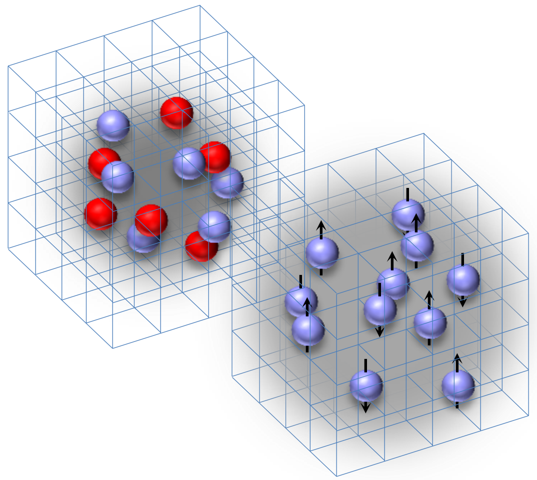
Lattice Effective Field Theory for Nuclei



Nuclear Lattice EFT Collaboration

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Study of Strongly Interacting Matter HadronPhysics3
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Outline

Introduction and motivation

What is lattice effective field theory?

Lattice interactions and scattering

Euclidean time projection and auxiliary fields

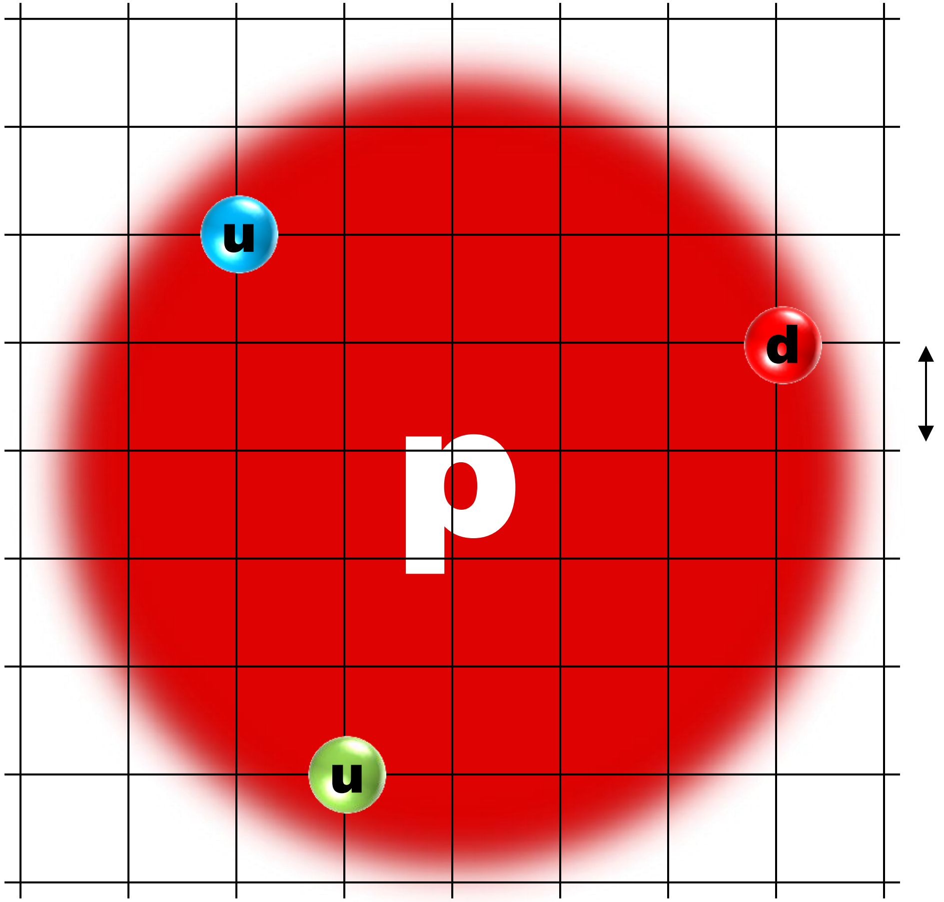
Carbon-12 and the Hoyle state

Light quark mass dependence of helium burning

Towards scattering and reactions on the lattice

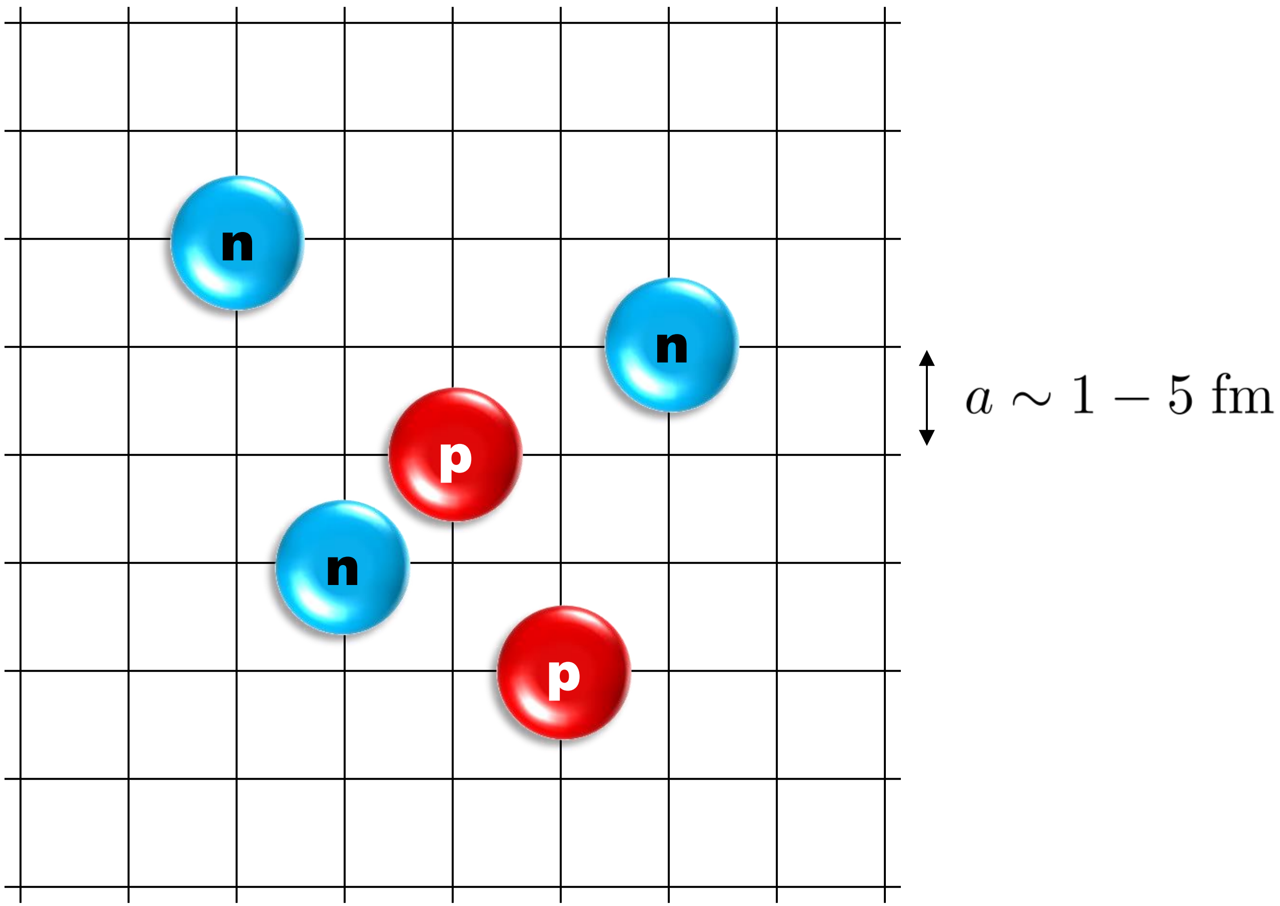
Summary and future directions

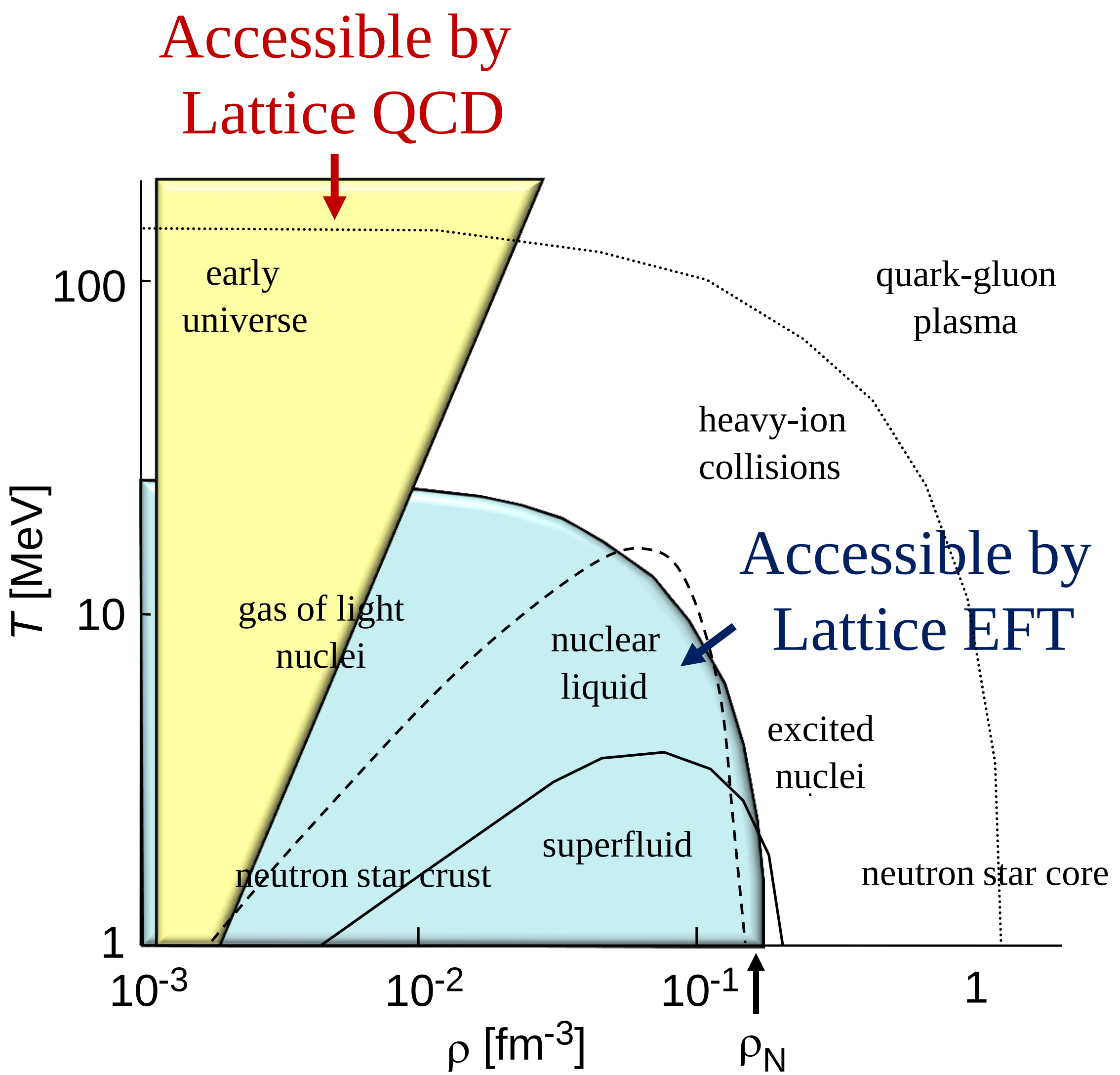
Lattice quantum chromodynamics



$a \sim 0.1$ fm

Lattice effective field theory

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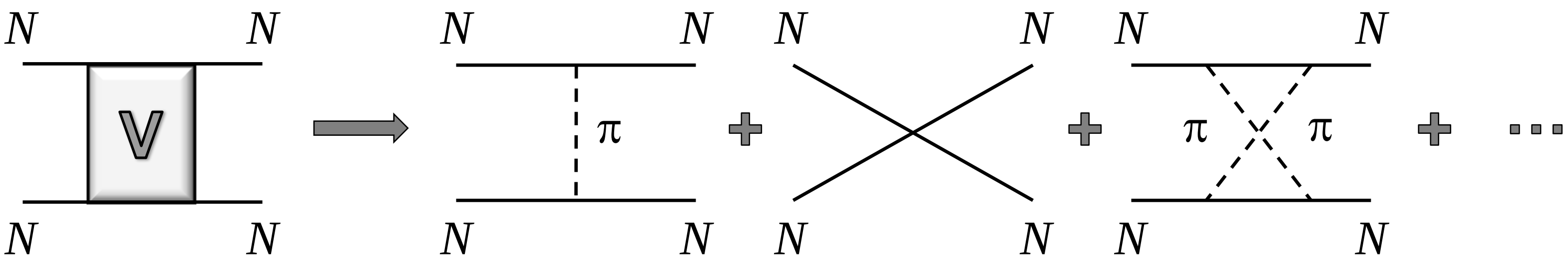
ρ [fm^{-3}]
ρ_N

