

Chiral Dynamics and Peripheral Transverse Densities

*Charge and Current Densities, Energy Momentum
Tensor and Orbital Angular Momentum*

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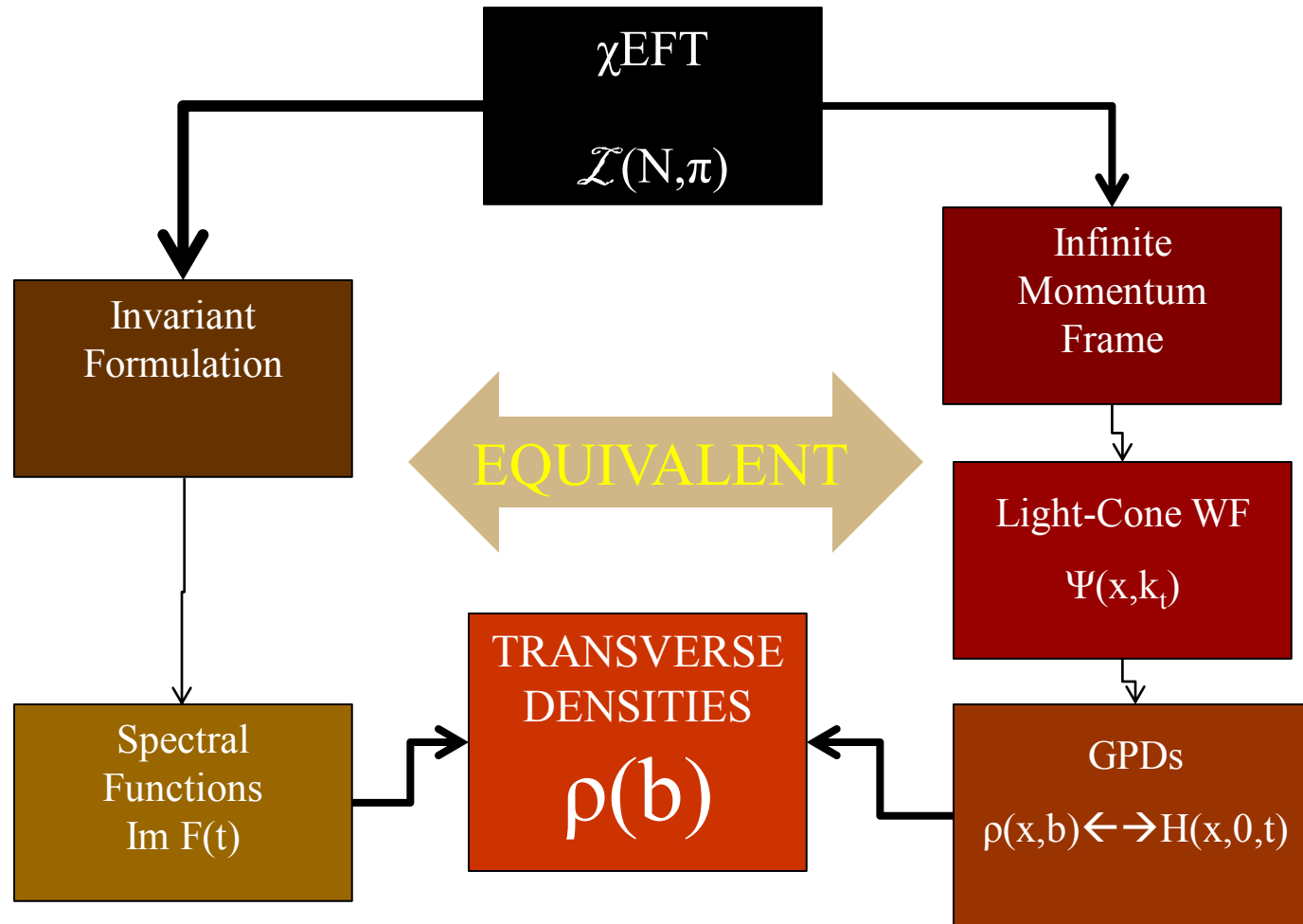
Aim and Context

- Spatial representation of hadrons as relativistic systems
 - GPDs, Transverse Densities
- Universality in large distance dynamics:
 - Chiral symmetry breaking, effective field theory
- Study of chiral periphery of transverse nucleon structure
 - Charge and current densities, EM form factors
 - Matter density and angular momentum, energy momentum tensor and GPDs

Motivation

- Methodology
 - Reveal spatial structure of χ EFT:
 - M_π^{-1} vs. short distance contributions
 - Explore different formulations of orbital angular momentum in field theory applied to a π N system
- Practical
 - Calculate model independent chiral components of the nucleon structure
 - Constrain form factors, peripheral GPDs
- Experiment
 - Form factors measurements in the low Q^2 region
 - JLab E12-11-106 $Q^2 \sim 10^{-2} - 10^{-4} \text{ GeV}^2$
 - Connect chiral dynamics with Peripheral Processes in High Energy ep and pp Reactions: EIC, LHC

Methodology



Transverse Charge and Current Densities

Definition

- Fourier Transform in Transverse Momentum

$$\rho(b) \equiv \int \frac{d^2 \Delta_T}{(2\pi)^2} e^{-i(\Delta_T \cdot \mathbf{b})} F(-\Delta_T^2) \longrightarrow \rho(b) = \int_0^\infty \frac{d\Delta}{2\pi} J_0(\Delta b) F(t = -\Delta^2)$$

- From Electromagnetic (EM) Form Factors (FF)

$$\langle N_2 | \mathbf{J}^\mu | N_1 \rangle = \bar{U}_2 \left[\gamma^\mu \mathbf{F}_1(\mathcal{A}^2) + i \sigma^{\mu\nu} \frac{\Delta_\nu}{2M} \mathbf{F}_2(\mathcal{A}^2) \right] U_1$$

- From Energy-Momentum Tensor (EMT) FF

$$\begin{aligned} \langle N_2 | \Theta_{\mu\nu}^{N\pi} | N_1 \rangle = & \bar{U}_2 \left[\gamma_{(\mu} P_{\nu)} \mathbf{A}(\mathcal{A}^2) + P_{(\mu} i \sigma_{\nu)\alpha} \frac{\Delta_\alpha}{2M} \mathbf{B}(\mathcal{A}^2) \right. \\ & \left. + \left(\frac{\Delta_\mu \Delta_\nu - \Delta^2 g_{\mu\nu}}{M} \right) \mathbf{C}(\mathcal{A}^2) + M g_{\mu\nu} \tilde{\mathbf{C}}(\mathcal{A}^2) \right] U_1 \end{aligned}$$

