# Consistency constraints on pion momentum distributions in the nucleon

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# Introduction

- Purpose:
  - Investigate effects of the nucleon's pion cloud on nucleon structure
    - excess of $\bar{d}$ over $\bar{u}$ in the proton sea
  - Seek consistent treatment of pion cloud within chiral field theory
- Method: calculation of vertex corrections to electromagnetic coupling of a photon to a nucleon in pseudovector, as compared to pseudoscalar, theory

- We also look at the effect of phenomenological pion-nucleon form factors

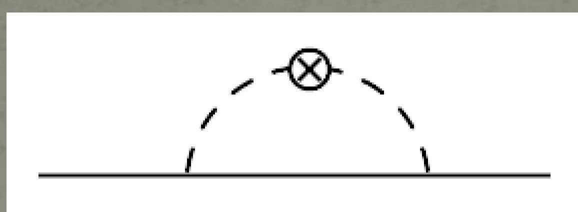
# Background

- Vertex renormalization constant $Z_1$ is defined:

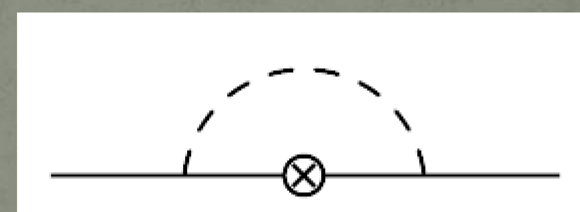
$$(Z_1^{-1} - 1)\,\bar{u}(p)\gamma^\mu u(p) = \bar{u}(p)\Lambda^\mu u(p)$$

where $\Lambda_i^\mu$ corresponds to a particular photon coupling:

$$\Lambda_\pi^\mu$$



$$\Lambda_N^\mu$$



- For small $(1-Z_1)$,

$$Z_1^{-1} - 1 \approx 1 - Z_1$$

# Pion LC Momentum Distributions

- It is convenient to define light-cone (LC) distribution function f(y):

$$(1 - Z_1^i) = \int dy\, f(y)_i$$

where $$y = \frac{k^+}{p+}$$

- Also note that if *y* is the pion momentum fraction, *y'* is the nucleon momentum fraction:

$$y + y' = 1 \Rightarrow y' = 1 - y$$

- We would like to interpret f(y) as a probability distribution – it should be positive to have physical meaning.

# Pseudoscalar Theory

- At lowest order in the pion field,

$$\mathcal{L}_{\pi N}^{PS} = -g_{\pi NN}\bar{\psi}_N i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}\psi_N$$

- Frequently used due to relative simplicity
- Not invariant under chiral transformation (without introduction of scalar field)
- Gauge invariance requires that $1 - Z_1^{\pi} = 1 - Z_1^{N}$
- After k_ integration, we have

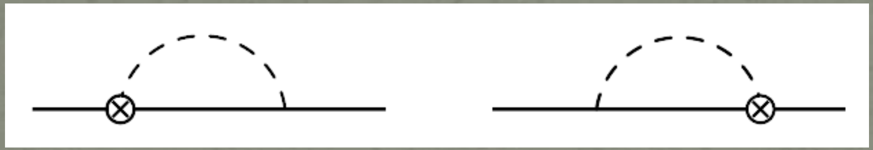
$$f(y)_N = f(y)_\pi = \frac{g_A^2 M^2}{(4\pi f_\pi)^2}\int dk_\perp^2 \frac{y(k_\perp^2 + y^2M^2)}{[k_\perp^2 + y^2M^2 + (1-y)m_\pi^2]^2}$$

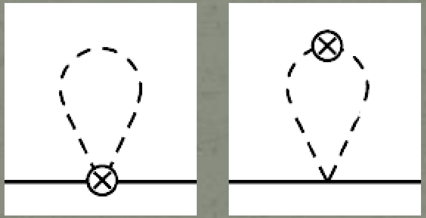
# Pseudovector Theory

- At lowest order in the pion field,

$$\mathcal{L}_{\pi N}^{PV} = \frac{g_A}{2f_\pi}\bar{\psi}_N\gamma^\mu\gamma_5\,\boldsymbol{\tau}\cdot\partial\boldsymbol{\pi}\psi_N - \frac{1}{(2f_\pi)^2}\bar{\psi}_N\gamma^\mu\,\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\psi_N$$

- Explicitly respects chiral symmetry.
- Introduces three new contributions: Weinberg-Tomozawa and two "tadpole" contributions:





- Gauge invariance must now be satisfied by finding an equality using all five contributions.

