$M_n - M_p$

Theory Seminar

Monday, February 11, 2013

André Walker-Loud

Introduction: M_n - M_p

- Nature: $M_n - M_p = 1.29333217(42)\ \mathrm{MeV}$ CODATA

- Standard Model has two sources of isospin breaking

$$\hat{Q} = \frac{1}{6}\mathbb{1} + \frac{1}{2}\tau_3 \qquad m_q = \hat{m}\mathbb{1} - \delta\tau_3$$

- Given only electro-static forces, one would predict

$$M_p > M_n$$

- The contribution from $m_d - m_u$ is comparable in size but opposite in sign

Introduction: M_n - M_p

- M_n - M_p plays an extremely significant role in the evolution of the universe as we know it

  Initial conditions for Big Bang Nucleosynthesis (BBN)

$$\frac{X_n}{X_p} = e^{-\frac{M_n - M_p}{T}}$$

- The neutron lifetime is highly sensitive to the value of this mass splitting

$$\frac{1}{\tau_n} = \frac{(G_F \cos\theta_C)^2}{2\pi^3} m_e^5 (1 + 3g_A^2)\, f\left(\frac{M_n - M_p}{m_e}\right)$$

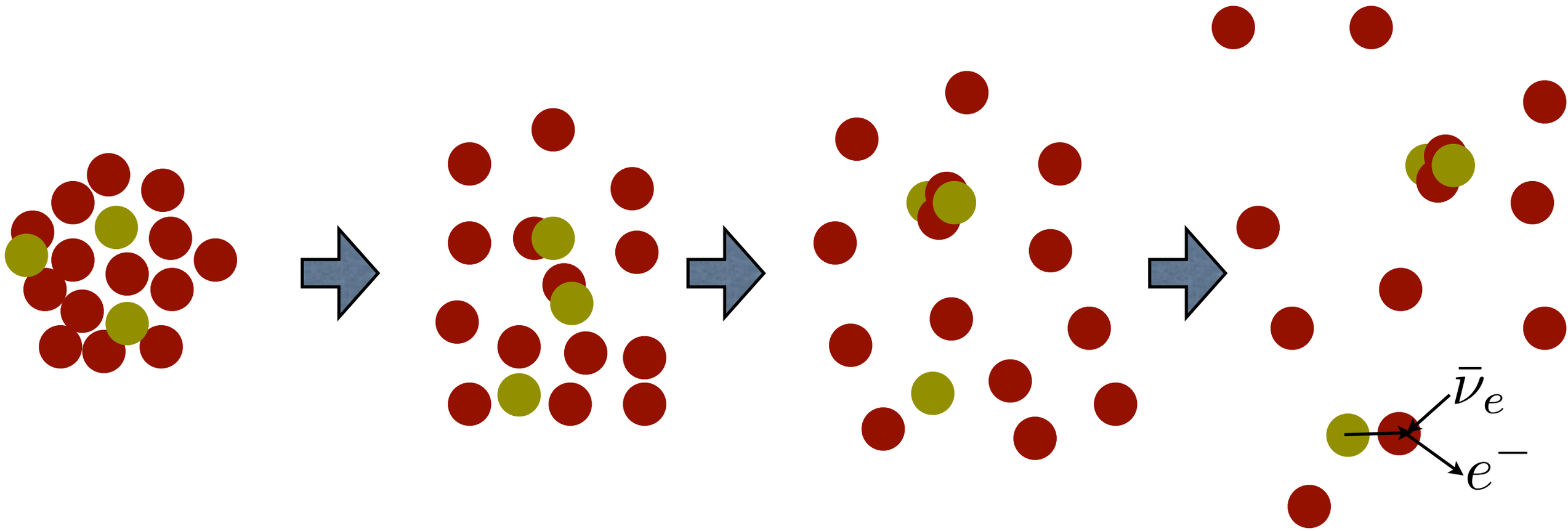
Point Nucleons $\quad f(a) \simeq \frac{1}{15}\left(2a^4 - 9a^2 - 8\right)\sqrt{a^2-1} + a \ln\left(a + \sqrt{a^2-1}\right)$

Griffiths "Introduction to Elementary Particles"

10% change in $M_n - M_p$ corresponds to ~100% change neutron lifetime

Introduction: M_n - M_p

Big Bang Nucleosynthesis



$p + n \longleftrightarrow d + \gamma$ $\quad$ $p + n \longrightarrow d + \gamma$

$t \sim 1$ sec $\quad$ $t \sim 3$ min $\quad$ $t \sim 3^+$ min $\quad$ $t \sim 15$min

$T \sim 1$ MeV $\quad$ $T \sim 0.1$ MeV $\quad$ $T \sim 0.1^-$ MeV $\quad$ $T \sim 0.01$ MeV

$$\frac{X_n}{X_p} = e^{-\frac{M_n - M_p}{T}}$$

Initial conditions

B_d

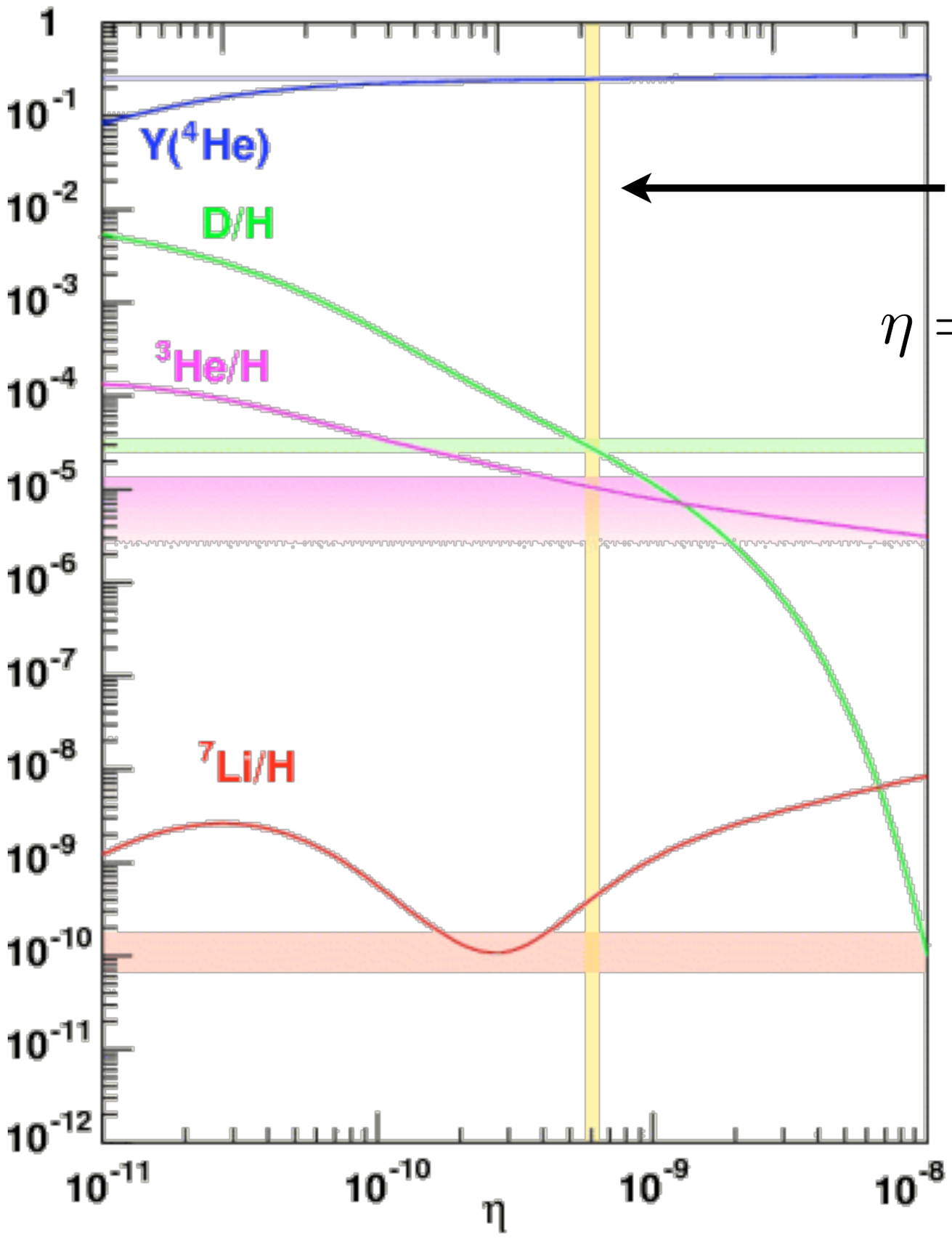
deuterium binding energy

τ_n

neutron lifetime

Introduction: M_n - M_p Primordial Universe (Mass Fraction)

~75% H
~25% ^{4}He

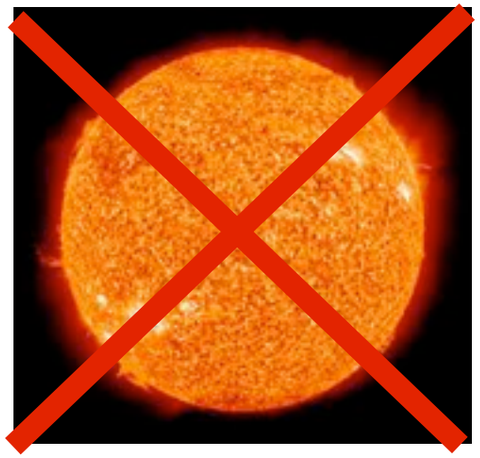


CMB

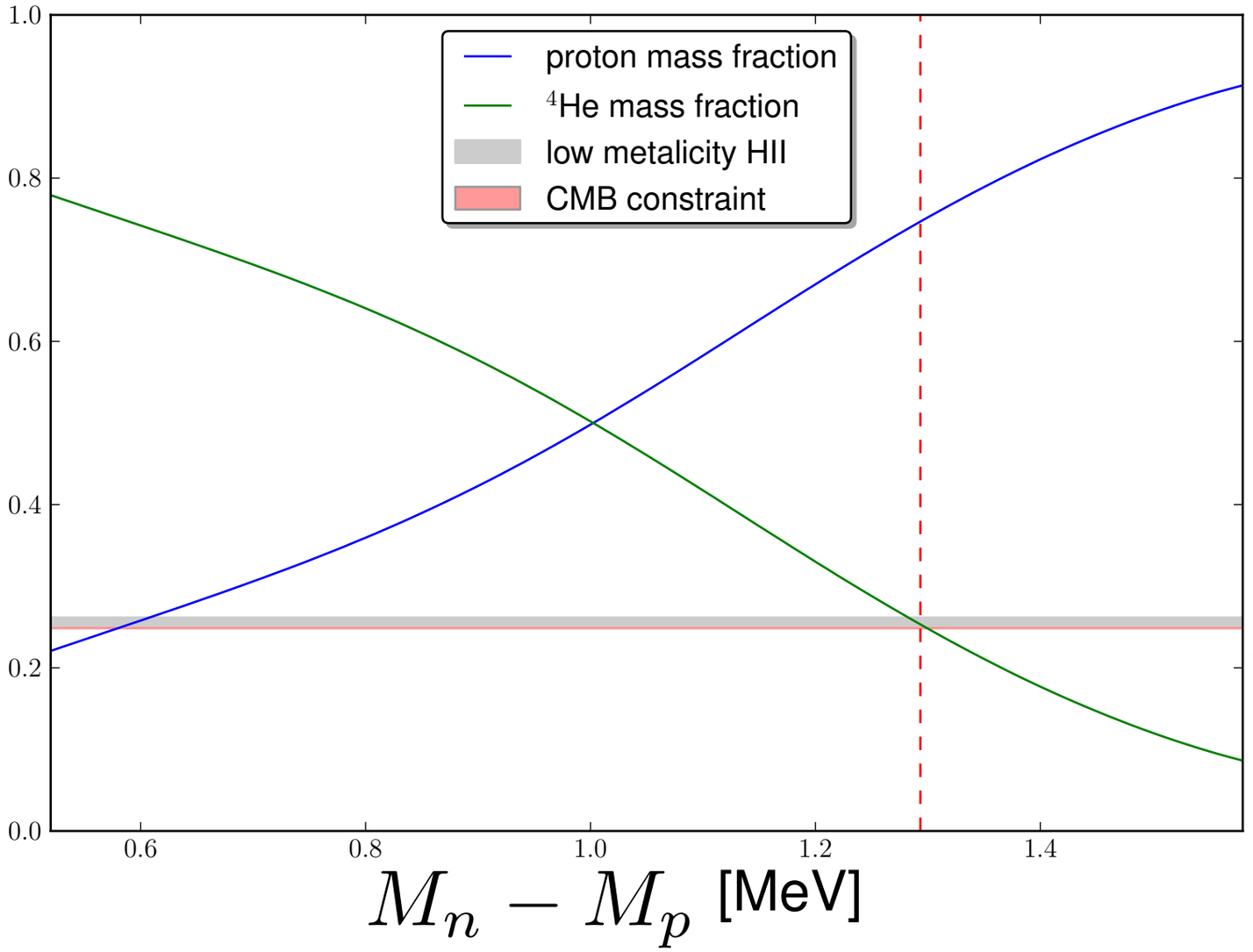
$$\eta = 6.19(15) \times 10^{-10}$$

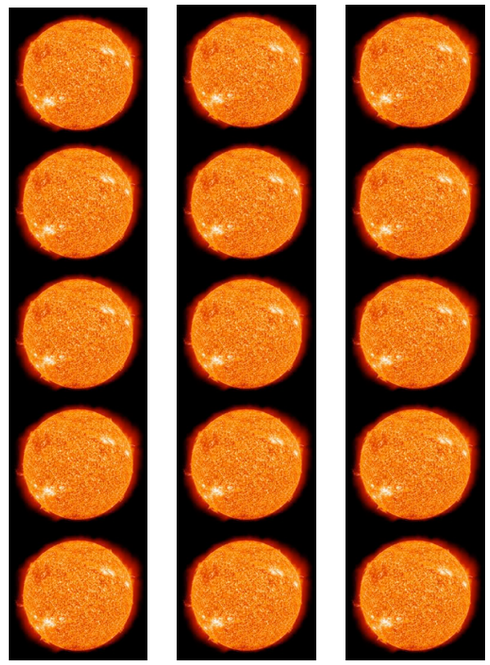
$$\eta \equiv \frac{X_N}{X_\gamma}$$

Introduction: M_n - M_p



No Sun!





Too many suns?

Introduction: M_n - M_p What do we know?

- We would like to understand the Neutron-Proton mass splitting from first principles

- $M_n - M_p = \delta M^{\gamma} + \delta M^{m_d - m_u}$ Separation only valid at LO in isospin breaking

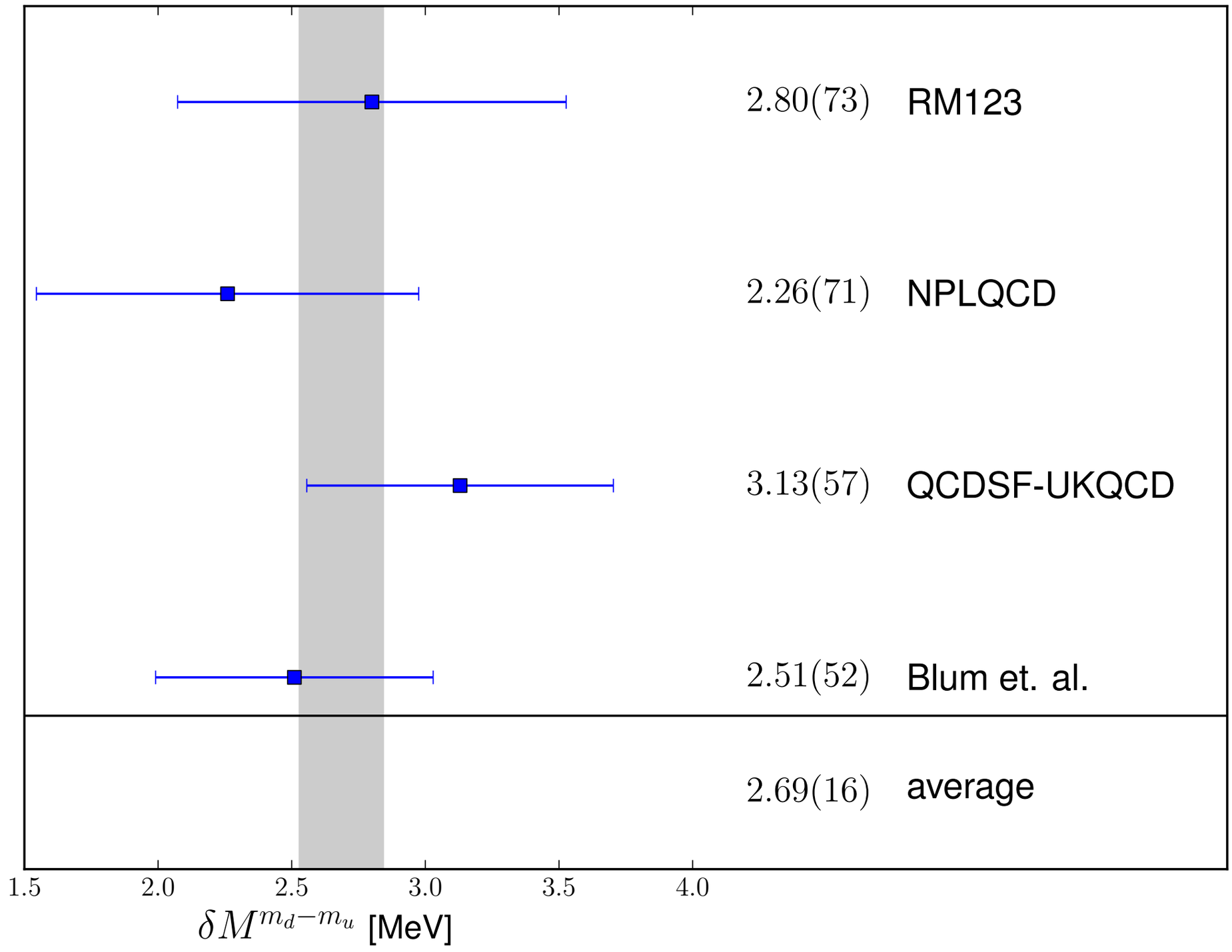
- $\delta M^{m_d - m_u}$ Well understood from lattice QCD

  δM^{γ} Disparate scales relevant for QCD and QED make this a very challenging problem to solve with LQCD: large systematic uncertainties

- Alternative means to determine δM^{γ}
  Cottingham Formulation

Introduction: $M_n - M_p$ What do we know?

