Cake Seminar
Oct 30, 2013

From QCD to Nuclear Physics

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Jefferson Lab

In collaboration with:
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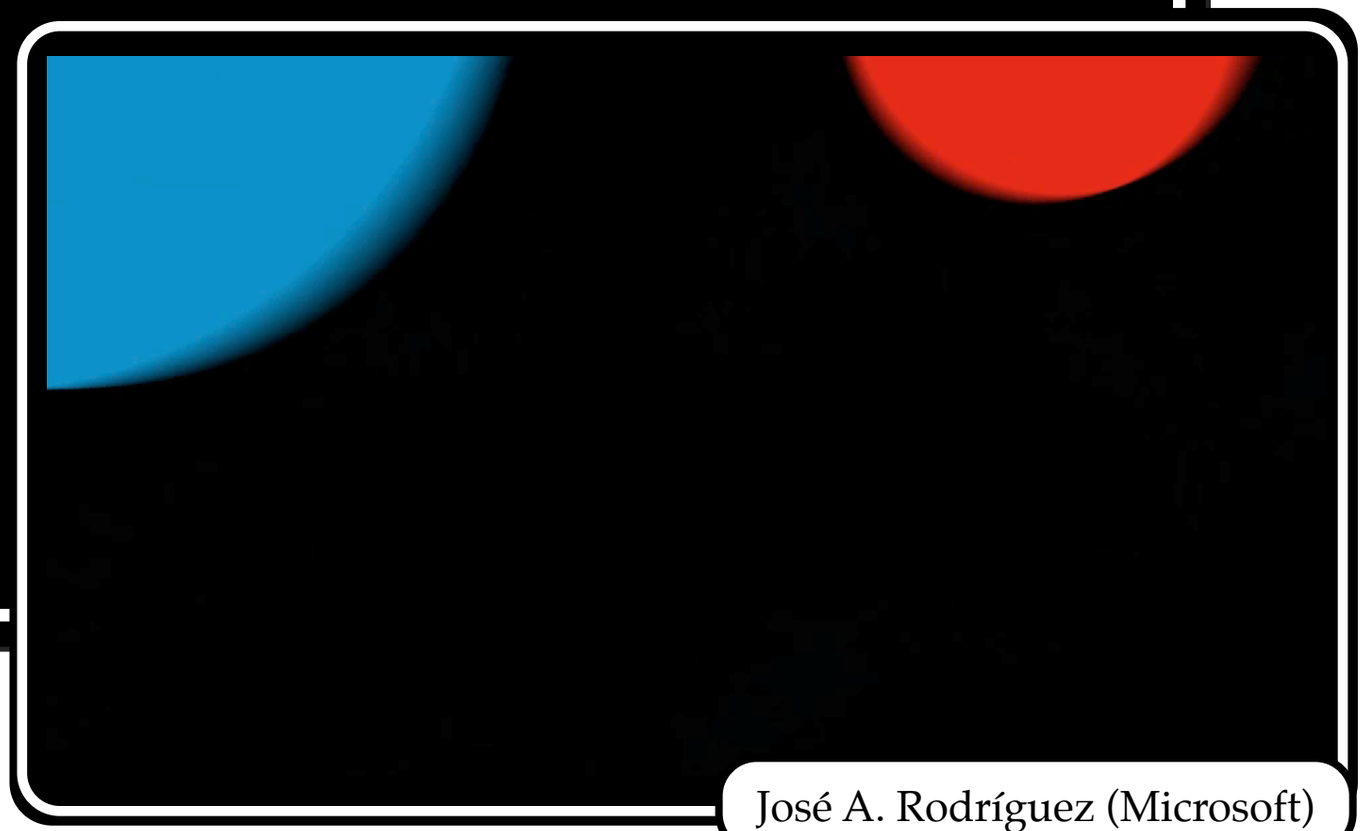
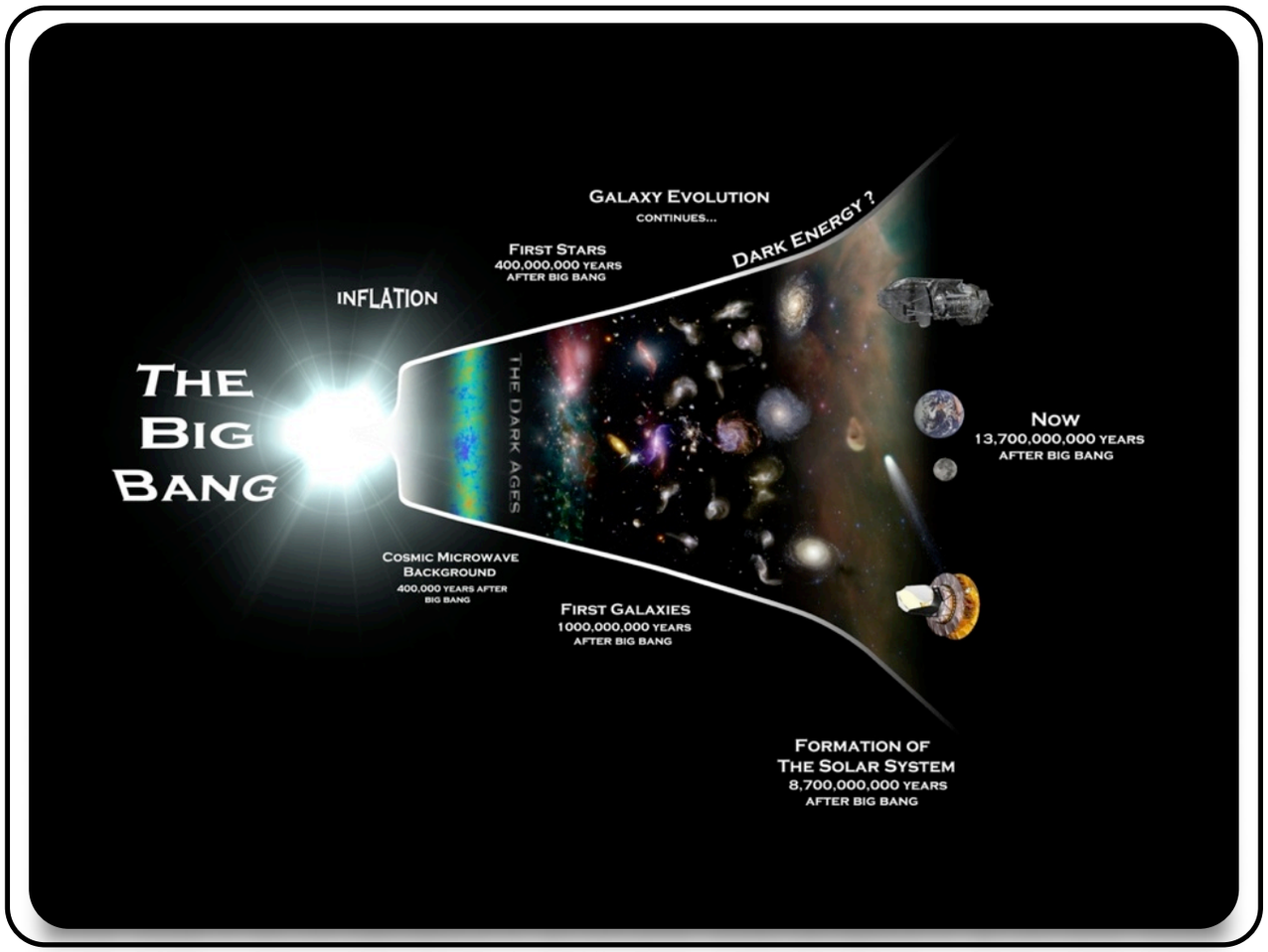
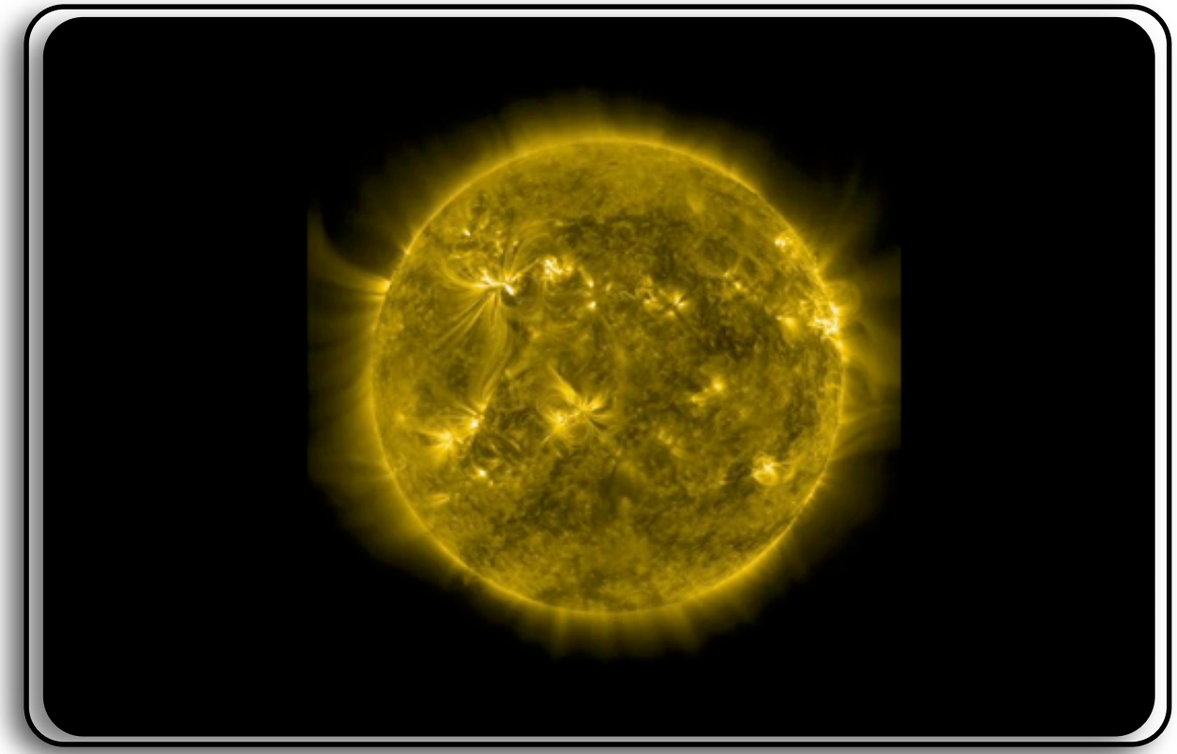


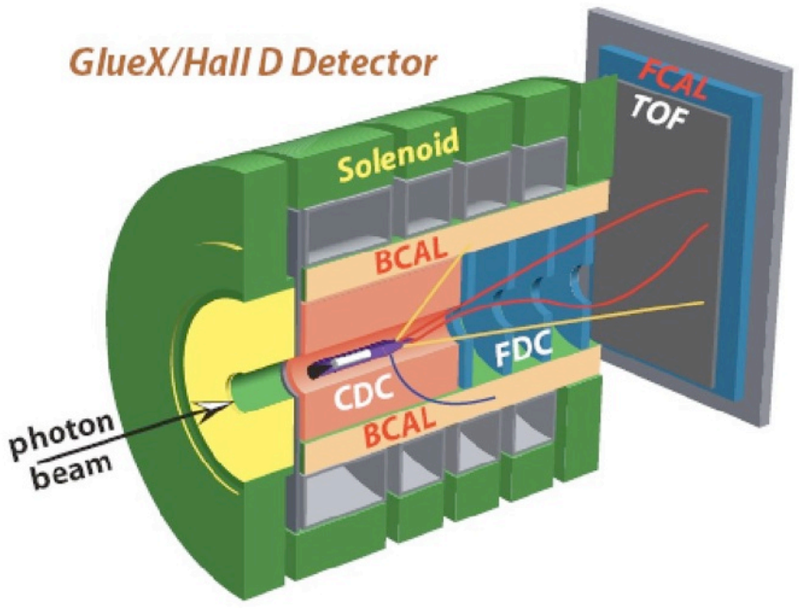
José A. Rodríguez (Microsoft)

arXiv:1204.1110 [hep-lat]
arXiv:1305.4903 [hep-lat]
arXiv:1309.3556 [hep-lat]
arXiv:1309.7923 [hep-lat]

From the cosmological to the subatomic





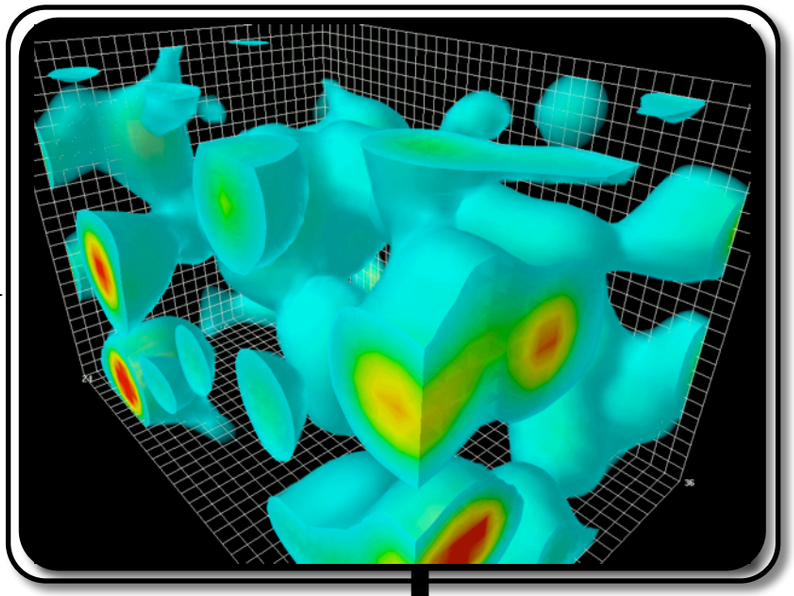


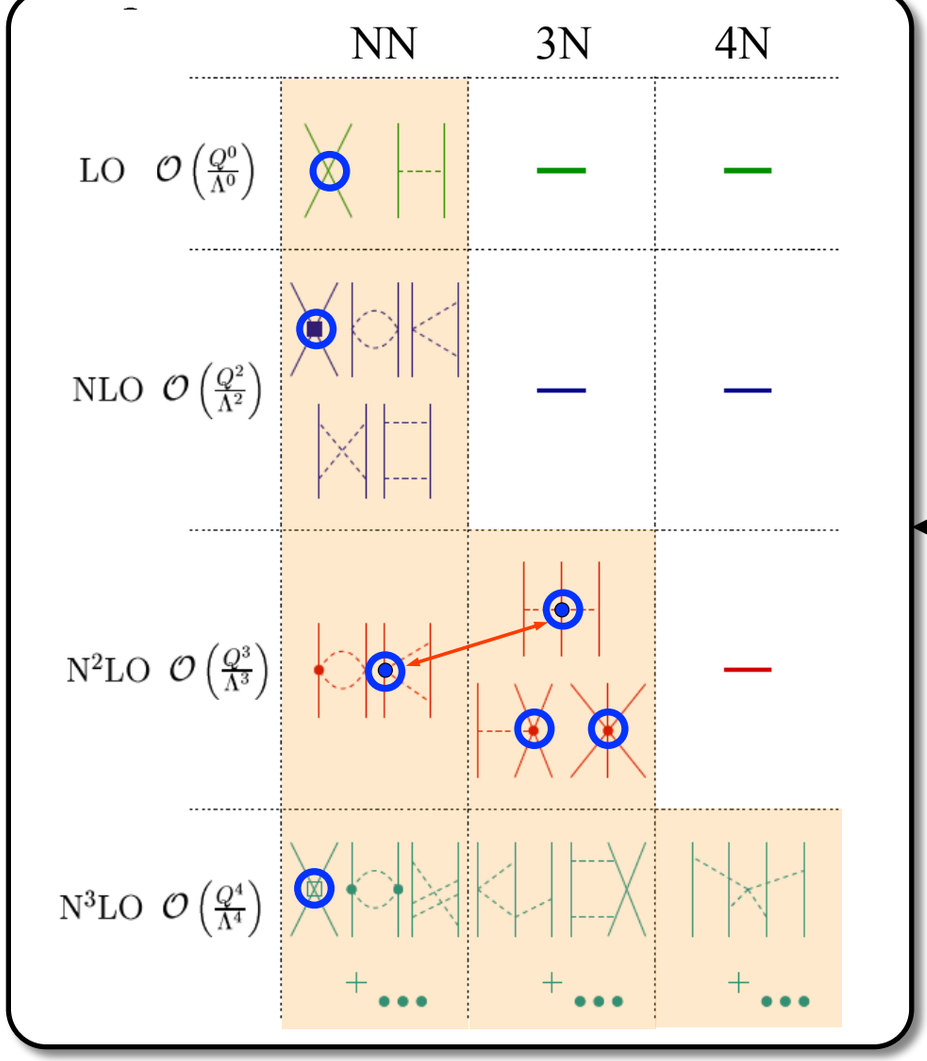
Quantum Chromodynamics

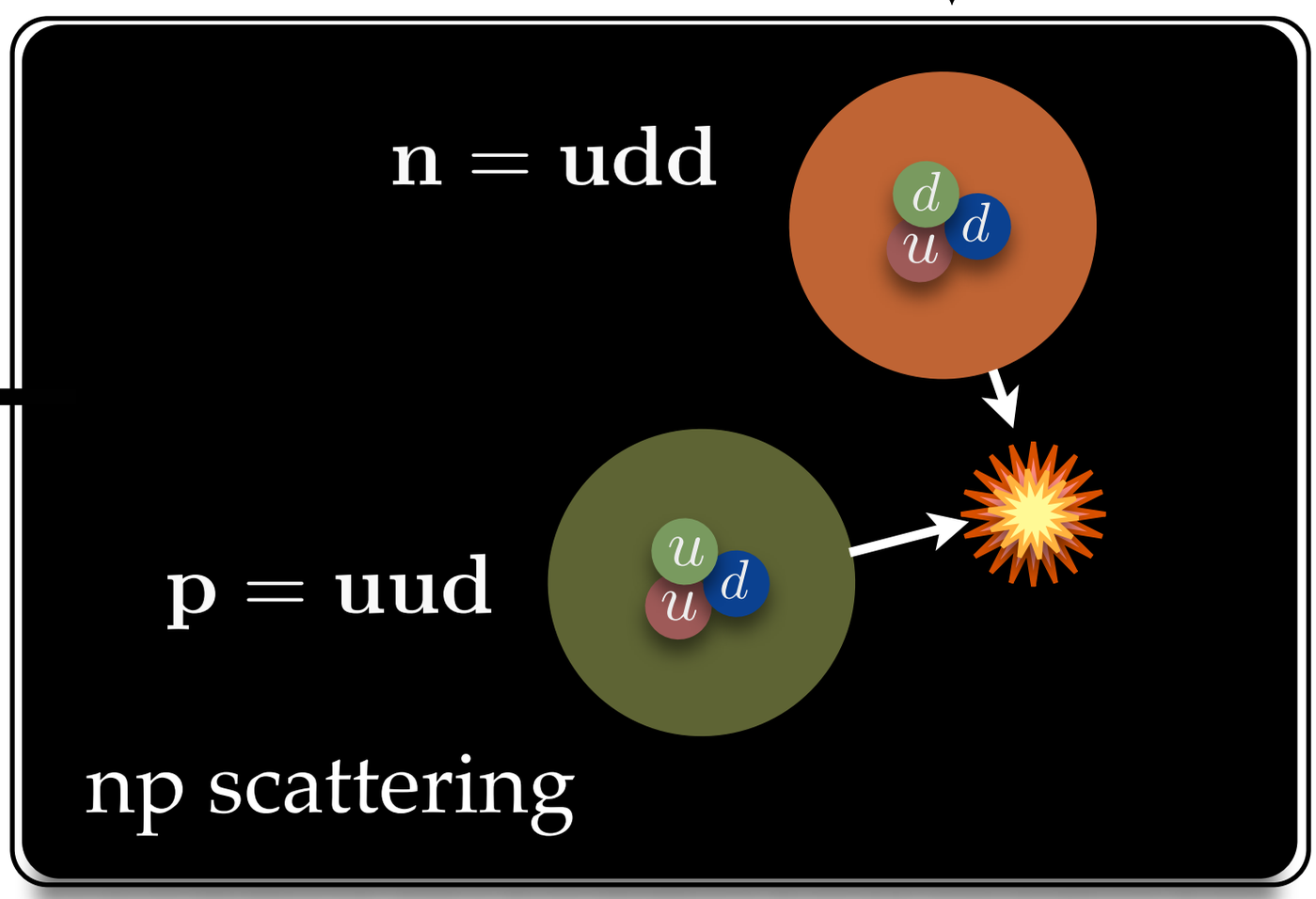
$$\mathcal{L}_{\mathbf{QCD}} = \bar{\psi}_{\mathbf{f}} \left(\mathbf{i} \not{\mathbf{D}} - \mathrm{m}_{\mathrm{f}} \right) \psi_{\mathbf{f}} - \frac{1}{4} \mathrm{tr} \left(\mathrm{GG} \right)$$