# Pseudovector Theory

- After k_ integration, we have

$$f(y)_N = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \left\{ \boxed{\frac{y(k_\perp^2 + y^2 M^2)}{[k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2]^2}} \boxed{- \frac{y}{k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2}} \right.$$
$$\left. \boxed{- \frac{1}{4M^2} \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y)} \right\}$$

$$f(y)_\pi = \boxed{\frac{g_A^2 \cancel{M^2}}{\cancel{(4\pi f_\pi)^2}}} \int dk_\perp^2 \left\{ \boxed{\frac{y(k_\perp^2 + y^2 M^2)}{[k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2]^2}} + \boxed{\boxed{\frac{1}{\cancel{4M^2}}} \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y)} \right\}$$

$$f(y)_{WT} = -\frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \left\{ \boxed{\frac{y}{k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2}} + \boxed{\frac{1}{2M^2} \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y)} \right\}$$

$$f(y)_{N_{tad}} = -f(y)_{\pi_{tad}} = \boxed{\frac{1}{2\cancel{(4\pi f_\pi)^2}}} \int dk_\perp^2 \boxed{\log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y)}$$

# Regularization

- Momentum integrals are formally divergent
  - Requires introduction of ultraviolet regularization (form factors, dimensional regularization, etc)
  - We chose to use form factors to be consistent with the existing literature
- $k_-$ integration used a cutoff $\mu$, which we kept constant at 1 GeV
- $k_+$ integration uses a cutoff $\Lambda$, along with a variety of commonly used form factors

# Regularization

- Cutoff parameter, $\Lambda$, has different effects with each form factor
  - Found "soft" $\Lambda$ at $(1\text{-}Z_1^{\pi}) = 0.25$ and "hard" $\Lambda$ at $(1\text{-}Z_1^{\pi}) = 0.50$ for each form factor to ensure meaningful comparisons
  - Calculated f(y) and $(1\text{-}Z_1)$ contributions at those values of $\Lambda$ using pseudovector theory

| | $k_\perp$-dep sc | k-dip (N) | k-dip ($\pi$) | $s_{\pi N}$-dep sc | $s_{\pi N}$-dip | $s_{\pi N}$- exp |
|---|---|---|---|---|---|---|
| $(1-Z_1^\pi) = 0.25$ | 693 | 1981 | 774 | 1806 | 1685 | 1665 |
| $(1-Z_1^\pi) = 0.50$ | 1063 | 2443 | 1315 | 2441 | 2964 | 2615 |