- $\delta M_{LQCD}^{m_d - m_u} = 2.69(16)\ \mathrm{MeV}$



Introduction: M_n - M_p What do we know?

- $\delta M_{LQCD}^{m_d - m_u} = 2.69(16)\ \mathrm{MeV}$

- $\delta M^{\gamma} = -0.76(30)\ \mathrm{MeV}$

Gasser & Leutwyler
Nucl. Phys. B94 (1975)
Phys. Rept. 87 (1982) "Quark Masses"

central value from elastic contribution

uncertainty from estimates of inelastic contributions

- Experiment & lattice QCD

$$\delta M_{n-p}^{phys} - \delta M_{LQCD}^{m_d - m_u} = -1.40(16)\ \mathrm{MeV}$$

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Can we improve our understanding of these contributions?

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Can we improve our understanding of these contributions?

Of course!

Electromagnetic Self Energy: Cottingham Formula

$$\delta M^{\gamma}_{p-n} = \alpha_{f.s.} \times f_{p-n}(QCD, QED)$$

Electromagnetic Self Energy: Cottingham Formula

Walker-Loud, Carlson, Miller PRL 108 (2012) [arXiv:1203.0254]

$$\delta M^\gamma = \delta M^{el} + \delta M^{inel} + \delta M^{sub} + \delta M^{ct}$$

elastic — precisely determined

inelastic — newly determined (precisely)

unknown subtraction — newly determined (imprecisely)

counter-term renormalization — determined by J.C. Collins

$$\delta M^\gamma_{p-n} = 1.30(03)(47)\ \mathrm{MeV}$$

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~~$\delta M^\gamma_{p-n} = 0.76(30)\ \text{MeV}$~~ Gasser & Leutwyler

$$\delta M^{phys}_{p-n} - \delta M^{m_d - m_u}_{LQCD} = 1.40(16)\ \text{MeV}$$ Experiment & LQCD

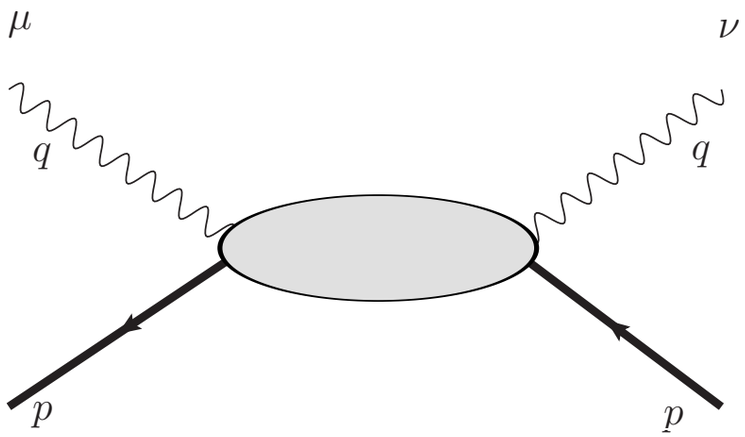
Electromagnetic Self Energy: Cottingham Formula

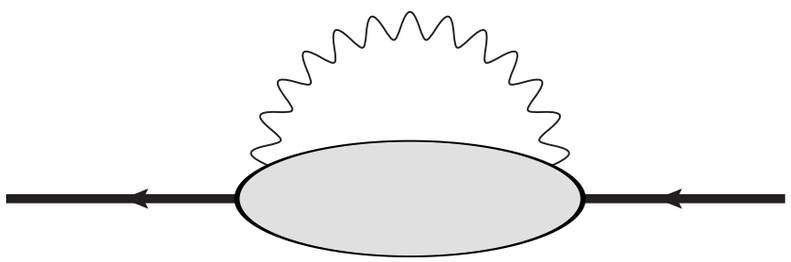
- Updating G&L result uncovered a "technical oversight"
  - The application of the Cottingham Formula requires the use of a subtracted dispersion integral.
  - Gasser & Leutwyler had an argument to evade the unknown subtraction function.
  - The argument was based on incorrect assumptions about scaling violations of the parton model
  - this has gone (mostly) unnoticed since 1982

Electromagnetic Self Energy: Cottingham Formula

electromagnetic correction

determined from Compton Scattering





$$\alpha = \frac{e^2}{4\pi}$$

Cini, Ferrari, Gato PRL 2 (1959)
Cottingham Annals Phys 25 (1963)
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Collins Nucl. Phys. B149 (1979)
Gasser, Leutwyler Phys. Rept 87 (1982)
AWL, C.Carlson, G.Miller PRL 108 (2012)

$$T_{\mu\nu} = \frac{i}{2}\sum_{\sigma}\int d^4\xi\; e^{iq\cdot\xi}\langle p\sigma|T\left\{J_\mu(\xi)J_\nu(0)\right\}|p\sigma\rangle$$

$$\delta M^\gamma = \frac{i}{2M}\frac{\alpha}{(2\pi)^3}\int_R d^4q\frac{T^\mu_\mu(p,q)}{q^2+i\epsilon}$$

Electromagnetic Self Energy: Cottingham Formula

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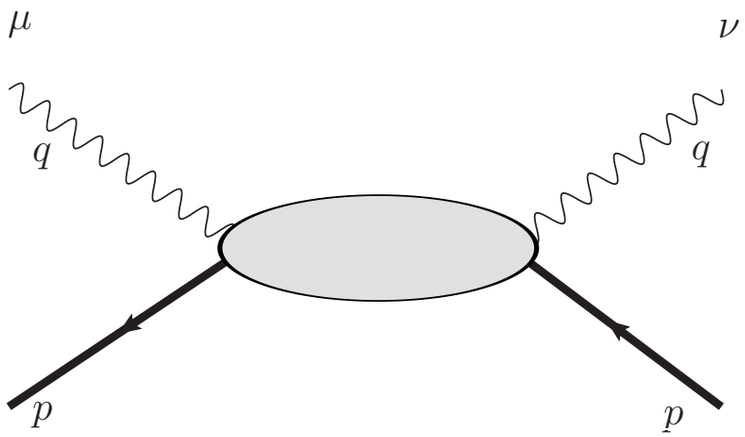

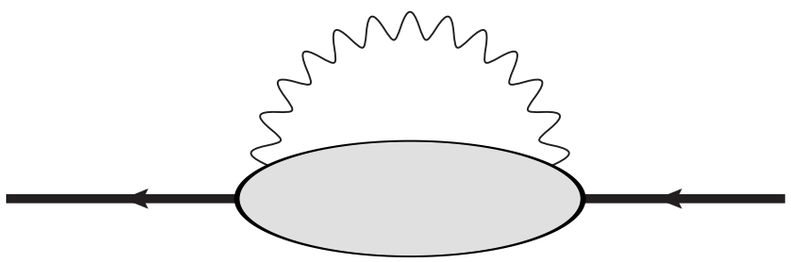

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$$\delta M^\gamma = \frac{i}{2M}\frac{\alpha}{(2\pi)^3}\int_R d^4q\frac{T^\mu_\mu(p,q)}{q^2+i\epsilon}$$

Integral diverges and must be renormalized

Electromagnetic Self Energy: Cottingham Formula

electromagnetic correction

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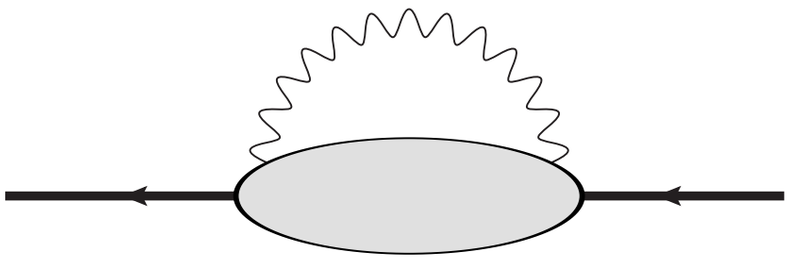


$$\delta M^\gamma = \frac{i}{2M}\frac{\alpha}{(2\pi)^3}\int_R d^4q \frac{T^\mu_\mu(p,q)}{q^2+i\epsilon}$$

- Wick rotate $q^0 \to i\nu$ variable transform $Q^2 = \mathbf{q}^2 + \nu^2$

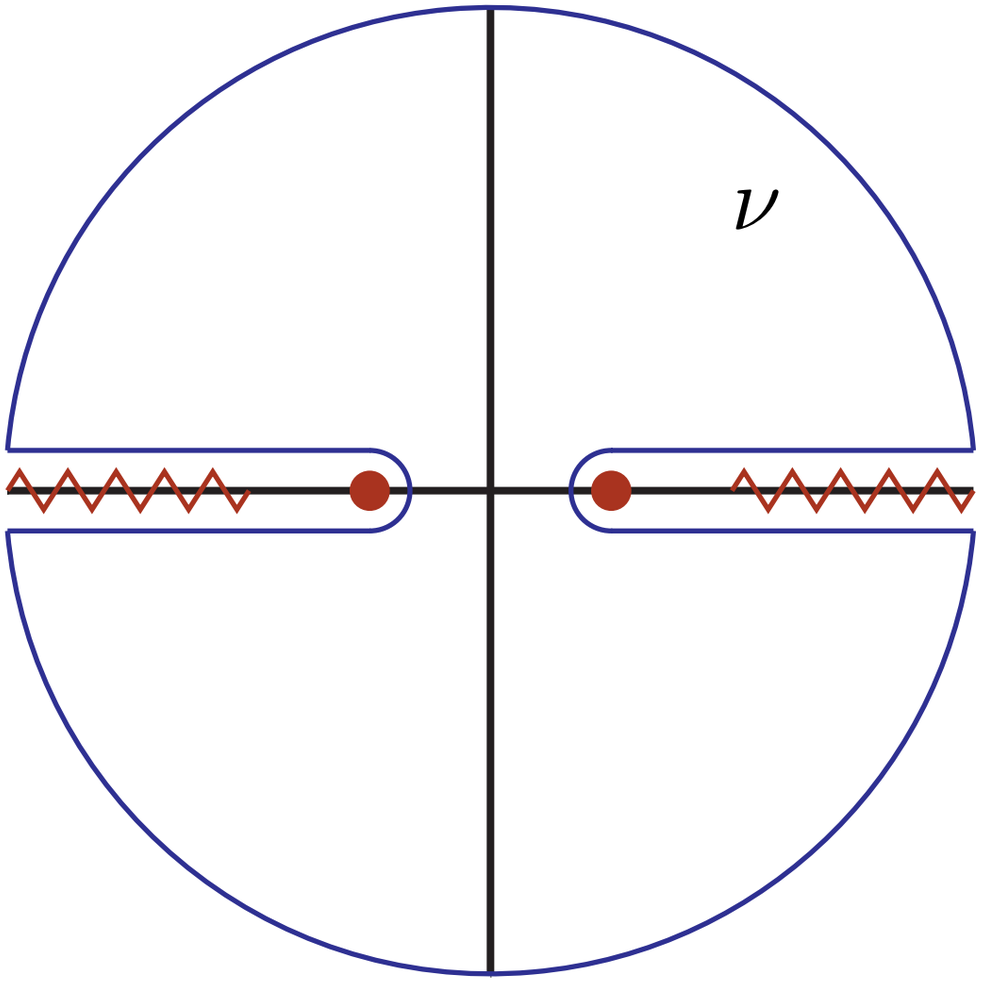
$$\delta M^\gamma = \frac{\alpha}{8\pi^2}\int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2-\nu^2}}{Q^2}\frac{T^\mu_\mu}{M} + \delta M^{ct}(\Lambda)$$

$$T^\mu_\mu = -3\,T_1(i\nu,Q^2) + \left(1-\frac{\nu^2}{Q^2}\right)T_2(i\nu,Q^2)\,, \qquad (7a)$$

$$= -3Q^2\,t_1(i\nu,Q^2) + \left(1+2\frac{\nu^2}{Q^2}\right)Q^2 t_2(i\nu,Q^2)\,. \qquad (7b)$$

use dispersion integrals to evaluate scalar functions $T_{1,2}(i\nu,Q^2)$ $[t_{1,2}(i\nu,Q^2)]$

Electromagnetic Self Energy: Cottingham Formula



dispersion integral = Cauchy contour integral

$$T_i(\nu, Q^2) = \frac{1}{2\pi} \oint d\nu' \frac{T_i(\nu', Q^2)}{\nu' - \nu}$$

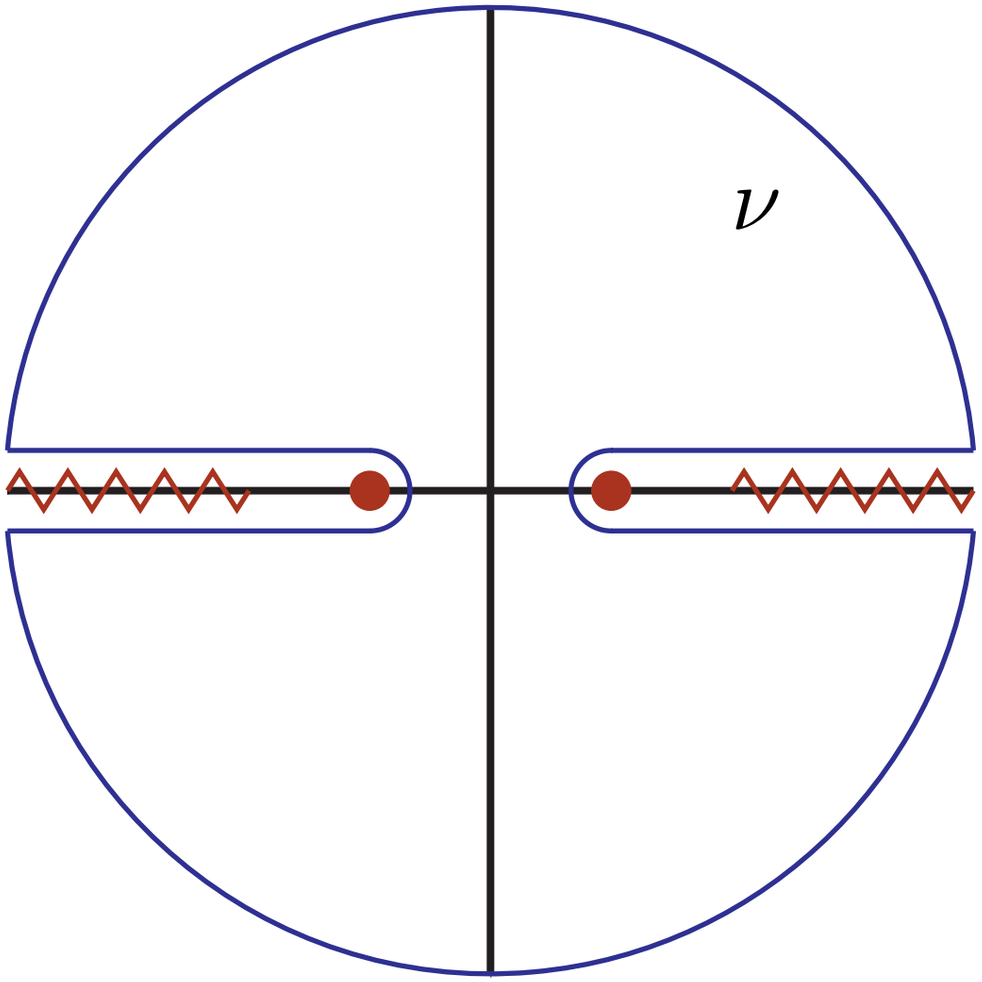
Crossing Symmetric

$$T_i(\nu, Q^2) = T_i(-\nu, Q^2)$$

$$T_i(\nu, Q^2) = \frac{1}{2\pi} \int_{\nu_t}^{\infty} d\nu' \frac{2\nu'}{(\nu')^2 - \nu^2} 2\mathrm{Im} T_i(\nu' + i\epsilon, Q^2)$$

(provided contour and infinity vanishes)

Electromagnetic Self Energy: Cottingham Formula



if contour at infinity does not vanish

subtracted dispersion integral

$$g(\nu) = \frac{T_i(\nu, Q^2)}{\nu^2}$$

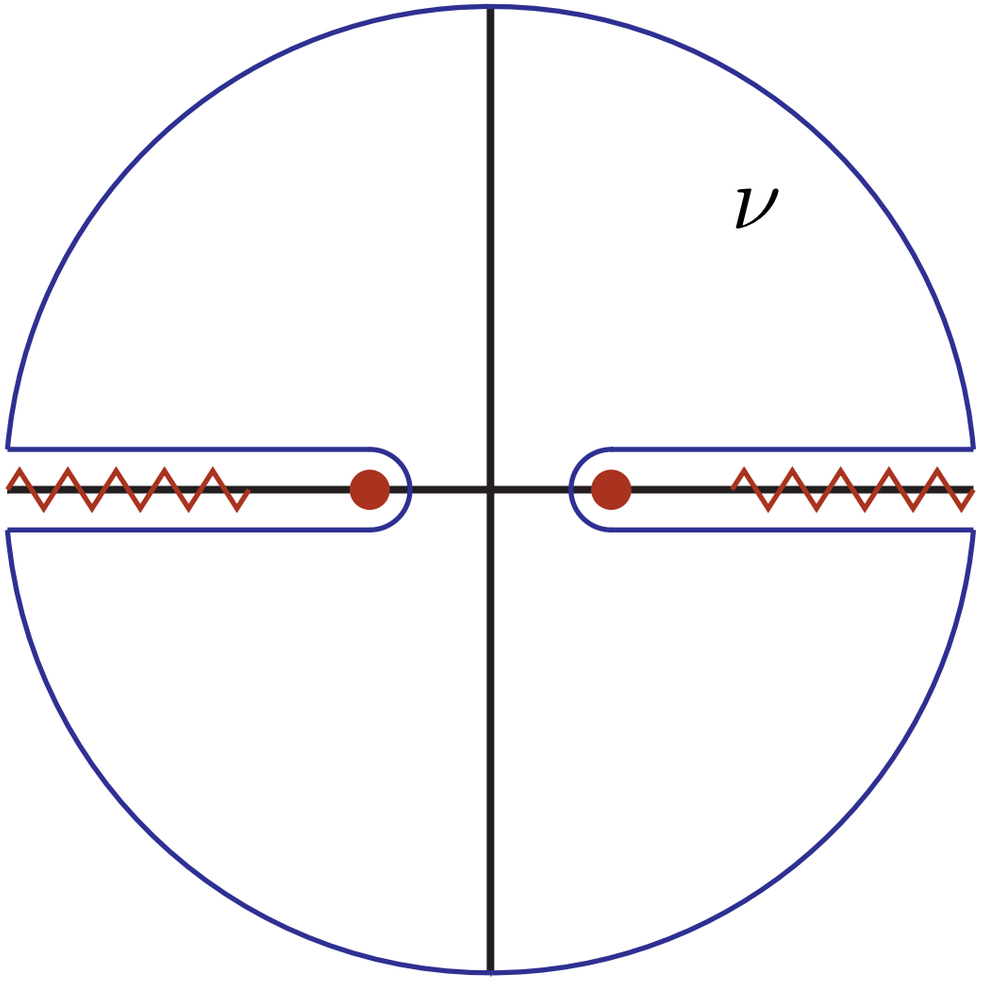
introduces new pole at $\nu = 0$
which you need to subtract

$$T_i(\nu, Q^2) = \frac{\nu^2}{2\pi} \int_{\nu_t}^{\infty} d\nu' \frac{2\nu'}{\nu'^2(\nu'^2 - \nu^2)} 2\mathrm{Im} T_i(\nu' + i\epsilon, Q^2) + T_i(0, Q^2)$$

measured experimentally

unknown function

Electromagnetic Self Energy: Cottingham Formula



It is known that

$$T_2(\nu, Q^2) \quad [t_2(\nu, Q^2)]$$

satisfies unsubtracted dispersion integral while

$$T_1(\nu, Q^2) \quad [t_1(\nu, Q^2)]$$

requires a subtraction

Regge behavior

$$\mathrm{Im} t_1[T_1]\Big|_{p-n} \propto \nu^{1/2}$$

H. Harari: PRL 17 (1966)

H.D. Abarbanel S. Nussinov: Phys.Rev. 158 (1967)

Electromagnetic Self Energy: Cottingham Formula

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

at the time, introducing an unknown subtraction function would be disastrous for getting a precise value:

they provided an argument based upon various assumptions to avoid the subtracted dispersive integral

$$\delta M^{\gamma}_{p-n} = 0.76(30)\ \mathrm{MeV}$$

central value: from elastic contribution

uncertainty: estimates of inelastic structure contributions

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however, one can show their arguments are incorrect:
one must face the subtraction function

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what is the flaw in the argument?