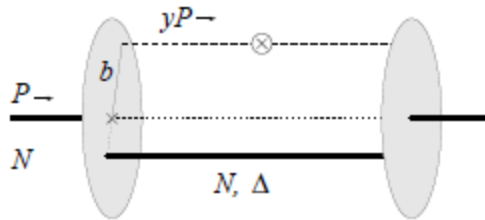
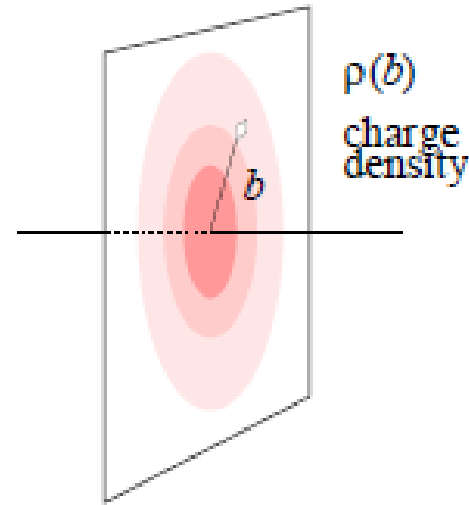
Transverse Charge and Current Densities

Definition

$$\rho(b) \equiv \int \frac{d^2 \Delta_T}{(2\pi)^2} e^{-i(\Delta_T \cdot b)} F(-\Delta_T^2)$$

G.Miller(PRL99)

$$|x_{iT}\rangle \equiv \int \frac{d^2 p_T}{(2\pi)^2} e^{-ip_T x_{iT}} |p_T\rangle$$



Parton current picture

$$\langle N_2 | J^\mu | N_1 \rangle = \langle N_2 | \left[\gamma^\mu F_1(\Delta^2) + i\sigma^{\mu\nu} \frac{\Delta_\nu}{2M} F_2(\Delta^2) \right] | N_1 \rangle$$

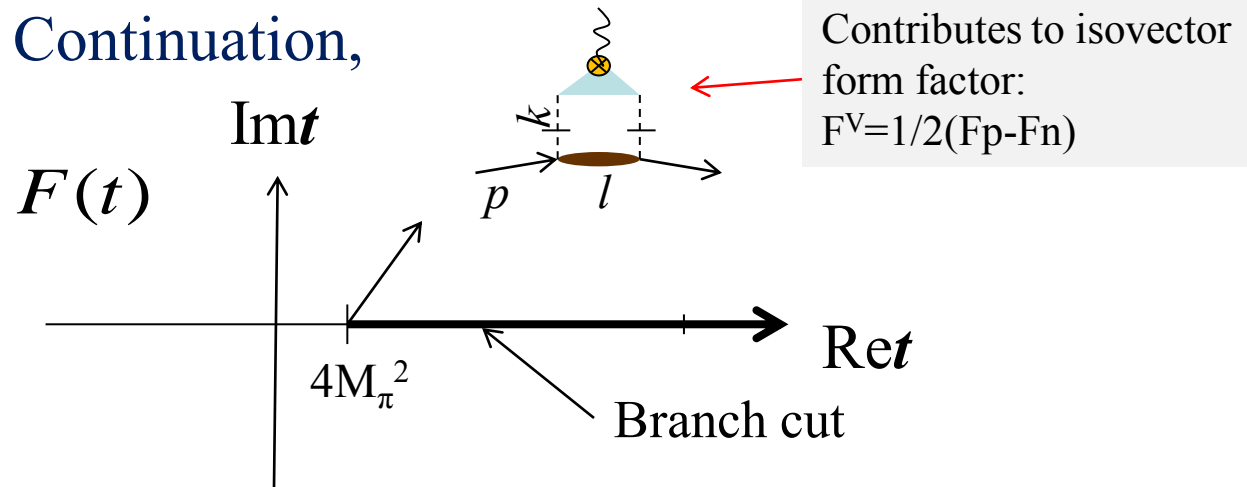
$$\langle P, x_{2T}, \sigma_2 | J^0(0, x_T, 0) | P, x_{1T}, \sigma_1 \rangle = [2P\delta^{(2)}(x_{2T} - x_{1T})] \delta_{\sigma_1\sigma_2} \rho_1(b)$$

$$\langle P, x_{2T}, \sigma_2 | J^3(0, x_T, 0) | P, x_{1T}, \sigma_1 \rangle = [\dots] \frac{S_{\sigma_1\sigma_2}}{M_N} \left(e_3 \times \frac{\partial}{\partial x} \right) \rho_2(b)$$

Transverse Charge and Current Densities

Dispersion Representation, Spectral Functions

- Form Factors
 - Analytic Continuation,



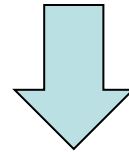
- Dispersion Relation : For $\text{Re } t < 0$,

$$F(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{t' - t} \frac{\text{Im } F(t' + i0)}{\pi}$$

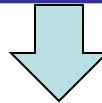
Transverse Charge and Current Densities

Dispersion Representation, Spectral Functions and Analytic Structure Near Threshold

$$F(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{t'-t} \frac{\text{Im} F(t'+i0)}{\pi}$$



$$\rho(b) = \int_0^{\infty} \frac{d\Delta}{2\pi} J_0(\Delta b) F(t = -\Delta^2)$$



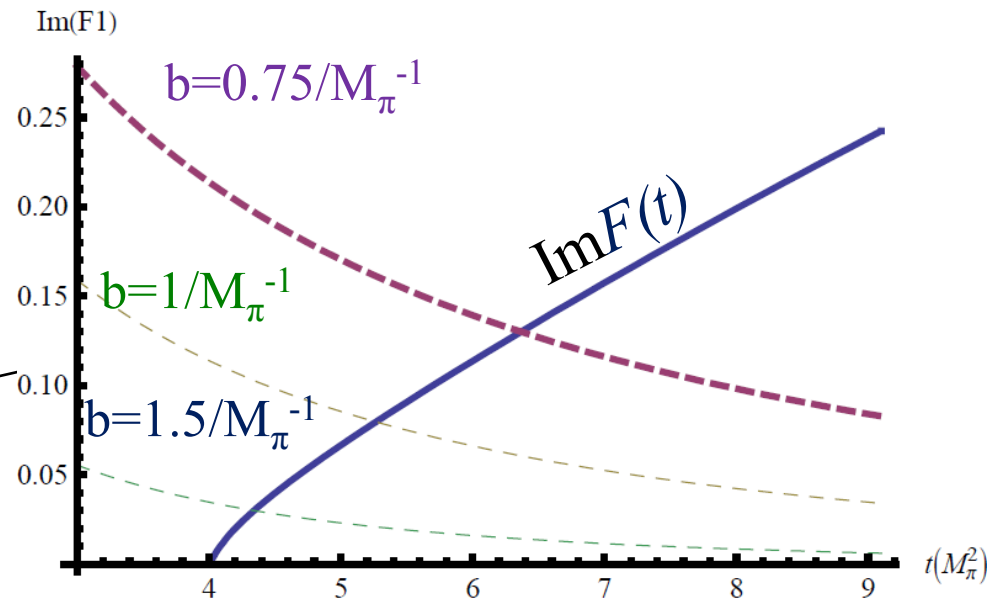
$$\rho(b) = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{tb}) \frac{1}{\pi} \text{Im} F(t + i0)$$

Transverse Charge and Current Densities

Dispersion Representation, Spectral Functions

$$\rho(b) = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{tb}) \frac{1}{\pi} \text{Im} F(t+i0)$$

$$K_0(\sqrt{tb}) \approx \sqrt{\frac{\pi}{2}} \frac{e^{-\sqrt{tb}}}{(\sqrt{tb})^{1/2}}$$