Low energy nucleons: Chiral effective field theory

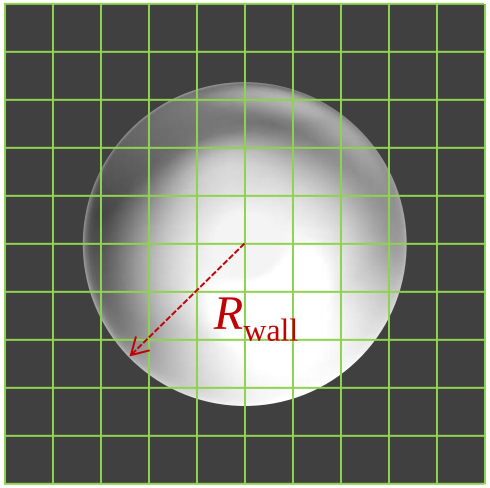
Construct the effective potential order by order



V^{OPEP} Contact interactions V^{TPEP}

Leading order (LO) Next-to-leading order (NLO)

Physical scattering data → Unknown operator coefficients

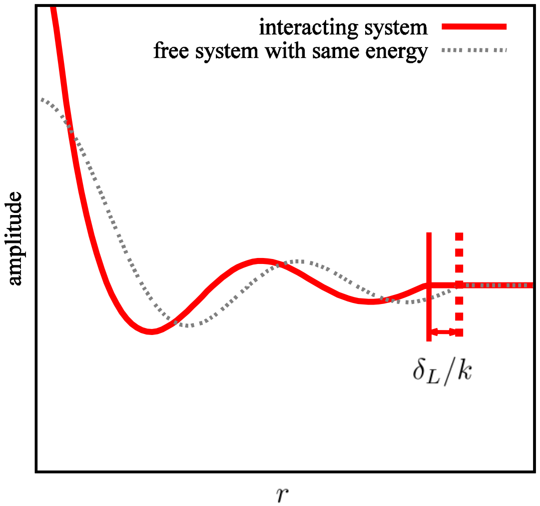


Spherical wall method

Borasoy, Epelbaum, Krebs, D.L., Meißner,
EPJA 34 (2007) 185

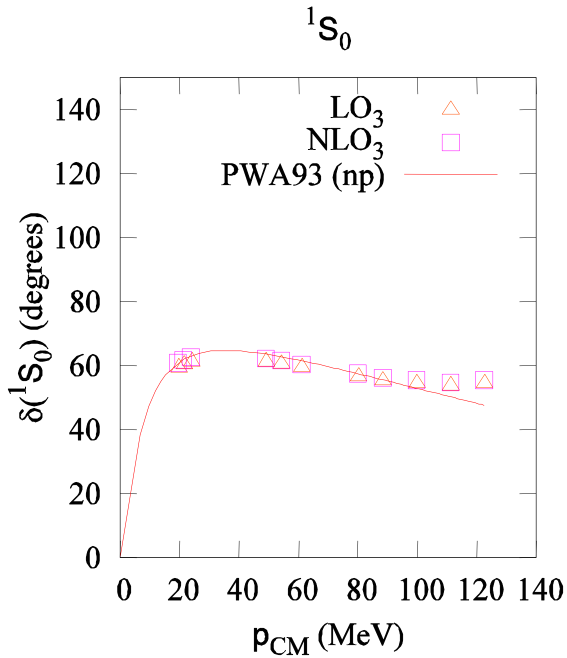
Spherical wall imposed in the center of mass frame

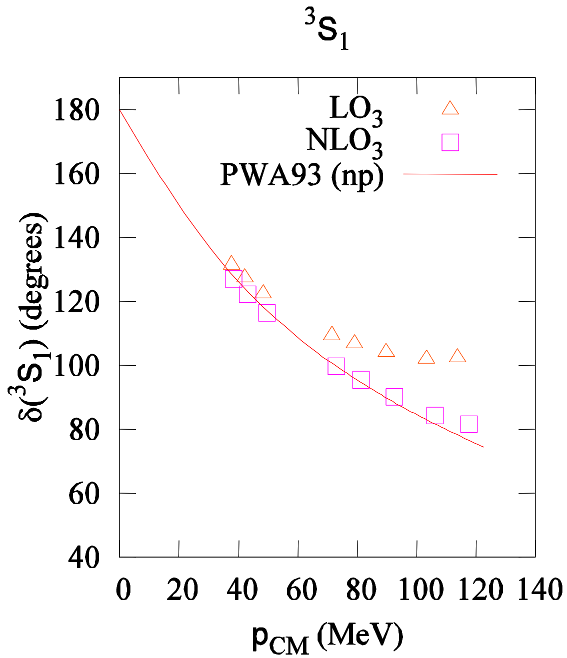
Representation	J_z	Example
A_1	$0 \bmod 4$	$Y_{0,0}$
T_1	$0, 1, 3 \bmod 4$	$\{Y_{1,0}, Y_{1,1}, Y_{1,-1}\}$
E	$0, 2 \bmod 4$	$\left\{Y_{2,0}, \frac{Y_{2,-2}+Y_{2,2}}{\sqrt{2}}\right\}$
T_2	$1, 2, 3 \bmod 4$	$\left\{Y_{2,1}, \frac{Y_{2,-2}-Y_{2,2}}{\sqrt{2}}, Y_{2,-1}\right\}$
A_2	$2 \bmod 4$	$\frac{Y_{3,2}-Y_{3,-2}}{\sqrt{2}}$



LO_3: S waves

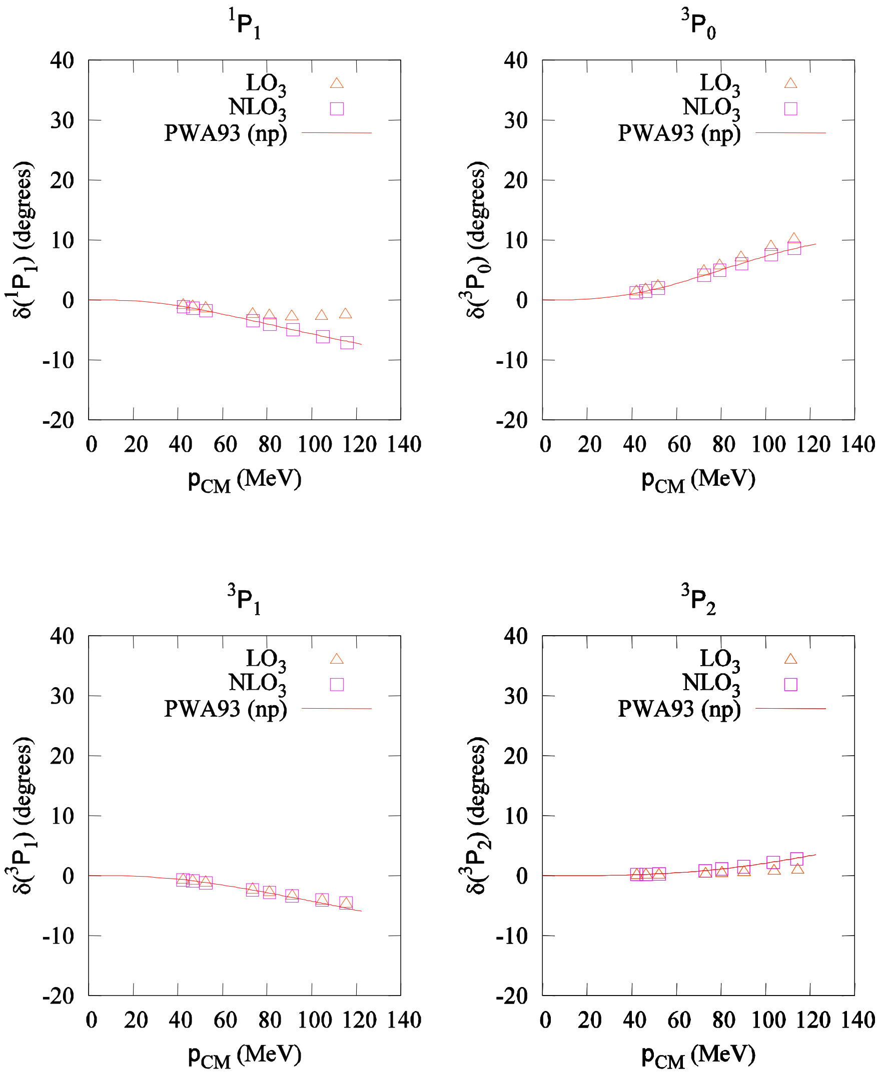
$a = 1.97$ fm





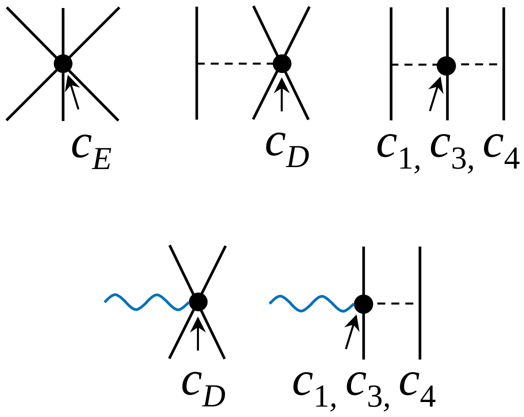
LO_3: P waves

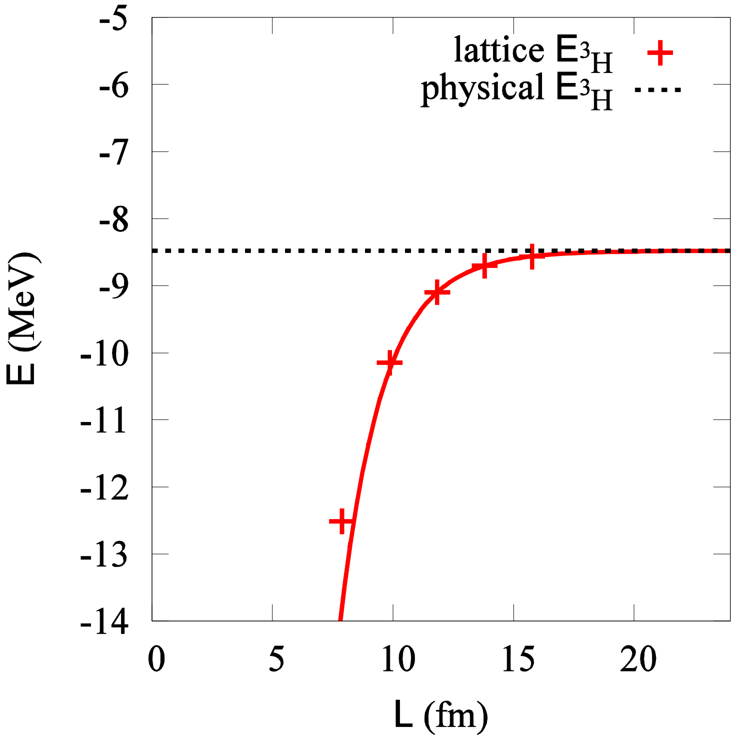
$$a = 1.97 \text{ fm}$$



Three nucleon forces

Two unknown coefficients at NNLO from three-nucleon forces. Determine c_D and c_E using ^{3}H binding energy and the weak axial current at low cutoff momentum.



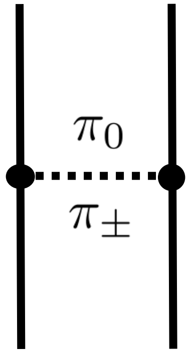


Park, et al., PRC 67 (2003) 055206,
Gårdestig, Phillips, PRL 96 (2006) 232301,
Gazit, Quaglioni, Navratil, PRL 103 (2009) 102502

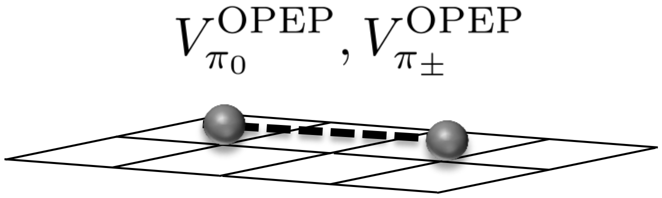
Neutrons and protons: Isospin breaking and Coulomb

Isospin-breaking and power counting *[Friar , van Kolck, PRC 60 (1999) 034006; Walzl, Meißner, Epelbaum NPA 693 (2001) 663; Friar, van Kolck, Payne, Coon, PRC 68 (2003) 024003; Epelbaum, Meißner, PRC72 (2005) 044001…]*