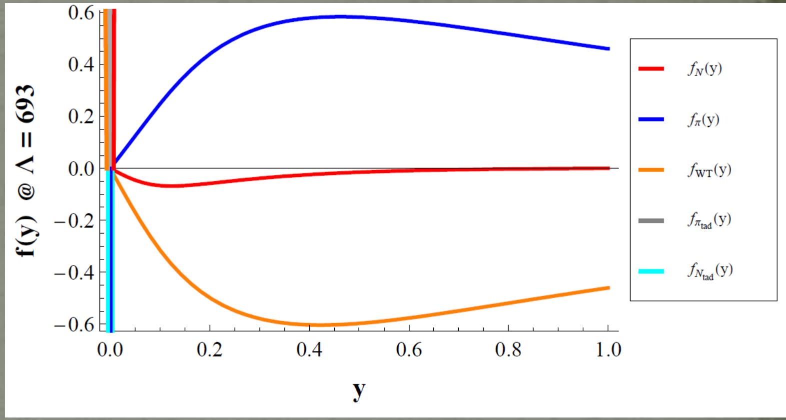
# Form Factors / Results

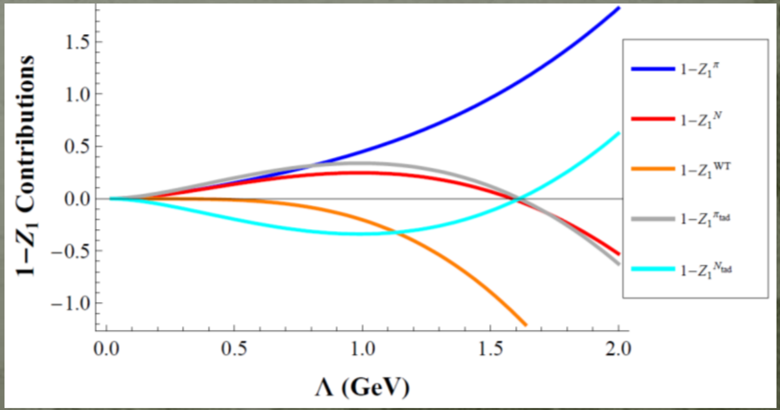
*$k_\perp$ -dependent sharp cutoff*

$$F_{k_\perp} = \begin{cases} 1 \text{ if } k_\perp^2 \leq \Lambda^2 \\ 0 \text{ if } k_\perp^2 > \Lambda^2 \end{cases}$$

- Simplest form factor
- Each contribution to f(y) has spike at $y = 0$ due to δ(y)
- δ(y) terms have significant effect on $(1\text{-}Z_1)$ contributions
- Signs of f(y) suggest grouping:

$$f_{\pi+\pi_{tad}}(y) = f_{N\text{-}WT\text{-}N_{tad}}(y)$$



# Form Factors / Results

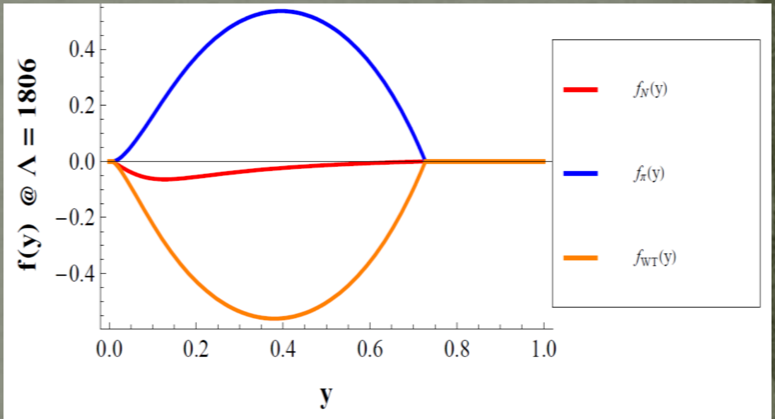
*s –dependent sharp cutoff*

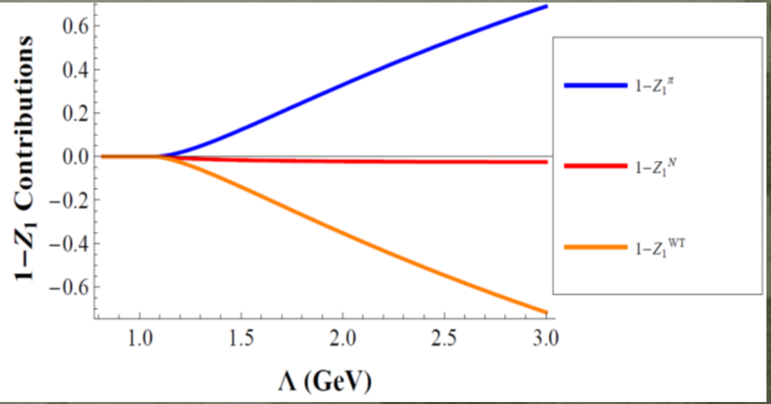
$$s_{\pi N} = \frac{k_\perp^2 + m_\pi^2}{y} + \frac{k_\perp^2 + M^2}{1-y}$$

$$F_{s_{\pi N}} = \begin{cases} 1 & \text{if } s_{\pi N} \le \Lambda^2 \\ 0 & \text{if } s_{\pi N} > \Lambda^2 \end{cases}$$