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is there some motivation to pick t_i vs T_i ?

Electromagnetic Self Energy: Cottingham Formula

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in the point limit (electron) $t_1(\nu, Q^2) = 0!$

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for the nucleon (with motivated resummations) the elastic contribution is

$$t_1(\nu, Q^2) = \frac{2}{Q^2}\left[\frac{Q^4\,\frac{G_M^2 - G_E^2}{1+\tau}}{(Q^2 - i\epsilon)^2 - 4M^2\nu^2} - \underbrace{\left(F_1^2 - \frac{G_E^2 + \tau G_M^2}{1+\tau}\right)}\right]$$

$$\tau = \frac{Q^2}{4M^2}$$

"Fixed-Pole" missed by unsubtracted dispersion relation

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$$\tau = \frac{Q^2}{4M^2}$$

numerically, this term is negligible

Electromagnetic Self Energy: Cottingham Formula

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real problem comes in the Regge limit: Q^2 fixed, $\nu \to \infty$

$$\mathrm{Im} t_1(\nu, Q^2) = \frac{\pi M \nu}{Q^4}\left[2xF_1(x, Q^2) - F_2(x, Q^2)\right] \qquad x = \frac{Q^2}{2M\nu}$$

in the strict DIS limit: Callan-Gross relation

$$2xF_1(x) - F_2(x) = 0$$

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Gasser and Leutwyler assumed

$$2xF_1(x, Q^2) - F_2(x, Q^2) = \frac{H_1(x)}{\nu}$$

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Gasser and Leutwyler assumed

$$2xF_1(x, Q^2) - F_2(x, Q^2) = \frac{H_1(x)}{\nu}$$

if this were true, their argument would go through, however...

Electromagnetic Self Energy: Cottingham Formula

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Zee, Wilczek and Treiman Phys.Rev. D10 (1974)

$$2xF_1(x) - F_2(x) = \frac{-32}{9}\frac{\alpha_s(Q^2)}{4\pi}F_2(x)$$

Both IR and UV safe

This criticism first given by J.C. Collins: Nucl. Phys. B149 (1979)

Electromagnetic Self Energy: Cottingham Formula

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

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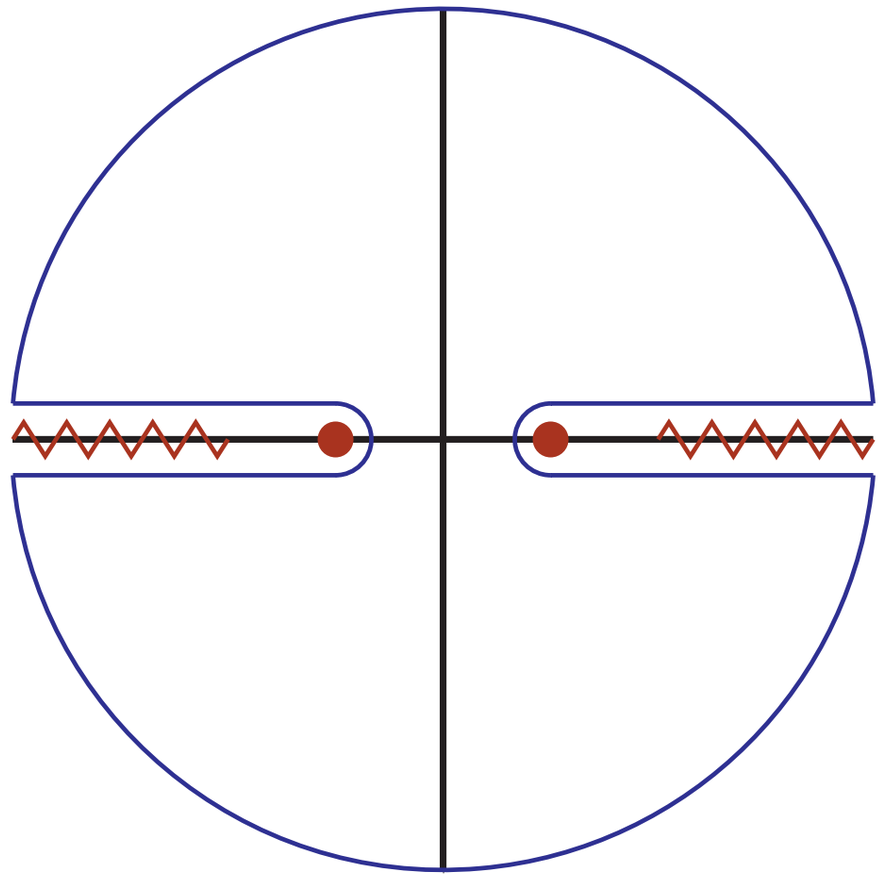
$$\lim_{x\to 0} F_2^{p-n}(x) \propto x^{1/2} \qquad x = \frac{Q^2}{2M\nu}$$

$$\mathrm{Im} t_1^{p-n}(\nu, Q^2) \propto \alpha_s(Q^2)\frac{\sqrt{M\nu}}{Q^3}$$

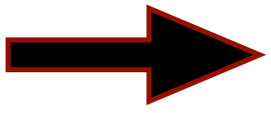
Electromagnetic Self Energy: Cottingham Formula

$$t_1(\nu, Q^2) = \frac{1}{2\pi} \int_{\nu_t}^{\infty} 2\nu' d\nu' \frac{2\mathrm{Im}t_1(\nu' + i\epsilon, Q^2)}{(\nu')^2 - \nu^2}$$



Regge Limit
fixed Q^2
$\nu \to \infty$

$$\mathrm{Im}t_1^{p-n}(\nu, Q^2) \propto \alpha_s(Q^2) \frac{\sqrt{M\nu}}{Q^3}$$



subtracted dispersion integral is unavoidable

Electromagnetic Self Energy: Cottingham Formula

evaluation of various contributions

Electromagnetic Self Energy: Cottingham Formula

perform once subtracted dispersion integral for $T_1(t_1)$
and unsubtracted dispersion integral for $T_2(t_2)$

$$\delta M^{\gamma} = \delta M^{el} + \delta M^{inel} + \delta M^{sub} + \delta \tilde{M}^{ct}$$

Electromagnetic Self Energy: Cottingham Formula

perform once subtracted dispersion integral for $T_1(t_1)$
and unsubtracted dispersion integral for $T_2(t_2)$

$$\delta M^{\gamma} = \delta M^{el} + \delta M^{inel} + \delta M^{sub} + \delta \tilde{M}^{ct}$$

$$\delta M^{el} = \frac{\alpha}{\pi} \int_0^{\Lambda_0^2} dQ \left\{ \frac{3\sqrt{\tau_{el}} G_M^2}{2(1+\tau_{el})} + \frac{\left[G_E^2 - 2\tau_{el}\, G_M^2\right]}{1+\tau_{el}} \left[(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}} \right] \right\}$$

$$\delta M^{inel} = \frac{\alpha}{\pi} \int_0^{\Lambda_0^2} \frac{dQ^2}{2Q} \int_{\nu_{th}}^{\infty} d\nu \left\{ \frac{3F_1(\nu, Q^2)}{M} \left[\frac{\tau^{3/2} - \tau\sqrt{1+\tau} + \sqrt{\tau}/2}{\tau} \right] + \frac{F_2(\nu, Q^2)}{\nu} \left[(1+\tau)^{3/2} - \tau^{3/2} - \frac{3}{2}\sqrt{\tau} \right] \right\},$$

$$\tau_{el} = \frac{Q^2}{4M^2} \qquad \tau = \frac{\nu^2}{Q^2}$$

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M} \int_0^{\Lambda_0^2} dQ^2\; T_1(0, Q^2)\,,$$

$$\delta \tilde{M}^{ct} = -\frac{3\alpha}{16\pi M} \int_{\Lambda_0^2}^{\Lambda_1^2} dQ^2 \sum_i C_{1,i} \langle \mathcal{O}^{i,0} \rangle\,,$$

OPE: operators and Wilson coeffic.
J.C. Collins: Nucl. Phys. B149 (1979)

Electromagnetic Self Energy: Cottingham Formula

elastic contribution: use well measured form factors

$$\delta M^{el} = \frac{\alpha}{\pi}\int_0^{\Lambda_0^2} dQ \left\{ \frac{3\sqrt{\tau_{el}}G_M^2}{2(1+\tau_{el})} + \frac{\left[G_E^2 - 2\tau_{el}\, G_M^2\right]}{1+\tau_{el}} \left[(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}} \right] \right\}$$

$$\delta M^{el}\Big|_{p-n} = 1.39(02)\ \mathrm{MeV}$$

- insensitive to value of Λ_0 since form factors fall as $1/Q^4$
- uncertainty from Monte Carlo evaluation of parameters describing form factors

central values: $\Lambda_0^2 = 2\ \mathrm{GeV}^2$

uncertainties: $1.5\ \mathrm{GeV}^2 \leq \Lambda_0^2 \leq 2.5\ \mathrm{GeV}^2$

Electromagnetic Self Energy: Cottingham Formula

inelastic terms: use modern knowledge of structure functions to improve determination of inelastic contributions

$$\delta M^{inel} = \frac{\alpha}{4\pi M}\int_0^{\Lambda_0^2}\frac{dQ^2}{Q}\int_{W_{th}^2}^{\infty} dW^2 \left\{\frac{3F_1(\nu,Q^2)}{M}\left[\frac{\tau^{3/2}-\tau\sqrt{1+\tau}+\sqrt{\tau}/2}{\tau}\right]\right.$$
$$\left. + \frac{F_2(\nu,Q^2)}{\nu}\left[(1+\tau)^{3/2}-\tau^{3/2}-\frac{3}{2}\sqrt{\tau}\right]\right\}$$

$$\delta M^{inel}\Big|_{p-n} = 0.057(16)\ \mathrm{MeV}$$

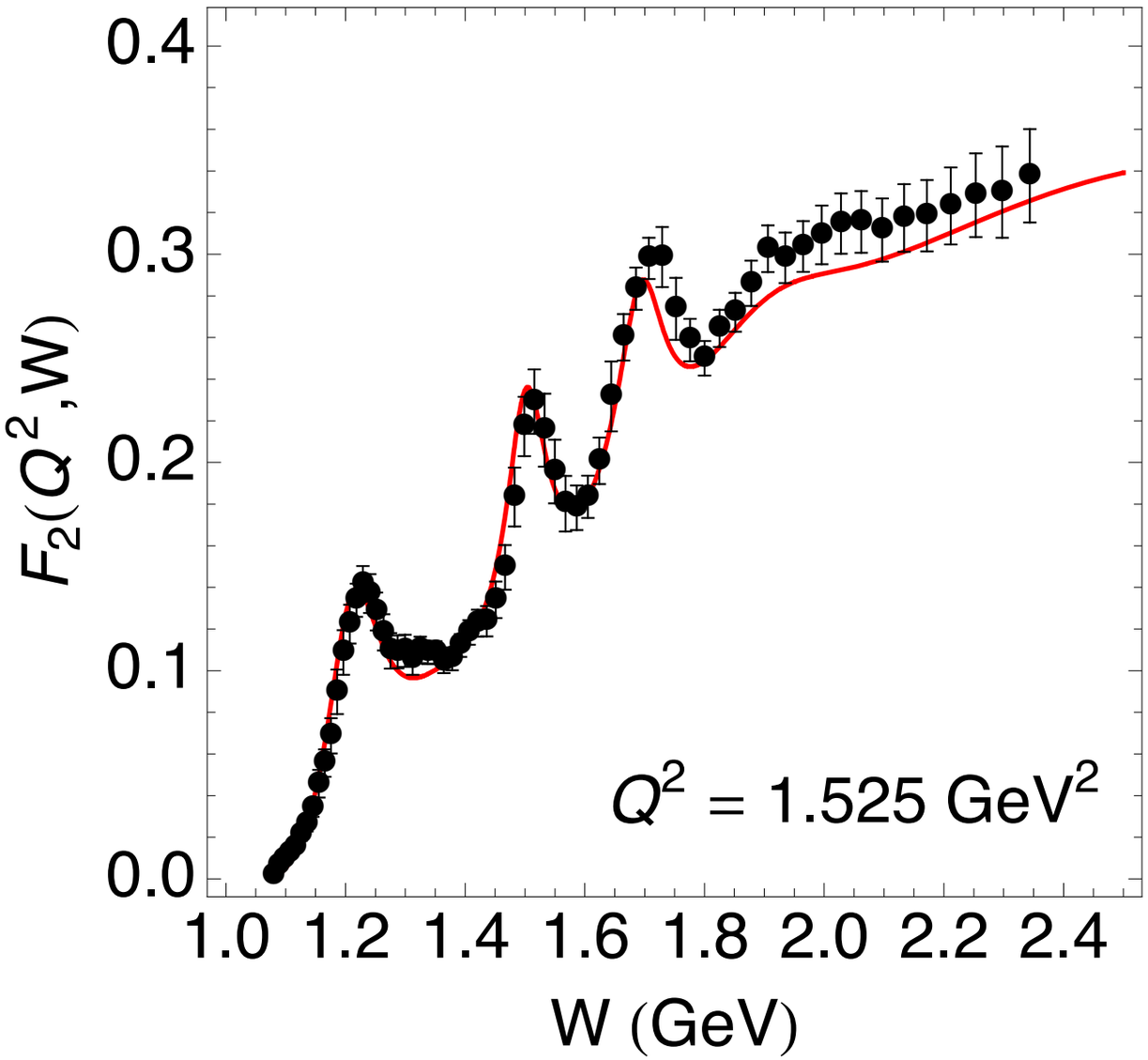
- contributions from two regions:
  - resonance region — Bosted and Christy: Phys.Rev. C77, C81
  - scaling region — Capella et al: PLB 337; Sibirtsev et al: Phys. Rev. D82
- uncertainty dominated by choice of transition between two regions

Electromagnetic Self Energy: Cottingham Formula

inelastic terms: use modern knowledge of structure functions to improve determination of inelastic contributions

$$\delta M^{inel} = \frac{\alpha}{4\pi M}\int_0^{\Lambda_0^2}\frac{dQ^2}{Q}\int_{W_{th}^2}^{\infty}dW^2\left\{\frac{3F_1(\nu,Q^2)}{M}\left[\frac{\tau^{3/2}-\tau\sqrt{1+\tau}+\sqrt{\tau}/2}{\tau}\right]\right.$$

$$\left.+\frac{F_2(\nu,Q^2)}{\nu}\left[(1+\tau)^{3/2}-\tau^{3/2}-\frac{3}{2}\sqrt{\tau}\right]\right\}$$



F_2 data from JLAB

resonance fit:

Bosted and Christy: Phys.Rev. C77, C81

$$\tau = \nu^2/Q^2 \qquad W_{th}^2 = (M+m_\pi)^2$$

$$W^2 = M^2 + 2M\nu - Q^2$$

Electromagnetic Self Energy: Cottingham Formula

renormalization: [J.C. Collins Nucl. Phys. B149 (1979)] one can show the contribution from the operator is numerically second order in isospin breaking with Naive Dimensional Analysis and suitable renormalization (dim. reg.)

quark mass operator renormalizes EM self-energy: can not cleanly separate these two contributions (but mixing is higher order in isospin breaking)

Electromagnetic Self Energy: Cottingham Formula

renormalization: [J.C. Collins Nucl. Phys. B149 (1979)]

$$\delta M_{UV}^{\gamma} \sim \frac{3\alpha_{f.s.}}{16\pi M}\int_{\Lambda^2}^{\infty}\left[\frac{M^2}{Q^2}\int_0^1 dx\Big(2xF_1(x)+F_2(x)\Big) - T_1(0,Q^2)\right]$$

subtraction function

- use OPE to connect to perturbative QCD
- log divergence arising from $2xF_1(x)+F_2(x)$ exactly cancels against log divergence from $T_1(0,Q^2)$
- counter term comes entirely from subtraction function

