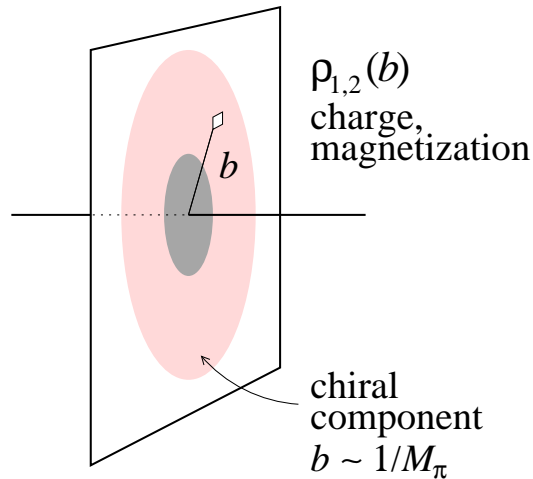


Chiral dynamics and peripheral transverse nucleon structure

C. Weiss (JLab), JLab Theory Seminar, 07–Oct–13 [Granados, CW, arXiv:1308.1634](#)



Universal element of
nucleon structure!

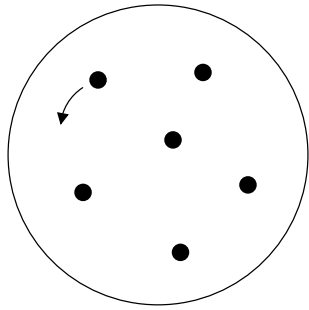
New arena for χ EFT:
Space–time picture,
 b as parameter

Low– t elastic FFs
JLab 12 E12-11-106 Gasparian et al.

Connection with GPDs,
peripheral ep/pp processes

- Light–front view of nucleon
 - Transverse densities from elastic FFs
 - Connection with GPDs
- Peripheral transverse densities
 - Dispersion representation
 - Peripheral densities from chiral EFT
 - Mechanical picture in light–front EFT,
charge vs. magnetization
 - Δ isobar and large– N_c QCD
 - Chiral vs. non–chiral component
- Experimental tests and extensions
 - Chiral component in low– $|t|$ FFs
 - GPDs and peripheral high–energy processes

Nucleon structure: Light–front view



- Non–relativistic quantum system

Particle number fixed, time absolute

$\psi(\mathbf{x}_1, \dots, \mathbf{x}_N; t)$ Schrödinger WF

$\rho(\mathbf{x}) = \sum \psi^\dagger(\dots; t)\psi(\dots; t)$ Densities

- Relativistic quantum system

Vacuum fluctuations: Particles appear/disappear

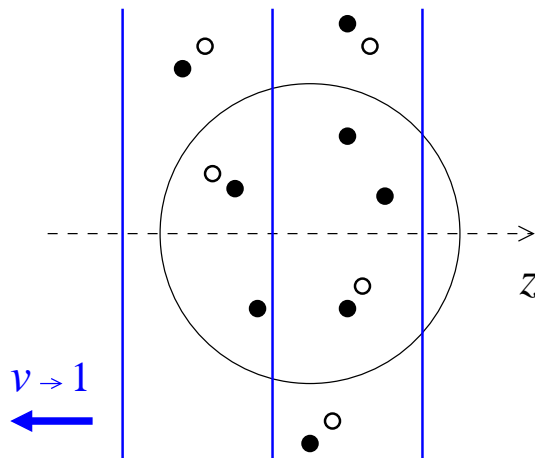
Time not absolute: How to synchronize clocks?

Light–front time $x^+ = x^0 + x^3$:

Observer moving with velocity $v \rightarrow 1$

Wave function at fixed x^+ : Components with different particle number

Densities at fixed x^+ : Boost–invariant!



$$x^+ = x^0 + x^3 = \text{const.}$$

- Advantages of light–front view

Objective notion of spatial structure

Connection with high–energy scattering

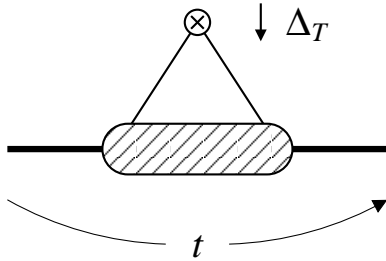
Probes system at fixed LF time. Cf. parton picture in QCD

Alt. view: Observer at rest, system moves with $v \rightarrow 1$. Infinite–momentum frame

Nucleon structure: Transverse densities

- Current matrix element parametrized by invariant form factors

$$\langle N' | J_\mu | N \rangle \rightarrow F_1(t), F_2(t) \quad \text{Dirac, Pauli}$$