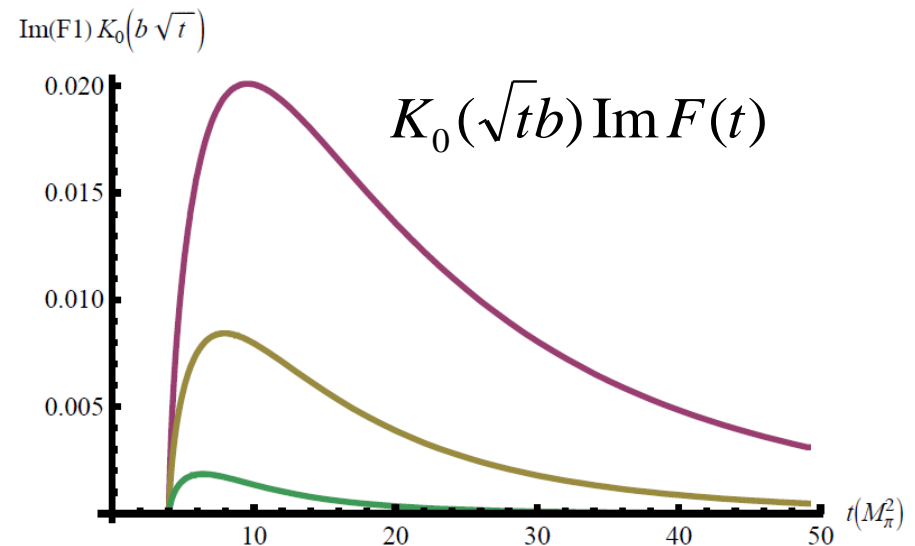
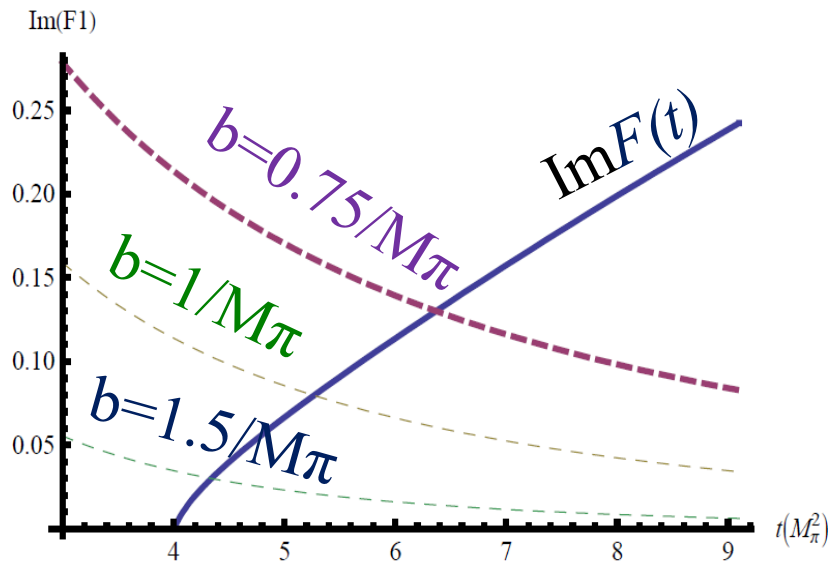


- As b grows, $F(t)$ is sampled closer to threshold ($t_{\text{thr}}=4M_\pi^2$)

Transverse Charge and Current Densities

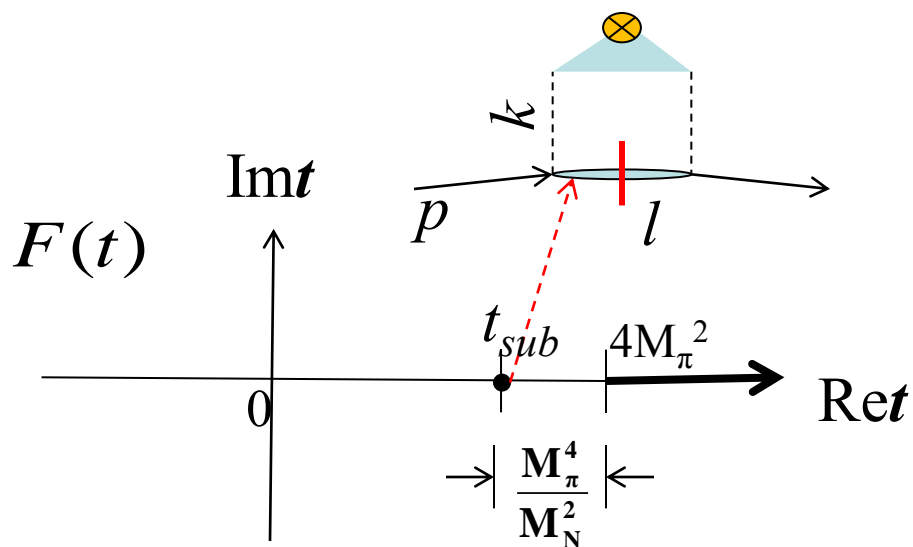
Dispersion Representation, Spectral Functions

- For large b , Transverse densities are dominated by near threshold values of spectral functions



Transverse Charge and Current Densities

Analytic Structure Near Threshold



$$\sim \frac{1}{l^2 - M_N^2 + i0} = -\frac{1}{A(t) - iB(t) \cos \theta}$$

$$\text{Im } F = \int_{-1}^1 \frac{f(t, \cos \theta)}{A(t) - iB(t) \cos \theta} d(\cos \theta)$$

$$A(t_{sub}) = iB(t_{sub}) \Rightarrow t_{sub} = 4M_\pi^2 - \frac{M_\pi^2}{M_N^2}$$

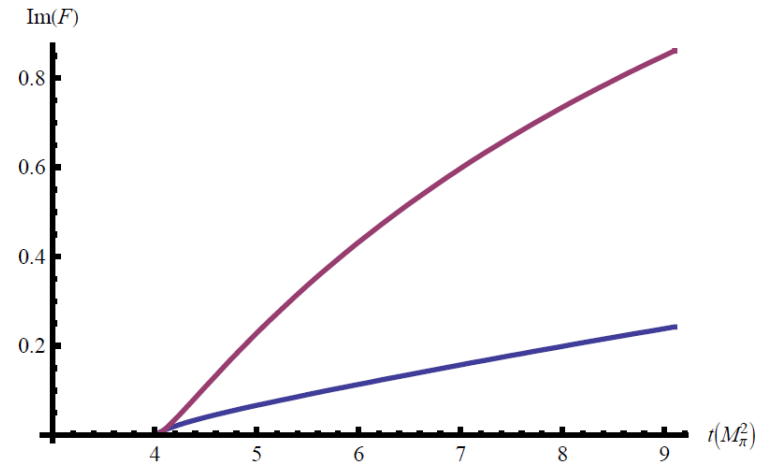
- Sub-threshold singularity
 - End-point singularity
 - Intermediate nucleon on-shell
 - Limits convergence of expansion near threshold
 - Controls large $b(\sim M_N^2 M_\pi^{-3})$ behavior of transverse densities

