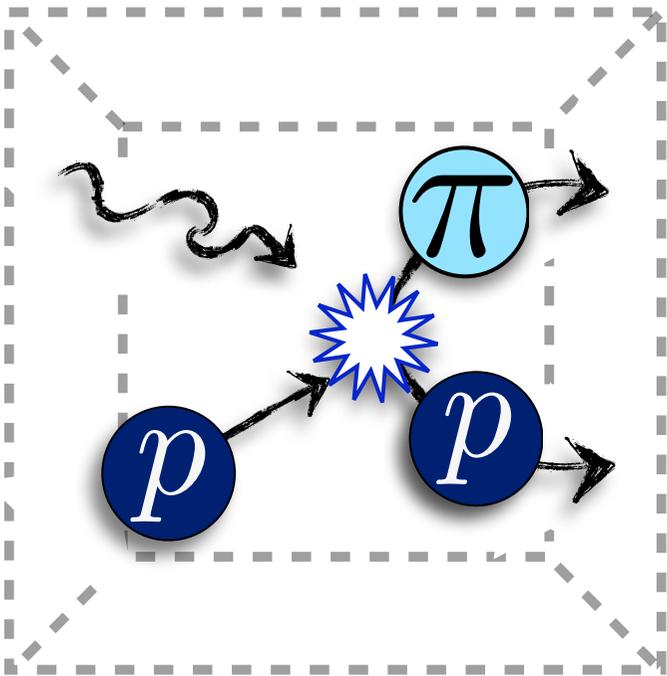


# Lattice QCD, form factors, resonances, scattering, bound states...

in 15 min or less!

Raúl Briceño

 Jefferson Lab



# Goals

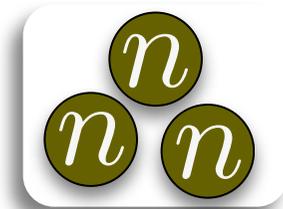
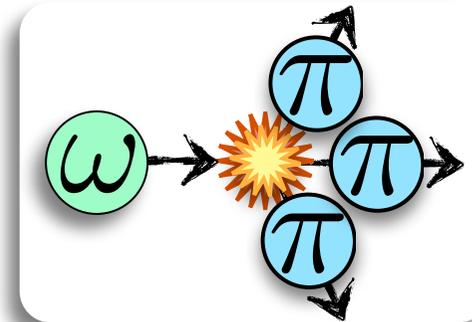
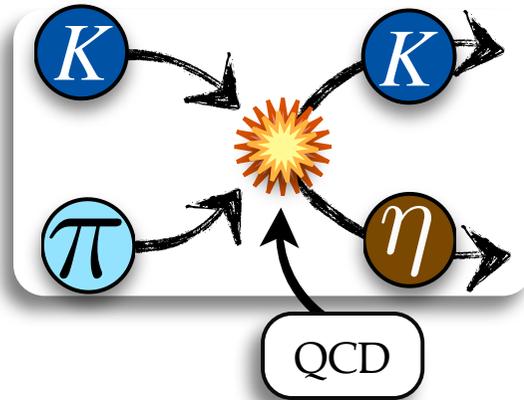
☛ Spectroscopy:

☛ Hadron structure-  
*transition form factors:*

☛ Fundamental  
symmetries:

# Goals

☞ Spectroscopy:

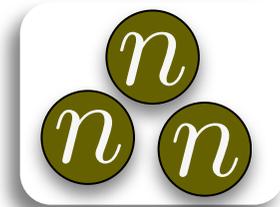
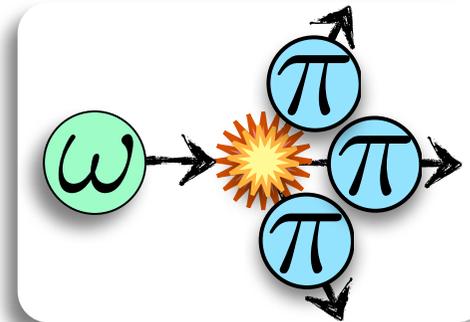
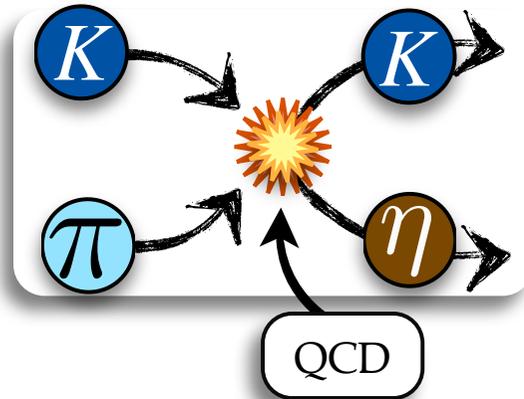


☞ Hadron structure-  
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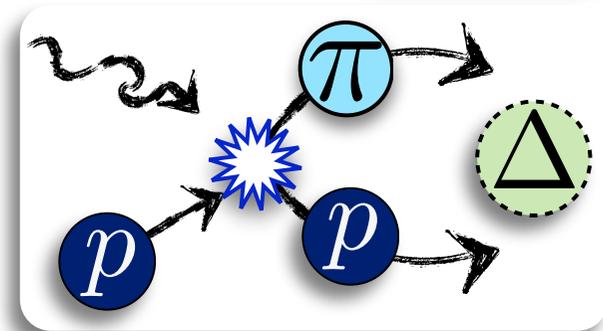
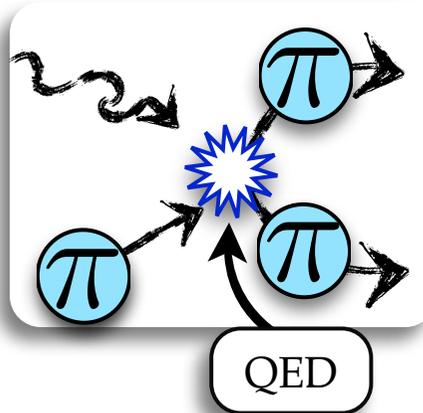
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# Goals

Spectroscopy:



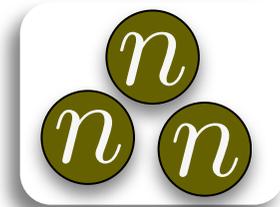
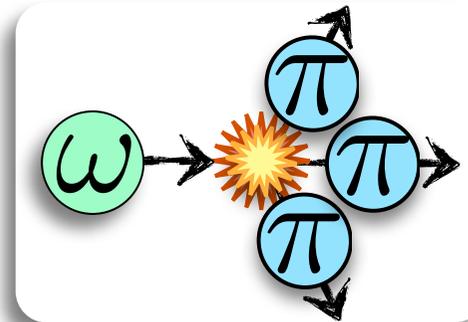
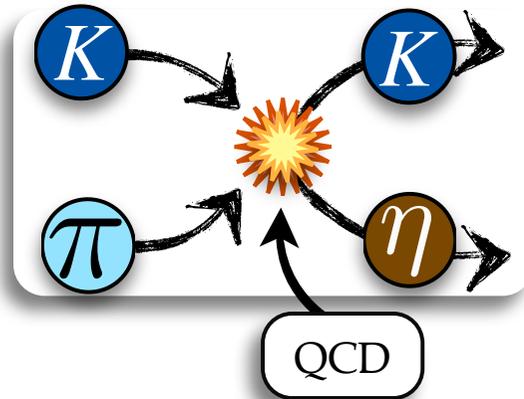
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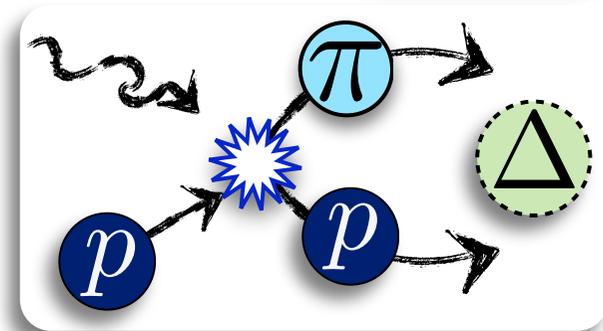
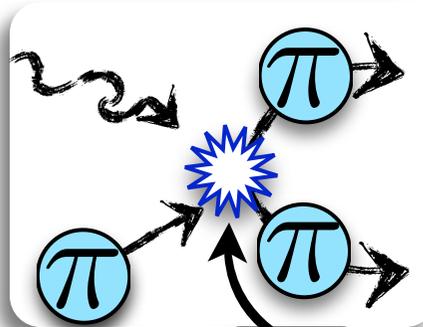
Fundamental  
symmetries:

# Goals

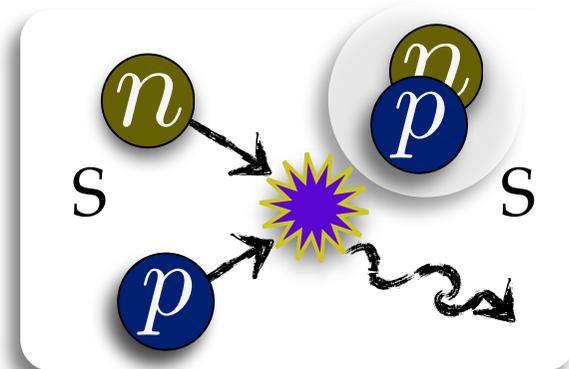
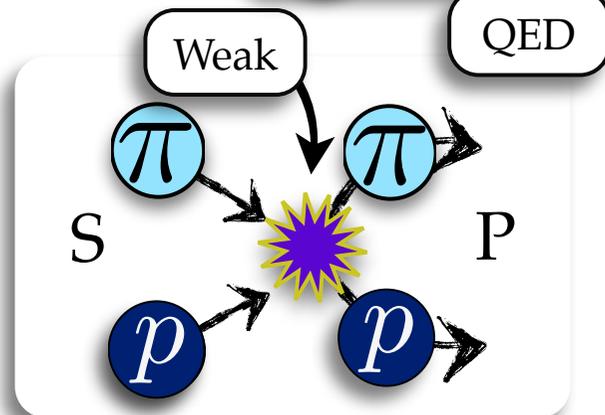
Spectroscopy:



Hadron structure-  
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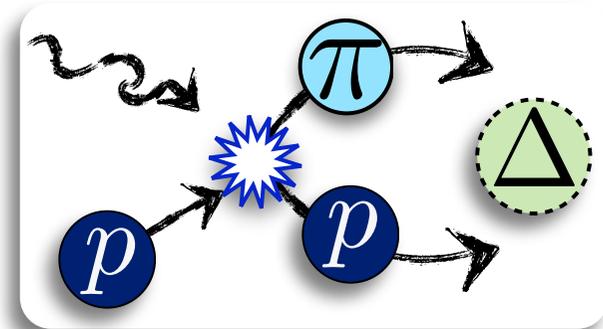
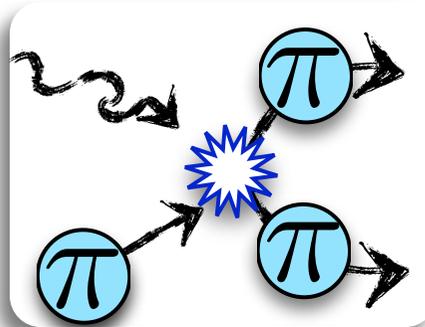
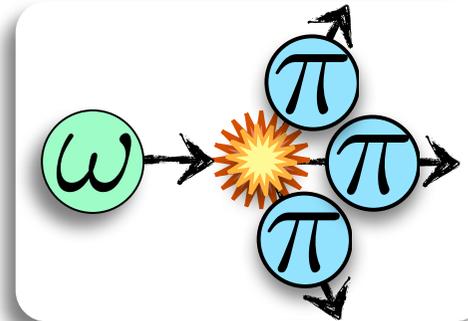
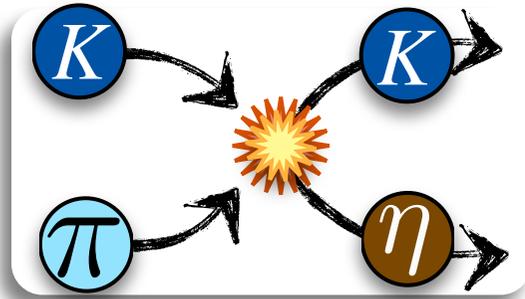


Fundamental  
symmetries:



# Goals

Spectroscopy:

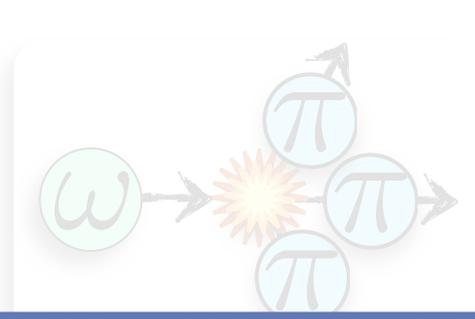


 **Jefferson Lab**



# Goals

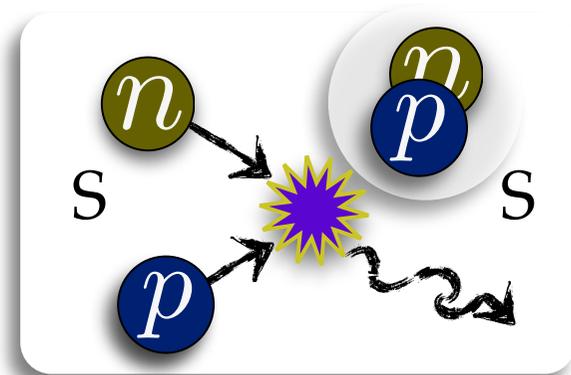
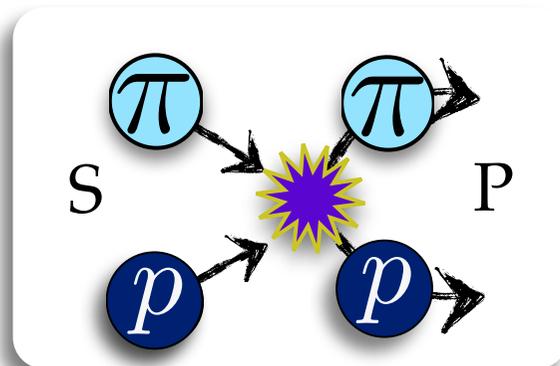
Spectroscopy:



Hadron structure  
transition form factors:



Fundamental  
symmetries:



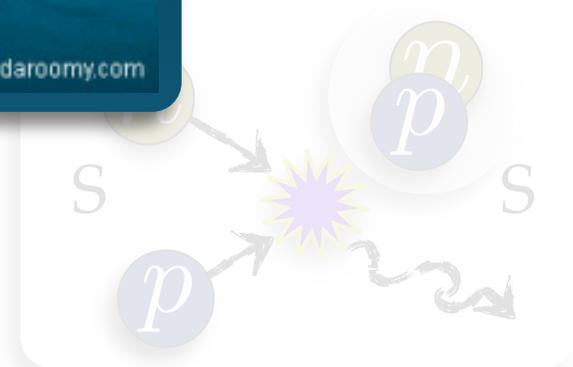
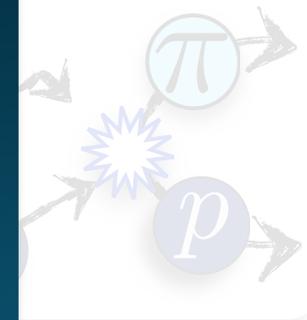
# Goals



Spectro

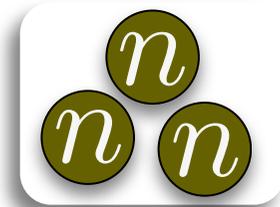
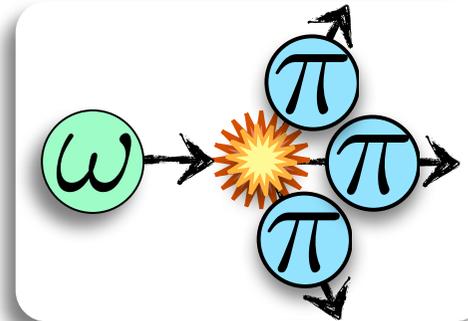
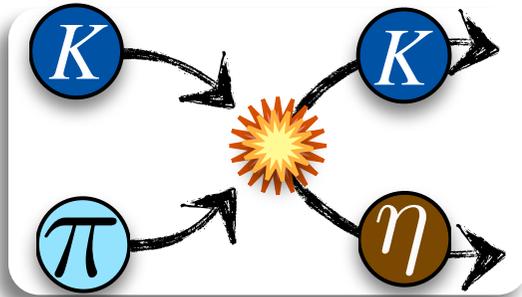
Hadron  
transiti

Fundamental  
symmetries:



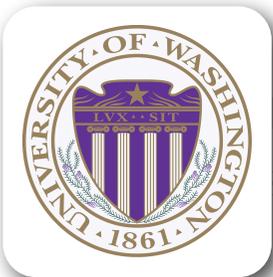
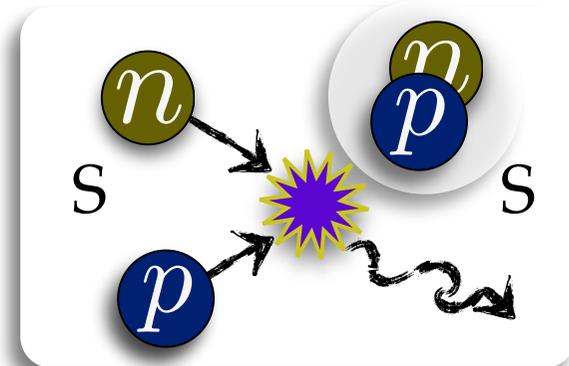
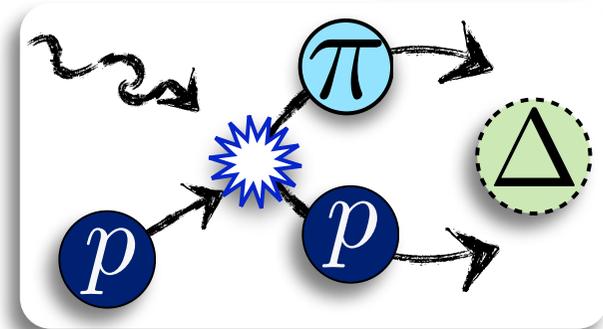
# Goals

Spectroscopy:



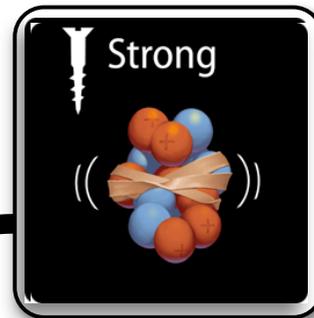
Institute for Nuclear Theory  
Workshop INT-13-53W

**Nuclear Reactions from Lattice QCD**  
March 11-12, 2013  
Organizers: RB, Zohreh Davoudi, Thomas Luu

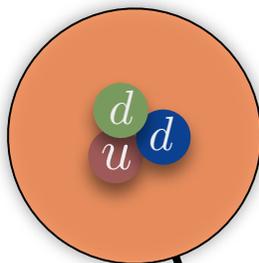


# Lattice QCD

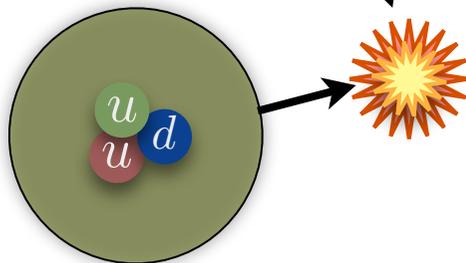
$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_f (i \not{D} - m_f) \psi_f - \frac{1}{4} \text{tr} (GG)$$