Four inputs : $\Lambda_{\mathbf{QCD}}, \frac{\mathbf{m_u}}{\mathbf{\Lambda_{QCD}}}, \frac{\mathbf{m_d}}{\mathbf{\Lambda_{QCD}}}, \frac{\mathbf{m_s}}{\mathbf{\Lambda_{QCD}}}$

QCD Vacuum (Leinweber)







Infinite Continuous Spacetime

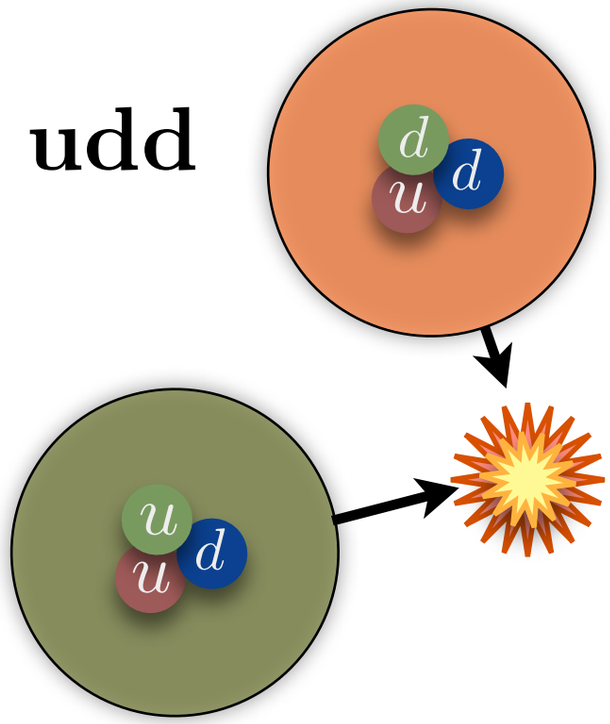
Cannot be solved analytically

Can be studied numerically

Doesn't fit in a computer

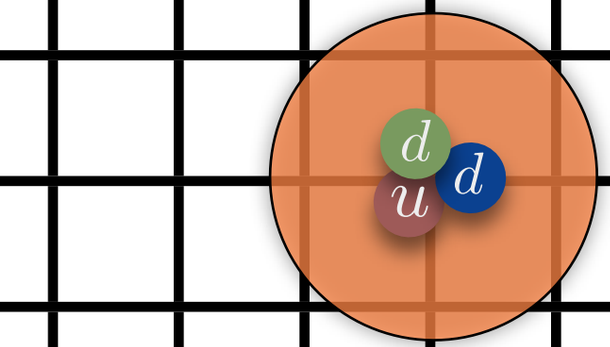
$\mathbf{n = udd}$

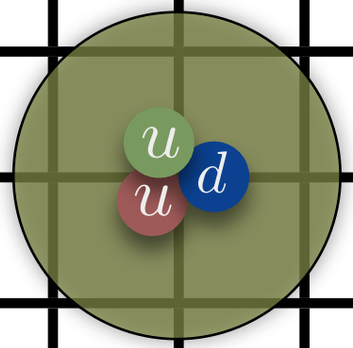
$\mathbf{p = uud}$



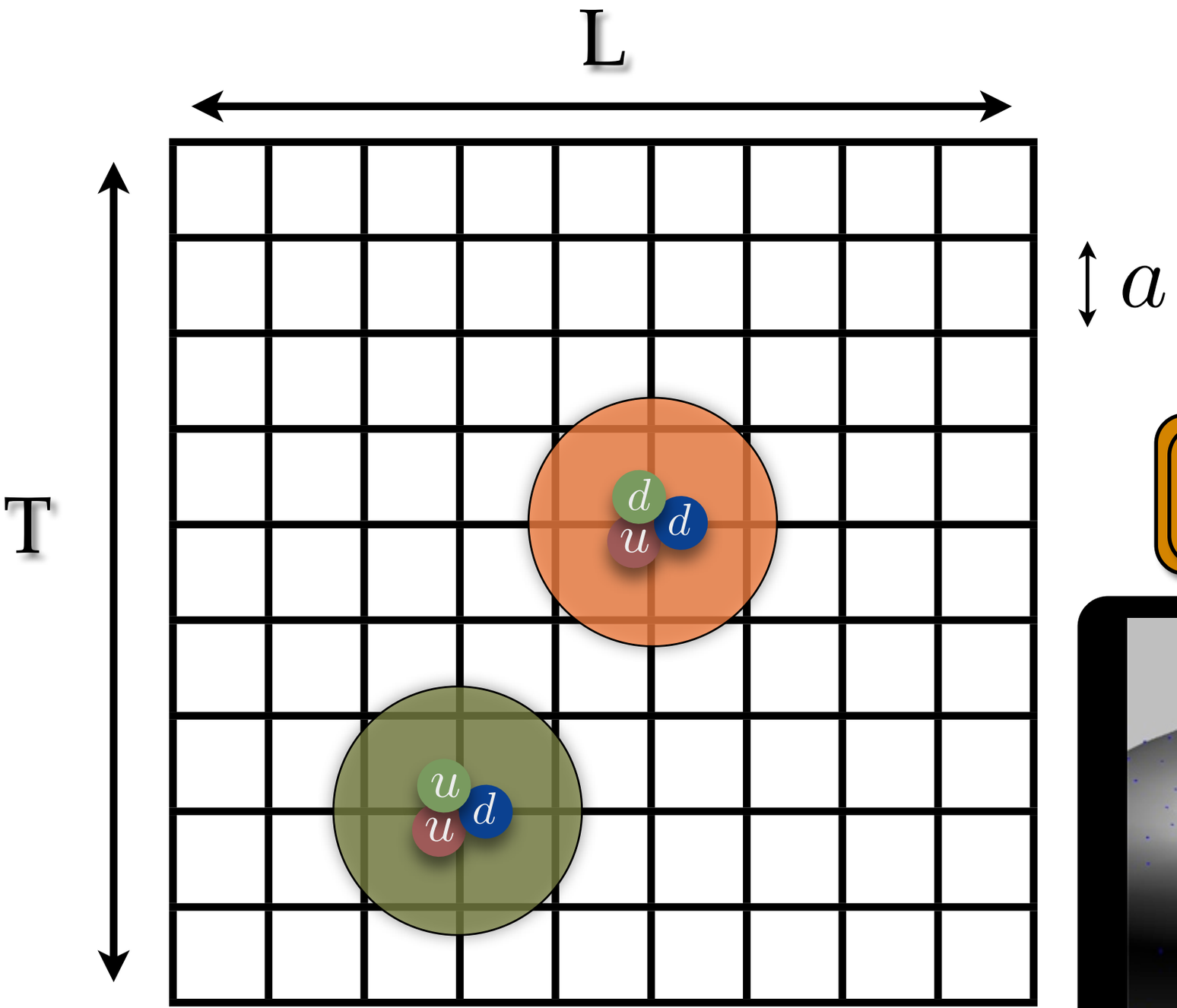
np scattering

Infinite Discretized Spacetime





LQCD: Finite Euclidean Spacetime

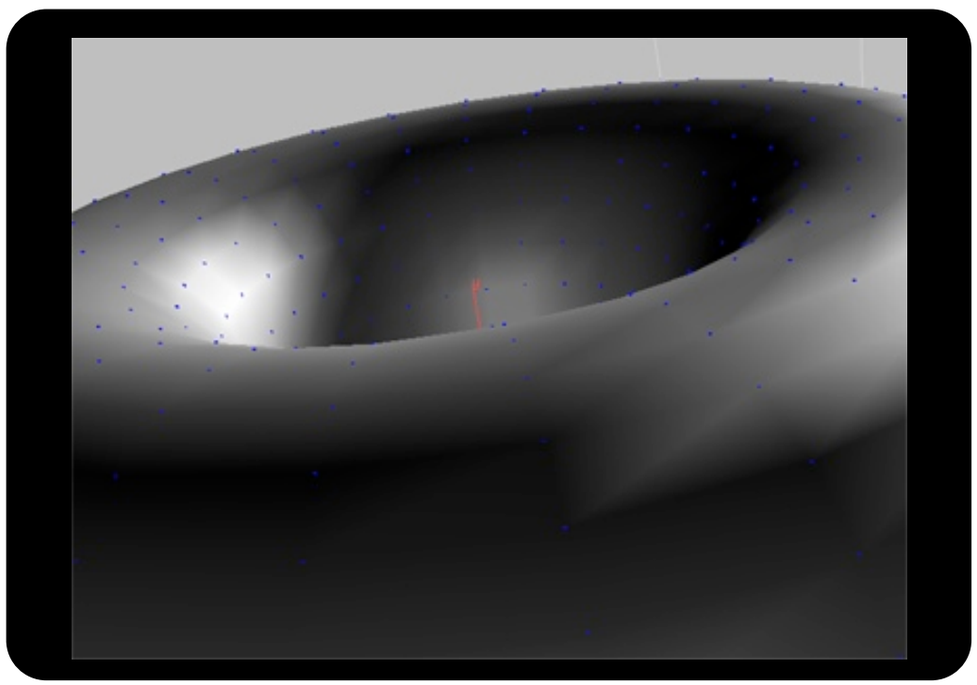


neutron-proton in a 4D torus

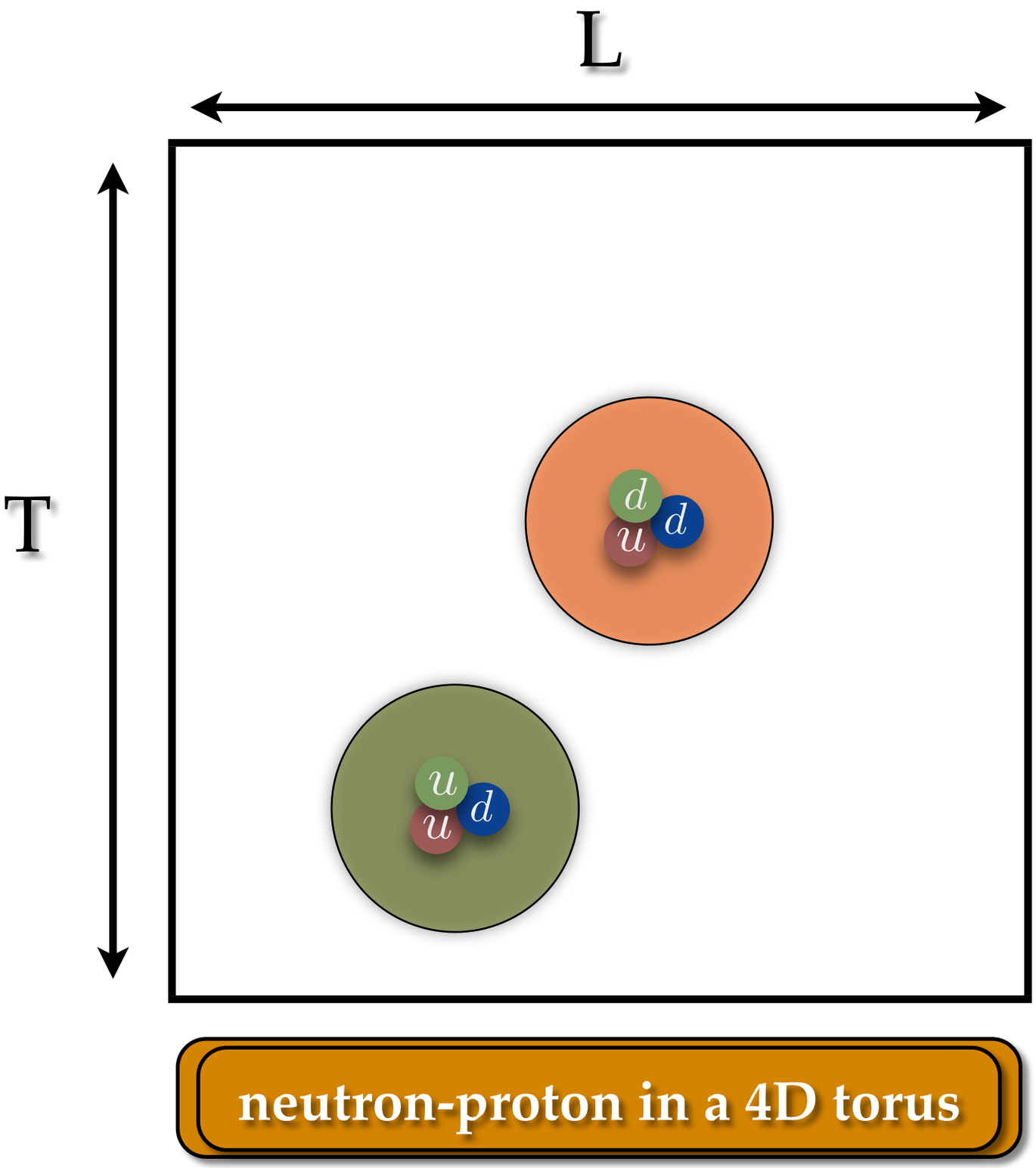
$$t \to i\tau$$

~~LSZ~~

2D torus



LQCD: Finite Euclidean Spacetime



neutron-proton in a 4D torus

$$a \to 0$$
$$\mathrm{T} \to \infty$$

- **Lüscher (1986) : scalar bosons**
- **Beane, Bedaque, Parreño & Savage(2003):**

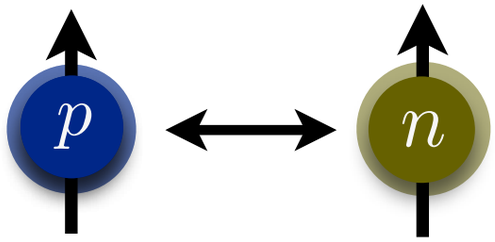
  S-wave NN-system (CM-frame)
- **Ishizuka (2009):**

  NN-system (CM-frame)

Rich Structure of NN

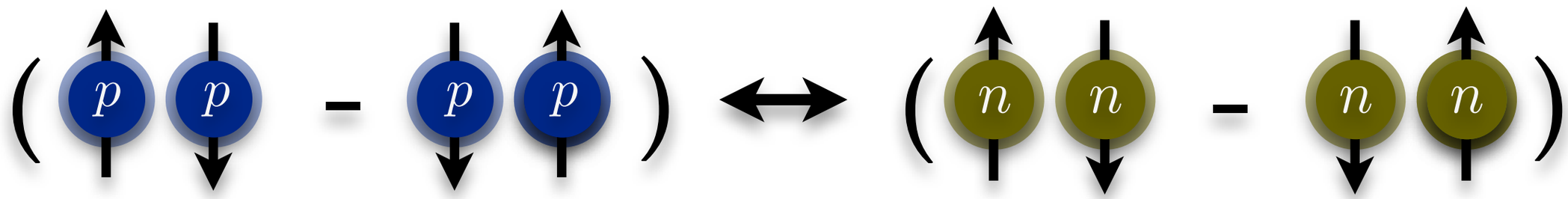
- Turn off QED
- $m_u = m_d$

} Exact Isospin symmetry

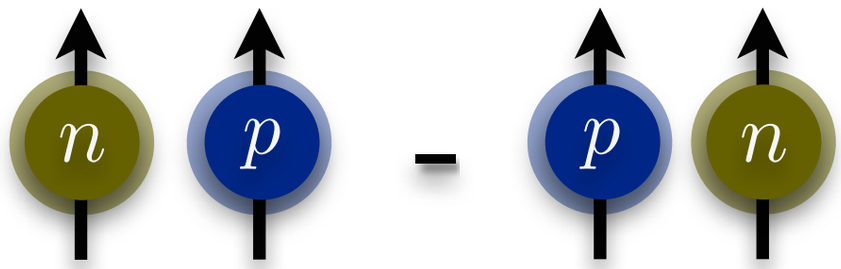


$m_N \sim 940$ MeV

- Wavefunction must be antisymmetric
- l=2n, I=1, S=0 :



- l=2n, I=0, S=1:



Deuteron channel (S-D)

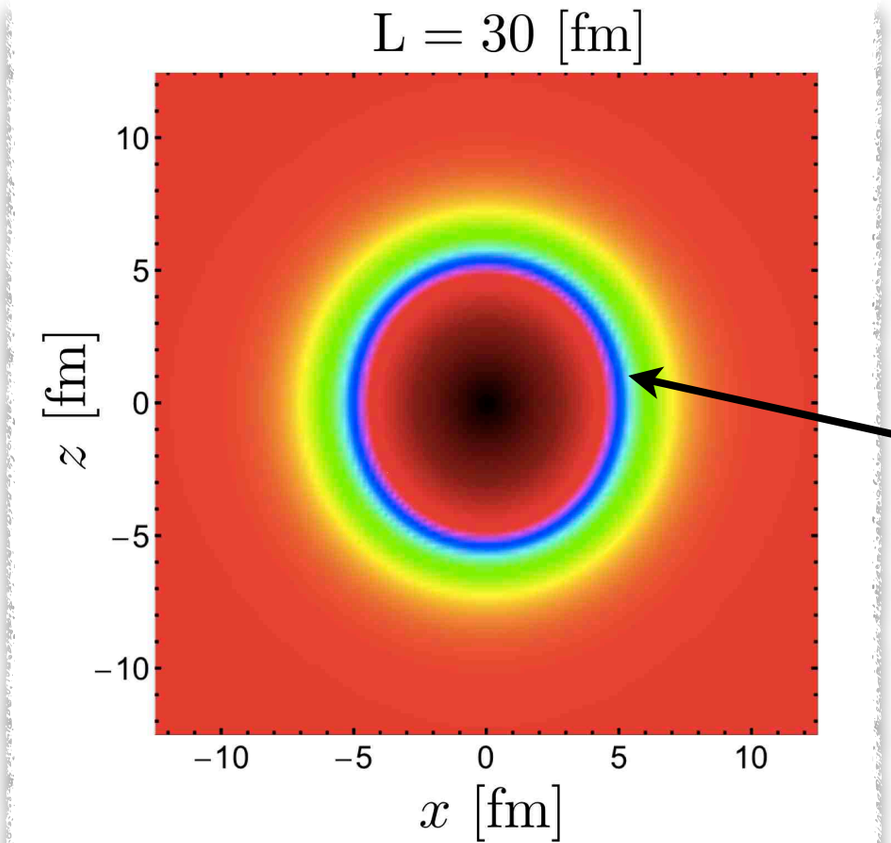
- Can also have (l=2n+1, I=0, S=0), (l=2n+1, I=1, S=1)

