Nucleon structure near the physical pion mass

Jeremy Green

Center for Theoretical Physics
Massachusetts Institute of Technology

January 4, 2013

Biographical information

Undergraduate: 2003–2007, University of Toronto

Hon. B. Sc., Math and Physics
Summer research:

- 2004, 2005: knot theory
- 2006: particle physics (ATLAS)

Graduate: 2007–2013 (expected), MIT

Ph. D, Department of Physics
Research (advisor: John Negele): Lattice QCD

- spatial diquark correlations
- nucleon structure

Outline

- Introduction
  - Isovector nucleon observables
  - State of past lattice calculations
- Lattice methodology
  - Challenges
  - BMW action; ensembles
  - Systematic errors
- Results
  - Vector form factors; radii and magnetic moment
  - Momentum fraction
  - Axial charge
  - Tensor and scalar charges
- Conclusions

Nucleon observables

Electromagnetic form factors:

$$\langle p(P', s')|\overline{q}\gamma^\mu q|p(P, s)\rangle = \overline{u}(p', s')\left(\gamma^\mu F_1^q(Q^2) + i\sigma^{\mu\nu}\frac{\Delta_\nu}{2m_p}F_2^q(Q^2)\right)u(p, s),$$

where $\Delta = P' - P$, $Q^2 = -\Delta^2$.
Quark momentum fraction:

$$\langle p(P, s')|\overline{q}\gamma_{\{\mu}D_{\nu\}}q|p(P, s')\rangle = \langle x\rangle_q\overline{u}(P, s')\gamma_{\{\mu}P_{\nu\}}u(P, s)$$

Axial, tensor, and scalar charges:

$$\langle p(P, s')|\overline{u}\gamma^\mu\gamma_5 d|n(P, s)\rangle = g_A\overline{u}_p(P, s')\gamma^\mu\gamma_5 u_n(P, s)$$
$$\langle p(P, s')|\overline{u}d|n(P, s)\rangle = g_S\overline{u}_p(P, s')u_n(P, s)$$
$$\langle p(P, s')|\overline{u}\sigma^{\mu\nu}d|n(P, s)\rangle = g_T\overline{u}_p(P, s')\sigma^{\mu\nu}u_n(P, s)$$

Vector form factors

Dirac and Pauli form factors:

$$\langle p(P',s')|\overline{q}\gamma^\mu q|p(P,s)\rangle = \overline{u}(p',s')\left(\gamma^\mu F_1^q(Q^2) + i\sigma^{\mu\nu}\frac{\Delta_\nu}{2m_p}F_2^q(Q^2)\right)u(p,s),$$

where $\Delta = P' - P$, $Q^2 = -\Delta^2$.

- Isovector combination:

$$F_{1,2}^v = F_{1,2}^u - F_{1,2}^d = F_{1,2}^p - F_{1,2}^n,$$

  where $F_{1,2}^{p,n}$ are form factors of the electromagnetic current in a proton and in a neutron.
- Dirac and Pauli radii defined via slope at $Q^2 = 0$:

$$F_{1,2}(Q^2) = F_{1,2}(0)(1 - \tfrac{1}{6}r_{1,2}^2 Q^2 + O(Q^4));$$

  $F_2(0) = \kappa$, the anomalous magnetic moment.
- Proton charge radius, $(r_E^2)^p = (r_1^2)^p + \frac{3\kappa^p}{2m_p^2}$, has 7σ discrepancy between measurements from e–p scattering and from Lamb shift in muonic hydrogen.

Quark momentum fraction

Measure forward matrix element of quark energy-momentum operator:

$$\langle p(P,s')|\overline{q}\gamma_{\{\mu}D_{\nu\}}q|p(P,s)\rangle = \langle x\rangle_q \overline{u}(P,s')\gamma_{\{\mu}P_{\nu\}}u(P,s).$$

$\langle x\rangle_q$ is the average momentum fraction carried by quarks q and $\overline{q}$; focus on isovector combination $\langle x\rangle_{u-d}$.

- Phemonenological values come from measurements of parton distribution functions in deep inelastic scattering experiments.

Axial charge

"Benchmark" observable:

$$\langle p(P,s')|\overline{u}\gamma^\mu\gamma_5 d|n(P,s)\rangle = g_A \overline{u}_p(P,s')\gamma^\mu\gamma_5 u_n(P,s).$$