- $s_{\pi N}$ is a natural choice in infinite momentum frame (IMF)
- Suppresses δ(y) terms:
  - absence of tadpole contributions
  - very different (1-$Z_1$) contributions
- Signs of f(y) suggest grouping:

$$f_\pi(y) = f_{N\text{-}WT}(y)$$



# Form Factors / Results

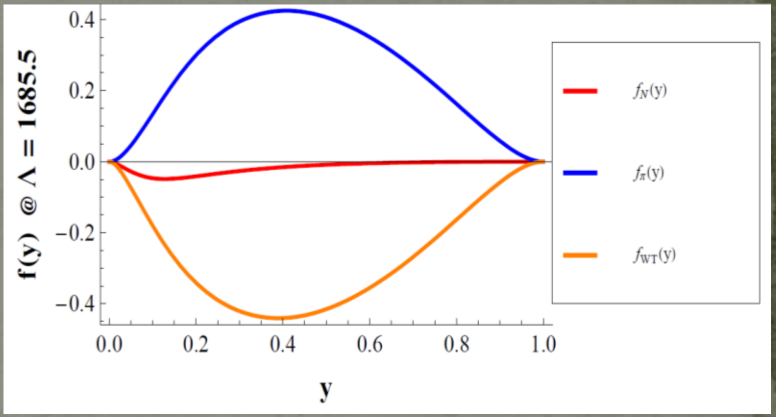
<u>*s –dependent sharp cutoff*</u>

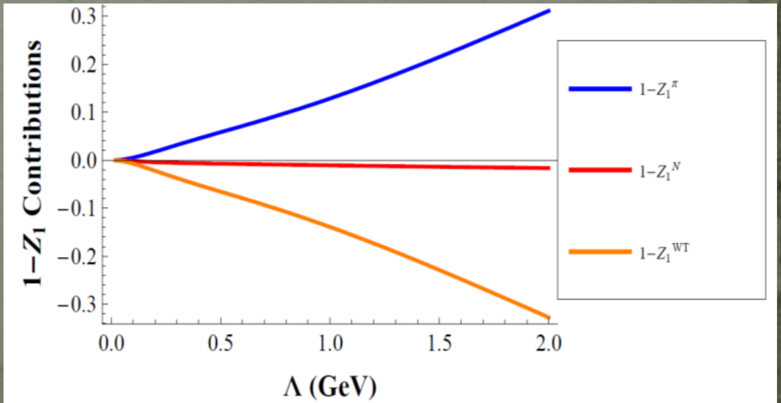
$$s_{\pi N} = \frac{k_\perp^2 + m_\pi^2}{y} + \frac{k_\perp^2 + M^2}{1-y}$$

$$F_{s-dip} = \left(\frac{\Lambda^2 + M^2}{\Lambda^2 + s_{\pi N}}\right)^2$$

- $s_{\pi N}$ is a natural choice in infinite momentum frame (IMF)
- Suppresses δ(y) terms:
  - absence of tadpole contributions
  - very different $(1\text{-}Z_1)$ contributions
- Signs of f(y) suggest grouping:

$$f_\pi(y) = f_{N\text{-}WT}(y)$$



# Form Factors / Results

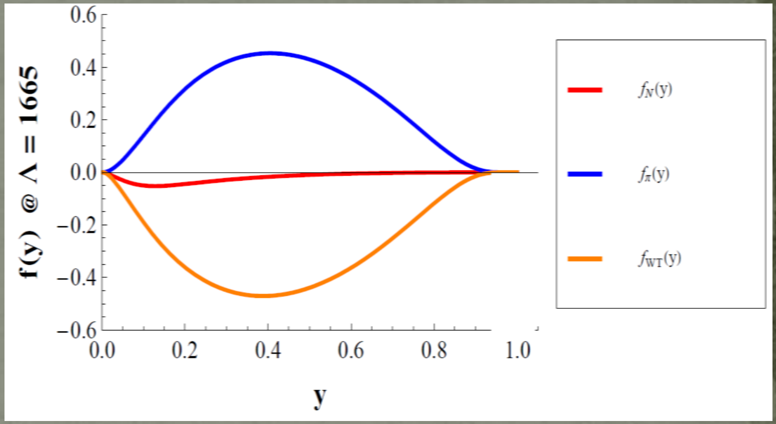
<u>*s –dependent sharp cutoff*</u>

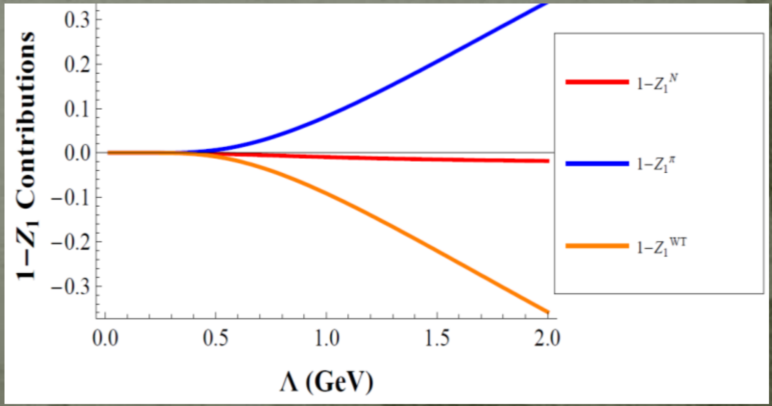
$$s_{\pi N} = \frac{k_\perp^2 + m_\pi^2}{y} + \frac{k_\perp^2 + M^2}{1-y}$$

$$F_{exp} = e^{\frac{M^2 - s_{\pi N}}{\Lambda^2}}$$

- $s_{\pi N}$ is a natural choice in infinite momentum frame (IMF)
- Suppresses δ(y) terms:
  - absence of tadpole contributions
  - very different $(1\text{-}Z_1)$ contributions
- Signs of f(y) suggest grouping:

$$f_\pi(y) = f_{N\text{-}WT}(y)$$



# Form Factors / Results

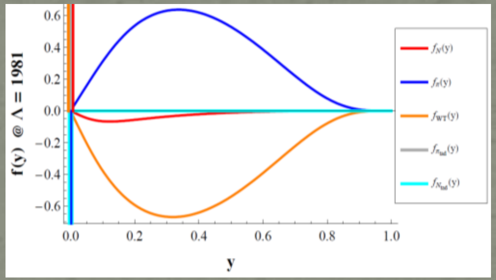
## $\vec{k}$-*dependent dipole*

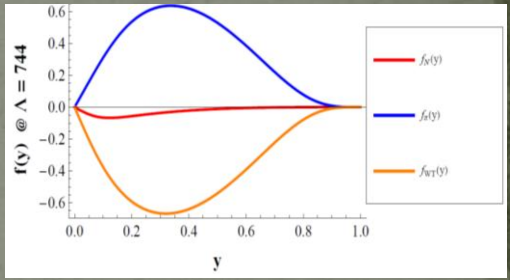
$$F_{k-dip} = \left(\frac{\Lambda^2}{\vec{k}^2 + \Lambda^2}\right)^2$$

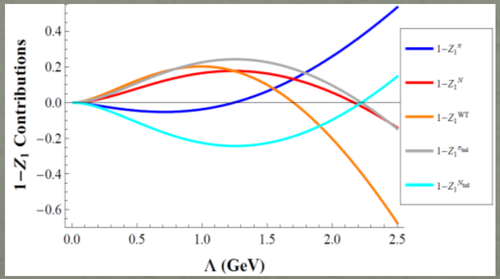
- $\vec{k}$ takes a different form depending on whether the nucleon or pion pole is taken in k_ integration
- Pion on mass shell: δ(y) terms suppressed, form similar to *s*-dep form factors
- Nucleon on mass shell: δ(y) terms *not* suppressed, form similar to $k_\perp$-dep form factor

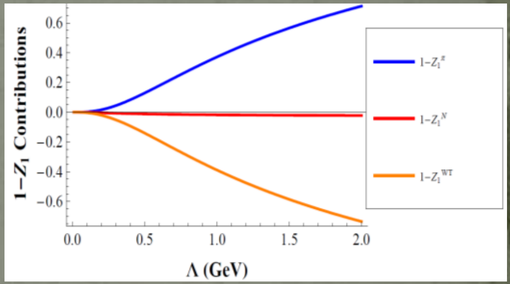
$$\vec{k}^2 = k_\perp^2 + \frac{1}{4}\left(\frac{k_\perp^2 + M^2}{M(1-y)} - M(1-y)\right)^2$$

$$\vec{k}^2 = k_\perp^2 + \frac{1}{4}\left(yM - \frac{k_\perp^2 + m_\pi^2}{yM}\right)^2$$