Coupled-Channels

- Nuclear sector:

$$\mathcal{M}_{J=1} = \begin{pmatrix} \mathcal{M}^{S} & \mathcal{M}^{SD} \\ \boxed{\mathcal{M}^{DS}} & \mathcal{M}^{D} \end{pmatrix}$$

The nuclear force has non-central components



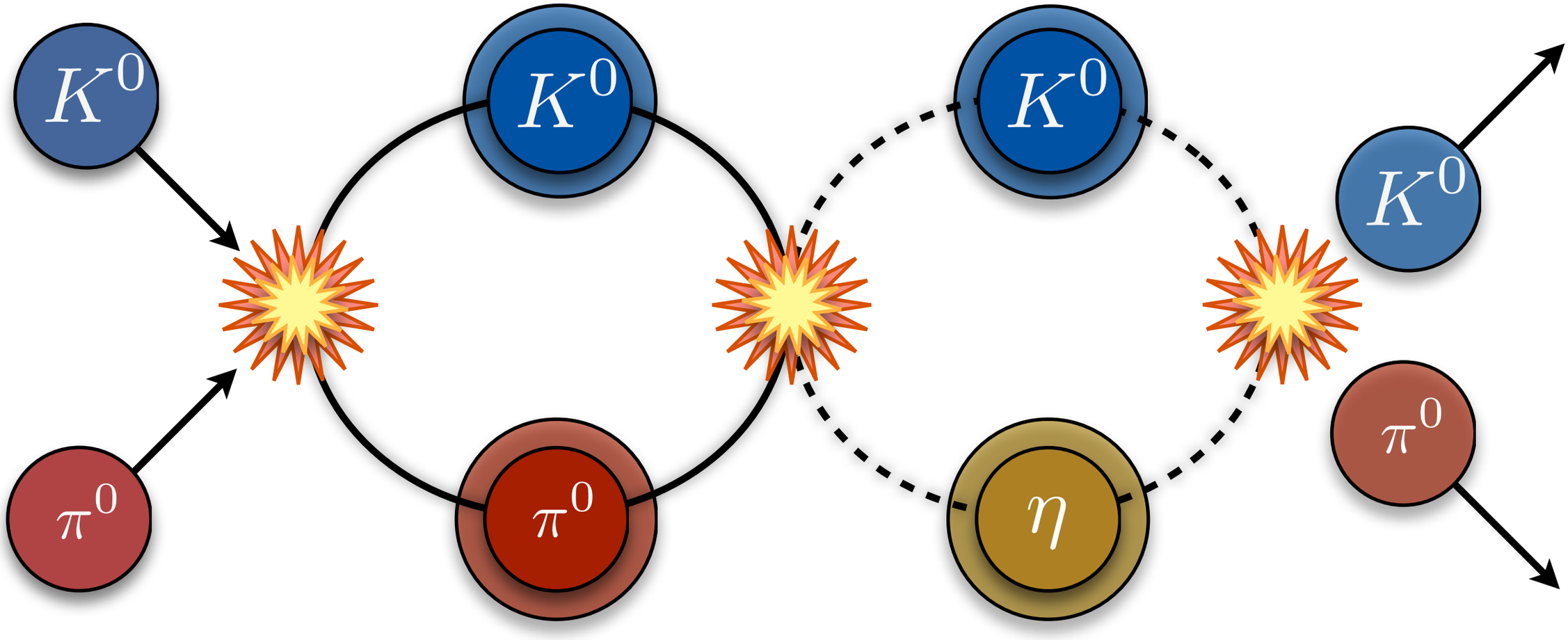
The deuteron is prolate-shaped

Bears some resembles to....

- Scalar sector:

$$\mathcal{M}_{I=1/2} = \begin{pmatrix} \mathcal{M}_{K\pi-K\pi} & \mathcal{M}_{K\pi-K\eta} \\ \mathcal{M}_{K\eta-K\pi} & \mathcal{M}_{K\eta-K\eta} \end{pmatrix}$$

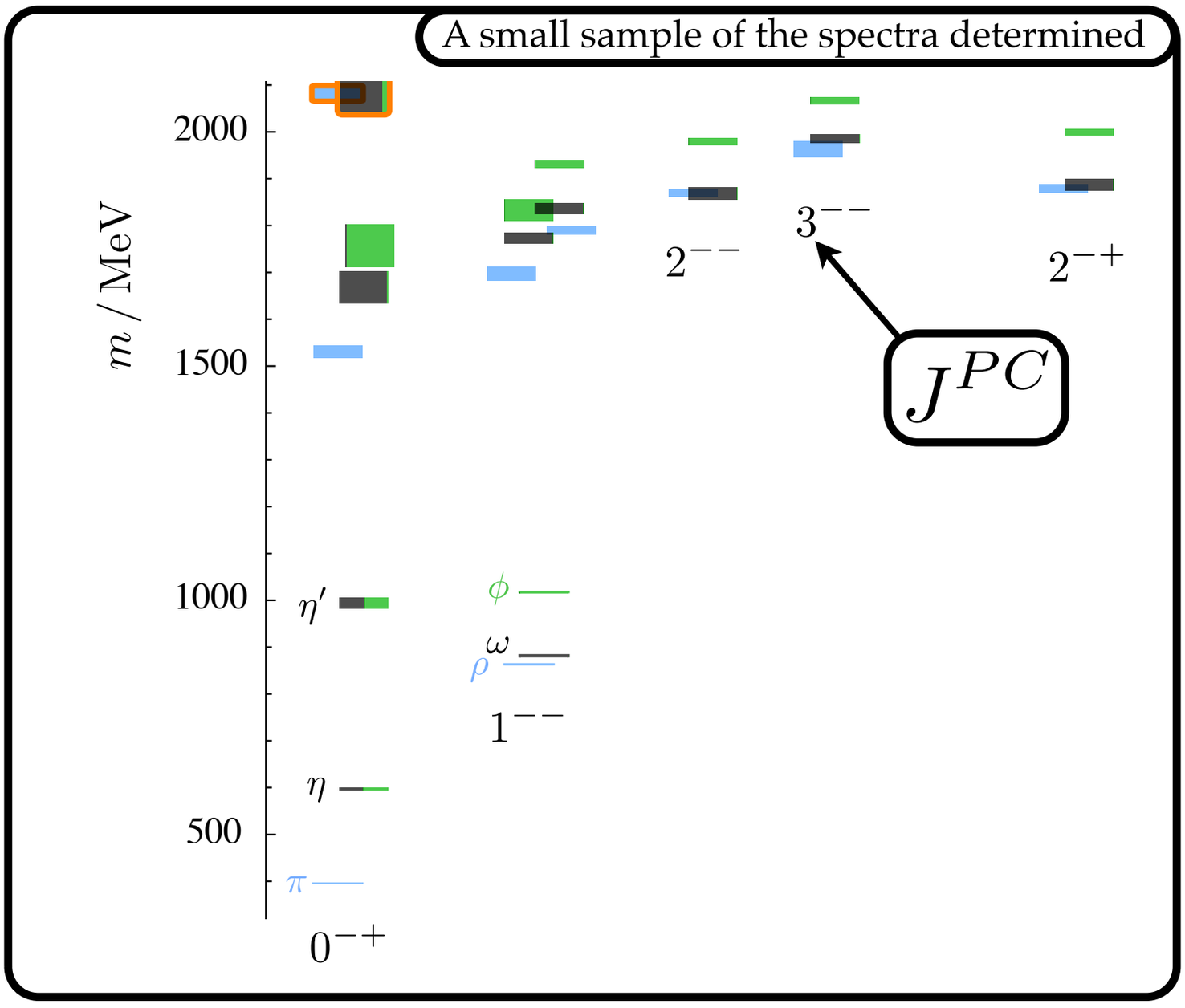
Coupled Channels in the Scalar Sector



arXiv:1204.1110 [hep-lat]

RB & Zohreh Davoudi (UW)

The state of the art!



$m_\pi = 392\,\mathrm{MeV}$

$24^3 \times 128$

$\ell\, s$

isoscalar

isovector

arXiv:1309.2608 [hep-lat]

J. Dudek, R. Edwards, P. Guo & C. Thomas (2013)

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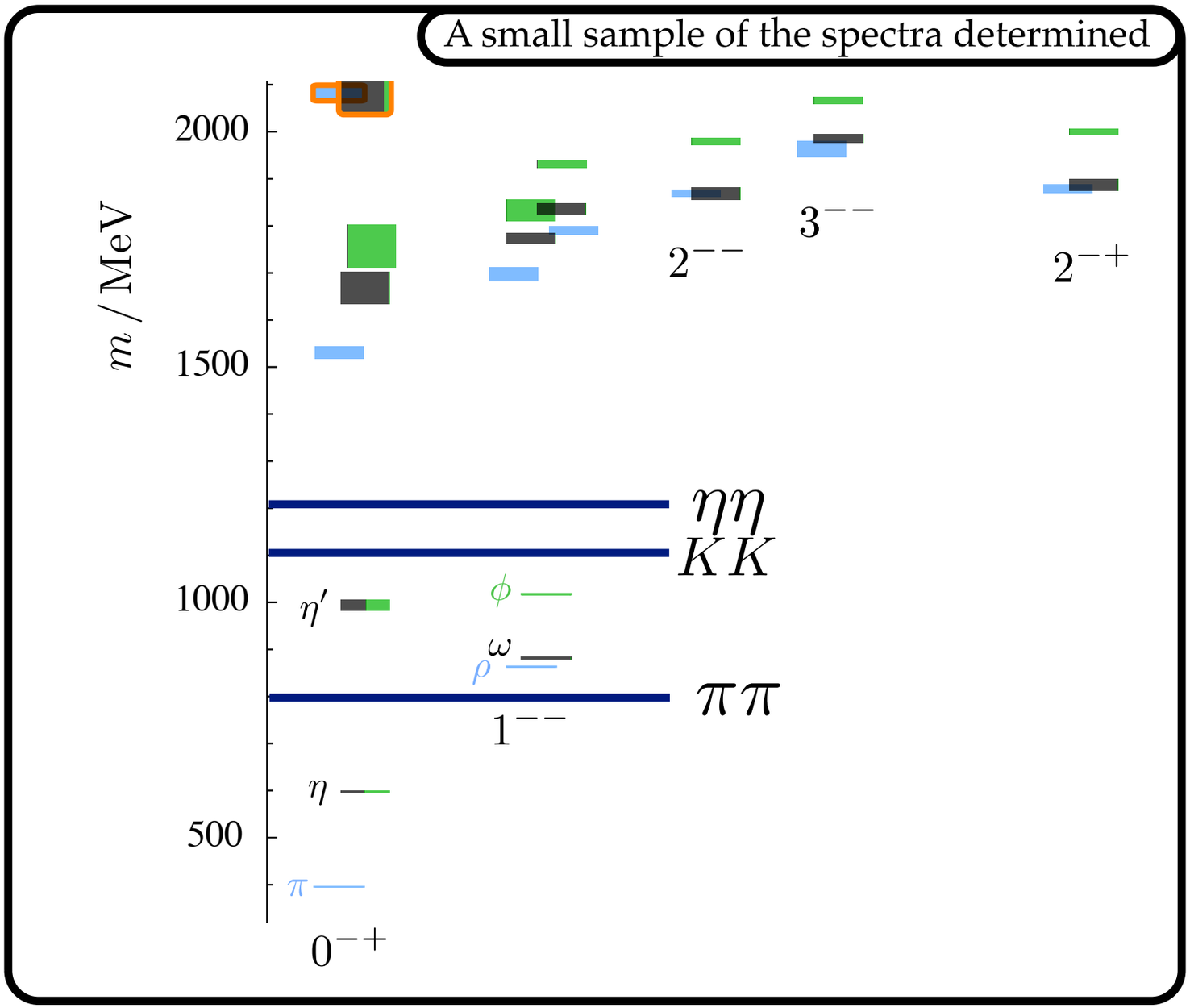

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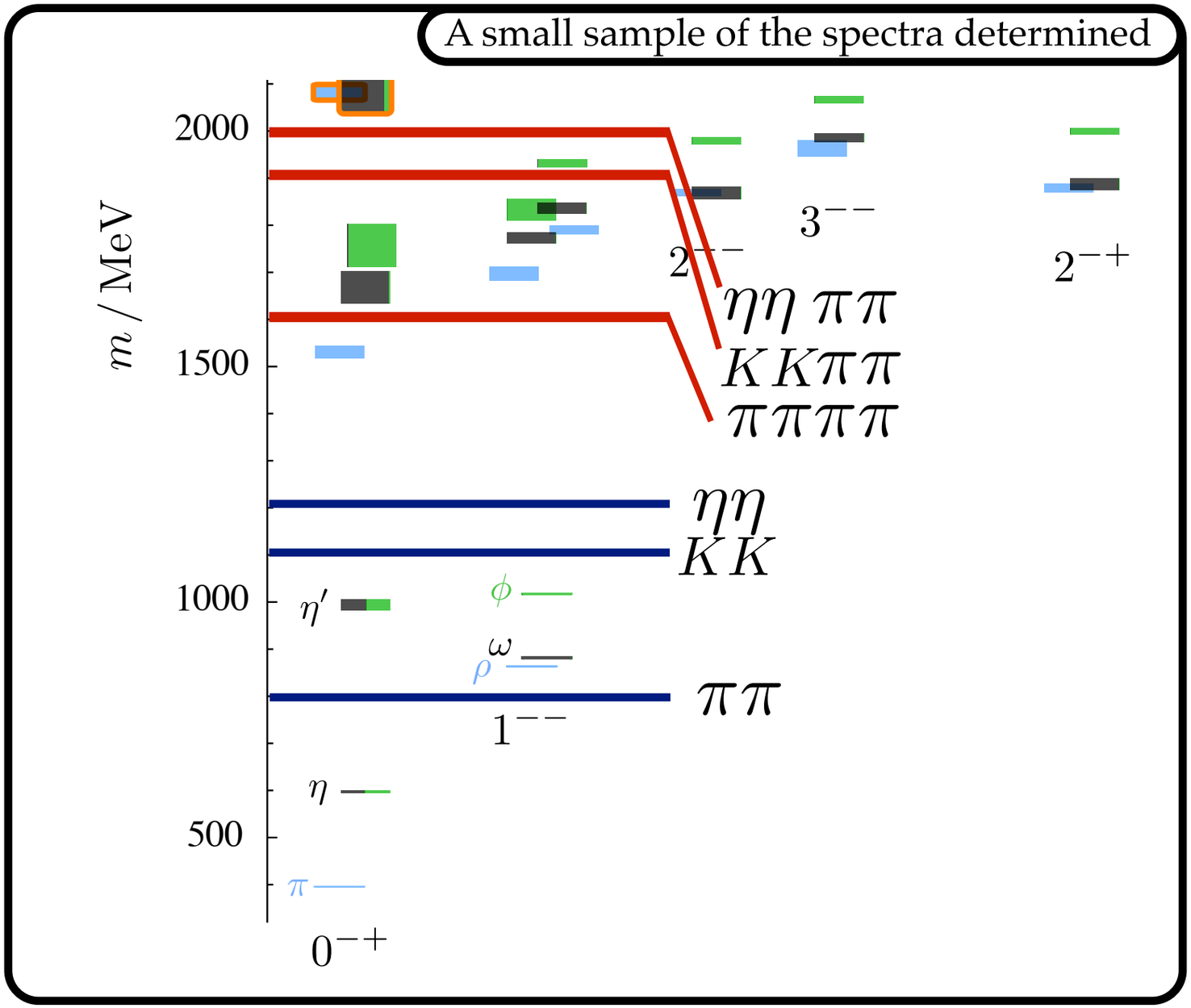

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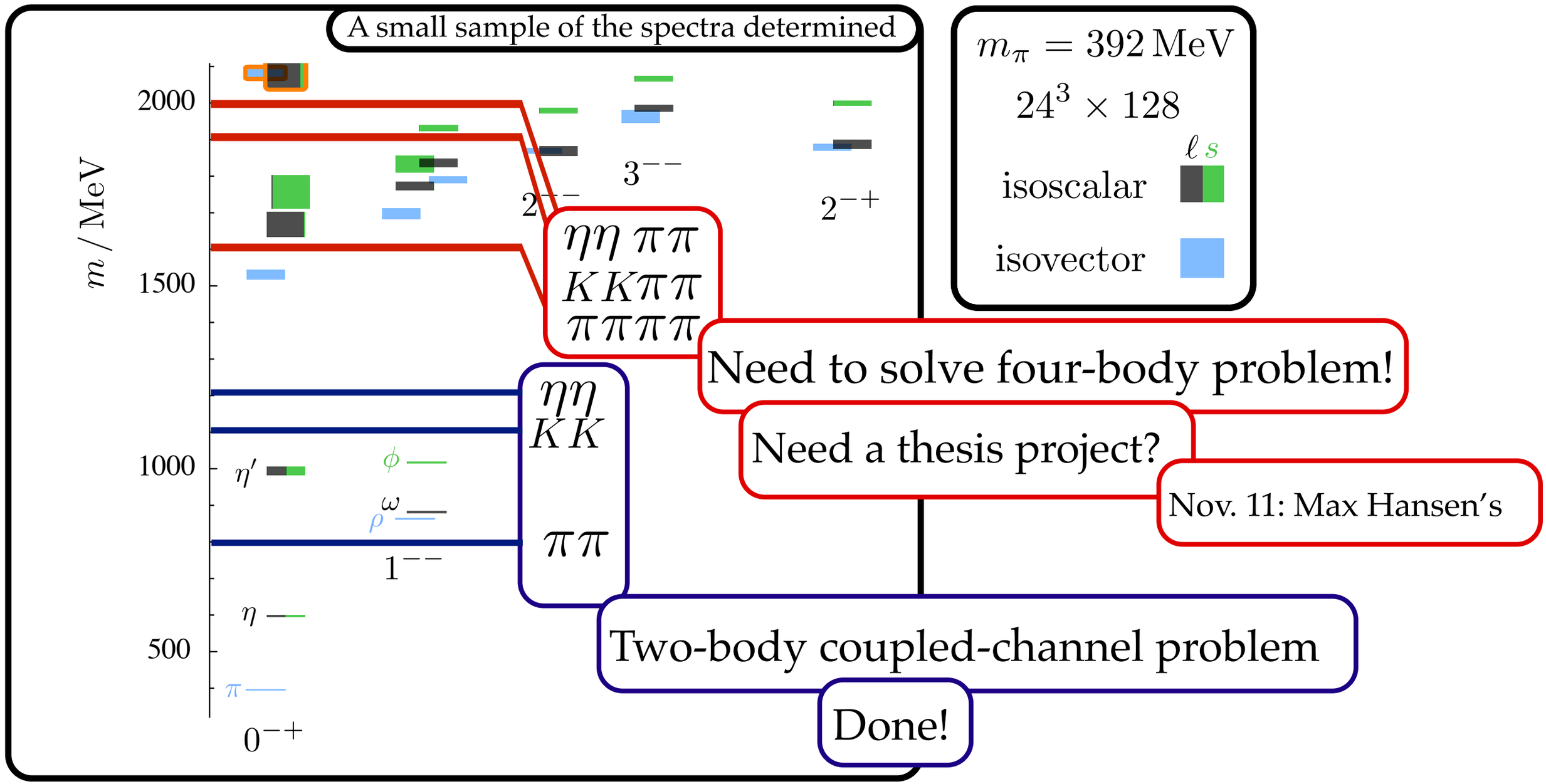

