Renormalization and power counting of chiral nuclear forces

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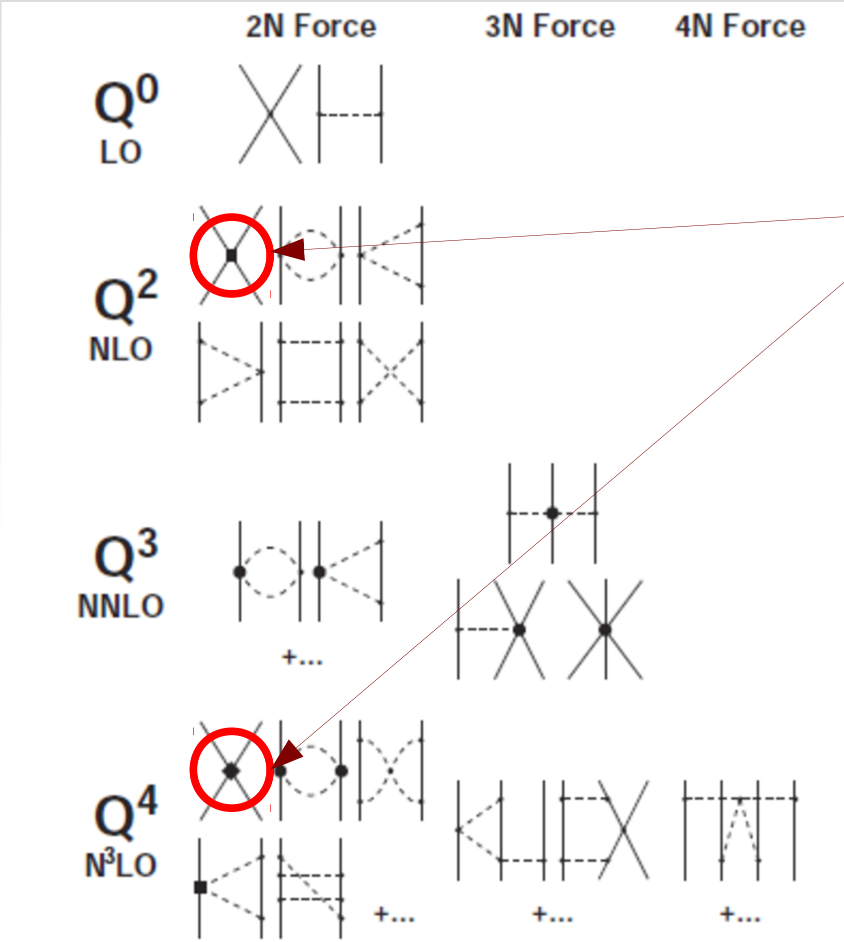
What are we really doing?



Correcting Weinberg's scheme about NN contact interactions using renormalization group invariance, (cutoff independence) as the guideline

Outline

- Intro to chiral effective field theory
- Dr. W's prescription for chiral nuclear forces
- What went wrong
- What need to be modified
- Summary

Why study low-energy nuclear EFT

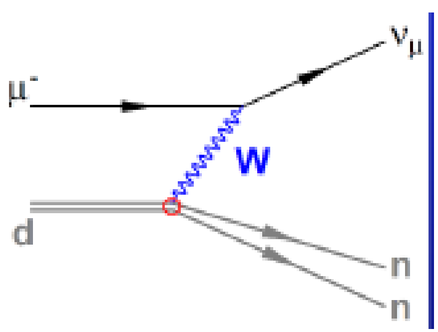
- Useful

Most nuclei bound below momentum scale Q ~ 140 MeV, no need to know the details of QCD

For many reactions, only near-threshold region interested

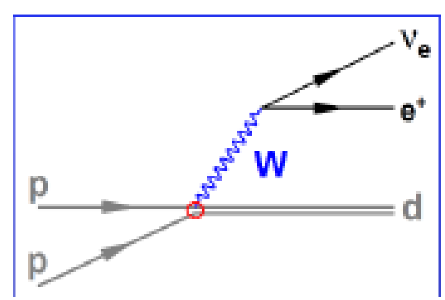
$$d + {}^3H \to {}^4He + n(14\ \mathrm{MeV})$$
$$d + d \to {}^3He + n(2.5\ \mathrm{MeV})$$
$$\ldots$$

Controlled fusion, big-bang nucleosynthesis



$\mu^- + d \to n + n + \nu_\mu$

effective field theory



$p + p \to d + e^+ + \nu_e$

Dominating reaction to produce solar neutrinos, but unobservable in lab

Pros and cons

Pros

- Most general Lagrangian w/ chiral symmetry
  - A unified framework to study strong interactions and electroweak probes
- Can estimate theoretical error, but power counting must be consistent

$$\mathcal{M} = \sum_n \left(\frac{Q}{M_{hi}} \right)^n \mathcal{F}_n \left(\frac{Q}{M_{lo}} \right)$$

