Single transverse spin asymmetry: progress and puzzles

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Outline

- Progress
  - Transverse momentum dependent approach
  - Collinear twist-3 factorization approach
  - Connection and evolutions

- Puzzles
  - Sivers functions extracted from SIDIS
  - ETQS twist-3 matrix element extracted from pp
  - “Sign mismatch”
  - Inconsistency?!?

- Summary
Experimental data on single transverse spin asymmetry

- Single transverse spin asymmetries (SSAs) have been observed in various experiments at different CM energies
  - PHENIX, BRAHMS, COMPASS, JLAB, too

\[ A_N = \frac{\Delta \sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})} \]

\[ \ell + p^\uparrow \rightarrow \ell' + \pi + X \]  
**HERMES**

\[ p + p^\uparrow \rightarrow \pi + X \]  
**STAR**

\[ p + p \rightarrow \Lambda^\uparrow + X \]  
**E704**

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SSA vanishes at leading twist in collinear factorization

- At leading twist formalism: partons are collinear

\[ \sigma(s_T) \sim \left| \begin{array}{c}
\begin{array}{c}
\text{(a)}
\end{array}
\end{array} \right| + \left| \begin{array}{c}
\begin{array}{c}
\text{(b)}
\end{array}
\end{array} \right| + \ldots \right|^2 \Rightarrow \Delta \sigma(s_T) \sim \text{Re}(a) \cdot \text{Im}(b) \]

- generate phase from loop diagrams, proportional to \( \alpha_s \)
- helicity is conserved for massless partons, helicity-flip is proportional to current quark mass \( m_q \)

Therefore we have

\[ A_N \sim \alpha_s \frac{m_q}{\sqrt{s}} \rightarrow 0 \]

- \( A_N \neq 0 \): result of parton’s transverse motion or correlations!
Two mechanisms to generate SSA in QCD

- **Collinear twist-3 factorization approach**
  \[ \sigma(p_h, s_\perp) \propto f_{a/A}^s(x) \otimes D_{h/c}(z) \otimes \hat{\sigma}_{\text{parton}} \]
  - Twist-3 three-parton correlation functions (PDFs)
  - Twist-3 three-parton fragmentation functions

- **TMD approach**: Transverse Momentum Dependent distributions probe the parton’s intrinsic transverse momentum
  \[ \sigma(p_h, s_\perp) \propto f_{a/A}(x, k_\perp) \otimes D_{h/c}(z, p_\perp) \otimes \hat{\sigma}_{\text{parton}} \]
  - Sivers function: in Parton Distribution Function (PDF)
    - Sivers 90
  - Collins function: in Fragmentation Function (FF)
    - Collins 93
Relation between twist-3 and TMD approaches

- They apply in different kinematic domain:
  - TMD approach: need TMD factorization, applies for the process with two observed momentum scales: DY at small $Q_T << Q$
    \[
    Q_1 \gg Q_2
    \]
    
    $Q_1$ necessary for pQCD factorization to have a chance
    
    $Q_2$ sensitive to parton’s transverse momentum
  - Collinear factorization approach: more relevant for single scale hard process inclusive pion production at high $p_T$ in pp collision

- They generate same results in the overlap region when they both apply:
  - Twist-3 three-parton correlation in distribution \[\mathcal{S} \text{ivers function}\]
    
    Ji, Qiu, Vogelsang, Yuan, 2006, ...
  - Twist-3 three-parton correlation in fragmentation \[\mathcal{C} \text{ollins function}\]
    
    Zhou, Yuan, 2009, Kang, Yuan, Zhou, 2010
A unified picture for Drell-Yan (leading $Q_T/Q$)
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TMD

$Q \gg Q_T \gtrsim \Lambda_{QCD}$

$\Lambda_{QCD} << Q_T << Q$
A unified picture for Drell-Yan (leading $Q_T/Q$)

TMD

$Q \gg Q_T \gtrsim \Lambda_{QCD}$

Collinear/twist-3

$Q, Q_T \gg \Lambda_{QCD}$
A unified picture for Drell-Yan (leading $Q_T/Q$)