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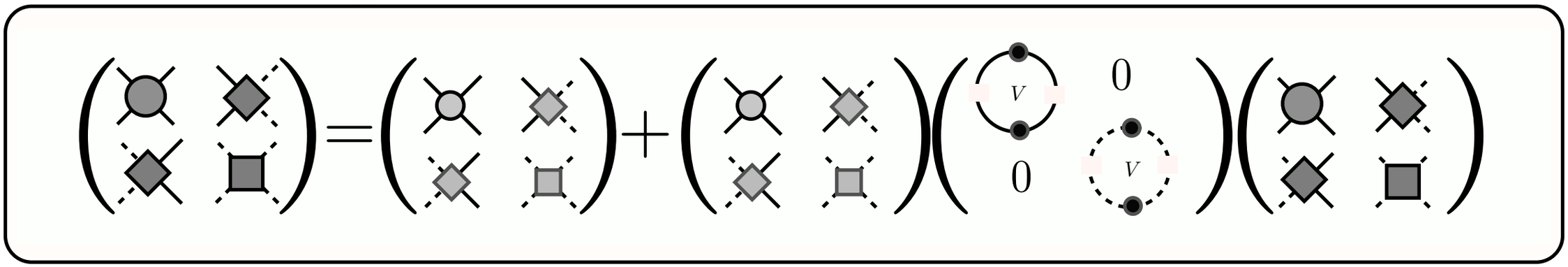
J. Dudek, R. Edwards, P. Guo & C. Thomas (2013)

Coupled Channels Spectrum

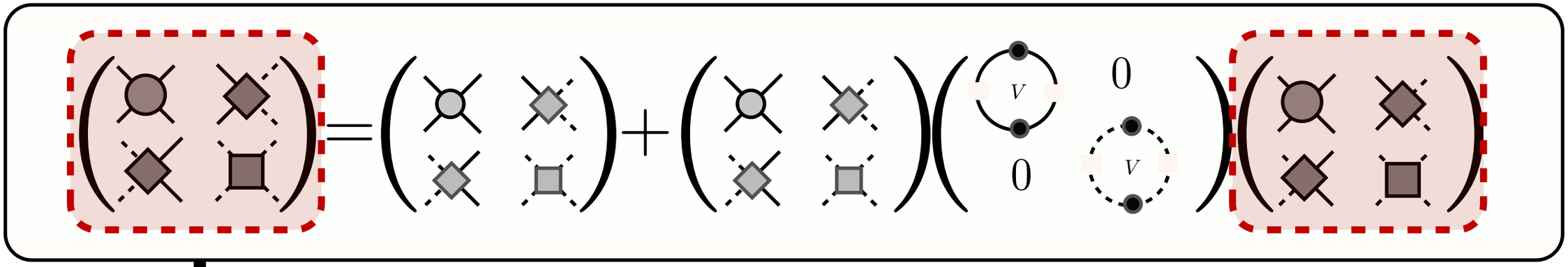
- Spectrum from poles of:



à la mode de Kim, Sachrajda & Sharpe (2005) arXiv:hep-lat/0507006

Coupled Channels Spectrum

- Spectrum from poles of:

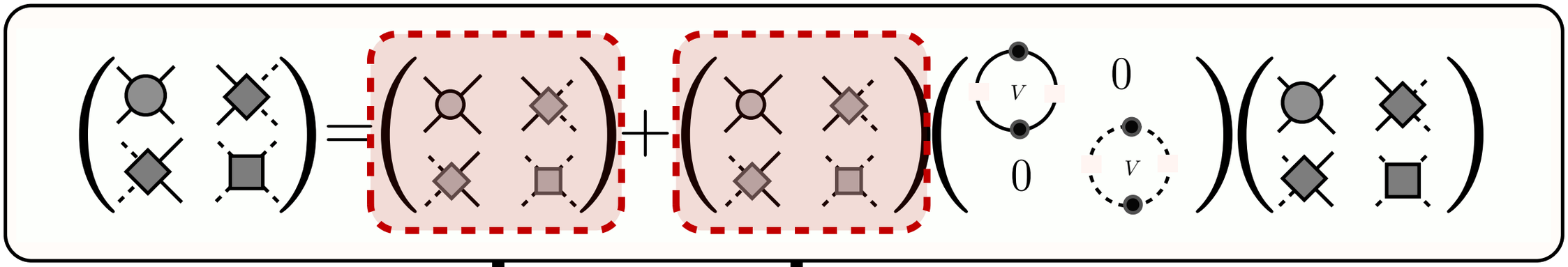


Finite volume two-particle amputated propagator

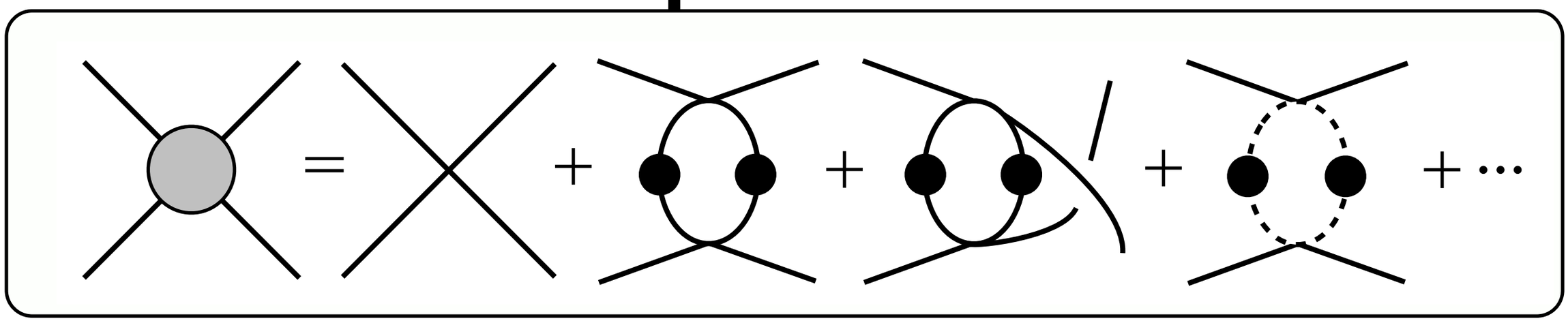
Sum over all 2 to 2 diagrams

Coupled Channels Spectrum

- Spectrum from poles of:

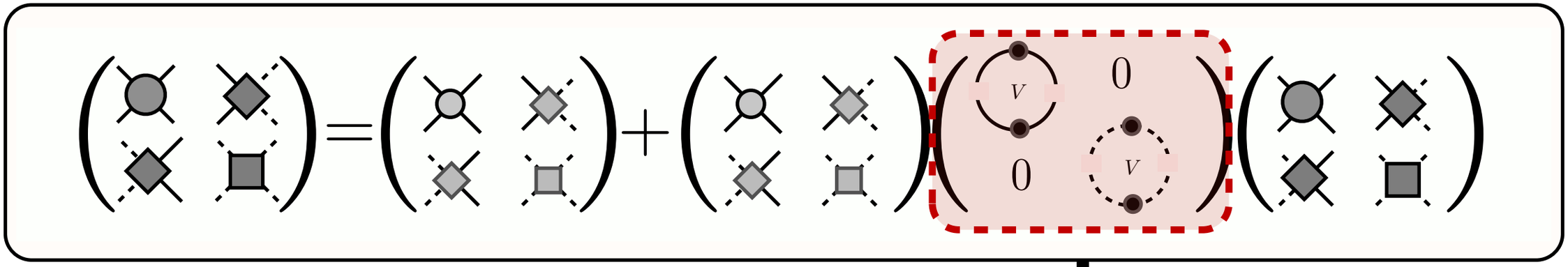


Bethe-Salpeter kernel
s-channel 2PI

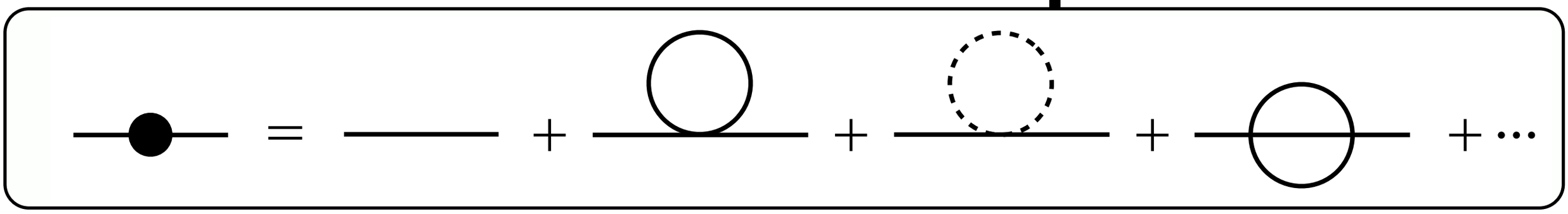


Coupled Channels Spectrum

- Spectrum from poles of:

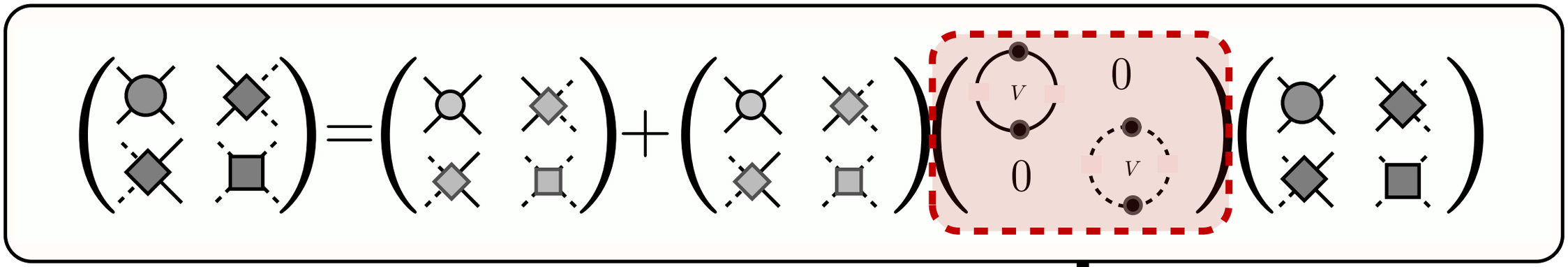


relativistic single particle propagator:
all 1PI diagrams



Coupled Channels Spectrum

- Spectrum from poles of:

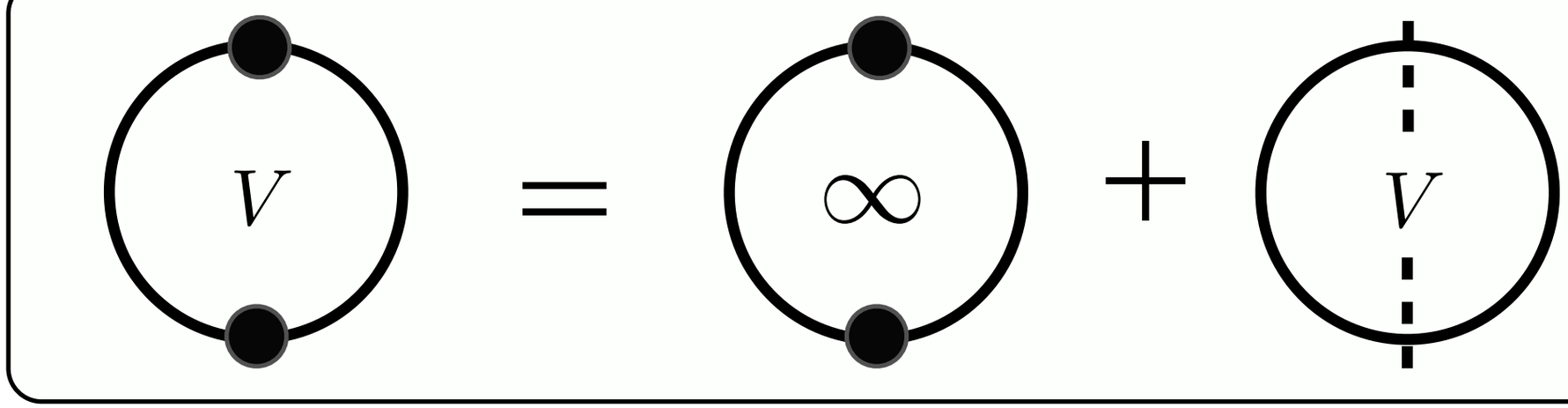


Finite volume loops

Momentum is discretrized: p = 2πn/L

Sum over momenta

Infinite volume loop, integral over continuous momentum

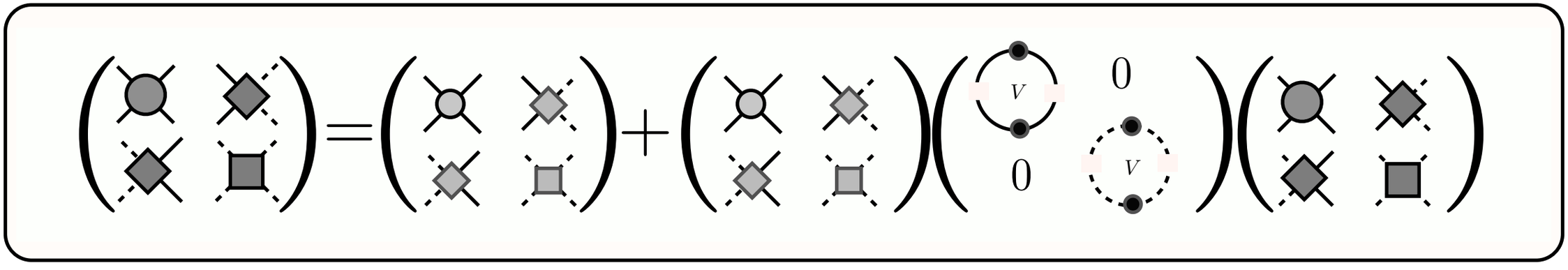


***On-shell* sum**

Power-law volume dependence

Coupled Channels Spectrum

- Spectrum from poles of:



- Quantization condition:

Holds for arbitrary numbers of channels

$$\det\left((\mathcal{M}^{\infty})^{-1} + \delta\mathcal{G}^{V}\right) = 0$$

Scattering amplitude
Diagonal in angular momentum
Couples channels

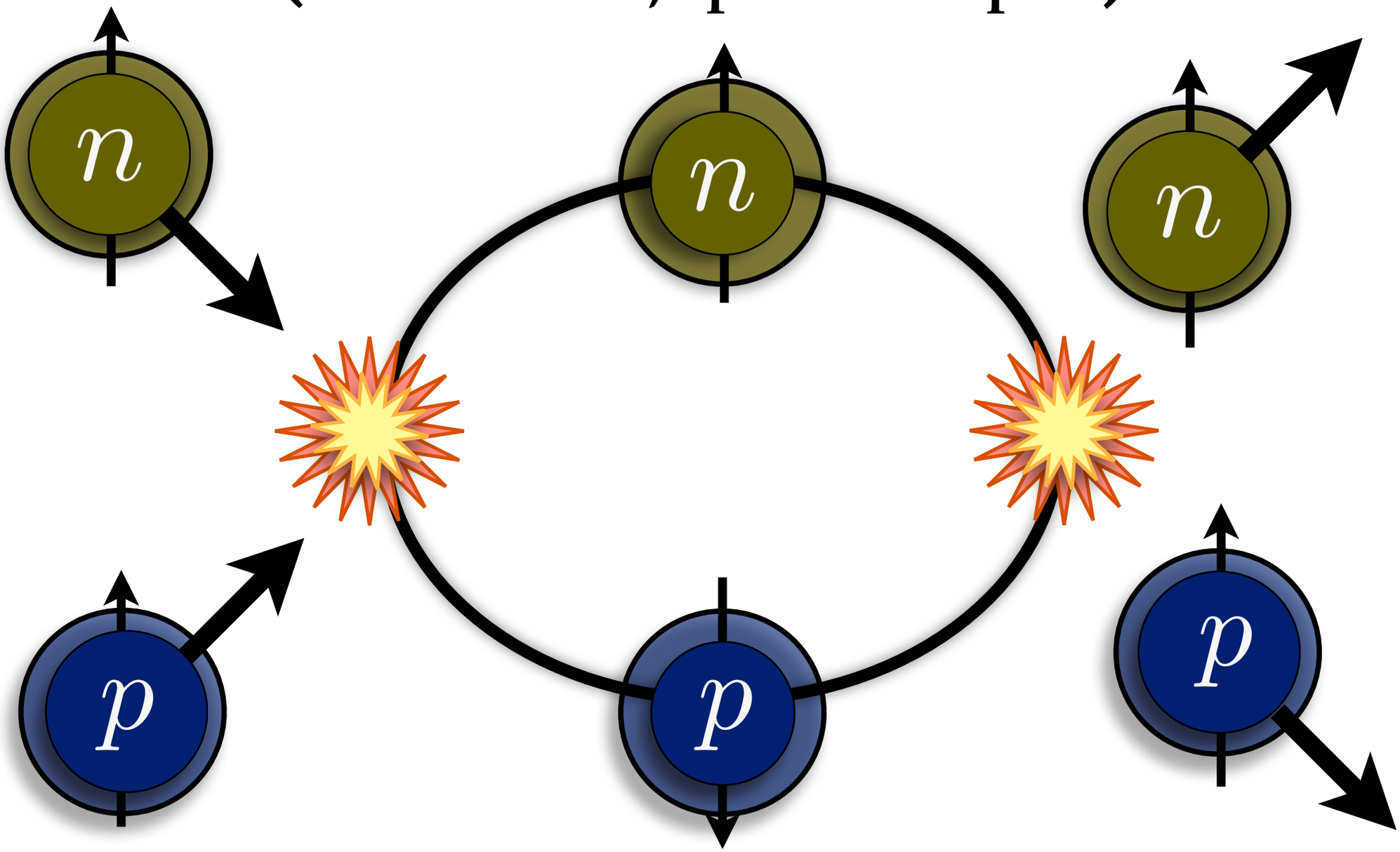
Kinematic function of (L, E_L)
Mixes angular momentum
Diagonal in flavor

- Derived independently by Max Hansen and Steve Sharpe (2012)

NN-Scattering

Onto nuclear physics....