$n = udd$



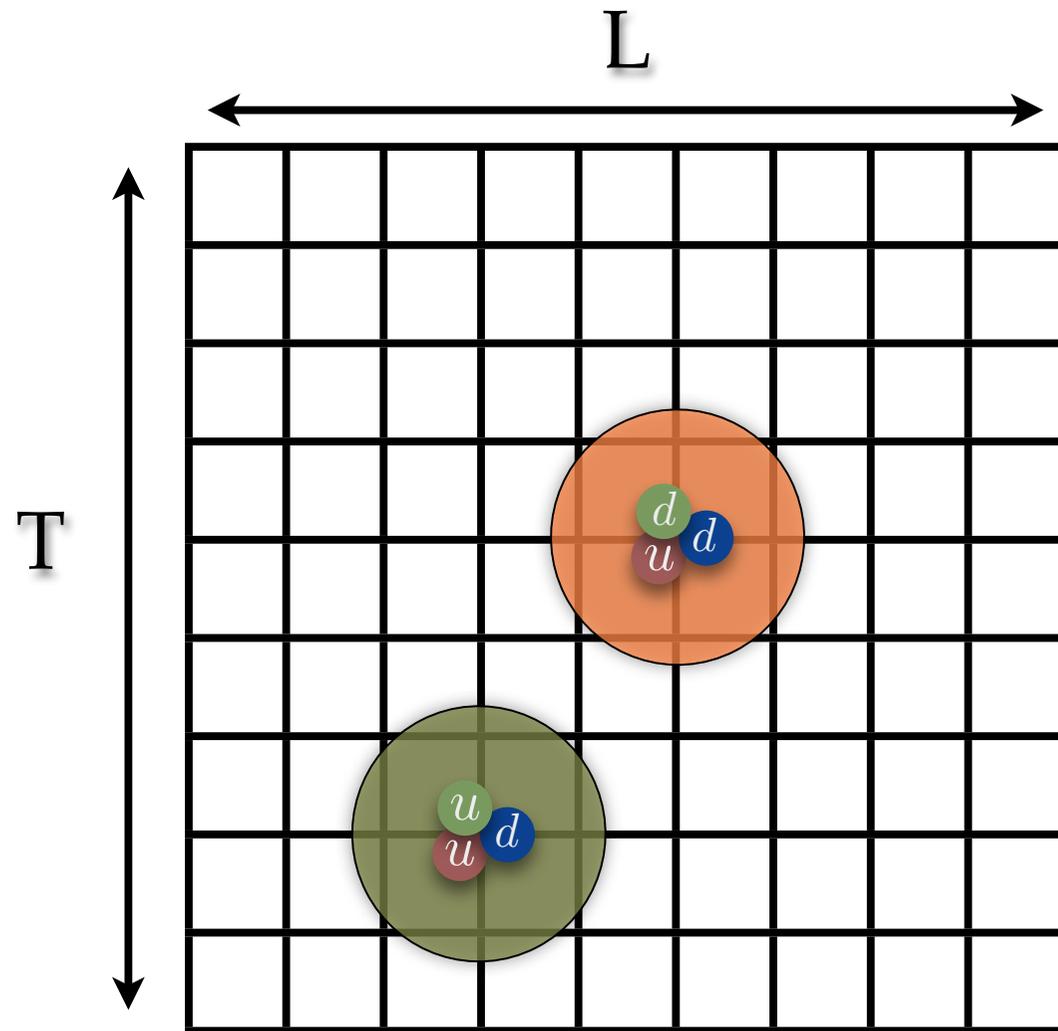
$p = uud$



np scattering



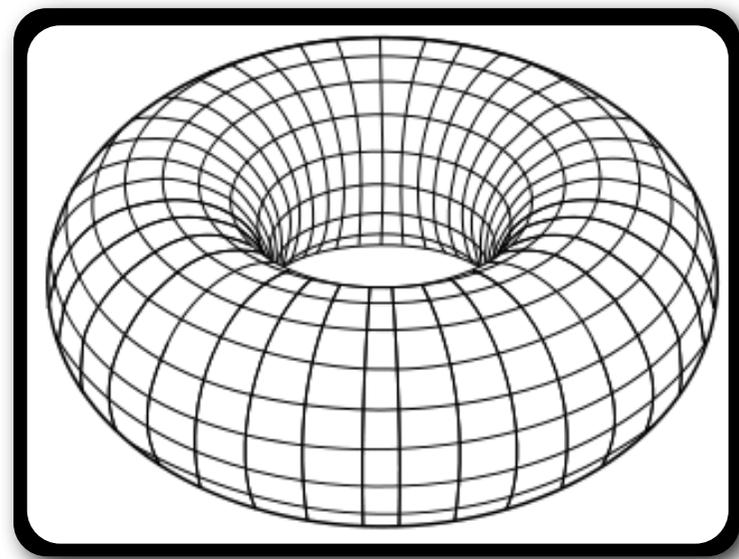
# LQCD: Finite Euclidean Spacetime



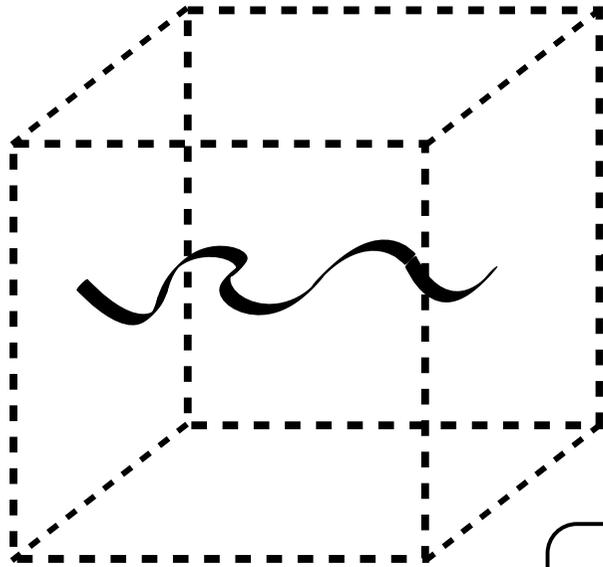
neutron-proton in a 4D torus

$t \rightarrow i\tau$

2D torus



# LQCD: Finite Euclidean Spacetime



Euclidean Spacetime

- Spectra:  $E(a, L, T, m_q)$
- Matrix elements:  $\mathcal{A}(a, L, T, m_q)$

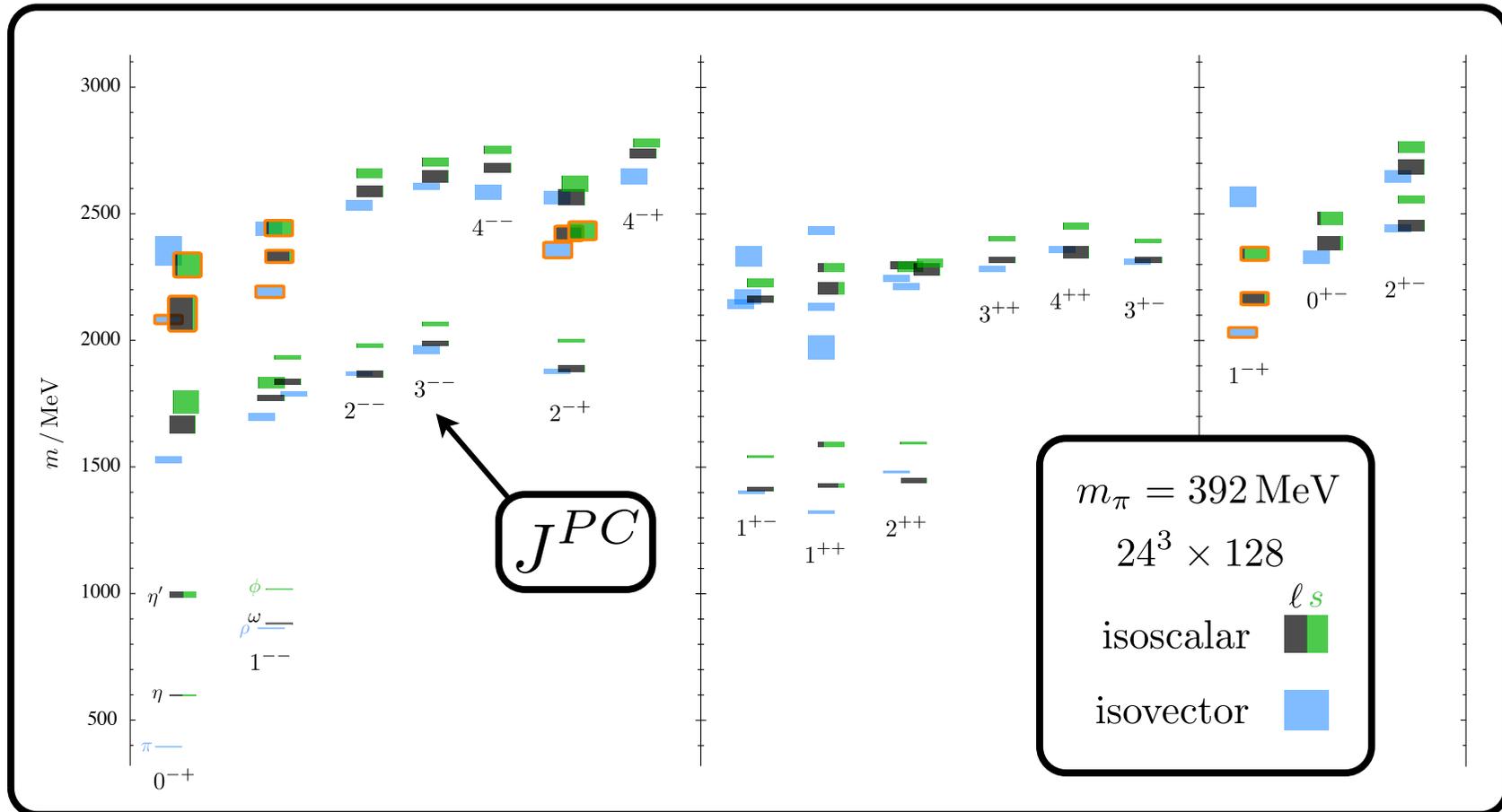
- **Formalism**

- Limits:  $(a \rightarrow 0, L \rightarrow \infty, T \rightarrow \infty, m_q \rightarrow m_q^{\text{phys}})$

- Physics: hadron masses, decay constants, scattering parameters, form factors,...

Minkowski Spacetime

# The state of the art!



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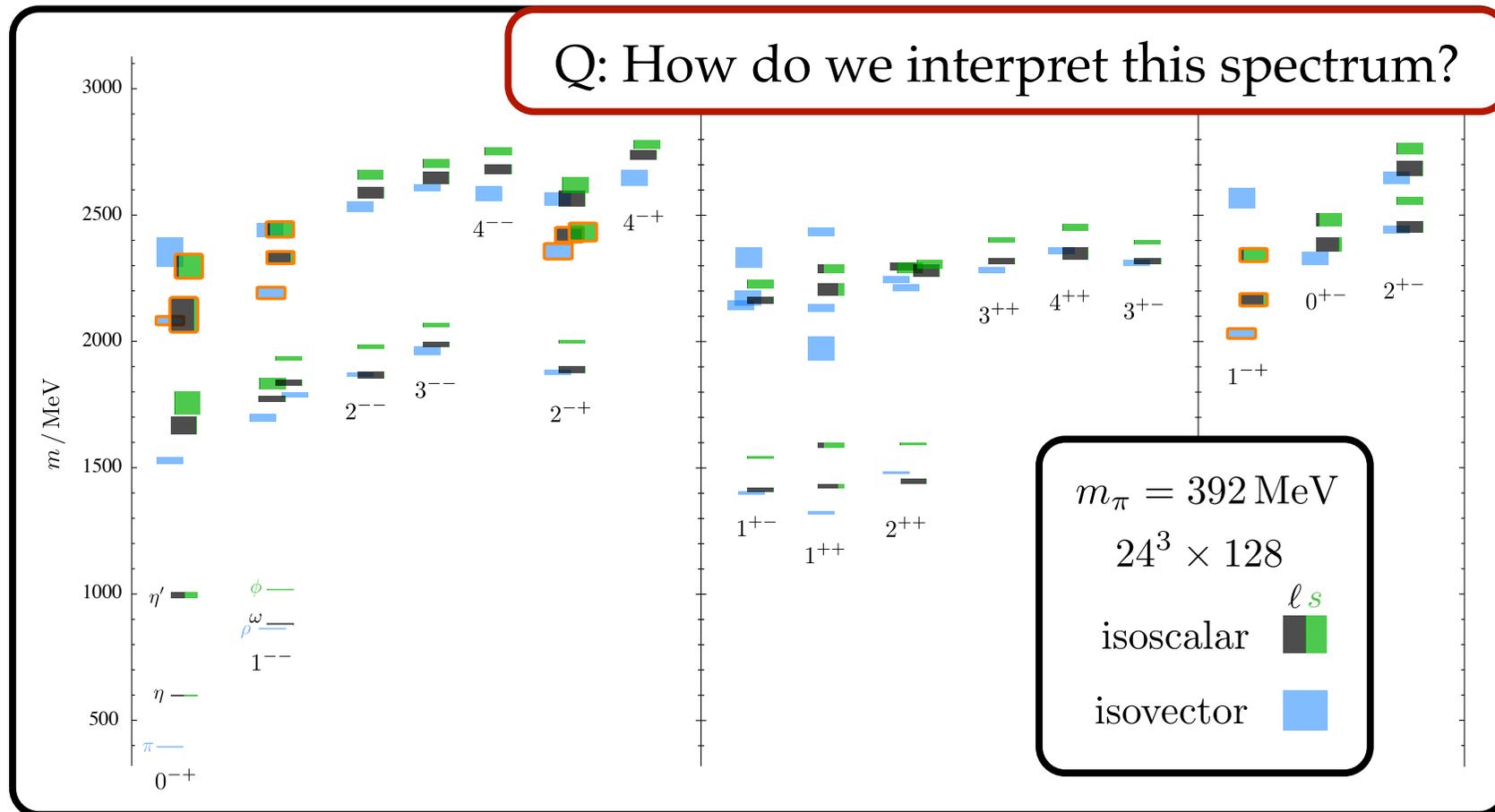
  
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Hadron Spectrum Collaboration: [PRD] [arXiv:1309.2608](https://arxiv.org/abs/1309.2608) [hep-lat]

J. Dudek, R. Edwards, P. Guo & C. Thomas (2013)

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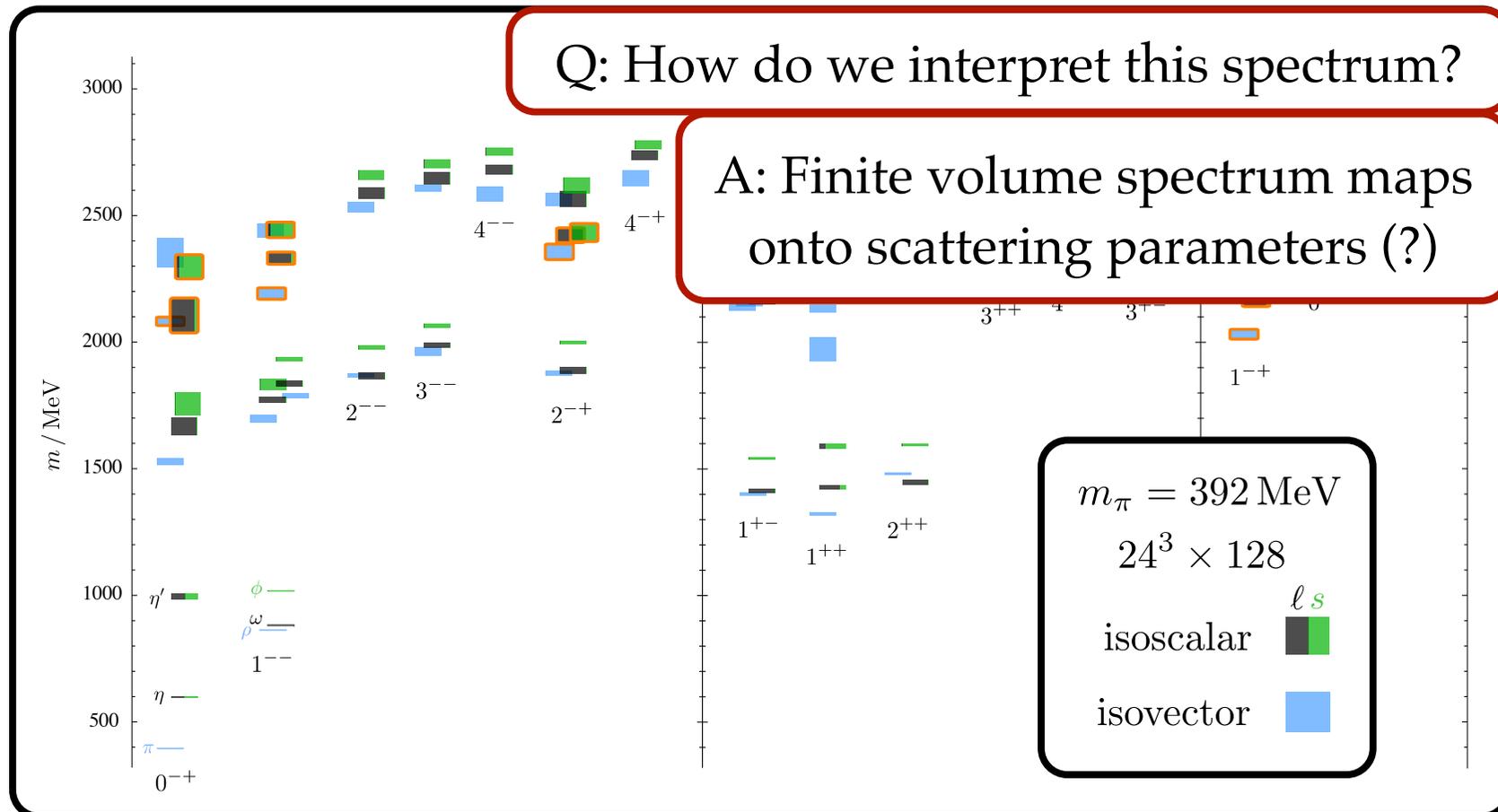
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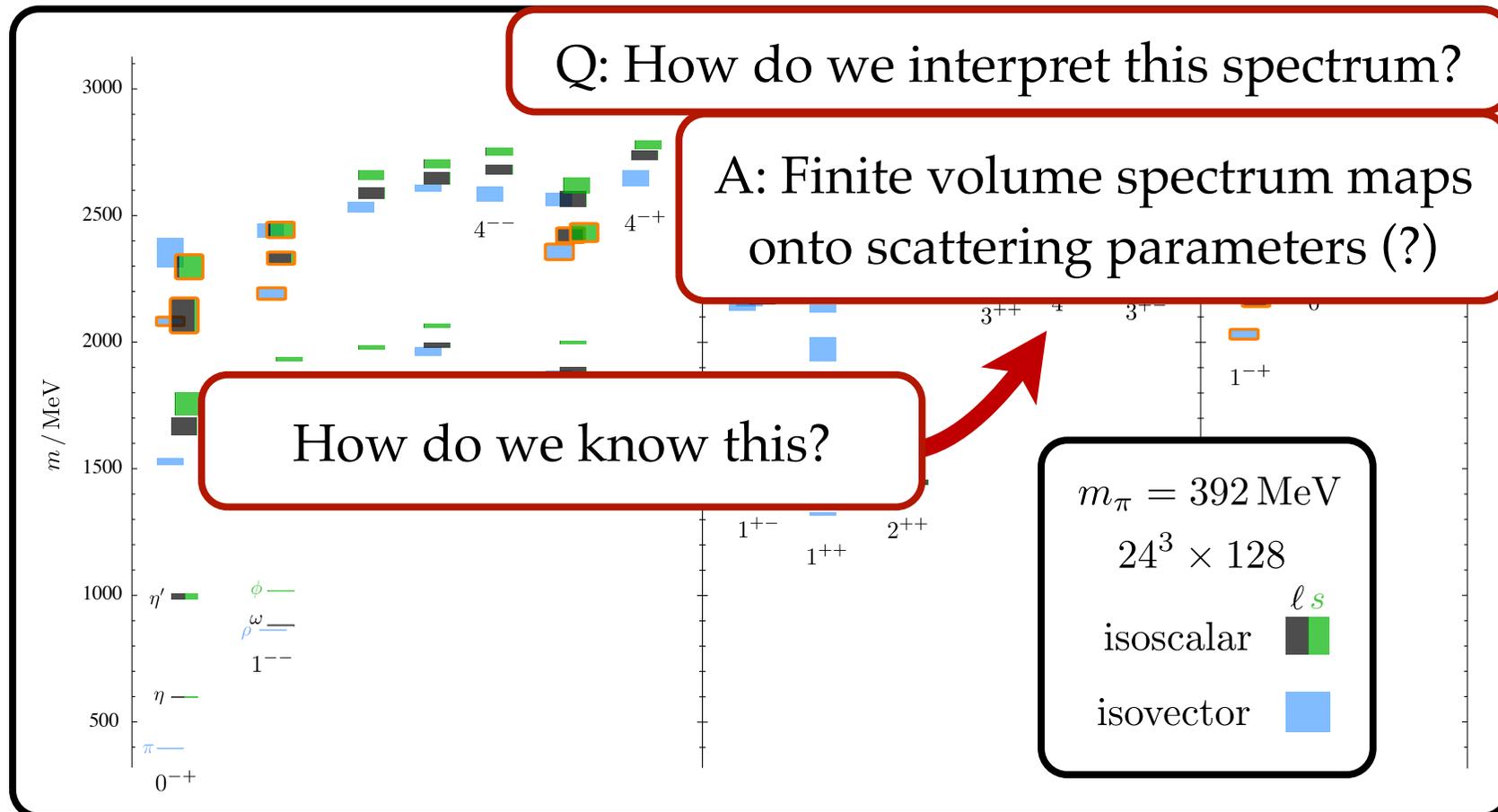
  