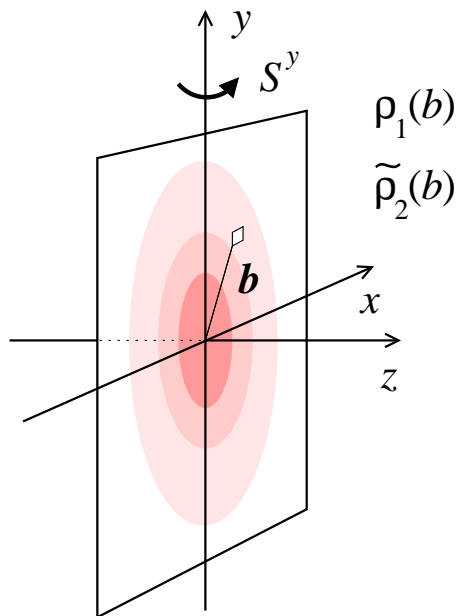


- Transverse densities $t = -\Delta_T^2$
Soper 76, Burkardt 00, Miller 07

$$F_{1,2}(t) = \int d^2b e^{i\Delta_T b} \rho_{1,2}(b) \quad \text{2D Fourier}$$

$\rho_{1,2}(b)$ charge/magnetization density

\mathbf{b} displacement from transverse C.M.

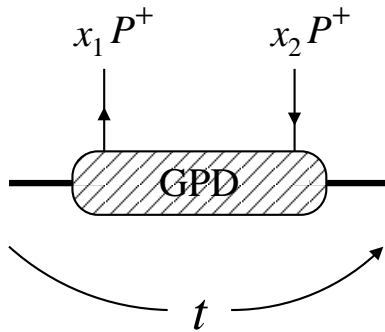


- Interpretation in polarized nucleon

$$\langle J^+(\mathbf{b}) \rangle_{y\text{-pol}} = \rho_1(b) \quad \text{spin-indep.}$$

$$+ (2S^y) \cos \phi \underbrace{\frac{d}{db} \left[\frac{\rho_2(b)}{2M_N} \right]}_{\tilde{\rho}_2(b)} \quad \text{spin-dep.}$$

Nucleon structure: Connection with GPDs



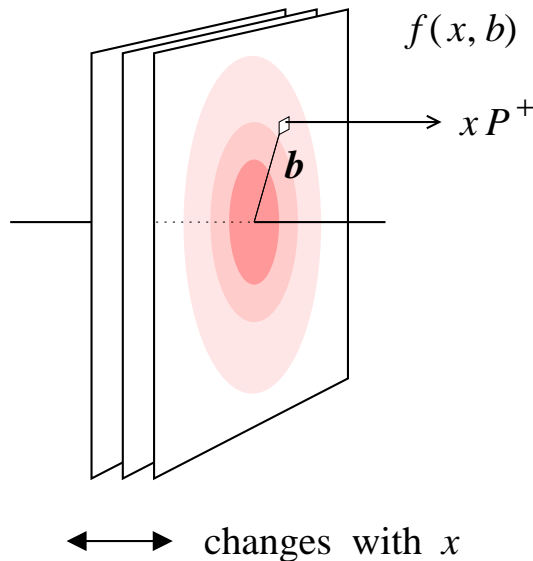
- Generalized parton distribution

$$\langle N' | \underbrace{\bar{\psi}(0) \dots \psi(z)}_{\text{QCD light-ray operator, } z^2 = 0} | N \rangle \rightarrow H(x_1, x_2; t), E(\dots), \dots$$

- Transverse distribution of partons $x_1 = x_2 = x$
Burkardt 00

$$H(x, x; t) = \int d^2b e^{i\Delta_T b} f(x, b)$$

Transverse spatial distribution of partons with LC momentum xP^+ : “Tomography”