(arbitrary angular momentum, momentum, spin & isospin)



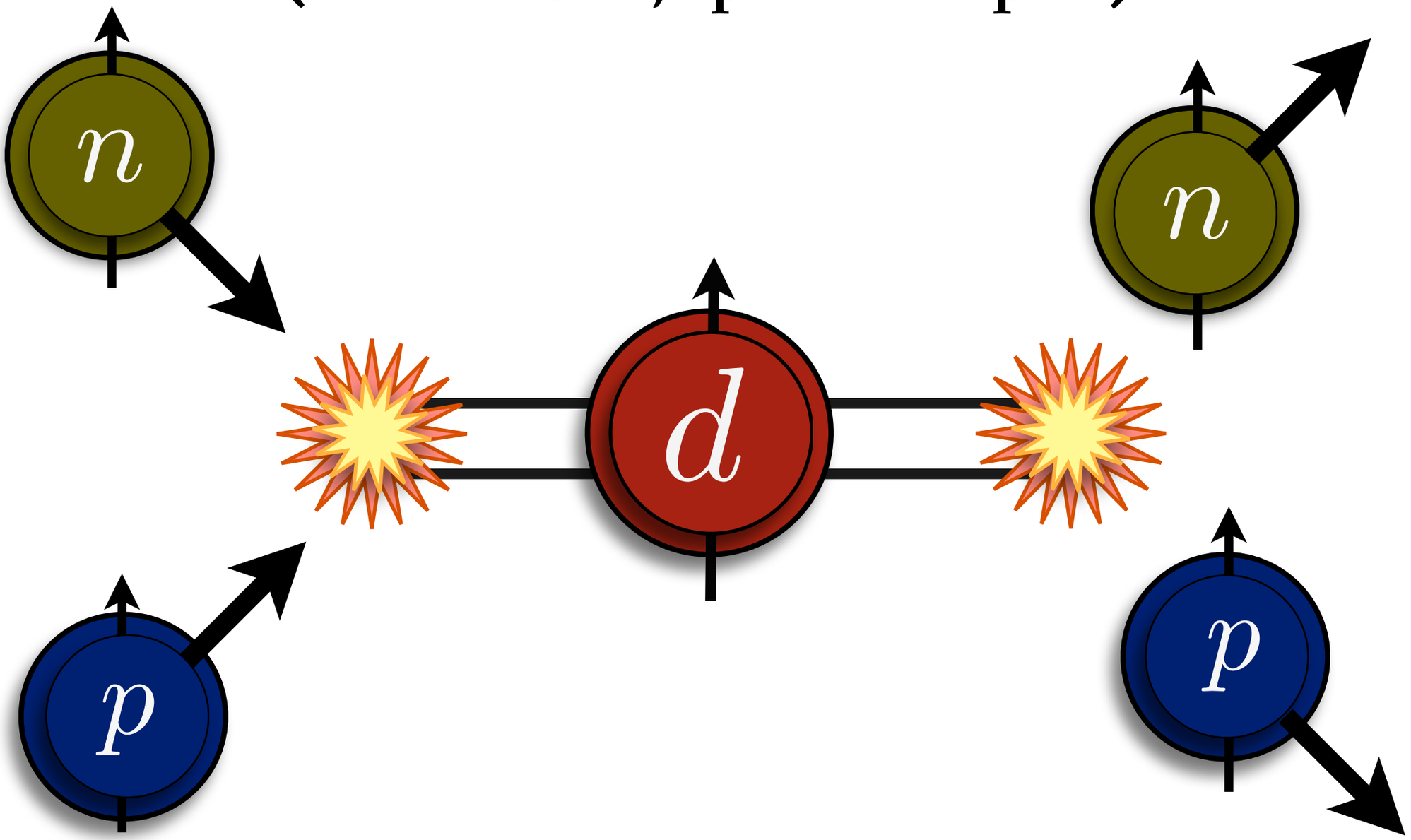
arXiv:1305.4903 [hep-lat]

RB, Zohreh Davoudi (UW) & Tom Luu (LLNL)

NN-Scattering

Onto nuclear physics....

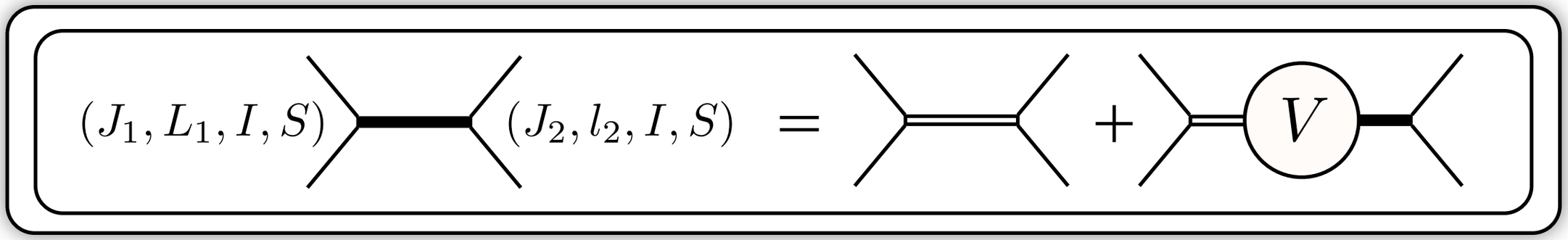
(arbitrary angular momentum, momentum, spin & isospin)



arXiv:nucl-th/9610052

à la mode de David Kaplan (1997)

Quantization Condition



$$\det\left((\mathcal{M}^{\infty})^{-1} + \delta\mathcal{G}^{V}\right) = 0$$

Scattering amplitude
Diagonal in J-basis
mixes *l*-states

Diagonal in
Spin & Isospin

Kinematic function of (L, E_L)
Mixes angular momentum

Diagonal in
Spin & Isospin

arXiv:1305.4903 [hep-lat]

RB, Zohreh Davoudi (UW) & Tom Luu (LLNL)

Quantization Condition

sanity check

$$\det\left((\mathcal{M}^{\infty})^{-1} + \delta\mathcal{G}^{V}\right) = 0$$

Isospin indices suppressed

$c_{lm} \sim Z_{lm}$

$$\left[\delta\mathcal{G}^{\mathbf{V}}\right]_{\mathbf{J}\mathbf{m}_\mathbf{J};\mathbf{J}'\mathbf{m}'_\mathbf{J}} = \sum_{\mathbf{m}_\mathbf{l},\mathbf{m}'_\mathbf{l},\mathbf{m}_\mathbf{S}} \langle \mathbf{J}\mathbf{m}_\mathbf{J}|\mathbf{l}\mathbf{m}_\mathbf{l},\mathbf{S}\mathbf{m}_\mathbf{S}\rangle\langle \mathbf{l}'\mathbf{m}'_\mathbf{l},\mathbf{S}\mathbf{m}_\mathbf{S}|\mathbf{J}'\mathbf{m}'_\mathbf{J}\rangle \left[\delta\tilde{\mathcal{G}}^{\mathbf{V}}\right]_{\mathbf{l}\mathbf{m}_\mathbf{l};\mathbf{l}'\mathbf{m}'_\mathbf{l}}$$

S = 0

Agrees with Lüscher, Gottlieb & Rummukainen

S = 1/2

Agrees with Bernard et al. & Göckeler et al.

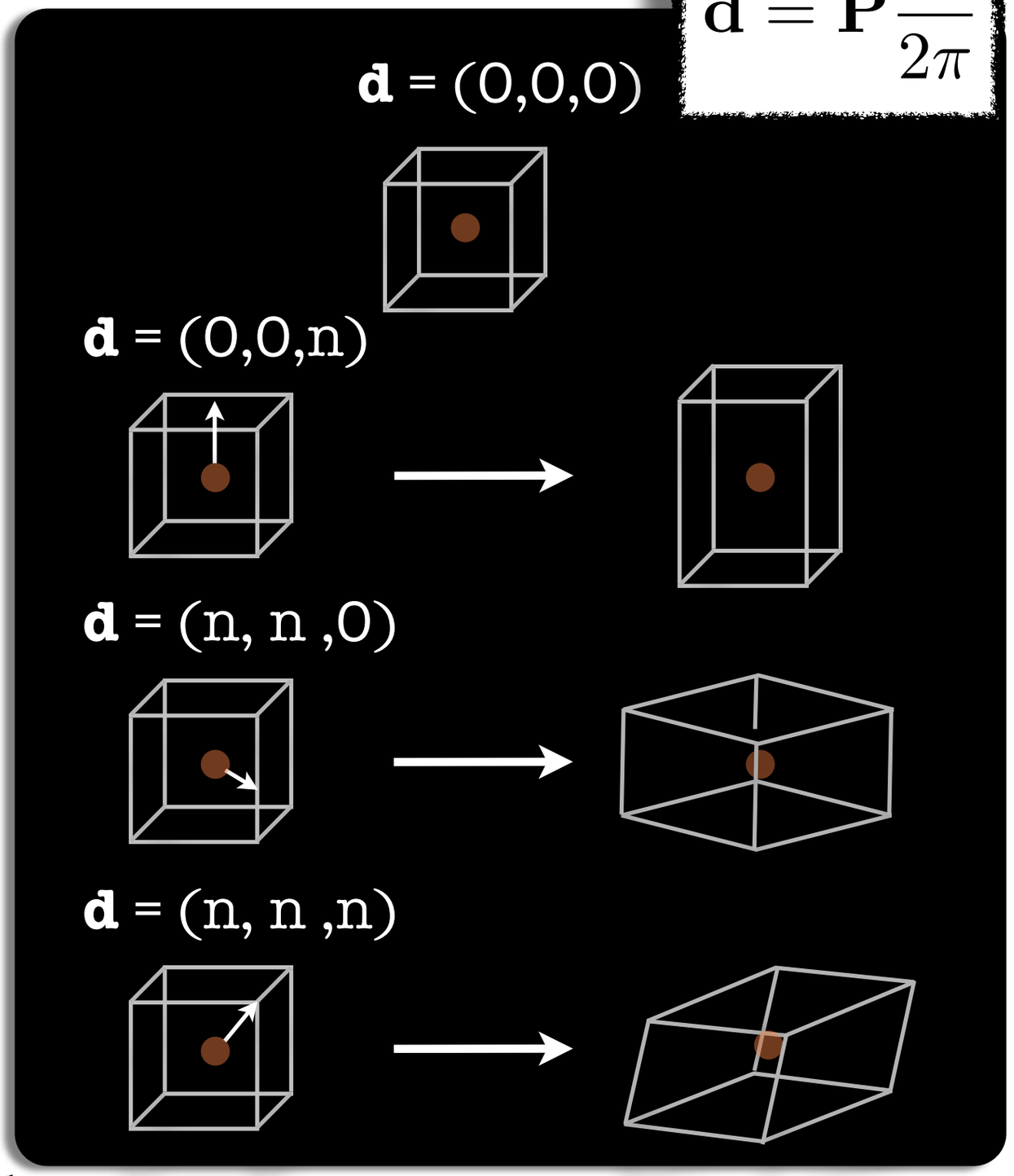
S = 0 ⊕ 1

Agrees with N. Ishizuka & Beane et al.

Boosts & Symmetry

$$\mathbf{d} = \mathbf{P}\frac{\mathrm{L}}{2\pi}$$

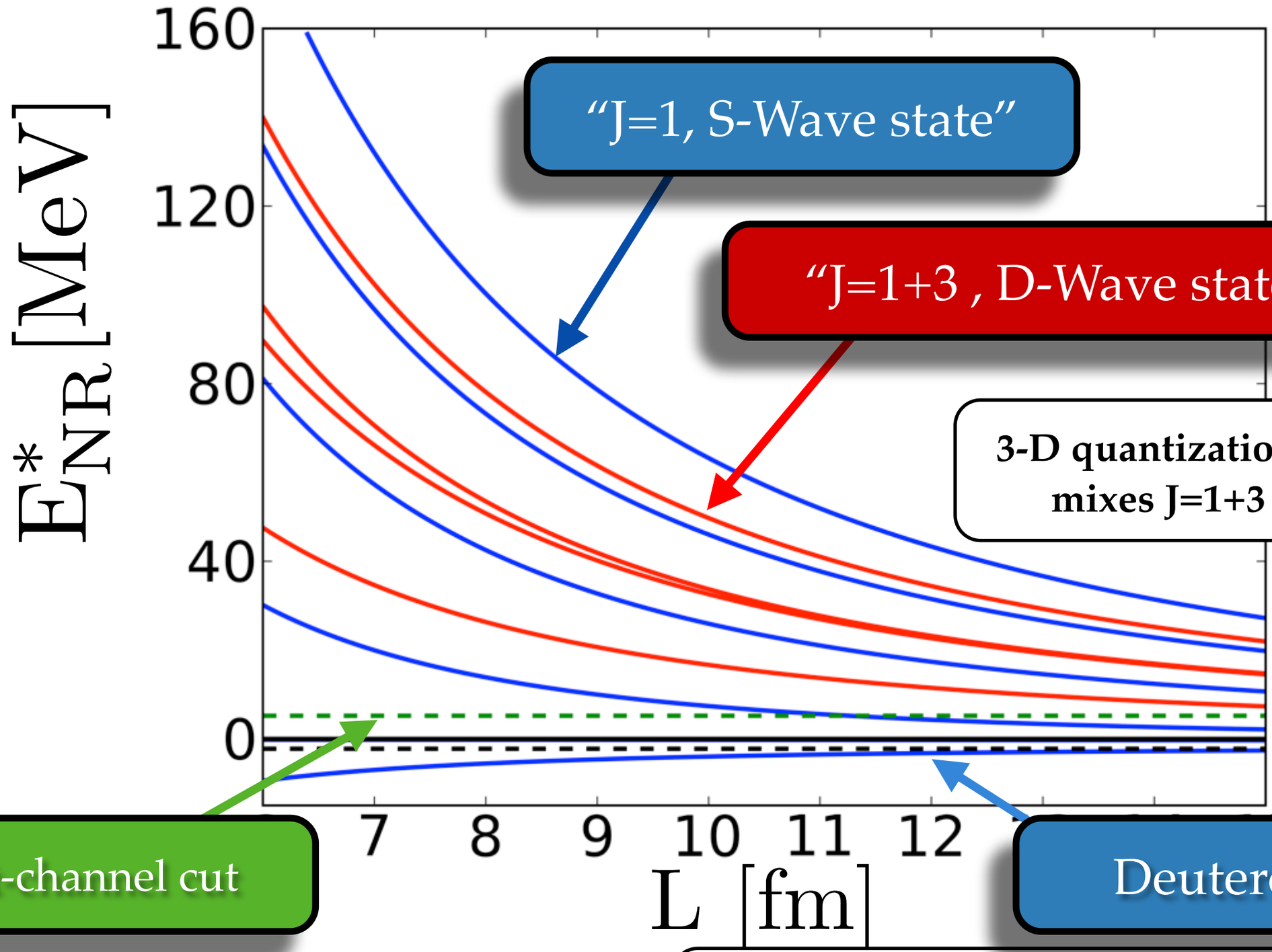
- O : Cubic
- D_4 : Tetragonal
- D_2 : Orthorhombic
- D_3 : Trigonal
- Isospin, Spin, Parity, l_{max}=3



Predicted T_1 Spectrum

$\mathbf{d} = (0, 0, 0)$

m_π ~140 MeV



"J=1, S-Wave state"

"J=1+3 , D-Wave state"

3-D quantization condition, mixes J=1+3 channels

t-channel cut

Deuteron

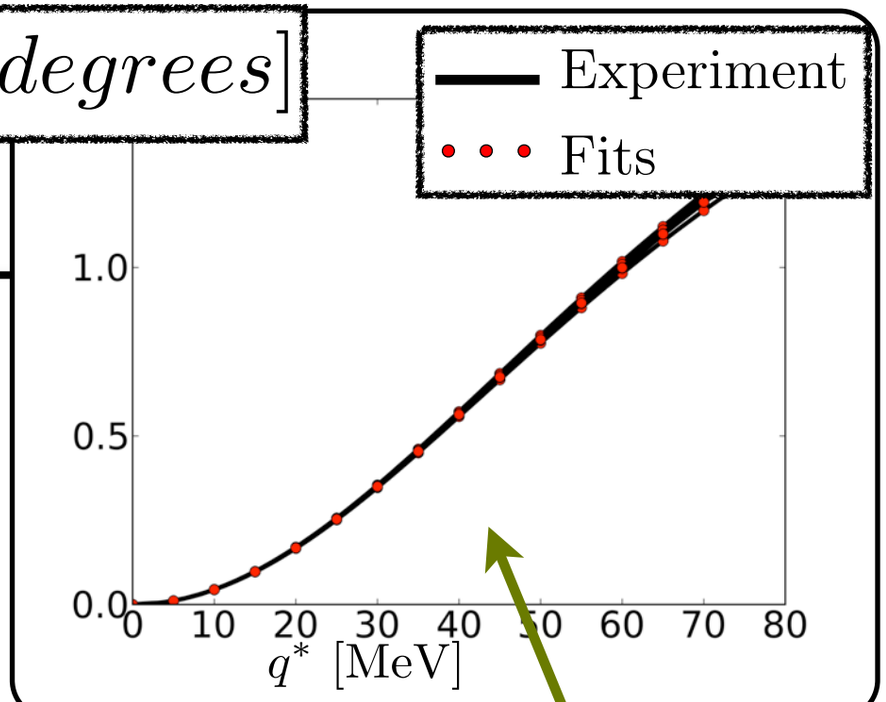
Input physical phase shifts [http://nn-online.org]

T_1 Excited states

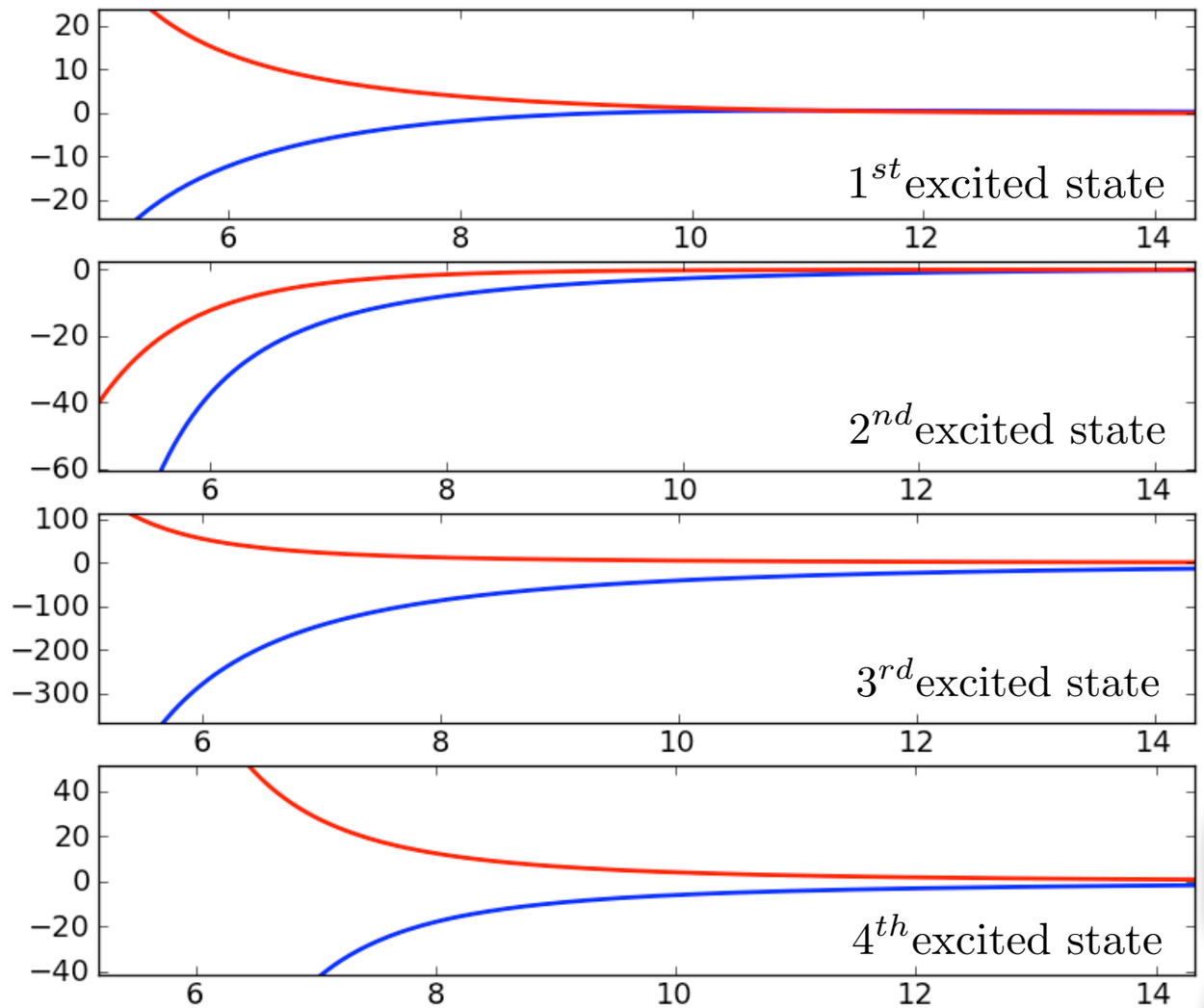
$\mathbf{d} = (0,0,0)$

$$\mathcal{M}_{J=1} = \begin{pmatrix} \mathcal{M}^{S} & \mathcal{M}^{SD} \\ \mathcal{M}^{DS} & \mathcal{M}^{D} \end{pmatrix}$$

