Radiative Corrections in DVCS and SIDIS

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Outline

- Radiative Corrections to BH with Leading Accuracy
  - Background, Justification, and State-of-Art;
  - Details of Analytic Calculation;
  - Numeric Results.
- Radiative Corrections to BH with Next-to-Leading Accuracy
  - Details of Analytic Calculation
    - Loop Contribution
    - Two-gamma contribution
    - Infrared divergence and the Total cross section
  - Monte Carlo Generator and Other Further Steps.
- Radiative Corrections to DVCS
  - Contributions to RC and Covariant Hadronic Tensor
  - Calculation in with Leading Accuracy and Numerical Results
- Radiative Corrections to SIDIS
  - Semi-inclusive and Exclusive Contributions to RC
  - Covariant Hadronic Tensor
  - Kinematic region of SFs

Part I:

Radiative Corrections to BH with Leading Accuracy

Our Aims and the State-of-Art in RC to DVCS

Our Aims follow from needs of current experiments at JLab: we would like to have

- A code capable of estimating RC to all contributions to the DVCS cross sections (e.g., BH, interference term);
- This code has to be constructed on the basis of clear and transparent results of calculation;
- This code has to be quite flexible to be generalized and quickly adopted for distinct experimental needs (e.g., to simulate radiative events or calculate RC to $< \sin\phi >$);
- All steps of the calculation have to be clear allowing to analyze assumptions and control accuracy.
- Avoiding assumptions accuracy of which is difficult to control (e.g., soft-photon approximation)

The message from literature review:

There are no available calculations that can be directly applied for our Aims.

Available information about RC to DVCS

One-loop correction and soft photon emission, Vanderhaeghen et al. PR C62(2000)025501

- *Detailed consideration of one-loop correction. Using dimensional regularization both for infrared and ultraviolet divergences. Reduction of the correction to the Feynman-parameter integrals and their numerical evaluation, i.e., not fully analytical calculation of loop effects*
- *Approximated evaluation of the radiative tail corresponding to photon emission processes, where the photon energy is very small compared to the lepton momenta.*

One-loop correction and hard photon emission in leading approximation, Butev, Kuraev, Tomasi-Gustafsson, PR C77, 055206 (2008)

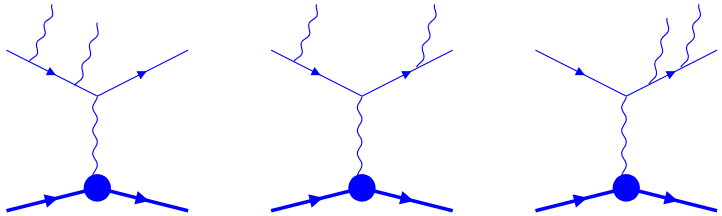
- *The process $e^- \mu^+$ is chosen as a model process*
- *The electron structure functions are used for calculation of the hard photon emission in leading approximation.*
- *Focus on the correction to the helicity-odd part of cross section, i.e., the interference between BH DVCS amplitudes.*
- *No explicit formulas are done for the kinematics we deal with (they integrated over the energy fraction of scattered electron.).*

The calculations of the next order corrections to the radiative tails from elastic peaks, e.g., Akhundov, Bardin, and Shumeiko, Yad. Fiz. 44, 1517 1986 (Sov. J. Nucl. Phys. 44, 988 1986), Akushevich, Kuraev, Shaikhatdenov Phys.Rev.D62:053016,2000.

- *Only results for the cross section integrated over t and ϕ.*

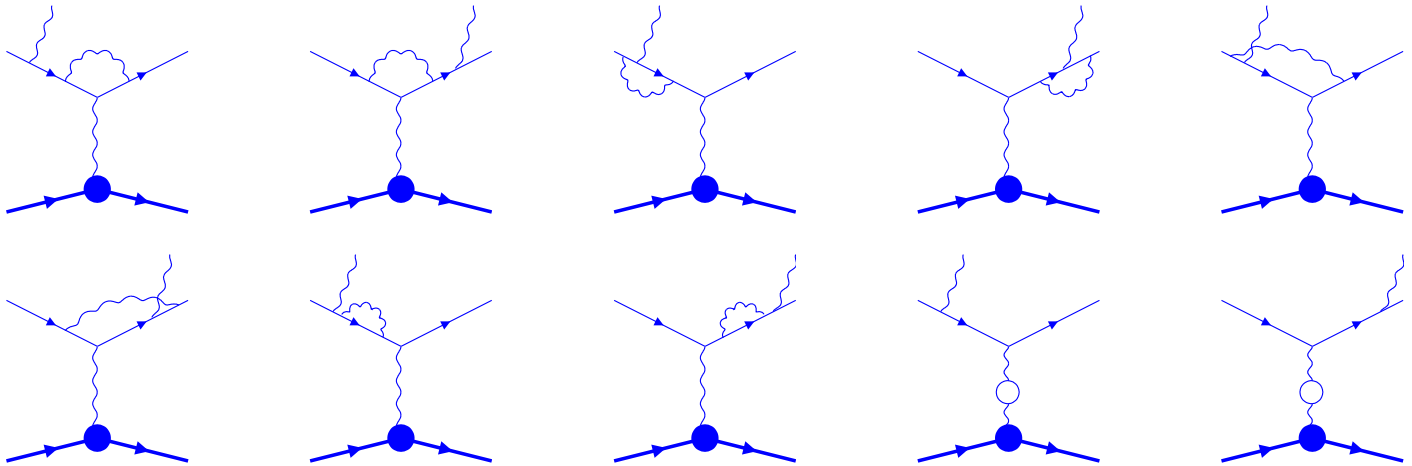
Steps in RC calculation to BH process

- Cross section of Two-Photon Emission process



  - Matrix element squared.
  - Phase space parametrization and integration over part of kinematical variables of an additional photon

- Loops and infrared divergence



Two-photon emission: Matrix elements

Six matrix elements of the process are denoted $\mathcal{M}_{1-6} = e^4 t^{-1} J^h_\mu J_{1-6,\mu}$, where

$$J_{1\mu} = \bar{u}_2 \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa} + m}{-2\kappa k_1 + V^2} \hat{\epsilon}_2 \frac{\hat{k}_1 - \hat{\kappa}_1 + m}{-2k_1\kappa_1} \hat{\epsilon}_1 u_1 \qquad J_{2\mu} = \bar{u}_2 \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa} + m}{-2\kappa k_1 + V^2} \hat{\epsilon}_1 \frac{\hat{k}_1 - \hat{\kappa}_2 + m}{-2k_1\kappa_2} \hat{\epsilon}_2 u_1$$

$$J_{3\mu} = \bar{u}_2 \hat{\epsilon}_2 \frac{\hat{k}_2 + \hat{\kappa}_2 + m}{2k_2\kappa_2} \hat{\epsilon}_1 \frac{\hat{k}_2 + \hat{\kappa} + m}{2\kappa k_2 + V^2} \gamma_\mu u_1 \qquad J_{4\mu} = \bar{u}_2 \hat{\epsilon}_1 \frac{\hat{k}_2 + \hat{\kappa}_1 + m}{2k_2\kappa_1} \hat{\epsilon}_2 \frac{\hat{k}_2 + \hat{\kappa} + m}{2\kappa k_2 + V^2} \gamma_\mu u_1$$

$$J_{5\mu} = \bar{u}_2 \hat{\epsilon}_1 \frac{\hat{k}_2 + \hat{\kappa}_1 + m}{2k_2\kappa_1} \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa}_2 + m}{-2k_1\kappa_2} \hat{\epsilon}_2 u_1 \qquad J_{6\mu} = \bar{u}_2 \hat{\epsilon}_2 \frac{\hat{k}_2 + \hat{\kappa}_2 + m}{2k_2\kappa_2} \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa}_1 + m}{-2k_1\kappa_1} \hat{\epsilon}_1 u_1$$

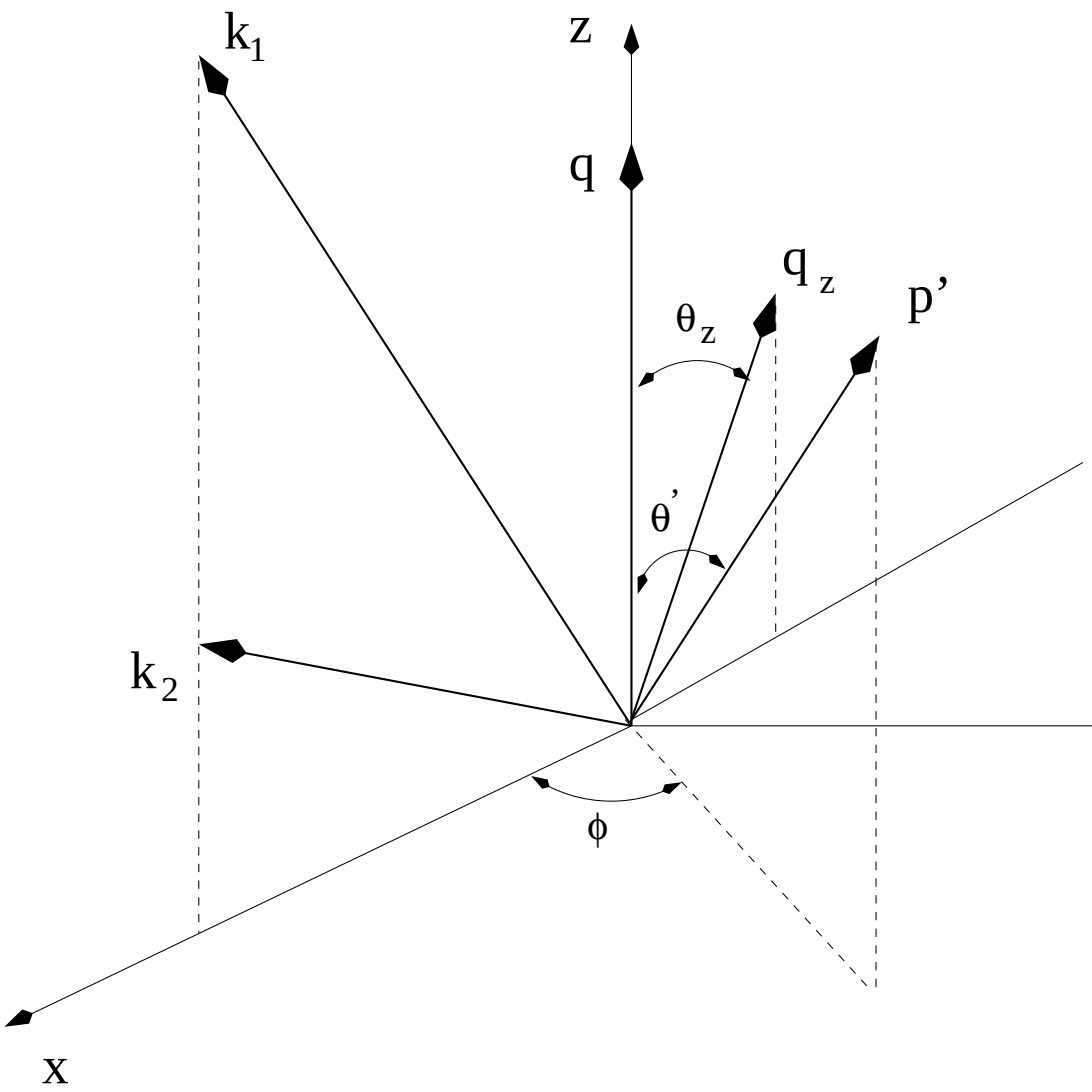
where $V^2 = \kappa^2 = (\kappa_1 + \kappa_2)^2$.

For s-peak (p-peak) the additional unobserved photon is emitted in the direction of the initial (final) lepton. Therefore,

$$\Big(\sum_{i=1}^{6} \mathcal{M}_i\Big)^2 = \mathcal{M}_{1s}^2 + \mathcal{M}_{1p}^2 + \mathcal{M}_{2s}^2 + \mathcal{M}_{2p}^2$$

where indices correspond to the unobserved photon, e.g., $1s$ means that the photon with momentum κ_1 is unobserved and in the s-peak.

Definitions of vectors and angles in the Lab. frame



The direction of q_z defines new polar ($\bar{\theta}$) and azimuthal ($\bar{\phi}$) angles of the final proton (so-called shifted kinematics):

$$\cos\bar{\theta} = \cos\theta' \cos\theta_z - \sin\theta' \sin\theta_z \cos\phi$$

$$\cos\theta' = \frac{A\cos\theta_z + \sqrt{\mathcal{D}_0}\sin\theta_z\cos\phi}{\cos^2\theta_z + \sin^2\theta_z\cos^2\phi}$$

$$\sin\theta' = \frac{\cos\theta_z\sqrt{\mathcal{D}_0} - A\sin\theta_z\cos\phi}{\cos^2\theta_z + \sin^2\theta_z\cos^2\phi}$$

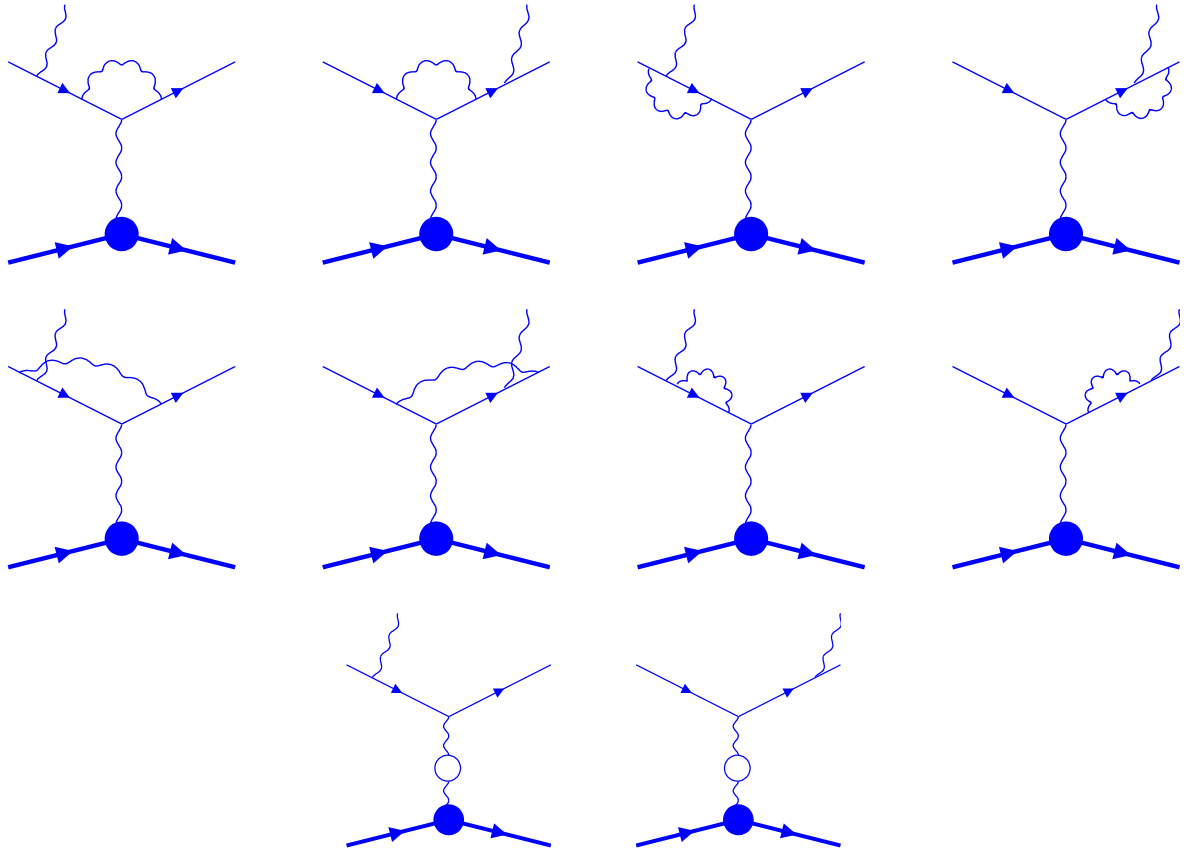
$$\mathcal{D}_0 = \cos^2\theta_z + \sin^2\theta_z\cos^2\phi - A^2$$

$$\sin\bar{\phi} = \frac{\sin\theta'\sin\phi}{\sin\bar{\theta}}$$

The “shifted” kinematics is completely calculatable (Akushevich, Ilyichev, PR D85 (2012) 053008)