Intermediate $Q_T$

$Q \gg Q_T \gg \Lambda_{QCD}$

TMD

$Q \gg Q_T \gtrsim \Lambda_{QCD}$

Collinear/twist-3

$Q, Q_T \gg \Lambda_{QCD}$

$\Lambda_{QCD}$  $<<$  $Q_T$  $<<$  $Q$  $Q_T$
TMD approach: Sivers function

- An asymmetric parton distribution in a polarized hadron (kt correlated with the spin of the hadron)

\[ f_{q/h\uparrow}(x, k_{\perp}, \vec{S}) \equiv f_{q/h}(x, k_{\perp}) + \frac{1}{2} \Delta^N f_{q/h\uparrow}(x, k_{\perp}) \vec{S} \cdot \hat{p} \times \hat{k}_{\perp} \]

Spin-dependent

Spin-independent
The existence of Sivers function

- **1990: Sivers function**
  - introduce $kt$ dependence of PDFs, generate the SSA through a correlation between the hadron spin and the parton $kt$

- **1993: Collins**
  - show Sivers function vanishes due to time-reversal invariance

- **2002: Brodsky, Hwang, Schmidt**
  - explicit model calculation show the existence of the Sivers function
  - the existence of Sivers function relies on the initial- and final-state interactions between the active parton and the remnant of the polarized hadron

- **2002: Ji, Yuan, Belitsky**
  - the initial- and final-state interaction presented by Brodsky, et.al. is equivalent to the color gauge links in the definition of the TMD distribution functions
  - since the details of the initial- and final-state interaction depend on the specific scattering process, the gauge link thus the Sivers function could be process-dependent
Sivers functions are process-dependent

- The existence of Sivers function relies on the initial- and final-state interaction, which could be process-dependent
  - SIDIS: final-state interaction
    - $\sigma \sim + + + \ldots$
    - $\approx$ PDFs with SIDIS gauge link
  - DY: initial-state interaction
    - $\sigma \sim + + + \ldots$
    - $\approx$ PDFs with DY gauge link
Non-universality of the Sivers function

- Different gauge link for gauge-invariant TMD distribution in SIDIS and DY

\[
f_{q/h^\uparrow}(x, k_\perp, \vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{i y^- - i k_\perp \cdot y_\perp} \langle p, \vec{S} | \overline{\psi}(0^-, 0^-) \rangle \text{Gauge link} \gamma^+ \frac{1}{2} \psi(y^-, y_\perp) | p, \vec{S} \rangle
\]

- SIDIS: \( \Phi_n^\dagger(\{+\infty, 0\}, 0_\perp) \Phi_{n_\perp}^\dagger(\{0_\perp\}, 0) \Phi_n(\{+\infty, y^-\}, y_\perp) \)
- DY: \( \Phi_n^\dagger(\{-\infty, 0\}, 0_\perp) \Phi_{n_\perp}^\dagger(-\infty, \{0_\perp\}) \Phi_n(\{-\infty, y^-\}, y_\perp) \)

**Wilson Loop** \( \sim \exp \left[ -i g \int \sigma^{\mu\nu} F_{\mu\nu} \right] \) Area is NOT zero

**Most critical test for TMD approach to SSA**

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Process-dependence: TMD vs collinear twist-3

- TMD approach: the process-dependence of the SSAs is completely absorbed into the process-dependence of the Sivers function
  - Sivers function is process-dependent

- Collinear twist-3 approach: the process-dependence of the SSAs is completely absorbed into the hard-part functions, thus the relevant collinear twist-3 correlation functions are universal
  - twist-3 correlation function is universal
One example to show process dependence in twist-3

- For the process $pp \rightarrow \pi + X$, one of the partonic channel: $qq' \rightarrow qq'$

\[
E_h \frac{d\Delta \sigma}{d^3 p_h} \propto e^{P_{T}\Delta n} \sum_{a,b,c} D_{h/c}(z_c) \otimes f_{b/B}(x_b) \otimes T_{a,F}(x, x) \otimes H_{ab \rightarrow c}^{Siv}
\]

- The effects of initial- and final-state interaction are absorbed to $H_{ab \rightarrow c}^{Siv}$
- ETQS function $T_{q,F}(x, x)$ is universal
- Since TMD and collinear twist-3 approaches provide a unified picture for the SSAs, ETQS function and Sivers function are closely related to each other
Both TMD and collinear twist-3 approaches are successful phenomenologically in their own kinematic domain

- **TMD approach:** semi-inclusive hadron production at low $p_T$ in deep inelastic scattering (SIDIS) - two scales $p_T$ and $Q$

\[ \ell + p^\uparrow \rightarrow \ell' + \pi(p_T) + X : p_T \ll Q \]