$\epsilon_1[degrees]$



$$\delta \mathrm{E}^{*(\mathbb{T}_1)}[keV] = \mathrm{E}^{*(\mathbb{T}_1)} - \mathrm{E}^{*(\mathbb{T}_1)}(\epsilon_1 = 0)$$

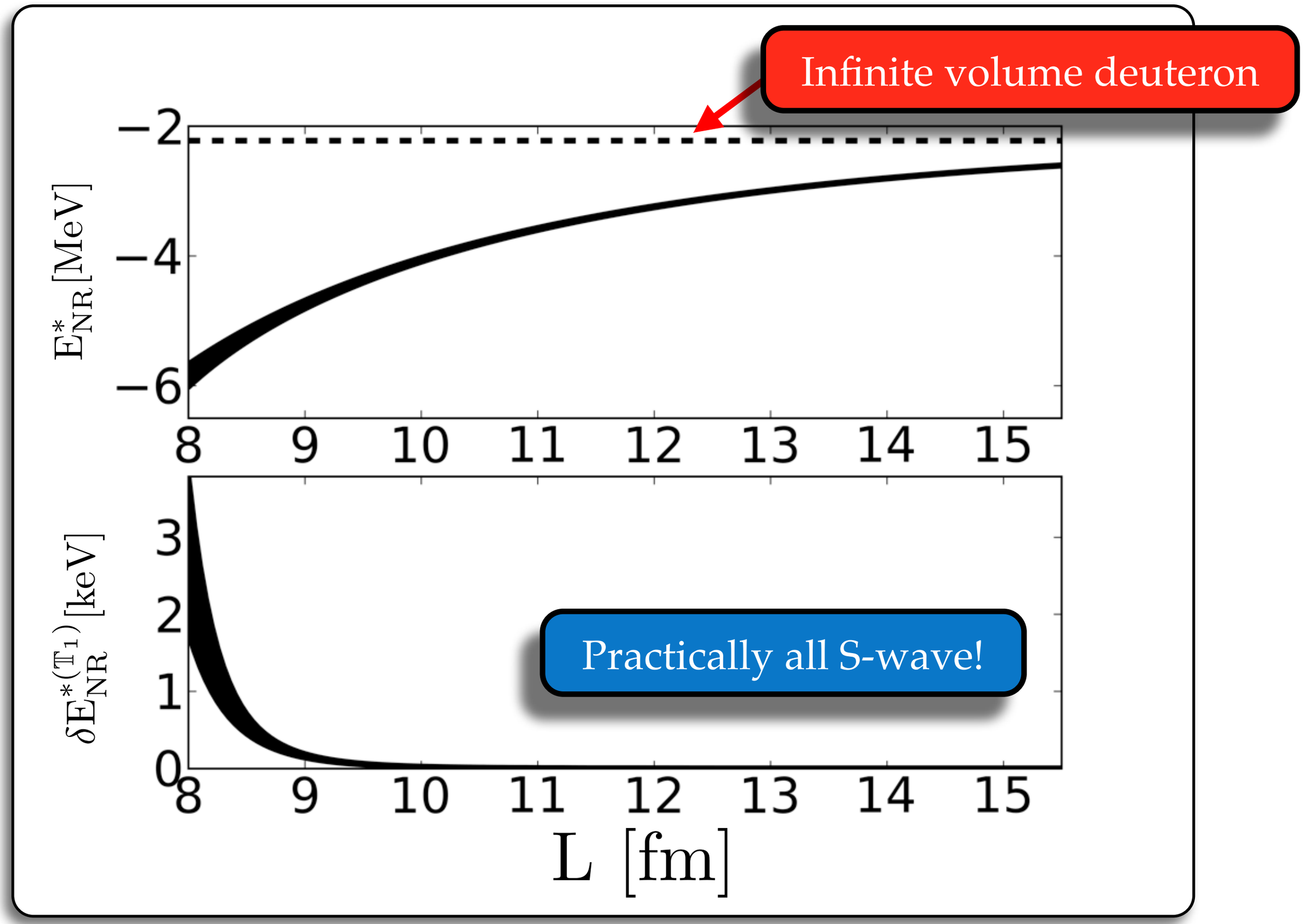


Can we expect to extract such small mixing angle?

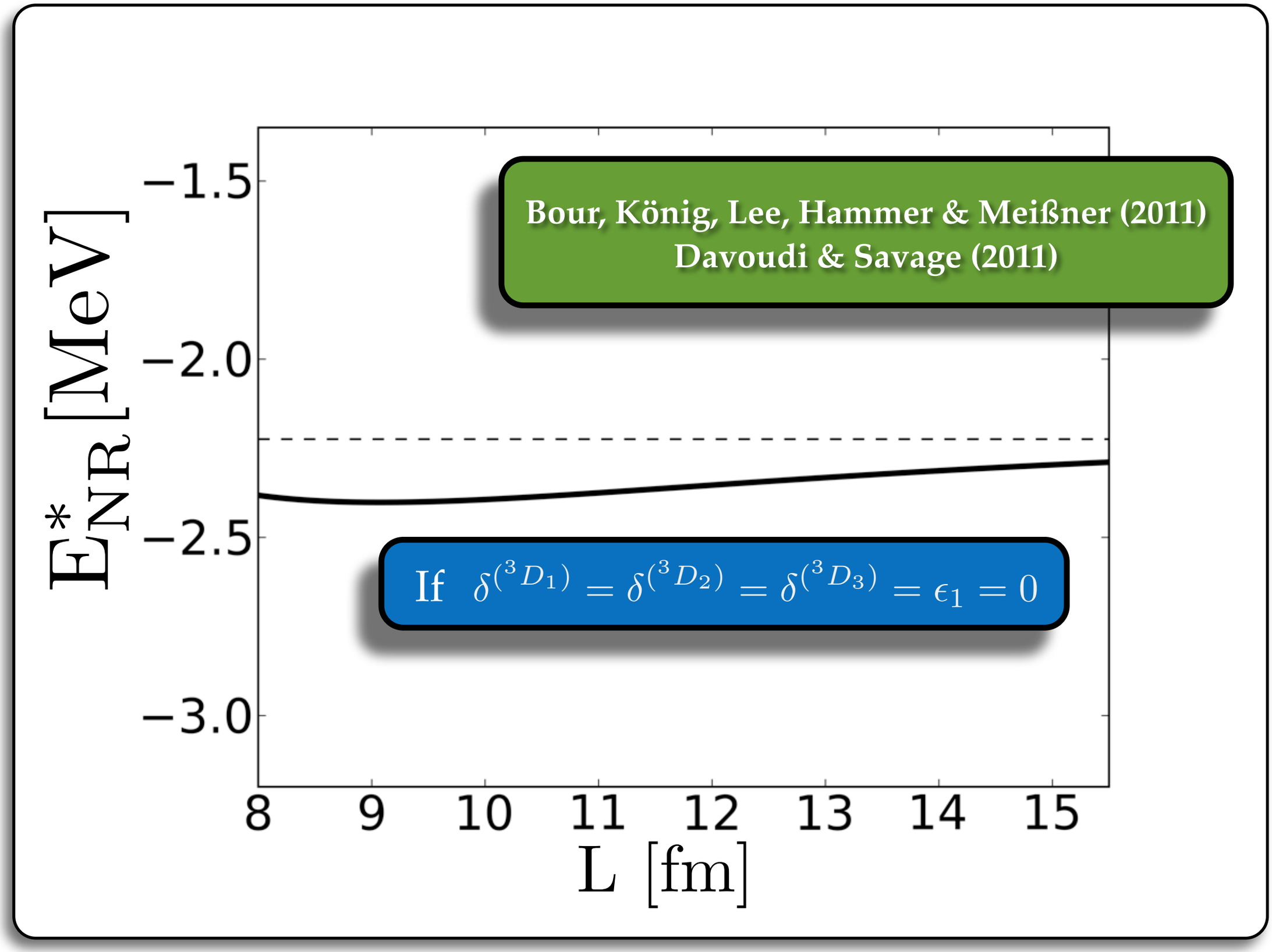
S-Wave

D-Wave

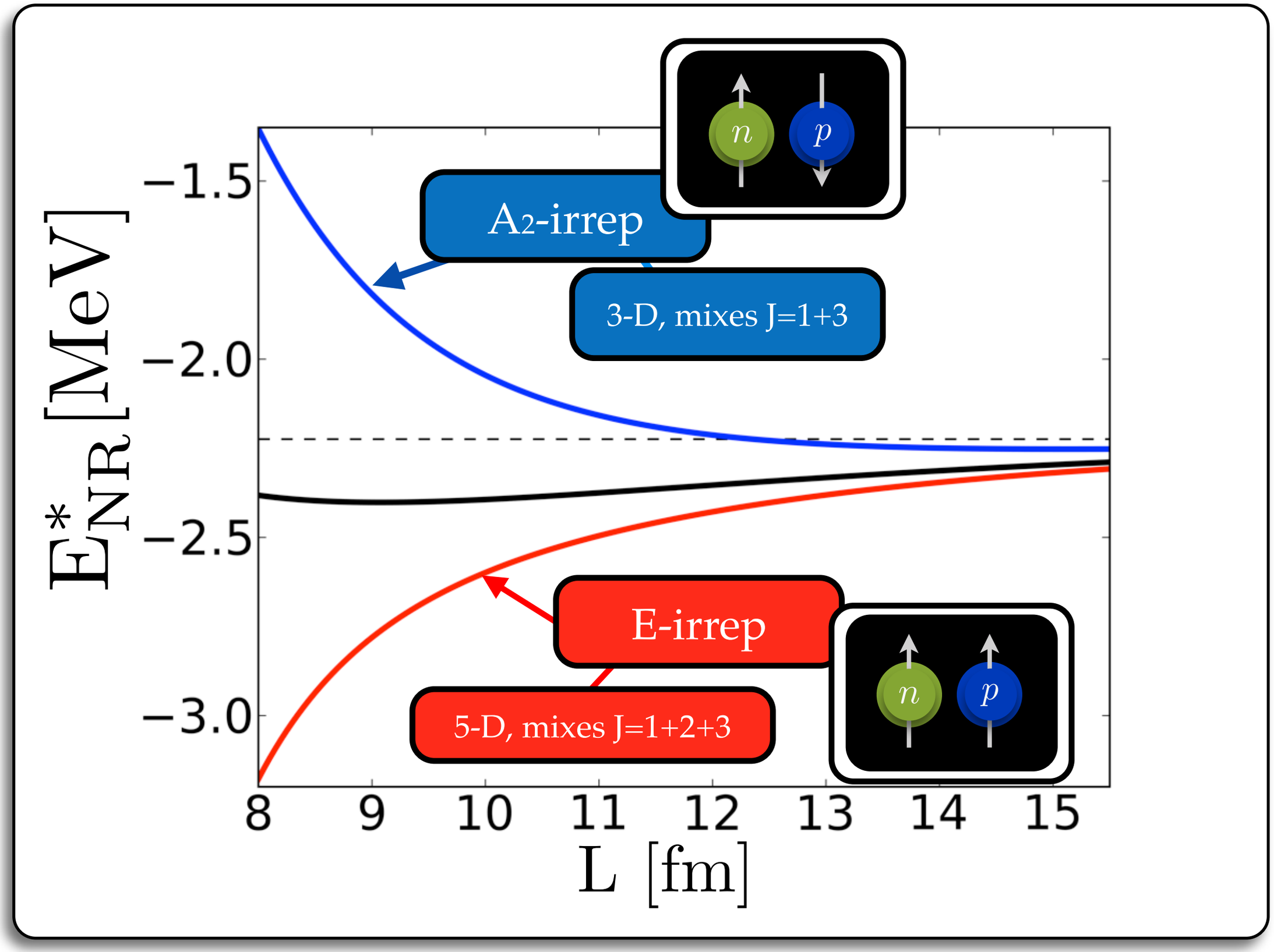
Deuteron at rest $\mathbf{d} = (0, 0, 0)$



Boosted deuteron $\mathbf{d} = (0, 0, 1)$



Boosted deuteron $\mathbf{d} = (0, 0, 1)$



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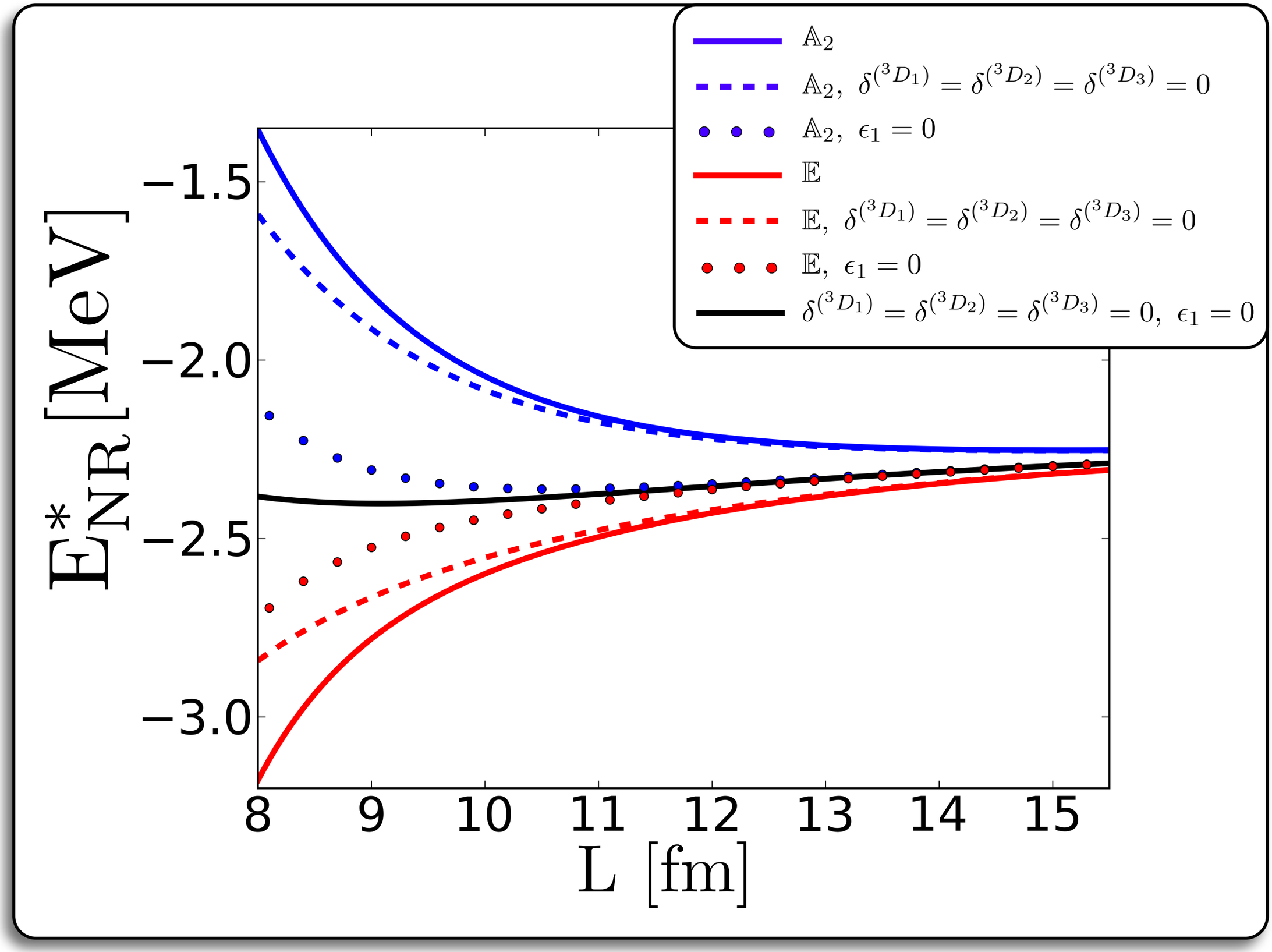
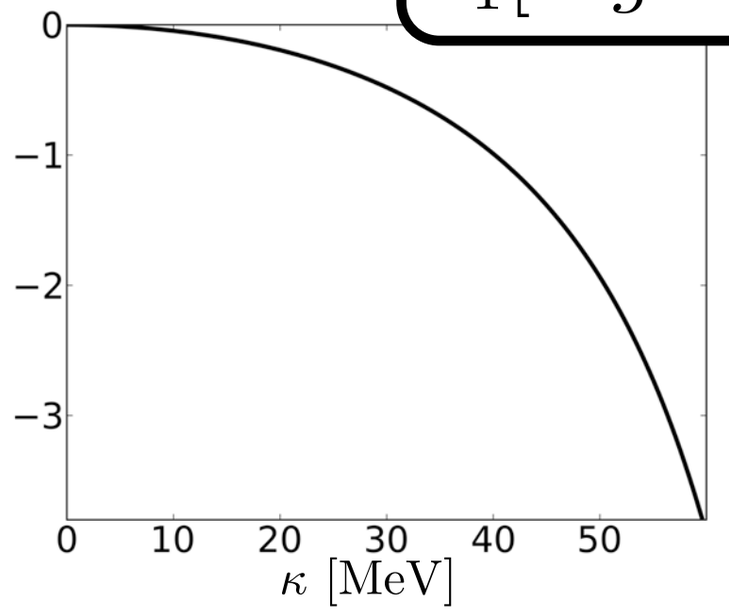
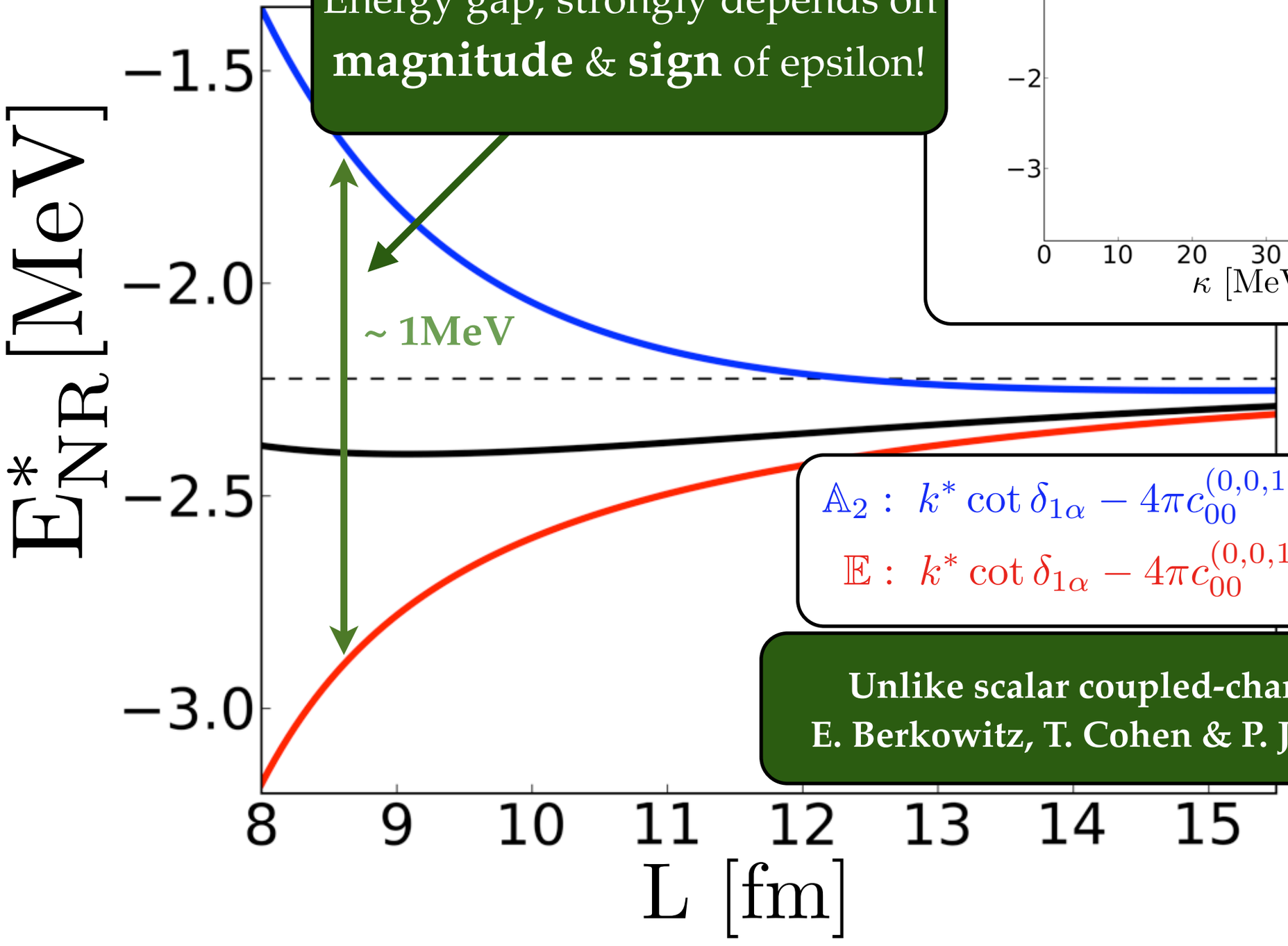