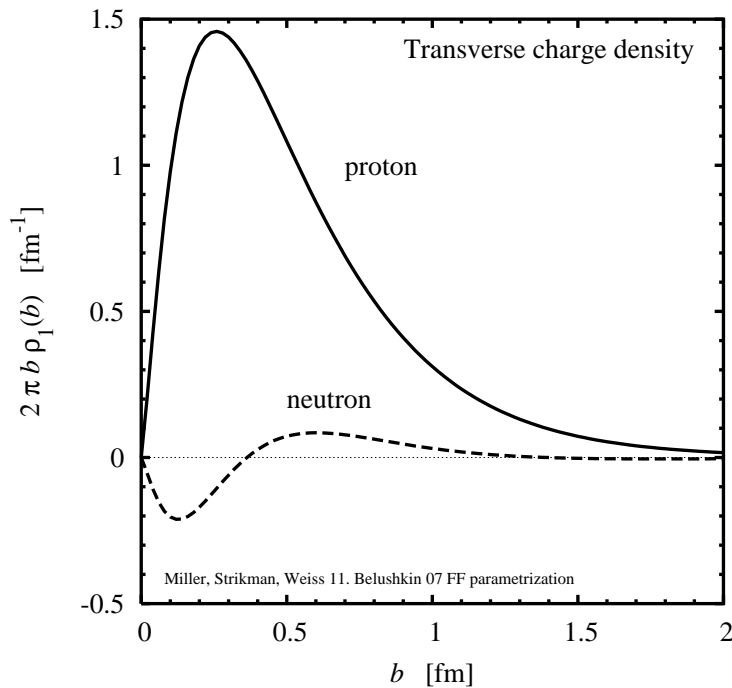


- Transverse densities as reduction

$$\rho_1(b) = \sum_q e_q \int_0^1 dx [f_q(x, b) - f_{\bar{q}}(x, b)] \quad \text{etc.}$$

Dual role of transverse densities:
Accessible through low-energy elastic FFs,
interpretable in context of QCD partons

Nucleon structure: Peripheral densities



- Empirical transverse densities from elastic form factor data

Experimental and incompleteness errors estimated [Venkat, Arrington, Miller, Zhan 10](#)

Recent low- and high- $|t|$ data incorporated [MAMI: Vanderhaeghen, Walcher 10](#). [JLab Hall A Riordan et al.](#)

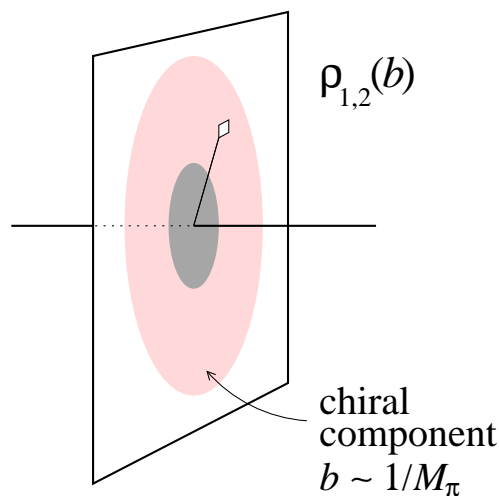
Many interesting questions: Neutron, flavor structure, charge vs. magnetization
 Also $N \rightarrow \Delta$, deuteron: [Carlson, Vanderhaeghen 08](#)

- Peripheral densities $b = O(M_\pi^{-1})$

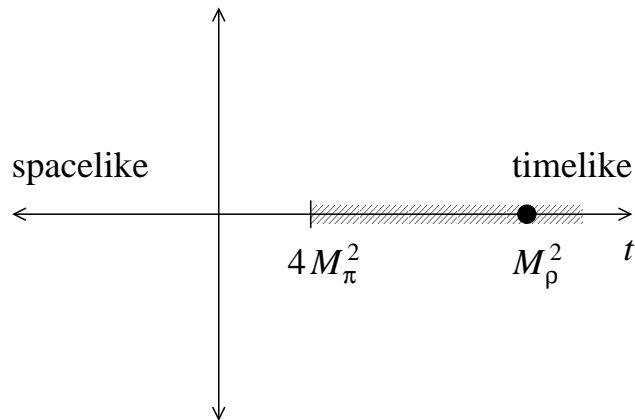
Governed by chiral dynamics: universal, model-independent, calculable using EFT methods

Theoretical interest: Parametric control, space-time picture of EFT dynamics, chiral vs. non-chiral contributions

Practical interest: Low- $|t|$ form factors, connection w. peripheral quark/gluon structure



Peripheral densities: Dispersion representation

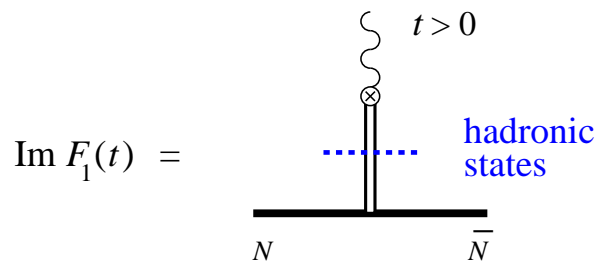


- Dispersion representation of form factor

$$F_{1,2}(t) = \int_{4m_\pi^2}^{\infty} \frac{dt'}{t' - t - i0} \frac{\text{Im } F_{1,2}(t')}{\pi}$$

Spectral function $\text{Im } F_{1,2}(t')$ describes “process”
current \rightarrow hadronic states $\rightarrow N\bar{N}$