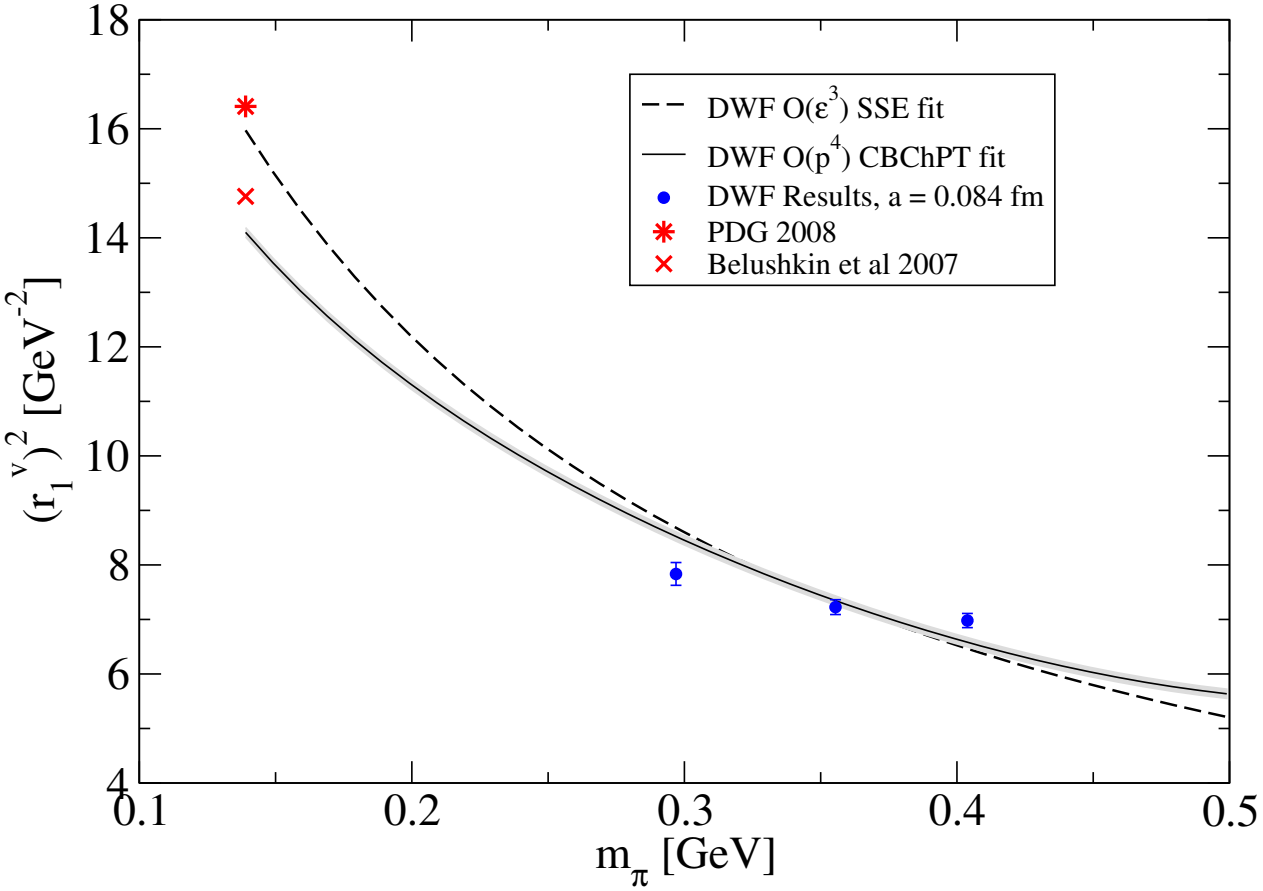
- Naturally isovector observable; PDG: $g_A/g_V = 1.2701(25)$ from β decay of polarized neutrons.
- Is also the difference between contributions from the spin of u and d quarks to the total proton spin.

Scalar and tensor charges

$$\langle p(P,s')|\overline{u}d|n(P,s)\rangle = g_S \overline{u}_p(P,s')u_n(P,s)$$
$$\langle p(P,s')|\overline{u}\sigma^{\mu\nu}d|n(P,s)\rangle = g_T \overline{u}_p(P,s')\sigma^{\mu\nu}u_n(P,s)$$

- Not measured experimentally.
- Recent interest because they are needed to know leading contributions to neutron β decay from BSM physics.
- There are plans to measure the tensor charge at JLab using the upcoming 12 GeV upgrade.

Past calculations