Boosted deuteron $\mathbf{d} = (0, 0, 1)$

$\epsilon_1 [degrees]$



Energy gap, strongly depends on **magnitude** & **sign** of epsilon!

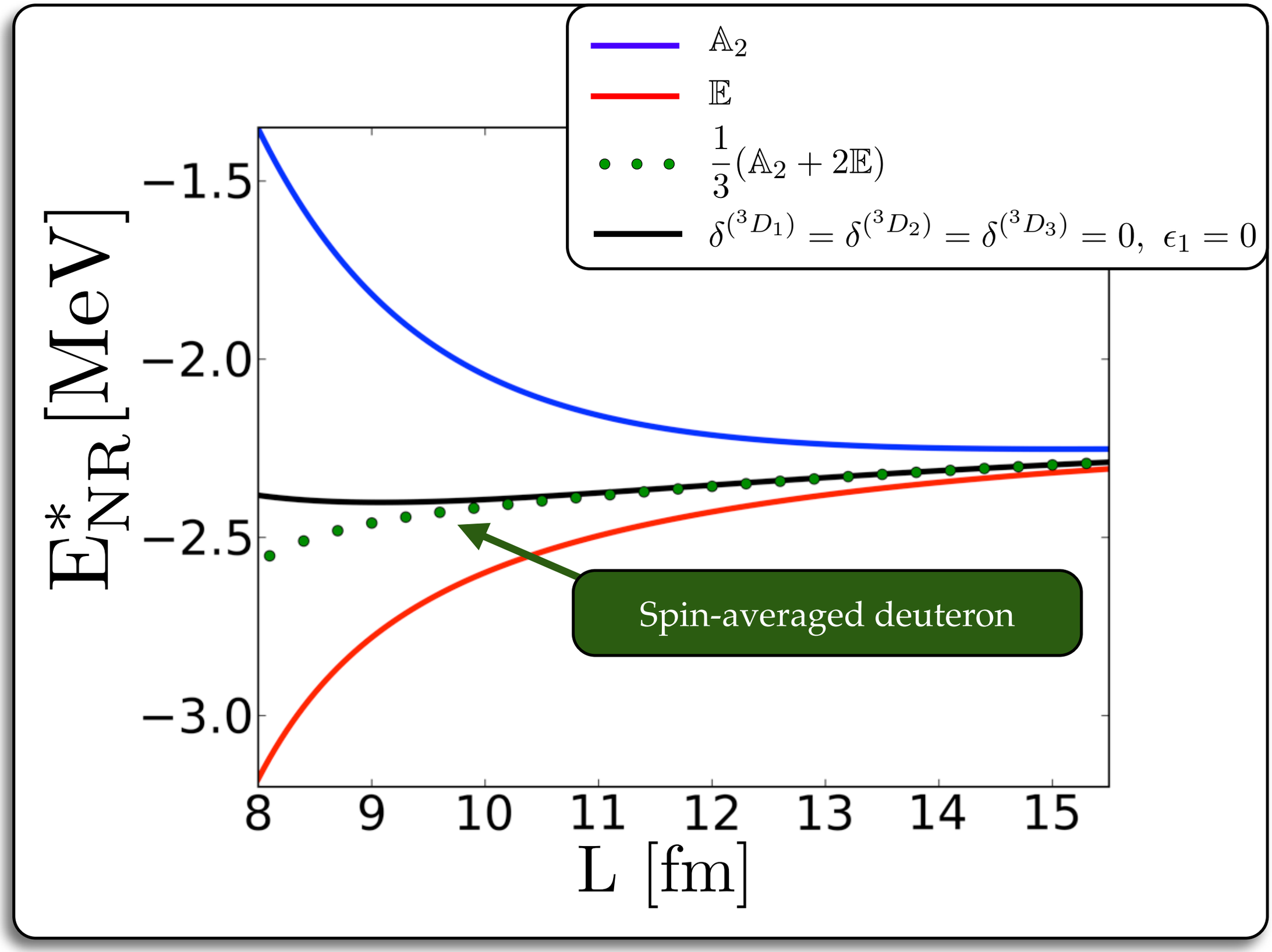


$$\mathbb{A}_2 : \; k^* \cot \delta_{1\alpha} - 4\pi c_{00}^{(0,0,1)}(k^{*2}; \mathrm{L}) \propto -\epsilon_1$$

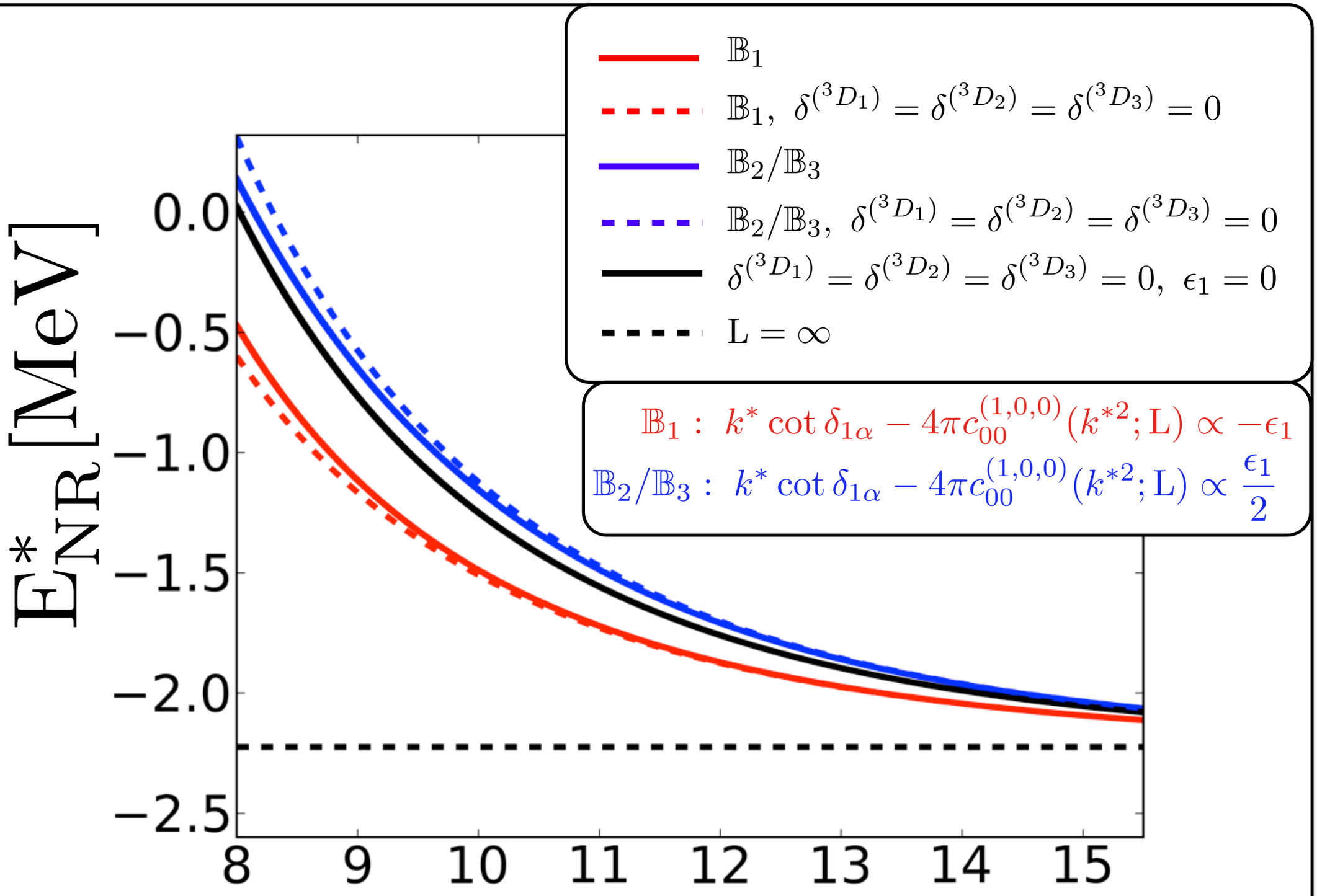
$$\mathbb{E} : \; k^* \cot \delta_{1\alpha} - 4\pi c_{00}^{(0,0,1)}(k^{*2}; \mathrm{L}) \propto \frac{\epsilon_1}{2}$$

Unlike scalar coupled-channels system
E. Berkowitz, T. Cohen & P. Jefferson (2012)

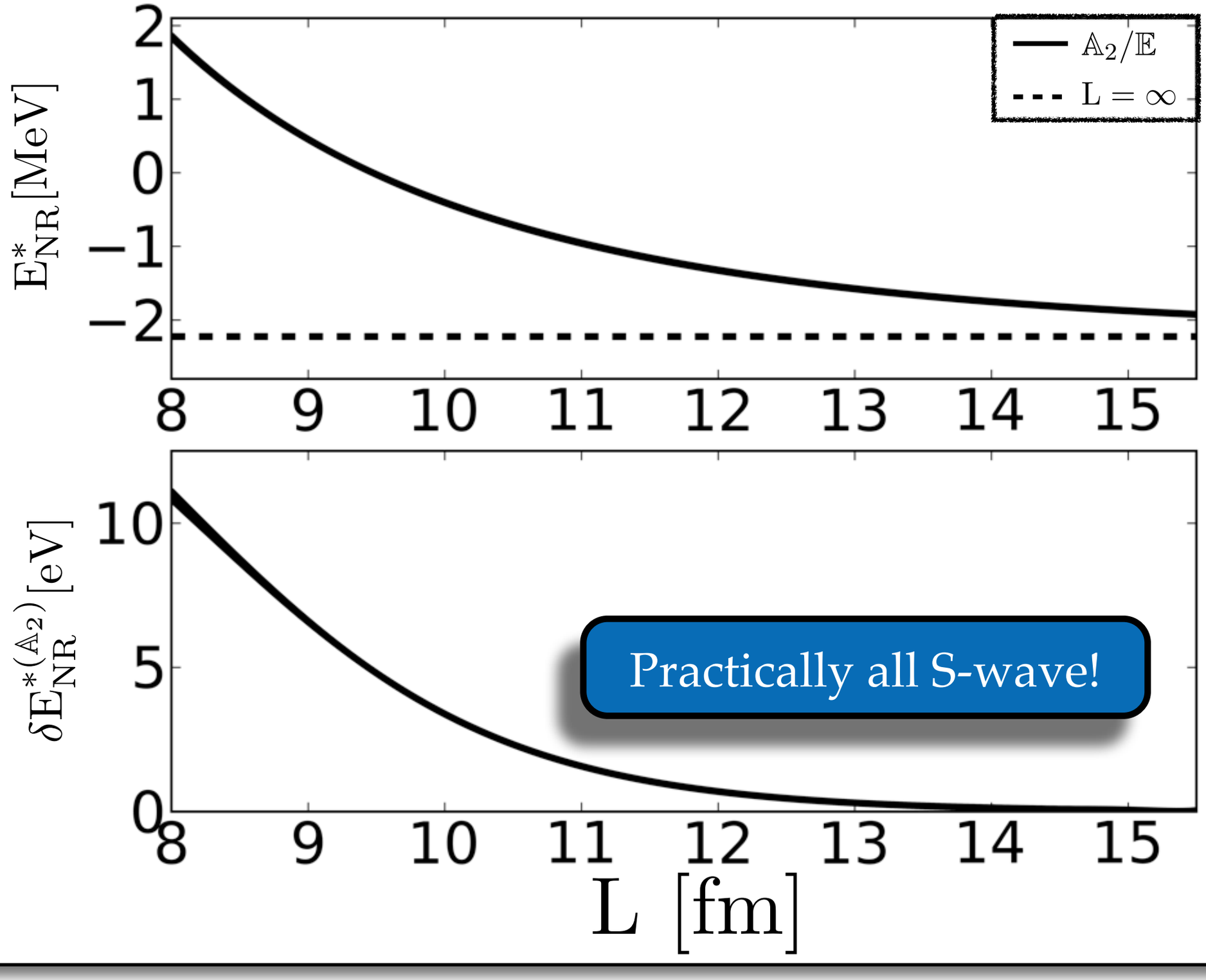
Boosted deuteron $\mathbf{d} = (0, 0, 1)$