Electromagnetic Self Energy: Cottingham Formula

renormalization: [J.C. Collins Nucl. Phys. B149 (1979)]

$$\delta M^\gamma = \frac{3\alpha_{f.s.}}{16\pi M}\left\{ \int_0^{\mu^2} \frac{dQ^2}{Q^2} f(Q^2) + \lim_{\Lambda^2\to\infty}\left[\int_{\mu^2}^{\Lambda^2} \frac{dQ^2}{Q^2}\left(f(Q^2) + \sum_i C^0_{1,i}\langle \mathcal{O}^{i,0}\rangle\right)\right]\right\}$$

$$\langle N|\sum_i C^0_{1,i}\mathcal{O}^{i,0}|N\rangle_{p-n} = \frac{2}{Q^2}(e_u^2 m_u - e_d^2 m_d)\langle p|\bar{u}u - \bar{d}d|p\rangle$$

- $\ln(\Lambda^2)$ divergence exactly cancels
- residual dependence on scale μ
- use Naive Dimensional Analysis to estimate size

Electromagnetic Self Energy: Cottingham Formula

renormalization: [J.C. Collins Nucl. Phys. B149 (1979)]

$$\delta\tilde{M}^{ct} = -\frac{3\alpha}{4\pi}\sigma_{\pi N}\ln\left(\frac{\Lambda_1^2}{\Lambda_0^2}\right)\frac{3\hat{m}-5\delta}{9\hat{m}}\frac{\langle p|\bar{u}u-\bar{d}d|p\rangle}{\langle p|\bar{u}u+\bar{d}d|p\rangle}$$

$$\sigma_{\pi N} = \frac{1}{2M}\langle p|\hat{m}(\bar{u}u+\bar{d}d)|p\rangle \simeq 45\ \mathrm{MeV}$$

- saturate matrix elements in valence limit

$$\frac{\langle p|\bar{u}u-\bar{d}d|p\rangle}{\langle p|\bar{u}u+\bar{d}d|p\rangle} \leq \frac{1}{3}$$

- vary arbitrary scales in scaling region

$$\Lambda_0^2 = 2\ \mathrm{GeV}^2\,, \quad \Lambda_1^2 = 100\ \mathrm{GeV}^2$$

$$|\delta\tilde{M}^{ct}| \lesssim 0.02\ \mathrm{MeV}$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2\; T_1(0,Q^2)\,,$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2\; T_1(0,Q^2)\,,$$

- low energy: constrained by effective field theory

$$T_1(0,Q^2) = 2\kappa(2+\kappa) - Q^2\left\{\frac{2}{3}\left[(1+\kappa)^2 r_M^2 - r_E^2\right] + \frac{\kappa}{M^2} - 2M\frac{\beta_M}{\alpha}\right\} + \mathcal{O}(Q^4)\,,$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

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most of these contributions come from Low Energy Theorems and are "elastic" (arising from a photon striking an on-shell nucleon)

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most of these contributions come from Low Energy Theorems and are "elastic" (arising from a photon striking an on-shell nucleon)

intimately related to the proton size puzzle which suffers from the same subtracted dispersive problem

K. Pachucki: Phys. Rev. A53 (1996); A. Pineda: Phys. Rev. C67 (2003); Phys. Rev. C71 (2005);
R.J. Hill, G. Paz: PRL 107 (2011); C. Carlson, M. Vanderhaeghen: Phys.Rev.A84 (2011); arXiv1109.3779;
M.. Birse, J. McGovern: arXiv:1206.3030

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2\; T_1(0,Q^2)\,,$$

- high energy: OPE (perturbative QCD) constrains

$$\lim_{Q^2\to\infty} T_1(0,Q^2) \propto \frac{1}{Q^2}$$

$$T_1(0,Q^2) \simeq 2G_M^2(Q^2) - 2F_1^2(Q^2) + Q^2 2M\frac{\beta_M}{\alpha}\left(\frac{m_0^2}{m_0^2+Q^2}\right)^2$$

$\mathcal{O}(Q^4)$ inelastic terms known

Birse and McGovern Eur.Phys.J A48 (2012) [arXiv:1206.3030]

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M_{el}^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2\left[2G_M^2 - 2F_1^2\right], \qquad \delta M_{el}^{sub}\Big|_{p-n} = -0.62 \text{ MeV}$$

$$\delta M_{inel}^{sub} = -\frac{3\beta_M}{8\pi}\int_0^{\Lambda_0^2} dQ^2 Q^2\left(\frac{m_0^2}{m_0^2+Q^2}\right)^2$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

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$$\beta_M^{p-n} = -1.0 \pm 1.0 \times 10^{-4} \text{ fm}^3$$

H.W. Griesshammer, J.A. McGovern, D.R. Phillips, G. Feldman: Prog.Nucl.Part.Phys. (2012)

Cottingham's Formula

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$$\beta_M^{p-n} = -1.0 \pm 1.0 \times 10^{-4} \text{ fm}^3$$

H.W. Griesshammer, J.A. McGovern, D.R. Phillips, G. Feldman: Prog.Nucl.Part.Phys. (2012)

taking $m_0^2 = 0.71 \text{ GeV}^2$

$$\delta M_{inel}^{sub}\Big|_{p-n} = 0.47 \pm 0.47 \text{ MeV}$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

adding it all up:

$$\begin{aligned}\delta M^{\gamma}|_{p-n} = &+ 1.39(02) \\ &- 0.62(02) \\ &+ 0.057(16) \\ &+ 0.47(47)\ \text{MeV} \\ \hline &= 1.30(03)(47)\ \text{MeV}\end{aligned} \quad > \quad = 0.77(03)\ \text{MeV}$$

elastic terms

inelastic terms

unknown subtraction term

recall the fixed pole in the elastic contribution makes a negligible contribtion

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

adding it all up:

$$\delta M^{\gamma}\Big|_{p-n} = 1.30(03)(47)\ \mathrm{MeV}$$

AWL, C.Carlson, G.Miller:
PRL 108 (2012)

$$= 0.76(30)\ \mathrm{MeV}$$

J. Gasser and H. Leutwyler:
Nucl Phys B94 (1975)

We reduced the uncertainty from structure by an order of magnitude! But we uncovered an oversight that dominates the uncertainty :(

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

adding it all up:

$$\delta M^{\gamma}\Big|_{p-n} = 1.30(03)(47)\ \mathrm{MeV}$$

$$= 0.76(30)\ \mathrm{MeV}$$

AWL, C.Carlson, G.Miller:
PRL 108 (2012)

J. Gasser and H. Leutwyler:
Nucl Phys B94 (1975)

expectation from experiment + lattice QCD

$$\delta M^{\gamma}_{p-n} = -1.29333217(42) + 2.69(16)\ \mathrm{MeV}$$

$$= \quad 1.40(16)\ \mathrm{MeV}$$

average of 4 independent lattice results

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M_{inel}^{sub} = -\frac{3\beta_M}{8\pi}\int_0^{\Lambda_0^2} dQ^2 Q^2 \left(\frac{m_0^2}{m_0^2+Q^2}\right)^2$$

$$\beta_M^{p-n} = -1.0 \pm 1.0 \times 10^{-4}\ \mathrm{fm}^3$$

H.W. Griesshammer, J.A. McGovern, D.R. Phillips, G. Feldman: Prog.Nucl.Part.Phys. (2012)

computing $\beta_M^{p,n}$ from lattice QCD

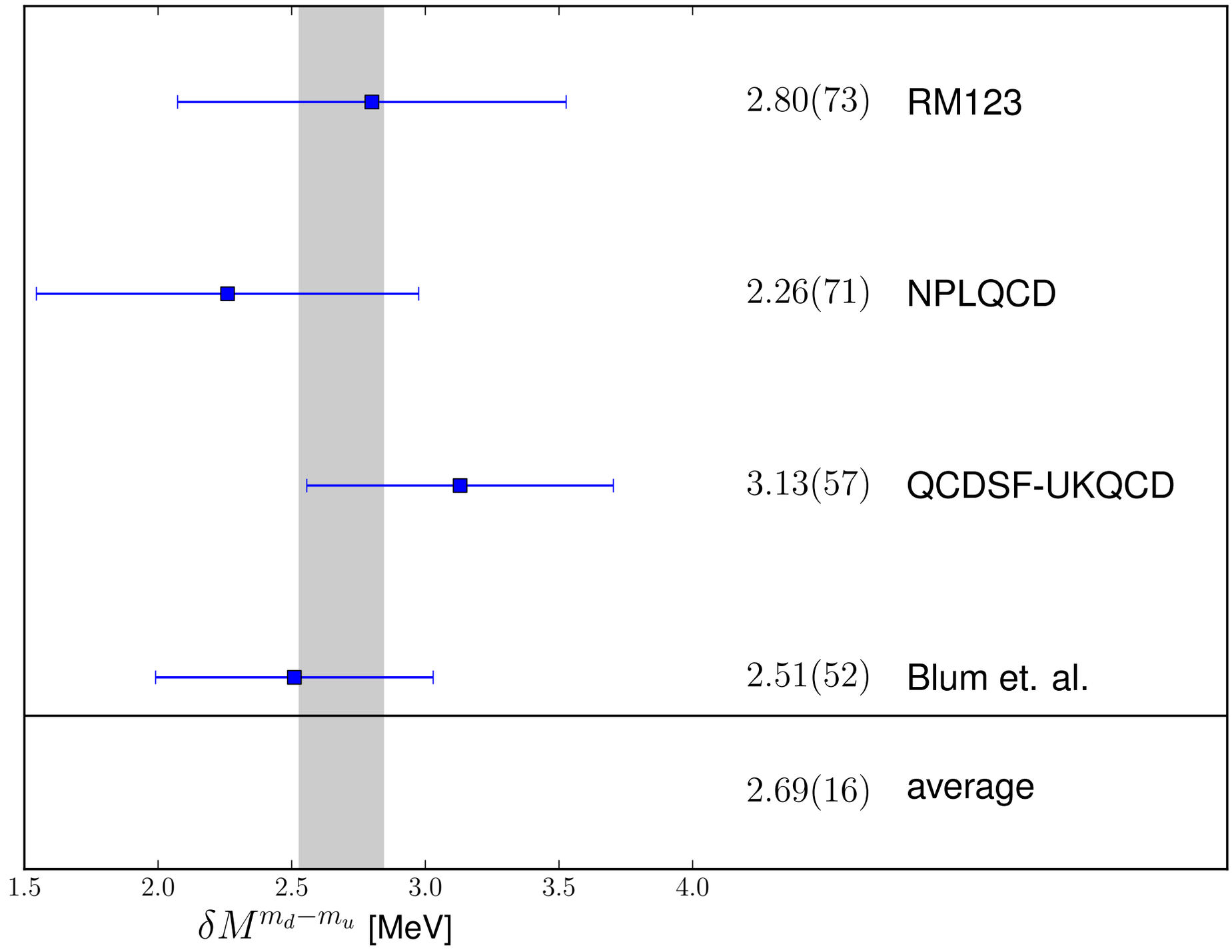
W. Detmold, B. Tiburzi, AWL

Strong Isospin Breaking: m_d - m_u

$$\delta M_{n-p}^{m_d - m_u} = \alpha(m_d - m_u)$$

Introduction: $M_n - M_p$ What do we know?

- $\delta M_{LQCD}^{m_d - m_u} = 2.69(16)\ \mathrm{MeV}$



Strong Isospin Breaking: m_d - m_u

strong isospin breaking correction

$$\delta M_{n-p}^{m_d - m_u} = \alpha(m_d - m_u)$$

ideal problem for lattice QCD

$$\delta M_{n-p}^{m_d - m_u} = 2.69(16)\ \mathrm{MeV}$$

B. Tiburzi, AWL Nucl. Phys. A764 (2006)
Beane, Orginos, Savage Nucl. Phys. B768 (2007)
AWL arXiv:0904.2404

lattice average

Blum, Izubuchi, etal Phys. Rev. D82 (2010)
AWL PoS Lattice2010 (2010)
de Divitiis etal JHEP 1204 (2012)
Horsley etal Phys. Rev. D86 (2012)

But in lattice calculation $m_u = m_d = m_l$?

Strong Isospin Breaking: m_d - m_u

strong isospin breaking correction

$$\delta M_{n-p}^{m_d - m_u} = \alpha(m_d - m_u)$$

ideal problem for lattice QCD

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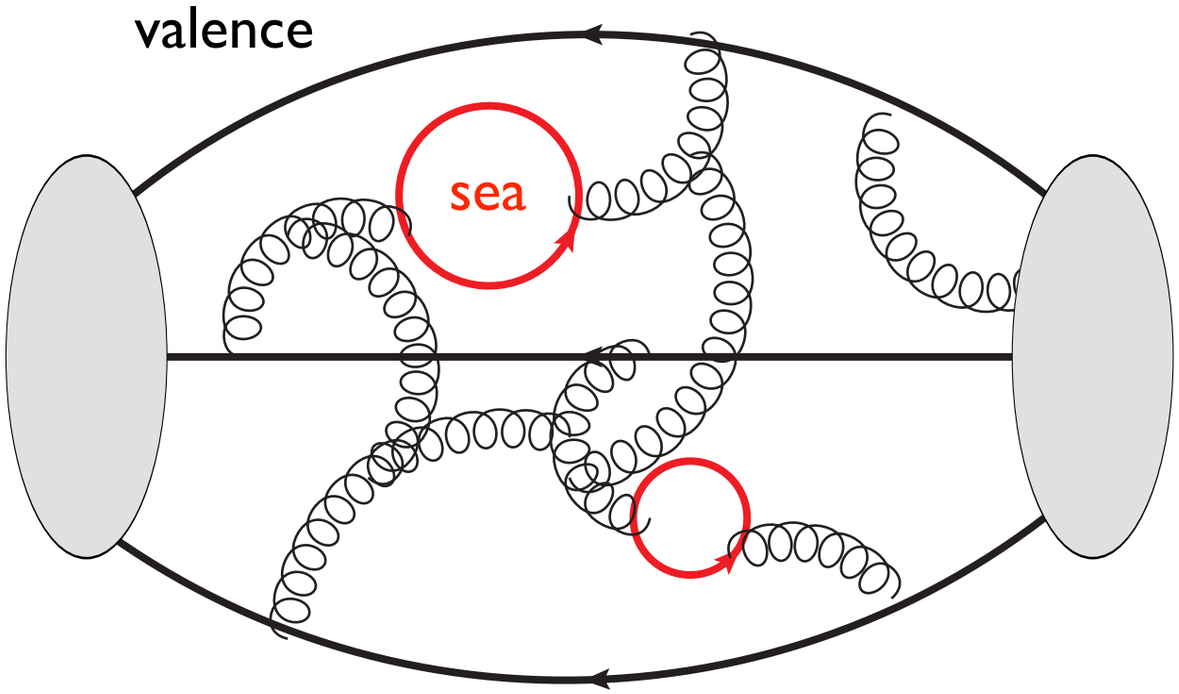

$$m_{u,d}^{valence} \neq m_l^{sea}$$

"partially quenched" lattice QCD trick that works on the computer but introduces error which must be corrected

Strong Isospin Breaking: m_d - m_u

strong isospin breaking correction

$$\delta M_{n-p}^{m_d - m_u} = \alpha(m_d - m_u)$$

ideal problem for lattice QCD

$$\delta M_{n-p}^{m_d - m_u} = 2.69(16)\ \mathrm{MeV}$$

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de Divitiis etal JHEP 1204 (2012)
Horsley etal Phys. Rev. D86 (2012)

can we improve this method?

of course!

"Symmetric breaking of isospin symmetry" AWL arXiv:0904.2404

Strong Isospin Breaking: m_d - m_u

AWL arXiv:0904.2404

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

$$\mathcal{Z}_{u,d} = \int DU_\mu \, \mathrm{Det}(D + m_l - \delta\tau_3) \, e^{-S[U_\mu]}$$

$$= \int DU_\mu \, \mathrm{Det}(D + m_l) \, \det\left(1 - \frac{\delta^2}{(D + m_l)^2}\right) \, e^{-S[U_\mu]}$$

Strong Isospin Breaking: m_d - m_u

AWL arXiv:0904.2404

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Isospin symmetric quantities: error $\mathcal{O}(\delta^2)$

Isospin violating quantities: error $\mathcal{O}(\delta^3)$

Strong Isospin Breaking: m_d - m_u

AWL arXiv:0904.2404

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

Pion Chiral Lagrangian

$$\mathcal{L} = \frac{f^2}{8}\,\mathrm{tr}\left(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger\right) - \frac{f^2}{8}\,\mathrm{tr}\left(\chi'^\dagger \Sigma + \Sigma^\dagger \chi'\right) - \frac{l_1}{4}\left[\mathrm{tr}\left(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger\right)\right]^2 - \frac{l_2}{4}\,\mathrm{tr}\left(\partial_\mu \Sigma \partial_\nu \Sigma^\dagger\right)\mathrm{tr}\left(\partial_\mu \Sigma \partial_\nu \Sigma^\dagger\right)$$
$$- \frac{l_3 + l_4}{16}\left[\mathrm{tr}\left(\chi'^\dagger \Sigma + \Sigma^\dagger \chi'\right)\right]^2 + \frac{l_4}{8}\,\mathrm{tr}\left(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger\right)\mathrm{tr}\left(\chi'^\dagger \Sigma + \Sigma^\dagger \chi'\right) - \frac{l_7}{16}\left[\mathrm{tr}\left(\chi'^\dagger \Sigma - \Sigma^\dagger \chi'\right)\right]^2$$

$$m_{\pi^\pm}^2 = 2Bm_l\left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2}\ln\left(\frac{m_\pi^2}{\mu^2}\right) + \frac{4m_\pi^2}{f_\pi^2} l_4^r(\mu)\right\} - \frac{\Delta_{PQ}^4}{2(4\pi f_\pi)^2}$$

$$m_{\pi^0}^2 = m_{\pi^\pm}^2 + \frac{16B^2\delta^2}{f_\pi^2} l_7$$

$$\Delta_{PQ}^2 = 2B\delta$$

Strong Isospin Breaking: m_d - m_u

AWL arXiv:0904.2404

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

Can also construct the partially quenched baryon chiral Lagrangian

Strong Isospin Breaking: m_d - m_u

AWL arXiv:0904.2404

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

Can also construct the partially quenched baryon chiral Lagrangian

$$m_p = M_0 - \alpha\delta + m_l(\alpha + \sigma_N) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu) + \frac{3\pi \Delta_{PQ}^4 (g_A + g_1)^2}{8 m_\pi (4\pi f_\pi)^2}$$

$$m_n = M_0 + \alpha\delta + m_l(\alpha + \sigma_N) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu) + \frac{3\pi \Delta_{PQ}^4 (g_A + g_1)^2}{8 m_\pi (4\pi f_\pi)^2}$$

Strong Isospin Breaking: m_d - m_u

AWL arXiv:0904.2404

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

Can also construct the partially quenched baryon chiral Lagrangian

$$m_p = M_0 - \alpha\delta + m_l(\alpha + \sigma_N) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2}\mathcal{F}(m_\pi, \Delta, \mu) + \frac{3\pi\Delta_{PQ}^4 (g_A + g_1)^2}{8 m_\pi (4\pi f_\pi)^2}$$

$$m_n = M_0 + \alpha\delta + m_l(\alpha + \sigma_N) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2}\mathcal{F}(m_\pi, \Delta, \mu) + \frac{3\pi\Delta_{PQ}^4 (g_A + g_1)^2}{8 m_\pi (4\pi f_\pi)^2}$$

$$M_n - M_p = \alpha(m_d - m_u) + \mathcal{O}(\delta^2, m_\pi^2\delta)$$

$$(2\delta = m_d - m_u)$$

Problematic terms exactly drop out of expansion for mass difference!