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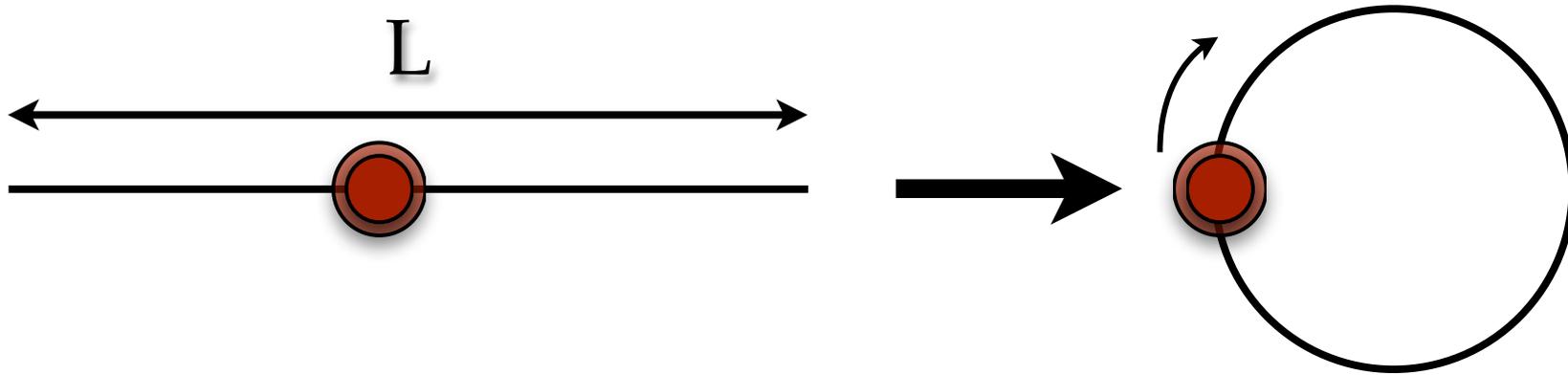
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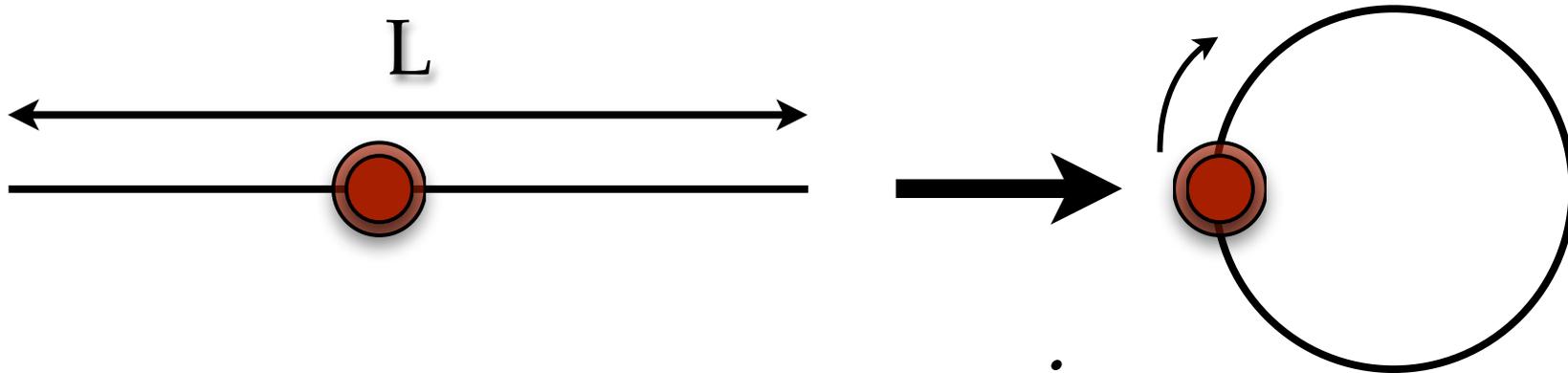
# Reinventing the *quantum-mechanical* wheel

(in 1+1 dimensions)



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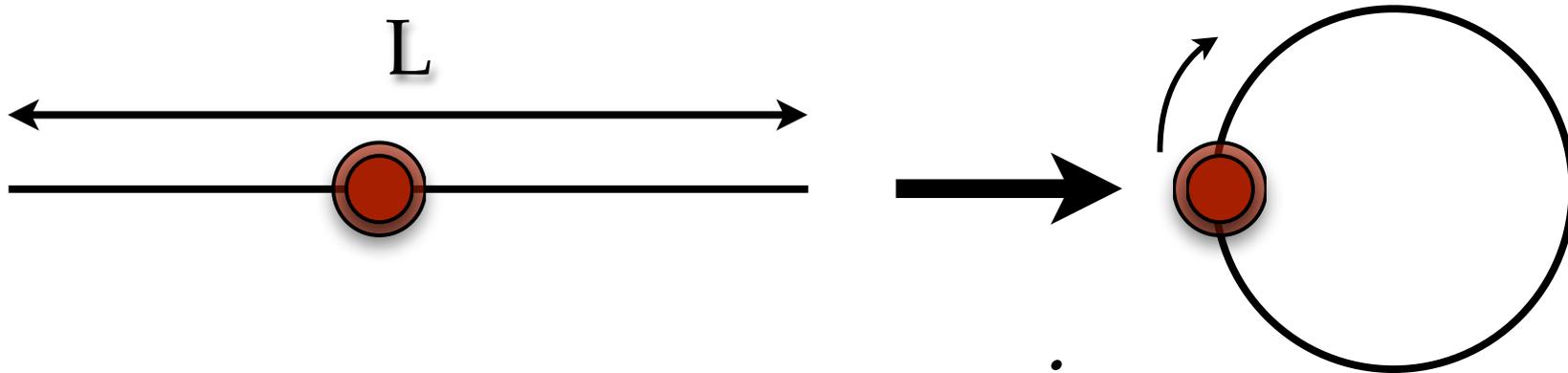
(in 1+1 dimensions)



$$\phi(x) \sim e^{ipx}$$

# Reinventing the *quantum-mechanical* wheel

(in 1+1 dimensions)



$$\phi(x) \sim e^{ipx}$$

**Periodicity:**

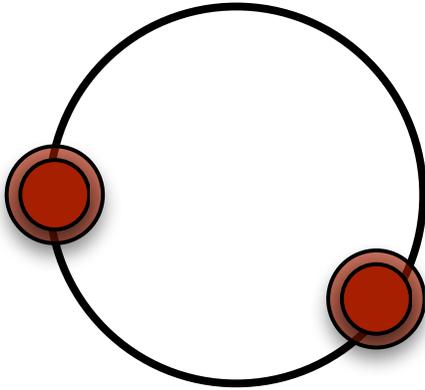
$$\phi(L) = \phi(0)$$

**Quantization condition:**

$$L p_n = 2\pi n$$

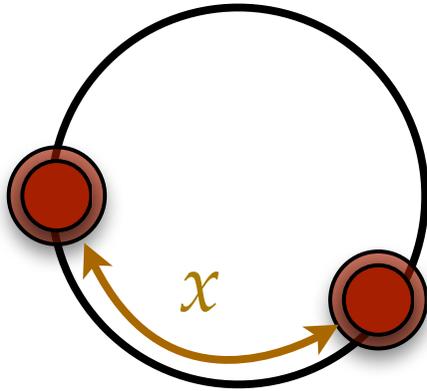
# Reinventing the *quantum-mechanical* wheel

Two particles:



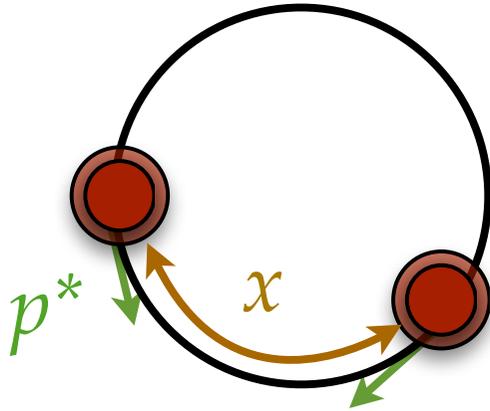
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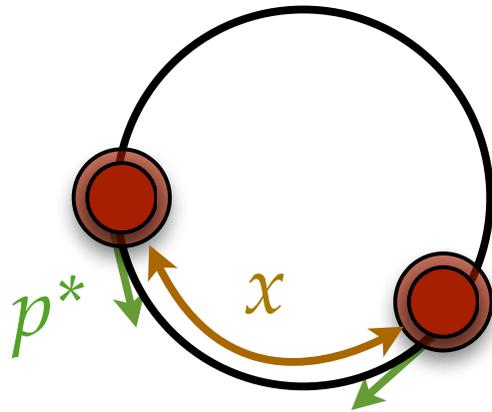
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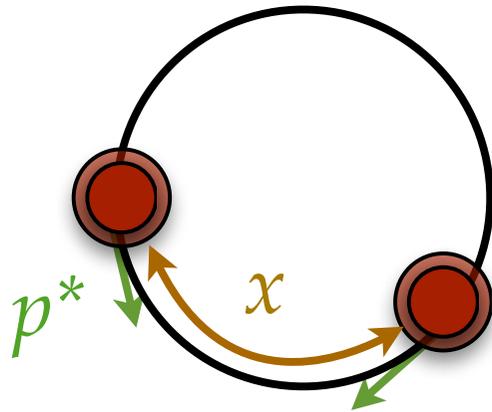
infinite volume  
scattering phase shift