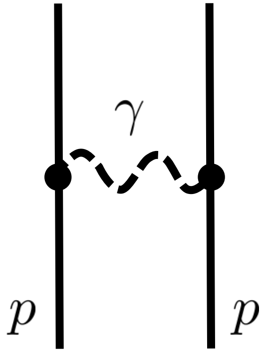
Pion mass difference

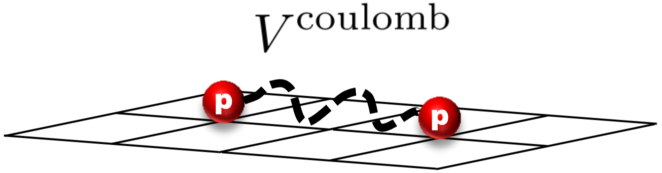


$$m_{\pi_0} \neq m_{\pi_\pm}$$

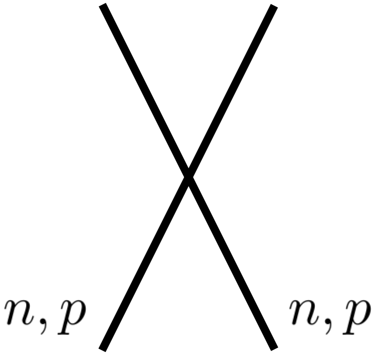


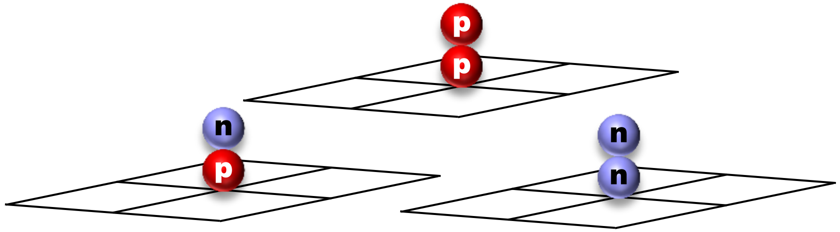
Coulomb potential





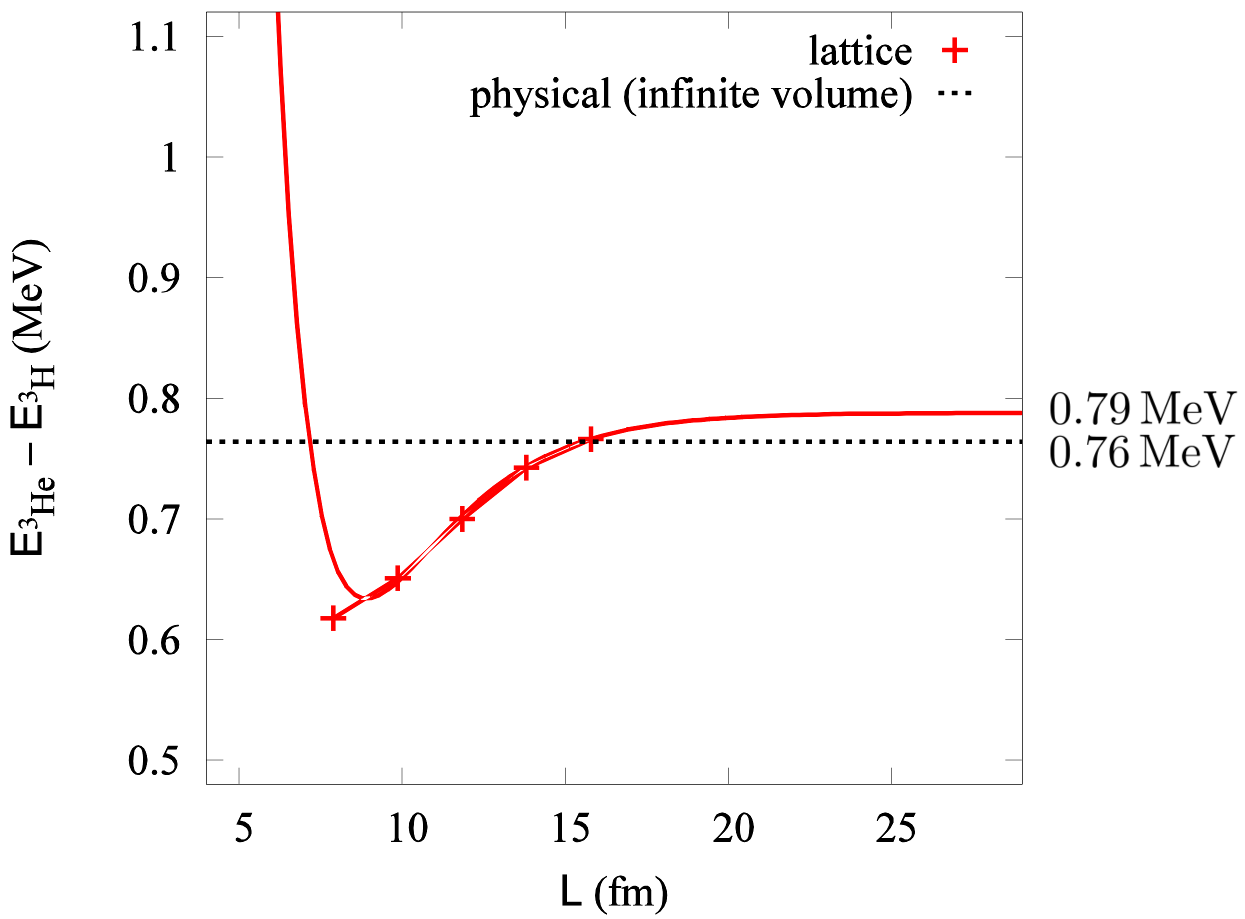
Charge symmetry breaking
Charge independence breaking



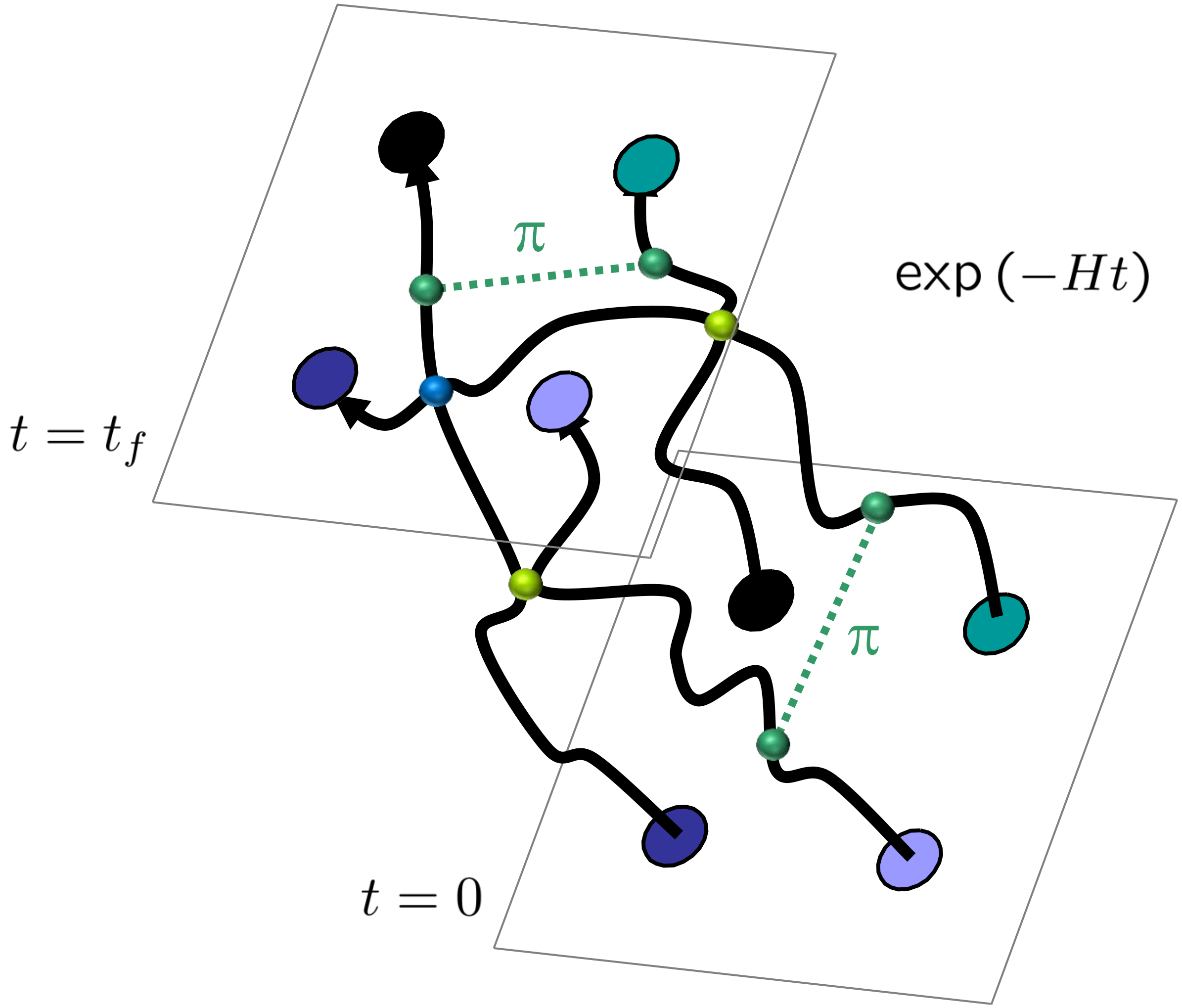


Triton and Helium-3

$$E_{^3\mathrm{He}} - E_{\mathrm{triton}} = 0.79(5)\ \mathrm{MeV}$$



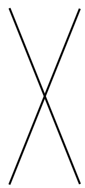
Euclidean time projection



Auxiliary field method

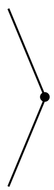
We can write exponentials of the interaction using a Gaussian integral identity

$$\exp\left[-\frac{C}{2}(N^\dagger N)^2\right]$$



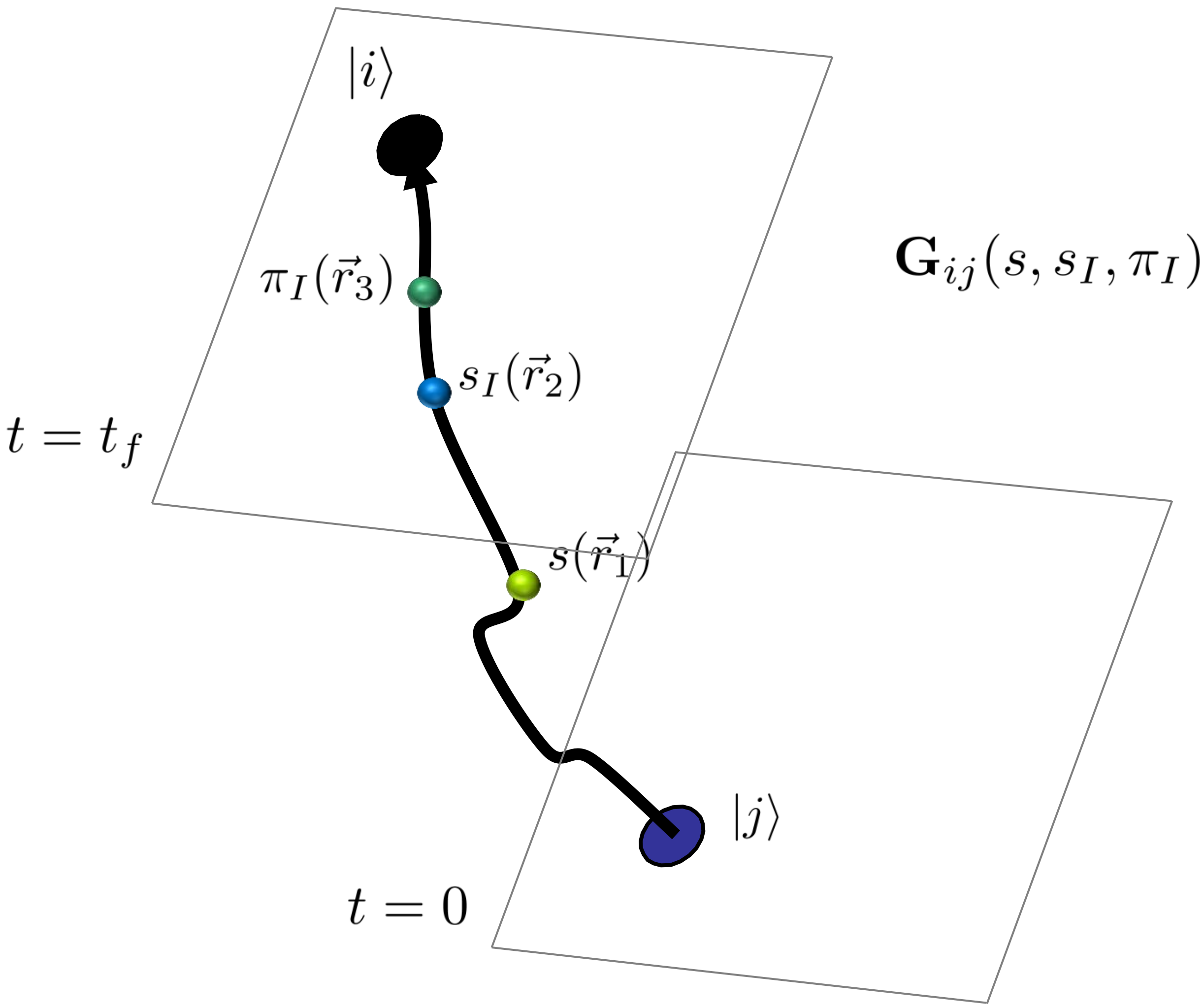
$(N^\dagger N)^2$

$$= \sqrt{\frac{1}{2\pi}}\int_{-\infty}^{\infty} ds \exp\left[-\frac{1}{2}s^2 + \sqrt{-C}\, s(N^\dagger N)\right]$$