Strong Isospin Breaking: m_d - m_u

PRELIMINARY

performed the lattice calculation using the Spectrum Collaboration anisotropic clover-Wilson gauge ensembles (developed here @ JLAB)

C.Aubin, W.Detmold, E.Mereghetti, K.Orginos, S.Syritsyn, B.Tiburzi, AWL

ensemble				m_π	m_K	$a_t\delta$ $[N_{cfg} \times N_{src}]$			
L	T	$a_t m_l$	$a_t m_s$	[MeV]	[MeV]	0.0002	0.0004	0.0010	0.0020
16	128	-0.0830	-0.0743	449	581	207×16	207×16	207×16	207×16
16	128	-0.0840	-0.0743	390	546	166×25	166×25	166×25	166×50
20	128	-0.0840	-0.0743	390	546	120×25	–	–	–
24	128	-0.0840	-0.0743	390	546	97×25	–	193×25	–
32	256	-0.0840	-0.0743	390	546	291×10	291×10	291×10	–
24	128	-0.0860	-0.0743	225	467	118×26	–	–	–
32	256	-0.0860	-0.0743	225	467	842×11	–	–	–

Strong Isospin Breaking: m_d - m_u

PRELIMINARY

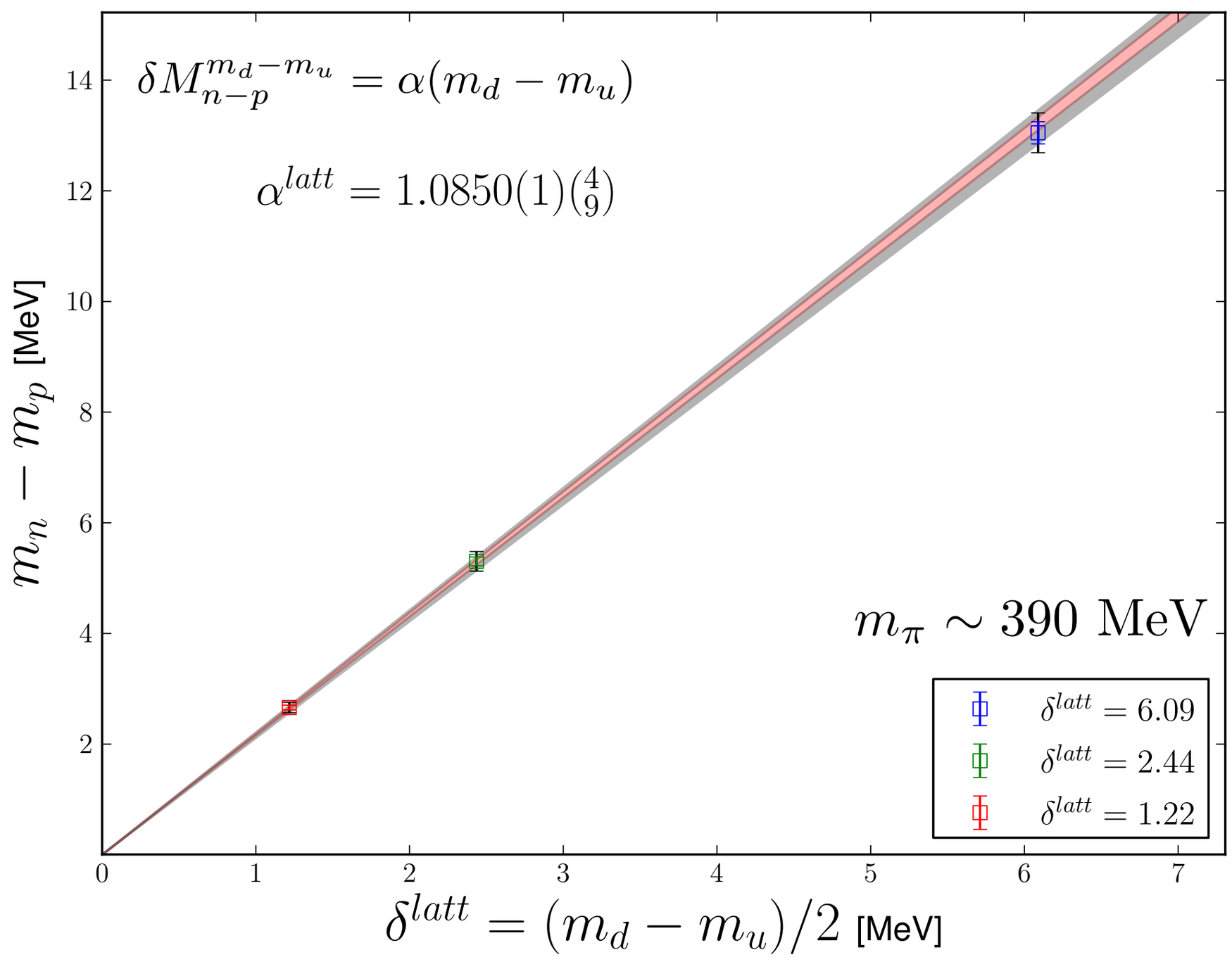
calculations performed at

Jefferson Lab HPC Center

Sporades Cluster

Strong Isospin Breaking: m_d - m_u

PRELIMINARY



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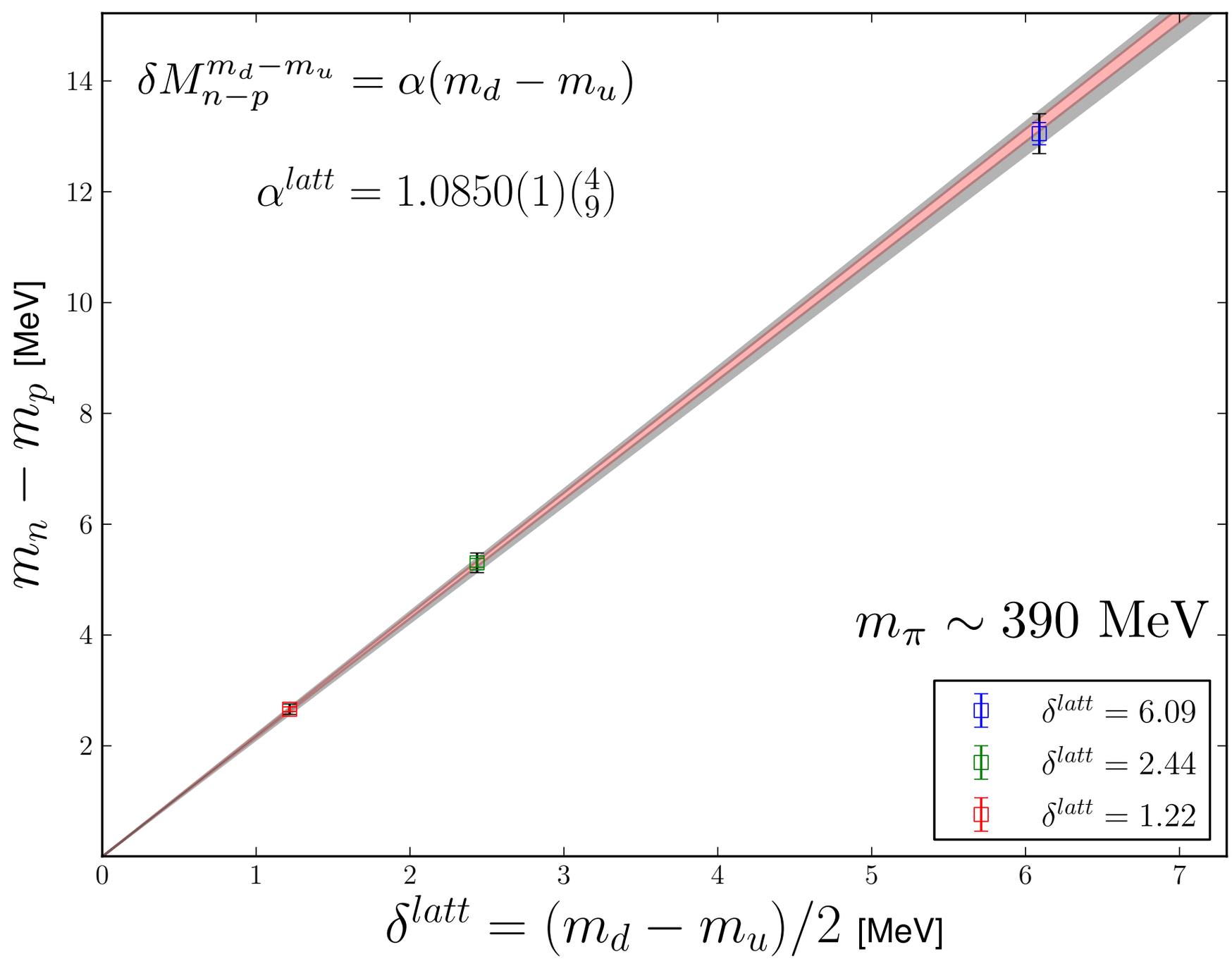

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AWL

This technique orders of magnitude less numerically expensive than using lattice QCD with full $m_u^{sea} \neq m_d^{sea}$

Big Bang Nucleosynthesis and $M_n - M_p$

$$M_n - M_p = \delta M^{\gamma}_{n-p} + \delta M^{m_d - m_u}_{n-p}$$
$$= -178(04)(64)\ \text{MeV} \times \alpha_{f.s.} + 1.08(6)(9) \times (m_d - m_u)$$

(lattice average)

my value orders of magnitude more precise

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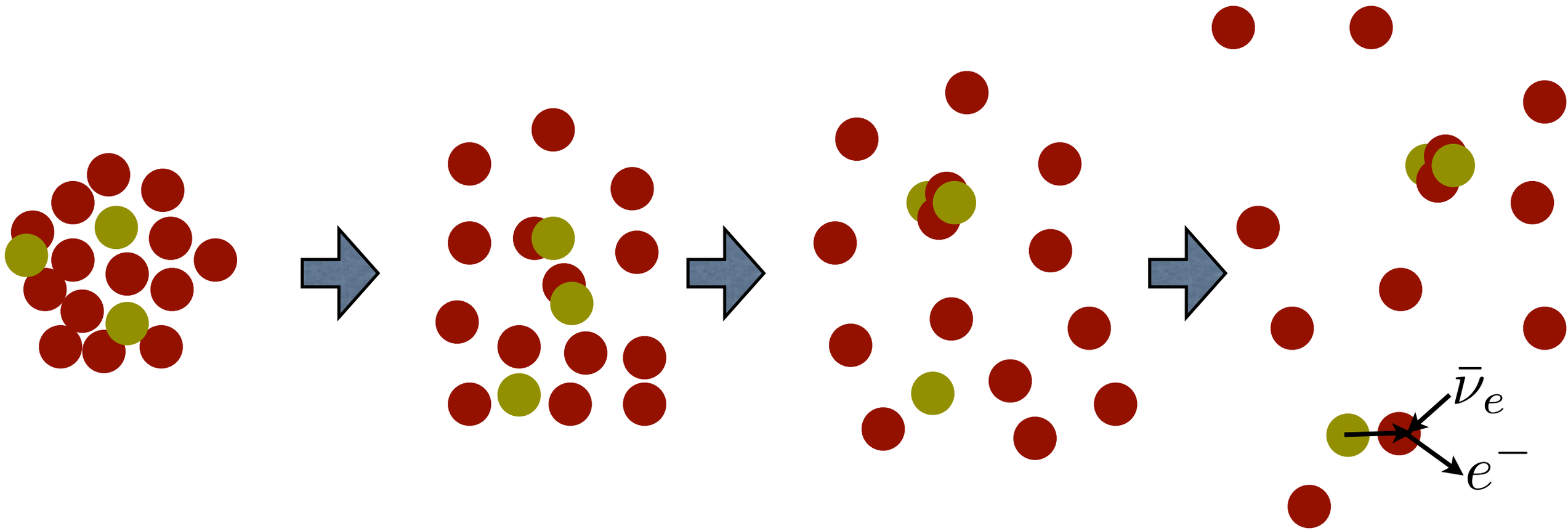
Big Bang Nucleosynthesis highly constrains variation of $M_n - M_p$ and hence variation of fundamental constants

considering $\alpha_{f.s.}$ and $m_d - m_u$ simultaneously relaxes constraints (not yet simultaneously considered)

for now - freeze electromagnetic coupling and just look at effects of quark mass splitting

Big Bang Nucleosynthesis and $M_n - M_p$



$p + n \longleftrightarrow d + \gamma$ $\quad$ $p + n \longrightarrow d + \gamma$

$t \sim 1$ sec $\quad$ $t \sim 3$ min $\quad$ $t \sim 3^+$ min $\quad$ $t \sim 15$min

$T \sim 1$ MeV $\quad$ $T \sim 0.1$ MeV $\quad$ $T \sim 0.1^-$ MeV $\quad$ $T \sim 0.01$ MeV

