Anatomy of Hadronic Parity Violation on the Lattice

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The Anatomy of Hadronic Parity Violation

• Parity Violation, Nuclear Parity Violation, Hadronic Parity Violation
  Weak interactions between quarks

• New Experiments, New Motivation
  Fundamental neutron physics beamline

• From Quarks to Nuclei
  Impossibility in non-perturbative QCD
  Opportunity

... especially for those interested in multi-hadrons

Disclaimer: P odd and CP even processes
Historical Introduction

• Parity Violation in the Weak Interaction ca. 1956

Maximal violation 100%

• Parity Violation in Nuclear Interactions

THE failure of parity conservation recently observed in β decay has raised the question of how accurately parity is conserved in nuclear reactions. A quite sensitive test is to be found in certain (p,α) reactions which are rigorously forbidden by angular momentum and parity conservation. The particular
55 Years Later: Standard Model

- Parity Violation in the Weak Interaction

\[ G_F = \frac{\sqrt{2}g^2}{8M_W^2} = 10^{-5}/\text{GeV}^2 \]

\[ \mathcal{H} = \frac{G_F}{\sqrt{2}} (\bar{u}_L \gamma_\mu d_L) (\bar{\nu}_L \gamma^\mu e_L) \]

\[ \langle p|V|n \rangle \sim g_V \quad \langle p|A|n \rangle \sim g_A \]

- Long-Range Nuclear Force from Strong Interactions

\[ \sim \left( \frac{g_A}{f_\pi} \right)^2 \frac{q \cdot \sigma_1 q \cdot \sigma_2}{q^2 + m_\pi^2} \]

**Nucleon-Nucleon Weak Interactions**

\[ G_F f_\pi^2 \sim 10^{-7} \]
Nuclear Parity Violation

• (Many) Parity Violating Nuclear reactions have been seen starting in 1967

• 1989 From one in ten million to one in ten... $^{139}$La $|P_+\rangle \rightarrow |P_+\rangle + \epsilon |P_-\rangle$

$$\epsilon = \langle P_+ \bigg| \frac{1}{E_+ - E_-} \mathcal{H}_{PV} \bigg| P_- \rangle$$

$$\Delta E/E \sim 10^{-6} \text{ for } \Delta E \sim 0.7 \text{ keV}$$

• Same ideas are being applied in Atomic Parity Violation expts. atoms, molecules, solids

• Forthcoming: $n + ^4$He $\vec{n}p \rightarrow d\gamma$

Parity Violating Nuclear Force
Panoply of Parity Violation

- PV nuclear transitions, PV photo-nuclear transitions (anapole moment), PV nucleon-nucleon interaction, PV nucleon-meson couplings, ...

- **Organize with Effective Theory mindset**

  - Weak Scale PV \( M_Z, M_W \)
  - Fermi Theory PV \( m_Q \)
  - QCD Scale PV \( \Lambda_{\text{QCD}} \)
  - Few Body

  boson exchange between quarks
  four quark operators
  below heavy quark thresholds
  non-perturbative
Panoply of Parity Violation

- PV nuclear transitions, PV photo-nuclear transitions (anapole moment), PV nucleon-nucleon interaction, PV nucleon-meson couplings, ... , ...

- **Organize with Effective Theory mindset**

\[ M_Z, M_W \]
\[ m_Q \]
\[ \Lambda_{QCD} \]
\[ h_{\gamma N}, h_{\pi N}, h_{NN}, \epsilon_{PV} \]

**Weak Scale PV**
**Fermi Theory PV**
**QCD Scale PV**

- hadronic PV
- NN PV
- Nuclear PV

**Few Body**
**Many Body**

**Non-perturbative**

**Boson exchange between quarks**

**Four quark operators below heavy quark thresholds**
Parallel to Parity Violation

Lattice QCD: Connect Quarks to Hadrons, Few Body
Quantum Many Body: Connect Few Body to Nuclei