$$\psi(x) \sim e^{ip^*x + i2\delta(p^*)}$$

Asymptotic  
wavefunction

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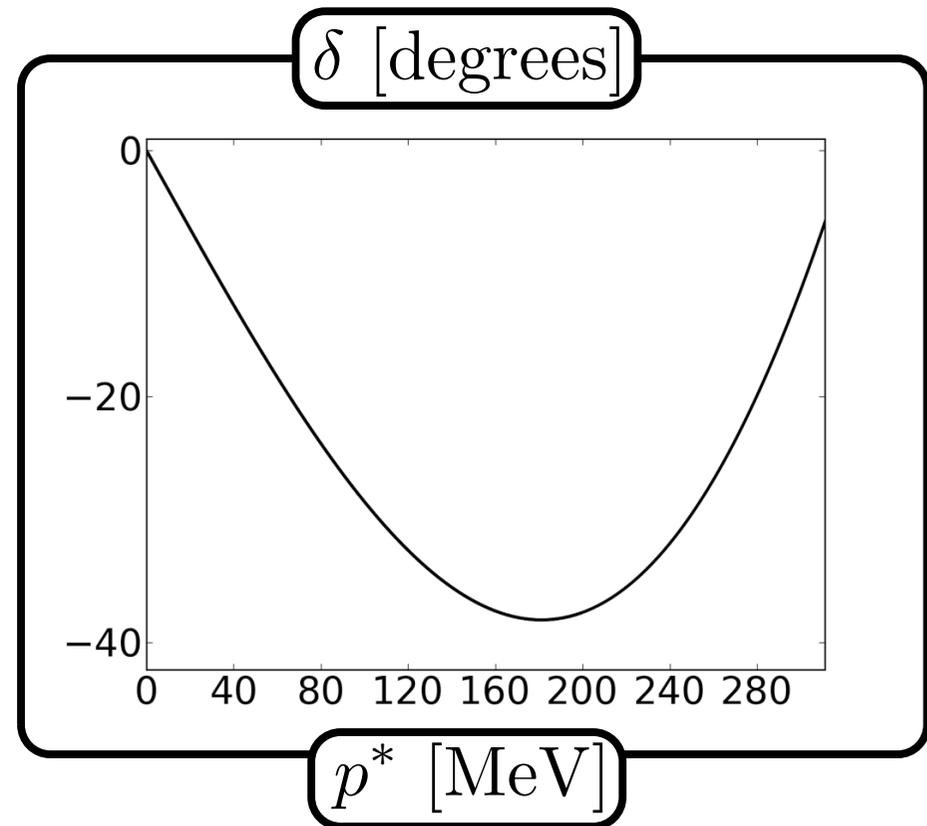
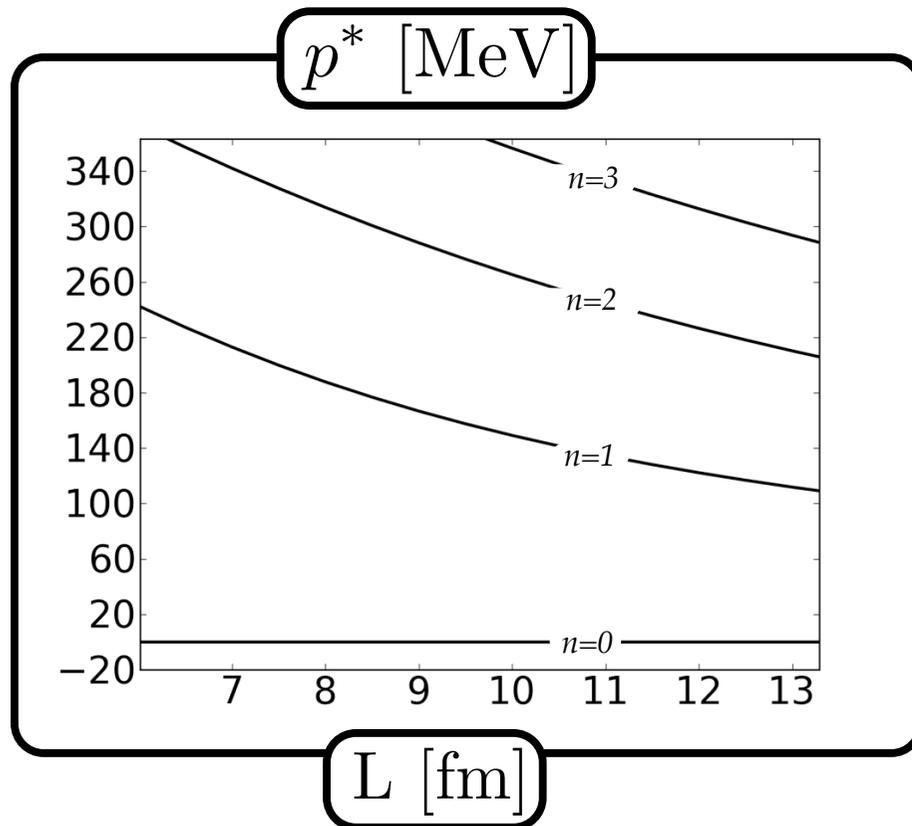
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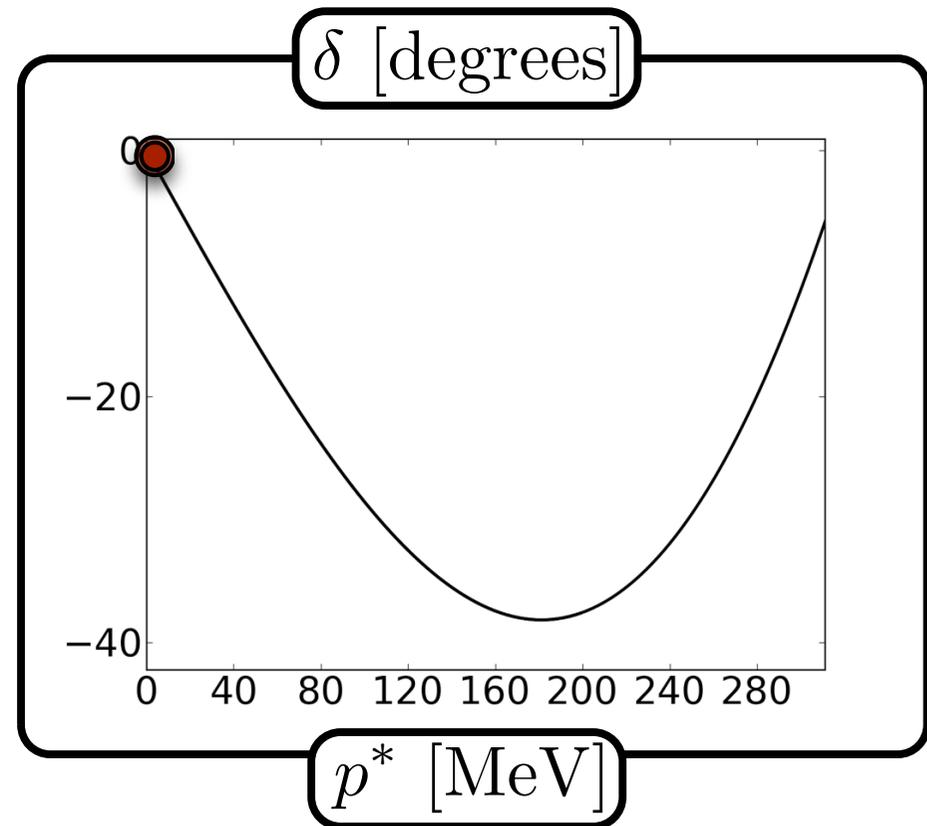
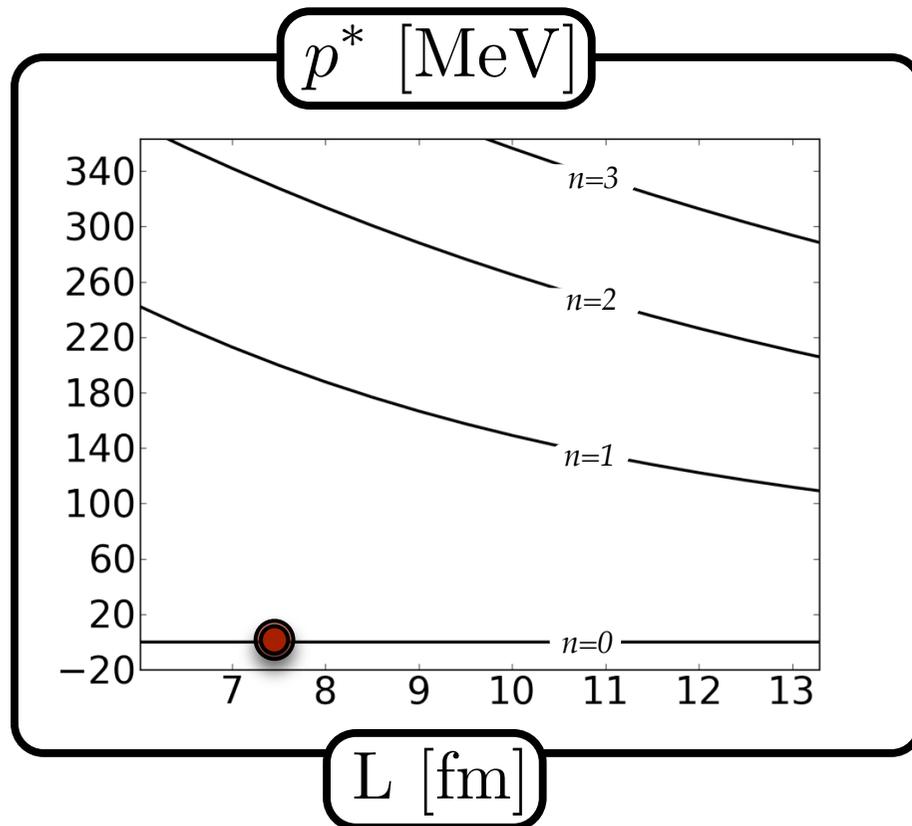
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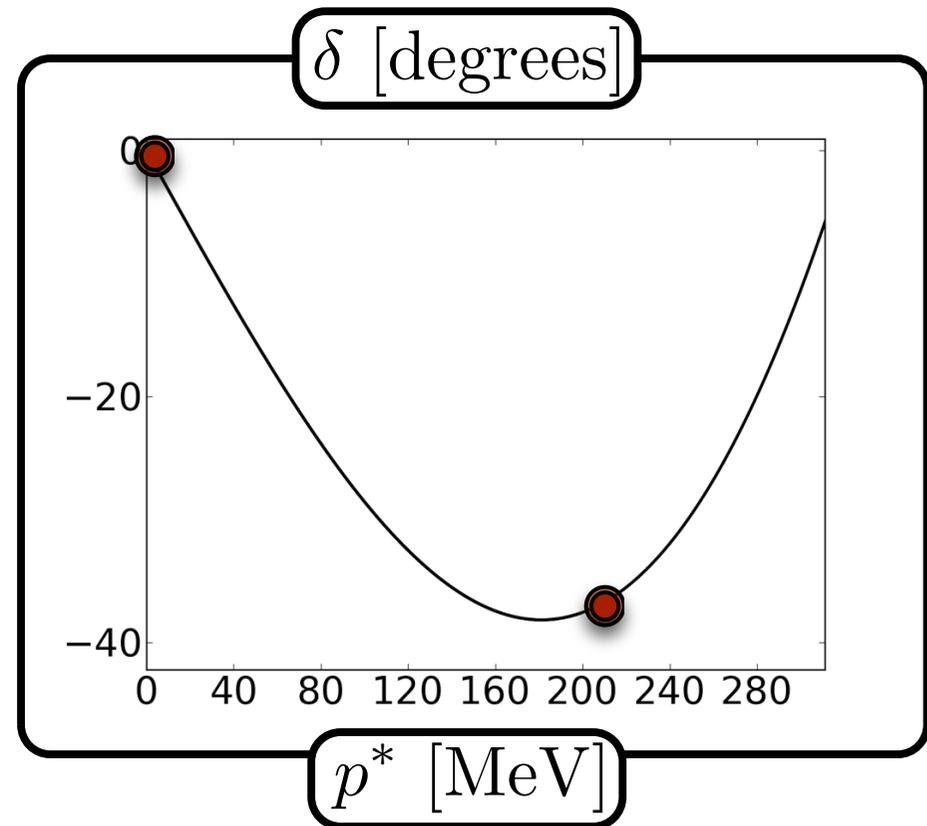
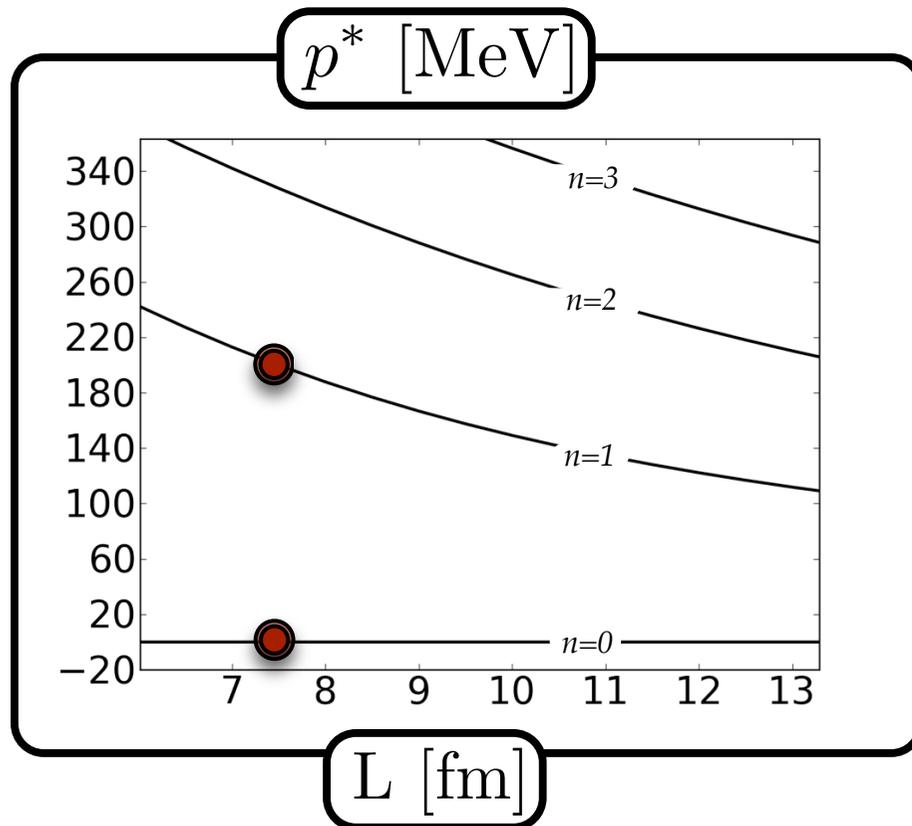
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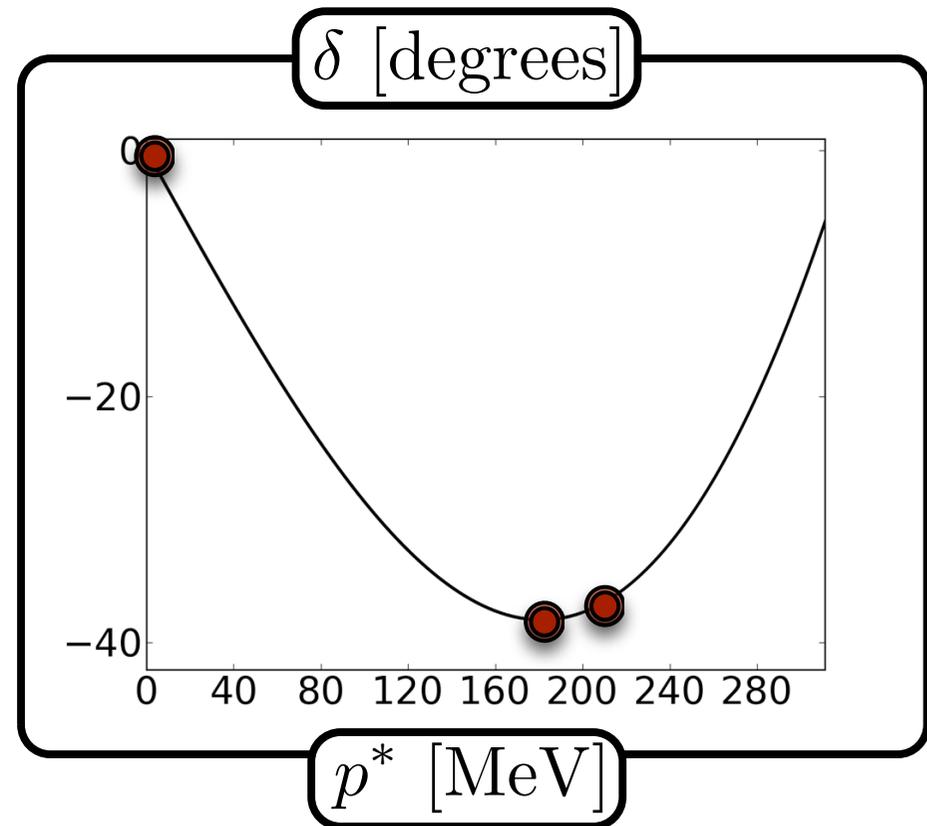
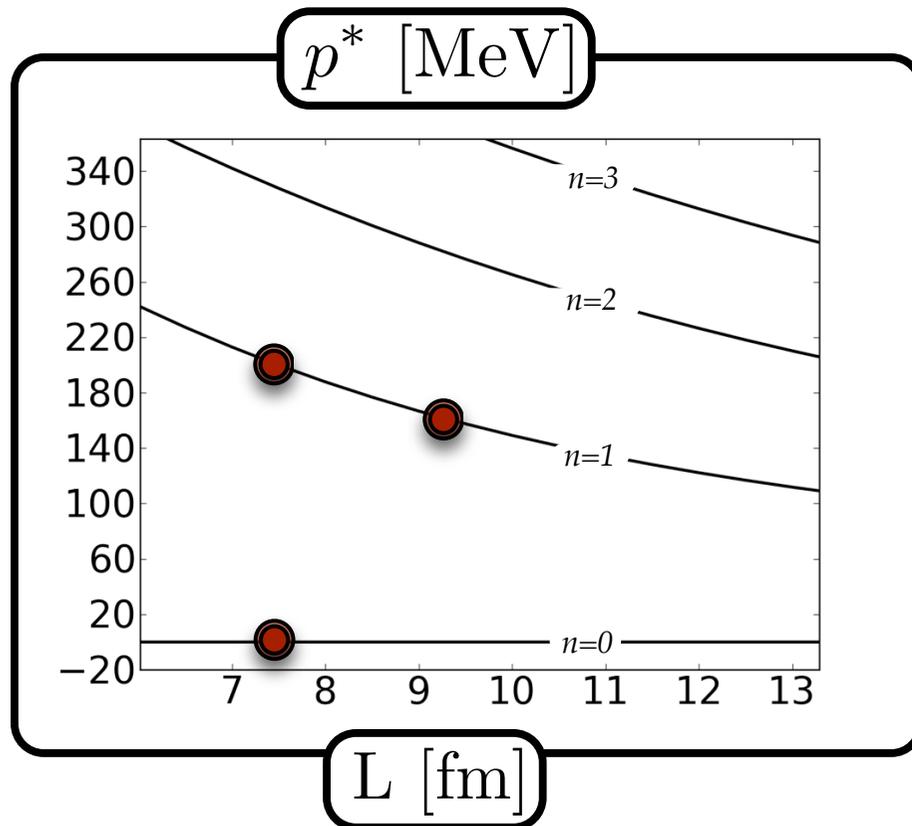
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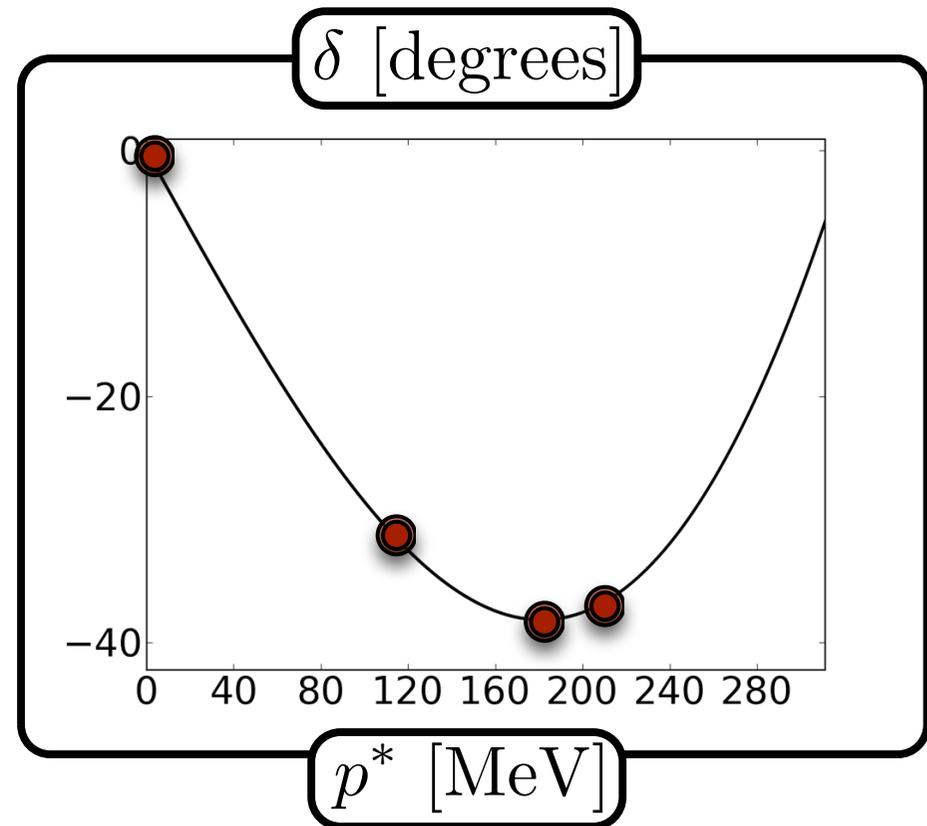
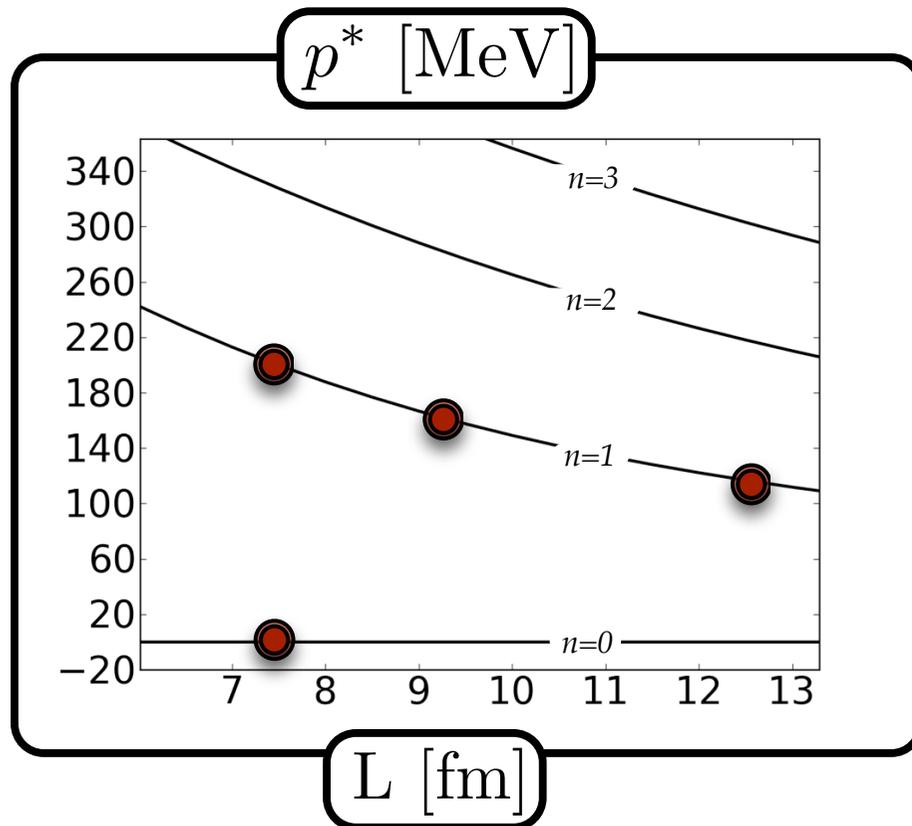
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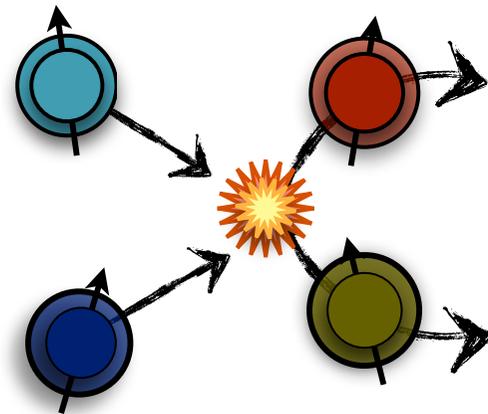
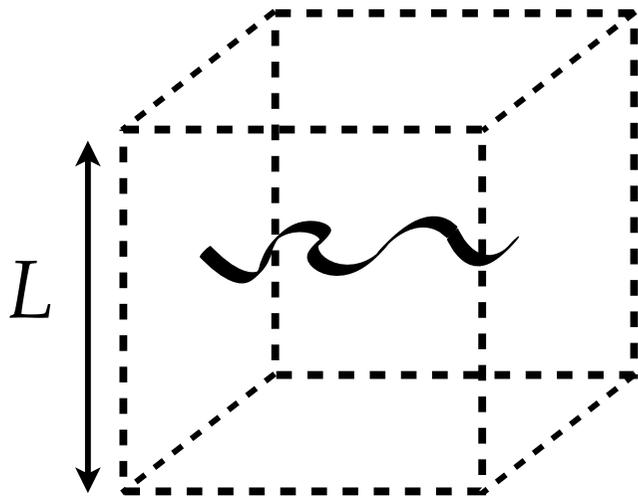
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# 3+1D result