Boosted deuteron $\mathbf{d} = (1,1,0)$

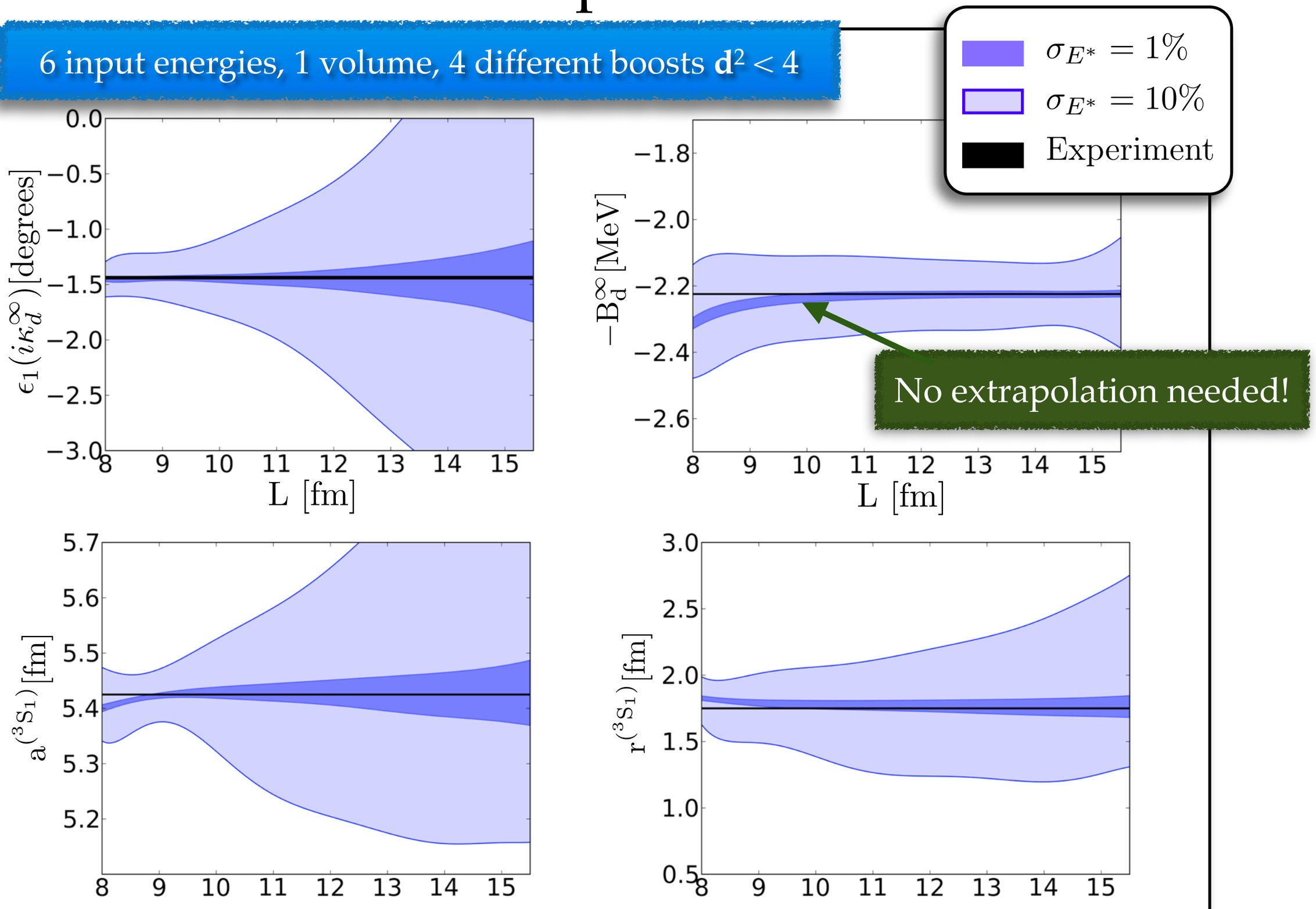


Boosted deuteron $\mathbf{d} = (1,1,1)$

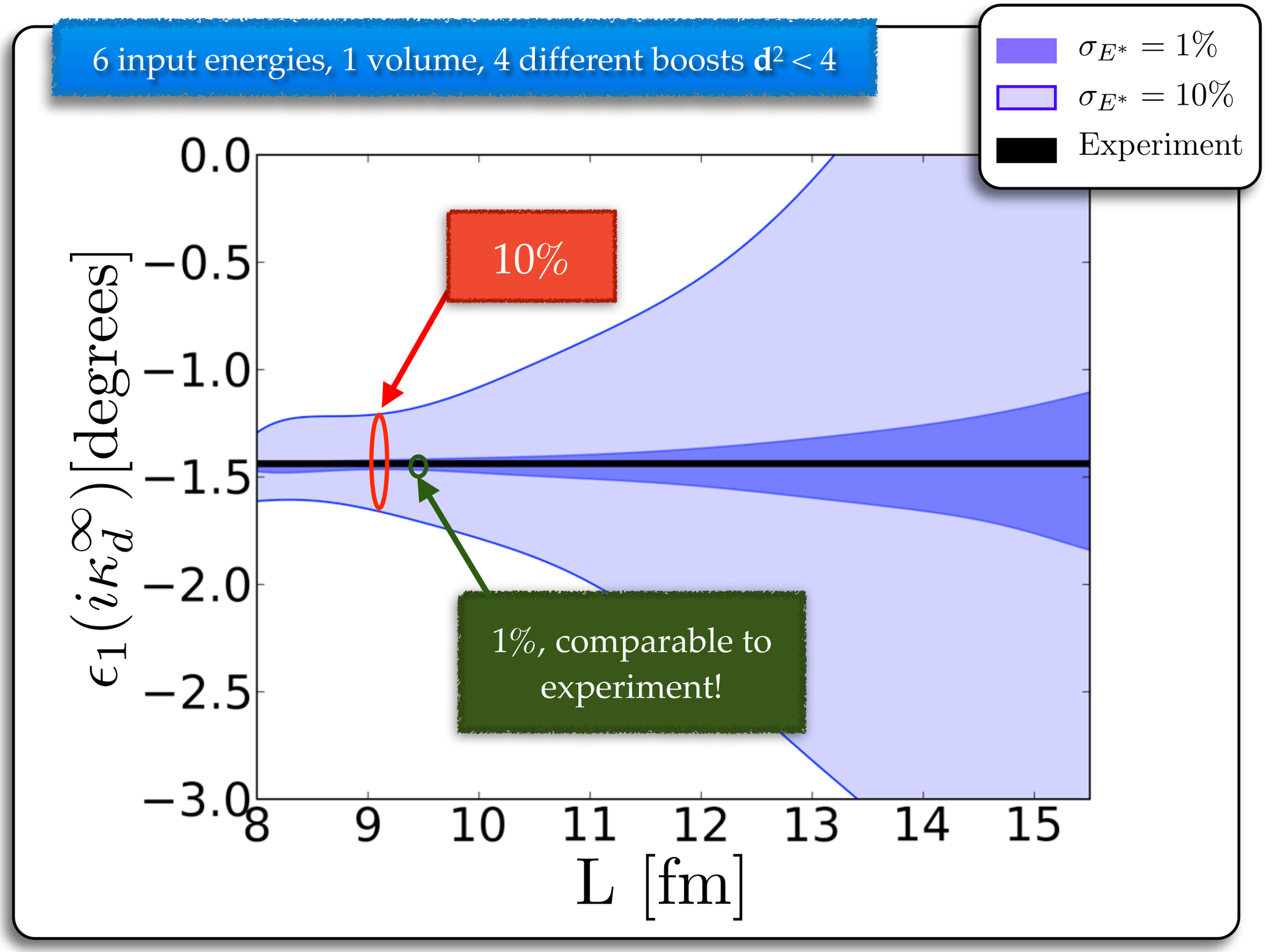


What can be determined with limited computational resources?

6 input energies, 1 volume, 4 different boosts $\mathbf{d}^2 < 4$



Extraction Estimates



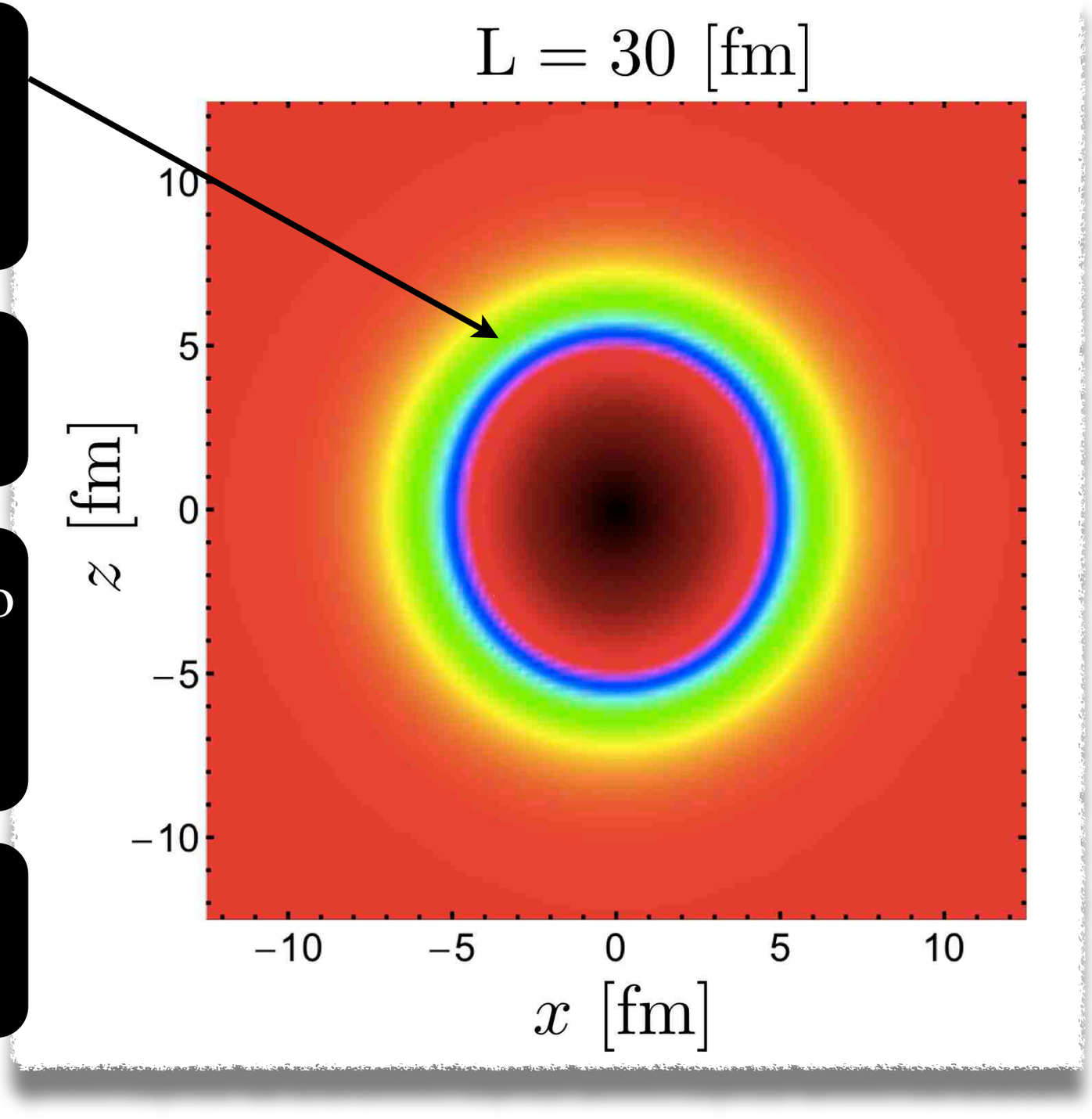
Why does a sign matter?

The tensor force causes the deuteron to be prolate

A change in sign leads to an oblate deuteron

A difference in shape leads to different interaction with mirror imagines

This leads to different spectrum!



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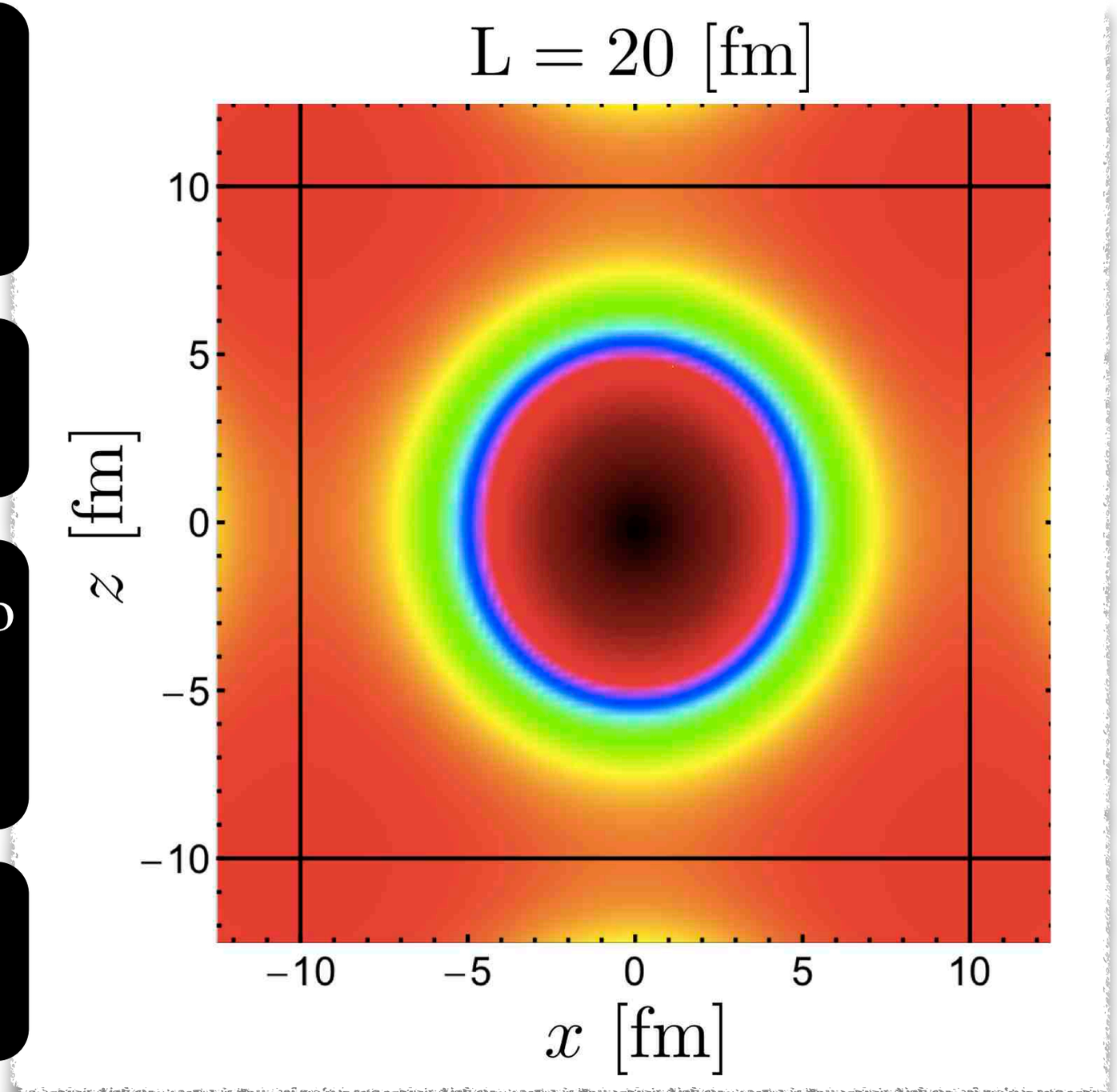

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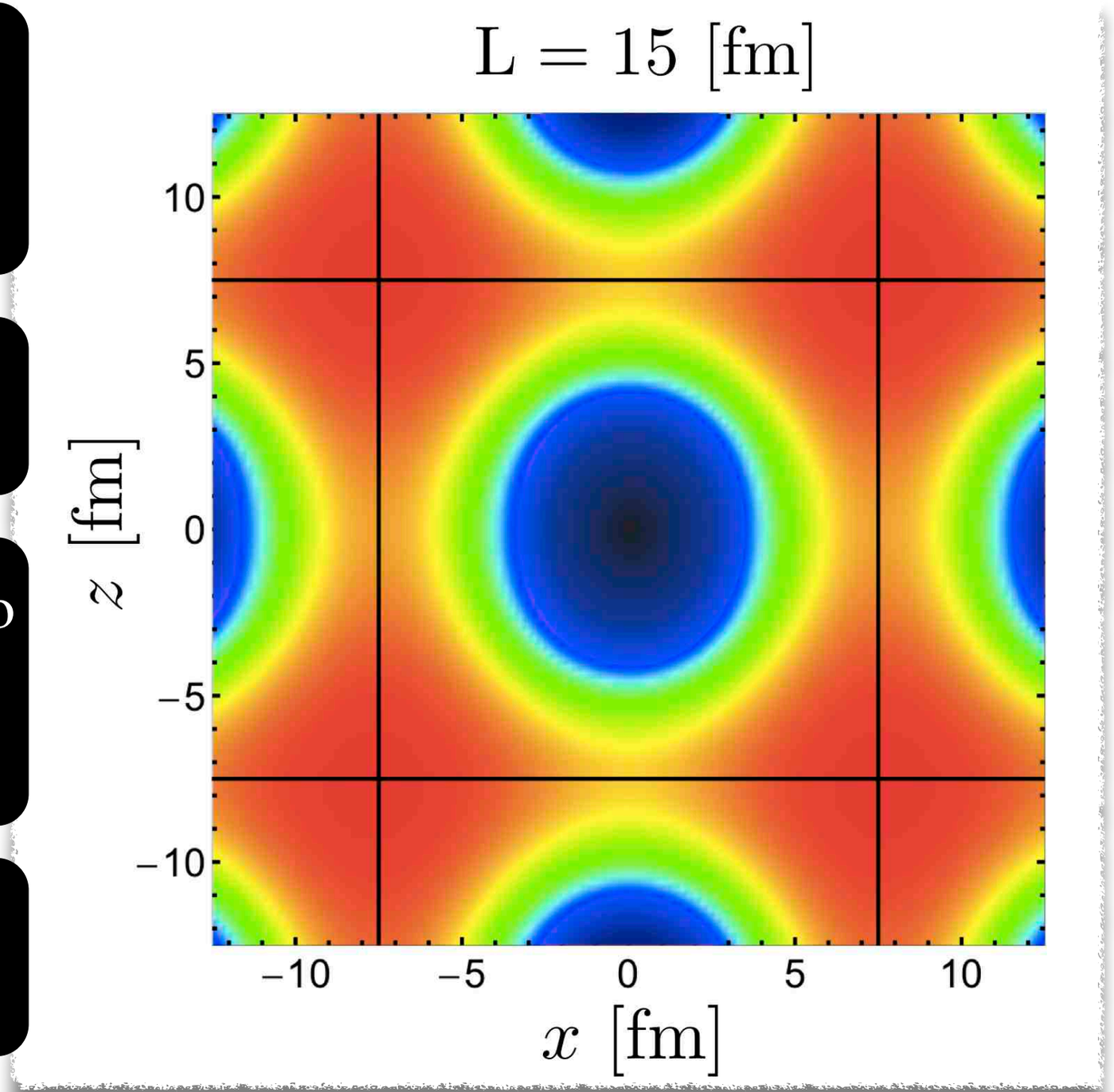

When studying finite volume wavefunctions, proceed with caution!

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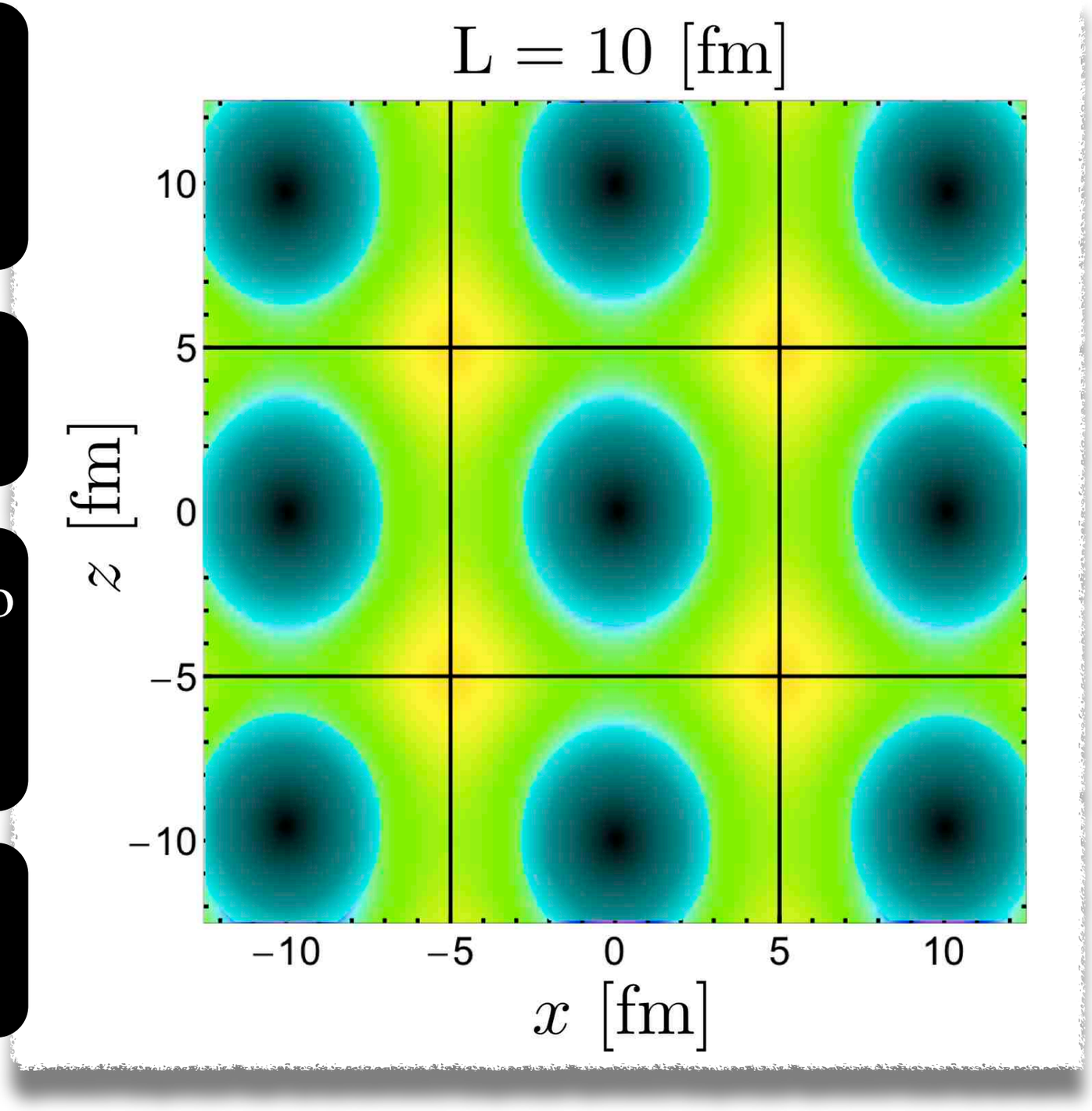

Final remarks

- Master QC for Coupled-channels & NN systems
- All J, S, I, L, and boosts
- Can study the tensor force from LQCD!
- No ambiguity on the sign S-matrix elements
- Thanks to collaborators: Zohreh Davoudi, Tom Luu,

  & Martin Savage