Loop diagrams

Feynman graphs of one-loop effects for the BH cross section



$$\sigma_V = \frac{\alpha}{\pi}\left(\log\frac{4M^2\omega_{min}^2}{SX} + \frac{3}{2}\right)L\sigma_{BH} = -\frac{\alpha L}{2\pi}\sigma_{BH}\left(\int\limits_0^{1-\Delta_1} dz_1 \frac{1+z_1^2}{1-z_1} + \int\limits_0^{1-\Delta_2} dz_2 \frac{1+z_2^2}{1-z_2}\right)$$

The lowest order RC

$$\sigma_{obs}(S,x,Q^2,t,\phi) = (1+2\Pi(t))\sigma_{BH}(S,x,Q^2,t,\phi) + \frac{\alpha}{2\pi}L\Bigg[$$

$$\int_0^1 dz_1 \left(\frac{1+z_1^2}{1-z_1}\right)\left(\frac{\sin\theta_s'}{\mathcal{D}_{0s}^{1/2}}\theta(z-z_1^m)\left(\frac{x_s}{x}\right)^2\sigma_{BH}(z_1S,x_s,z_1Q^2,t,\bar{\phi}_s) - \sigma_{BH}(S,x,Q^2,t,\phi)\right)$$

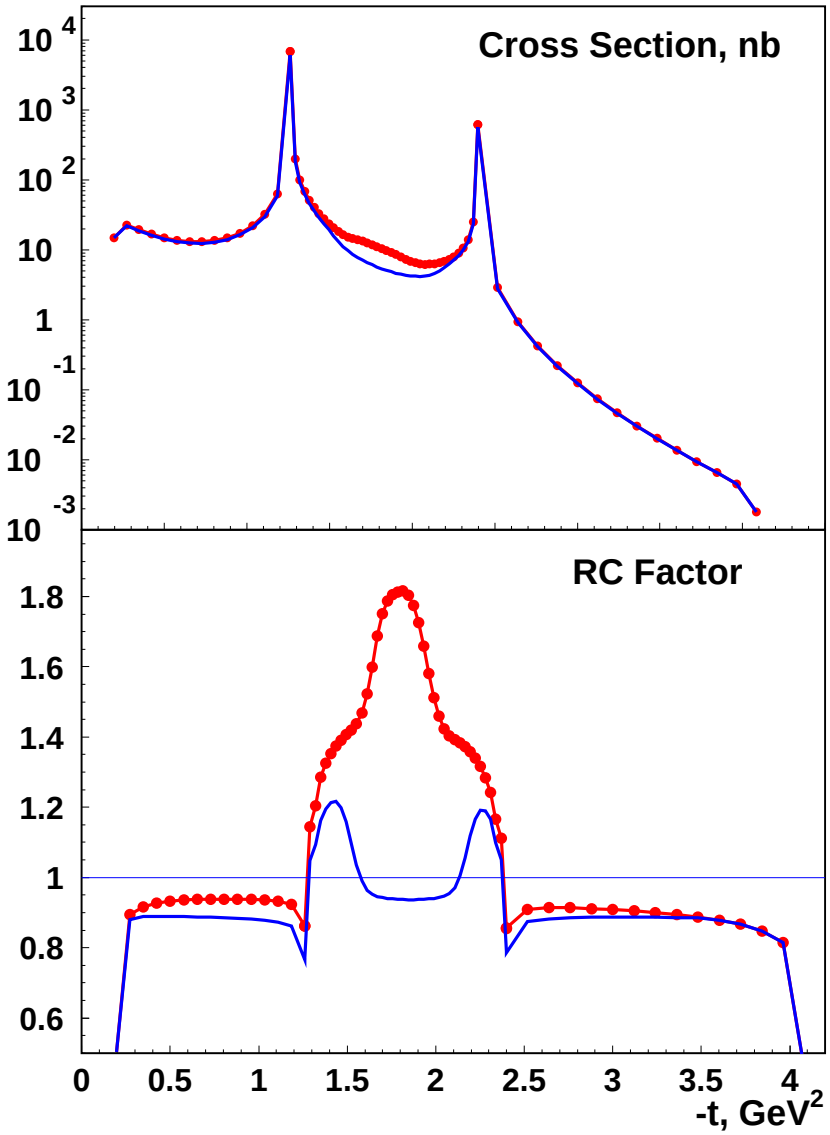
$$+\int_0^1 dz_2 \left(\frac{1+z_2^2}{1-z_2}\right)\left(\frac{\sin\theta_p'}{\mathcal{D}_{0p}^{1/2}}\theta(z-z_2^m)\frac{1}{z_2}\left(\frac{x_p}{x}\right)^2\sigma_{BH}(S,x_p,z_2^{-1}Q^2,t,\bar{\phi}_p) - \sigma_{BH}(S,x,Q^2,t,\phi)\right)\Bigg]$$

where $x_s = z_1Q^2/(z_1S-X)$ and $x_p = Q^2/(z_2S-X)$ are Bjorken x in shifted kinematics
Integration limits do not depend on ϕ ($\xi^2 = \lambda_t/\lambda_Y$):

$$z_1^m = \frac{Xt - 2M^2t + \xi(XS_x - 2M^2Q^2)}{St - 2M^2Q^2 + \xi(SS_x + 2M^2Q^2)}$$

$$z_2^m = \frac{Xt + 2M^2Q^2 + \xi(XS_x - 2M^2Q^2)}{St + 2M^2t + \xi(SS_x + 2M^2Q^2)}.$$

Numerical results: Cross section and RC factor

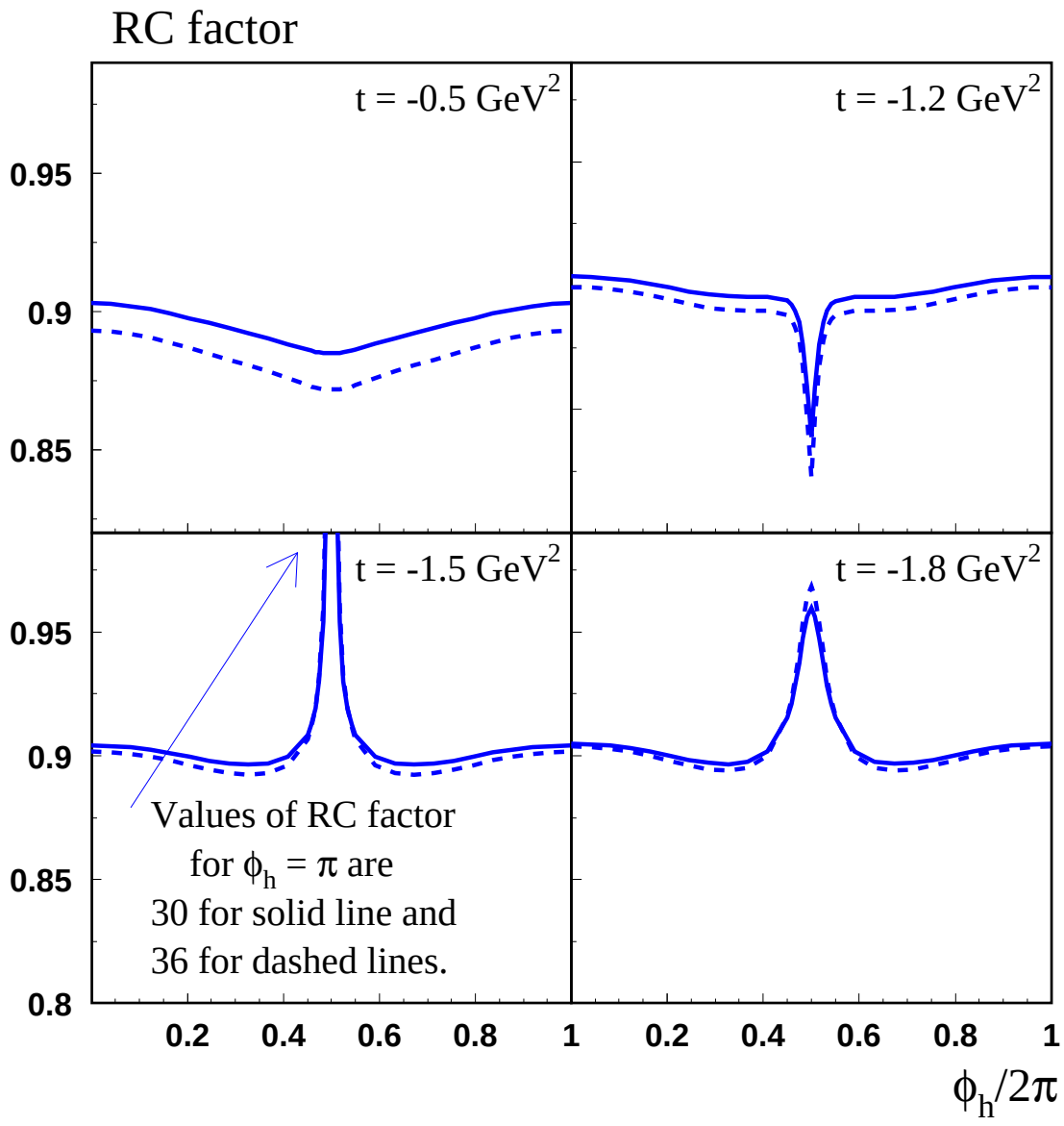


The observed cross sections of the BH process (upper plot) and respective RC factors (lower plot) for beam energy 5.77 GeV, x=0.4, and Q^2=1.8GeV^2.

The red (blue) line shows the results of calculation without (with) the cut on missing energy (E_γ <0.3 GeV).

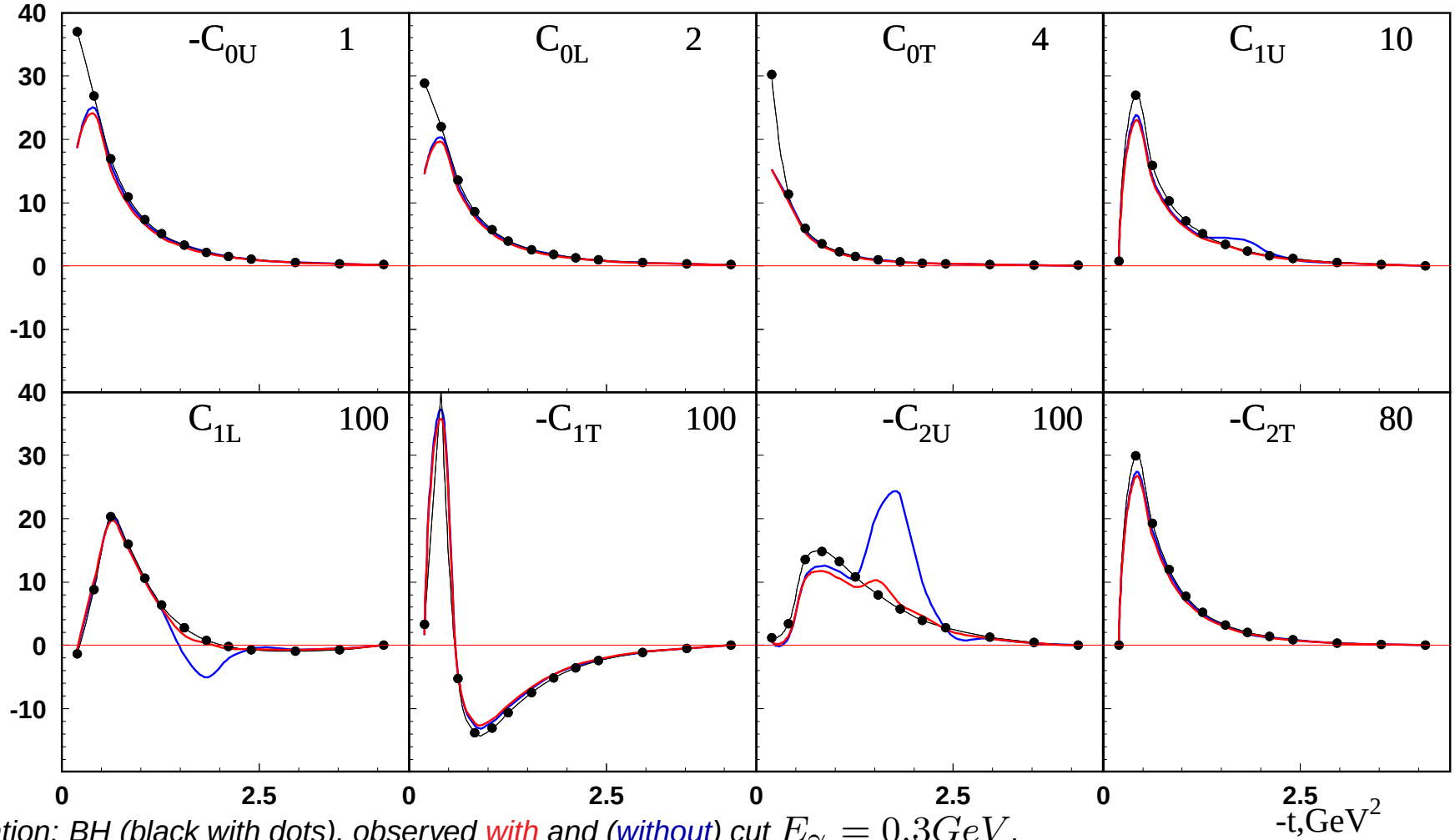
$$\mathrm{RCfactor} = \frac{\sigma_{observed}}{\sigma_{BH}}$$