Non-analytical functions from loops

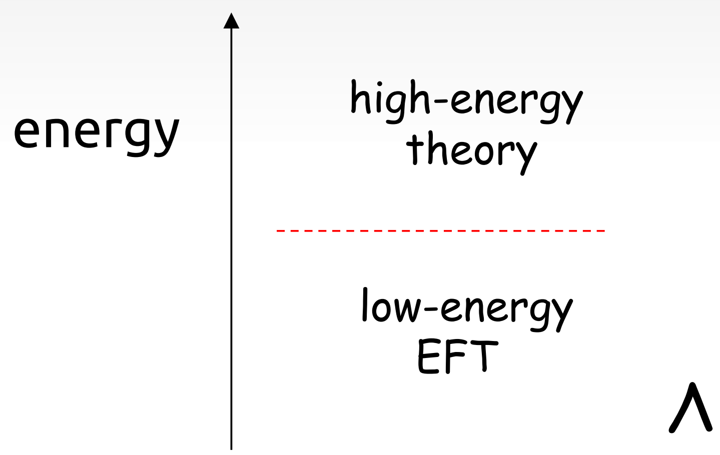
Cons

- Breaks down for Q > 500 MeV

estimate diagrams
before calculation

EFT = effective Lagrangian + power counting

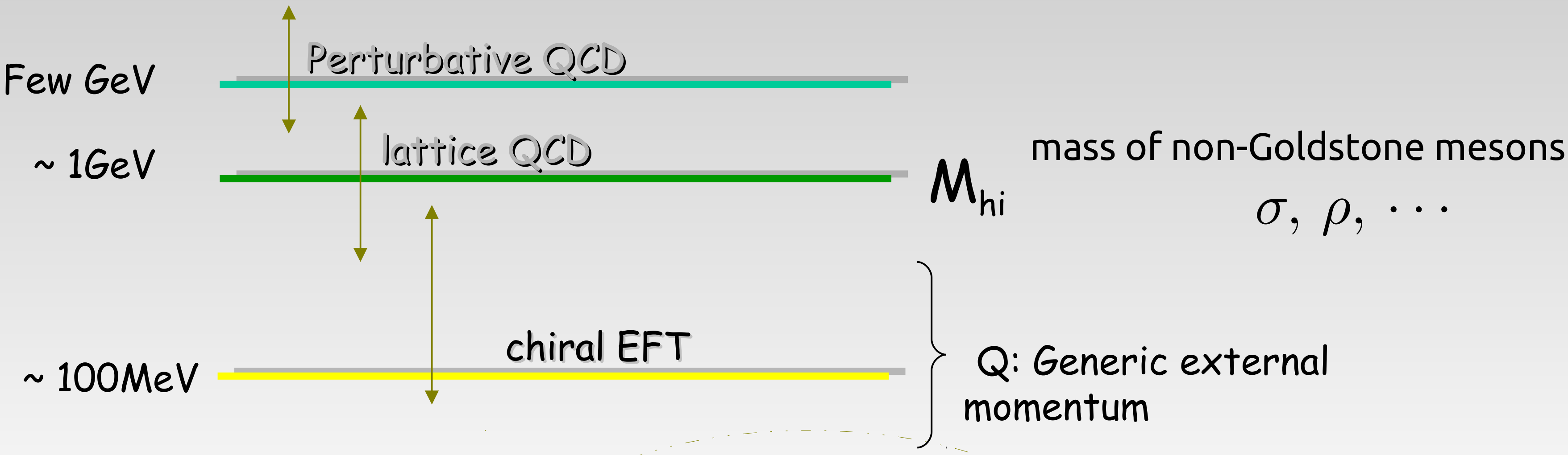
. low-energy dofs
. symmetries



renormalization

observables
independent of Λ → Model independence

What does chiral effective field theory look like



D.o.f.s

Nucleons

Pions

chiral symmetry of QCD

$SU(2)_L \times SU(2)_R$

- spontaneously broken
- approximate $m_q > 0$

$$\frac{Q}{M_{hi}}$$

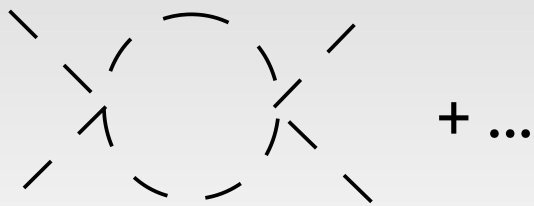
$$\mathcal{L}_{int} = N^\dagger \left(-\frac{1}{4f_\pi^2}\epsilon_{abc}\tau_a\pi_b\dot{\pi}_c - \frac{g_A}{2f_\pi}\tau_a\vec{\sigma}\cdot\vec{\nabla}\pi_a \right) N - \frac{1}{2}C_S(N^\dagger N)^2 - \frac{1}{2}C_T(N^\dagger\vec{\sigma}N)^2$$

pion coupling p dep.