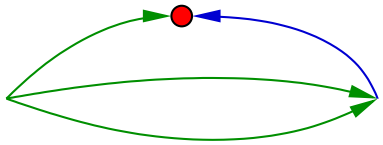
S. N. Syritsyn *et al.* (LHPC), Phys. Rev. D **81**, 034507 (2010), arXiv:0907.4194

Past calculations required large $m_\pi \to m_\pi^{\text{phys}}$ extrapolations; chiral perturbation theory allows for rapid changes, e.g. log-divergence in Dirac radius as $m_\pi \to 0$.

Lattice calculations

Two kinds of quark contractions for $\langle N(t_s)\mathcal{O}(t)\overline{N}(0)\rangle$:

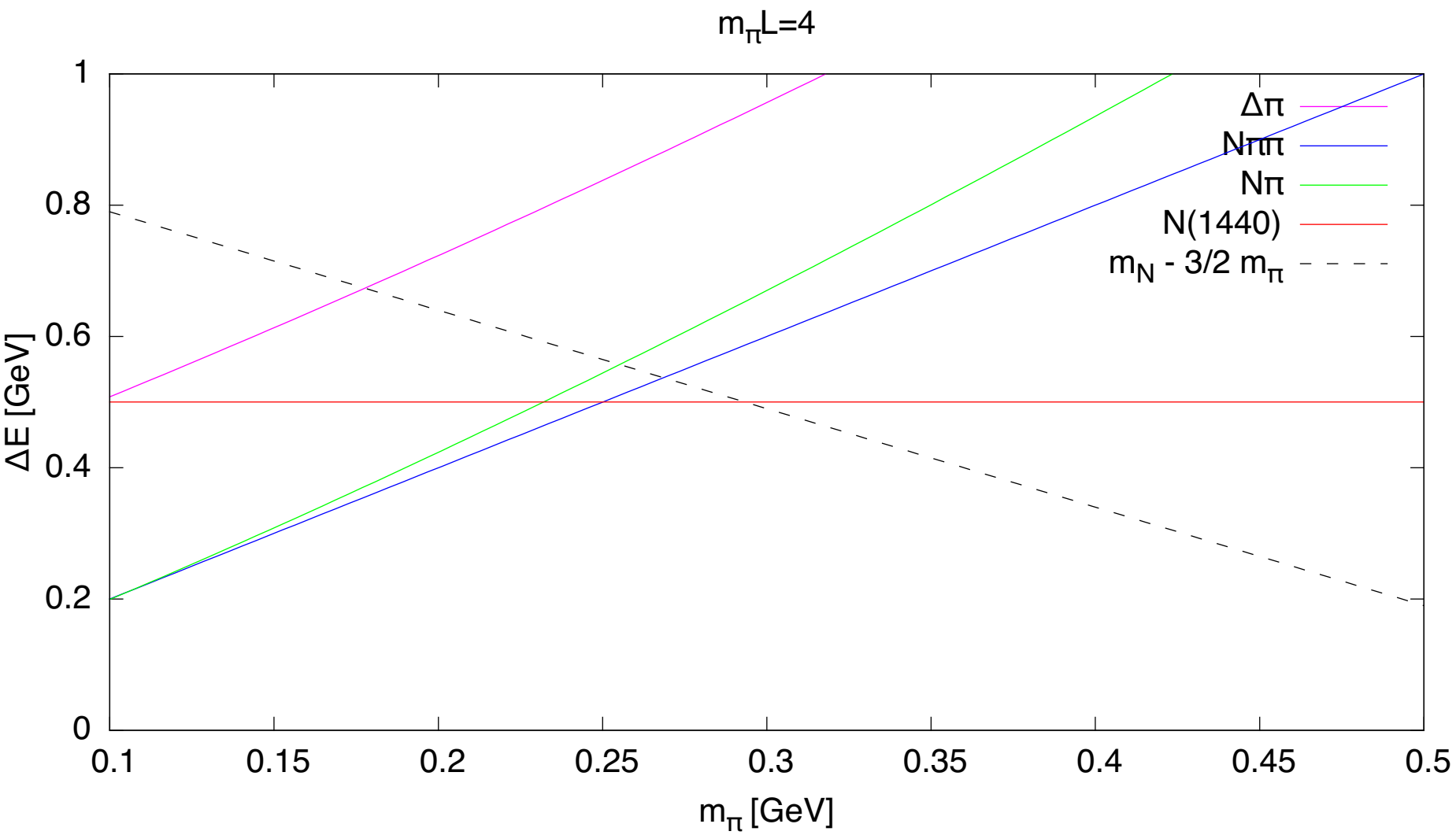
Computationally-expensive disconnected diagrams cancel for isovector quantities, so we are focusing on those for now.
Use sequential propagator through the sink: propagators from fixed source and sink allow for measurement at all intermediate times.