$$\det (\mathbb{F}(E, L) + S(E)) = 0$$



RB (PhD Thesis) : [arXiv:1311.6032](https://arxiv.org/abs/1311.6032) [hep-lat]

RB, Zohreh Davoudi\* [PRD] : [arXiv:1204.1110](https://arxiv.org/abs/1204.1110) [hep-lat]

RB, Zohreh Davoudi\* [PRD] : [arXiv:1212.3398](https://arxiv.org/abs/1212.3398) [hep-lat]

RB, Zohreh Davoudi\*, Tom Luu\*\* [PRD] : [arXiv:1305.4903](https://arxiv.org/abs/1305.4903) [hep-lat]

RB, Zohreh Davoudi\*, Tom Luu\*\*, Martin Savage\* [PRD] : [arXiv:1309.3556](https://arxiv.org/abs/1309.3556) [hep-lat]

RB, Zohreh Davoudi\*, Tom Luu\*\*, Martin Savage\* : [arXiv:1311.7686](https://arxiv.org/abs/1311.7686) [hep-lat]

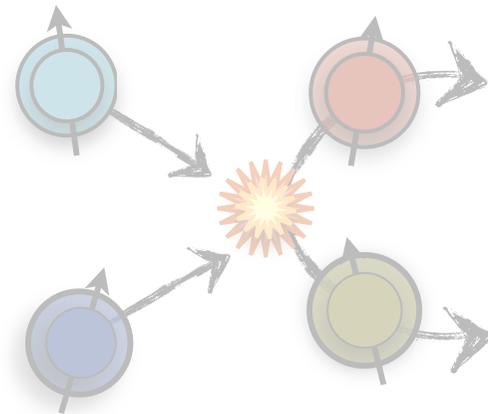
RB [paper is underway!] : most general 2-Body formalism

\*UW, \*\* Jülich

# 3+1D result

$$\det (\mathbb{F}(E, L) + S(E)) = 0$$

determinant over partial waves



RB (PhD Thesis) : [arXiv:1311.6032](https://arxiv.org/abs/1311.6032) [hep-lat]

RB, Zohreh Davoudi\* [PRD] : [arXiv:1204.1110](https://arxiv.org/abs/1204.1110) [hep-lat]

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RB, Zohreh Davoudi\*, Tom Luu\*\* [PRD] : [arXiv:1305.4903](https://arxiv.org/abs/1305.4903) [hep-lat]

RB, Zohreh Davoudi\*, Tom Luu\*\*, Martin Savage\* [PRD] : [arXiv:1309.3556](https://arxiv.org/abs/1309.3556) [hep-lat]

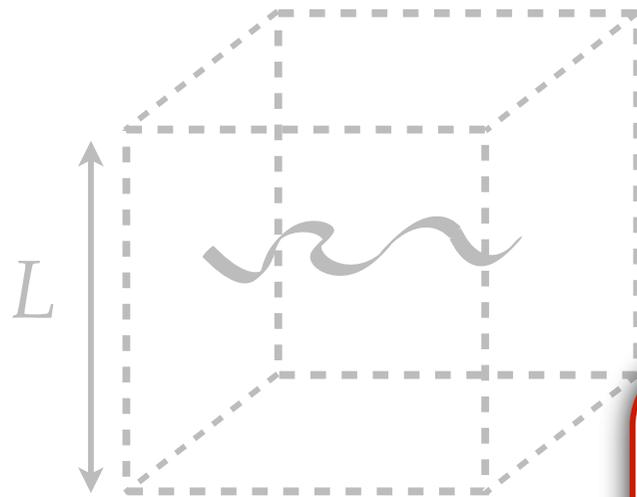
RB, Zohreh Davoudi\*, Tom Luu\*\*, Martin Savage\* : [arXiv:1311.7686](https://arxiv.org/abs/1311.7686) [hep-lat]

RB [on going!] : most general 2-Body formalism

\*UW, \*\* Jülich

# 3+1D result

$$\det (\mathbb{F}(E, L) + S(E)) = 0$$



S-matrix

$$\begin{pmatrix} e^{i2\delta_0} & & & & \\ & e^{i2\delta_1} & & & \\ & & e^{i2\delta_2} & & \\ & & & \ddots & \\ & & & & \ddots \end{pmatrix}$$

[11.6032 \[hep-lat\]](#)

[1204.1110 \[hep-lat\]](#)

[1212.3398 \[hep-lat\]](#)

[1305.4903 \[hep-lat\]](#)

[RB, Zohreh Davoudi\\*, Tom Luu\\*\\*, Martin Savage\\* : arXiv:1309.3556 \[hep-lat\]](#)