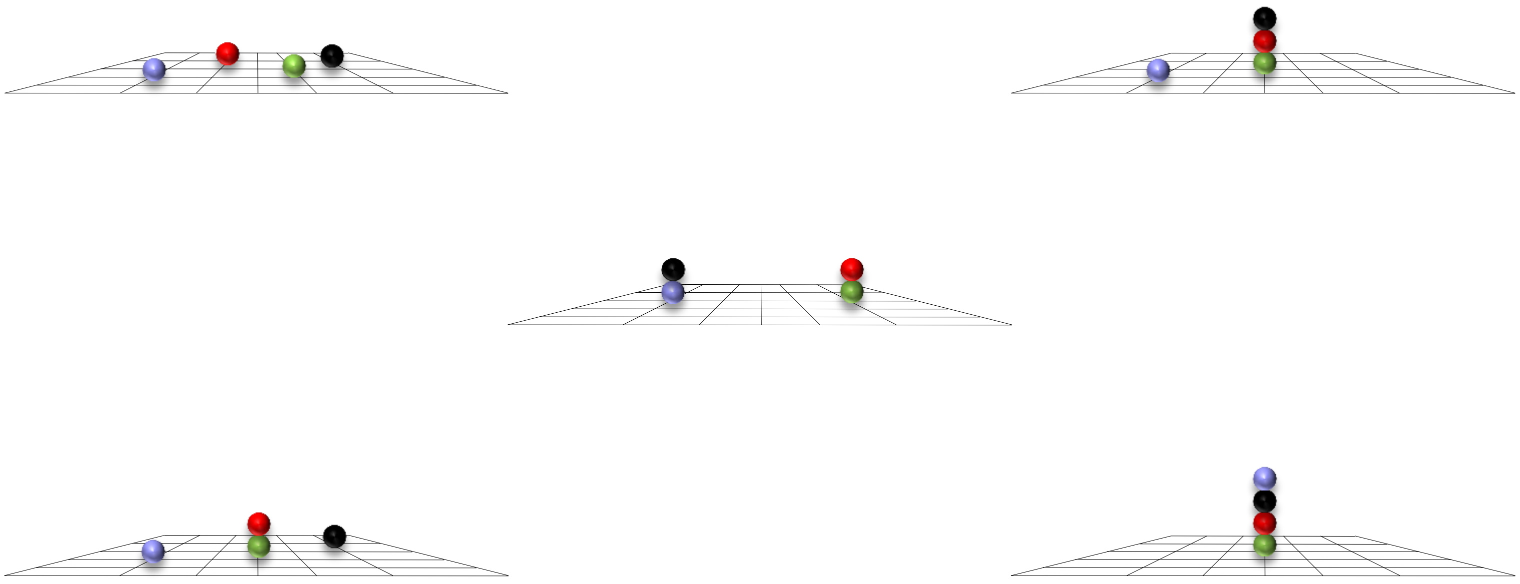


$sN^\dagger N$

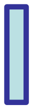
We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.

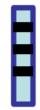

$|i\rangle$
$\pi_I(\vec{r}_3)$
$s_I(\vec{r}_2)$
$t = t_f$
$s(\vec{r}_1)$
$|j\rangle$
$t = 0$
$\mathbf{G}_{ij}(s, s_I, \pi_I)$

Particle clustering included automatically

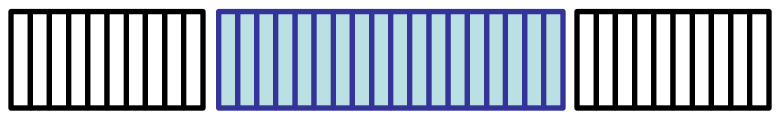


Schematic of lattice Monte Carlo calculation

 $= M_{\mathrm{LO}}$  $= M_{\mathrm{approx}}$  $= O_{\mathrm{observable}}$

 $= M_{\mathrm{NLO}}$  $= M_{\mathrm{NNLO}}$

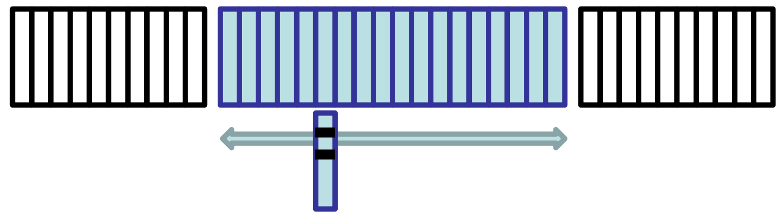
Hybrid Monte Carlo sampling

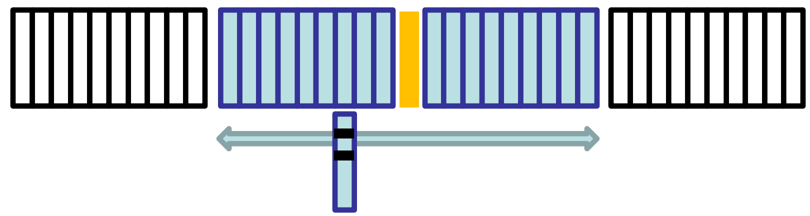
$Z_{n_t,\mathrm{LO}} = \langle \psi_{\mathrm{init}} |$  $| \psi_{\mathrm{init}} \rangle$

$Z^{\langle O \rangle}_{n_t,\mathrm{LO}} = \langle \psi_{\mathrm{init}} |$  $| \psi_{\mathrm{init}} \rangle$

$$e^{-E_{0,\mathrm{LO}} a_t} = \lim_{n_t \to \infty} Z_{n_t+1,\mathrm{LO}} / Z_{n_t,\mathrm{LO}}$$

$$\langle O \rangle_{0,\mathrm{LO}} = \lim_{n_t \to \infty} Z^{\langle O \rangle}_{n_t,\mathrm{LO}} / Z_{n_t,\mathrm{LO}}$$

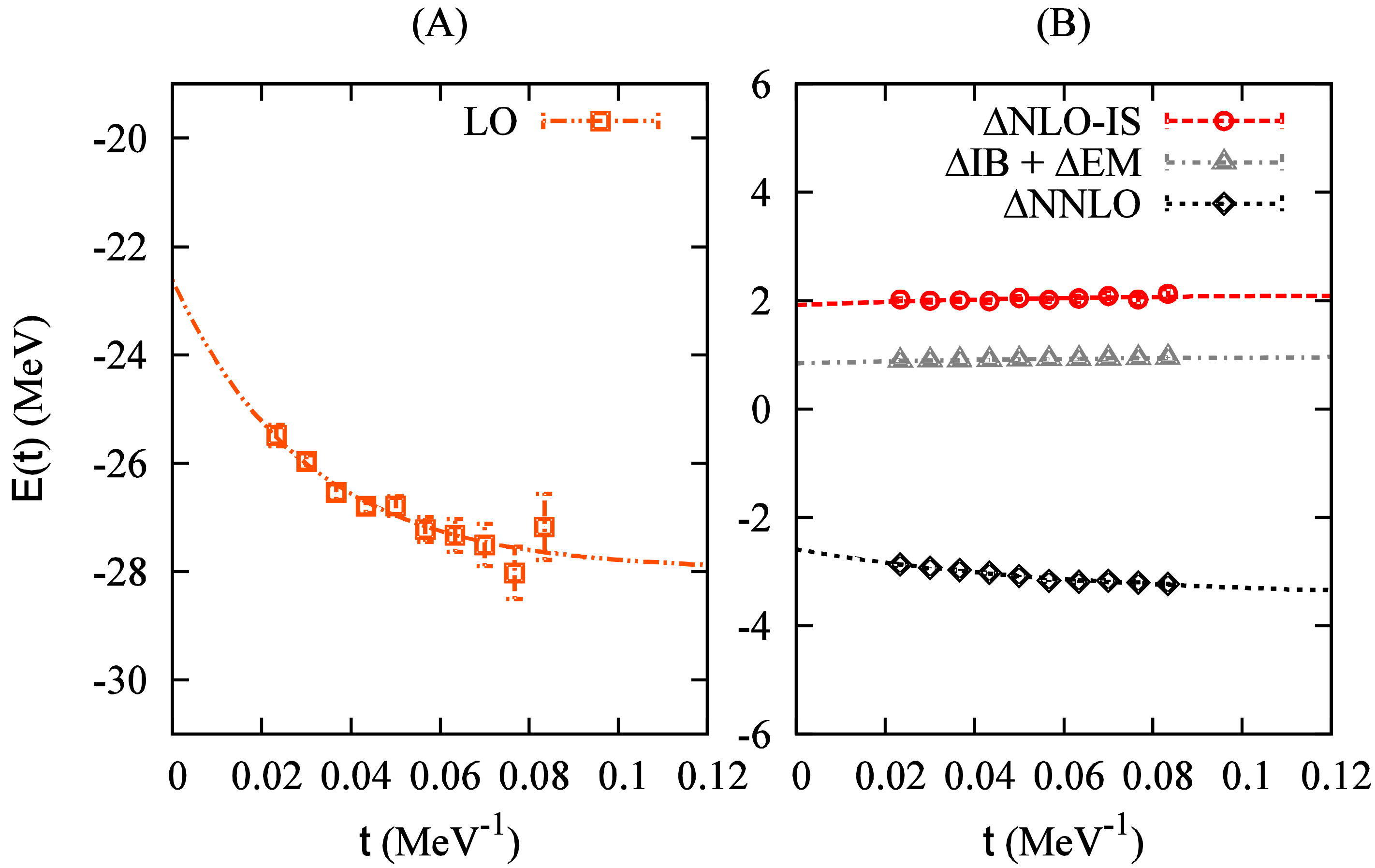
$Z_{n_t,\mathrm{NLO}} = \langle \psi_{\mathrm{init}} |$  $| \psi_{\mathrm{init}} \rangle$

$Z^{\langle O \rangle}_{n_t,\mathrm{NLO}} = \langle \psi_{\mathrm{init}} |$  $| \psi_{\mathrm{init}} \rangle$

$$\langle O \rangle_{0,\mathrm{NLO}} = \lim_{n_t \to \infty} Z^{\langle O \rangle}_{n_t,\mathrm{NLO}} / Z_{n_t,\mathrm{NLO}}$$

Ground state of Helium-4

$L = 11.8\,\mathrm{fm}$



Epelbaum, Krebs, D.L, Meißner, PRL 106 (2011) 192501

Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 (2012) 252501

Ground state of Helium-4

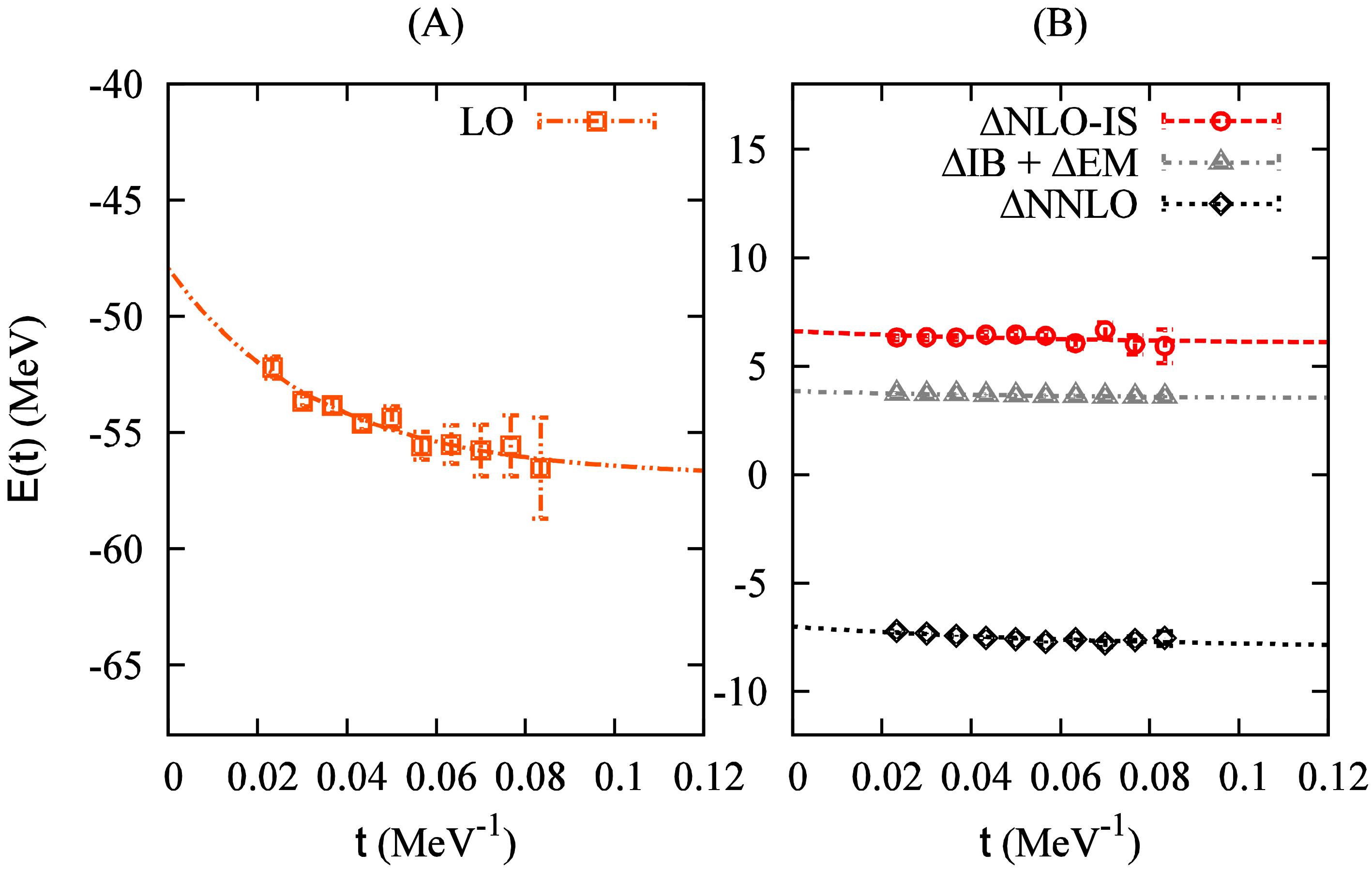
$$L = 11.8\,\mathrm{fm}$$

LO* ($O(Q^0)$)	–28.0(3) MeV
NLO ($O(Q^2)$)	–24.9(5) MeV
NNLO ($O(Q^3)$)	–28.3(6) MeV
Experiment	–28.3 MeV