$$\frac{X_n}{X_p} = e^{-\frac{M_n - M_p}{T}}$$

Initial conditions

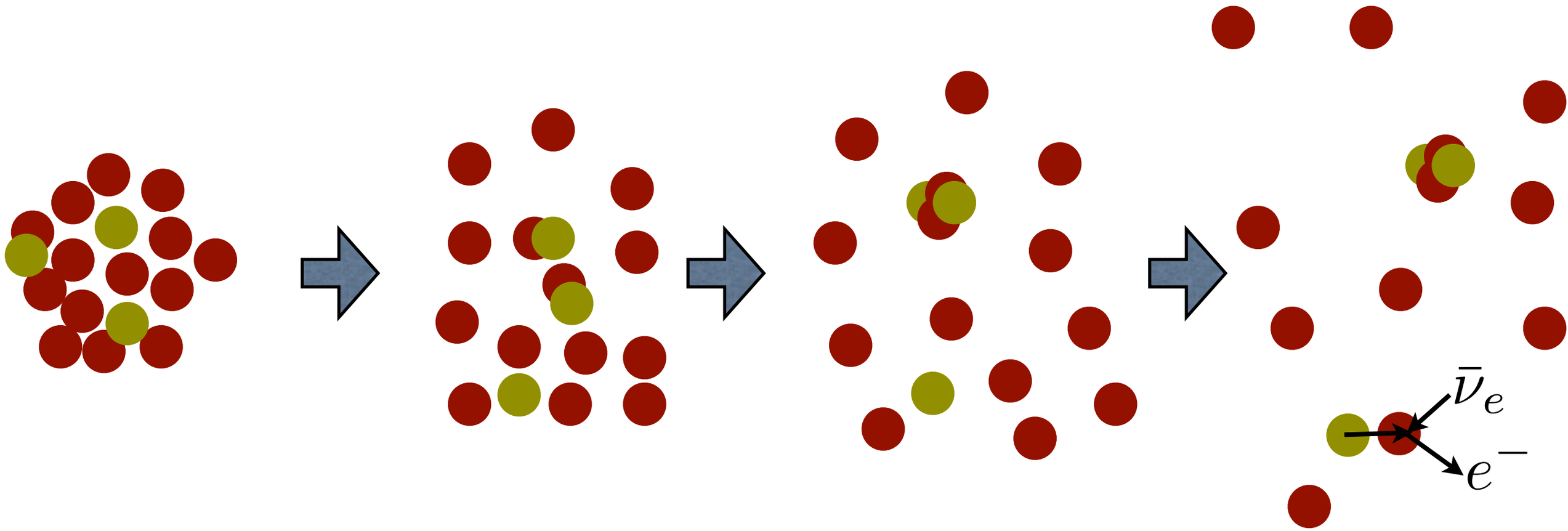
B_d

deuterium binding energy

τ_n

neutron lifetime

Big Bang Nucleosynthesis and $M_n - M_p$



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Initial conditions

focus on leading isospin breaking

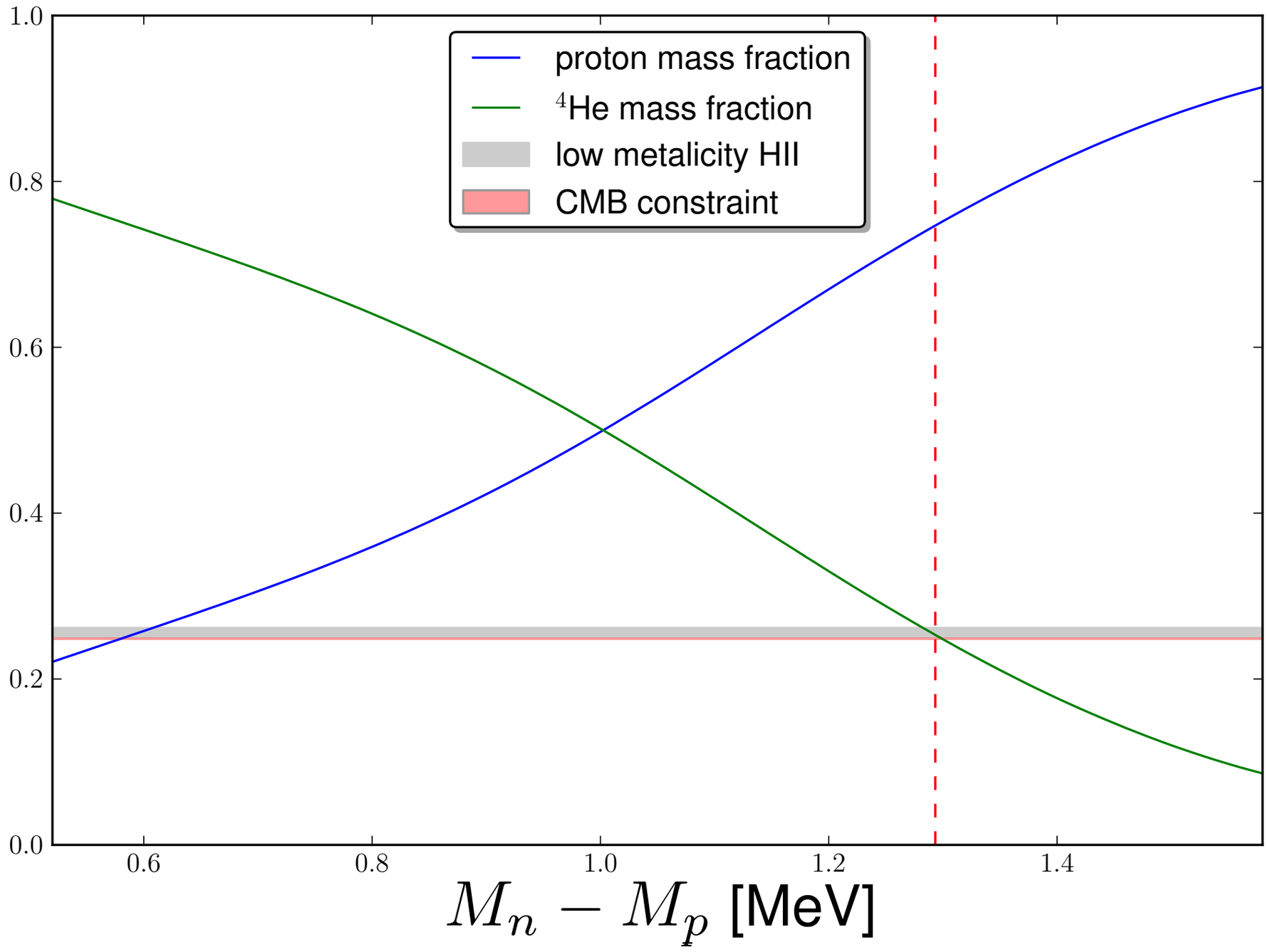
τ_n

neutron lifetime

Big Bang Nucleosynthesis and $M_n - M_p$

P. Banerjee, T. Luu, AWL

PRELIMINARY



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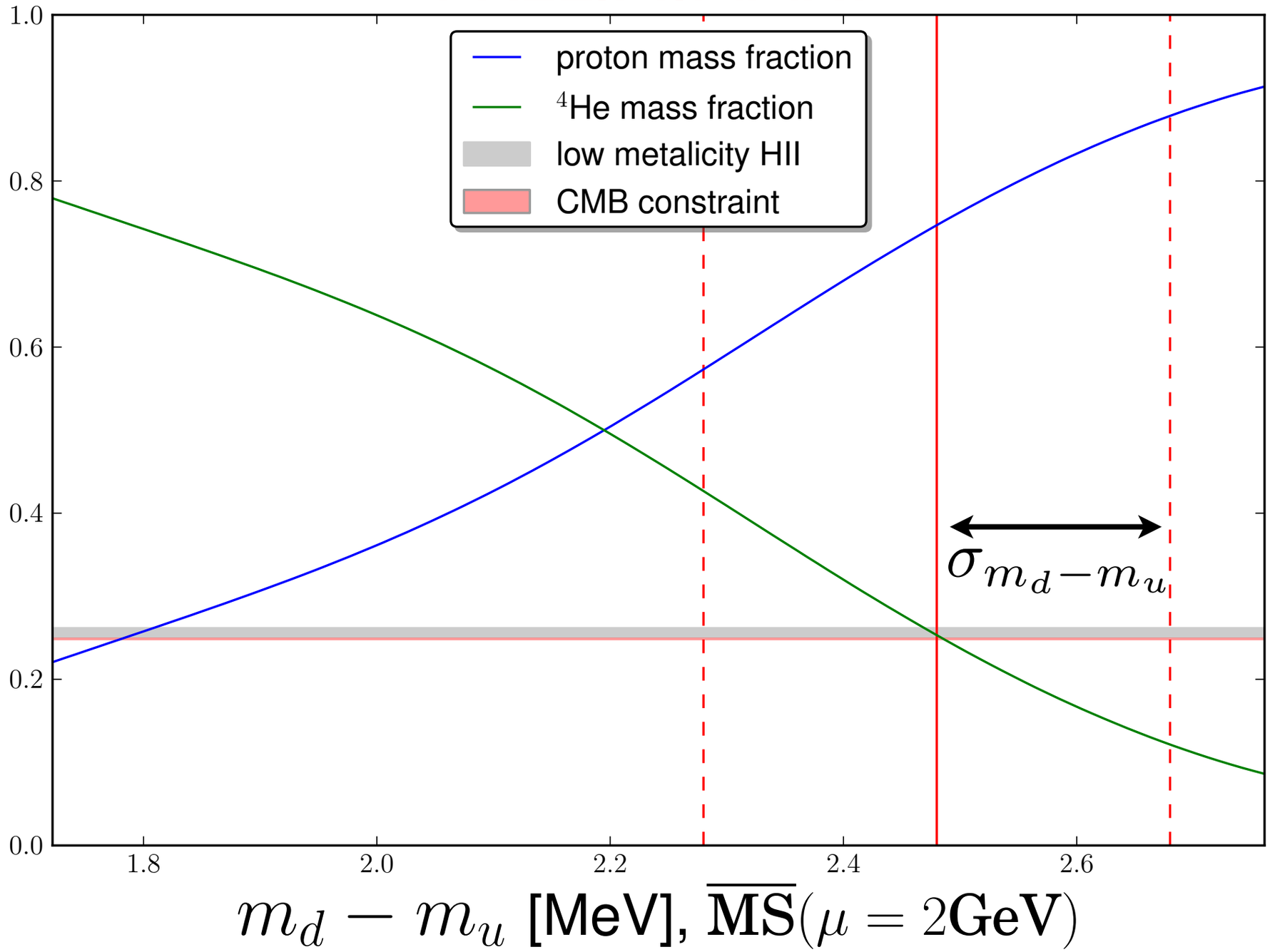
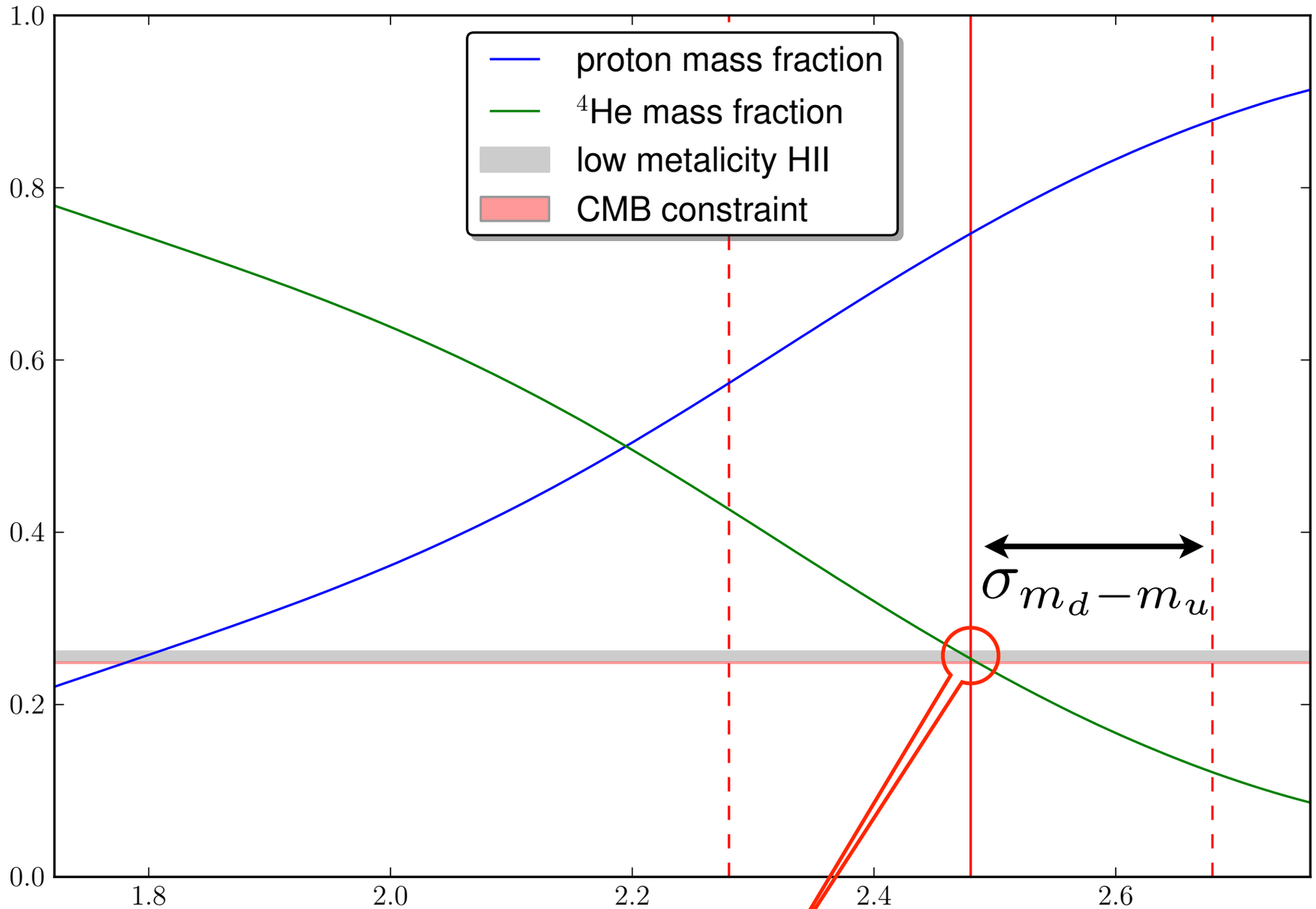


Lattice QCD

Big Bang Nucleosynthesis and $M_n - M_p$

P. Banerjee, T. Luu, AWL

PRELIMINARY



Lattice QCD

$m_d - m_u$ [MeV], $\overline{\rm MS}(\mu = 2{\rm GeV})$

A precise determination of α + BBN can constrain $m_d - m_u$

$$\delta M_{n-p}^{m_d - m_u} \equiv \alpha (m_d - m_u)$$

CONCLUSIONS

- related a simple quantity $M_n - M_p$ to the primordial abundance of light nuclear elements, formed in the first few minutes after the Big Bang

- showed how modern knowledge of nucleon structure can be used to determine the electromagnetic self-energy contribution

  improvements will come with a determination of the iso-vector nucleon magnetic polarizability - either experimentally of from lattice QCD

- the strong contribution $(m_d - m_u)$ can only be determined with lattice QCD: I showed you what we know now and a calculation I am performing that will hopefully improve the precision an order of magnitude

- this was just a simple example of exciting connections we can now make between the universe and QCD because of the tremendous growth of lattice QCD as a tool for non-perturbative QCD phenomena

FUTURE RESEARCH PLANS

- Beyond the Standard Model (BSM) physics is one of the most exciting areas of physics research today

  we know it must be there

    neutrino masses, Dark Matter, Dark Energy

  we have no confirmed ideas what it is

- A principle means for searching for BSM physics is through precision tests of Standard Model symmetries in low-energy observables

  hadronic-parity violation eg. Q-Weak

  electric dipole moments (EDM)s

  direct Dark Matter searches

- Due to the non-perturbative nature of low-energy QCD, lattice QCD is an essential tool

  very successful with HEP observables

  now being applied to NP observables

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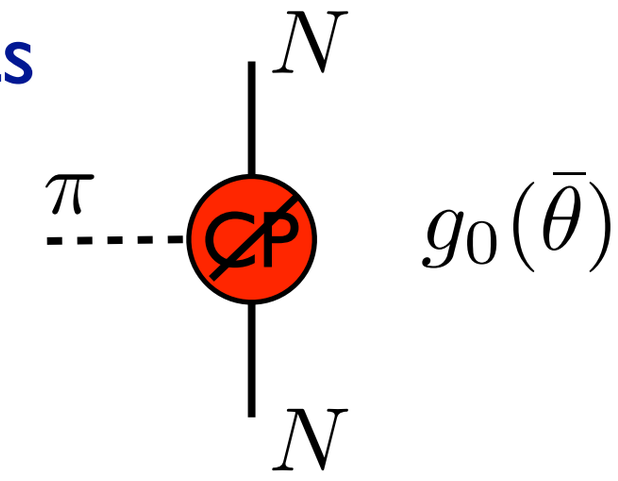
  now being applied to NP observables $m_s\langle N|\bar{s}s|N\rangle$ & Dark Matter

  P. Junnarkar and AWL arXiv:1301.1114

FUTURE RESEARCH PLANS

Electric Dipole Moments

long range ~~CP~~ interaction dominates nuclear EDMs



$$g_0(\bar{\theta}) = \delta M_{n-p}^{m_d - m_u} \frac{2 m_d m_u \sin(\bar{\theta})}{(m_d + m_u)(m_d - m_u)} \qquad \delta M_{n-p}^{m_d - m_u} \equiv \alpha (m_d - m_u)$$

$$= \sin(\bar{\theta})\, \alpha \frac{2 m_d m_u}{(m_d + m_u)}$$

I am computing this with lattice QCD

The world's most stringent constraint on an EDM from Atomic measurement competitive constraint on $\bar{\theta}$

Griffith, Swallows, Loftus, Romalis, Heckel, Fortson
PRL 102 101601 (2009)

FUTURE RESEARCH PLANS

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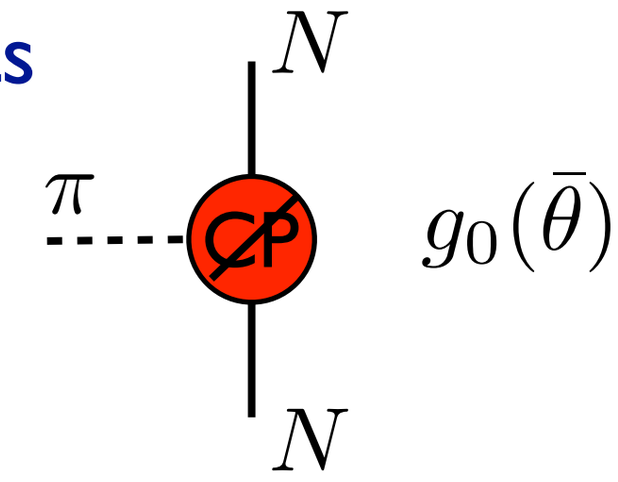

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Will compute nuclear-EDMs for generic quark-EDMs

Research is not done in a vacuum

Light Nuclei from (lattice) QCD

S. Beane
E. Chang
S. Cohen
W. Detmold
P. Junnarkar
H-W. Lin
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Fundamental Symmetries PV, EDMs

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+ AM + CS

Electromagnetic and Weak properties of hadrons/nuclei

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Robert Edwards, Balint Joo,
Chip Watson & JLAB Computing Support

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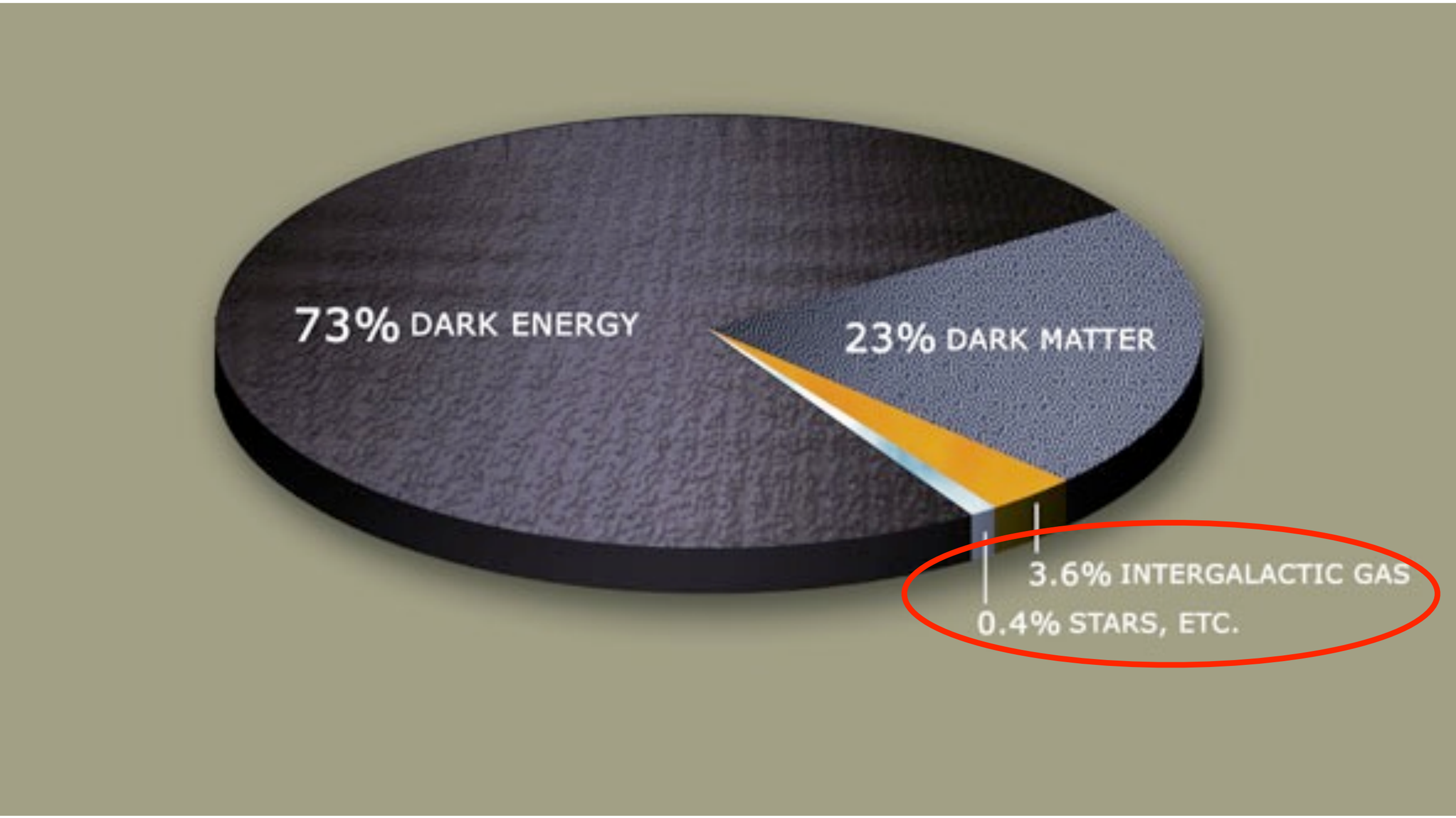
Thank You