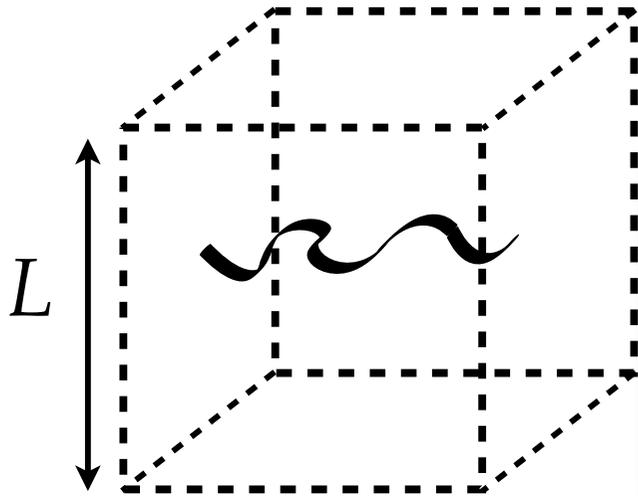
[RB, Zohreh Davoudi\\*, Tom Luu\\*\\*, Martin Savage\\* : arXiv:1311.7686 \[hep-lat\]](#)

RB [on going!] : most general 2-Body formalism

\*UW, \*\* Jülich

# 3+1D result

$$\det (\mathbb{F}(E, L) + S(E)) = 0$$



$$\begin{pmatrix} \mathbb{F}_{00} & \mathbb{F}_{01} & \mathbb{F}_{02} \\ \mathbb{F}_{10} & \mathbb{F}_{11} & \mathbb{F}_{12} \\ \mathbb{F}_{20} & \mathbb{F}_{21} & \mathbb{F}_{22} \\ & & \ddots \end{pmatrix}$$

RB (PhD Thesis) : [arXiv:1311.6032 \[hep-lat\]](https://arxiv.org/abs/1311.6032)

Typically a sparse matrix

[arXiv:1204.1110 \[hep-lat\]](https://arxiv.org/abs/1204.1110)

[arXiv:1212.3398 \[hep-lat\]](https://arxiv.org/abs/1212.3398)

[arXiv:1305.4903 \[hep-lat\]](https://arxiv.org/abs/1305.4903)

RB, Zohreh Davoudi\*, Tom Luu\*\*, Martin Savage\* [PRD] : [arXiv:1309.3556 \[hep-lat\]](https://arxiv.org/abs/1309.3556)

RB, Zohreh Davoudi\*, Tom Luu\*\*, Martin Savage\* : [arXiv:1311.7686 \[hep-lat\]](https://arxiv.org/abs/1311.7686)

RB [on going!] : most general 2-Body formalism

\*UW, \*\* Jülich

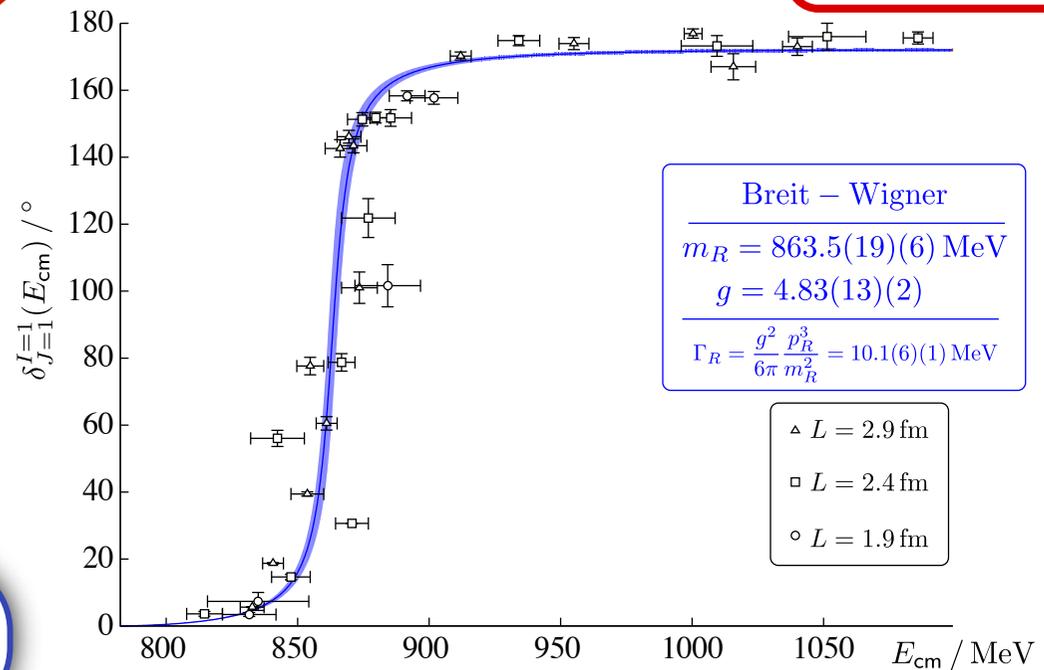
# 3+1D result

$$\det (\mathbb{F}(E, L) + S(E)) = 0$$

In goes finite  
volume spectra

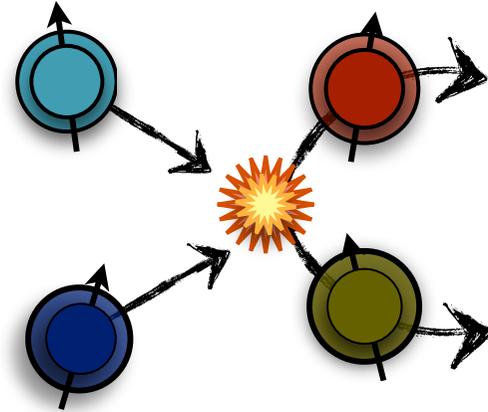
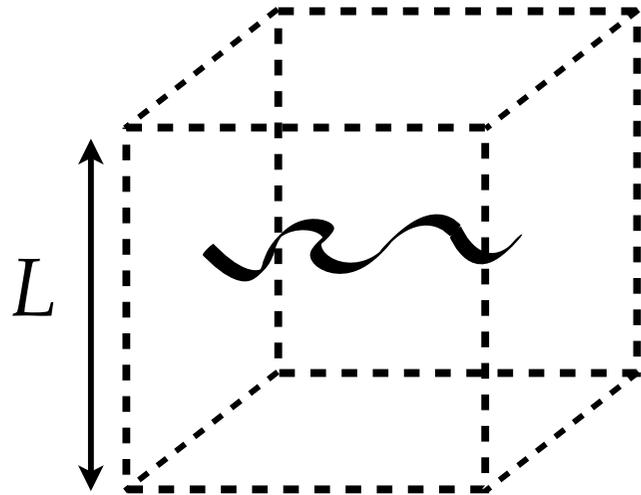
out goes...

$$m_\pi = 392 \text{ MeV}$$



[PRD] arXiv:1212.0830 [hep-ph] [PRD]  
J. Dudek, R. Edwards & C. Thomas (2012)

# 3+1D result



*"just a consequence of quantum mechanics"*

- Model independent & non-perturbative
- Universal: nuclear physics, atomic physics, etc
- Arbitrary quantum numbers: relativity, spin, masses, momenta, angular momentum, inelasticities, etc
- General boundary conditions: periodic, anti-periodic, or any linear combination

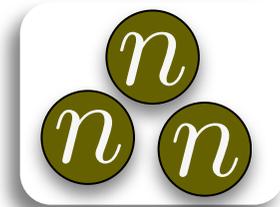
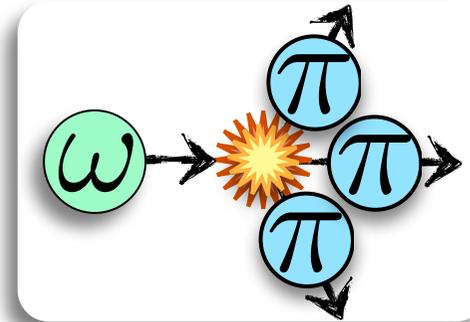
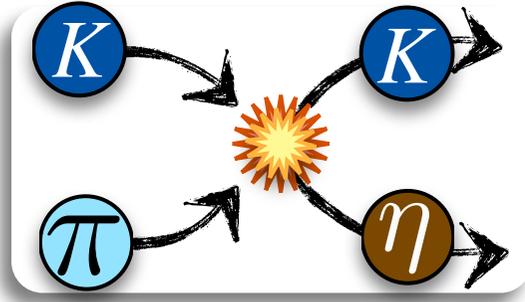
**RB [paper is underway!] : most general 2-Body formalism**

# Status

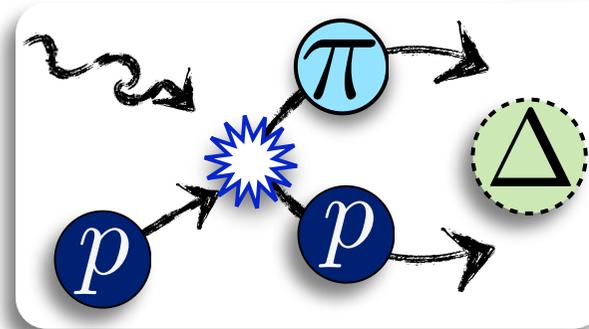
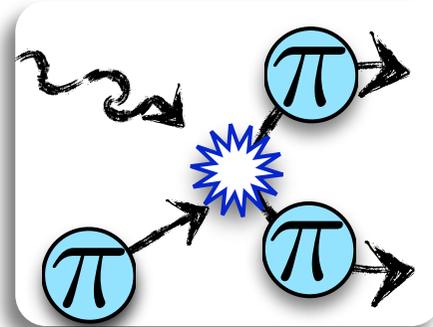
□ Formalism, systematics, ...

○ Code development, LQCD calculations

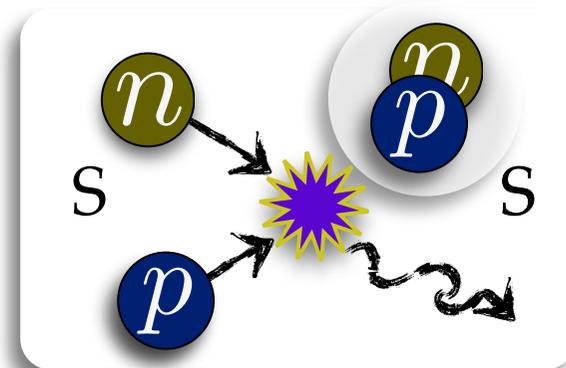
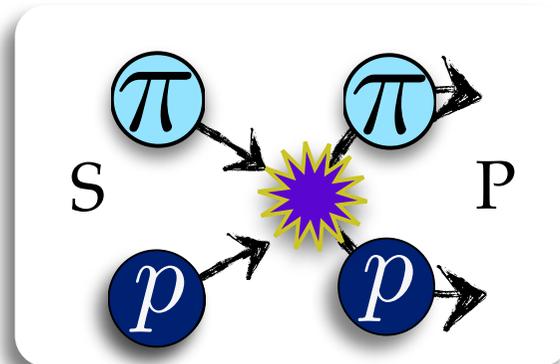
☪ Spectroscopy:



☪ Hadron structure-  
transition form factors:



☪ Fundamental symmetries:

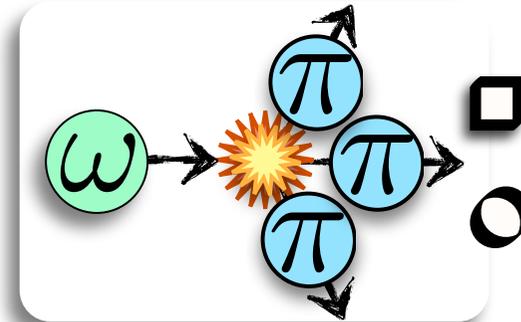
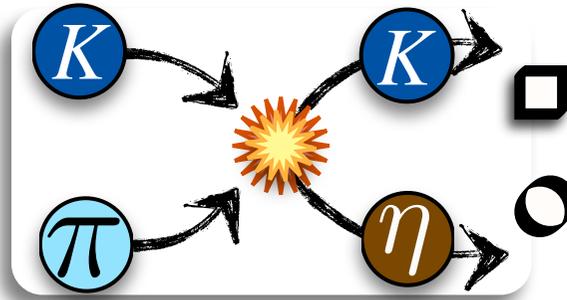


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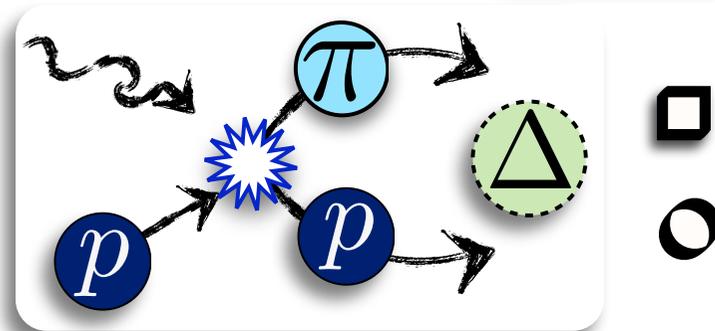
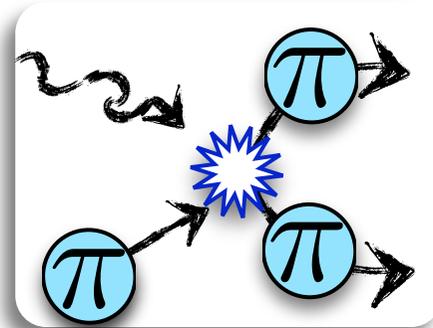
□ Formalism, systematics, ...