+ ...

Power of counting

Q: generic external momenta



pion propagator ~ $\frac{1}{Q^2}$

pi-pi vertex~ $\frac{Q^2}{f_\pi^2}$

$\int d^4 l \sim \frac{Q^4}{16\pi^2}$

actual cal. $= \frac{Q^2}{f_\pi^2} \frac{\# Q^2}{16\pi^2 f_\pi^2} \ln\left(\frac{Q}{\mu}\right) + \frac{Q^4}{16\pi^2 f_\pi^4} \ln\left(\frac{\mu}{\Lambda}\right) + CQ^4$

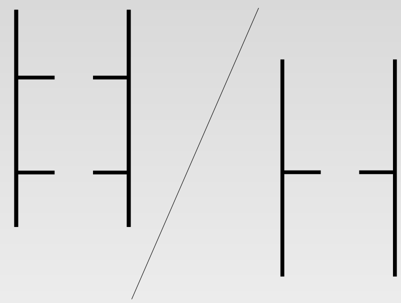
dim. analysis $\longrightarrow$ $\frac{Q^2}{f_\pi^2} \frac{Q^2}{(4\pi f_\pi)^2}$

Dim. analysis captures long-range phys (non-analytic in Q) very well

Imposing naturalness → counterterms and logs contribute about the same

$$\frac{Q^2}{f_\pi^2} \frac{\# Q^2}{16\pi^2 f_\pi^2} \ln\left(\frac{Q}{\mu}\right) \simeq \frac{Q^4}{16\pi^2 f_\pi^4} \ln\left(\frac{\mu}{\Lambda}\right) + CQ^4 \qquad C \sim \frac{1}{f_\pi^2 (4\pi f_\pi)^2}$$

Mass scale of OPE's strength



$$\sim \frac{m_N}{4\pi f_\pi}\frac{Q}{a(l) f_\pi}$$

l: ang. momentum

For lower p.w.
where a(l)~1: $Q \sim a(l) f_\pi \sim 100\,\mathrm{MeV}$ → nonperturbative OPE

This is a good thing
→ no need to put in by hand low-energy mass scale in order to generate bound states

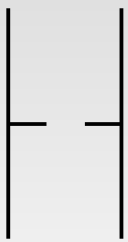
This is a bad thing, too
→ always have to choose between two mass scales in power counting
→ NDA no longer reliable

$$M_{hi} = 4\pi f_\pi \sim 1\mathrm{GeV} \qquad M_{lo} = a(l) f_\pi \sim 0.1\mathrm{GeV}$$

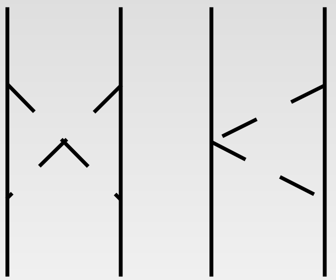
Two scales differ only by a numerical factor!

Dr. Weinberg's prescription for NN "potentials"

OPE



Leading irreducible TPE



....

$$w \equiv \sqrt{4m_\pi^2 + q^2}$$

$$V_{1\pi} = \frac{g_A^2}{4f_\pi^2}\frac{\vec{q}\cdot\vec{\sigma}_1\vec{q}\cdot\vec{\sigma}_2}{m_\pi^2 + q^2}$$

$$V_{2\pi} = -\frac{3g_A^4}{4f_\pi^2(4\pi f_\pi)^2}\frac{w}{q}\ln\frac{w+q}{2m_\pi}\vec{q}\cdot\vec{\sigma}_1\vec{q}\cdot\vec{\sigma}_2 + \cdots$$

$$+\mathcal{A}q^2 + \mathcal{B}k^2$$ primordial c.t.