Unphysical region: $\text{Im } F_{1,2}(t')$ from theory, FF fits
Höhler et al. 76; Belushkin, Hammer, Meissner 06



- Transverse densities

$$\rho_{1,2}(b) = \int_{4m_\pi^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{t}b) \frac{\text{Im } F_{1,2}(t)}{\pi}$$

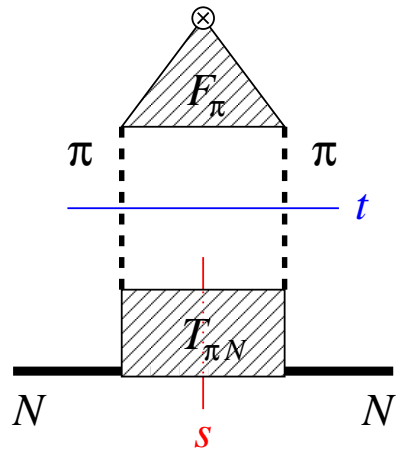
$K_0 \sim e^{-b\sqrt{t}}$ exponential suppression of large t

Distance b selects masses $\sqrt{t} \sim 1/b$: “Filter”
Cf. Borel transformation in QCD sum rules. Strikman, CW 10

Isvector: $\pi\pi, \rho, \rho', \dots$
Isoscalar: $\omega, \phi, K\bar{K}, \dots$

Peripheral $\rho(b) \longleftrightarrow$ low-mass hadronic states

Peripheral densities: Spectral function



- Spectral function near threshold

Two-pion exchange with $t - 4M_{\pi}^2 = O(M_{\pi}^2)$

Subthreshold singularity on unphysical sheet from N pole in πN scattering amplitude

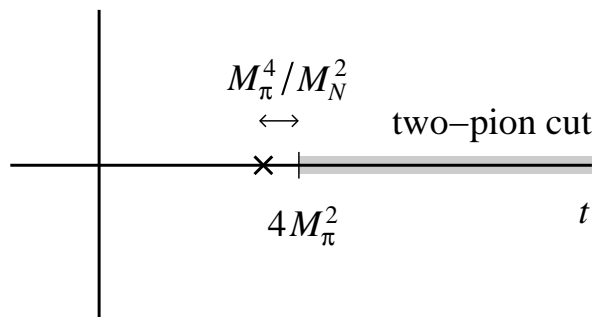
Anomalously small scale M_{π}^4/M_N^2

Dominates behavior of spectral function near threshold!

- Parametric regions of distances

$$b \sim M_{\pi}^{-1} \quad t - 4M_{\pi}^2 \sim M_{\pi}^2 \quad \text{“chiral”} \leftarrow$$

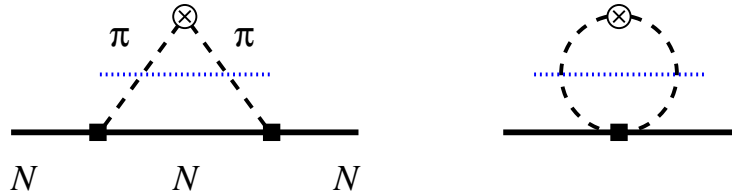
$$\sim \frac{M_N^2}{M_{\pi}^3} \quad \sim \frac{M_{\pi}^4}{M_N^2} \quad \text{“molecular”}$$



Distances in molecular region extremely large, \sim several 10 fm. Practical use?

Cf. NN potential Robilotta 96. Review Epelbaum

Peripheral densities: Chiral component



- Spectral functions from chiral EFT

Gasser et al. 87; Bernard et al. 96, Kubis, Meissner 00, Kaiser 03

Expansion in $k_\pi/\Lambda_\chi \ll 1$. Lagrangian from chiral symmetry + phenom. constants

Relativistic nucleon: Analytic structure, subthreshold singularity [Becher, Leutwyler 99](#)

Efficient calculation: t -channel cut only, Cutkosky rules, no regularization

Compact analytic expressions

- Chiral component of isovector densities

[Strikman, CW 10](#); [Granados CW 13](#)

$$\rho_1^V, \tilde{\rho}_2^V(b) = e^{-2M_\pi b} \times \text{function}(M_N, M_\pi; b)$$