# Form Factors / Results

$$F_{t_N} = \left( \frac{\Lambda^2 - M^2}{\Lambda^2 - t_N(y')} \right)^2$$

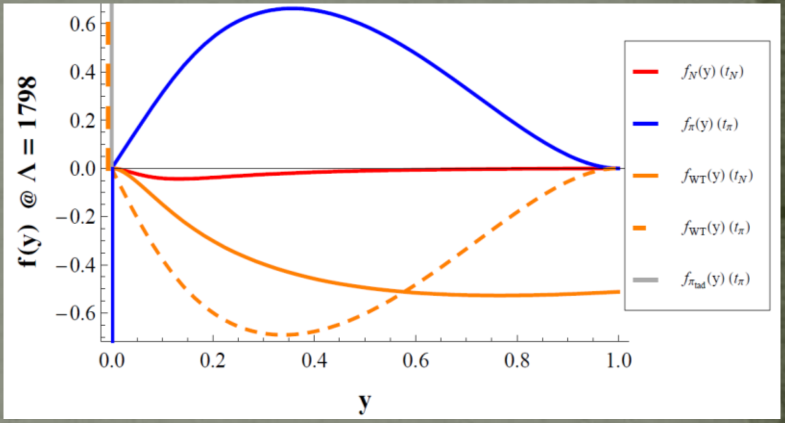
$$F_{t_\pi} = \left( \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - t_\pi(y)} \right)^2$$

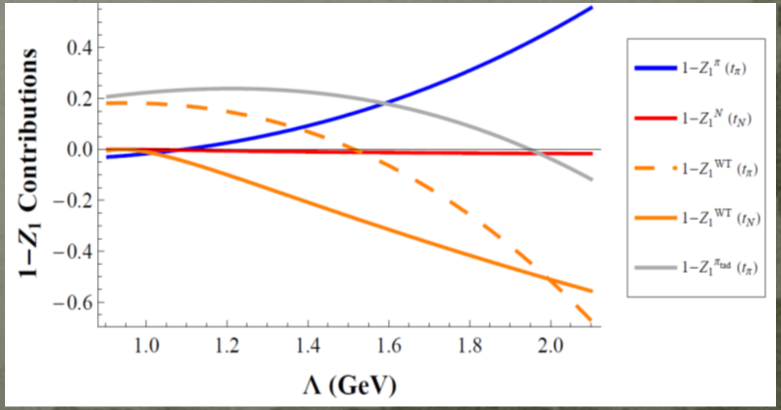
*t-dependent covariant dipole*

$$t_N(y') = \frac{-k_\perp^2}{1-y'} - \frac{(m_\pi^2-(1-y')M^2)y'}{1-y'}$$

$$t_\pi(y) = \frac{-k_\perp^2}{1-y} - \frac{M^2y^2}{1-y}$$

- Uses different form factor for pion and nucleon contributions
- It is unclear which form factor to use for the WT and tadpole contributions