Numerical results: ϕ dependence of RC factor



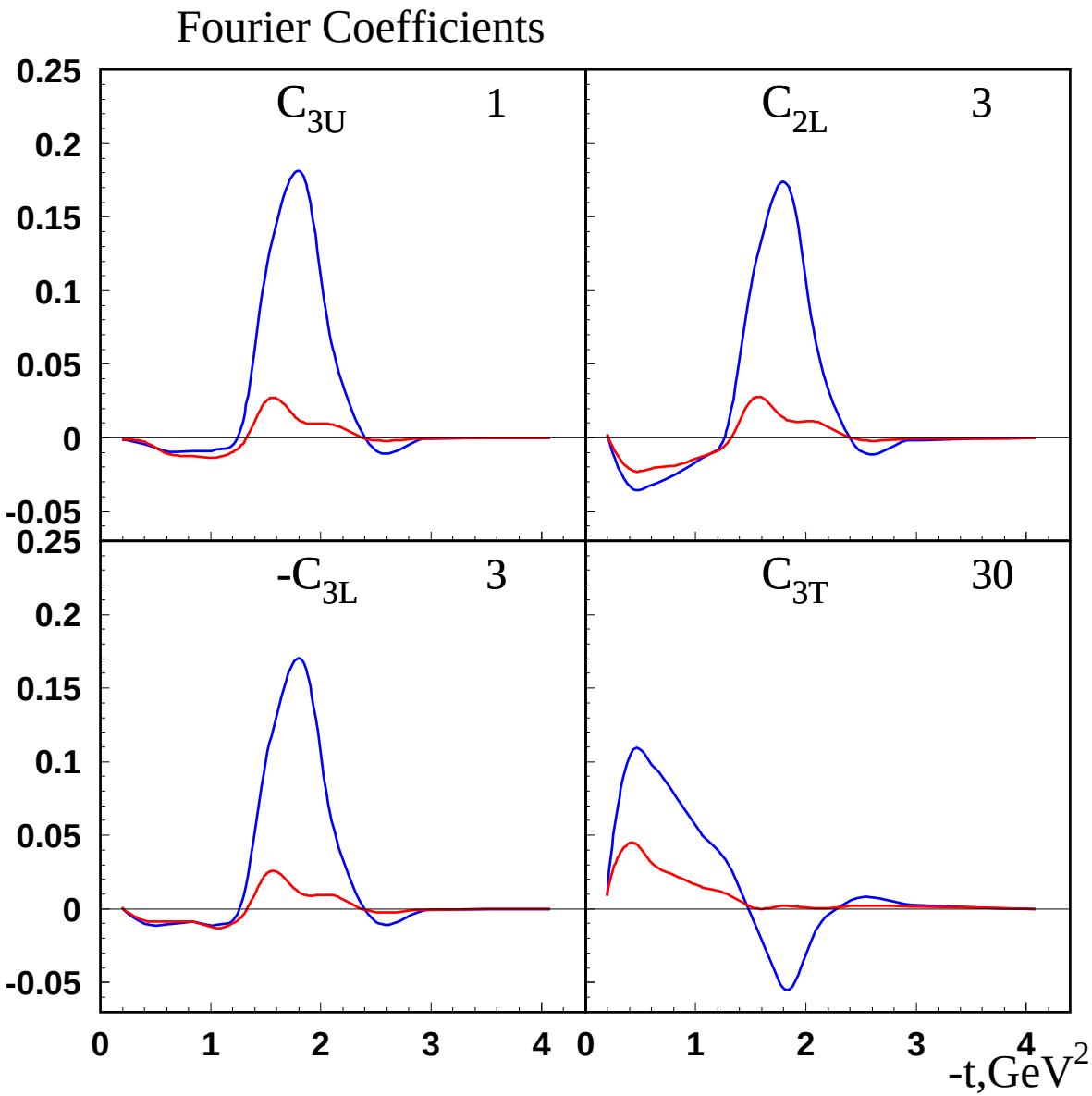
Numerical results: Fourier Coefficients

$$C_{nPol} = \frac{1}{2\pi f}\int_0^{2\pi} d\phi \cos(n\phi)\ \ \mathcal{P}_1\mathcal{P}_2\ \ \sigma_{BH,Pol}, \qquad Pol = U, L, P$$



Notation: BH (black with dots), observed with and (without) cut $E_\gamma = 0.3GeV.$

Numerical results: ϕ dependence



Monte Carlo Generator to BH: Approach

- To represent the observed cross section as a sum of positively definite contributions:
  - Original formula: $\sigma_{obs} = (1+\delta)\sigma_{BH} + \int_{z_s}^{1} \frac{K_s(z)-K_s(1)}{1-z} + \int_{z_p}^{1} \frac{K_p(z)-K_p(1)}{1-z}$,
  - Define Δ as a minimal energy of the photon we want to generate (i.e., calorimeter resolution),
  - Split each integral as

$$\int_{z_s}^{1} \frac{K_s(z)-K_s(1)}{1-z} = \int_{z_s}^{1-\frac{\Delta}{E}} \frac{K_s(z)}{1-z} - \int_{z_s}^{1-\frac{\Delta}{E}} \frac{K_s(1)}{1-z} + \int_{1-\frac{\Delta}{E}}^{1} \frac{K_s(z)-K_s(1)}{1-z},$$

  - Calculate analytically second integral and neglect third integral resulting in

$$\int_{z_s}^{1} \frac{K_s(z)-K_s(1)}{1-z} = \sigma_s(\Delta) + \delta_s(\Delta)\sigma_{BH}$$

    and

$$\int_{z_p}^{1} \frac{K_p(z)-K_p(1)}{1-z} = \sigma_p(\Delta) + \delta_p(\Delta)\sigma_{BH}$$

  - combining all together $\sigma_{obs} = (1+\delta(\Delta))\sigma_{BH} + \sigma_s(\Delta) + \sigma_p(\Delta)$, where each contribution is positively definite. *The price for this representation is the dependence on* Δ

Monte Carlo Generator to BH: Numeric Implementation

Generation of an event by BHRADGEN

- Calculate observed cross section in the kinematical point (external x, Q^2, t, and ϕ_h and uniformly simulated ϕ_e) according $\sigma_{obs} = (1 + \delta(\Delta))\sigma_{BH} + \sigma_s(\Delta) + \sigma_p(\Delta)$
- calculate probabilities of all three channels: nonradiated, radiated in s-peak, and radiated in p-peak:

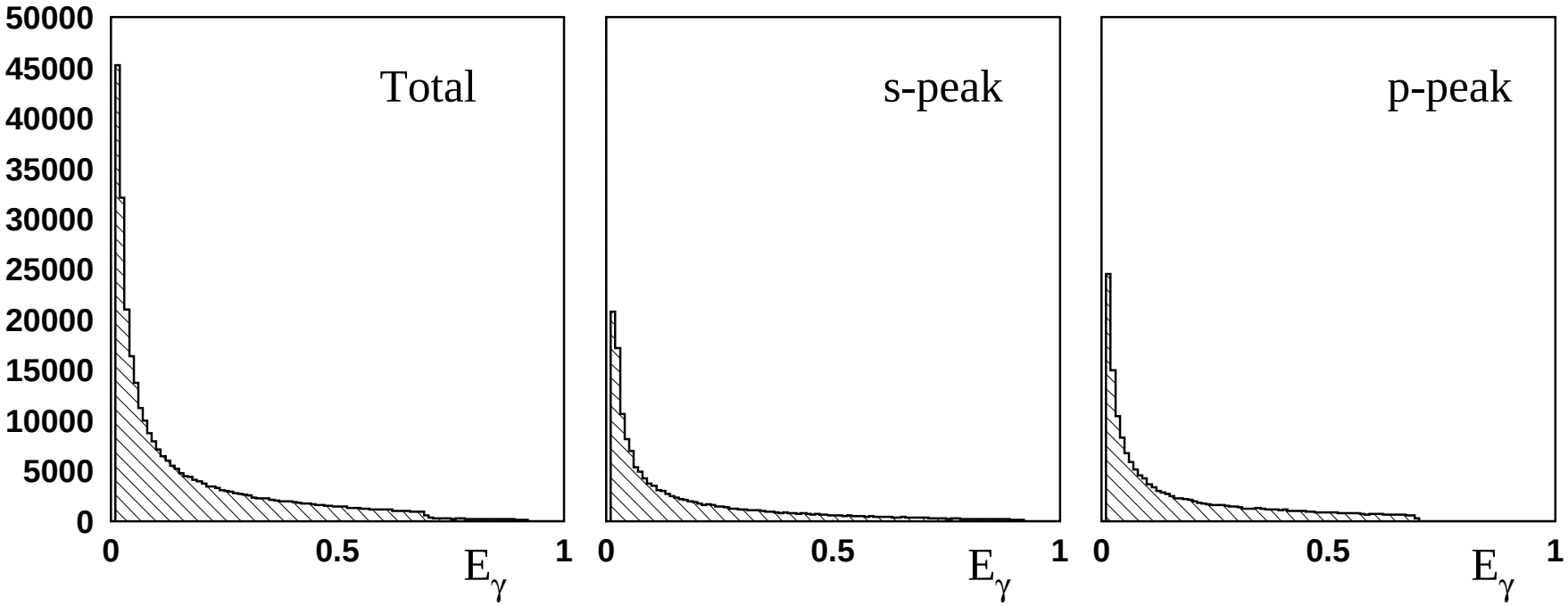
$$p_{nonrad} = \frac{(1 + \delta(\Delta))\sigma_{BH}}{\sigma_{obs}}, \qquad p_{s-peak} = \frac{\sigma_s(\Delta)}{\sigma_{obs}}, \qquad p_{p-peak} = \frac{\sigma_p(\Delta)}{\sigma_{obs}}.$$

- Generate the scattering channel according these probabilities
  - if base BH event (i.e., with one photon in the final state) is simulated then no additional variables are needed to be simulated
  - if two-photon event is simulated (i.e., s-peak or p-peak events), then kinematical variables of an additional photon are needed to be simulated
    - Photon energy is simulated through the variable $z_{1,2}$ according their distributions in integrand of expressions for s- and p-peak contributions.
    - Photon angles are simulated in s- or p-peaks.

Monte Carlo Generator to BH: Numeric Example

- Kinematical point: x=0.4, Q^2=1.8 GeV^2, t=-0.2 GeV^2, ϕ_h=160°.
- Probabilities:

$$p_{nonrad} = 0.686, \qquad p_{s-peak} = 0.152, \qquad p_{p-peak} = 0.162.$$



Conclusion: Leading Log RC to DVCS

- Large effects are possible when both photons are collinear (one is collinear to the initial lepton and another is collinear to the final lepton). Since the photon in respective BH process is not collinear (its momentum is the sum of two collinear photons), the BH cross section is not so large.
- Important to keep mass corrections in the lepton propagators w_0 and u_0 and terms like m^2/u_0^2 and m^2/w_0^2.
- Further Research includes calculation of i) next-to-leading contributions to the BH cross section and ii) RC to the interference term.
- Higher order Fourier coefficients at $\cos(n\phi)$ (i.e., not contributed to the BH cross section) can give noticeable contributions, e.g., coefficient C_{U3} can reach 10% from the effect of main contribution represented by C_{U0}. No significant contributions from $\sin(n\phi)$ at the level of observed cross section were found
- All conclusions are valid for the specific way of reconstruction of kinematic variables: leptonic and hadronic momenta are used to reconstruct the kinematics of the BH process. Kinematical variables of the photon were assumed to be unmeasured. Universal way to avoid multiple calculation to cover all possibilities for data analysis designs is the development of the Monte Carlo generator of the BH process with the additional process with two photons.

Part II:

Radiative Corrections to BH with Next-to-Leading Accuracy

Why we need an exact calculation

By “exactly” calculated RC we understand the estimation of the lowest order RC contribution with any predetermined accuracy.

The structure of the dependence on the electron mass in RC cross section:

$$\sigma_{RC} = A \log \frac{Q^2}{m^2} + B + O(m^2/Q^2)$$

where A and B do not depend on the electron mass.

$\log\left(\frac{Q^2}{m^2}\right) \sim 15$ for $Q^2 \sim$ 1GeV^2

If only A is kept, this is the leading log approximation.

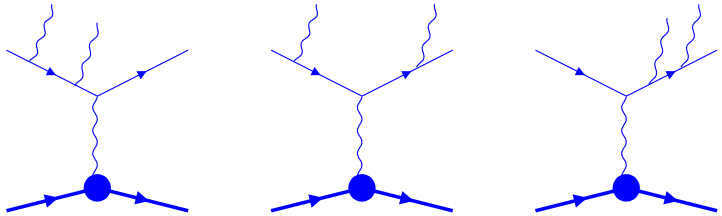
Unpleasant feature is that $B > A$ or even $B \gg A$, especially for the contribution with additional photon emission.

Approximate Calculation: Strengths and Limitations

- Strengths:
  - Expressions for RC are clear, transparent and easy to code.
  - The results are independent on the model of hadronic structure in the larger extent. Therefore they allow for calculations for all contributions to the total cross sections (e.g., for BH squared contribution, the interference term) when respective code for the Born cross section is available.
  - The formulas allow for generalization for the next-to-leading order and multiple photon emission.
- Limitations:
  - Only leading contribution is reconstructed exactly, i.e., if the correction is of the form $\sigma_{RC} = A \log(Q^2/m^2) + B + O(m^2/Q^2)$ then the approach reconstruct A exactly and B in some approximation. The quality of this approximation is difficult to control without exact formulas.
  - Implementation of experimental cuts usually focused on suppression of main contribution could lead to inexact results for RC evaluation.

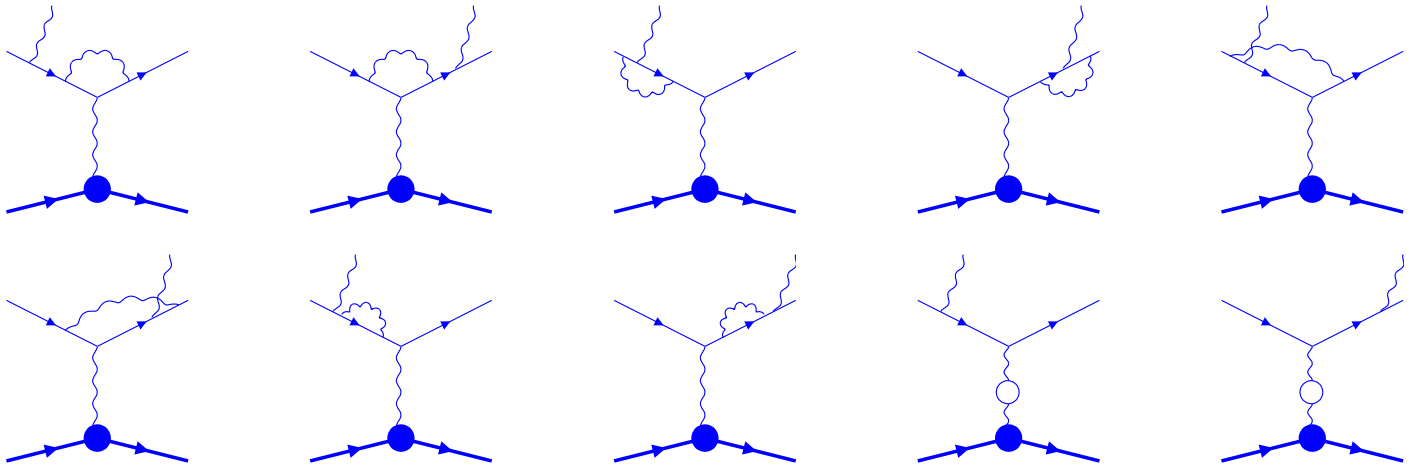
Steps in RC calculation to BH process

- Cross section of Two-Photon Emission process



  - Matrix element squared.
  - Phase space parametrization and integration over part of kinematical variables of an additional photon

- Loops and infrared divergence



Two-photon emission: Phase Space

The phase space for the cross section $d\sigma_r = \frac{M_r^2}{2S(2\pi)^8} d\Gamma$ of the process $e + p \to e' + p' + \gamma_1 + \gamma_2$ is parametrized as

$$d\Gamma = \frac{dp_2}{2E_2} \frac{dk_1}{2\omega_1} \frac{dk_2}{2\omega_2} \frac{dp'}{2E'} \delta(p + p_1 - p_2 - p' - k_1 - k_2) = d\Gamma_0 dV^2 d\Gamma_{2\gamma}$$

where V is invariant mass of two photons and

$$d\Gamma_{2\gamma} = \frac{dk_1}{2\omega_1} \frac{dk_2}{2\omega_2} \delta(p + p_1 - p_2 - p' - k_1 - k_2) = \frac{1}{8} d\Omega_R = \frac{1}{8} \cos(\theta_R) d\phi_R$$

The four kinematical variables to describe the kinematics (and phase space) of one-photon emission process are usual:

$$Q^2, t, x_B, \phi_h$$

Three additional variables to describe the kinematics of the additional photon is

$$V^2, \phi_R, \theta_R.$$

Two-photon emission: Matrix elements

Six matrix elements of the process are denoted $\mathcal{M}_{1-6} = e^4 t^{-1} J^h_\mu J_{1-6,\mu}$, where

$$J_{1\mu} = \bar{u}_2 \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa} + m}{-2\kappa k_1 + V^2} \hat{\epsilon}_2 \frac{\hat{k}_1 - \hat{\kappa}_1 + m}{-2k_1\kappa_1} \hat{\epsilon}_1 u_1 \qquad J_{2\mu} = \bar{u}_2 \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa} + m}{-2\kappa k_1 + V^2} \hat{\epsilon}_1 \frac{\hat{k}_1 - \hat{\kappa}_2 + m}{-2k_1\kappa_2} \hat{\epsilon}_2 u_1$$

$$J_{3\mu} = \bar{u}_2 \hat{\epsilon}_2 \frac{\hat{k}_2 + \hat{\kappa}_2 + m}{2k_2\kappa_2} \hat{\epsilon}_1 \frac{\hat{k}_2 + \hat{\kappa} + m}{2\kappa k_2 + V^2} \gamma_\mu u_1 \qquad J_{4\mu} = \bar{u}_2 \hat{\epsilon}_1 \frac{\hat{k}_2 + \hat{\kappa}_1 + m}{2k_2\kappa_1} \hat{\epsilon}_2 \frac{\hat{k}_2 + \hat{\kappa} + m}{2\kappa k_2 + V^2} \gamma_\mu u_1$$

$$J_{5\mu} = \bar{u}_2 \hat{\epsilon}_1 \frac{\hat{k}_2 + \hat{\kappa}_1 + m}{2k_2\kappa_1} \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa}_2 + m}{-2k_1\kappa_2} \hat{\epsilon}_2 u_1 \qquad J_{6\mu} = \bar{u}_2 \hat{\epsilon}_2 \frac{\hat{k}_2 + \hat{\kappa}_2 + m}{2k_2\kappa_2} \gamma_\mu \frac{\hat{k}_1 - \hat{\kappa}_1 + m}{-2k_1\kappa_1} \hat{\epsilon}_1 u_1$$

where $V^2 = \kappa^2 = (\kappa_1 + \kappa_2)^2$.