“Yukawa tail” with range $2M_\pi$, pre-exponential factor with rich structure

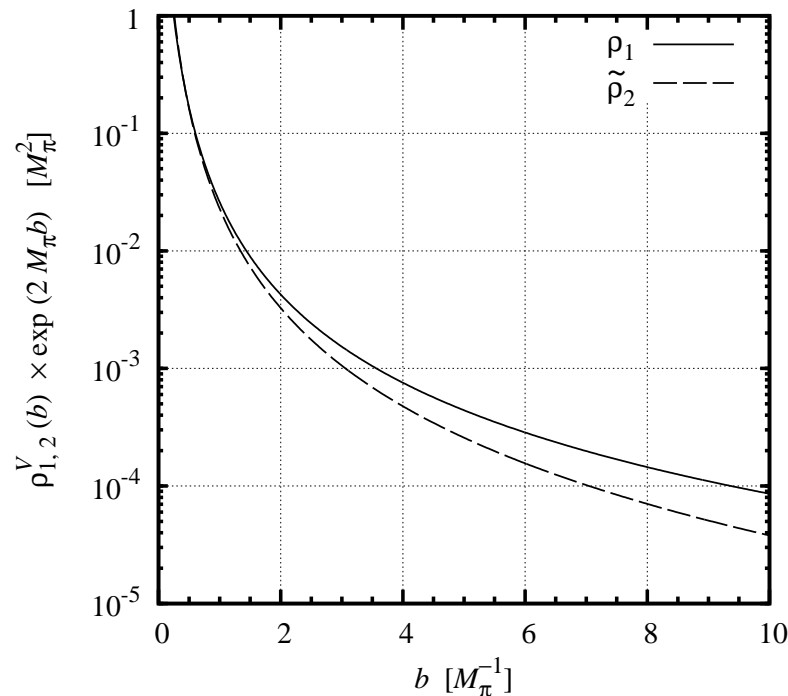
Heavy-baryon expansion:

$\rho_1, \tilde{\rho}_2$ of same order in M_π/M_N

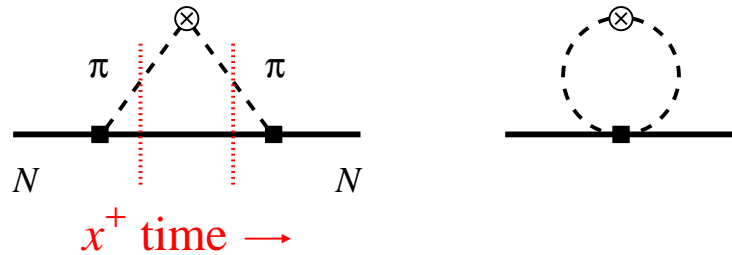
Convergence, numerical accuracy of HBE: [Granados CW 13](#).

Interesting inequality: $\tilde{\rho}_2^V(b) < \rho_1(b)$

Explanation?



Peripheral densities: Time-ordered formulation



- Time-ordered formulation of χ EFT
Follow evolution in LF time $x^+ = x^0 + x^3$

- Wave function of chiral πN system

Describes transition $N \rightarrow N\pi$ in χ EFT, calculable from chiral Lagrangian

Universal, frame-independent
Also in high-energy processes, $\bar{u} - \bar{d}$, etc.

Pion momentum fraction $y \sim M_\pi/M_N$,
transverse distance $r_T \sim M_\pi^{-1}$

Orbital angular momentum $L_z = 0, 1$

- Densities as wave function overlap

Explains inequality $|\rho_2^V| < \rho_1$ [Granados, CW 13](#)

Contact terms $\delta(y)$ represent high-mass interm. states. Coefficient $(1 - g_A^2)$

Equivalent to invariant formulation
[Granados, CW 13](#). Cf. also [Ji, Melnitchouk et al. 09+](#)

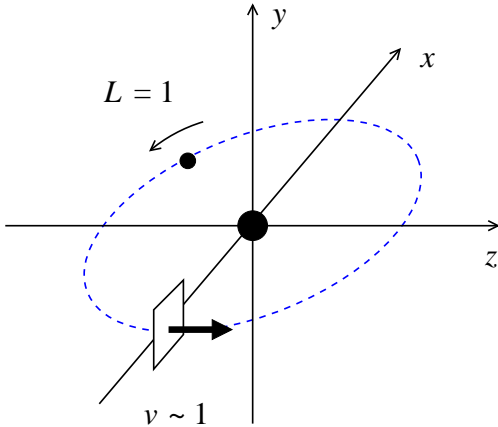
$$\psi_{L=0,1}^{\pi N}(y, \mathbf{r}_T) = \frac{\langle \pi N | \mathcal{L}_\chi | N \rangle}{\underbrace{p_\pi^- + p_{N'}^- - p_N^-}_{\text{energy denominator}}}$$

$$\rho_1^V(b) = \int_0^1 dy \left[|\psi_0|^2 + |\psi_1|^2 \right]_{r_T=b/\bar{y}}$$

+ contact term

$$\tilde{\rho}_2^V(b) = \dots \quad \psi_0^* \psi_1 + \psi_1^* \psi_0$$

Peripheral densities: Rest frame



- Rest frame picture Granados, CW 13

LF formulation boost-invariant!