Transverse Charge and Current Densities

Parametric Regions

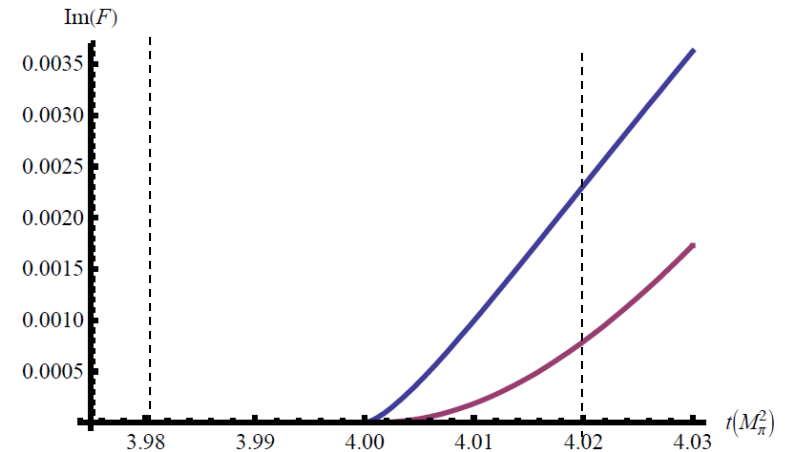
- Chiral Region

$$- \Delta b \sim \frac{1}{(\Delta t_\chi)^{1/2}} \sim \frac{1}{M_\pi} \approx 1.5 \text{fm}$$



- Molecular Region

$$- \Delta b > \frac{M_N^2}{M_\pi^2} \frac{1}{M_\pi} \approx 56 \frac{1}{M_\pi} \approx 90 \text{fm}$$



Peripheral Densities from Invariant χ PT

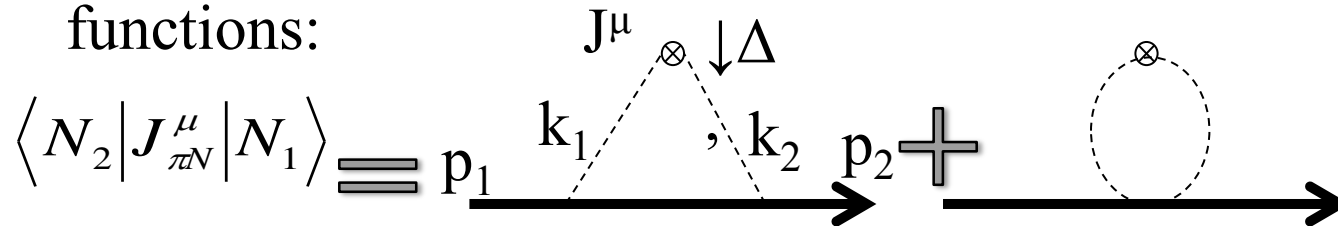
- Chiral EFT Lagrangian

- Relativistic formulation of pion-nucleon dynamics

$$\mathcal{L}_{int} = -\frac{g_A}{2F_\pi} \bar{\psi} \gamma^\mu \gamma_5 \tau^a \psi \partial_\mu \pi^a - \frac{1}{4F_\pi^2} \bar{\psi} \gamma^\mu \tau^a \psi \varepsilon^{abc} \pi^b \partial_\mu \pi^c + \dots$$

- Axial Vector coupling and contact terms

- EM current. Leading π contributions to isovector spectral functions:



$$\langle N_2 | J^\mu | N_1 \rangle = \bar{U}_2 \left[\gamma^\mu F_1(\Delta^2) + i \sigma^{\mu\nu} \frac{\Delta_\nu}{2M} F_2(\Delta^2) \right] U_1$$

Peripheral Densities from Invariant χ PT

$$F_1 = \frac{2}{F_\pi^2} (1 - g_A^2) I^{(1)} + \frac{8M_N^2 g_A^2}{F_\pi^2} I^{(2)}$$

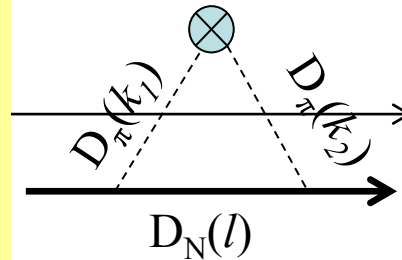
$$F_2 = \frac{8M_N^2 g_A^2}{F_\pi^2} I^{(3)}$$

$$\boxed{\begin{aligned} k &\equiv \frac{k_1 + k_2}{2} \\ P &\equiv \frac{p_1 + p_2}{2} \end{aligned}}$$

- Find Spectral Functions ($\text{Im}F(t)$) instead,
 - Cutkosky Rules
 - Pion on mass-shell

$$I^{(1)} = i \int \frac{d^4 k}{(2\pi)^2} D_\pi(k_1) D_\pi(k_2) N^{(1)}$$

$$I^{(2,3)} = i \int \frac{d^4 k}{(2\pi)^2} D_\pi(k_1) D_\pi(k_2) D_N(l) N^{(2,3)}$$



$$\frac{1}{k_{1,2}^2 - m_\pi^2 + i\epsilon} \rightarrow -2\pi i \delta(k_{1,2}^2 - m_\pi^2)$$

$$N^{(1)} = \frac{1}{3} \left[k^2 - \frac{(k\Delta)^2}{\Delta^2} \right]$$

$$N^{(2)} = \frac{1}{2} \left[k^2 - \frac{(kP)^2}{P^2} - \frac{(k\Delta)^2}{\Delta^2} \right] + \frac{1}{2} \left[-k^2 + 3 \frac{(kP)^2}{P^2} + \frac{(k\Delta)^2}{\Delta^2} \right] \frac{M_N^2}{P^2}$$

$$N^{(3)} = -\frac{1}{2} \left[-k^2 + 3 \frac{(kP)^2}{P^2} + \frac{(k\Delta)^2}{\Delta^2} \right] \frac{M_N^2}{P^2}$$

$$I(t) = -i \int \frac{d^4 k}{(2\pi)^2} D_\pi(k_1) D_\pi(k_2) \Phi(k)$$

$$\frac{1}{\pi} \text{Im} I(t) = \frac{k_{cm}^3}{16\pi^2 \sqrt{t}} \int_{-1}^1 d(\cos \theta) \Phi(k^0 = 0, |\mathbf{k}| = k_{cm})$$