- Organize with Effective Theory mindset

μ

QCD Scale PV

Λ_{QCD}

hadronic PC

NN PC

Nuclear PC

\begin{align*}
g_V \\
g_A \\
\epsilon_d \\
\epsilon_N
\end{align*}

non-perturbative

\begin{align*}
h_{\gamma N} \\
h_{\pi N} \\
h_{NN} \\
\epsilon_{PV}
\end{align*}

Few Body

Many Body
Parallels to Parity Violation

- Organize with Effective Theory mindset

Weak Scale PV $M_Z, M_W$
Fermi Theory PV
QCD Scale PV $m_Q$
- hadronic PV $h_{\gamma N}$
- NN PV $h_{\pi N}$
- Nuclear PV $h_{NN}$

$\Lambda_{QCD}$

boson exchange between quarks below heavy quark thresholds
four quark operators non-perturbative

$\mu$ 0 Few Body Many Body
Parallels to Parity Violation

- Organize with Effective Theory mindset

Perturbative QCD: Connect Standard Model to QCD scale
Lattice QCD: Connect Four Quark Ops. to Observables

Four-Quark Correlations
deep inside hadrons

\[ \Delta F = 1, 2 \]

\[ \Delta I = 1/2 \]

\[ K \rightarrow \pi\pi \]

\[ \sim 1/100 \text{ fm} \]
Hadronic Parity Violation in QCD

Perturbative QCD: Connect Standard Model to QCD scale
Lattice QCD: Connect Four Quark Ops. to Observables

• QCD renormalization of PV

\[ \mathcal{L}_{PV}^{I=1} = \sum_i C_i(\mu)O_i(\mu) \]

\[ \langle p|\mathcal{L}_{PV}^{I=1}|\pi n\rangle = h_{\pi}^1 \]

• (First) Lattice QCD calculation of PV

B Tiburzi, PRD 85 054020 (2012)

J Wasem, PRC 85 022501(R) (2012)

• In tandem: program to remove model dependence in NN, NNN, ...

Zhu Maekawa Holstein Ramsey-Musolf van Kolck, Phillips Schindler Springer Grießhammer, Shin Ando Hyun, Vanasse, ...
Isovector Parity Violation in QCD

Why Isovector?

- QCD renormalization of PV
- (First) Lattice QCD calculation of PV

\[ \mathcal{L}_{PV}^{I=1} = \sum_i C_i(\mu) \mathcal{O}_i(\mu) \]

\[ \langle p | \mathcal{L}_{PV}^{I=1} | \pi n \rangle = h_\pi^{1} \]

\[ \sim \frac{h_\pi^{1}}{f_\pi^{2}} \frac{g_A}{q^2 + m_\pi^2} \]

Alleged: Longest range piece of PV NN interaction
Isovector Parity Violation in QCD

Why Isovector?

• QCD renormalization of PV

\[ L_{PV}^{I=1} = \sum_i C_i(\mu) O_i(\mu) \]

\[ W^\pm : \Delta I = 0, 2 \propto |V_{ud}|^2 \]
\[ \Delta I = 1 \propto |V_{us}|^2 \]

\[ Z^0 : \Delta I = 0, 1, 2 \]

• (First) Lattice QCD calculation of PV

\[ \langle p | L_{PV}^{I=1} | \pi n \rangle = h_\pi^1 \]

\[ J_{\mu}^{W^-} = \bar{U}_L \gamma_\mu V D_L \]

\[ J_{\mu}^{Z^0} = \frac{1}{c_W} \left[ \bar{\Psi}_L \gamma_\mu T_3 \Psi_L - s_W^2 \bar{\Psi} \gamma_\mu Q \Psi \right] \]

Alleged: 95% probe of hadronic neutral current
QCD Renormalization of Isovector Parity Violation

Alleged: 95% probe of hadronic neutral current

\[ \mathcal{L}^{I=1}_{\text{PV}} = \frac{G_F}{\sqrt{2}} \frac{1}{3} s_W^2 (\bar{u}u - \bar{d}d)_A (\bar{u}u + \bar{d}d)_V \]

Tree Level

\[ \log \frac{M_Z^2}{p^2} \]

\[ \log \frac{\mu^2}{p^2} \]

One Loop

\[ \log \frac{M_Z^2}{p^2} = \log \frac{\mu^2}{p^2} - \log \frac{\mu^2}{M_Z^2} \]

\[ \mathcal{L}^{I=1}_{\text{PV}} = \sum_i C_i(\mu) \mathcal{O}_i(\mu) \]

\[ \delta C(\mu) \sim -\alpha_s(\mu) \log \frac{\mu^2}{M_Z^2} \]