*contains some interactions promoted from NLO

Ground state of Beryllium-8

$L = 11.8\,\mathrm{fm}$



Epelbaum, Krebs, D.L, Meißner, PRL 106 (2011) 192501
Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 (2012) 252501

Ground state of Beryllium-8

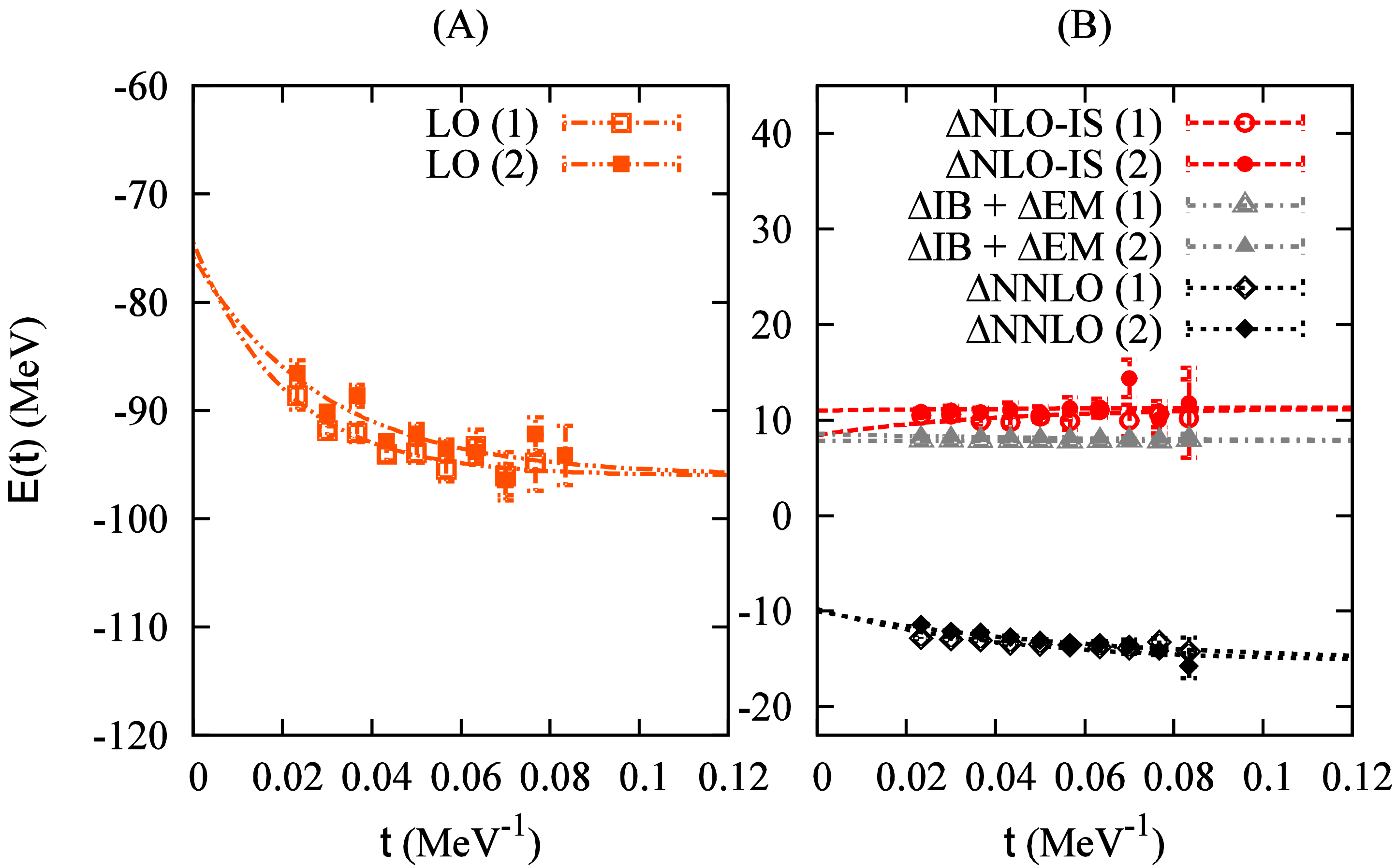
$$L = 11.8\,\mathrm{fm}$$

LO* ($O(Q^0)$)	–57(2) MeV
NLO ($O(Q^2)$)	–47(2) MeV
NNLO ($O(Q^3)$)	–55(2) MeV
Experiment	–56.5 MeV

*contains some interactions promoted from NLO

Ground state of Carbon-12

$$L = 11.8\,\mathrm{fm}$$



Epelbaum, Krebs, D.L, Meißner, PRL 106 (2011) 192501
Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 (2012) 252501

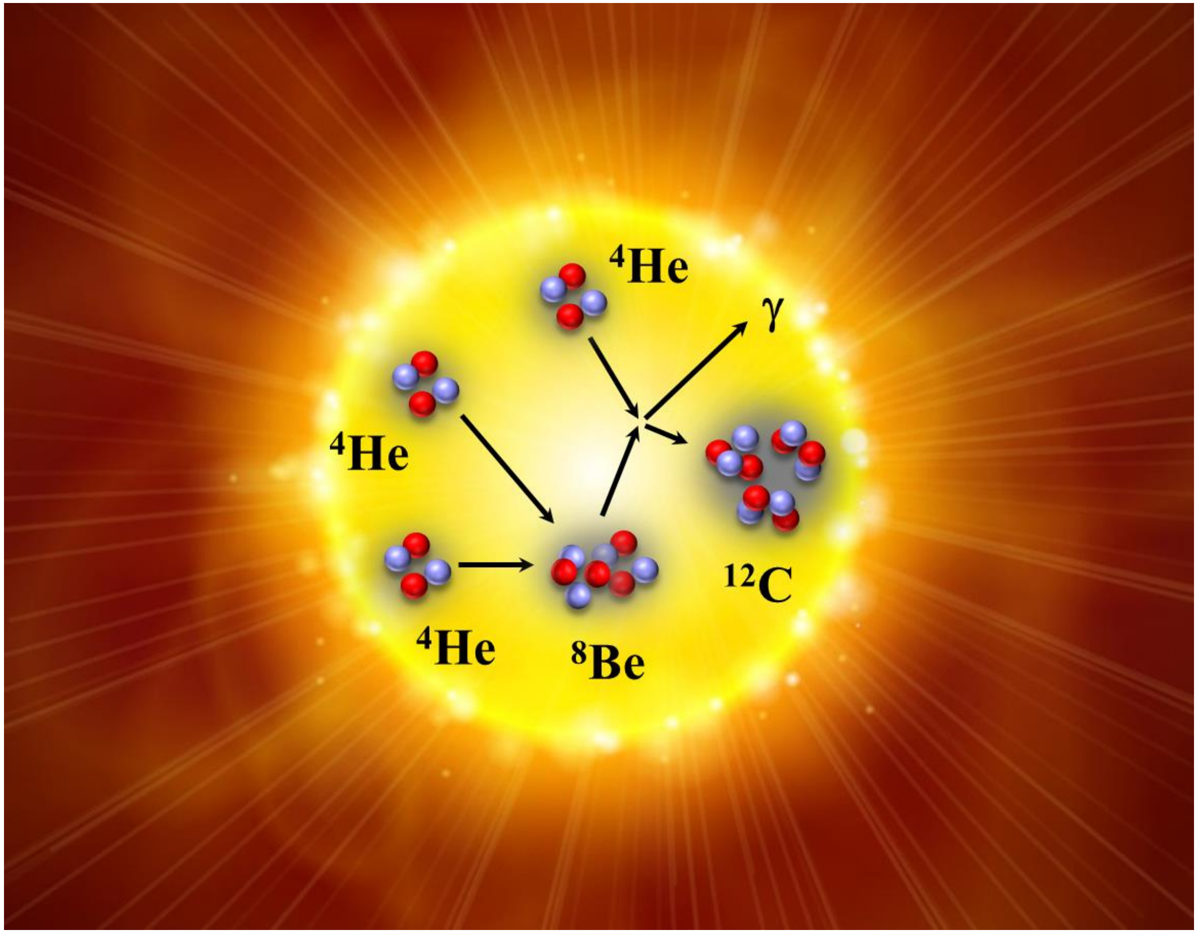
Ground state of Carbon-12

$$L = 11.8\,\mathrm{fm}$$

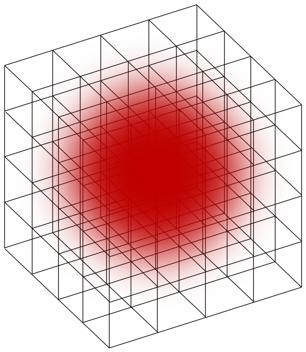
LO* ($O(Q^0)$)	–96(2) MeV
NLO ($O(Q^2)$)	–77(3) MeV
NNLO ($O(Q^3)$)	–92(3) MeV
Experiment	–92.2 MeV

*contains some interactions promoted from NLO

Carbon-12 spectrum and the Hoyle state



Simulations using general initial/final state wavefunctions



$$\bigwedge_{j=1,\cdots,A} |\psi_j(\vec{n})\rangle$$

Construct states with well-defined momentum using all possible translations.

$$L^{-3/2} \sum_{\vec{m}} e^{i\vec{P}\cdot\vec{m}} \bigwedge_{j=1,\cdots,A} |\psi_j(\vec{n}-\vec{m})\rangle$$

Shell model wavefunctions

$$\psi_j(\vec{n}) = \exp(-c\vec{n}^2)$$
$$\psi'_j(\vec{n}) = n_x \exp(-c\vec{n}^2)$$
$$\psi''_j(\vec{n}) = n_y \exp(-c\vec{n}^2)$$
$$\psi'''_j(\vec{n}) = n_z \exp(-c\vec{n}^2)$$
$$\vdots$$

Alpha cluster wavefunctions

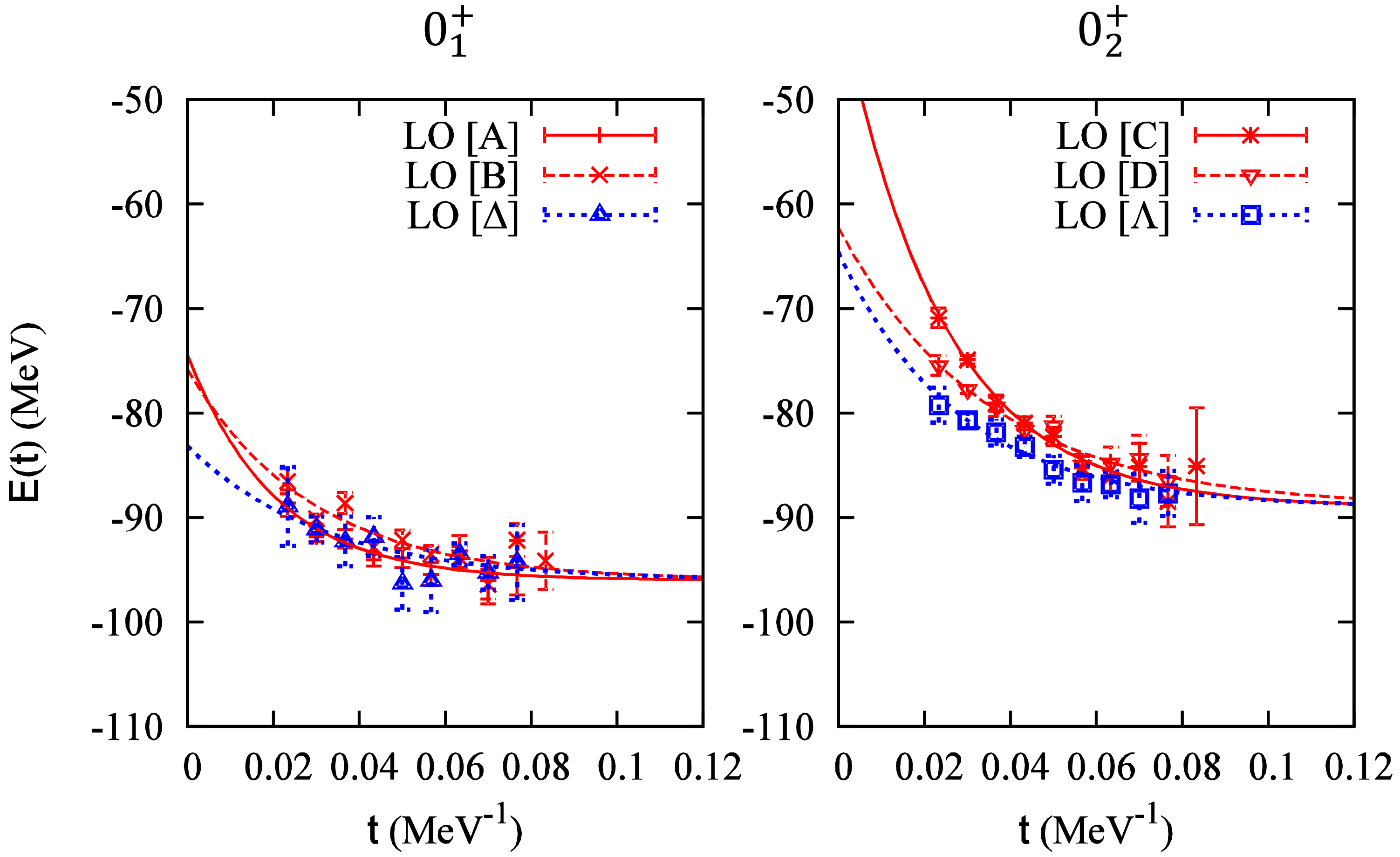
$$\psi_j(\vec{n}) = \exp[-c(\vec{n}-\vec{m})^2]$$
$$\psi'_j(\vec{n}) = \exp[-c(\vec{n}-\vec{m}')^2]$$
$$\psi''_j(\vec{n}) = \exp[-c(\vec{n}-\vec{m}'')^2]$$
$$\vdots$$

Shell model wavefunctions by themselves do not have enough local four nucleon correlations,

$$< (N^\dagger N)^4 >$$

Needs to develop the four nucleon correlations via Euclidean time projection.