BACKUP

Energy Budget of the Universe



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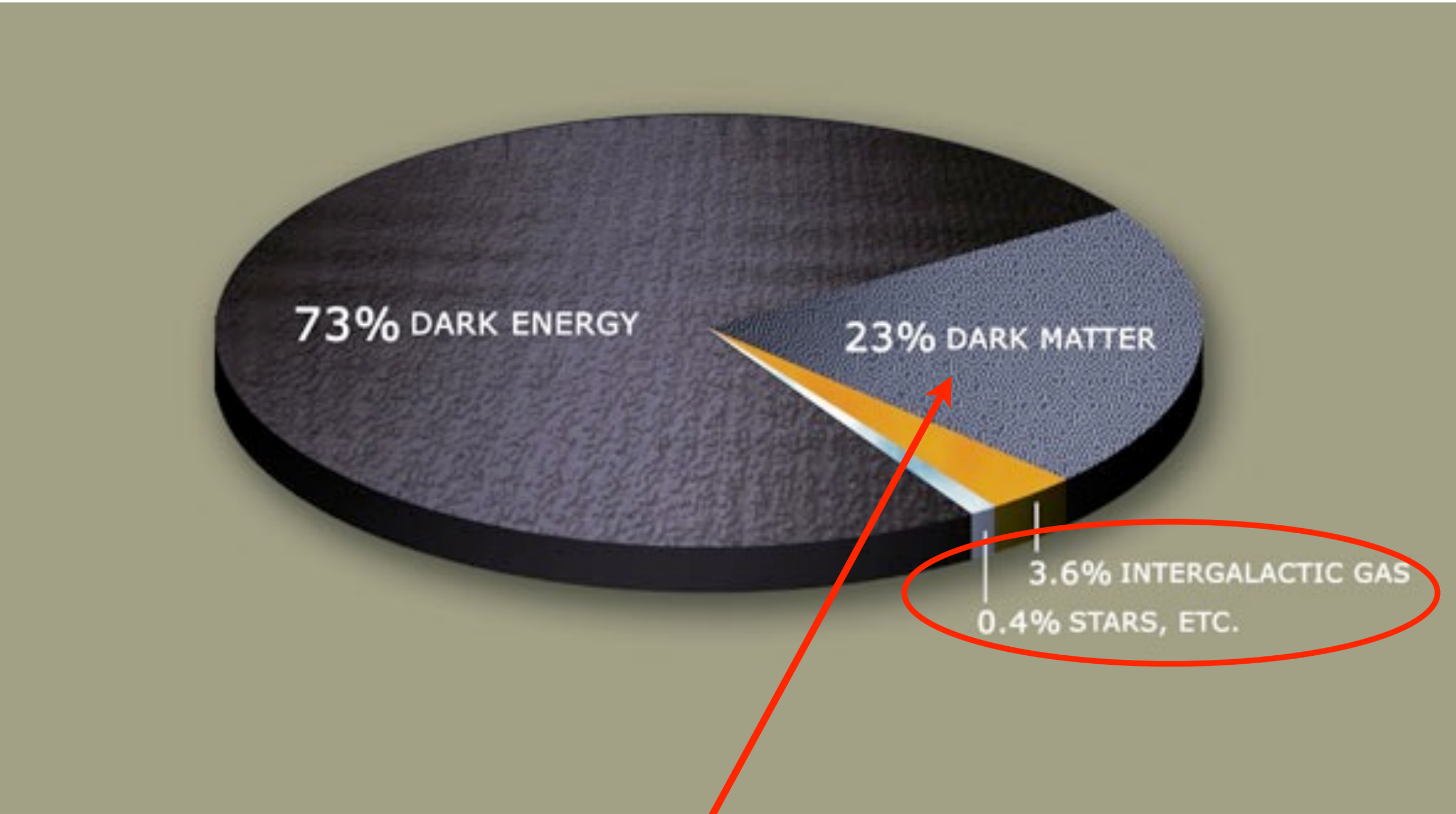
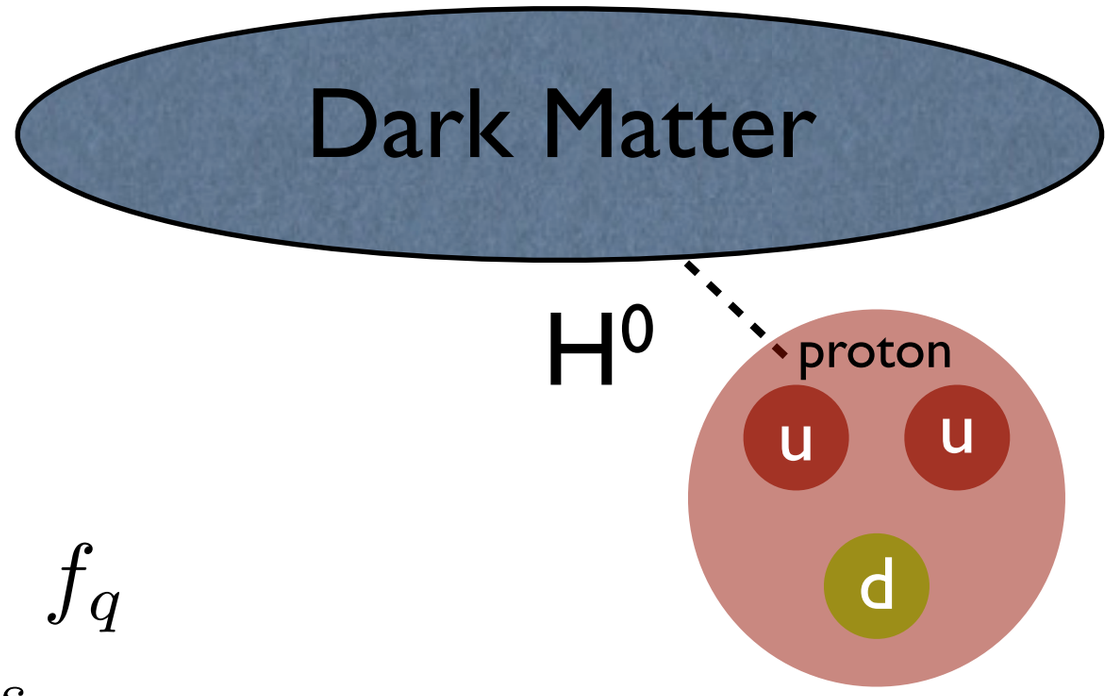


quark mass dependence of nucleon mass helps us understand sensitivities of direct dark matter detection

If Dark Matter couples to the scalar current of the nucleon (eg via Higgs) Spin Independent cross section



$$\sigma \propto |f|^2 \qquad f = \frac{2}{9} + \frac{7}{9} \sum_{q=u,d,s} f_q$$

$$f_q \equiv \frac{\langle N|m_q \bar{q} q|N\rangle}{m_N}$$

see eg. Cheung, Hall, Pinner, Ruderman
arXiv:1211.4873

with enhancement of A^2 for nucleus (Xenon)

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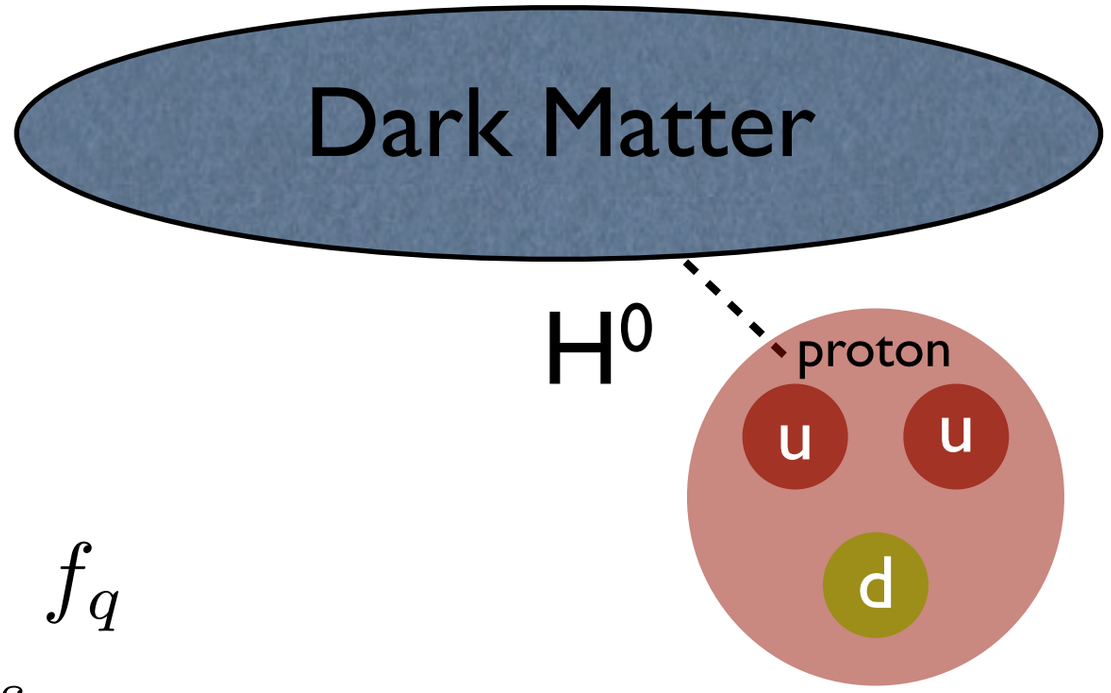

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with enhancement of A^2 for nucleus (Xenon)

scalar current difficult to measure experimentally

$f_{u,d}$ estimated from pion-nucleon scattering

f_s uncertainty dominates estimates of cross section

Ellis, Olive, Savage
Phys.Rev. D77 (2008)

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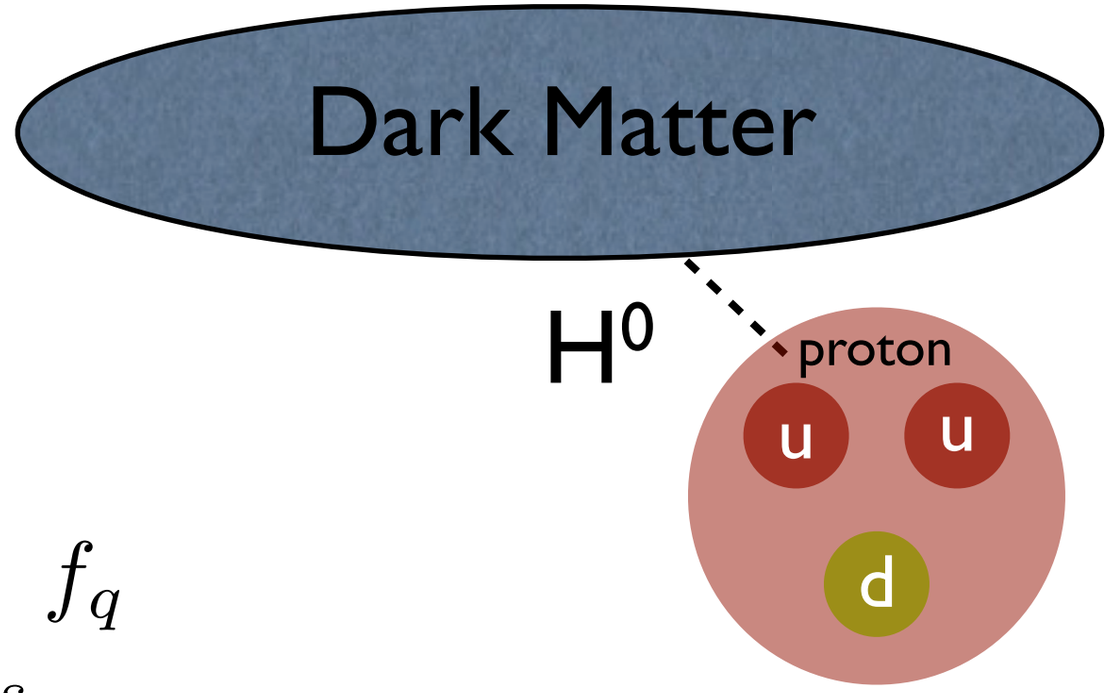

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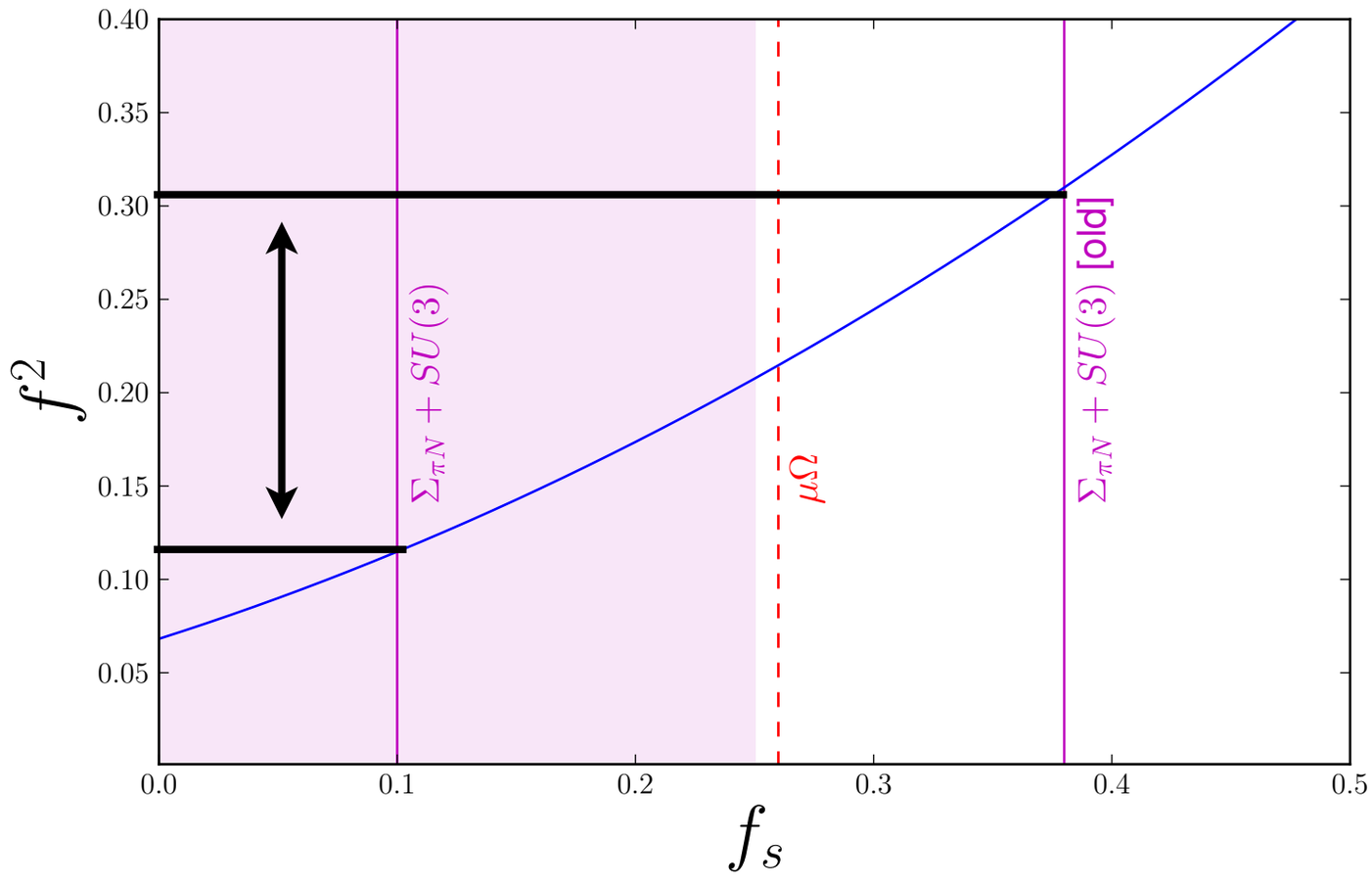
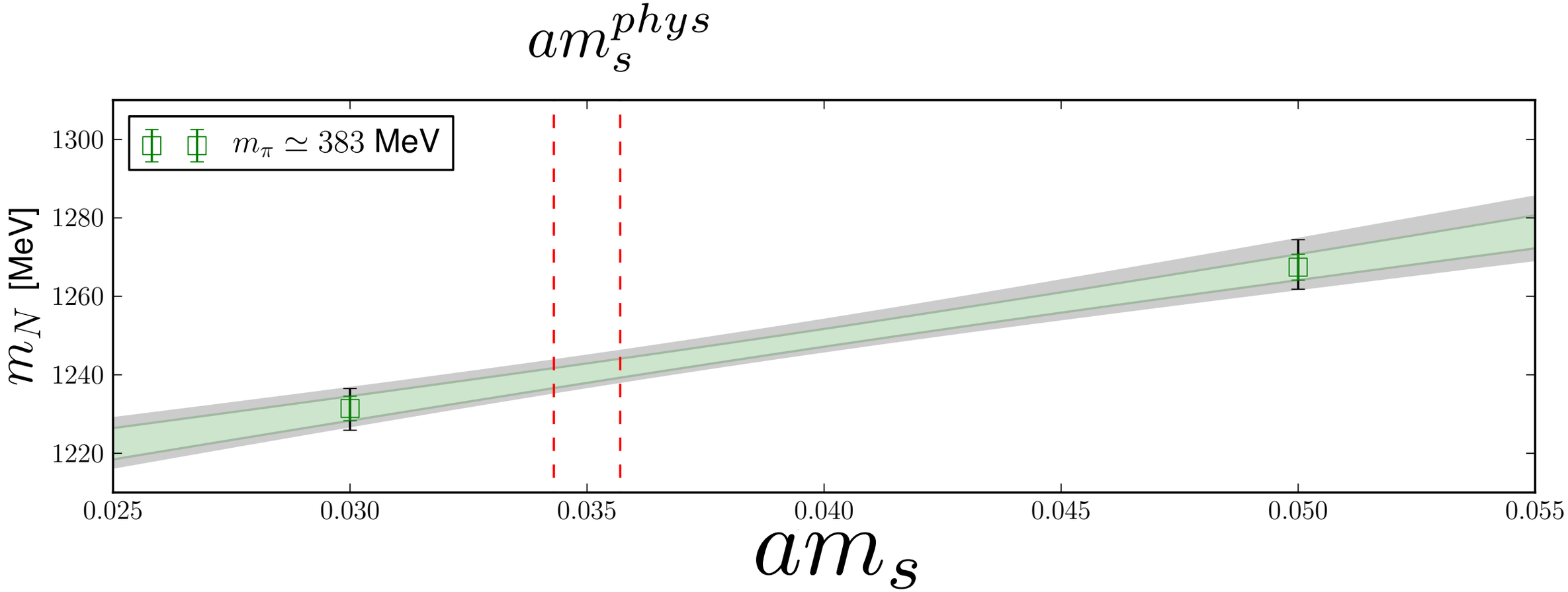


strange content of the nucleon

P. Junnarkar and AWL
arXiv:1301.1114

Lattice QCD perfect tool to compute strange content of nucleon $m_s \langle N|\bar{s}s|N\rangle$