Nucleon state polarized in y -direction.
Intermediate pion orbits with $L_y = 1$

- Explains peripheral densities

$\langle J^+(\mathbf{b}) \rangle = \rho_1(b) + (2S^y) \cos \phi \tilde{\rho}_2(b) \geq 0$
for current carried by quasi-real pion,
therefore $|\tilde{\rho}_2| \leq \rho_1$

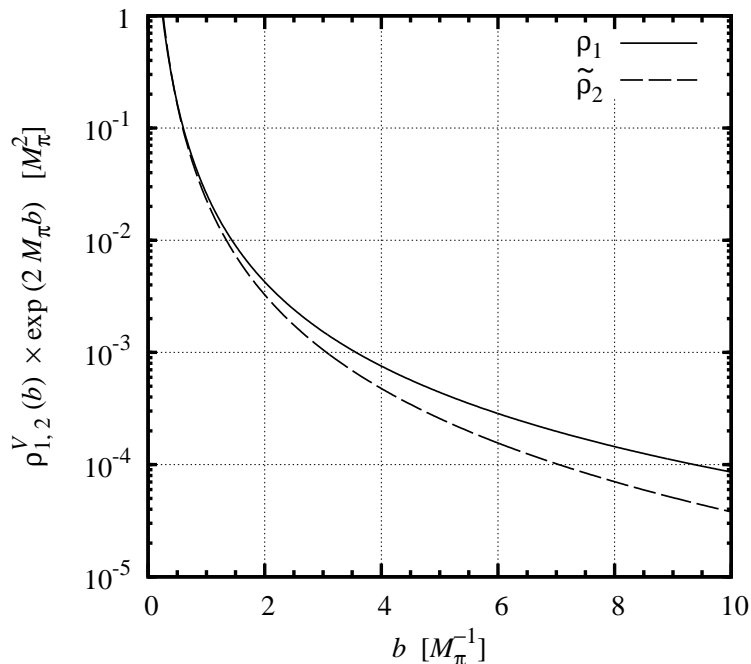
$\tilde{\rho}_2/\rho_1 \sim v_\pi$ pion velocity

- Mechanical interpretation of χ EFT

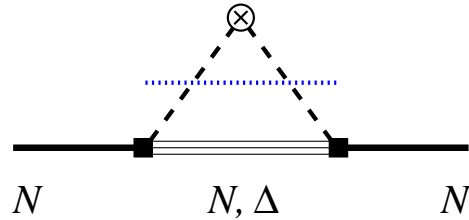
Bare N fluctuates into πN system
via χ EFT interaction

Peripheral densities result from charge/current
carried by pion at $b = O(M_\pi)$

Fully relativistic! Model-independent dynamics!



Peripheral densities: Δ isobar



- Two-pion component with intermediate Δ

Large coupling due to spin/isospin

N and Δ degenerate in large- N_c limit of QCD:
 $M_\Delta - M_N = O(N_c^{-1})$

Δ contribution to peripheral densities calculated in relativistic Rarita-Schwinger formalism

Strikman, CW 10, Granados, CW 13

- Peripheral densities in large- N_c limit of QCD

$$\rho_1^V(b) \sim N_c^0$$

General N_c -scaling in QCD, $b = O(N_c^0)$

$$\tilde{\rho}_2^V(b) \sim N_c$$

$$\rho_1^V(N \text{ alone}) \sim N_c$$

Wrong... too large!

$$\tilde{\rho}_2^V(N \text{ alone}) \sim N_c$$

$$\rho_1^V(N + \Delta) \sim N_c^0$$

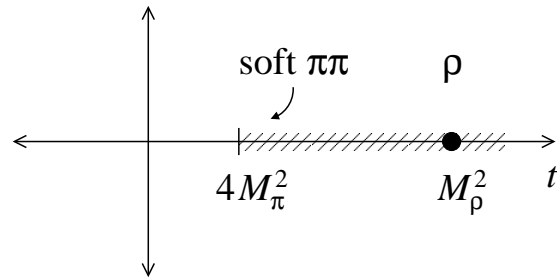
Δ restores correct N_c -scaling of ρ_1^V

$$\tilde{\rho}_2^V(N + \Delta) \sim N_c$$

Two-pion component has correct N_c scaling if Δ included

Cf. Isovector electric/magnetic radii. Cohen, Broniowski 92; Cohen 96

Peripheral densities: Chiral vs. non-chiral



- At what distances does the chiral component of densities become numerically dominant?