- **Collinear twist-3 approach:** single inclusive hadron production at high $p_T$ in $pp$ collisions - $p_T$ is the only large scale

\[ p + p^\uparrow \rightarrow \pi(p_T) + X : p_T \gg \Lambda_{QCD} \]
Current Sivers function from SIDIS

- Sivers and Collins can be separately extracted from SIDIS

\[ \Delta \sigma \propto A_{UT}^{\text{Collins}} \sin(\phi + \phi_S) + A_{UT}^{\text{Sivers}} \sin(\phi - \phi_S) \]
Includes HERMES Proton data and COMPASS Deuteron data

- Also seem to work fine for Collins effect
Twist-3 approach: initial success

- Describe both E704 and RHIC data simultaneously with a universal set of $T_{q,F}(x,x)$

$$T_{q,F}(x,x) = N_q x^{\alpha_q} (1 - x)^{\beta_q} \phi_q(x)$$

- Evolution has been derived for both twist-3 correlation function and fragmentation functions

Kouvaris, Qiu, Vogelsang, Yuan, 2006

What about the connections?

- Both seem to describe the data well (in their own kinematic region), but what about their connections?
  - At the operator level, ETQS function is related to the first kt-moment of the Sivers function
    \[ gT_{q,F}(x, x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f^{T}_{11}(x, k^2_\perp)|_{\text{SIDIS}} \]
  - Left hand side: from \( p + p^\uparrow \rightarrow \pi(p_T) + X \)
  - Right hand side: from \( \ell + p^\uparrow \rightarrow \ell' + \pi(p_T) + X \)
  - There is a “sign mismatch”

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Boer, Mulders, Pijlman, 2003
Ji, Qiu, Vogelsang, Yuan, 2006

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Parametrization of Sivers function

- To extract the Sivers function, the following parametrization is used
  - unpolarized PDFs: \( f_1^q(x, k_\perp) = f_1^q(x) g(k_\perp) \)
  - Sivers function: \( \Delta^N f_{q/h}^{\uparrow}(x, k_\perp) = 2N_q(x) f_1^q(x) h(k_\perp) g(k_\perp) \)

\( \mathcal{N}_q(x) \) is a fitted function

\[
g(k_\perp) = \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2 / \langle k_\perp^2 \rangle}
\]

old Sivers: \( h(k_\perp) = \frac{2k_\perp M_0}{k_\perp^2 + M_0^2} \)  
new Sivers: \( h(k_\perp) = \sqrt{2e} \frac{k_\perp}{M_1} e^{-k_\perp^2 / M_1^2} \)

- Using \( \Delta^N f_{q/A}^{\uparrow}(x, k_\perp) = -\frac{2k_\perp}{M} f_{1T}^{\perp q}(x, k_\perp^2) \), one can obtain

\[
g_{T_{q,F}}(x, x)|_{\text{old Sivers}} = 0.40 f_1^q(x) \mathcal{N}_q(x)|_{\text{old}}
\]

\[
g_{T_{q,F}}(x, x)|_{\text{new Sivers}} = 0.33 f_1^q(x) \mathcal{N}_q(x)|_{\text{new}}
\]
Indirectly obtained ETQS function

- The plot of indirectly obtained ETQS function $T_{q,F}(x, x)$

- ETQS function is positive for u-quark
- ETQS function is negative for d-quark
Directly obtained ETQS function

- ETQS function could be directly obtained from the global fitting of inclusive hadron production in hadronic collisions

- Directly obtained ETQS functions for both u and d quarks are opposite in sign to those indirectly obtained from the $kt$-moment of the quark Sivers function - “a sign mismatch”
Does this apparent sign “mismatch” indicate an inconsistency in our current QCD formalism for describing the SSAs?
Does this apparent sign “mismatch” indicate an inconsistency in our current QCD formalism for describing the SSAs?

The answer is possibly yes, but not necessarily.
Scenario I

- Let us assume the directly obtained ETQS function from inclusive hadron production reflects the true sign of these functions.