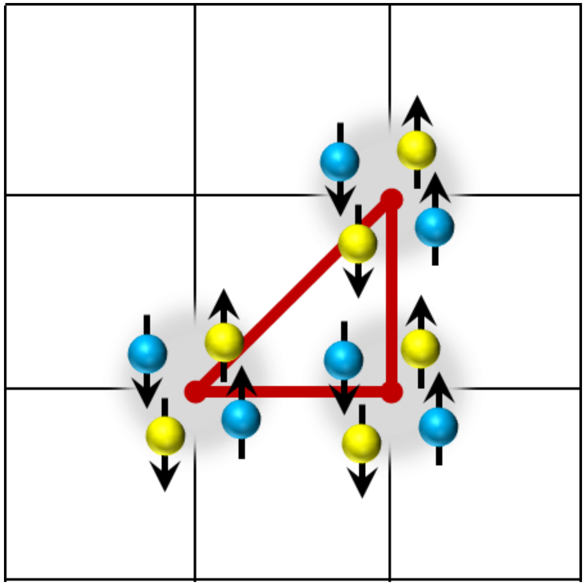
But can reproduce same results starting directly from alpha cluster wavefunctions [Δ and Λ in plots on next slide].



Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 252501 (2012)

Structure of ground state and first 2+

Strong overlap with compact triangle configuration

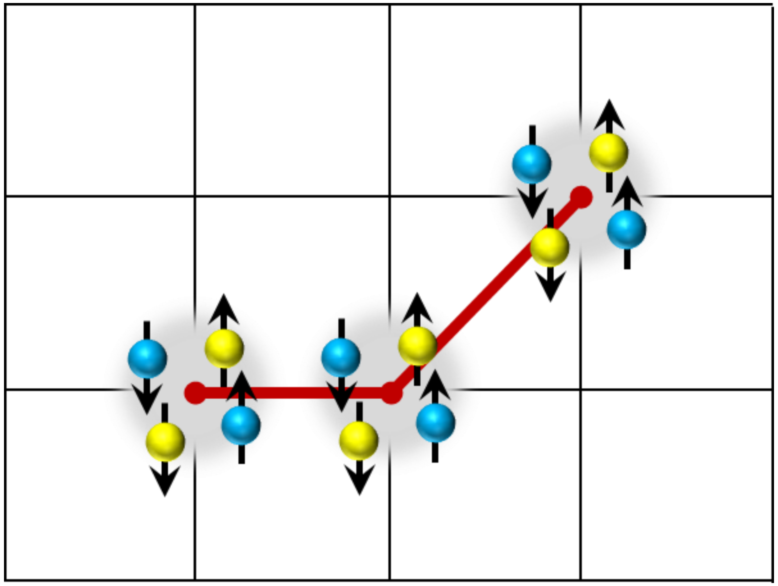


$$a = 1.97 \text{ fm}$$

12 rotational orientations

Structure of Hoyle state and second 2+

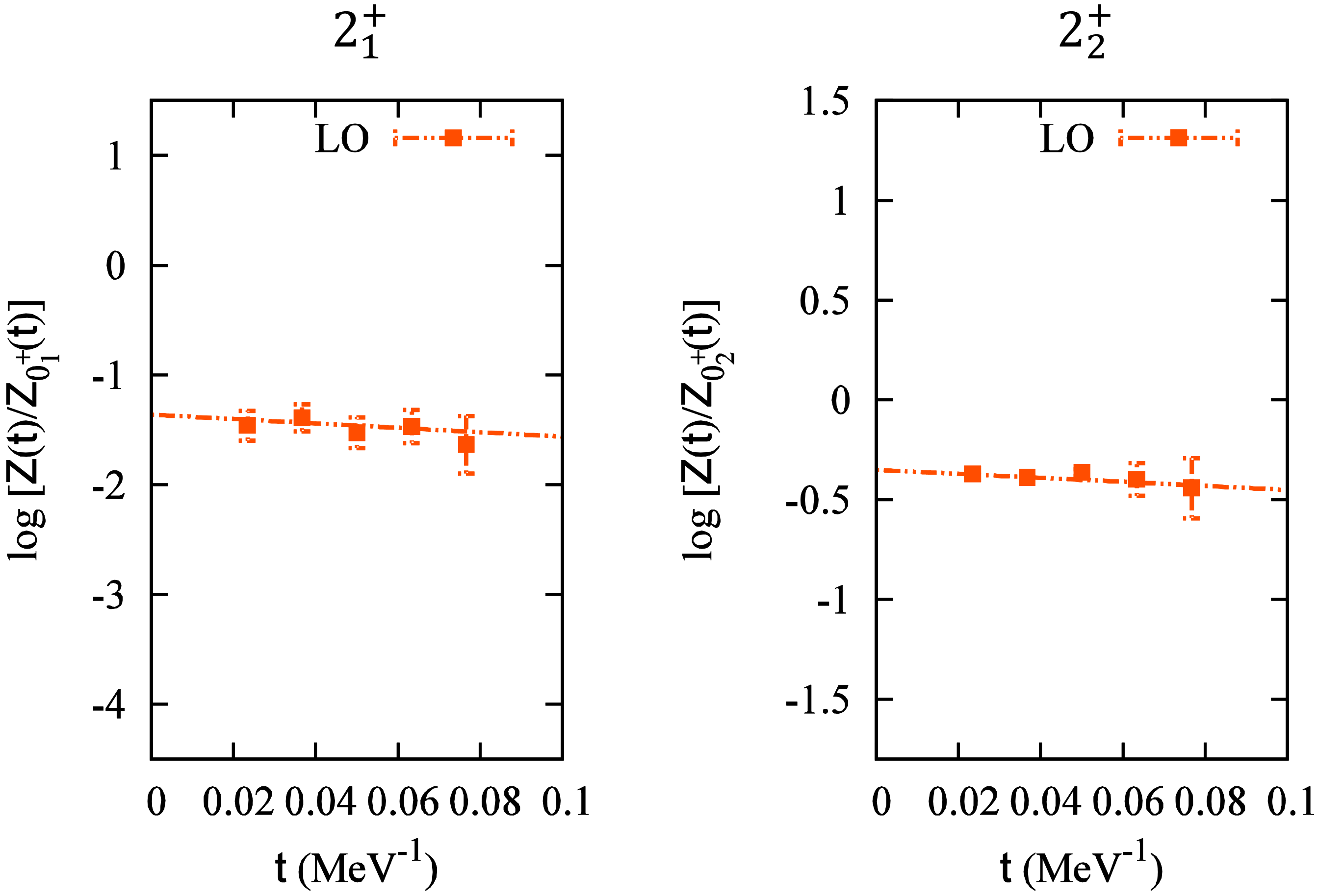
Strong overlap with bent arm configuration



24 rotational orientations

$a = 1.97\ \mathrm{fm}$

Rotational excitations – 2_1^+, 2_2^+



Excited state spectrum of carbon-12 (even parity)

	2_1^+	0_2^+	2_2^+
LO* ($O(Q^0)$)	–94(2) MeV	–89(2) MeV	–88(2) MeV
NLO ($O(Q^2)$)	–74(3) MeV	–72(3) MeV	–70(3) MeV
NNLO ($O(Q^3)$)	–89(3) MeV	–85(3) MeV	–83(3) MeV
Experiment	–87.72 MeV	–84.51 MeV	–82.6(1) MeV (*A*,*B*) –81.1(3) MeV (*C*) –82.13(11) MeV (*D*)

*contains some interactions promoted from NLO

A – Freer et al., PRC 80 (2009) 041303

B – Zimmerman et al., PRC 84 (2011) 027304

C – Hyldegaard et al., PRC 81 (2010) 024303

D – Itoh et al., PRC 84 (2011) 054308

Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 252501 (2012)

RMS charge radius

bound states at leading order

	LO	Experiment
$r_{0_1^+}$ [fm]	2.2(2)	2.47(2)
$r_{2_1^+}$ [fm]	2.2(2)	–
$r_{0_2^+}$ [fm]	2.4(2)	–
$r_{2_2^+}$ [fm]	2.4(2)	–

Schaller, et al.
NPA 379 (1982) 523

Quadrupole moment

	LO	Experiment
$Q_{2_1^+}$ $[e\ \mathrm{fm}^2]$	6(2)	6(3)
$Q_{2_2^+}$ $[e\ \mathrm{fm}^2]$	–7(2)	–

Vermeer, et al.
PLB 122 (1983) 23

Electromagnetic transition strengths

	LO	Experiment
$B(E2, 2_1^+ \to 0_1^+)$ $[e^2 \text{ fm}^4]$	5(2)	7.6(4)
$B(E2, 2_1^+ \to 0_2^+)$ $[e^2 \text{ fm}^4]$	1.5(7)	2.6(4)
$B(E2, 2_2^+ \to 0_1^+)$ $[e^2 \text{ fm}^4]$	2(1)	0.73(13)
$B(E2, 2_2^+ \to 0_2^+)$ $[e^2 \text{ fm}^4]$	6(2)	–
$m(E0, 0_2^+ \to 0_1^+)$ $[e \text{ fm}^2]$	3(1)	5.5(1)

Ajzenberg-Selove, NPA 506 (1990) 1

Zimmerman, et al., arXiv:1303.4326

Chernykh, et al., PRL 105 (2010) 022501

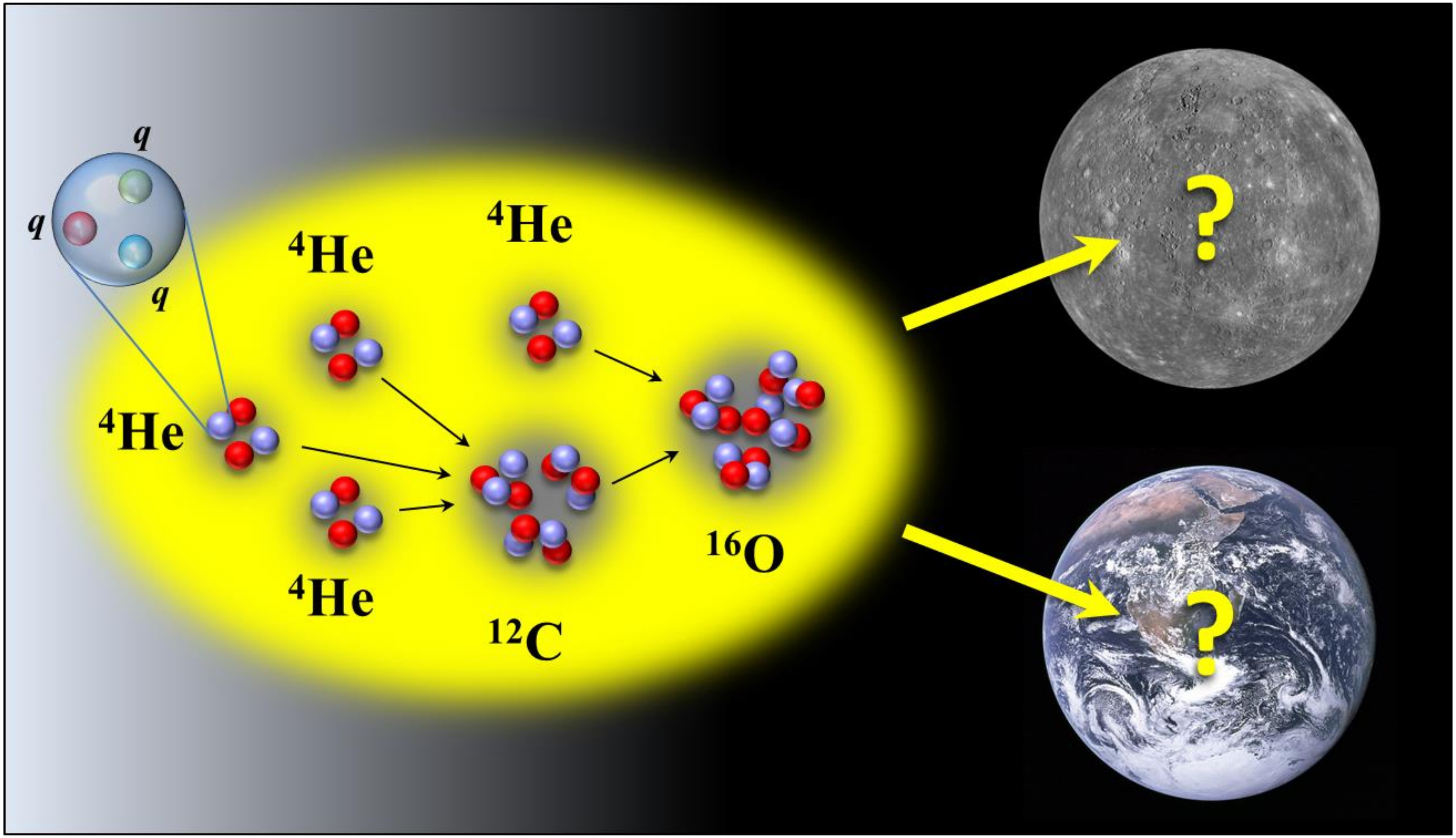
See also other recent calculations using fermionic molecular dynamics