k: relative momentum
q: momentum transfer

Long-range

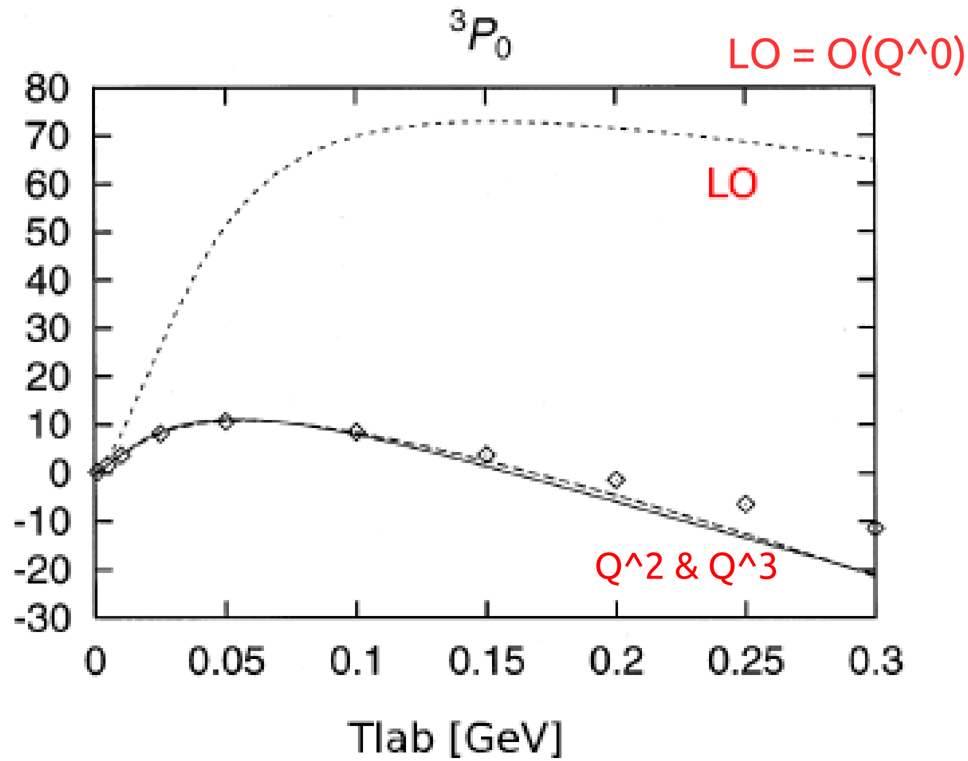
non-polynomials follow naïve dimensional analysis:

$$\frac{V_{2\pi}}{V_{1\pi}} \sim \frac{Q^2}{(4\pi f_\pi)^2}\mathcal{F}\left(\frac{Q}{m_\pi}\right)$$

Weinberg's prescription

→ assumming c.t. follow naïve dimensional analysis, too

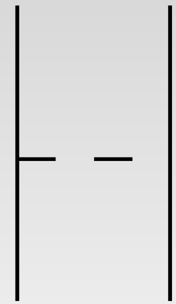
But, is there really a problem?



Epelgaum et at, NPA 671, 295

Large subleading corrections in 3P0

Let there be OPE



$$V_{1\pi}(\vec{r}) = \frac{m_\pi^3}{12\pi}\left(\frac{g_A^2}{4f_\pi^2}\right)\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left[T(r)S_{12} + Y(r)\vec{\sigma}_1 \cdot \vec{\sigma}_2\right]$$

$$T(r) = \frac{e^{-m_\pi r}}{m_\pi r}\left[1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right] \to 1/r^3 \text{ at } r \to 0$$

$$Y(r) = \frac{e^{-m_\pi r}}{m_\pi r} \to 1/r \text{ at } r \to 0$$

- tensor force (TF) acts on only triplet channels.
- due to S_{12}, TF could be attractive or repulsive in different channels.

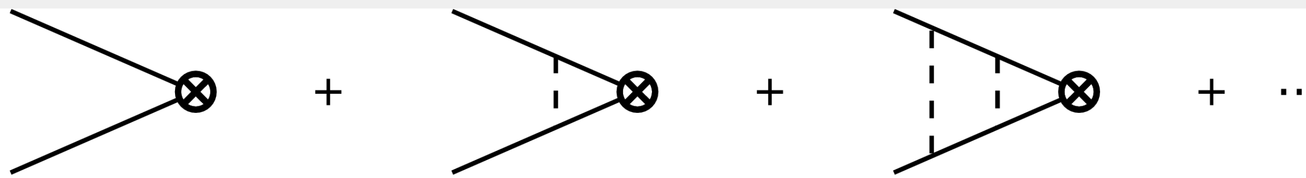
3S1, 3P0... 3P1...

The saga of 1S0

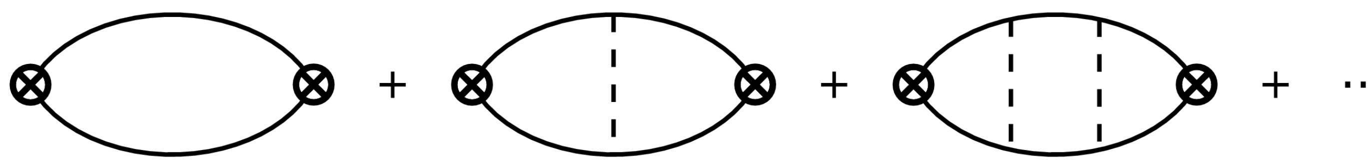
$$V^{(0)}_{1S0} = -\frac{g_A^2 m_\pi^2}{4 f_\pi^2}\,\frac{e^{-m_\pi r}}{r} + C_0\,\delta(\vec{r})$$