Sum leading logs

\[ \mu \frac{d}{d \mu} \tilde{C} = \frac{\alpha_s}{4\pi} \gamma^T \cdot \tilde{C} \]
QCD Renormalization of Isovector Parity Violation

\[ \alpha_s(1 \text{ GeV}) \sim 0.4 \]

Sum NLL

... renormalization scheme dependence (Good!)
QCD Renormalization of Isovector Parity Violation

Renormalization scheme: dimensional regularization

\[ \gamma_\mu \gamma_5 \otimes \gamma^\mu \]

\[ \gamma_\nu \gamma_\rho \gamma_\mu \gamma_5 \otimes \gamma^\nu \gamma^\rho \gamma^\mu \]

Four dimensions affords simplification

\[ \gamma_\nu \gamma_\rho \gamma_\mu = g_{\nu \rho} \gamma_\mu - g_{\nu \mu} \gamma_\rho + g_{\rho \mu} \gamma_\nu - i \varepsilon_{\nu \rho \mu \sigma} \gamma^\sigma \gamma_5 \]

Enlarge operator basis to include mixing with evanescent operators

\[ \Delta F \neq 0 \]

Experts: Buras Jamin Lautenbacher Weisz, Ciuchini Franco Lubicz Martinelli Reina Scimemi Silvestrini

(PV in Lattice QCD requires different scheme than dim. reg. ... one-loop matching = bookkeeping)
QCD Renormalization of Isovector Parity Violation

Renormalization scheme: dimensional regularization

E.g.

\[ \Delta S = 1 \]

\[ Q_1 = (\bar{s}d)_{V-A}(\bar{u}u)_{V-A} \]

Mass independent scheme & QCD flavor blind!

\[ \text{W-exchange} \]

\[ U\text{-spin} \]

\[ (\bar{s}s - \bar{d}d)_{V-A}(\bar{u}u)_{V-A} \]

\[ V\text{-spin} \]

\[ (\bar{u}u - \bar{d}d)_{V-A}(\bar{s}s)_{V-A} \]

Parity invariance

\[ \Delta I = 1 \]

\[ O = (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)_L(\bar{s}\gamma^\mu s)_L - (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)_R(\bar{s}\gamma^\mu s)_R \]

\[ \text{Z-exchange} \]

Full set of 5 operators

\[ \Delta I = 1 \]

follows from \[ \Delta S = 1 \] including QED penguins, and BSM operators

(remaining combinations of \( L \otimes R \))

(need orthogonal flavor combinations)

\[ \ldots \text{just different initial conditions in evolution} \]
QCD Renormalization of Isovector Parity Violation

Results ('t Hooft-Veltman scheme)

\[
\mathcal{L}_{PV}^{I=1} = \sum_i C_i(\mu) O_i(\mu)
\]

Non-Strange vs. Strange

\[O_1 = (\bar{u}u - \bar{d}d)_A (\bar{u}u + \bar{d}d)_V,\]
\[O_2 = (\bar{u}u - \bar{d}d)_A [\bar{u}u + \bar{d}d]_V,\]
\[O_3 = (\bar{u}u - \bar{d}d)_V (\bar{u}u + \bar{d}d)_A,\]
\[O_4 = (\bar{u}u - \bar{d}d)_V [\bar{u}u + \bar{d}d]_A,\]
\[O_5 = (\bar{u}u - \bar{d}d)_A (\bar{s}s)_V,\]
\[O_6 = (\bar{u}u - \bar{d}d)_A [\bar{s}s]_V,\]
\[O_7 = (\bar{u}u - \bar{d}d)_V (\bar{s}s)_A,\]
\[O_8 = (\bar{u}u - \bar{d}d)_V [\bar{s}s]_A.\]

Alleged: 95% probe of hadronic neutral current

\[\sin^2 \theta_W\]

\[\begin{array}{ccccc}
\text{No}\text{nt-Strange} & \text{vs.} & \text{Strange} \\
1 & \text{vs.} & 1 & 80 - 100\% & \text{Dynamical Question!}
\end{array}\]

\[C_i(\mu = 1 \text{ GeV})\]