$$\boxed{k_{cm} = \sqrt{\frac{t}{4} - M_\pi^2}}$$

Peripheral Densities from Invariant χ PT

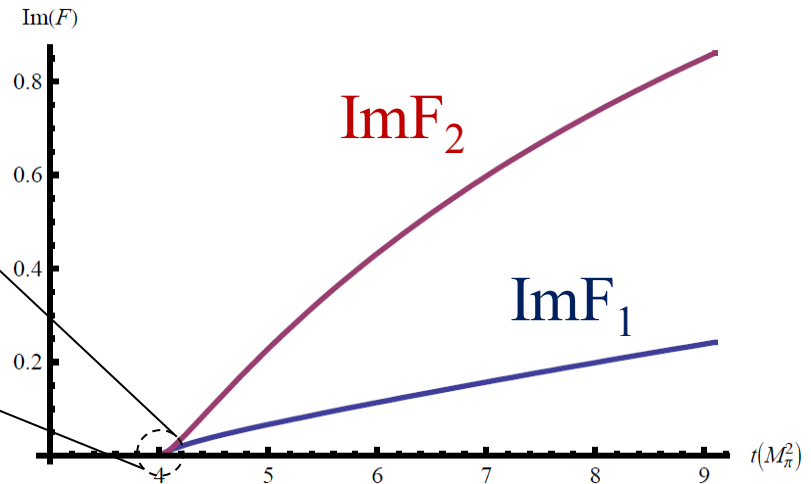
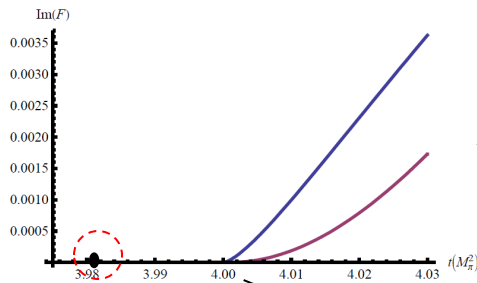
- Spectral Functions

$$\frac{\text{Im}F_1}{\pi} = 2g_{\pi N}^2 \frac{\left(\frac{t}{2} - M_\pi^2\right)^2}{(4\pi)^2 (P^2)^{\frac{5}{2}} \sqrt{t}} \left[-\frac{t}{8} x^2 \arctan(x) + \left(M_N^2 + \frac{t}{8}\right) (x + \arctan(x)) \right] + \frac{(1 - g_A^2)(t - 4M_\pi^2)^{\frac{3}{2}}}{6(4\pi F_\pi)^2}$$

$$\frac{\text{Im}F_2}{\pi} = g_{\pi N}^2 \frac{\left(\frac{t}{2} - M_\pi^2\right)^2}{(4\pi)^2 (P^2)^{\frac{5}{2}} \sqrt{t}} [(x^2 + 3)\arctan(x) - 3x]$$

$$x = \frac{2\sqrt{M_N^2 - \frac{t}{4}} \sqrt{\frac{t}{4} - M_\pi^2}}{\frac{t}{2} - M_\pi^2}$$

Strikman, Weiss, PRC82 (2010) 042201



- Sub-threshold singularity at $x(t_{sub}) = \pm i$

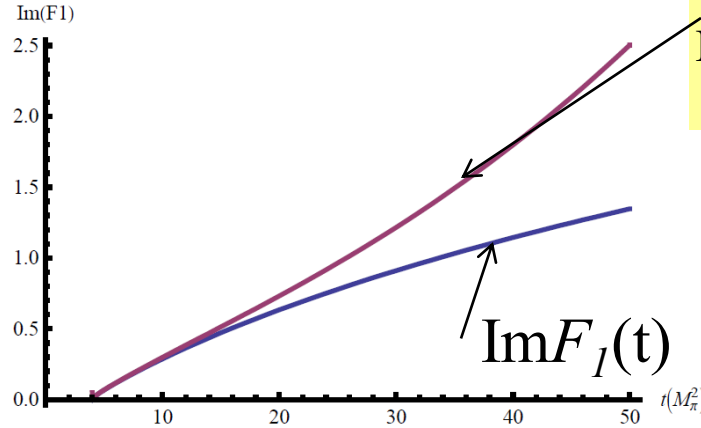
$$\rightarrow t_{sub} = 4M_\pi^2 - \frac{M_\pi^2}{M_N^2}$$

Peripheral Densities from Invariant χ PT

Heavy Baryon Expansion

$$\varepsilon \equiv \frac{M_\pi}{M_N}$$

$$\varepsilon \ll 1, \quad \frac{t}{M_\pi^2} \sim O(0)$$



$$\text{Im} F_1^{HB}(t) = \pi \frac{g_A^2 M_\pi^2}{(4\pi F_\pi)^2} \left[\sum_{i=0}^{\infty} \varepsilon^i C_i \left(\frac{\sqrt{t}}{2M_\pi} \right) \right]$$

$$C_0(\tau) = \frac{1}{\tau\sqrt{\tau^2-1}} \left[\tau^4 - \frac{3}{2}\tau^2 + \frac{1}{2} \right]$$

$$C_1(\tau) = -\frac{\pi}{2\tau^2} \left[2\tau^5 - 2\tau^3 + \frac{1}{4}\tau \right]$$

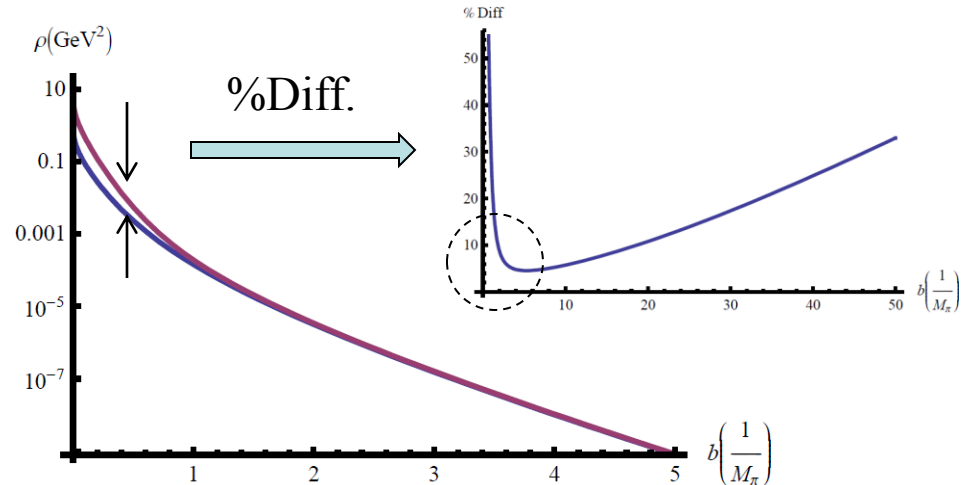
$$C_2(\tau) = \frac{1}{\tau\sqrt{\tau^2-1}} \left[4\tau^6 - 3\tau^4 + \frac{7}{4}\tau^2 - \frac{1}{8} \right]$$

$$\rho_1^{HB}(b) = \frac{1}{2\pi} \frac{4g_A^2 M_\pi^2}{(4\pi F_\pi)^2} \left[\sum_{i=0}^{\infty} \varepsilon^i f_i(2M_\pi b) \right]$$

$$f_0(\beta) = \frac{1}{16}(K_2(\beta))^2 + \frac{1}{8}(K_1(\beta))^2 + \frac{1}{16}(K_0(\beta))^2$$

$$f_1(\beta) \approx -\left(\frac{\pi}{2}\right)^{\frac{3}{2}} \left[2\beta^{-5}\Gamma\left(\frac{9}{2}, \beta\right) + 2\beta^{-3}\Gamma\left(\frac{5}{2}, \beta\right) + \frac{1}{4}\beta^{-1}\Gamma\left(-\frac{1}{2}, \beta\right) \right]$$

$$f_2(\beta) = \frac{1}{16}(K_3(\beta))^2 + \frac{3}{16}(K_2(\beta))^2 + \frac{7}{2}(K_1(\beta))^2 + \frac{1}{2}(K_0(\beta))^2$$