- OPE becomes regular near the origin ~ 1/r → no singular attraction
- Since T_yukawa is finite, renormalization can be easily understood

$\chi(p; k) =$



$I_k =$



$$V^{(0)} = V_{Yukawa} + C_0\,,\ T^{(0)}_{1S0} = T_{Yukawa} + \frac{\chi^2(k;k)}{\frac{1}{C_0} - I_k}\,,\ I_k \sim \#\Lambda + \#m_\pi^2 \ln\Lambda$$

(Kaplan et al, 1996)

O(Q) does not vanish in 1S0

LO residual cutoff variation ~ $\dfrac{k^2}{M_{lo}\Lambda}$

For comparison, in 3S1~ $\dfrac{k^2 M_{lo}^{1/2}}{\Lambda^{5/2}}$

→ LO theoretical error is at least O(Q)

→ can't be provided by TPE

→ $C_2 p^2$ must be O(Q), rather than O(Q^2) as suggested by WPC

→ $T^{(0)} + T^{(1)} = T_{\mathrm{Y}} + \dfrac{4\pi}{m_N} \dfrac{\chi_k^2}{-\frac{1}{\widetilde{a}(\mu)} + \frac{\widetilde{r}}{2}k^2 - \frac{4\pi}{m_N} I_k^R(\mu)}$ with $\dfrac{\widetilde{r}}{2} \sim \dfrac{1}{M_{hi}}$

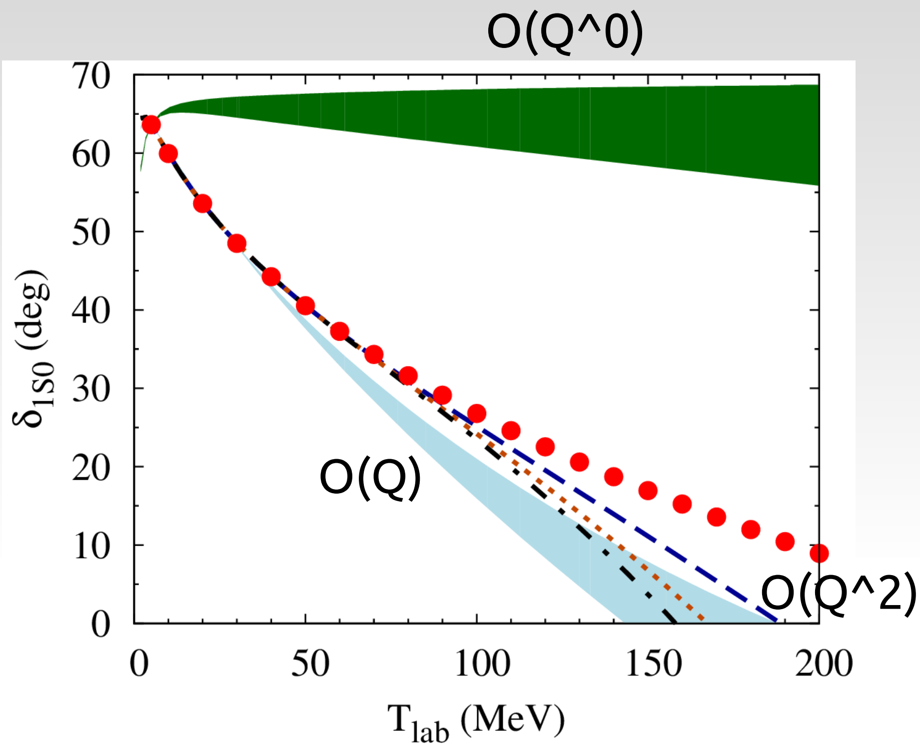
But, data say $\widetilde{r}$ is rather large...

$$\frac{\widetilde{r}}{2} = 1.55\,\mathrm{fm} = \frac{1}{127\,\mathrm{MeV}}$$

Steele & Furnstahl (1999)

Need to improve LO of 1S0

BwL & CJ Yang (2012)



- Convergence too slow
- Needs to promote $C_2\delta''(\vec{r})$ to LO → fine tuning
- Not so easy as far as renormalization is concerned

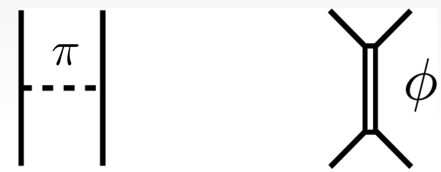
Red dots are PWA

$$V^{(0)} = V_Y + C_0\,,\; T^{(0)}_{1S0} = T_Y + \frac{\chi^2(k;k)}{\frac{1}{C_0+C_2k^2} - I_k}\,,\; I_k \sim \#\Lambda + \#m_\pi^2 \ln\Lambda$$