Cost grows linearly with number of source-sink separations T; past calculations typically used just one.

- Excited-state contamination $\sim \exp(-\Delta E T)$.
- Signal-to-noise $\sim \exp(-(m_N - \frac{3}{2}m_\pi)T)$.

Challenges at low pion mass

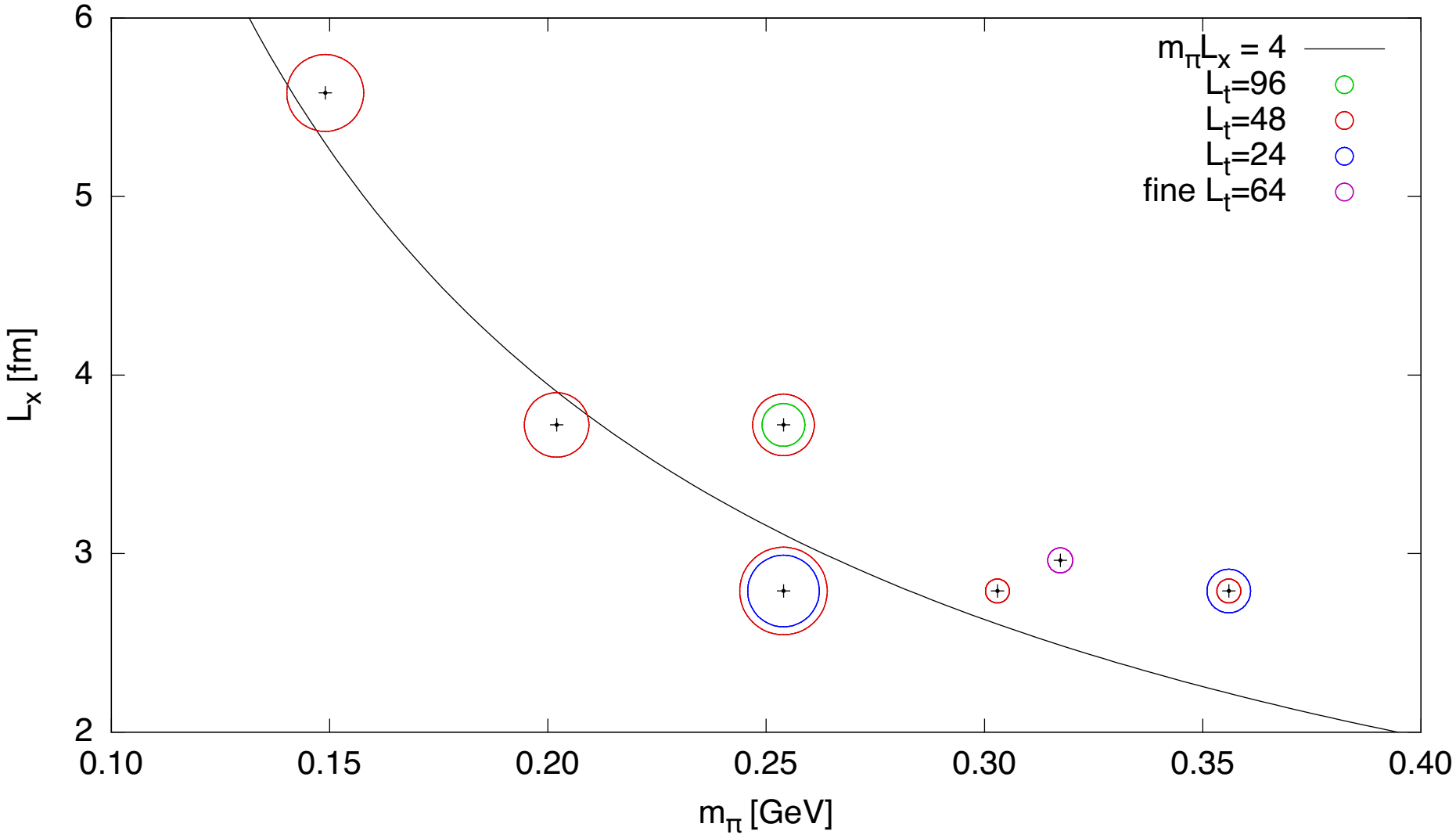


Solid curves show expected low-lying $N(\frac{1}{2}^+)$ excited states in a finite box.

BMW action and ensembles

- $N_f = 2 + 1$ tree-level $\mathcal{O}(a)$-improved Wilson fermions coupled to double-HEX-smeared gauge fields.
- Pion mass ranging from 149 MeV to 356 MeV.
- Nine coarse lattices with $a = 0.116$ fm;
  one fine lattice with $a = 0.09$ fm.
- No disconnected diagrams, so we focus on isovector observables.
- Three source-sink separations for controlling excited-state contributions: $T \in \{0.9, 1.2, 1.4\}$ fm.

Ensembles



Areas of circles scale with number of measurements: largest is 10,000.