Strikman, CW 10

Model higher mass states in spectral function by ρ meson pole

Refined estimates w. empirical spectral functions
Miller, Strikman, CW 11

Chiral component dominates only at $b > 2$ fm. Surprisingly large!

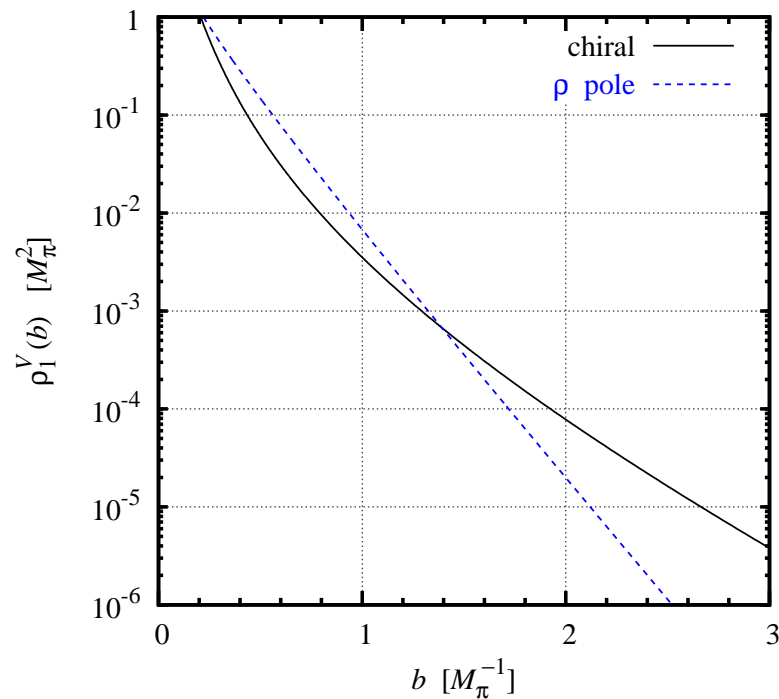
Reasons are strength of ρ ,
suppression of $\pi\pi$ near threshold

- Spatial representation as new way of identifying chiral component

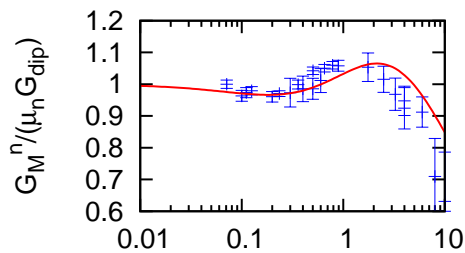
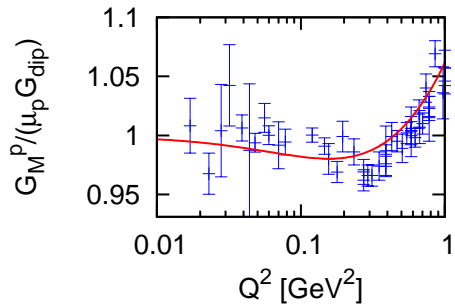
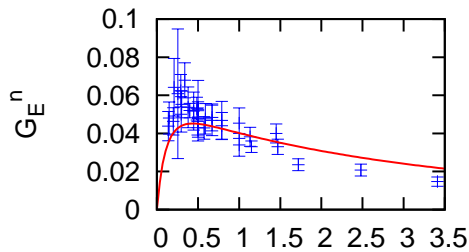
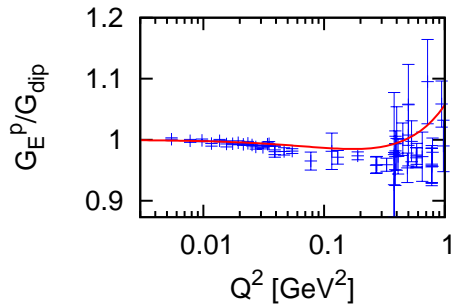
Model-independent, fully relativistic

Impact parameter b objectively defined,
observable in exclusive processes

\leftrightarrow Breit frame radius



Chiral component: Effect on low- t form factors



Dispersion fit: Lorenz, Hammer, Meissner 12.
Includes recent MAMI data

- Moments of transverse charge density

$$\begin{aligned} \langle b^2 \rangle &= \int d^2b b^2 \rho(b) = 4 F_1'(0) \\ \langle b^4 \rangle &= 32 F_1''(0) \end{aligned}$$

- Contribution of chiral component *isovector*

$$\langle b^2 \rangle_{\text{chiral}} \approx 0.2 \times \langle b^2 \rangle_{\text{fit}} \quad \text{small}$$

$$\langle b^4 \rangle_{\text{chiral}} \approx 1.5 \times \langle b^2 \rangle_{\text{fit}}^2 \quad \text{sizable}$$

Chiral component should be visible in “unnatural” second and higher derivatives of FF at $Q^2 = 0$

Can we extract it?