Improve LO of 1S0

- To introduce energy dependence in LO counterterm, use auxiliary field (only coupled to 1S0) → s-channel exchange
- Φ does not correspond to physical state

$$V^{(0)}_{1S0} =$$



(Kaplan, 1996, with a bit of modifciation by BwL)

$$V^{(0)} = V_{Yukawa} + \frac{\sigma y^2}{E + \Delta}\,, \qquad T^{(0)}_{1S0} = T_{Yukawa} + \frac{\chi^2(k;k)}{\frac{E+\Delta}{\sigma y^2} - I_k}$$

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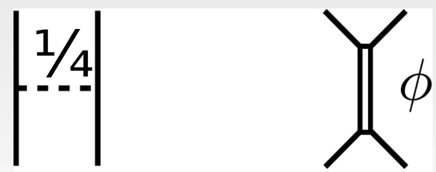

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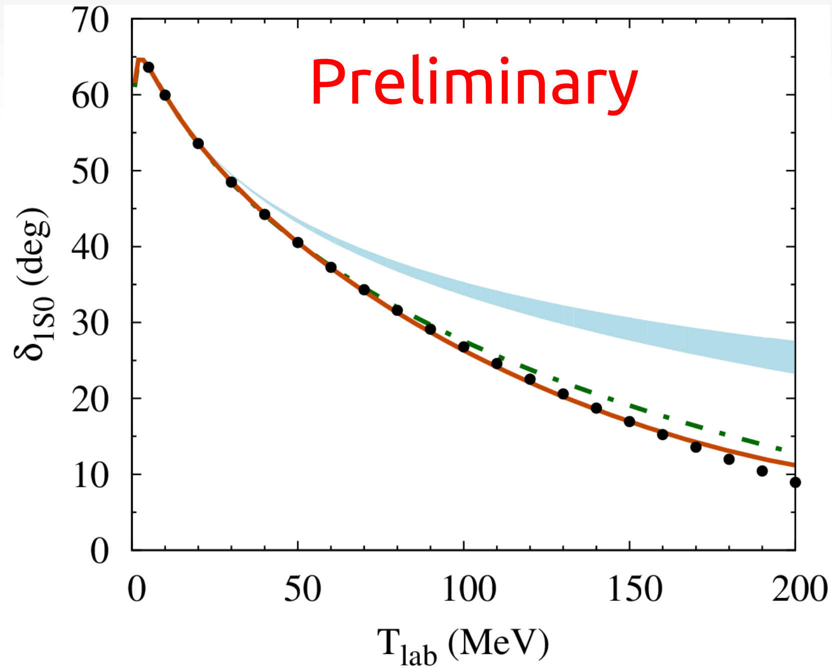
$$V^{(0)} = V_{Yukawa} + \frac{\sigma y^2}{E+\Delta}\,, \qquad T^{(0)}_{1S0} = T_{Yukawa} + \frac{\chi^2(k;k)}{\frac{E+\Delta}{\sigma y^2} - I_k}$$

$$\sigma = -1\,, \quad \Delta_R \sim 13\text{MeV}$$

low-energy dof → we're not modeling short-range physics

Subleading orders of 1S0

- Dibaryon Lagrangian doesn't need to be the most general one
- Convergence improved, with one more para.
- Fine-tuning incorporated systematically

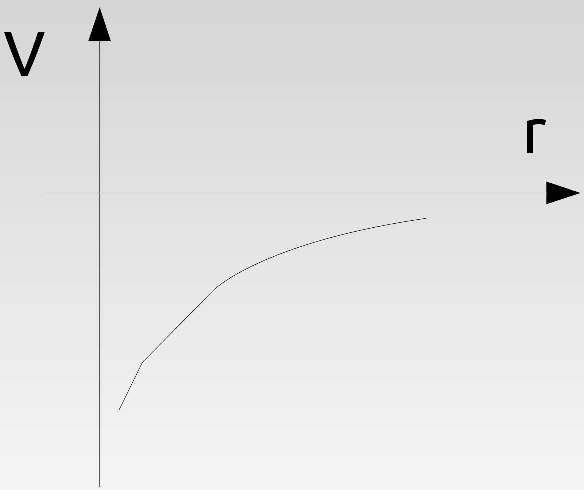


Blue: O(Q^0)
Green: O(Q)
Brown: O(Q^2)
Black: PWA

$-1/r^3$ is more interesting

V_T could be attrative,e.g. in 3P0

triplet orb. ang. mom. total J



- $-1/r^3$ dominates over kinetic energy (~ $+1/r^2$) and centrifugal barrier

  → unbounded from below, or equivalently, amplitude depends drastically on the cutoff