- In such case, to make everything consistent, we need to explain how the sign of the kt-moment of the Sivers function is different from the sign of the Sivers function.

\[
g_{T,q,F} (x,x) = - \int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^q (x, k^2_\perp) |_{\text{SIDIS}}
\]
What could go wrong - Scenario I

- To obtain ETQS function, one needs the full $k_t$-dependence of the quark Sivers function

$$g_{Tq,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1Tq}(x,k_\perp^2)|_{\text{SIDIS}}$$

- However, the Sivers functions are extracted mainly from HERMES data at rather low $Q^2 \sim 2.4 \text{ GeV}^2$, and TMD formalism is only valid for the kinematic region $k_t << Q$.
  - HERMES data only constrain the behavior (or the sign) of the Sivers function at very low $k_t \sim \Lambda_{\text{QCD}}$.
  - The current Torino group has assumed that a Gaussian-type $k_t$-dependence, thus low $k_t$ and high $k_t$ has the same sign, which needs not to be true.
Sivers function needs not to be positive

- Sivers function corresponds to the difference between two PDFs, and thus could be negative, or even have a node

\[ \Delta^N f_{q/h}^{\uparrow}(x, k_{\perp}) \vec{S} \cdot \hat{p} \times \hat{k}_{\perp} = f_{q/h}^{\uparrow}(x, k_{\perp}, \vec{S}) - f_{q/h}^{\uparrow}(x, k_{\perp}, -\vec{S}) \]

- Node situation: if \( f_{q/h}^{\uparrow}(x, k_{\perp}, \vec{S}) \) and \( f_{q/h}^{\uparrow}(x, k_{\perp}, -\vec{S}) \) are both Gaussian, but with different width, one could immediately find out the Sivers function could have the following kt-behavior

- The kt-moment of the Sivers function now will depends on the competition between low kt and high kt. It is possible that the high kt region wins, thus the sign of the kt-moment is actually opposite to the sign of the Sivers function itself at low kt.

\[ f_{q/h}^{\uparrow}(x, k_{\perp}^2) \]
Difference of distributions has a node is not new

- Current best fit for gluon helicity distribution function $\Delta g(x)$ seems to favor a $x$-distribution with a node.
Measure $kt$-dependence of Sivers function

- To test whether we have a sign change in the $kt$-distribution (or have a node), we need to expand the reach of $kt$ in the SIDIS
  - With a much broader $Q$ and energy coverage
  - A Electron Ion Collider might be ideal
Scenario II

- Let assume the indirectly obtained (from the kt-moment of the Sivers function) ETQS function reflects the true sign of these functions.

- In such case, to make everything consistent, we need to explain why we obtain a sign-mismatched ETQS function by analyzing the inclusive hadron data.

\[ g_{Tq,F}(x, x) = - \int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{+q}(x, k_\perp^2) |_{\text{SIDIS}} \]
Single inclusive hadron production is complicated

- There are two major contributions to the SSAs of the single inclusive hadron production in pp collisions
  - Twist-3 three-parton correlation functions (PDFs)
  - Twist-3 three-parton fragmentation functions

- So far the calculations related to three-parton correlation functions are more complete, while those related to the twist-3 fragmentation functions are available only very recently (not complete) Kang, Yuan, Zhou 2010
  - The current available global fittings are based on the assumptions that the SSAs mainly come from the twist-3 correlation functions, which might not be the case
  - If the contribution from the twist-3 fragmentation functions dominates, one might even reverse the sign of the ETQS function?

\[
A_N = A_N|^{PDFs} + A_N|^{FFs}
\]

If \(A_N|^{FFs} > A_N\), sign of \(A_N|^{PDFs}\) is opposite to \(A_N\)
Distinguish scenario I and II

- Scenario I and II are completely different from each other

To distinguish one from the other, in hadronic machine (like RHIC), one needs to find observables which are sensitive to twist-3 correlation function (not fragmentation function), such as single inclusive jet production, direct photon production.
Predictions for jet and direct photon

- at RHIC 200 GeV:

\[ A_N \]

\[ y = 3.3 \]

\[ \begin{array}{c}
\text{jets} \\
\text{new Sivers} \\
\text{old Sivers} \\
\text{directly obtained}
\end{array} \]
Summary

- Much progress has been made in understanding single transverse spin asymmetry.
  - Both TMD and collinear twist-3 approaches seem to be successful phenomenologically.

- Their connection seems to have a puzzle:
  - Directly obtained ETQS functions are opposite in sign to those indirectly obtained from the $k_t$-moment of the quark Sivers function.
  - This sign mismatch does not necessarily lead to any inconsistency in our current formalism for describing the SSAs.
  - Future experiments could help resolve different scenarios, which will help understand the SSAs and hadron structure better.
Backup
Use the ETQS function derived from the old Sivers and new Sivers functions, one could make predictions for the single inclusive hadron production. We find they are opposite to the experimental observations.
Definition of $A_N$ in experiments