Further simplifications were possible in the leading log approximation. For s-peak (p-peak) the additional unobserved photon is emitted in the direction of the initial (final) lepton. Therefore,

$$\Big(\sum_{i=1}^{6} \mathcal{M}_i\Big)^2 = \mathcal{M}_{1s}^2 + \mathcal{M}_{1p}^2 + \mathcal{M}_{2s}^2 + \mathcal{M}_{2p}^2$$

where indices correspond to the unobserved photon, e.g., $1s$ means that the photon with momentum κ_1 is unobserved and in the s-peak.

Analytical Integration

- Integration of the matrix element squared over $d\Gamma_{2\gamma}$ (or over ϕ_R and θ_R) can be performed analytically.
- 69 specific integrals including vector and tensor integrals were calculated and combined in the table of integrals.
- All integrals were calculated analytically and the results of analytical integration were tested numerically.
- Note, the integration performed exactly: even the approximation of the small lepton mass is not required.
- The integrals were calculated using the system of center-mass of two photons. The result of the integration is represented in covariant form, and therefore can be presented in Lab. system.

Example of Analytical Integration

$$\mathrm{J}[A] = \frac{2}{\pi}\int d\Gamma_{2\gamma}\; A = \frac{1}{4\pi}\int d\Omega_R\; A = \frac{1}{4\pi}\int d\cos\theta_R \phi_R\; A.$$

the angles θ_R and ϕ_R define the orientation of momenta of photons in the system where $\vec{k} = 0$, i.e., in the two-photon central mass system.

$$\mathrm{J}[1] = 1; \qquad \mathrm{J}\Big[\frac{1}{w_1}\Big] = \mathrm{J}\Big[\frac{1}{w_2}\Big] = L_1 = \frac{1}{\sqrt{\lambda_1}}\log\frac{w+\sqrt{\lambda_1}}{w-\sqrt{\lambda_1}}; \qquad \mathrm{J}\Big[\frac{1}{w_1^2}\Big] = \mathrm{J}\Big[\frac{1}{w_2^2}\Big] = \frac{1}{m^2V^2}$$

$$\mathrm{J}\Big[\frac{1}{u_1^2 w_2}\Big] = \mathrm{J}\Big[\frac{1}{u_2^2 w_1}\Big] = \frac{1}{\lambda_I}\Big(\frac{u_I}{m^2V^2} + w_I L_I\Big); \quad \mathrm{J}\Big[\frac{D}{u_1^2 w_2}\Big] = -\mathrm{J}\Big[\frac{D}{u_2^2 w_1}\Big] = \frac{1}{2\lambda_I}\left(\frac{\Phi_{I2}}{m^2V^2} - \Phi_{I1}L_I\right)$$

The new variables are functions of kinematical variables, e.g.,

$$w_I = w(wu - V^2Q^2) - 2m^2V^2(w+u), \quad V_I = wu - V^2Y_m, \quad \lambda_I = V_I^2 - 4m^4V^4,$$

$$W_{Ip} = w^2u - V^2w_Y, \quad U_{Ip} = u^2w - V^2u_Y, \quad L_I = \frac{1}{\sqrt{\lambda_I}}\log\frac{V_I+\sqrt{\lambda_I}}{V_I-\sqrt{\lambda_I}},$$

$$\Phi_{I1} = S_{xt}W_{Ip} - 2V^2(SV_I + 2m^2V^2X), \quad \Phi_{I2} = S_{xt}U_{Ip} - 2V^2(XV_I + 2m^2V^2S), \quad \ldots$$

Infrared Divergence

Two approaches are known in literature;

- Mo and Tsai approach. Contains hidden assumptions.
- Covariant approach of Bardin and Shumeiko.

The integral is divergent for $V^2 \to 0$. This is the infrared divergence which needs to be extracted and canceled with respective contributions from loops. All terms with infrared divergence are combined in the form

$$\delta_R^{IR} = \frac{1}{2\pi} \int_0^{V_m^2} dV^2 \int d\Gamma_{2\gamma} \left(\frac{Q^2 + 2m^2}{u_{1,2} w_{1,2}} - \frac{m^2}{u_{1,2}^2} - \frac{m^2}{w_{1,2}^2} \right)$$

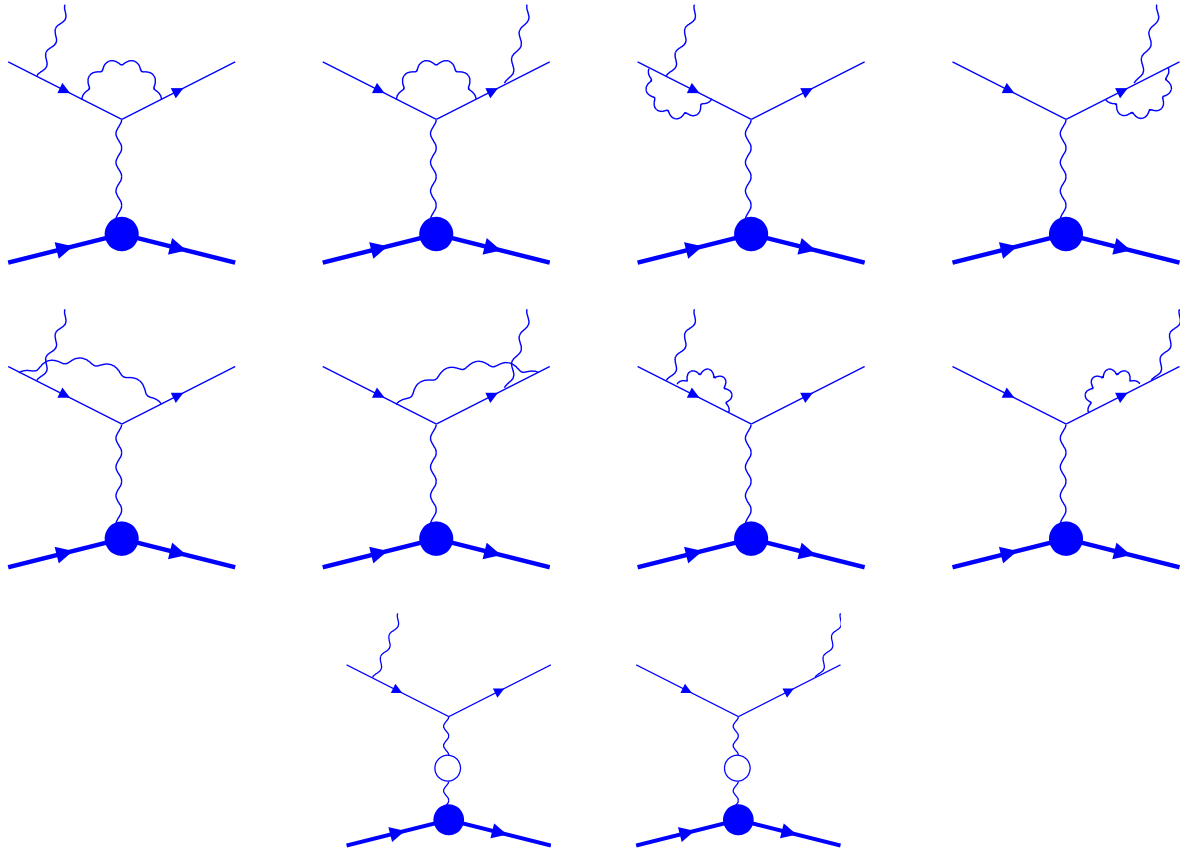
This integral needs to be calculated in dimensional regularization. The result is

$$\delta_R^{IR} = \left((Q^2 + 2m^2) L_m - 2 \right) \left(\frac{1}{n-4} + \frac{C}{2} - \log 2\sqrt{\pi} + \log \frac{v_m^2}{\sqrt{uw}} \right)$$

$$+ \frac{Q^2 + 2m^2}{\sqrt{Q^2(Q^2+4m^2)}} \left(\log \frac{Q^2 uw}{m^2 (Q^2+t)^2} + \text{terms with } \mathrm{Li}_2() \right)$$

Loop effects

Feynman graphs of one-loop effects for the BH cross section



In the case of leading log approximation we had

$$\sigma_V = \frac{\alpha}{\pi}\left(\log\frac{4M^2\omega_{min}^2}{SX} + \frac{3}{2}\right)L\sigma_{BH} = -\frac{\alpha L}{2\pi}\sigma_{BH}\left(\int\limits_0^{1-\Delta_1} dz_1\frac{1+z_1^2}{1-z_1} + \int\limits_0^{1-\Delta_2} dz_2\frac{1+z_2^2}{1-z_2}\right)$$

Loop effects: Details of Calculation

- For all of two-, three-, and four-denominator integrals we use Feynman parametrization, e.g.,

$$\int d^4l \frac{1}{A(l)B(l)C(l)} = \int\limits_0^1 d\alpha \int\limits_0^1 d\beta \int\limits_0^1 d\gamma \int d^4l \frac{2\delta(1-\alpha-\beta-\gamma)}{(\alpha A(l)+\beta B(l)+\gamma C(l))^3}$$

- Vector and tensor integration is performed for integrals containing l_μ and $l_\mu l_\nu$ in numerator.
- All integrals are calculated with next-to-leading accuracy within dimensional regularization and the Table of calculated integrals is created. For example

$$J_{012q} = -\frac{P_{IR}L_m}{w} + \frac{1}{wQ^2}\left(2L_mL_w - L_t^2 - \Phi\left(1-\frac{t}{Q^2}\right) - \frac{\pi^2}{6}\right)$$

$$J^\mu_{0q} = \left(-P+1-\frac{1}{2}L_w\right)(k_{1\mu}-k_\mu)$$

where $L_m = \log(Q^2/m^2)$, $L_w = \log(w/m^2)$, $L_t = \log(-t/m^2)$.

- Important is that all integrals can be calculated analytically.

Structure of the RC cross section

Combining all contributions we have

$$\frac{d\sigma}{d\Gamma_0} = A \log\frac{Q^2}{m^2} + B + \sum_{i=1}^{4} T_i^v \mathcal{F}_i + \int_0 \frac{dV^2}{V^2} \sum_{i=1}^{4} \Big(T_i^F(V^2) - T_i^F(0) \Big) \mathcal{F}_i$$

- A and B are resulted from the sum of (and infrared divergent) terms of the loop and two-gamma constitutions. They do not depend on the lepton mass.
- $\mathcal{F}_i$ are squared combinations of formfactors, (e.g., $\mathcal{F}_1 = (F_1 + F_2)^2$, $\mathcal{F}_2 = F_1^2 + \tau F_2^2$, $\mathcal{F}_3 = (F_1 + F_2)(F_1 + \tau F_2)$, and $\mathcal{F}_4 = (F_1 + F_2)F_2$. Terms with i=1,2 correspond to unpolarized case , and i=3,4—polarized case.

Structure of the RC cross section

Combining all contributions we have

$$\frac{d\sigma}{d\Gamma_0} = A\log\frac{Q^2}{m^2} + B + \sum_{i=1}^{4} T_i^v \mathcal{F}_i + \int_0 \frac{dV^2}{V^2}\sum_{i=1}^{4}\Big(T_i^F(V^2) - T_i^F(0)\Big)\mathcal{F}_i$$

- A and B are resulted from the sum of (and infrared divergent) terms of the loop and two-gamma constitutions. They do not depend on the lepton mass.
- $\mathcal{F}_i$ are squared combinations of formfactors.
- T_i^v came from nonfactorized (and lepton mass independent) part of the loop cross section:

$$T_i^v = T_{i0}^v + T_{i1}^v P_{yt} + T_{i2}^v P_w + T_{i3}^v P_u + T_{i4}^v F_{1y} + T_{i5}^v F_{2w} + T_{i6}^v F_{2u},$$

where where $\Phi(x)$ is the Spence function defined as $\Phi(x) = -\int_0^x t^{-1}\log|1-t|dt$, T_{ij}^v are rational functions of w_0, u_0, and t and $P_{yt} = \frac{\pi^2}{3} + 2\Phi\left(1-\frac{t}{Q^2}\right) - \log^2\frac{t}{Q^2}$, $P_w = \log\frac{t}{Q^2}\log\frac{w_0}{Q^2} - \Phi\left(1-\frac{w_0}{t}\right)$, $P_u = \log\frac{t}{Q^2}\log\frac{u_0}{Q^2} - \Phi\left(1+\frac{u_0}{t}\right)$, $F_{1y} = \frac{1}{t_y}\log\frac{t}{Q^2}$, $F_{1w} = \frac{1}{t_w}\log\frac{t}{w_0}$, $F_{1u} = \frac{1}{t_u}\log\frac{u_0}{t}$, $F_{2y} = \frac{F_{1y}-1}{t_y}$, $F_{2w} = \frac{F_{1w}-1}{t_w}$, and $F_{2u} = \frac{F_{1u}-1}{t_u}$.

Structure of the RC cross section

Combining all contributions we have

$$\frac{d\sigma}{d\Gamma_0} = A \log\frac{Q^2}{m^2} + B + \sum_{i=1}^{4} T_i^v \mathcal{F}_i + \int_0 \frac{dV^2}{V^2} \sum_{i=1}^{4} \Big(T_i^F(V^2) - T_i^F(0) \Big) \mathcal{F}_i$$

- A and B are resulted from the sum of (and infrared divergent) terms of the loop and two-gamma constitutions. They do not depend on the lepton mass.
- $\mathcal{F}_i$ are squared combinations of formfactors.
- T_i^v came from nonfactorized (and lepton mass independent) part of the loop cross section.
- $T_i^F(V^2)$ came from nonfactorized (can have mass-dependence) part of the contribution of two-gamma contribution:

$$T_i^F = \frac{T_{i1}^F}{w} \log\frac{w^2}{m^2 V^2} + \frac{T_{i2}^F}{u} \log\frac{u^2}{m^2 V^2} + T_{i3}^F \log\frac{V_{\rm I}}{m^2 V^2} + T_{i4}^F \log\frac{Q^2}{m^2} + T_{i5}^F$$

Quantities T_{ij}^F are rational (mass independent) functions of Q^2, u, w, and V^2.