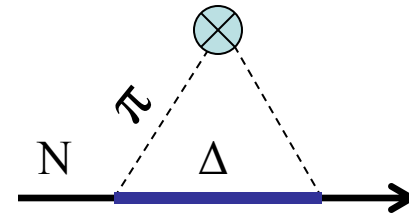
Peripheral Densities from Invariant χ PT

Contributions from Δ and Large N_c Limit

- Consistency with QCD in the Large N_c Limit

– At Large N_c

- $M_\pi \sim N_c^0$, $M_{N,\Delta} \sim N_c^1$
- $g_{\pi NN} \sim N_c^{3/2}$, $g_{\pi N\Delta} \sim N_c^{3/2}$
- $\rho \sim N_c^0$



- But in the Large N_c Limit, considering only nucleonic intermediate state χ PT contributions to $F(t)$ lead to

$$\rho_{(N\pi)N} \approx A_N N_c + B_N N_c^0$$

- Contributions from Δ -intermediate states remedy this discrepancy ,

$$\rho_{(\Delta\pi)N} \approx -A_N N_c + B_\Delta N_c^0$$

$$\rho_N = \rho_{(N\pi)N} + \rho_{(\Delta\pi)N}$$

$$\approx (B_N + B_\Delta) N_c^0$$

Peripheral Densities in Light-Front χ PT

- Develop a partonic formulation of chiral dynamics
 - Charge and current of pions in the chiral periphery
 - Orbital angular momentum decomposition of chiral π -N LCWF
- Demonstrate equivalence with invariant formalism
- Connect to GPD formalism
 - Compute model-independent χ GPDs

Peripheral Densities in Light-Front χ PT

- Form factors from the infinite momentum frame

$$F_1 = \left\langle P_{2,+} \left| \frac{J^3}{2P} \right| P_{1,+} \right\rangle$$

$$-\frac{\Delta_L}{2M} F_2 = \left\langle P_{2,+} \left| \frac{J^3}{2P} \right| P_{1,-} \right\rangle$$

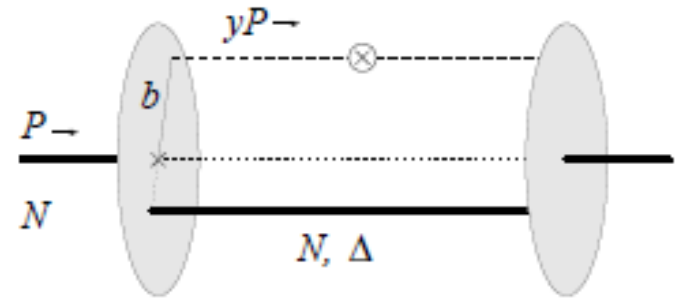
$$F_1 = 2 \int \frac{d^2 k_{\perp}}{(2\pi)^2} \frac{dy}{y(1-y)} \sum_{\lambda'} \Psi_{\pi N}^{\dagger}(y, k'_T, +, \lambda') \Psi_{\pi N}(y, k_T, +, \lambda') + \delta(y)(1-g_A^2) \mathbf{ct},$$

$$-\frac{\Delta_L}{2M} F_2 = 2 \int \frac{d^2 k_{\perp}}{(2\pi)^2} \frac{dy}{y(1-y)} \sum_{\lambda'} \Psi_{\pi N}^{\dagger}(y, k'_T, +, \lambda') \Psi_{\pi N}(y, k_T, -, \lambda')$$

- LCWF from $N\pi N$ pseudo scalar coupling

$$\Psi_{\pi N}(y, k_T, \lambda, \lambda') = -\frac{ig_A}{2F_{\pi}} \left[\frac{y(1-y)}{k_{\perp}^2 + \overline{M}_{\pi}^2(y)} \right] \overline{U}_{\lambda'} \gamma_5 U_{\lambda}$$

- In impact parameter space

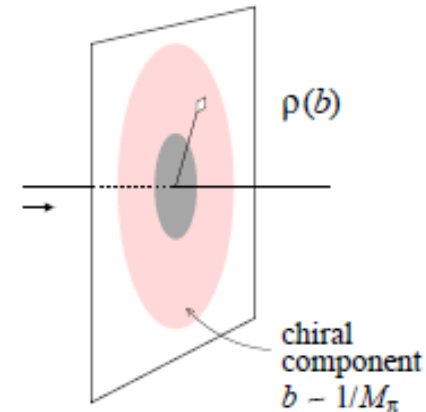


$$\Psi_{\pi N}(y, b) = \int \frac{d^2 k_T}{(2\pi)^2} e^{-i(k_T b)} \Psi_{\pi N}(y, k_T)$$

Peripheral Densities in Light-Front χ PT

$$\rho(b) = \int \frac{d^2 \Delta_T}{(2\pi)^2} e^{-i(\Delta_T \cdot b)} F(-\Delta_T^2)$$

- Transverse densities from form factors
- Charge and current densities from pion-nucleon light-cone wave functions with



$$\Psi_{\pi N}^{++}(y, \mathbf{b}) = \frac{ig_A}{F_\pi} \frac{y^2 \sqrt{1-y}}{2\pi} M_N^2 K_0(\overline{M}_\pi \mathbf{b})$$