- Analyticity of form factor fit is essential

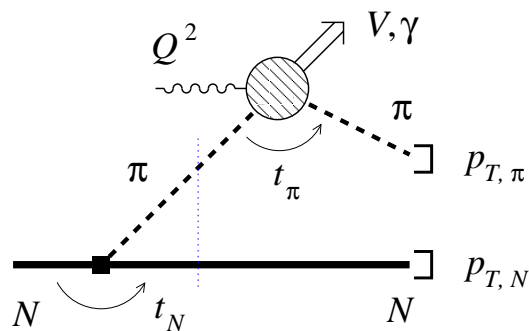
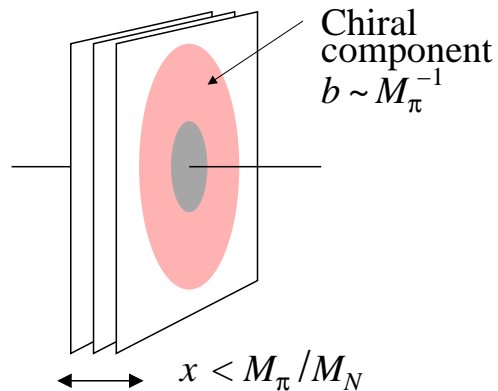
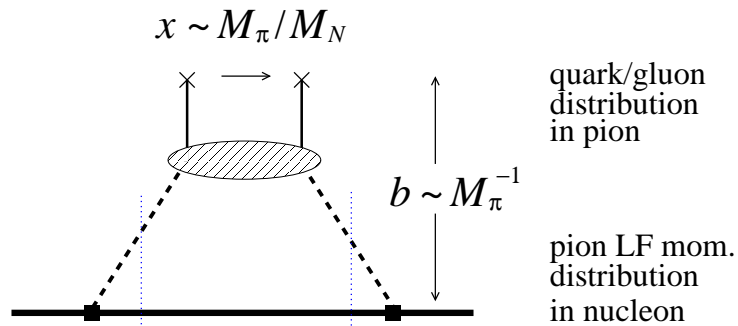
Needs dispersion analysis: Belushkin et al. 07; Lorenz et al. 12

- Affects extrapolation to $t \rightarrow 0$

CLAS/PRIMEX 12 GeV experiment at $Q^2 = 10^{-4} - 10^{-2} \text{ GeV}^2$

PR12-11-106 Gasparian et al.

Chiral component: Partonic structure



- Peripheral quark/gluon structure of nucleon
Strikman, CW PRD69:054012,2004; PRD80:114029,2009

Parton densities at $b \sim M_\pi^{-1}$ and $x \sim M_\pi / M_N$

Calculable from χ EFT πN wave functions and empirical quark/gluon densities in pion
Same πN WFs as in transverse charge/current densities!

Small fraction of total parton number:
Most partons sit at distances $b \lesssim 0.5$ fm

Increase of nucleon's transverse size below $x \sim M_\pi / M_N$

- Exclusive processes on peripheral pion

Soft peripheral pion requires
 $x \ll M_\pi / M_N \sim 0.1$

$p_{T, \pi} \sim 1$ GeV \gg $p_{T, N} \sim 100$ MeV
suppresses production on nucleon

Probes GPDs in pion at $|t_\pi| \sim 1$ GeV²
Fundamental interest. Moments calculable in Lattice QCD

Detection of low- p_T forward nucleon and moderate- p_T pion

Summary

- Light–front view provides concise spatial representation of relativistic system
 - Elastic FFs reveal transverse densities
 - Independent of dynamics — can be applied to QCD, χ EFT, ...
- Peripheral transverse densities from χ EFT
 - Chiral expansion justified by $b = O(M_\pi^{-1})$, new parameter
 - Chiral component dominant only at large $b \gtrsim 2$ fm
 - Inclusion of Δ ensures proper N_c scaling of densities
- Light–front time evolution of χ EFT offers new insights
 - Mechanical picture of low–energy chiral nucleon structure
 - Connection with quark/gluon structure and high–energy processes
- Many extensions and applications
 - FFs of energy–momentum tensor — transverse densities of mass, momentum, forces
Granados, CW, in preparation
 - Axial and pseudoscalar FFs — constraining spin–dependent quark GPDs
 - Transverse densities from amplitude analysis