Contributions to systematic error

From best-controlled to worst-controlled:

- Quark masses: smallest pion mass is 149 MeV, so $m_\pi \to 135$ MeV chiral extrapolation is under control.
- Excited states: use of multiple source-sink separations allows for clear identification of observables where excited states are a problem; the summation method seems to work well for reducing excited-state contamination.
- Finite volume: effects expected to be small with $m_\pi L \approx 4$, and two volumes at $m_\pi = 254$ MeV allow for fully-controlled test of finite-size effects.
- Finite temperature: at small m_π, L_t is smaller than the typically used $L_t \approx 2L_x$, but three different time extents at $m_\pi \approx 250$ MeV are useful for identifying possible problems.
- Discretization: one finer lattice at $m_\pi = 317$ MeV for consistency check, but no $a \to 0$ extrapolation.

Systematic error: excited states

Usual approach for extracting matrix elements (forward case):

$$C_{2\text{pt}}(t) = \langle N(t)\overline{N}(0)\rangle \qquad C_{3\text{pt}}(T,\tau) = \langle N(T)\mathcal{O}(\tau)\overline{N}(0)\rangle$$

- Ratio-plateau method: compute ratio

$$\begin{aligned} R(T,\tau) &= C_{3\text{pt}}(T,\tau)/C_{2\text{pt}}(T) \\ &= c_{00} + c_{10}e^{-\Delta E\tau} + c_{01}e^{-\Delta E(T-\tau)} + c_{11}e^{-\Delta ET} + \ldots, \end{aligned}$$

  where c_{00} is the desired ground-state matrix element. Then average a fixed number of points around $\tau = T/2$, yielding asymptotic errors that fall off as $e^{-\Delta ET/2}$.

- Summation method: compute sums

$$S(T) = \sum_{\tau} R(T,\tau) = b + c_{00}T + dTe^{-\Delta ET} + \ldots,$$

  then find their slope, which gives c_{00} with errors that fall off as $Te^{-\Delta ET}$.

Results

Collaborators:

- JRG, J. W. Negele, A. V. Pochinsky (MIT)
- S. N. Syritsyn (LBNL)
- M. Engelhardt (New Mexico State U.)
- S. Krieg (FZ Jülich & U. Wuppertal)

Nucleon structure from Lattice QCD using a nearly physical pion mass
arXiv:1209.1687

- $(r_1^2)^v$, κ^v, $(r_2^2)^v$, $\langle x \rangle_{u-d}$, g_A

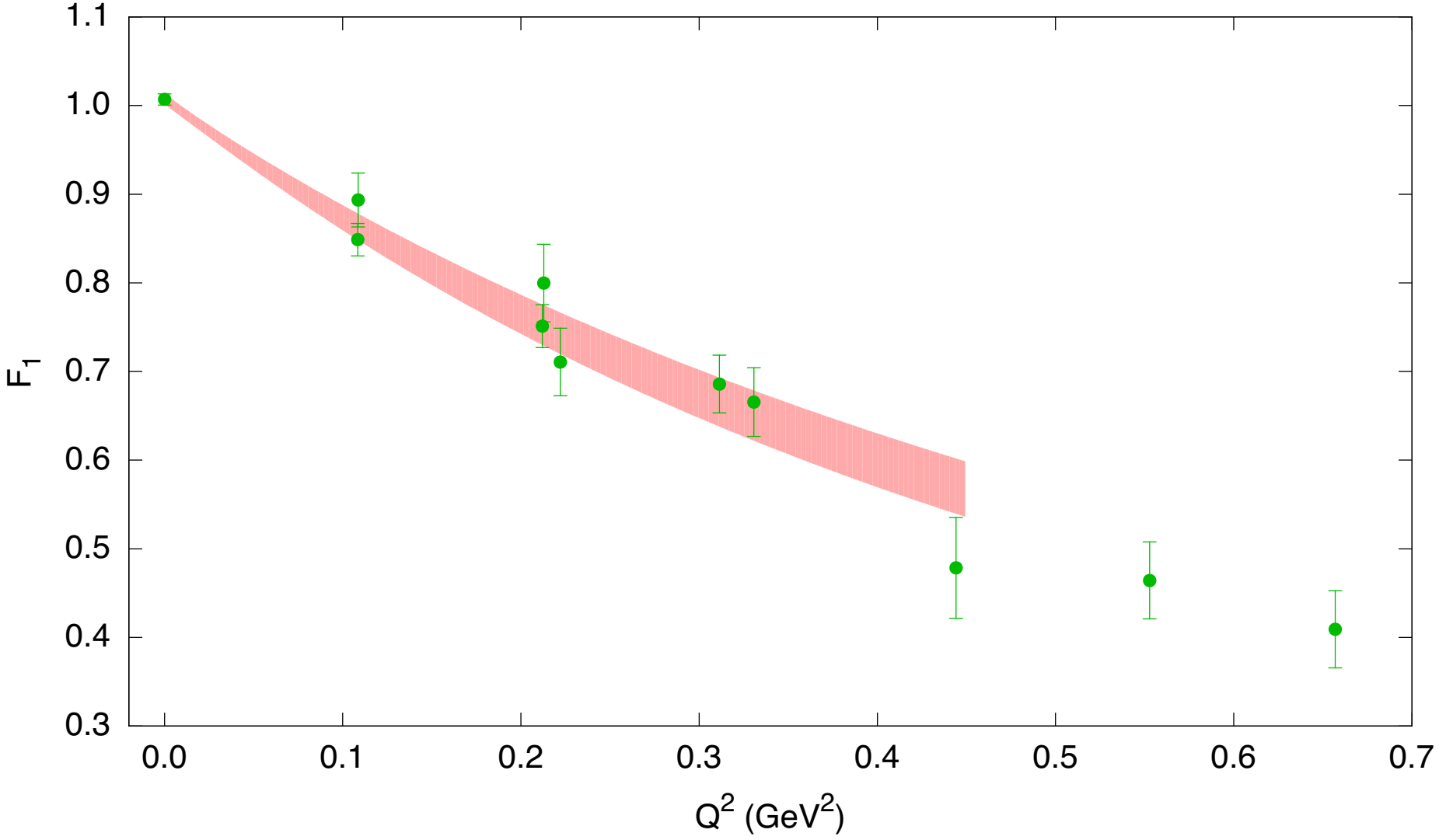
Nucleon scalar and tensor charges from Lattice QCD with light Wilson quarks
arXiv:1206.4527, Phys. Rev. D **86**, 114509