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<th>(i)</th>
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<th>NLO (Z + W)</th>
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</tr>
</tbody>
</table>


B Tiburzi, PRD 85 054020 (2012)
First Lattice Computation (J Wasem) \( \text{PRC } 85 \ 022501(R) \ (2012) \)

\[ p \sim \mathcal{O}_p(x) \quad (\pi n)_{s\text{-wave}} \sim \gamma_5 \mathcal{O}_p(x) \]

\[
\langle p | \mathcal{L}^{I=1}_{PV} | \pi n \rangle = h_\pi^1
\]

\[
h_{\pi NN}^{1,\text{con}} = (1.099 \pm 0.505^{+0.058}_{-0.064}) \times 10^{-7}
\]

Easy to pick on a first calculation (much harder to have done it, or improve it!)

- Multi-hadron overlap? \( \pi n \)
- No Lellouch-Lüscher factor
- Ignores regularization scheme
- Inexact kinematics \( \partial^2_\tau \) vs. \( m^2_\pi \)

Wilson coefficients of strange operators are enhanced \( 5 \times \)

Strangeness not-so suppressed \( N_c^{-1/2} \)
Auxiliary Fields for Isovector Parity Violation

- Perhaps only a Gedankenexperiment until exascale computers materialize

\[ \mathcal{O} = (\bar{q} \gamma_\mu \gamma_5 \tau^3 q) (\bar{q} \gamma_\mu q) \rightarrow -a [\bar{q} \gamma_\mu (\gamma_5 \tau^3 - b \cdot 1) q]^2 P \otimes \tau^1 \]

\( \tau^3 \)-chiral symmetry

Introduces PC and PV four-quark operators

Integrate in auxiliary field

\[ \Delta \mathcal{L} = \sigma^2 + i a \sigma [\bar{q} \gamma_\mu (\gamma_5 \tau^3 - b \cdot 1) q] \]

No sign problem \( \gamma_5 \otimes \tau^1 \)-Hermiticity

- Can implement all isovector PV operators in sign-problem-free ways

Continuum limit, parameter tuning (!?!?)

\[ \langle p | \mathcal{L}^{I=1}_{PV} | \pi n \rangle = h_\pi \quad \rightarrow \quad \langle p | \pi^+ (x) | n \rangle_\sigma \]

Other PV observables:

Nucleon anapole moment: just calculate anapole form factor

PV NN interactions from PV two-point functions

Bodies buried in gauge field generation
Isotensor Parity Violation

\[ \mathcal{O} = \langle \bar{q} \tau^3 q \rangle_A \langle \bar{q} \tau^3 q \rangle_V - \frac{1}{3} \langle \bar{q} \tau^3 q \rangle_A \cdot \langle \bar{q} \tau^3 q \rangle_V \]

- Only one operator & without self-contractions

\[ \mathcal{L}^{\Delta I=2}_{PV} = \frac{G_F}{\sqrt{2}} C'(\mu) \mathcal{O}(\mu) \]

Renormalization by bookkeeping

\[ \Delta I = 2 \longleftrightarrow \Delta S = 2 \]

Better proving ground for Lattice QCD?

\[ \mathcal{L}_{NN} = [\vec{n} p^\dagger \cdot \vec{\sigma} \sigma_2 p^*] \cdot [n^T \sigma_2 n] + \ldots \]

- s- to p-wave NN interaction

Operator matrix element between two hadrons (... bound states currently!)

\[ \pi N \text{ interactions} \]

\[ \mathcal{L}_{\pi\pi N} + \mathcal{L}_{\pi\gamma N} \]

External fields could "substitute" for hadrons

\[ \pi PV \]

Isotensor three-pion vertex exists (\(\pi PV\) very suppressed in other channels)
Anatomy of Parity Violation

- New neutron experiments will constrain PV in few-body systems
- Connecting few-body PV to many-body PV stringent test of methods NMB/NNEFT
- Connecting PV four quark operators to PV couplings between hadrons: test of non-pQCD
- Connection of nuclear PV to Standard Model