Chernykh, et al., PRL 98 (2007) 032501

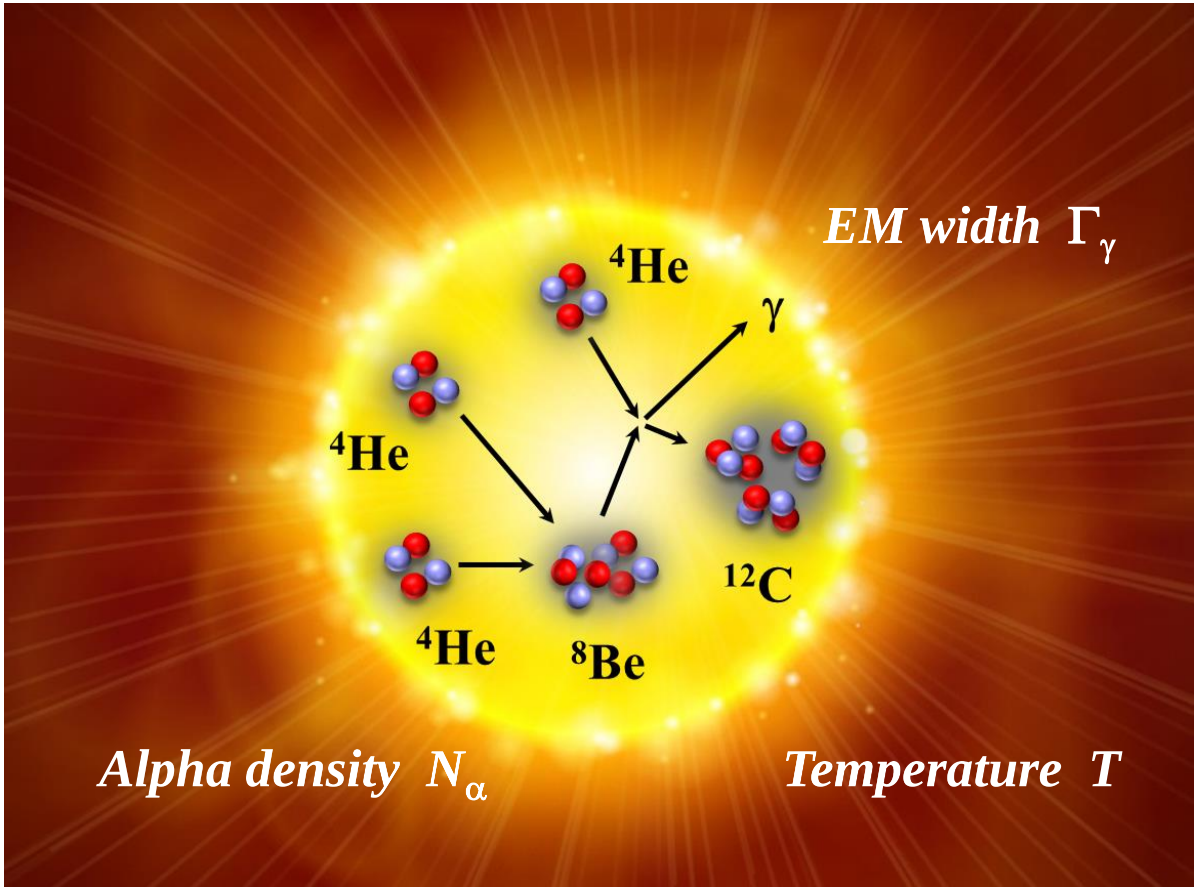
and no-core shell model

Forssen, Roth, Navratil, arXiv:1110.0634v2

Light quark mass dependence of helium burning



Triple alpha reaction rate



$$r_{3\alpha} \propto \Gamma_\gamma \, (N_\alpha / k_B T)^3 \times \exp(-\varepsilon / k_B T)$$

$\varepsilon = E_h - 3E_\alpha$ Hoyle relative to triple-alpha

Is nature fine-tuned?

$$\varepsilon = E_h - 3E_\alpha = 379\,\mathrm{keV}$$

$\varepsilon > 479\,\mathrm{keV}$

Less resonance enhancement.
Rate of carbon production smaller by several orders of magnitude.
Low carbon abundance is unfavorable for carbon-based life.

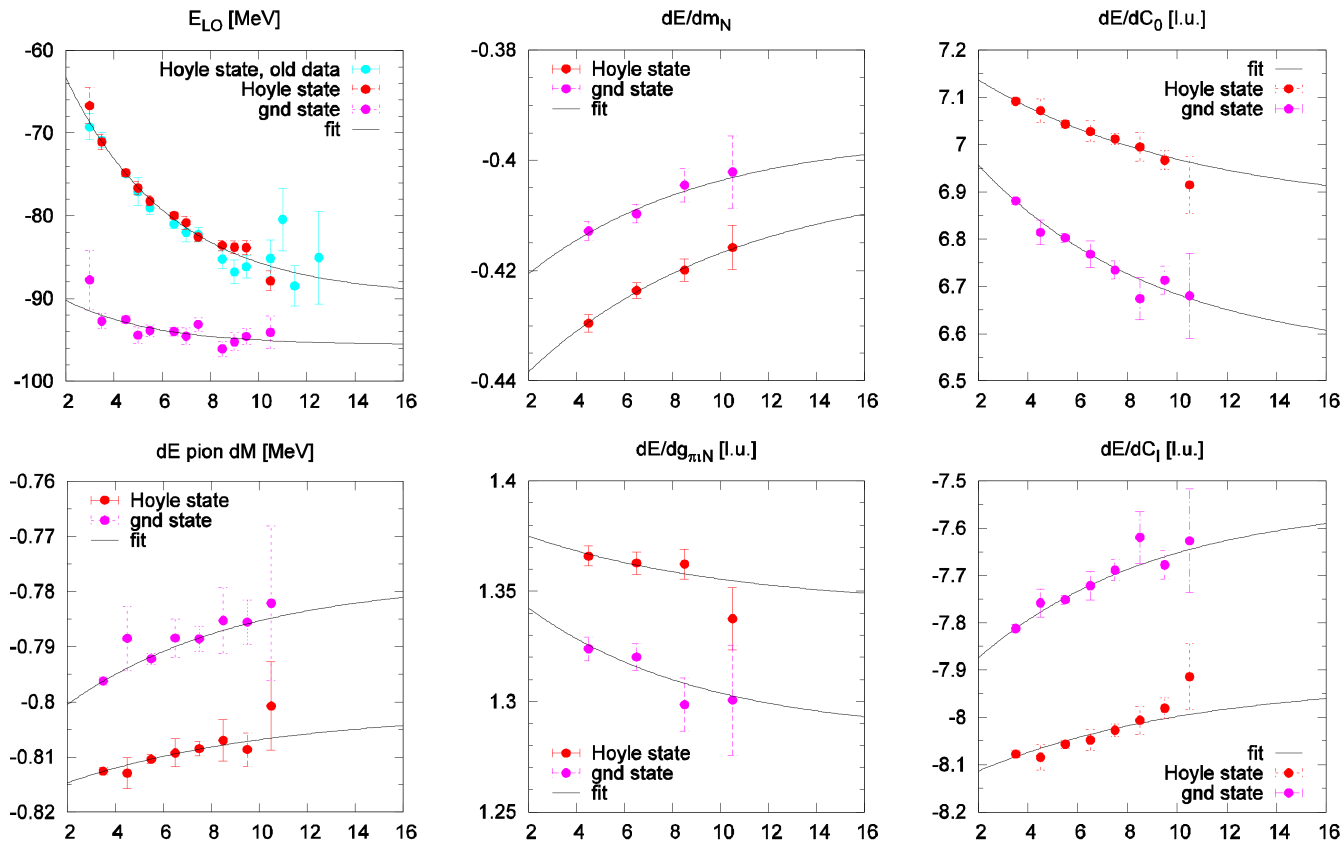
$\varepsilon < 279\,\mathrm{keV}$

Carbon production occurs at lower stellar temperatures and oxygen production greatly reduced.
Low oxygen abundance is unfavorable for carbon-based life.

Schlattl et al., Astrophys. Space Sci., 291, 27–56 (2004)

We investigate the dependence on the fundamental parameters of the standard model such as the light quark masses. Can be parameterized by the pion mass.

Lattice results for pion mass dependence



$\Delta E_h = E_h - E_b - E_\alpha$	Hoyle relative to Be-8-alpha
$\Delta E_b = E_b - 2E_\alpha$	Be-8 relative to alpha-alpha
$\varepsilon = E_h - 3E_\alpha$	Hoyle relative to triple-alpha

$$\left.\frac{\partial \Delta E_h}{\partial M_\pi}\right|_{M_\pi^{\mathrm{ph}}} = -0.455(35)\bar{A}_s - 0.744(24)\bar{A}_t + 0.051(19)$$

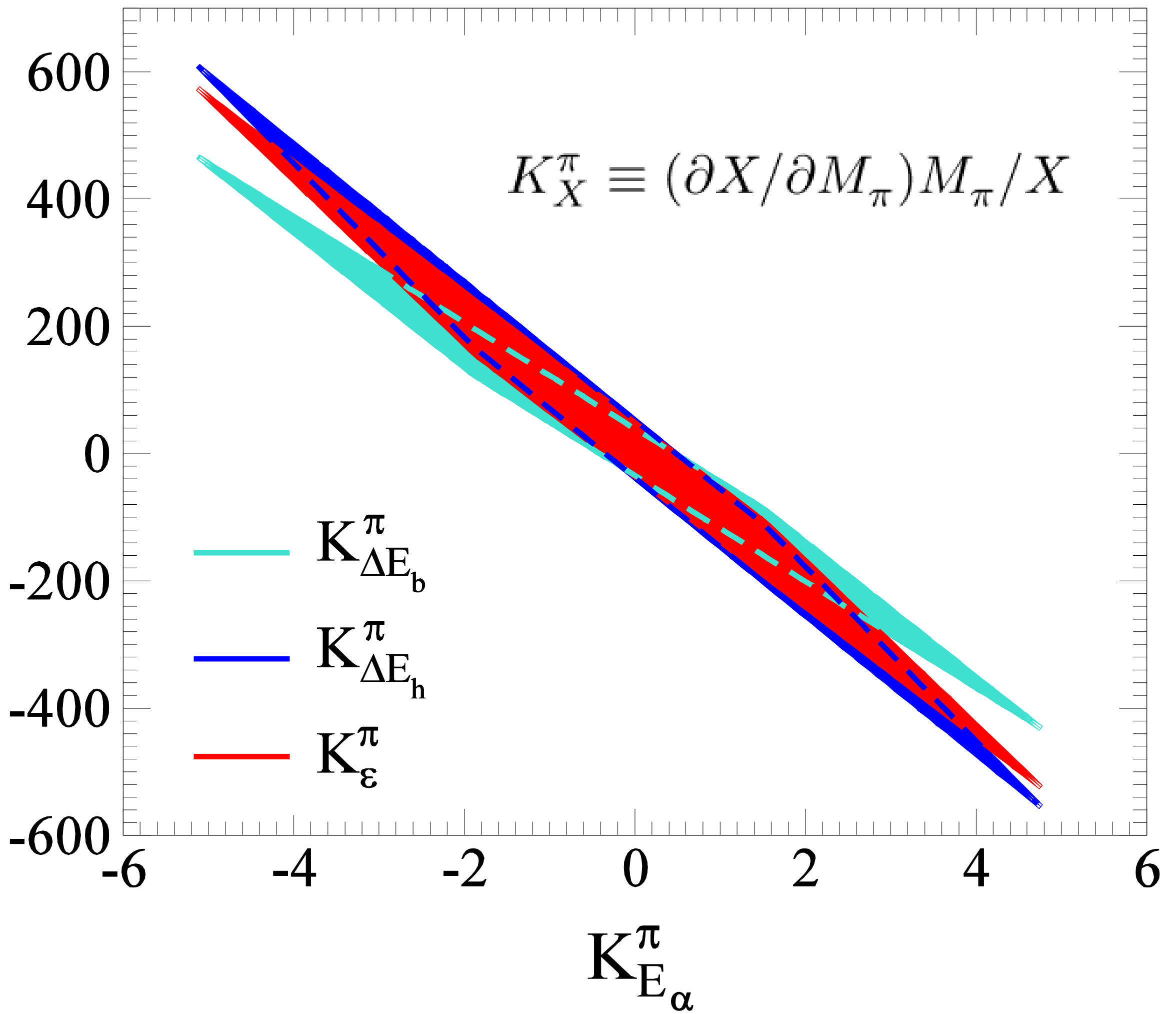
$$\left.\frac{\partial \Delta E_b}{\partial M_\pi}\right|_{M_\pi^{\mathrm{ph}}} = -0.117(34)\bar{A}_s - 0.189(24)\bar{A}_t + 0.013(12)$$

$$\left.\frac{\partial \varepsilon}{\partial M_\pi}\right|_{M_\pi^{\mathrm{ph}}} = -0.572(19)\bar{A}_s - 0.933(15)\bar{A}_t + 0.064(16)$$

$$\bar{A}_s \equiv \partial a_s^{-1}/\partial M_\pi\big|_{M_\pi^{\mathrm{ph}}} \qquad \bar{A}_t \equiv \partial a_t^{-1}/\partial M_\pi\big|_{M_\pi^{\mathrm{ph}}}$$