○ Code development, LQCD calculations

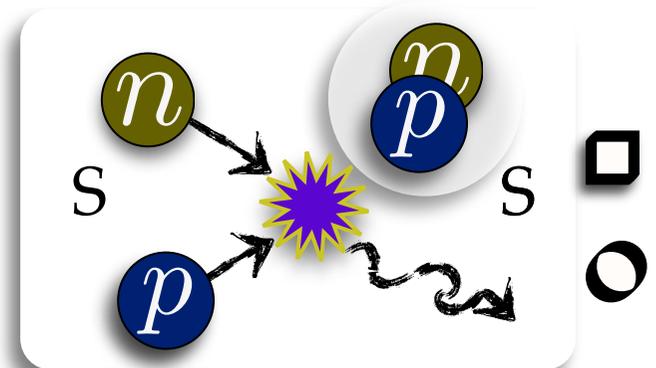
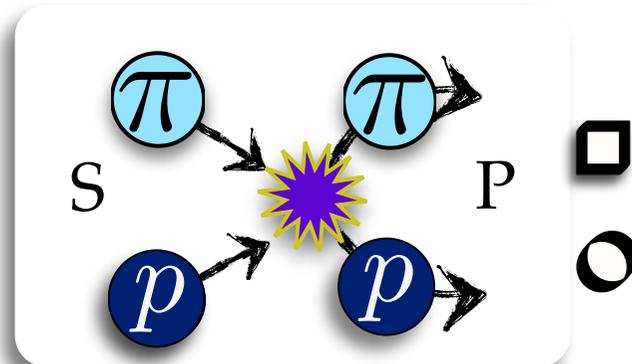
Spectroscopy:



Hadron structure-  
transition form factors:



Fundamental  
symmetries:



# Status

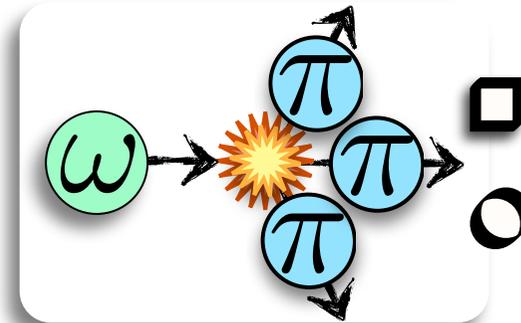
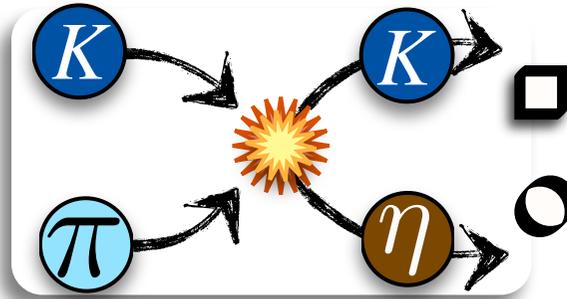
□ Formalism, systematics, ...

○ Code development, LQCD calculations

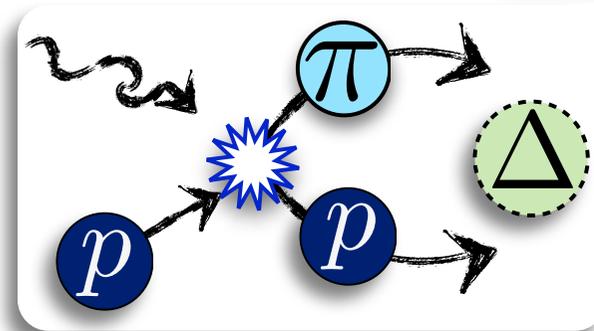
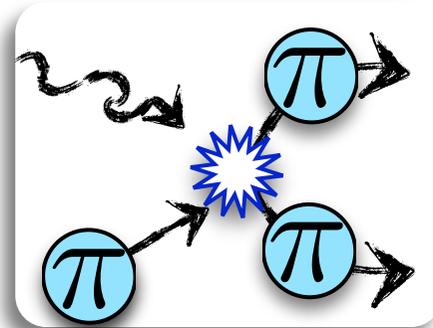
✓ : Under control

! : progress made  
? : open problem

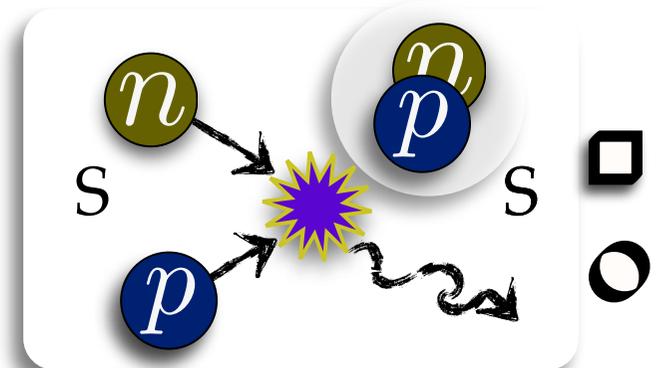
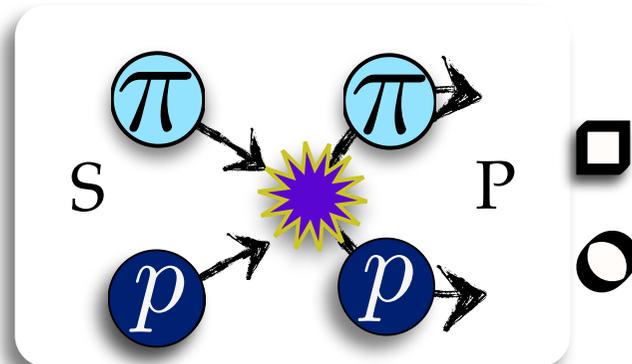
Spectroscopy:



Hadron structure-  
transition form factors:



Fundamental  
symmetries:



# Status

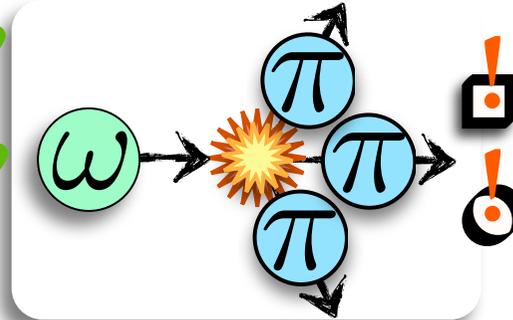
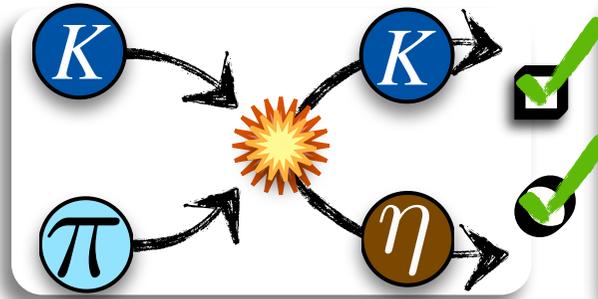
□ Formalism, systematics, ...

○ Code development, LQCD calculations

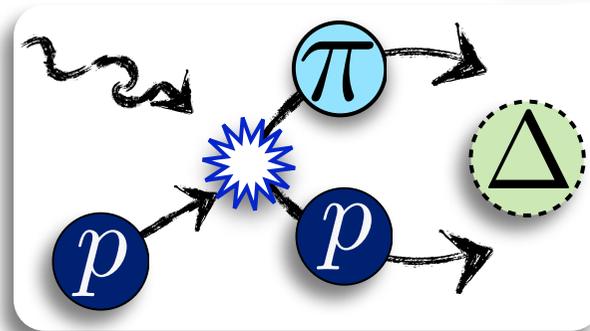
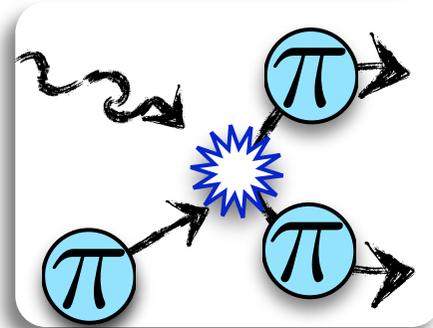
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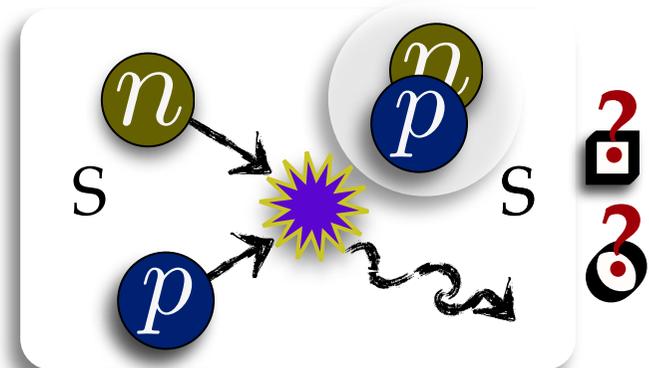
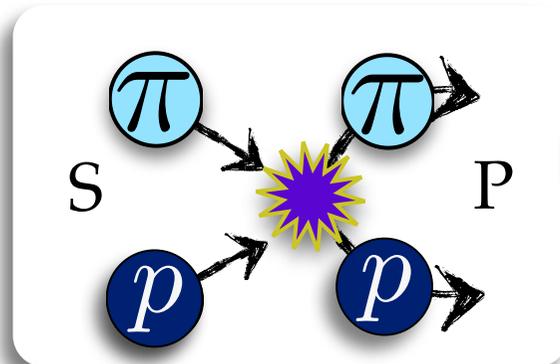
Spectroscopy:



Hadron structure-  
transition form factors:



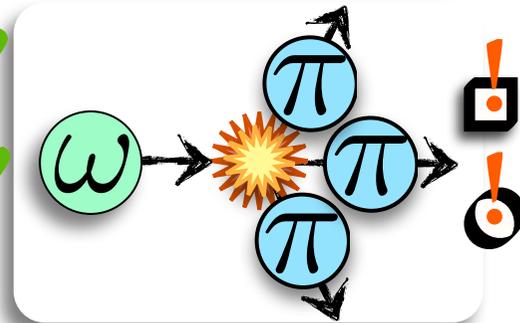
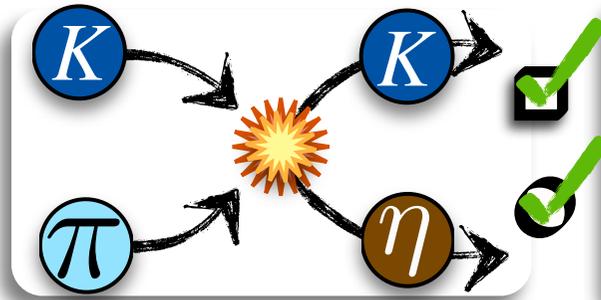
Fundamental  
symmetries:



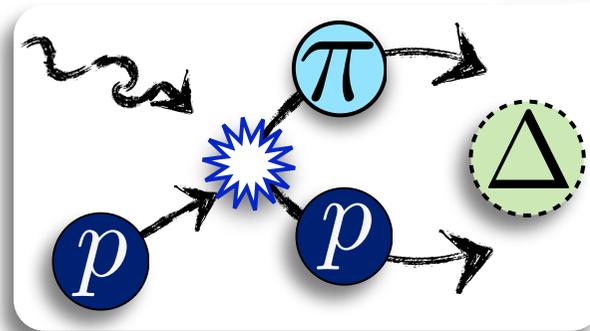
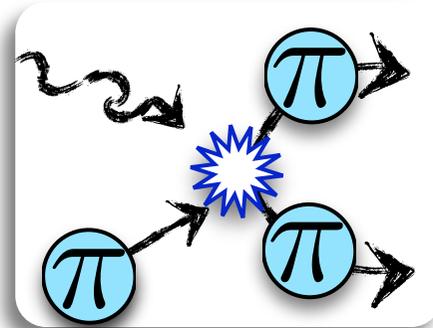
# 3 year plan:



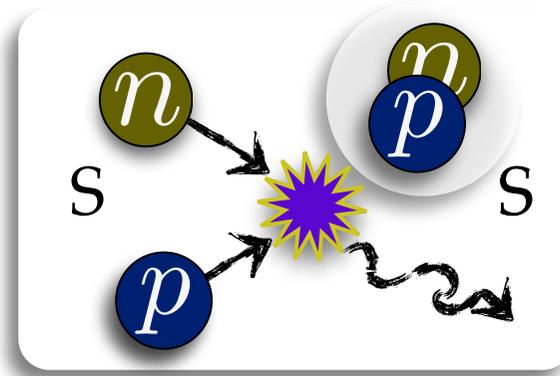
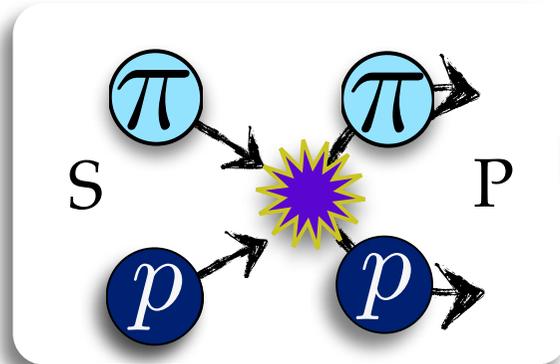
Spectroscopy:



Hadron structure-  
*transition form factors:*



Fundamental  
symmetries:



exciting work  
under  
construction



please come back later...

Jefferson Lab

OLD DOMINION  
UNIVERSITY

WILLIAM  
& MARY



## Not a one person job!

- JLab affiliates: Jozef Dudek, Robert Edwards, Chris Monahan, David Richards, Andre Walker-Loud, David Wilson,...
- Non-JLab affiliates: Daniel Bolton, Zohreh Davoudi, Max Hansen, Tom Luu, Martin Savage,...

Hansen, Tom Luu, Martin Savage,...