Feynman-Hellmann Theorem $\quad m_q \langle N|\bar{q}q|N\rangle = m_q \dfrac{\partial}{\partial m_q} m_N$



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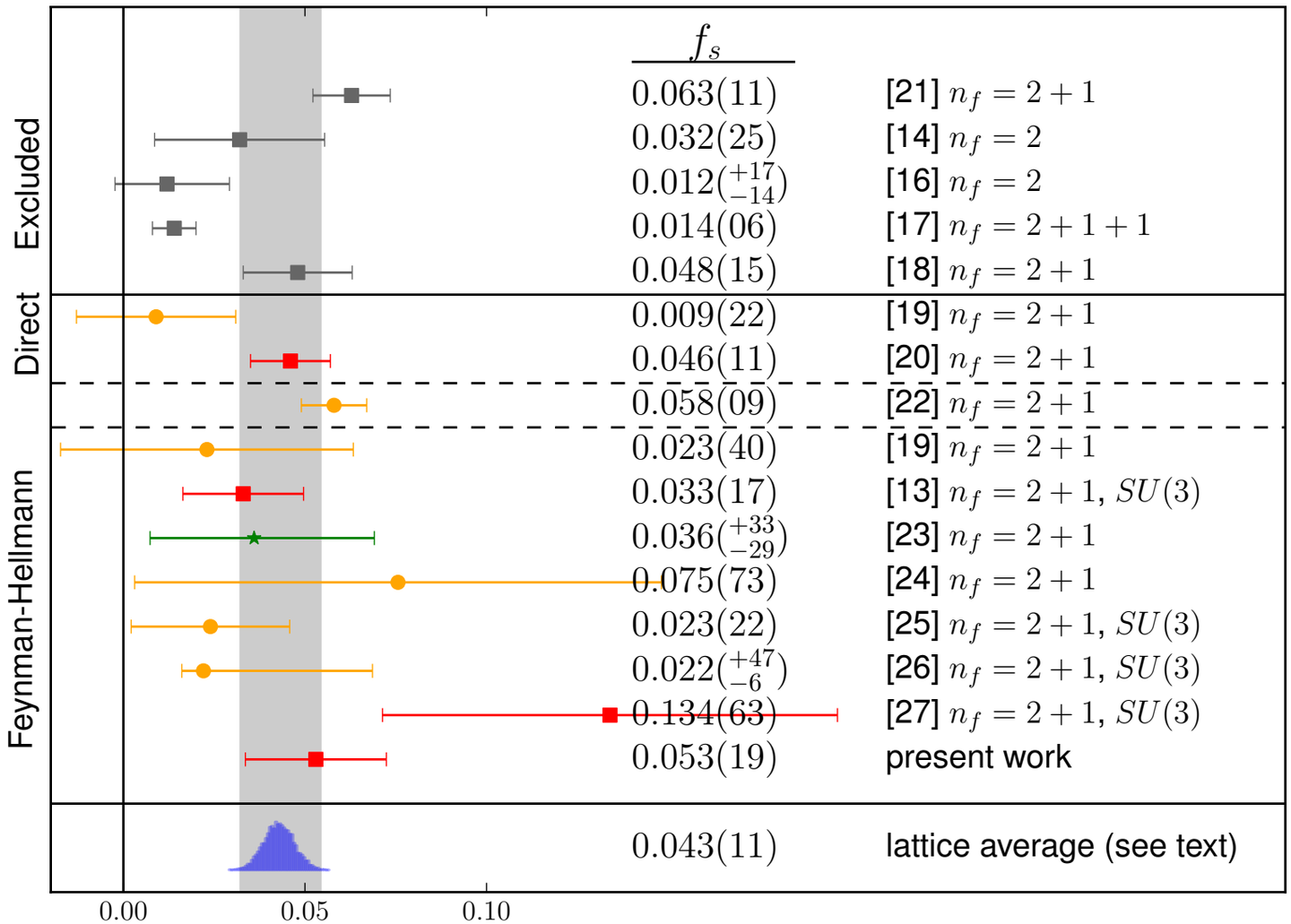

$$f_s = m_s\langle N|\bar{s}s|N\rangle / m_N$$

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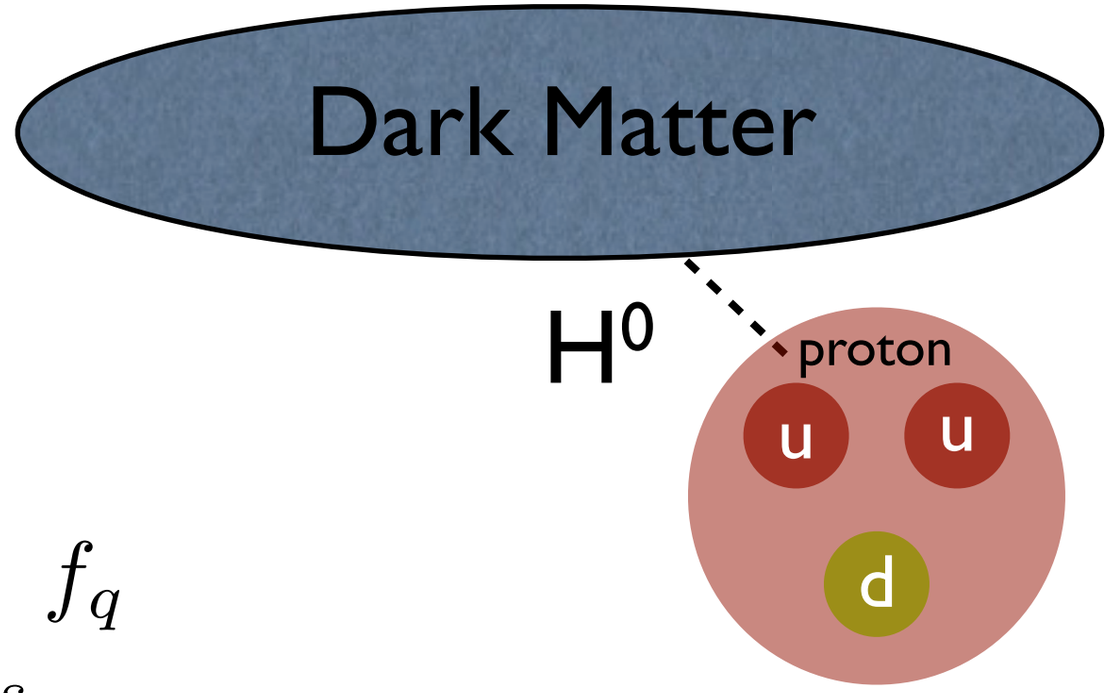

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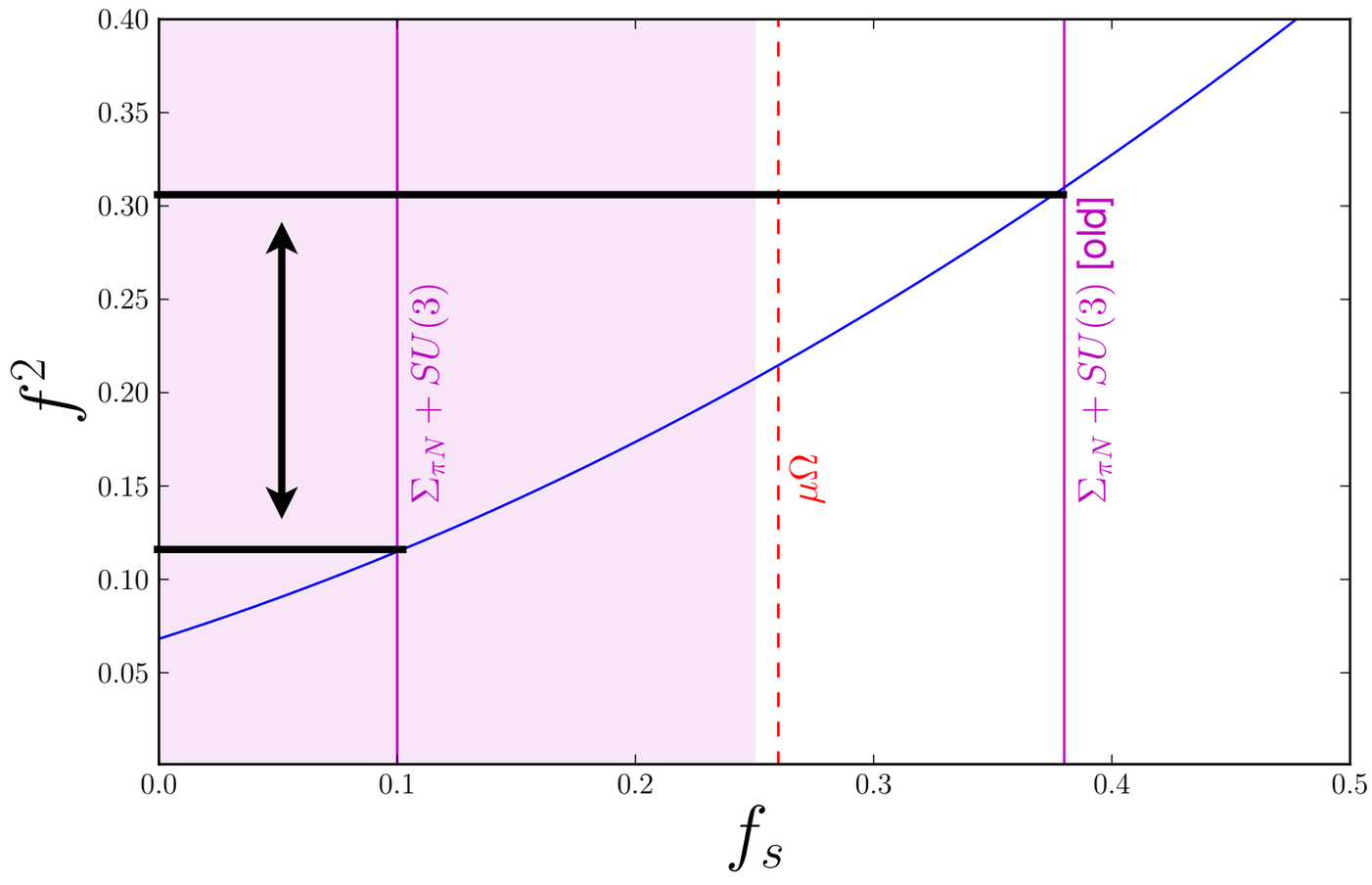

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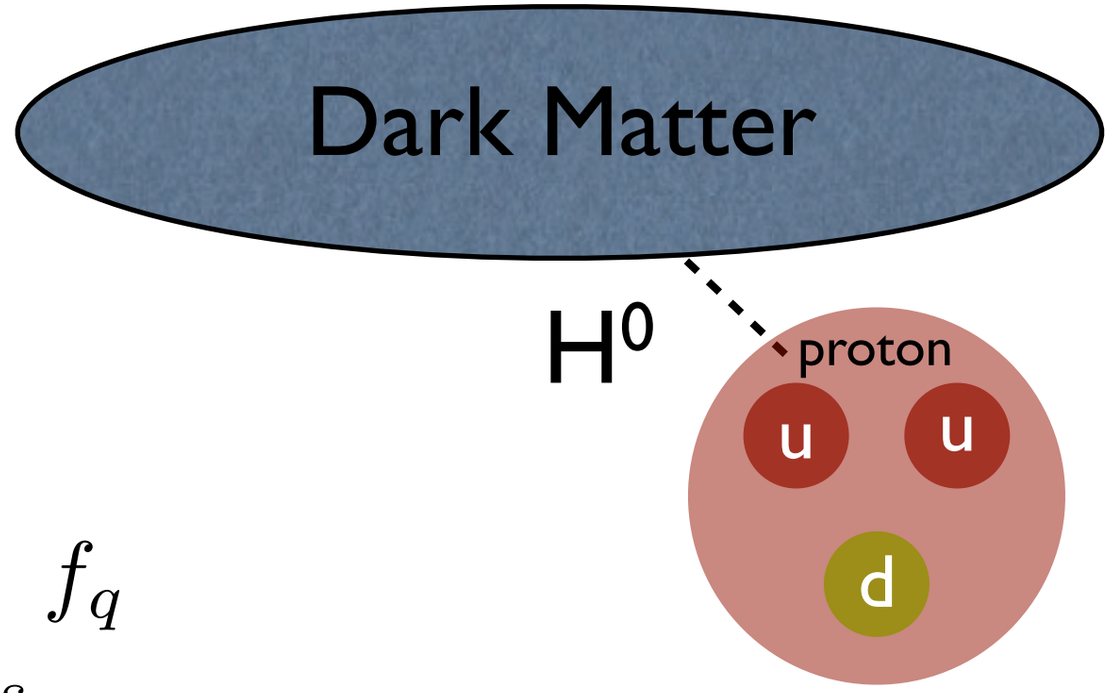

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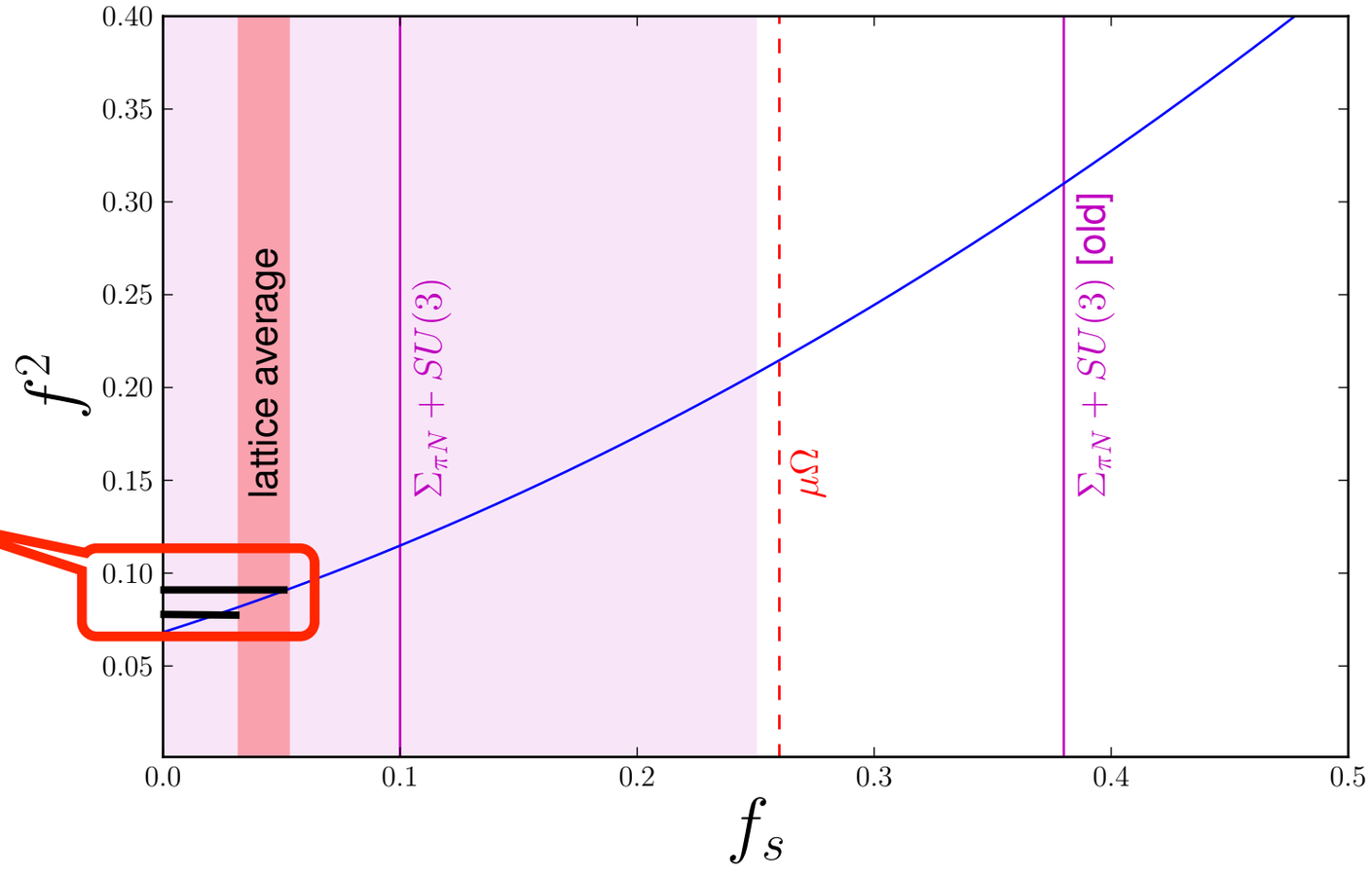

dramatic reduction in uncertainty of cross section

now $f_{u,d}$ gives larger uncertainty - but harder