- NN contact interaction (counterterm) needed → 4-fermion operators

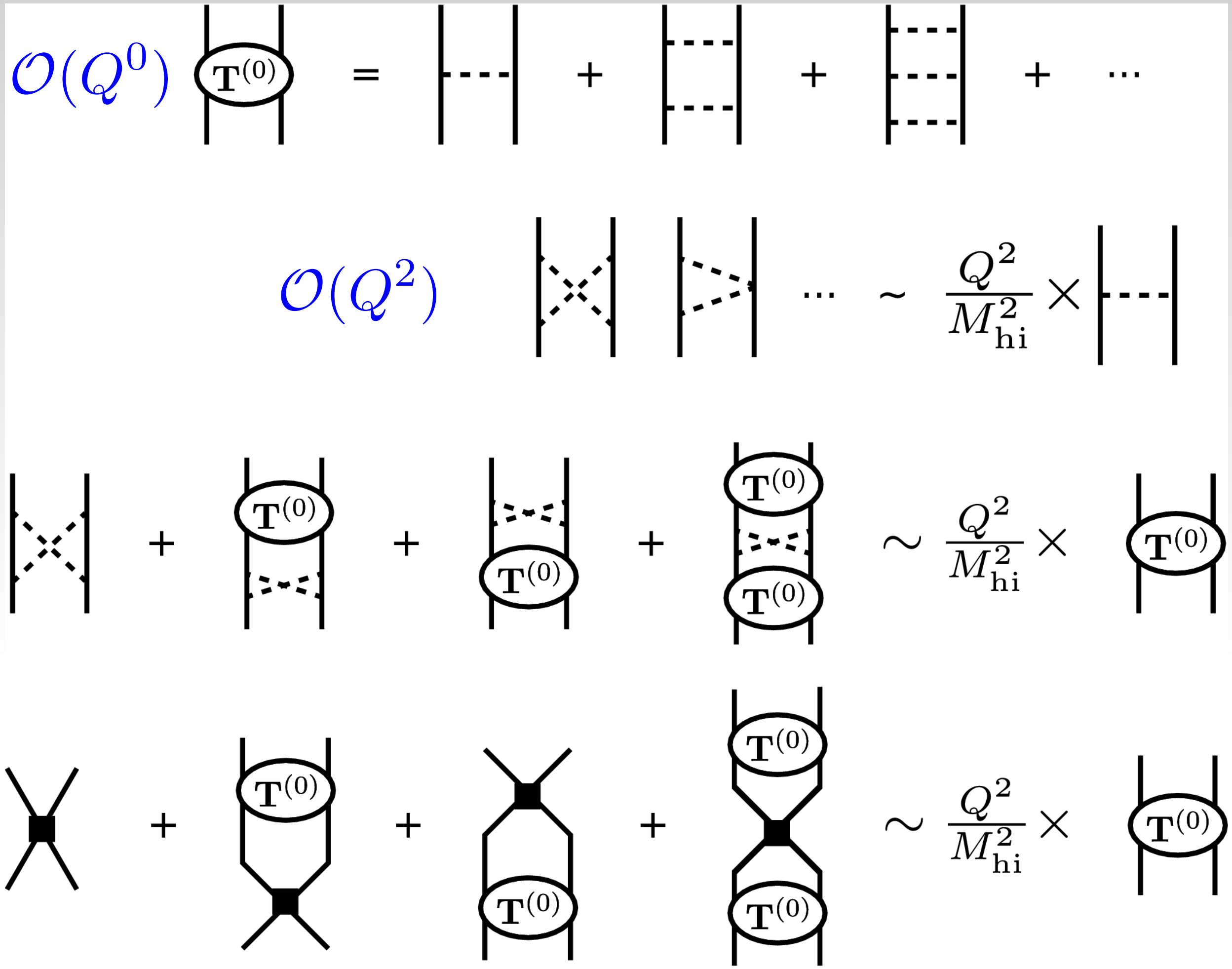
- 3P0 4-fermion operator has at least 2 derivatives, and yet has to appear in LO for renormalization purpose → not suppressed as in 1-N sector

Nogga et al (2005)

(only for illustration)

$$\mathcal{L}_{3P0} = D_0(N^\dagger \partial^2 N)(N^\dagger N) + \cdots\ ,\ D_0 \propto \frac{1}{M_{lo}^2} \qquad \cancel{D_0 \propto \frac{1}{M_{hi}^2}}$$

Subleading orders: triplet channels



$\mathcal{O}(Q)$ vanishes!

$$\mathcal{L}_{3P0} = D_0 (N^\dagger \partial^2 N)(N^\dagger N)$$

$$+ D_2 (N^\dagger \partial^4 N)(N^\dagger N) + \cdots$$

$$D_0 \propto \frac{1}{M_{lo}^2}\,, D_2 \propto \frac{1}{M_{lo}^2 M_{hi}^2}$$

- Insertion of TPE can be divergent → look for suitable counterterms to cancel
- Modified NDA → D_0, D_2(p^2) … are enhanced by the same amount