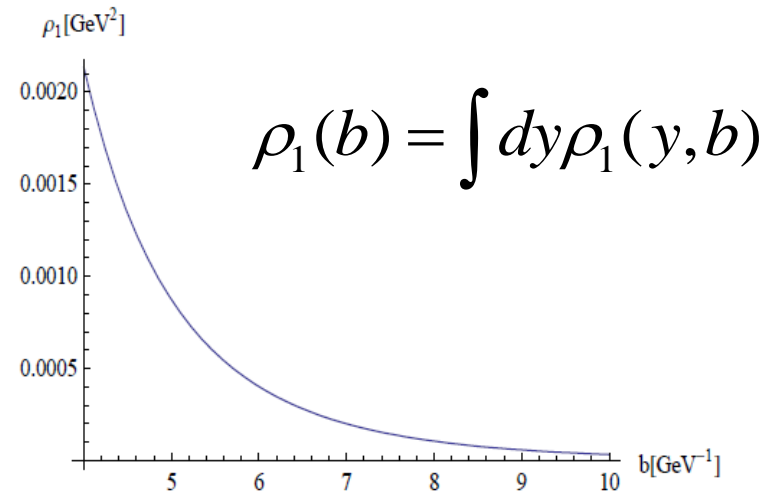
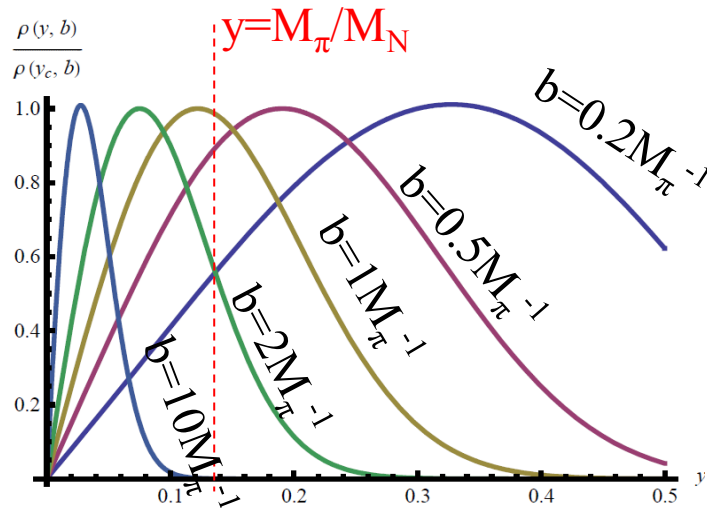
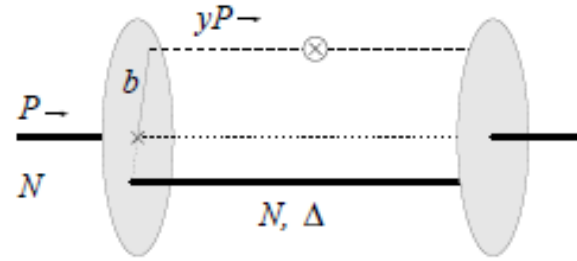
$$\Psi_{\pi N}^{+-}(y, \mathbf{b}) = \frac{g_A}{F_\pi} \frac{y \sqrt{1-y}}{2\pi} M_N \overline{M}_\pi K_1(\overline{M}_\pi \mathbf{b}) e^{-i\theta}$$

$$\rho_1(b) = 2 \sum_{\lambda'} \int \frac{dy}{2\pi} \left(\frac{|\Psi_{+\lambda'}(y, b')|^2}{y(1-y)} + (1 - g_A^2) \delta(y) \text{C.T.} \right)$$

$$\frac{\partial}{\partial b_R} \rho_2(b) = 2iM \sum_{\lambda'} \int \frac{dy}{2\pi} \frac{\Psi_{+\lambda'}(y, b') \Psi_{-\lambda'}(y, b')}{y(1-y)}$$

Peripheral Densities in Light-Front χ PT

$$\rho_1(y, b) \equiv 2 \sum_{\lambda'} \left(\frac{|\Psi_{\lambda'}(y, b')|^2}{y(1-y)} \right)$$



- Slower pions at larger impact parameter

$$\rho_1(b) = \int dy \frac{2g^2}{2\pi (2\pi)^2} \frac{yM_N^2}{(1-y)^2} \left[y^2 K_0^2(\overline{M}_\pi b') + \left(\frac{\overline{M}_\pi}{M_N} \right)^2 K_1^2(\overline{M}_\pi b') \right]$$

Energy-Momentum Tensor: Matter density and Orbital Angular Momentum

- Energy-Momentum tensor form factors (calculable in χ PT)

$$\langle N' | \Theta_{\mu\nu}^{N\pi} | N \rangle = \bar{u}_p \left[\gamma_{(\mu} P_{\nu)} A(\Delta^2) + P_{(\mu} i \sigma_{\nu)\alpha} \frac{\Delta_\alpha}{2M} B(\Delta^2) + \left(\frac{\Delta_\mu \Delta_\nu - \Delta^2 g_{\mu\nu}}{M} \right) C(\Delta^2) + M g_{\mu\nu} \tilde{C}(\Delta^2) \right] u_p$$

- Angular momentum of a pion-nucleon system,

$$J_{N\pi} = \frac{1}{2} (A(0) + B(0))$$

[X.Ji(PRD97)]

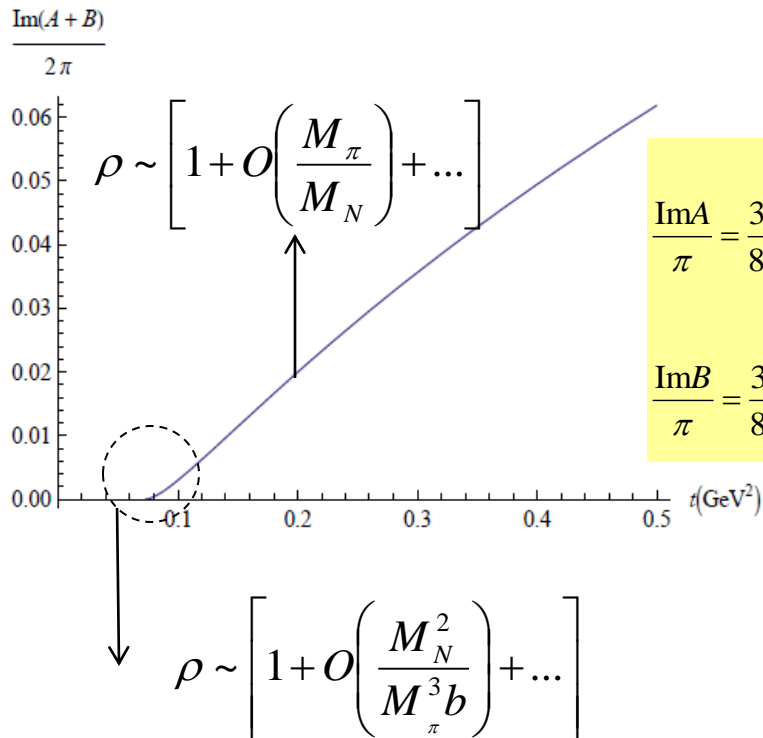
- Transverse densities ρ_A , ρ_B from form factors A and B, and A+B.

$$\rho_{A+B}(b) = \frac{1}{2\pi} \int_{thr}^{\infty} dt K_0(\sqrt{tb}) \frac{1}{\pi} \text{Im}[A(t+i0) + B(t+i0)]$$

- Calculated leading chiral contribution to spectral functions (Cutkosky Rules). No contact term diagrams!