Why we are sure that our calculation is correct

A series of analytical tests were performed,

- Terms with the electron mass in denominator completely cancel
- Infrared terms (i.e., terms with V^2 in denominator) are combined in an expected form and exactly cancel in the sum of loop and two-gamma contributions.
- Table of integrals for both contributions were numerically tested
- Leading log terms (i.e., terms containing $L = \log Q^2/m^2$) being combined analytically reproduce the cross section in the leading log approximation.
- All important stages of calculation were cross checked using symbolic manipulation software such as Mathematica 8 and Reduce 3.9.

Conclusion and Further plans

- Main conclusion is that the analytic calculation of the radiative corrections in the BH process for unpolarized, longitudinally polarized, and transversely polarized targets within next-to-leading accuracy (practically “exactly”) has been completed.
- The most important feature of the calculation is that complete integration for loop contributions and integration over angles of an additional photon for two-gamma contribution was performed analytically.

Further steps:

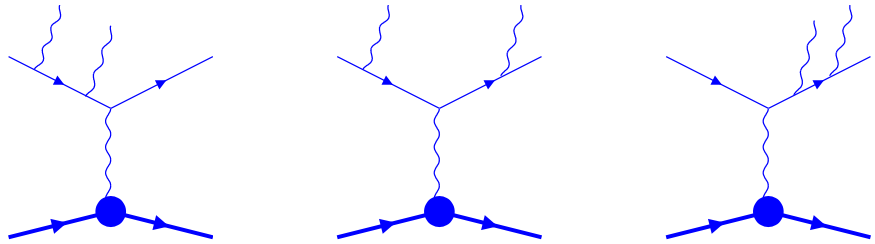
- develop a code capable of evaluating RC to BH numerically for unpolarized, longitudinally polarized, and transversely polarized targets.
- Perform numerical analysis and compare exact and leading log calculations
- Code Monte Carlo generator
  - V^2 *is simulated based on the distribution* $d\sigma_r/dV^2$ *that is evaluated numerically.*
  - *then* θ_R *is simulated according the conditional distribution:* $\rho(\theta_R|V^2)$ *(analytical formula is available).*
  - *then* ϕ_R *is simulated according the conditional distribution:* $\rho(\phi|\theta_R, V^2)$ *(formula is also available).*
- Detailed comparison with other calculations, e.g., with the calculation of Mark.
- Publish the results as a Jlab preprint and then as a peer-review publication

Part III:

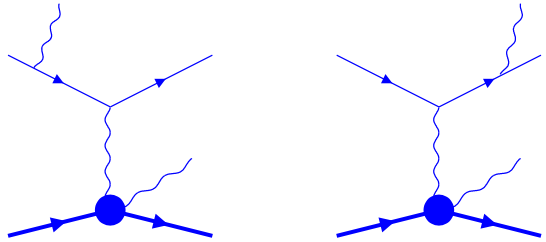
Radiative Corrections to DVCS

RC to DVCS: Emission of two hard photons

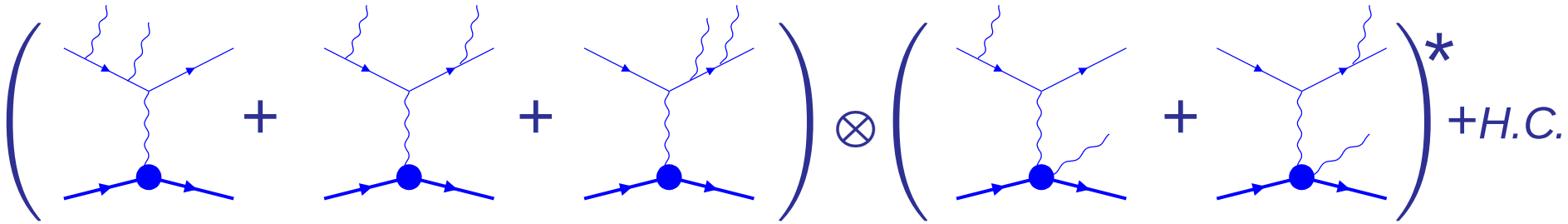
Two real photon emission from lepton line



One real photon emission from lepton line and one real photon emission from hadronic line

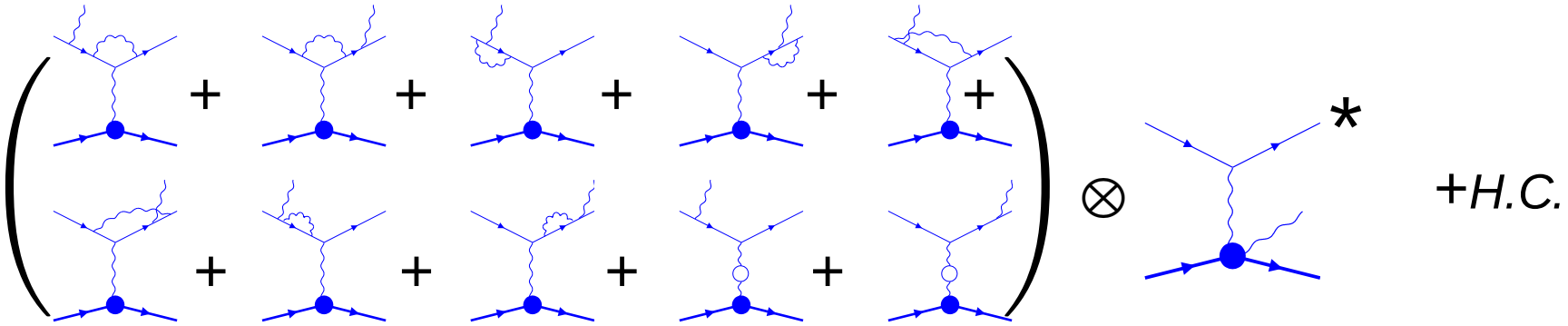


The contribution to the helicity dependent part of DVCS cross section

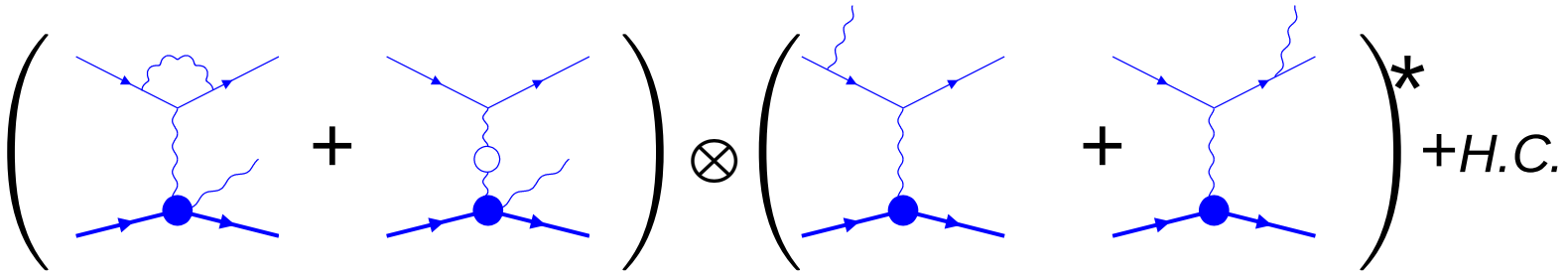


RC to DVCS: Loop Effects

Two contributions need to be taken into account



The second contribution



Calculation of RC to DVCS

Steps in the calculation of RC to DVCS:

- Make sure that the results of analytical calculation of the 1γ-emission cross section in the BMK-approximation involving the covariant hadronic tensor as an intermediate quentity coinside with results of the original paper
- Calculate matrix element squared controlling all assumptions and approximations.
- Parametrize the phase space of all final particles (use the shifted kinematics).
- Add the contribution of loop diagram.
- Implement and estimate the effects of higher order corrections.

The result is (quite expected):

- The expression of the observed cross section through integral (over additional photon energies) on the base DVCS cross cross is in exactly same form as for BH process.

Therefore, we can simply calculate the cross section for complete one-photon-emission process including BH and DVCS contributions and use this formula to calculate the next order RC.

The lowest order RC

$$\sigma_{obs}(S,x,Q^2,t,\phi) = (1+2\Pi(t))\sigma_{BH}(S,x,Q^2,t,\phi) + \frac{\alpha}{2\pi}L\Bigg[$$

$$\int\limits_0^1 dz_1 \left(\frac{1+z_1^2}{1-z_1}\right)\left(\frac{\sin\theta_s'}{\mathcal{D}_{0s}^{1/2}}\theta(z-z_1^m)\left(\frac{x_s}{x}\right)^2\sigma_{BH}(z_1S,x_s,z_1Q^2,t,\bar{\phi}_s) - \sigma_{BH}(S,x,Q^2,t,\phi)\right)$$

$$+\int\limits_0^1 dz_2 \left(\frac{1+z_2^2}{1-z_2}\right)\left(\frac{\sin\theta_p'}{\mathcal{D}_{0p}^{1/2}}\theta(z-z_2^m)\frac{1}{z_2}\left(\frac{x_p}{x}\right)^2\sigma_{BH}(S,x_p,z_2^{-1}Q^2,t,\bar{\phi}_p) - \sigma_{BH}(S,x,Q^2,t,\phi)\right)\Bigg]$$

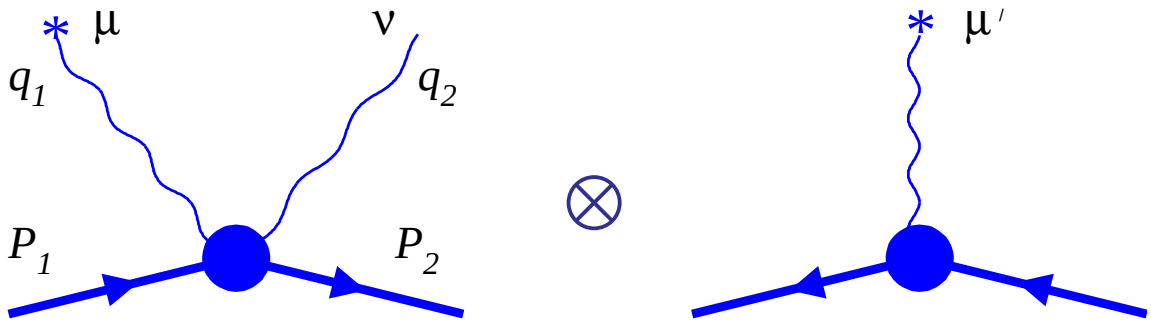
where $x_s = z_1Q^2/(z_1S-X)$ and $x_p = Q^2/(z_2S-X)$ are Bjorken x in shifted kinematics
Integration limits do not depend on ϕ ($\xi^2 = \lambda_t/\lambda_Y$):

$$z_1^m = \frac{Xt - 2M^2t + \xi(XS_x - 2M^2Q^2)}{St - 2M^2Q^2 + \xi(SS_x + 2M^2Q^2)}$$

$$z_2^m = \frac{Xt + 2M^2Q^2 + \xi(XS_x - 2M^2Q^2)}{St + 2M^2t + \xi(SS_x + 2M^2Q^2)}.$$

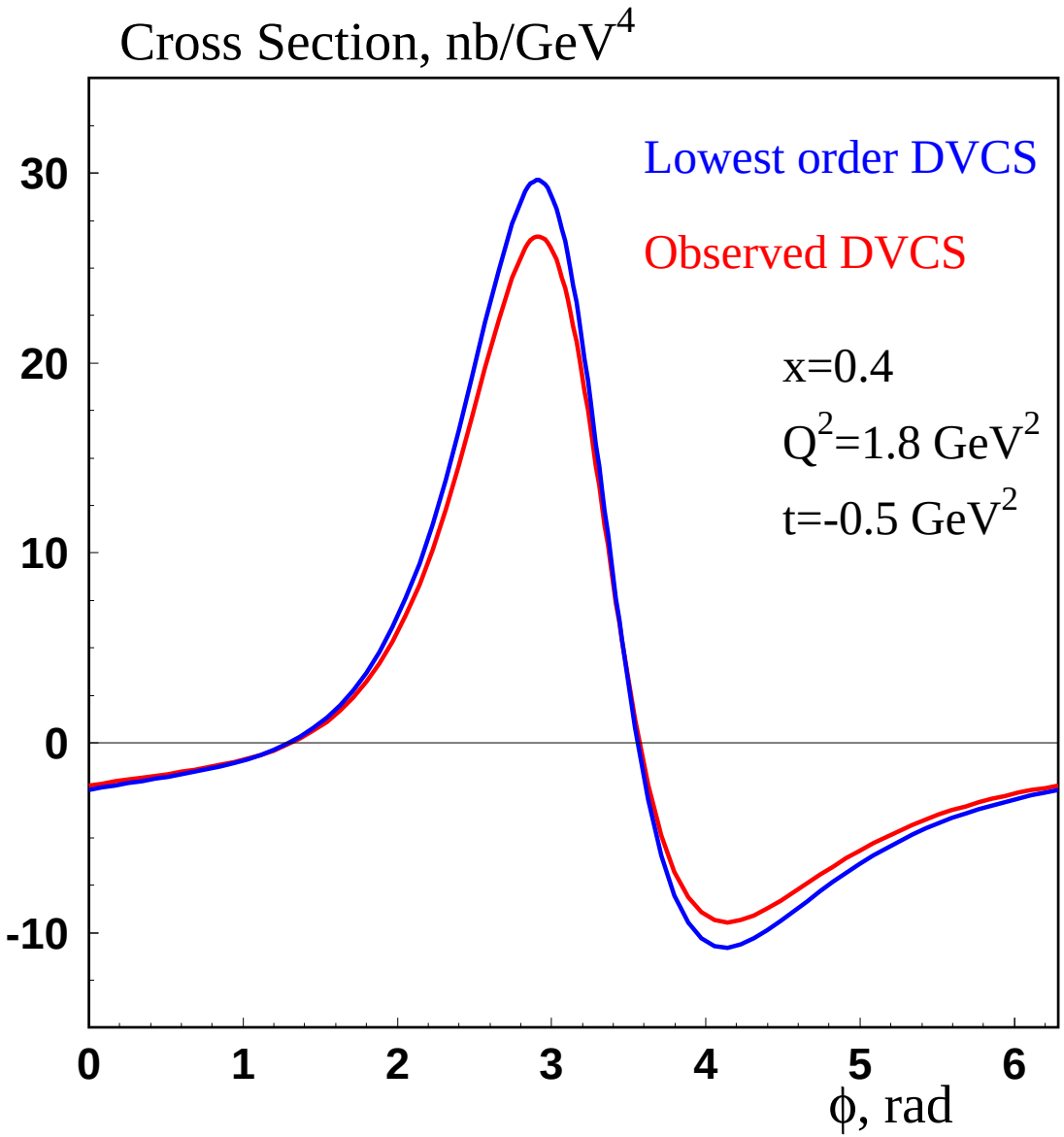
Exact Calculation of RC to DVCS

The calculation of RC for interference term with next-to-leading accuracy requires appealing the model for hadronic state.