- g_S, g_T

Nucleon structure with pion mass down to 149 MeV
arXiv:1211.0253, PoS(Lattice 2012)170

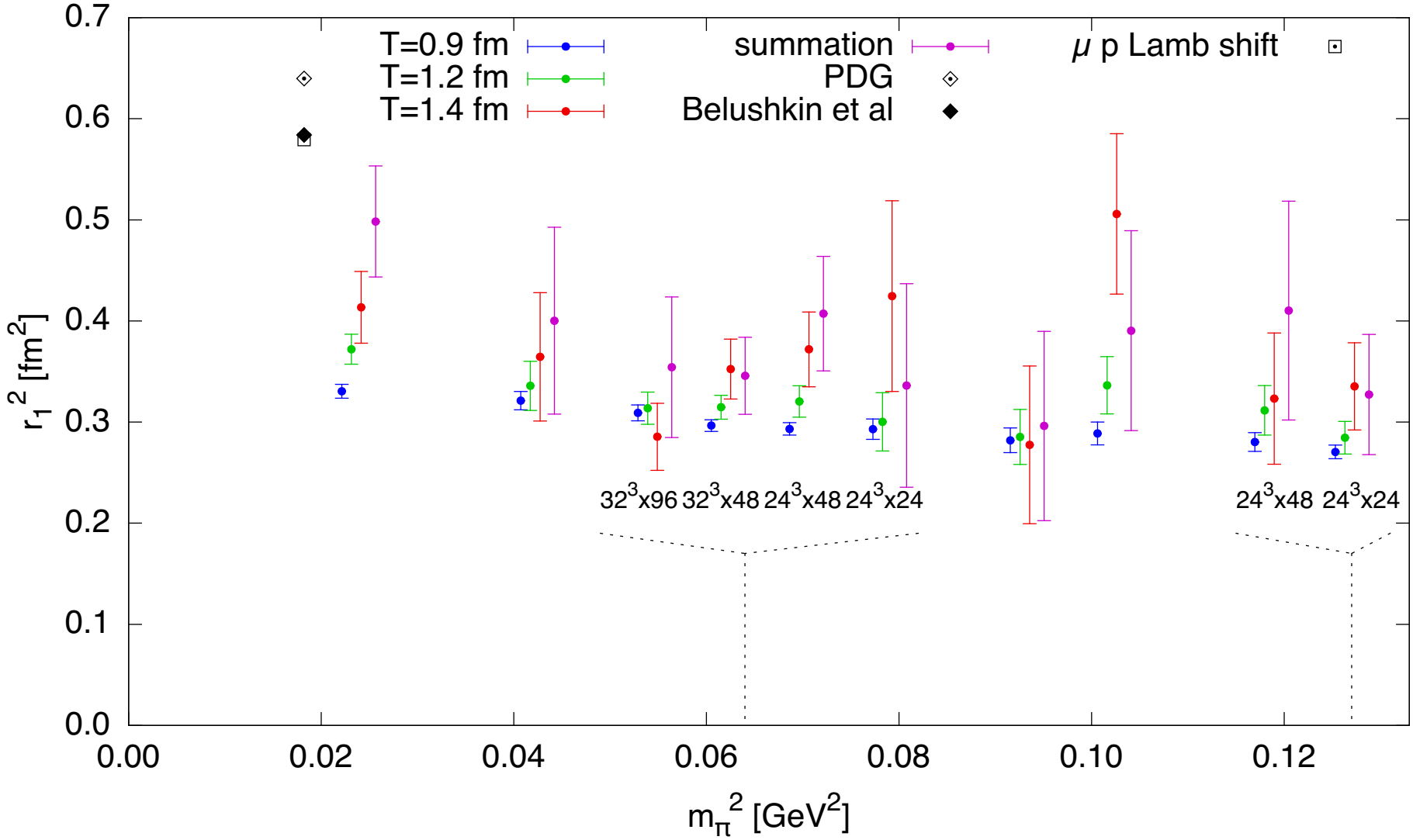
- $(r_1^2)^v$, g_A, g_S, g_T

$F_1^v(Q^2)$: $m_\pi = 254$ MeV, $32^3 \times 48$, summation

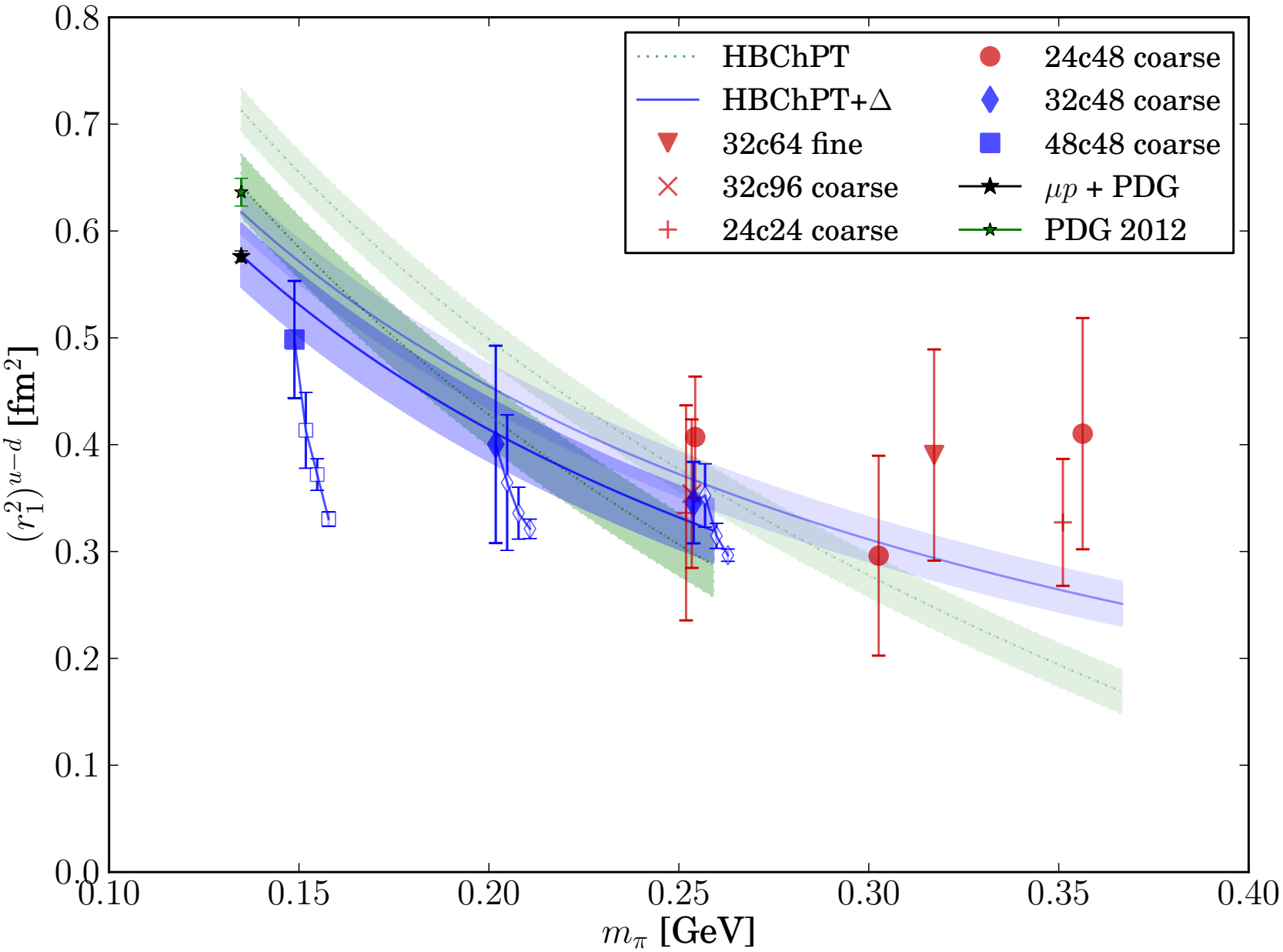


Dipole fit $F_1(Q^2) \sim \frac{F_1(0)}{(1+Q^2/M_D^2)^2}$ to range $0 \leq Q^2 \leq 0.5$ GeV^2.

