -21.2^\circ \quad \theta_1 = -9.2^\circ
\]
Results for $\eta\gamma$ and $\eta'\gamma$

data BaBar (10)
dashed: $\Phi_{AS}$ Feldmann-K.(97)
dotted: asymptotic behavior
solid:

$\sigma_8 = 0.84 \pm 0.14$ GeV$^{-1}$; $a_2^8(\mu_0) = -0.06 \pm 0.06$

$\sigma_1 = 0.74 \pm 0.05$ GeV$^{-1}$; $a_2^1(\mu_0) = -0.07 \pm 0.04$
Extension to $\eta_c\gamma$

$T_H = \frac{2\sqrt{6}e_c^2}{xQ^2 + (1 + 4x(1 - x))m_c^2 + k^2_{\bot}}$

$\Phi_{\eta_c} = Nx(1-x)\exp\left[-\sigma_{\eta_c}^2 M_{\eta_c}^2 \frac{(x - 1/2)^2}{x(1-x)}\right]$

Wirbel-Stech-Bauer(85)

$(N = 8.849, \sigma_{\eta_c} = 0.44 \text{ GeV}^{-1})$

Sudakov unimportant

solid (dashed, dotted) line:

$m_c = 1.35(1.49, 1.21) \text{ GeV}$

in contrast to $\pi\gamma$ form factor:

behavior predicted Feldmann-K(97)

NLO corrections at low $Q^2$:

Shifman-Vysotskii(81), Braaten(83)
The time-like region

collinear factorization to LO accuracy: time-like = space-like (at $s = Q^2$)

within m.p.a. (as proposed by Gousset-Pire(94) for pion elm. FF)

$$1/(xQ^2 + k_\perp^2) \longrightarrow 1/(-xs + k_\perp^2 - \nu \epsilon) \quad \text{or} \quad K_0(\sqrt{xQb}) \longrightarrow \frac{\nu \pi}{2} H_0^{(1)}(\sqrt{xs}b)$$

analytic continuation of Sudakov f. not well understood (Magnea-Sterman(90)) (probably leads to an oscillating phase)

Gousset-Pire: take space-like Sudakov factor

estimate:

$s = 112$ GeV$^2$: $s|F_{\eta\gamma}| \simeq 0.17$ GeV $s|F_{\eta'\gamma}| \simeq 0.28$ GeV

BaBar(06): $s|F_{\eta\gamma}| = 0.229 \pm 0.031$ GeV $s|F_{\eta'\gamma}| = 0.251 \pm 0.021$ GeV
Consequences for the pion elm. form factor

perturbative contribution:
with distr. amplitude from best fit to $\pi\gamma$ form fator
with $\Phi_{AS}$ Jakob-K.(93)
Summary

even this simple excl. observable, believed to be understood very well, is subject to strong power suppressed corrections visible even at $Q^2$ as large as $40 \text{GeV}^2$

casts severe doubts on any attempt to explain other excl. observables within coll. factorization frame work (e.g. pion or proton FF)

quark-transverse momenta and Sudakov suppressions is one way to estimate power corrections; existing data on $P\gamma$ trans. form factor ($P = \pi, \eta, \eta', \eta_c$) can well be described within that approach. One Gegenbauer coeff. of DA can be determined from data

preserves standard asymptotics $Q^2 F_{\pi\gamma} \rightarrow \sqrt{2} f_\pi$

$\pi\gamma$ form factor should be remeasured by BELLE
Comparison with Li-Mishima (09)

Gaussian wf. with $\Phi_\pi \equiv 1$ accompanied by threshold factor resummed $\alpha_s \ln^2(x)$ and $\alpha_s \ln^2(1-x)$ arising from endpoint singularities which occur for flat DA

$\Rightarrow$ effective DA

$$\Phi^r = \frac{\Gamma(2 + 2r)}{\Gamma^2(1 + r)} \left[ x(1-x) \right]^r$$

limiting cases: $\Phi_{AS}$ and $\Phi_\pi \equiv 1$

remains power-like under evolution with $r \to r(\mu)$

Fit to BaBar data: $r(2 \text{ GeV}) = 0.59 \pm 0.06$

comparison of evolution behavior

LM claim:

$r \simeq 1$ at low $Q^2$ ($\Phi \simeq \Phi_{AS}$)

$r$ small at $Q^2 \simeq 40 \text{ GeV}^2$ ($\Phi \simeq 1$)