Divergence of distorted-wave expansion

for LO potential $\sim -1/r^3$,

$$\psi_k^{(0)}(r) \sim \left(\frac{\lambda}{r}\right)^{\frac{1}{4}} \left[u_0(r/\lambda) + k^2 r^2 \sqrt{\frac{r}{\lambda}} u_1(r/\lambda) + \mathcal{O}(k^4)\right]$$

$$\lambda = \frac{3 g_A^2 m_N}{8\pi f_\pi^2} \qquad u_{1,\,2}(x) \sim \mathcal{O}(1)$$

$$V_{2\pi} \sim \frac{1}{r^5} \; r \to 0$$

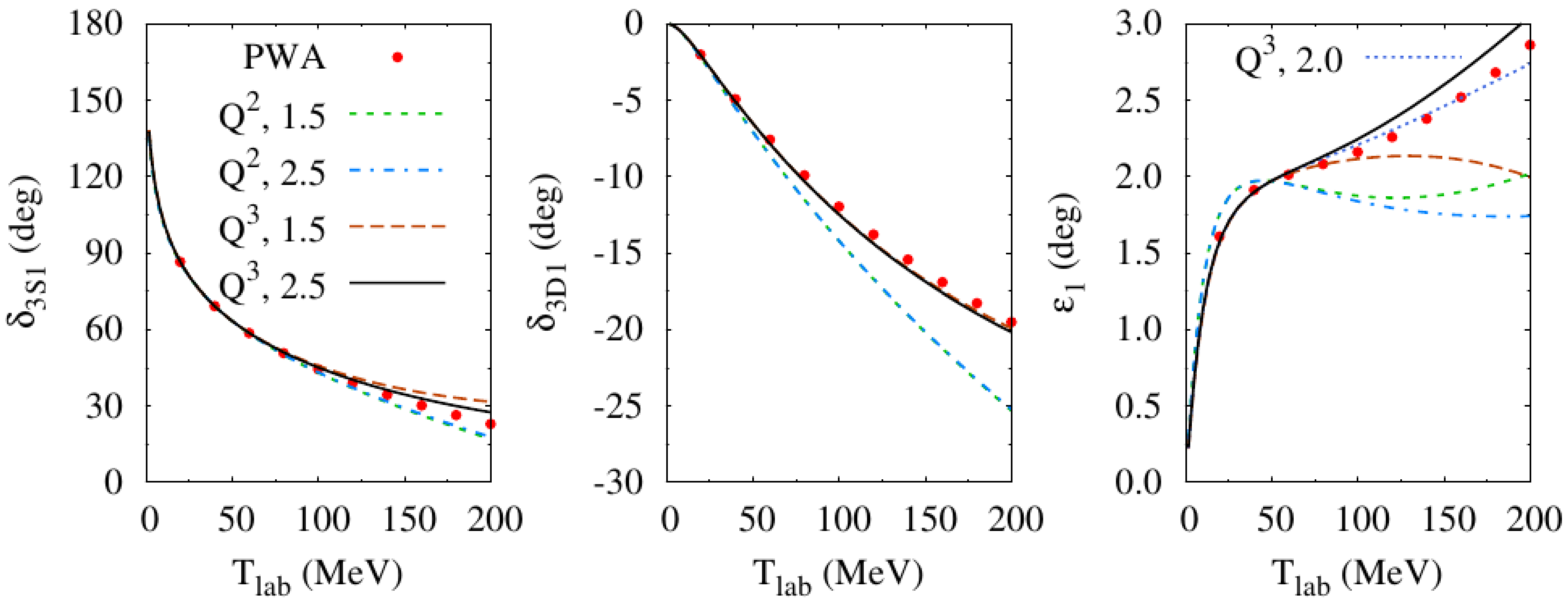
$$T^{(2)} = \langle \psi^{(0)} | V_{2\pi} | \psi^{(0)} \rangle$$

$$\sim \int_{\sim 1/\Lambda} dr r^2 |\psi^{(0)}(r)|^2 \frac{1}{r^5} \sim \alpha_0(\Lambda)\Lambda^{5/2} + \beta_0(\Lambda) k^2 + \mathcal{O}(k^4 \Lambda^{-5/2})$$

Two pieces of divergences suggest two counterterms in uncoupled channels: C & D terms in $^3\mathrm{P}_0$ ...

3S1 – 3D1 phase shifts

(BwL & Yang, 2011)



Q^2: leading TPE, Q^3: subleading TPE. "1.5": $\Lambda = 1.5$ GeV

Good agreement with partial-wave analysis up to T_lab ~ 100 MeV (k_cm ~ 200 MeV)

Summary

- Consistent power counting → meaningful theoretical error
- NDA may fail to capture short-range physics because of two mass scales
- RG invariance can constrain power-counting schemes
- Good fit to NN phase shifts up to T_lab ~ 100 MeV