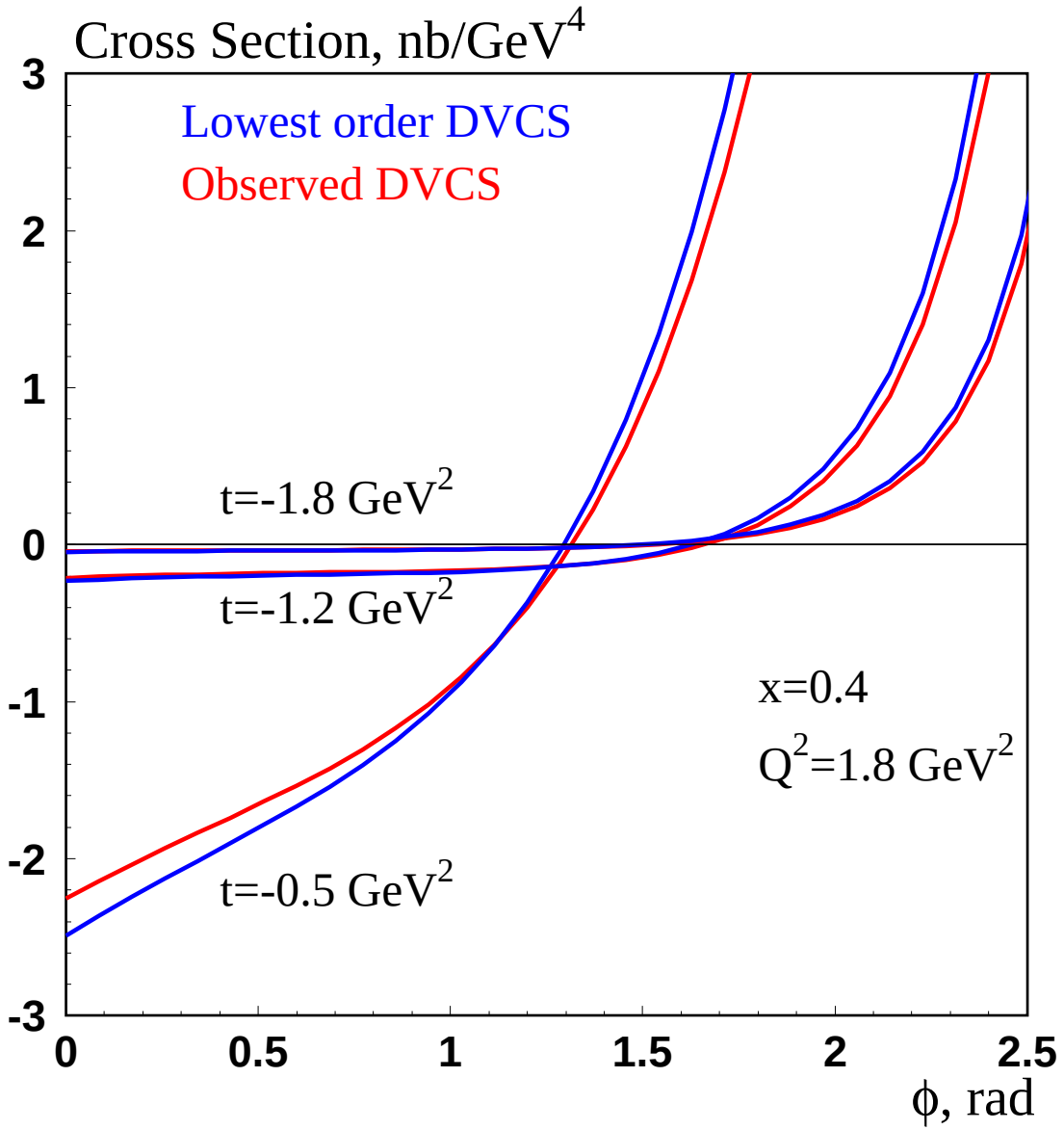


The theory of DVCS from Belitsky, Mueller, Kirchner (*Nucl. Phys. B629(2002)323*) are used to construct covariant hadronic tensor $T_{\mu\nu,\mu'}$ in terms of nucleon formfactors $F_{(p,d)}$, GDPs ($\mathcal{E}$, $\mathcal{H}$, $\tilde{\mathcal{E}}$, $\tilde{\mathcal{E}}^3_+$, etc.), four-vectors ($P = P_1 + P_2$, $\Delta = P_2 - P_1$, $q = (q_1 + q_2)/2$), and kinematic variables (ξ, η, Δ^2, Q^2).

Numerical results: observed cross sections

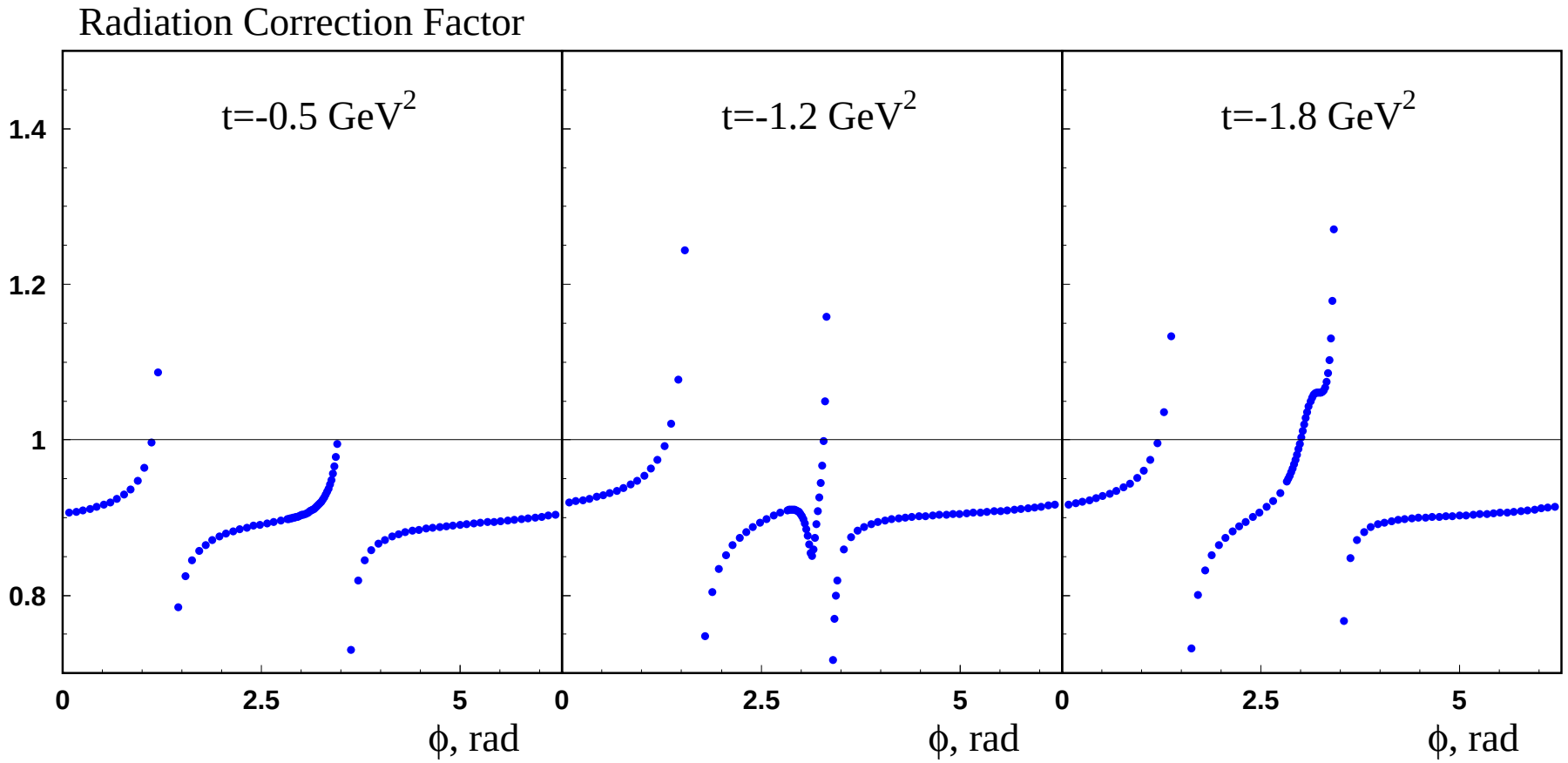


Numerical results: observed cross sections



Numerical results: observed cross sections

Kinematical point: $x = 0.4$, $Q^2 = 1.8\text{GeV}^2$.

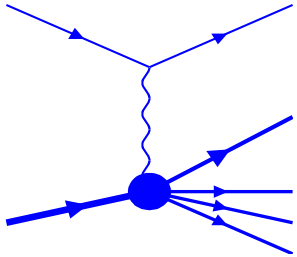


Part IV:

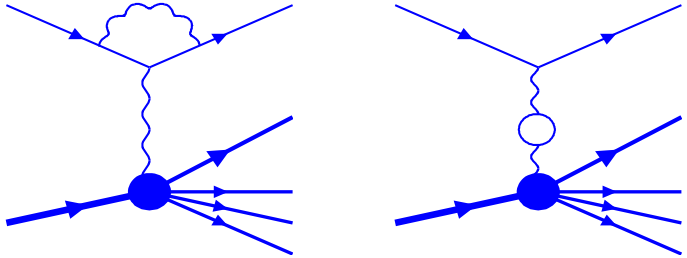
Semi-inclusive and Exclusive Contributions to RC

Contribution to RC in SIDIS

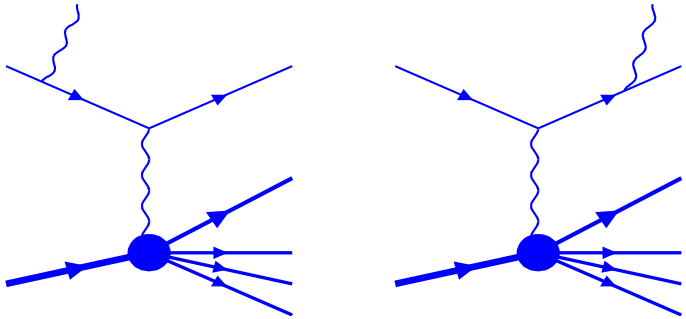
The Born cross section



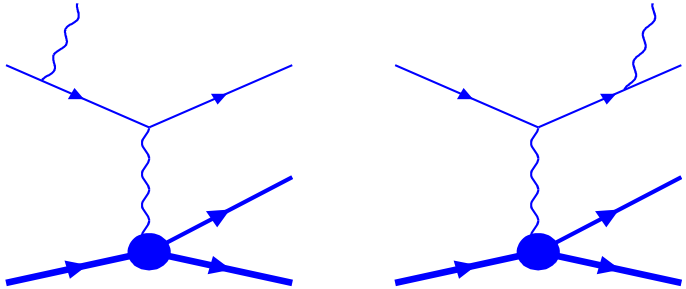
Loop diagrams



Emission of a radiated photon (semi-inclusive processes)



Emission of a radiated photon (exclusive processes)



Calculation of RC to SIDIS using POLRAD 2.0

- Option SIRAD has to be used.
- Simple QPM is assumed. The RC cross section is calculated in terms of parton distributions $f(x)$ and fragmentation functions $D(z)$.
- RC for unpolarized cross section and polarized part of the cross section are calculated.
- No exclusive radiative tail is separately calculated.
- Several models for parton distributions and fragmentation functions are implemented.
- RC has traditional form: a factorized part representing the contributions of loops and soft photon emission and the term in the form of two-dimensional integral representing the hard photon emission.
- Integration over p_t and ϕ_h is assumed, i.e., the RC to the three-fold cross section is calculated: $d\sigma/dxdydz$.

Numerical Estimates of RC to SIDIS using POLRAD 2.0

hadron	x	y	z	A_{born},%	A_{obs},%	δ_A	δ_u	δ_p
π^-	0.18	0.76	0.54	17.57	17.34	-1.34	-4.65	-5.93
	0.24	0.70	0.61	20.08	19.71	-1.87	-6.69	-8.43
	0.31	0.69	0.69	22.15	21.61	-2.40	-8.54	-10.74
	0.37	0.70	0.77	21.37	20.68	-3.24	-10.28	-13.18
π^+	0.18	0.76	0.54	39.70	40.08	0.95	-3.04	-2.11
	0.24	0.70	0.61	43.77	43.80	0.07	-5.07	-5.00
	0.31	0.69	0.69	49.07	48.83	-0.48	-7.03	-7.48
	0.37	0.70	0.77	53.90	53.34	-1.04	-8.92	-9.87
π^0	0.18	0.76	0.54	32.45	32.71	0.81	-3.57	-2.79
	0.24	0.70	0.61	37.13	37.13	0.00	-5.52	-5.52
	0.31	0.69	0.69	42.89	42.66	-0.53	-7.38	-7.87
	0.37	0.70	0.77	47.91	47.40	-1.07	-9.17	-10.14

δ_A *is the relative correction to polarization asymmetry,*

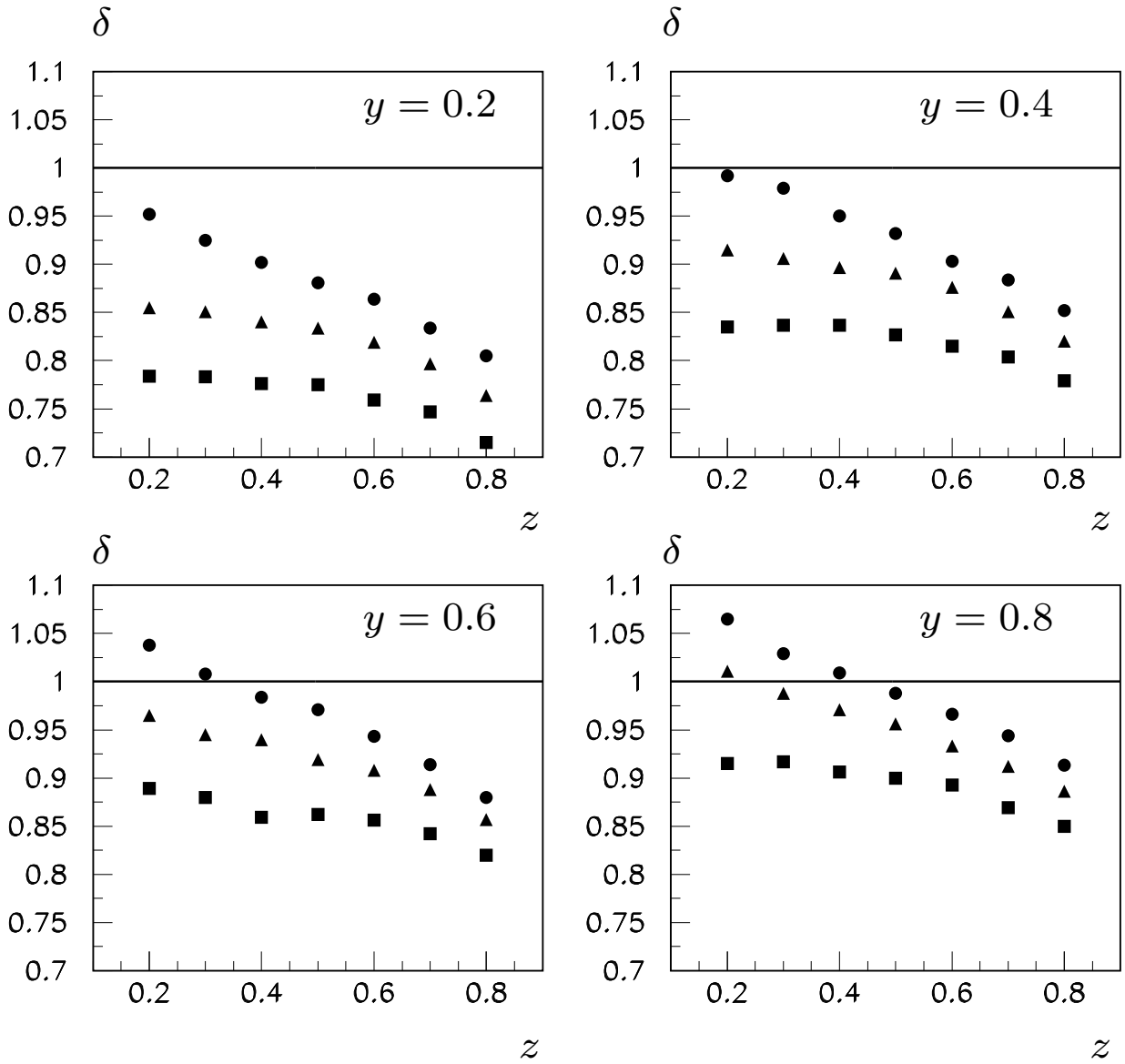
$\delta_{u,p}$ *are corrections to unpolarized and polarized parts of cross sections.*