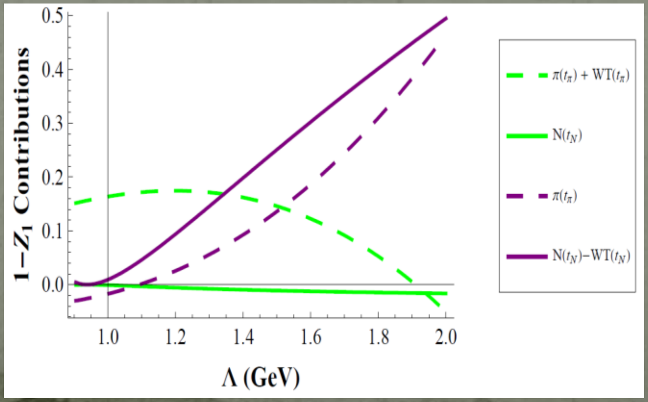


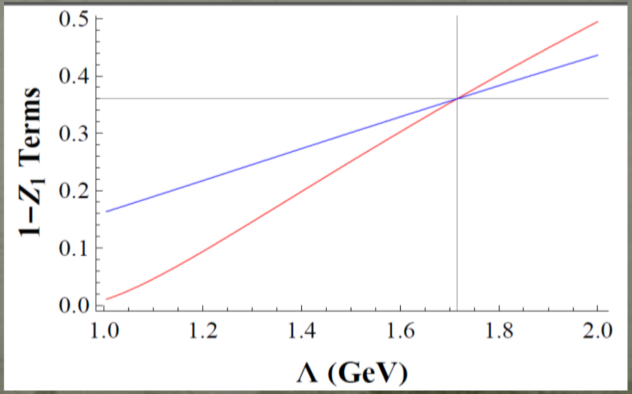



# Form Factors / Results

<u>*t-dependent covariant dipole*</u>

- The *t*-dependent form factor violates charge conservation at all but a single value of Λ, even when using the pseudoscalar formulation.
- Multiple attempts were made to find a form factor for the WT contribution that would restore gauge invariance without success



# Conclusions

→ We computed pion LC distributions in the nucleon for the first time in pseudovector theory.

→ Constraints of physical/probabilistic interpretations and gauge invariance requires grouping of contributions:

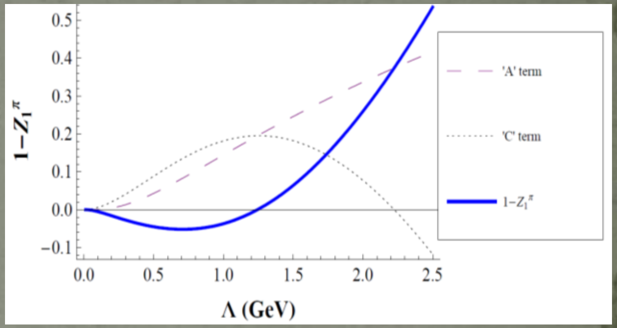
$$f_{\pi+\pi_{tad}}(y) = f_{N\text{-}WT\text{-}N_{tad}}(y)$$

→ *t* – dependent covariant form factor is irreconcilable with gauge invariance unless

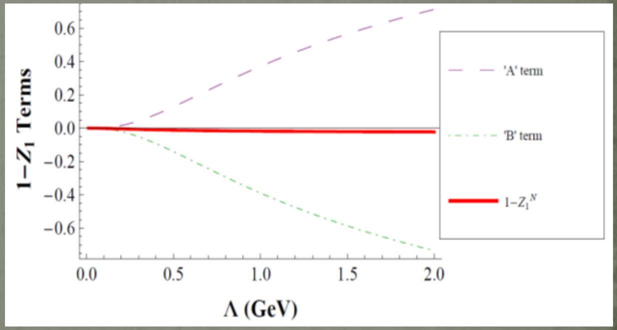
- a special form factor for the WT contribution can be devised
- the unique $\Lambda$ that preserves gauge invariance is strictly maintained.

# Conclusions

- Some choices of form factor ($s_{\pi N}$ -dep, $t_N$ -dep, $\vec{k}$ -dep pion pole) suppress end-point contributions ($y = 0$, $y = 1$)
  - Suppression of the $\delta(y)$ terms leads to significant difference in the resulting f(y) and ($1\text{-}Z_1$) values
  - Regardless of whether the $\delta(y)$ terms are suppressed, the additional terms from the pseudovector formulation significantly alter the shape of the functions.