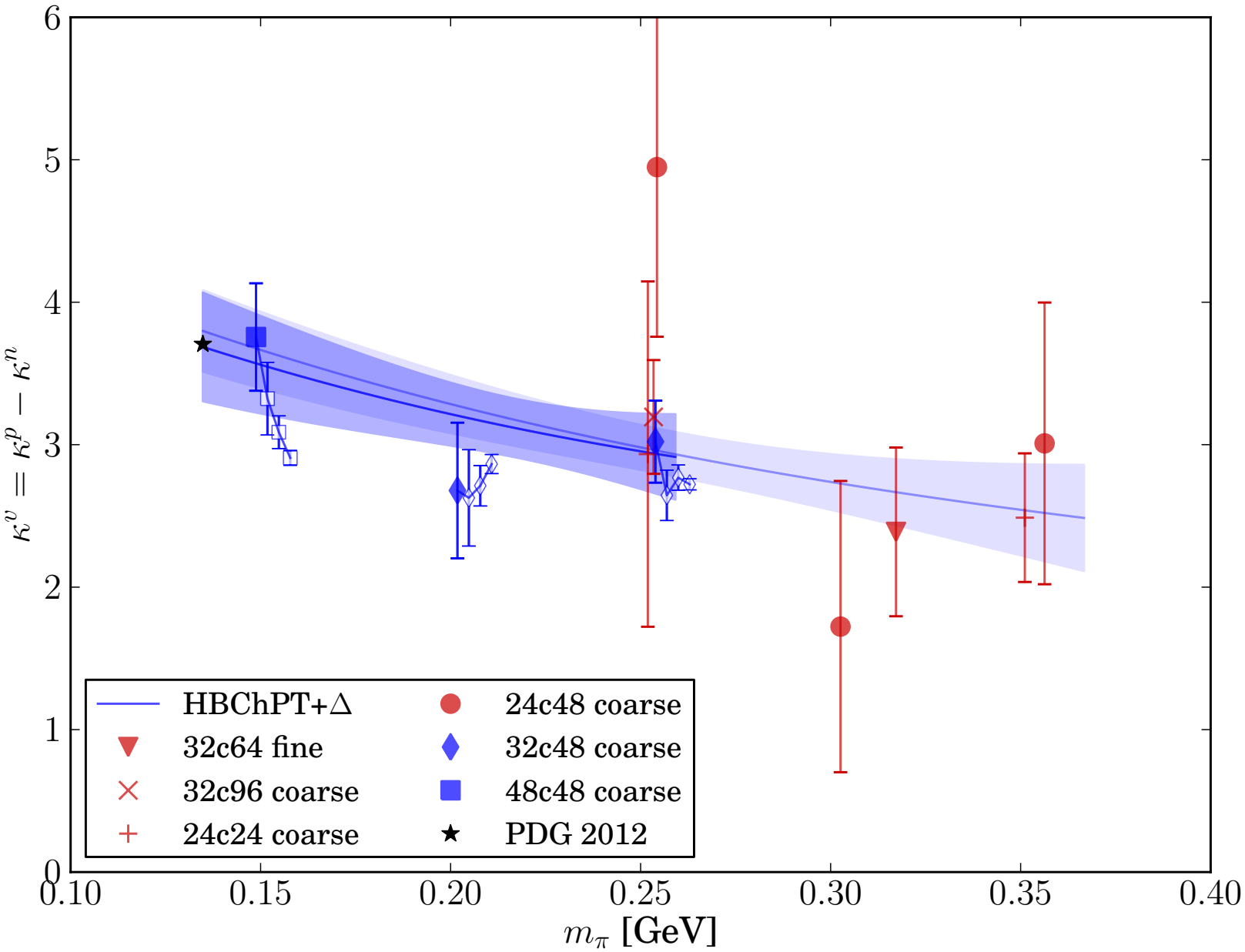
Isovector Dirac radius $(r_1^2)^v$



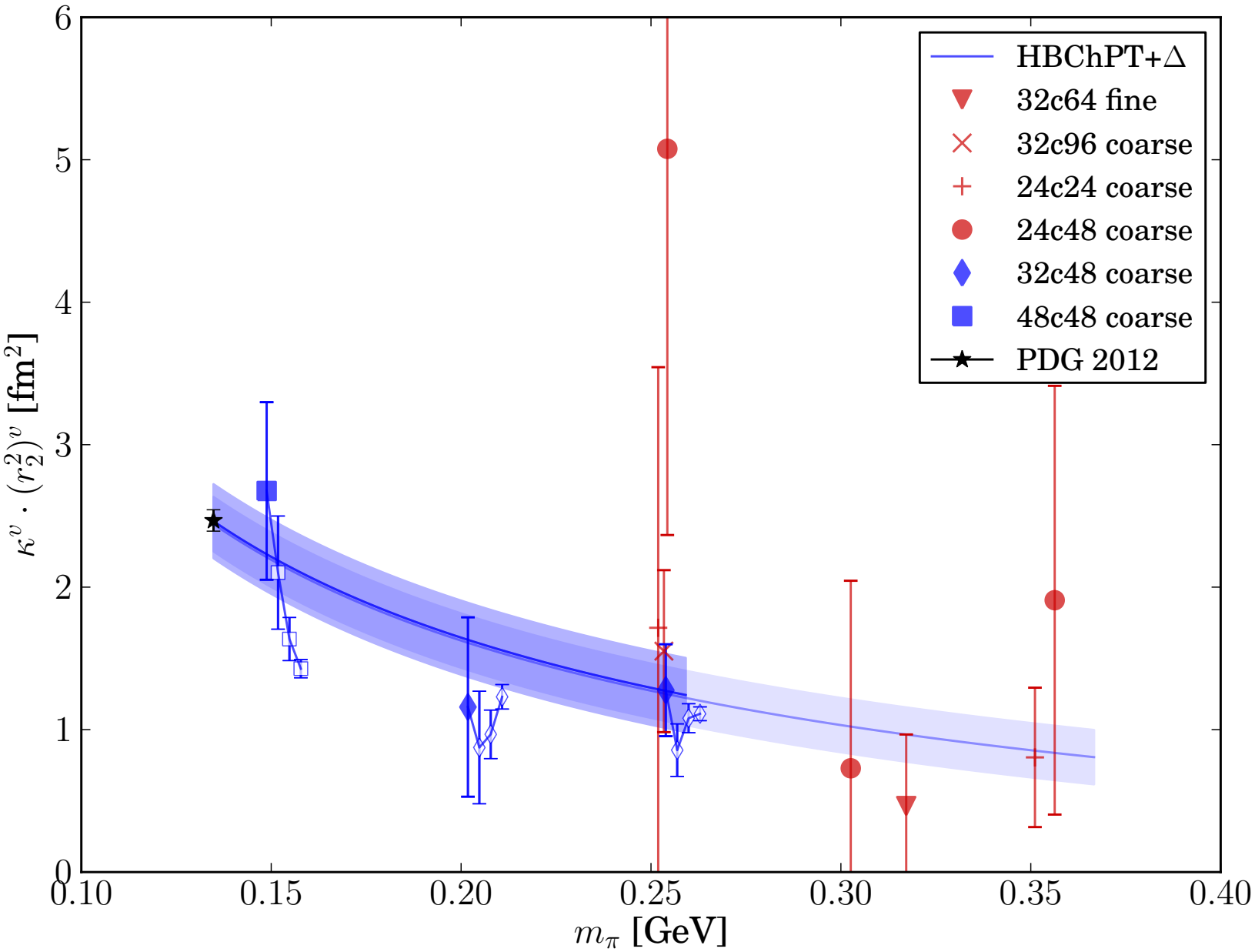
$(r_1^2)^v$: chiral extrapolation of summation points