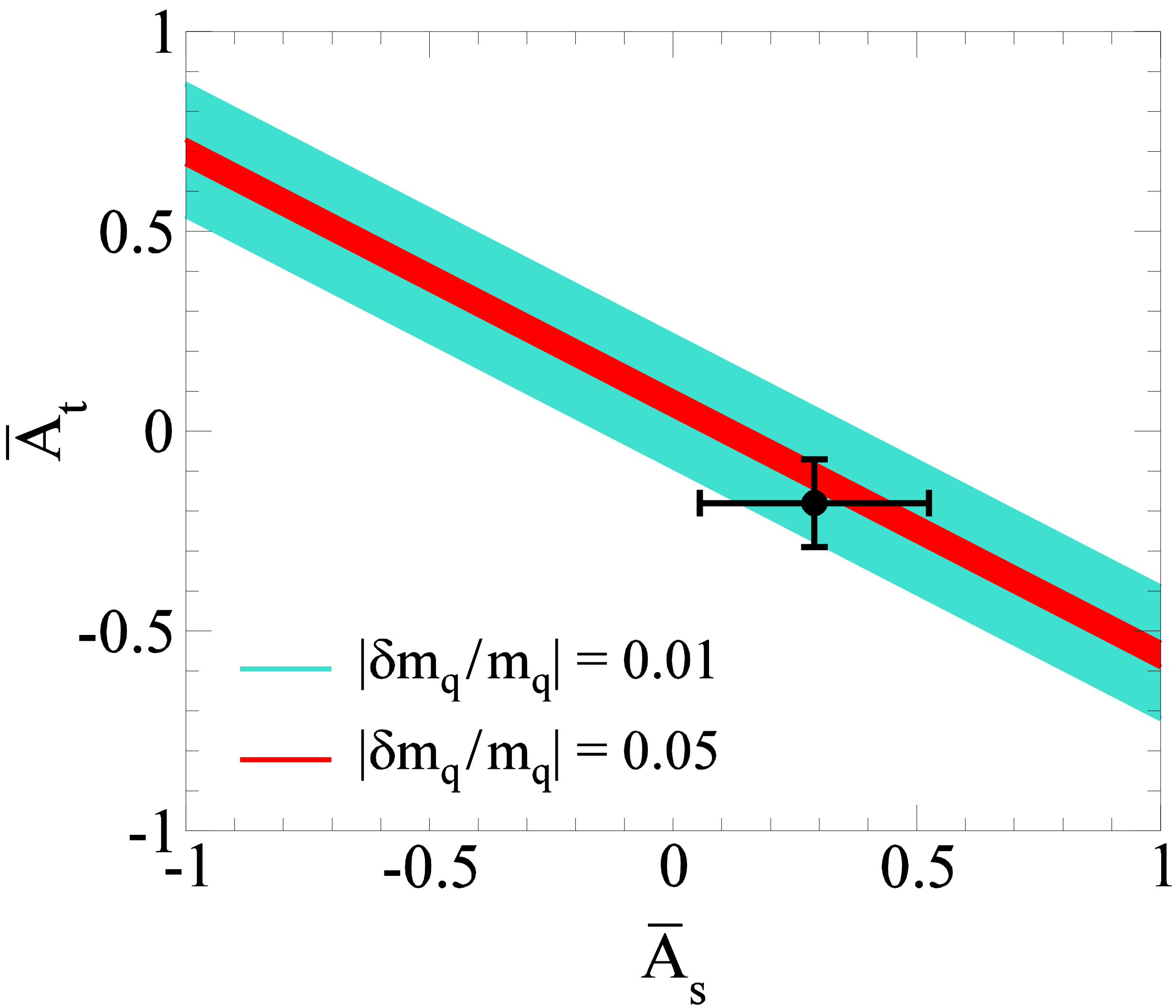
Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 110 (2013) 112502; ibid., arXiv:1303.4856
Berengut et al., arXiv:1301.1738

Evidence for correlation with alpha binding energy



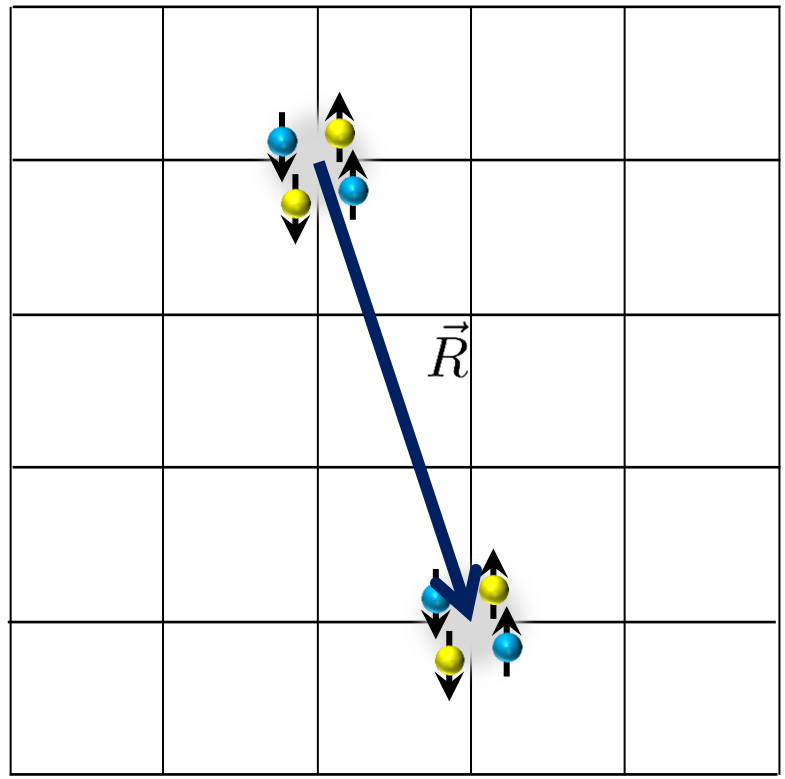
Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 110 (2013) 112502; ibid., arXiv:1303.4856

“End of the world” plot



Towards scattering and reactions on the lattice

Projected adiabatic matrix method

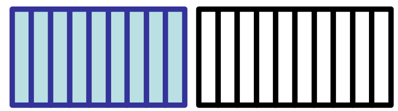


Using cluster wavefunctions for continuum scattering states

initial state $|\vec{R}>$

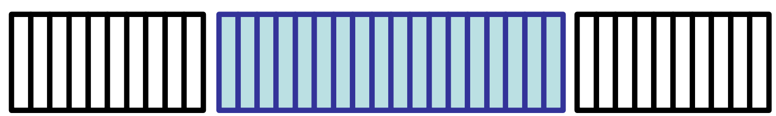
Use projection Monte Carlo to propagate cluster wavefunctions in Euclidean time

$$|\vec{R}>_t = e^{-Ht}|\vec{R}>$$

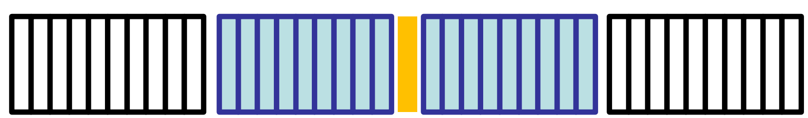
$$|\vec{R}>_t =$$  $$|\vec{R}>$$

Construct a norm matrix and matrix of expectation values

$$<N>_t = {}_t\!<\vec{R}'|\vec{R}>_t =$$

$$<\vec{R}'|$$  $$|\vec{R}>$$

$$<O>_t = {}_t\!<\vec{R}'|O|\vec{R}>_t =$$

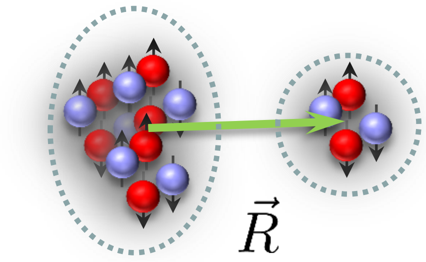
$$<\vec{R}'|$$  $$|\vec{R}>$$

Compute the projected adiabatic matrix

$$< O >_{\text{adiab}} =< N >_t^{-1/2} < O >_t < N >_t^{-1/2}$$

Projected adiabatic Hamiltonian is now an effective two-body Hamiltonian. Similar in spirit to no-core shell model with resonating group method.

But some differences. Distortion of the nucleus wavefunctions is automatic due to projection in Euclidean time.



Allow for several coupled channels to calculate capture reactions, exchange processes, and break up processes

$$1 + 2 \to 3 + \gamma$$

$$1 + 2 \to 3 + 4$$

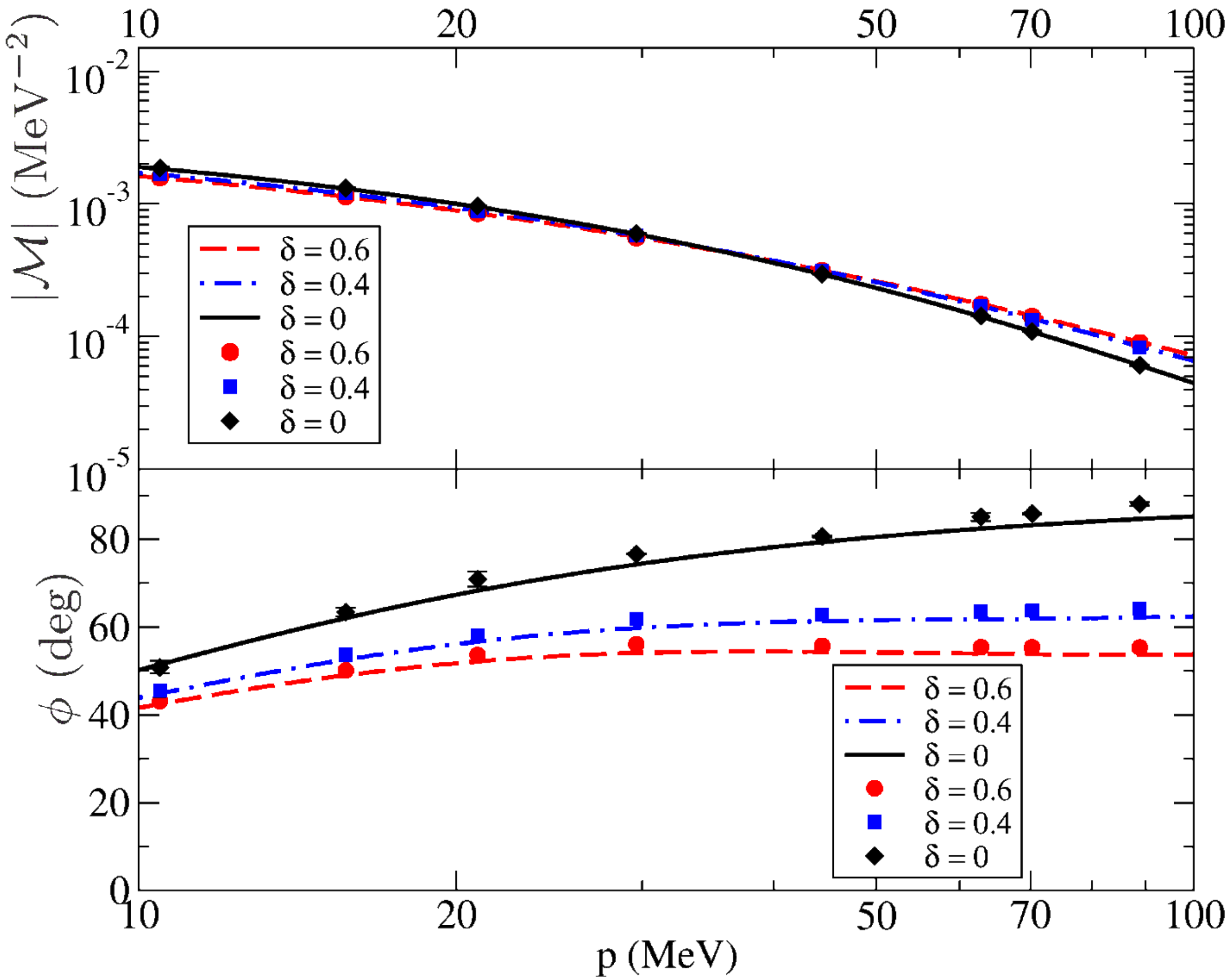
$$1 + 2 \to 3 + 4 + 5$$

We have tested the lattice capture reaction formalism by calculating the M1 amplitude for radiative neutron capture on proton

$$n + p \to d + \gamma$$

at leading order in pionless effective field theory.

$n + p \to d + \gamma$ (M1)



Rupak, D.L., arXiv:1302.4158 [nucl-th]

Summary and future directions

A golden age for nuclear theory from first principles. Big science discoveries being made and many more around the corner.

Lattice effective field theory is a relatively new and promising tool that combines the framework of effective field theory and computational lattice methods. May play a significant role in the future of *ab initio* nuclear theory.

Topics to be addressed in the near future…

Different lattice spacings, spectrum of oxygen-16, adiabatic Hamiltonians for scattering and reactions, alpha clustering in nuclei, transition from S-wave to P-wave pairing in superfluid neutron matter, weak matrix elements, etc.