*π contribution, k-dep dipole, N on mass shell*




*N contribution, k-dep dipole, π on mass shell*

# Future Work

- Repeat calculations for pion-Δ contributions
- Compute moments of f(y) and investigate leading non-analytic (LNA) behavior
- Compare results to data on $\bar{d}$-$\bar{u}$
- Add spin dependence

I will be able to continue this research at NC State during the fall 2012 semester under the mentorship of Dr. Chueng R. Ji

# Questions?

Jefferson Lab
Thomas Jefferson National Accelerator Facility

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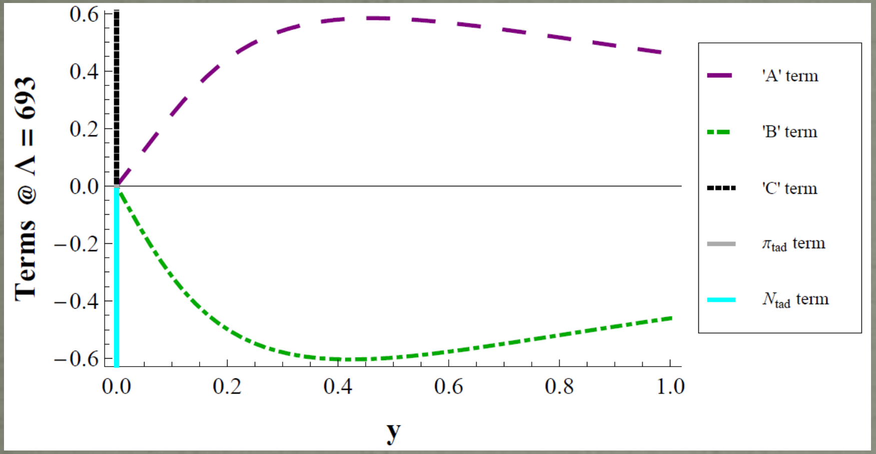
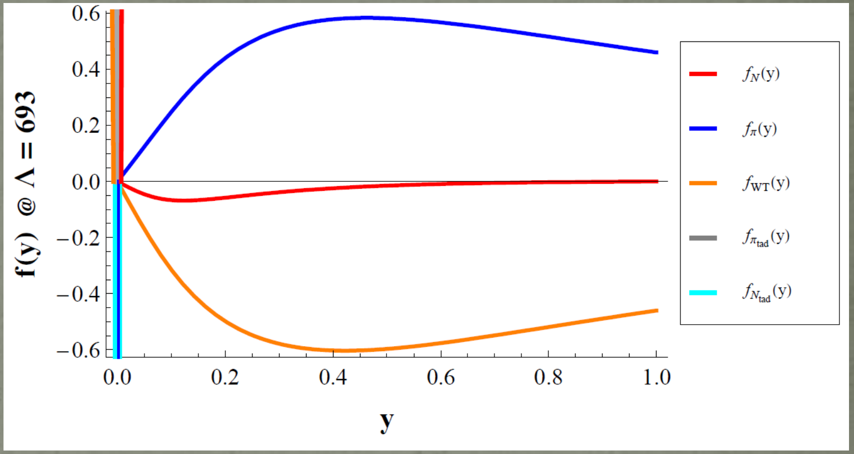
# Additional information




N = A+B+C

π = A-C

WT = B + 2C

Ntad = -πtad = -(2/$g_A^2$) * C ≈ -1.25 C

# Additional information

The interaction of pions and nucleons with the electromagnetic field is introduced by minimal substituion

$$\partial_\mu \to \partial_\mu + ieA_\mu$$

giving

$$\begin{aligned}\mathcal{L}^{PV}_{\gamma\pi N} &= -\bar{\psi}_N \gamma^\mu \hat{Q}_N \psi_N\, A_\mu + i(\partial^\mu(\boldsymbol{\pi}) \cdot (\hat{Q}_\pi \boldsymbol{\pi})\, A_\mu \\ &+ \frac{g_A}{2f_\pi} \bar{\psi}_N \gamma^\mu \gamma_5\, \boldsymbol{\tau} \cdot \hat{Q}_\pi \boldsymbol{\pi} \psi_N\, A_\mu - \frac{i}{(2f_\pi)^2}\, \bar{\psi}_N \gamma^\mu\, \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \hat{Q}_\pi \boldsymbol{\pi}) \psi_N\, A_\mu\end{aligned}$$

$g_A$ = 1.267 / nucleon axial vector charge

$f_\pi$ = 93 MeV / pion decay constant

$g_{\pi NN}$ = πNN coupling constant

$$\frac{g_A}{f_\pi} = \frac{g_{\pi NN}}{M}$$

(Goldberger-Treiman relation)