Calculation of RC to SIDIS using HAPRAD

- Original version is based on the calculation in Akushevich, Soroko, Shumeiko EPJ C10(1999)681. In this paper an approach to calculate radiative corrections to unpolarized cross section of semi-inclusive electroproduction is developed.
- calculate the RC to five-dimensional cross section $d^5\sigma/dxdydzdp_t^2 d\theta_h$ as well as to four- and three-dimensional cross sections (i.e., $d^4\sigma/dxdydzdp_t^2 d$ and $d^3\sigma/dxdydz$).
- For the three-dimensional cross section the results for RC are close to that given by POLRAD 2.0 (SIRAD)
- The correction is of standard form:

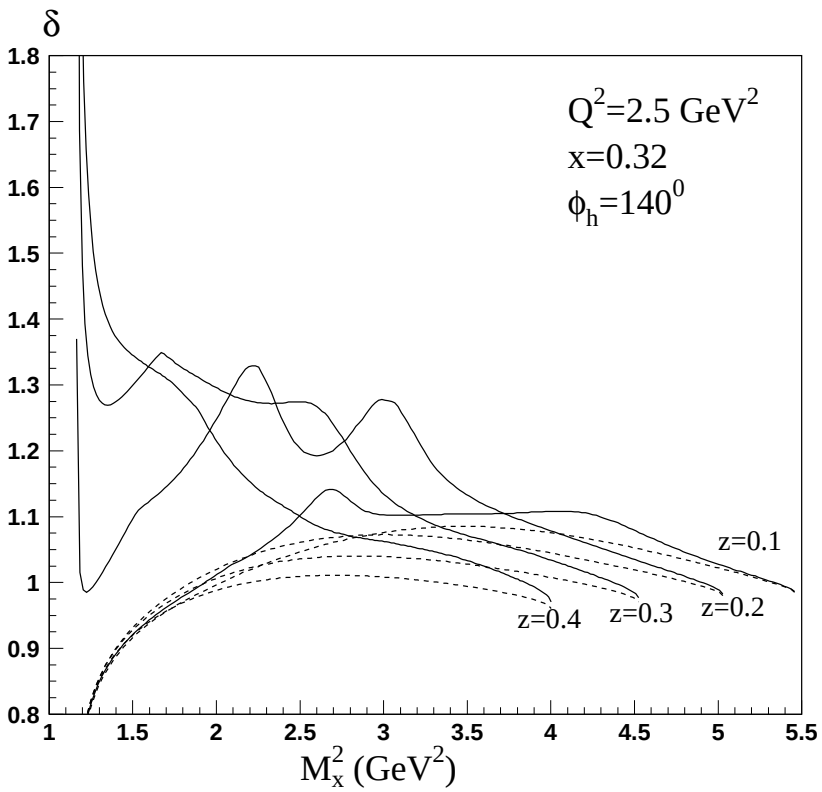
$$\sigma_{obs} = \sigma_0 e^{\delta_{inf}} (1 + \delta_{VR} + \delta_{vac}) + \sigma_F.$$

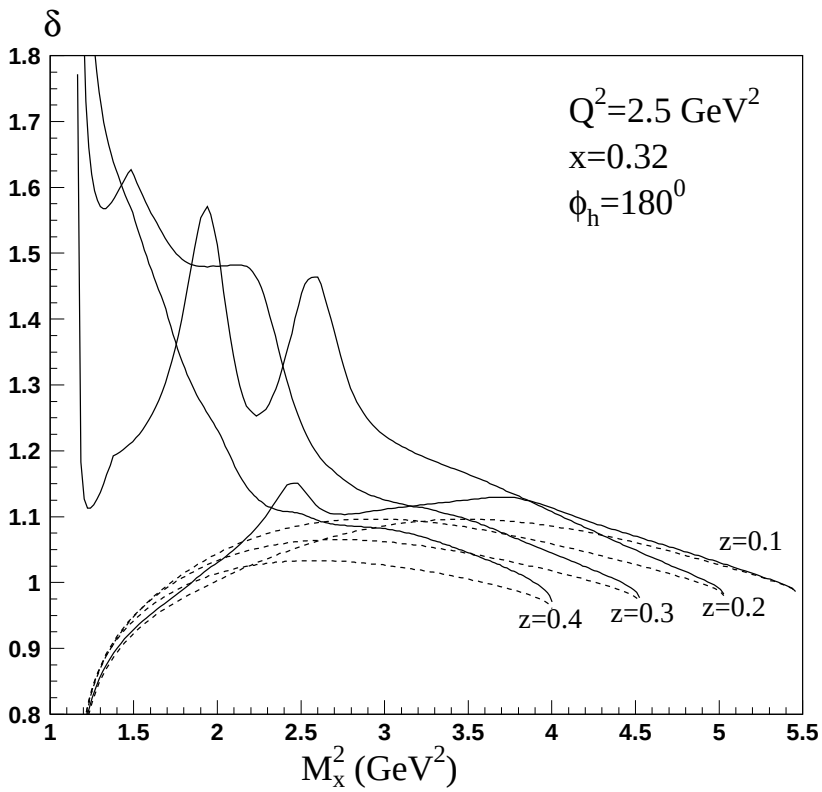
Numerical results



Radiative correction to the semi-inclusive cross section for kinematics of HERMES; $\sqrt{S}$*=7.19 GeV. Symbols from top to bottom correspond to the* x*=0.05, 0.45 and 0.7. The results for* x*=0.15 are skipped, because they practically coincide with ones for* x*=0.05.*

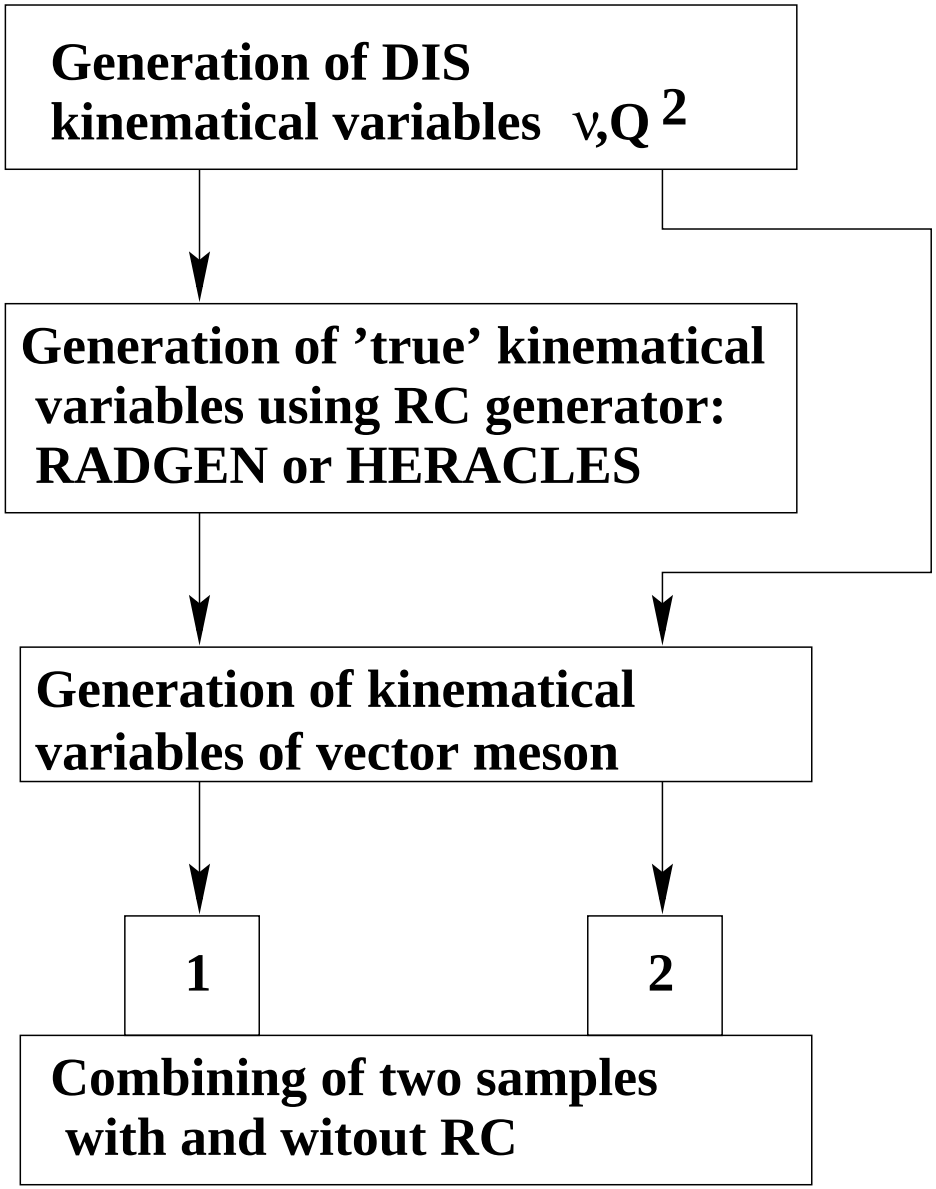
Importance of exclusive radiative tail





M_X^2-dependence of the RC factor for the semi-inclusive π^+ electroproduction at fixed proton for lepton beam energy 6 GeV: solid lines show the total correction, dashed lines represent the correction excluding the exclusive radiative tail (Akushevich, Ilyichev, Osipenko, Phys.Lett. B672(2009)35)

Calculation of RC to SIDIS using Monte Carlo generator



Possible scheme of Monte Carlo calculation of the RC factor (Akushevich, hep-ph/9906410)

General task for RC to SIDIS

General consideration of RC to SIDIS of polarized particles requires knowledge of analytic form of the covariant hadronic tensor $W_{\mu\nu}(p,p_h,q)+iS_\rho W_{\mu\nu\rho}(p,p_h,q)$ in terms of 18 structure functions $\mathcal{H}_i(x,z,Q^2,t)$ in both semi-inclusive and exclusive regions.

The expression for covariant hadron tensor for arbitrary polarization is not available in literature, therefore we need to construct it by ourselves:

According to Aram Kotzinian NP B441(1995)234:

$$l_{\mu\nu}W^{\mu\nu}=L^{ab}\big(H^{(0)}_{ab}+S^\rho e^i_\rho H^{(s)}_{abi}\big),\qquad L^{ab}=\epsilon^a_\mu\epsilon^b_\nu l_{\mu\nu}$$

Therefore

$$W^{\mu\nu}=\epsilon^a_\mu\epsilon^b_\nu\big(H^{(0)}_{ab}+S^\rho e^i_\rho H^{(s)}_{abi}\big)$$

Here

$$\epsilon^\mu_0=\tfrac{1}{Q}(q^3,0,0,q^0),\quad e^\mu_0=(1,0,0,0),$$
$$\epsilon^\mu_1=(0,1,0,0),\quad e^\mu_1=(0,1,0,0),$$
$$\epsilon^\mu_2=(0,0,1,0),\quad e^\mu_2=(0,0,1,0),$$
$$\epsilon^\mu_3=\tfrac{q_\mu}{Q},\quad e^\mu_3=(0,0,0,1).$$

Vectors ϵ^a_μ and ϵ^b_ν are given in Lab. system, so what we need is only to use covariant expressions for them (e.g., *from Arens et al. ZP C74(1997)651*) and explicitely calculate sums over a, b, and i in expression for $W_{\mu\nu}$.

Explicit Expression for SIDIS Hadronic Tensor

$$W_{\mu\nu} = n_\mu n_\nu T_1 + p_\mu p_\nu T_2 + p_{h\mu} p_{h\nu} T_3 + (p_\mu p_{h\nu} + p_{h\mu} p_\nu) T_4 + (p_\mu p_{h\nu} - p_{h\mu} p_\nu) T_5$$

$$+ (p_\mu n_\nu + n_\mu p_\nu) T_6 + (p_\mu n_\nu - n_\mu p_\nu) T_7 + (p_{h\mu} n_\nu + n_\mu p_{h\nu}) T_8 + (p_{h\mu} n_\nu - n_\mu p_{h\nu}) T_9$$

where $n_\mu = \frac{1}{M} \epsilon_{\mu p q p_h}$, $S_x = \frac{Q^2}{x}$, $\lambda_y = S_x^2 + 4M^2 Q^2$, $Z = z \lambda_Y^{1/2} - (z^2 S_x^2 - 4M^2 (p_t^2 + m_h^2))^{1/2}$

$$T_1 = \frac{4M^2}{\lambda_Y p_t^2} H_{22}^{(0)} - \frac{8(nS) M^3}{\lambda_Y^{3/2} p_t^3} H_{222}^{(S)}$$

$$T_2 = \frac{1}{4M^2 \lambda_Y p_t^2} (Z^2 S_x^2 H_{11}^{(0)} - 8(ZQS_x \Re H_{01}^{(0)} - 2M^2 p_t Q^2 H_{00}^{(0)}) M^2 p_t)$$

$$+ \frac{(nS)}{2M^3 \lambda_Y^{3/2} p_t^3} (-Z^2 H_{112}^{(S)} S_x^2 + 8(ZQS_x \Re H_{012}^{(S)} - 2M^2 p_t Q^2 H_{002}^{(S)}) M^2 p_t)$$

$$T_3 = \frac{H_{11}^{(0)}}{p_t^2} - \frac{2(nS) M}{\lambda_Y^{1/2} p_t^3} H_{112}^{(S)}$$

$$T_4 = \frac{1}{2M^2 \sqrt{\lambda_Y} p_t} (-ZS_x H_{11}^{(0)} + 4M^2 p_t Q \Re H_{01}^{(0)}) + \frac{(nS)}{M \lambda_Y p_t^3} (ZS_x H_{112}^{(S)} - 4M^2 Q p_t H_{012}^{(S)})$$

$$T_5 = i \frac{2Q}{\sqrt{\lambda_Y} p_t} \Im H_{01}^{(0)} - i \frac{4(nS) MQ}{\lambda_Y p_t^2} \Im H_{012}^{(S)}$$