Energy-Momentum Tensor: Matter density and Orbital Angular Momentum

- Asymptotic behavior of ρ controlled by $\text{Im}F$ at threshold and near threshold



$$\frac{\text{Im}A}{\pi} = \frac{3}{8} g^2 \frac{\left(\frac{t}{2} - M_\pi^2\right)^3}{(4\pi)^2 \sqrt{P^2}^5 \sqrt{t}} \left[\frac{4}{3} \left(1 - \frac{M_N^2}{P^2}\right) x^3 + 2x - \left(\left(2 - 3 \frac{M_N^2}{P^2}\right) x^2 - 3 \right) \arctan(x) \right]$$

$$\frac{\text{Im}B}{\pi} = \frac{3}{8} g^2 \frac{\left(\frac{t}{2} - M_\pi^2\right)^3}{(4\pi)^2 \sqrt{P^2}^5 \sqrt{t}} \left[\frac{4}{3} x^3 + 5x - (3x^2 + 5) \arctan(x) \right]$$

Energy-Momentum Tensor: Matter density and Orbital Angular Momentum

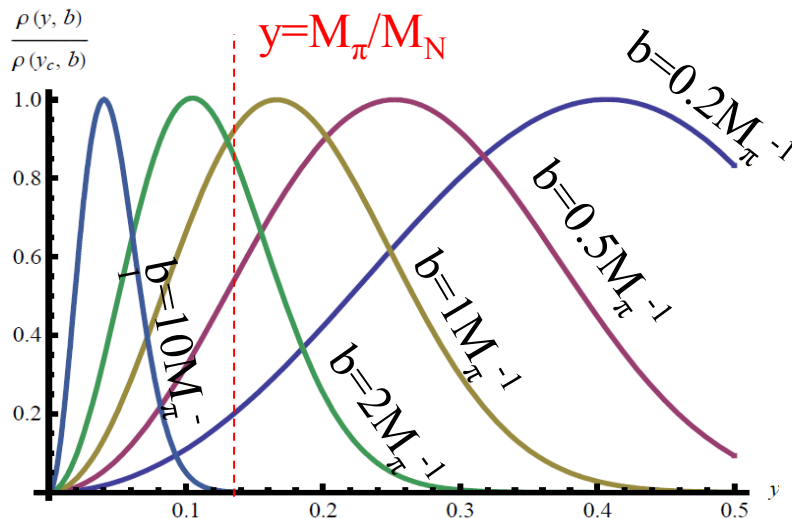
- EMT form factors in IMF
- Corresponding Transverse densities as overlap of LC- wave functions
- Matter distributions in impact parameter space:
 - Moments of parton distributions

$$A = \left\langle P_{2,+} \left| \frac{\Theta^{33}}{2P^2} \right| P_{1,+} \right\rangle$$

$$-\frac{\Delta_L}{2M} B = \left\langle P_{2,+} \left| \frac{\Theta^{33}}{2P^2} \right| P_{1,-} \right\rangle$$

$$\rho_A(b) = \frac{3}{2} \sum_{\lambda'} \int \frac{dy}{2\pi} y \left[\frac{|\Psi_{\lambda'}(y,b)|^2}{y(1-y)^3} \right]$$

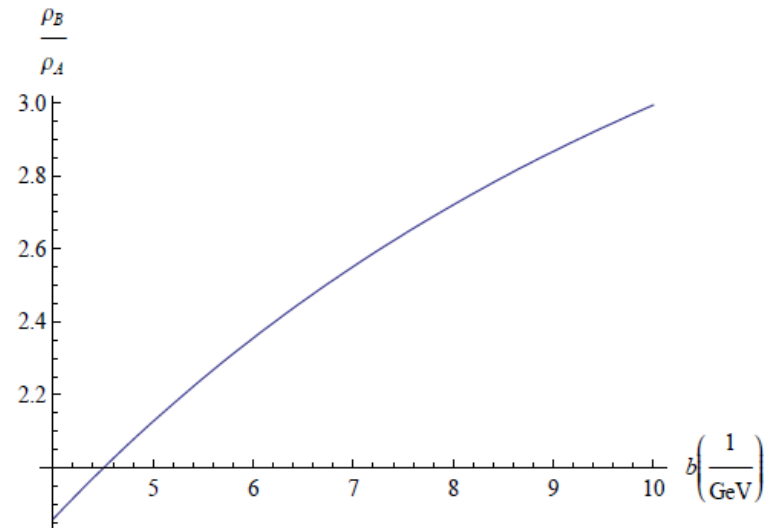
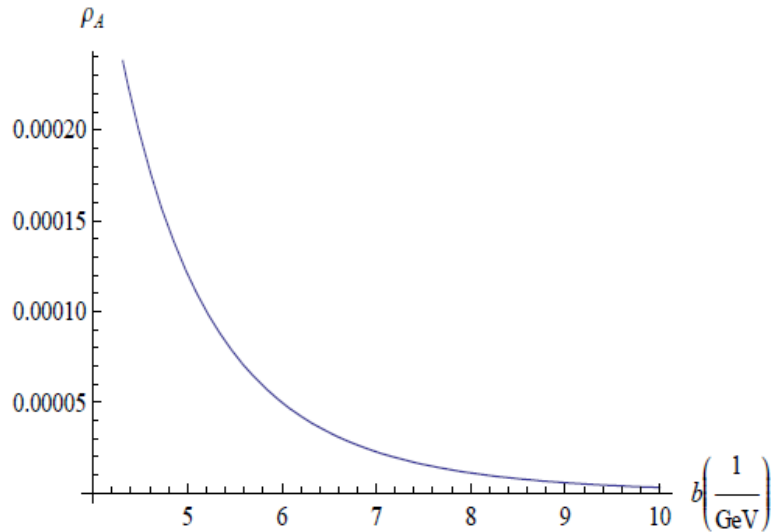
$$\frac{\partial}{\partial b_R} \rho_B(b) = \frac{3}{2} iM \sum_{\lambda'} \int \frac{dy}{2\pi} y \left[\frac{\Psi_{+\lambda'}(y,b) \Psi_{-\lambda'}(y,b)}{y(1-y)^3} \right]$$



$$\rho_A(y,b) \equiv \frac{3}{2} y \sum_{\lambda'} \left(\frac{|\Psi_{\lambda'}(y,b)|^2}{y(1-y)} \right)$$

$$\rho_A(y,b) = 3y\rho_1(y,b)/4$$

Energy-Momentum Tensor: Matter density and Orbital Angular Momentum



$$\rho_A(b) = \frac{3}{2} \int \frac{dy}{2\pi} \frac{g^2}{(2\pi)^2} \frac{y^2 M_N^2}{(1-y)^2} \left[y^2 K_0^2(\overline{M}_\pi b') + \left(\frac{\overline{M}_\pi}{M_N} \right)^2 K_1^2(\overline{M}_\pi b') \right]$$

$$\rho_B(b) = \frac{3}{2} \int \frac{dy}{2\pi} \frac{g^2}{(2\pi)^2} \frac{y^2 M_N^2}{(1-y)} \left[2y K_0^2(\overline{M}_\pi b') \right]$$

Summary

- Explored Nucleonic Structure in a setting that guarantees a model independent analysis of the dynamics governed by χ EFT.
- Derived EM and EMT transverse peripheral densities from corresponding FF
- From spectral functions (invariant formalism)
 - Distinguish parametrical regions (chiral and molecular scales)
 - Accuracy of the Heavy Baryon expansion
 - Consistency with the QCD Large N_c limit (add Δ -contribution)
- Light-front χ PT (IMF)
 - Equivalent to invariant formalism
 - Calculated transverse densities from LC-Wave Functions (Connection to GPD formalism)

Outlook

- Understand origin of contact term (higher mass states, nucleon compositeness)
- Use the chiral pion–nucleon system as a toy model for exploring the nature of orbital angular momentum OAM and other operators in field theory (moments of GPDs, Axial form factors)
- Test use π N-LCWF in experimental studies at Low and High energies.