Explicit Expression for SIDIS Hadronic Tensor

$$W_{\mu\nu} = n_\mu n_\nu T_1 + p_\mu p_\nu T_2 + p_{h\mu} p_{h\nu} T_3 + (p_\mu p_{h\nu} + p_{h\mu} p_\nu) T_4 + (p_\mu p_{h\nu} - p_{h\mu} p_\nu) T_5$$

$$+ (p_\mu n_\nu + n_\mu p_\nu) T_6 + (p_\mu n_\nu - n_\mu p_\nu) T_7 + (p_{h\mu} n_\nu + n_\mu p_{h\nu}) T_8 + (p_{h\mu} n_\nu - n_\mu p_{h\nu}) T_9$$

where $n_\mu = \frac{1}{M} \epsilon_{\mu p q p_h}$, $S_x = \frac{Q^2}{x}$, $\lambda_y = S_x^2 + 4M^2 Q^2$, $Z = z\lambda_Y^{1/2} - (z^2 S_x^2 - 4M^2(p_t^2 + m_h^2))^{1/2}$

$$\begin{aligned}
T_6 &= -\frac{1}{M {\lambda_Y}^{3/2} p_t^3} \big(4 p_t M^2 Q (2M p_t \Re H_{023}^{(S)} - \sqrt{\lambda_z} \Re H_{021}^{(S)}) - \sqrt{\lambda_z} S_x (2M p_t \Re H_{123}^{(S)} \\
&\quad - \sqrt{\lambda_z} \Re H_{121}^{(S)})) (qS) + (4M^2 Q p_t \Re H_{021}^{(S)} - S_x \sqrt{\lambda_z} \Re H_{121}^{(S)}) \sqrt{\lambda_Y} (p_h S)\big) \\
T_7 &= -i \frac{1}{M {\lambda_Y}^{3/2} p_t^3} \big(4 p_t M^2 Q (2M p_t \Im H_{023}^{(S)} - \sqrt{\lambda_z} \Im H_{021}^{(S)}) - \sqrt{\lambda_z} S_x (2M p_t \Im H_{123}^{(S)} \\
&\quad - \sqrt{\lambda_z} \Im H_{121}^{(S)})) (qS) + (4M^2 Q p_t \Im H_{021}^{(S)} - S_x \sqrt{\lambda_z} \Im H_{121}^{(S)}) \sqrt{\lambda_Y} (p_h S)\big) \\
T_8 &= -\frac{2M}{\lambda_Y p_t^3} \big((qS)(-\sqrt{\lambda_z} \Re H_{121}^{(S)} + 2M p_t \Re H_{123}^{(S)}) - (p_h S) \sqrt{\lambda_Y} \Re H_{121}^{(S)}\big) \\
T_9 &= i \frac{2M}{\lambda_Y p_t^3} \big((qS)(\sqrt{\lambda_z} \Im H_{121}^{(S)} - 2M p_t \Im H_{123}^{(S)}) - (p_h S) \sqrt{\lambda_Y} \Im H_{121}^{(S)}\big)
\end{aligned}$$

where $\lambda_z = z^2 S_x^2 - 4M^2(p_t^2 + m_h^2)$, so $Z = z\lambda_Y^{1/2} - \lambda_z^{1/2}$.

GCD Calculation of Hadronic Tensor

Alternatively, we can

- choose the leading contribution (e.g., twist-two contribution) to SIDIS cross section,
- use hadronic tensor defined as

$$2MW^{\mu\nu}(q,P,P_h) = \sum_q e_q \int \frac{d^2k_T}{(2\pi)^2}\frac{d^2p_T}{(2\pi)^2}\mathrm{Tr}(\Phi(p,P)\gamma^\mu\Delta(k,P_h)\gamma^\nu)\delta^{(2)}(zp_T+k_T-P_{h\perp})$$

where

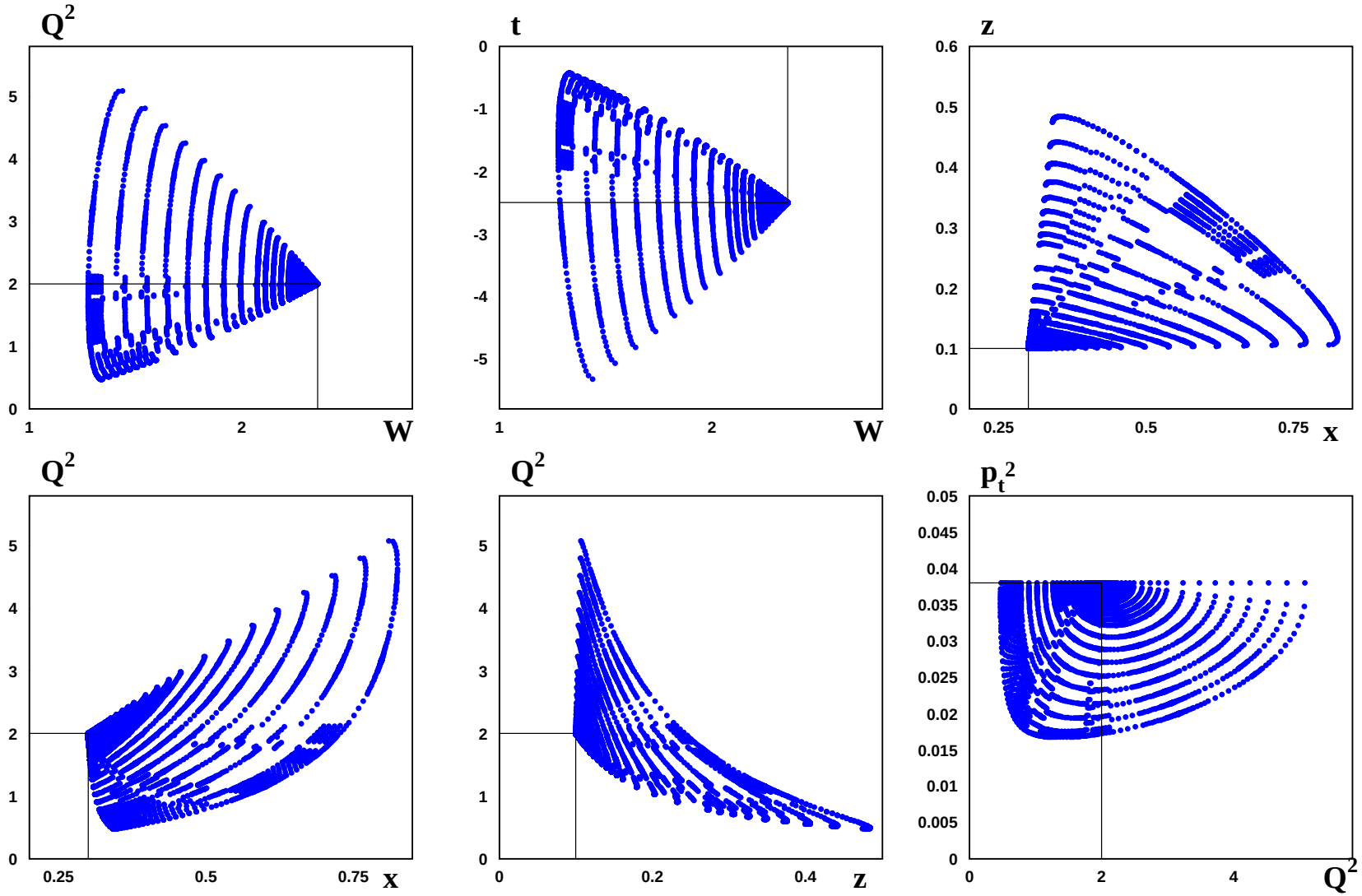
$$\Phi(p,P) = \frac{1}{2}f_1(x,p_T^2)\hat{p}|_{p^+=xP^+,p_-=0} + \dots$$

$$\Delta(k,P_h) = \frac{1}{2}D_1(z,k_T^2)\hat{k}|_{k^-=P_h^-/z,k_+=0} + \dots$$

- perform tensor integration to bring tensor structure out of the integral i.e., to have hadronic tensor in the form $W^{\mu\mu} = -g^{\mu\nu}H_1 + P^\mu P^\nu H_2 + (P^\mu P_h^\nu + P_h^\mu P^\nu)H_3 + \dots$
- and finally calculate $L_{\mu\nu}W^{\mu\nu}$, where $L_{\mu\nu}$ is the leptonic tensor with RC.

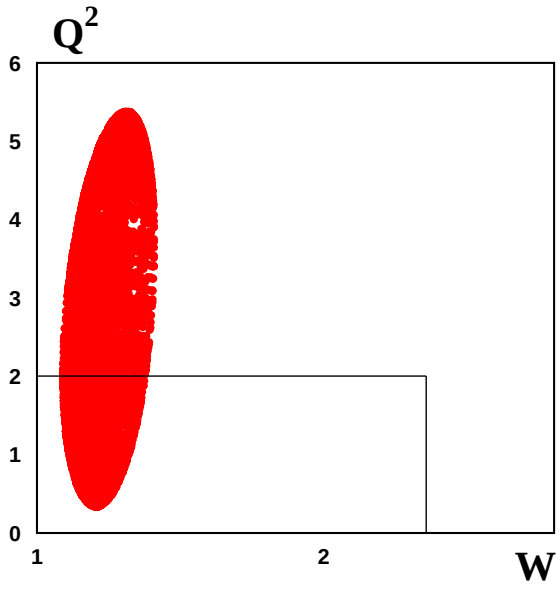
Integration area: Inelastic radiative tail

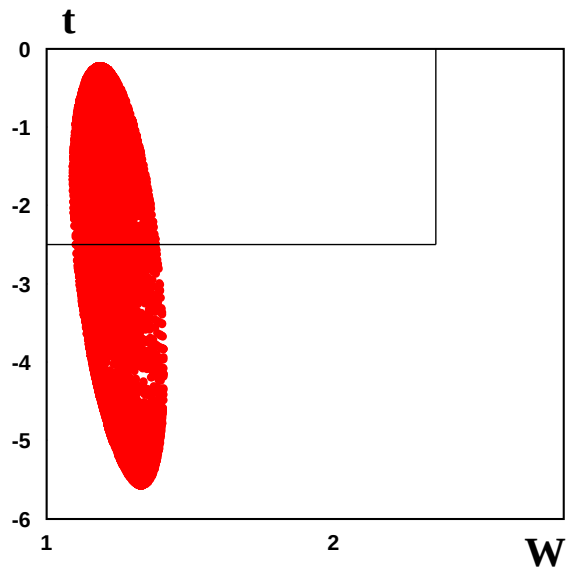
- Kinematical point: x=0.3, z=0.1, Q^2=2.0 GeV^2, t=-2.5 GeV^2.

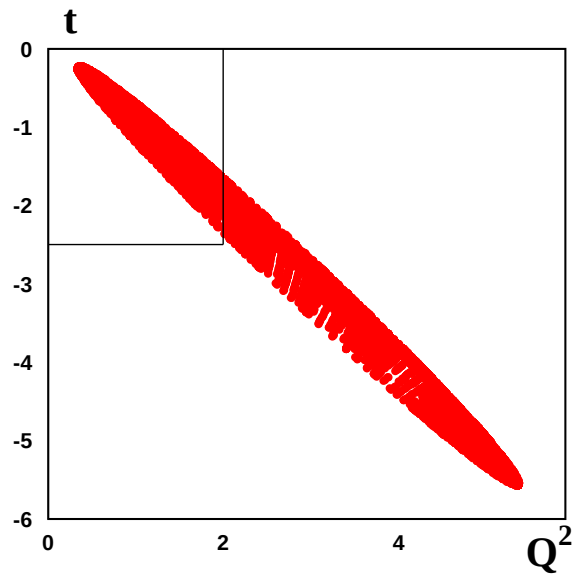


Integration area: Exclusive radiative tail

- Kinematical point: x=0.3, z=0.1, Q^2=2.0 GeV^2, t=-2.5 GeV^2.

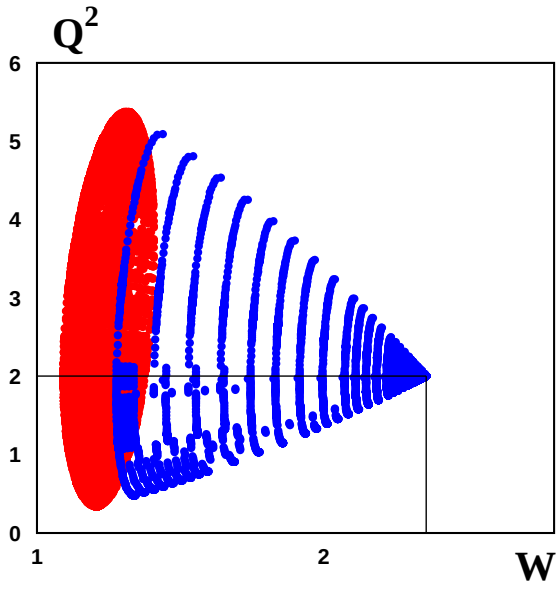


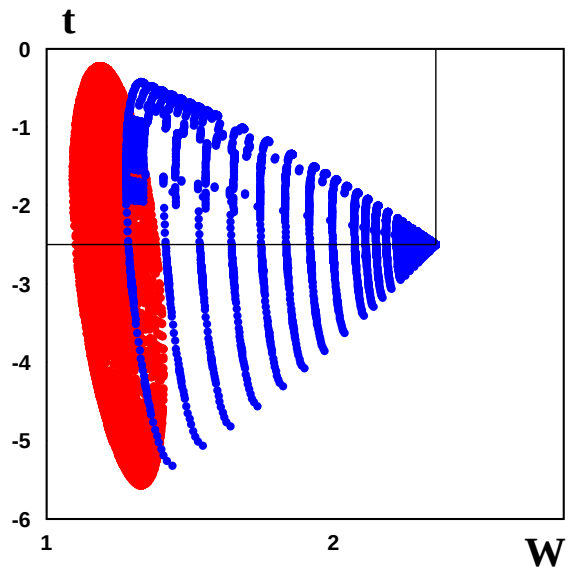


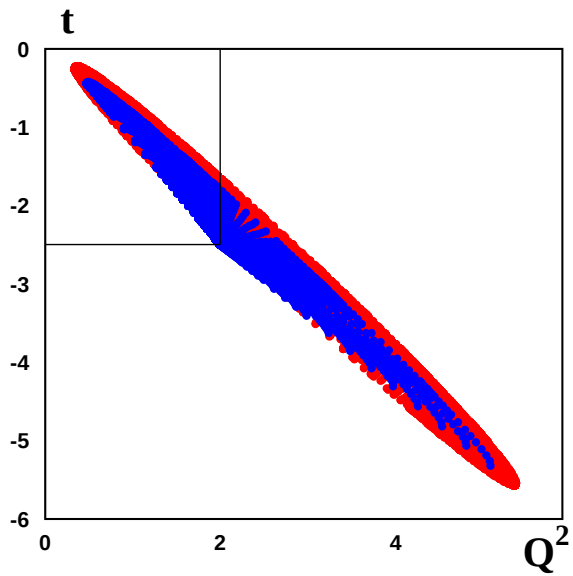


Integration area: Combined

- Kinematical point: x=0.3, z=0.1, Q^2=2.0 GeV^2, t=-2.5 GeV^2.







Conclusion: RC to SIDIS

- Two basic contributions to RC in SIDIS needs to be separately calculated: the contribution from continuous spectrum (i.e., analog of inelastic radiative tail in DIS RC) and exclusive radiative tail (i.e., analog of elastic radiative tail in DIS or radiative tail from elastic peak).
- RC from continuous spectrum to the three-dimensional cross section $d\sigma/dxdydz$ is of order 5-15% with relatively strong dependence of x and z.
- Large effects are possible from exclusive tail for small M_x.
- Large correction is expected for p_t-distribution.
- Dependence on the model of SIDIS SF is expected (e.g., models for p_t-distribution).
- Only RC to continuous spectrum was evaluated for polarization asymmetry. No large correction was found.
- Important contributions to be calculated:
  - RC to polarization asymmetries due to inelastic radiative tail
  - exclusive radiative tail for polarized case
  - RC to dihadron SIDIS
- Models for 18 structure functions are required in the broad kinematical region including the region of exclusive scattering.