κ^v: chiral extrapolation of summation points



$\kappa^v (r_2^2)^v$: chiral extrapolation of summation points

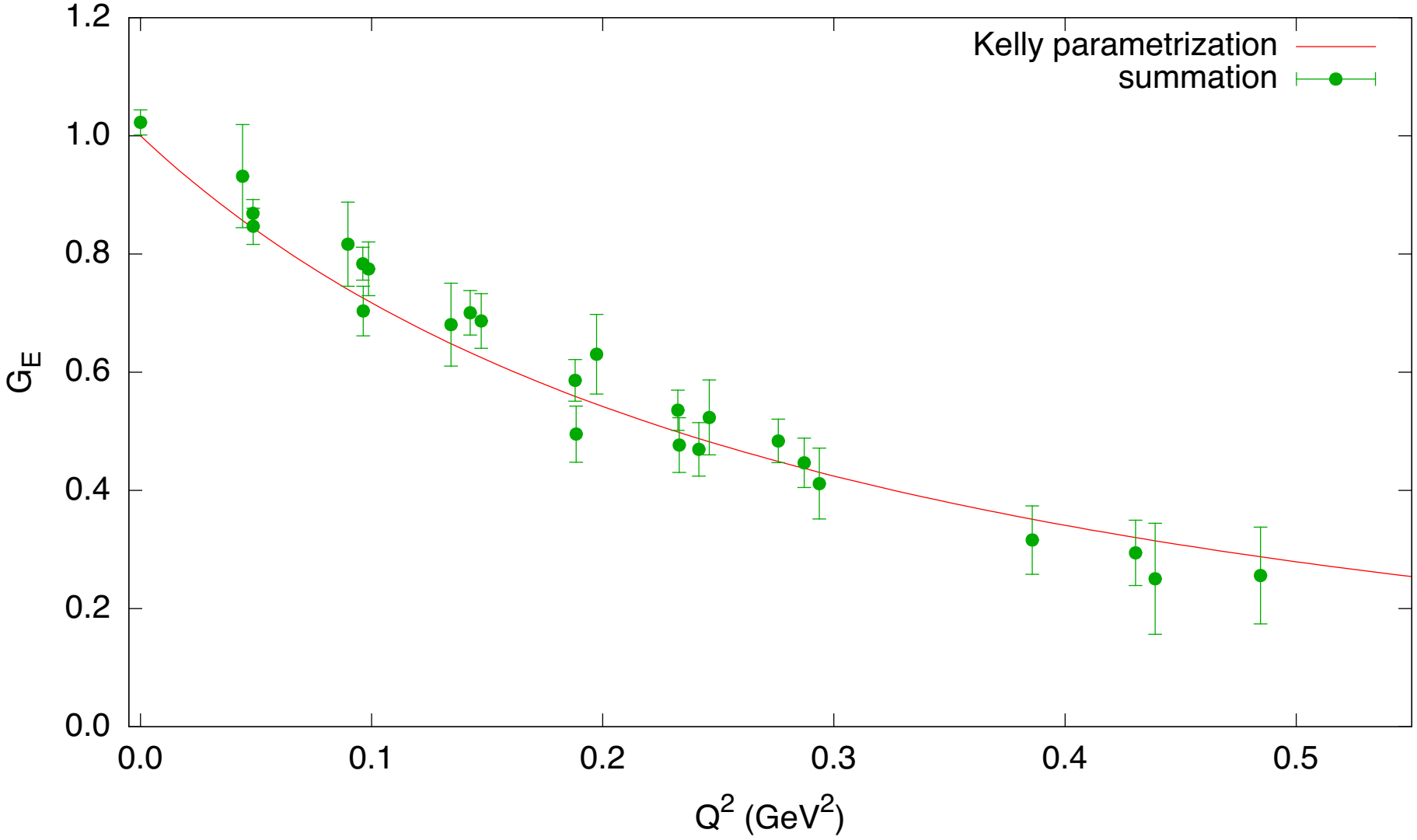


Sachs form factors

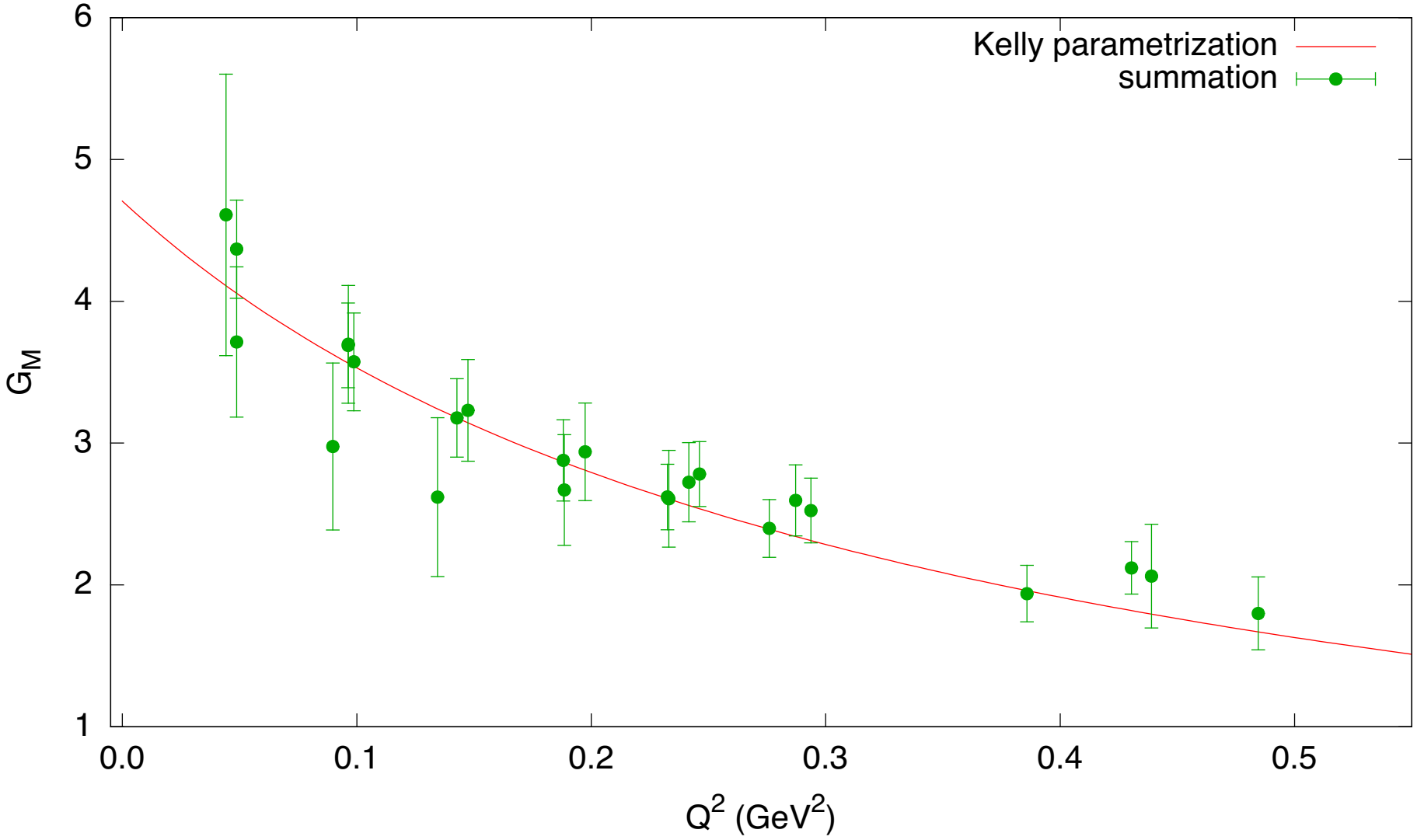
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{2m_N} F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

- Slopes at $Q^2 = 0$ give rms charge and magnetic radii.
- Compare:
  1. Isovector $m_\pi = 149$ MeV, summation data from lattice calculation.
  2. Parameterization of experimental data:
     J. J. Kelly, "Simple parametrization of nucleon form factors," Phys. Rev. C **70**, 068202 (2004).
     4 parameters for each of G_{Ep}, G_{Mp}, G_{Mn}; 2 parameters for G_{En}: determined from fit to experiment.

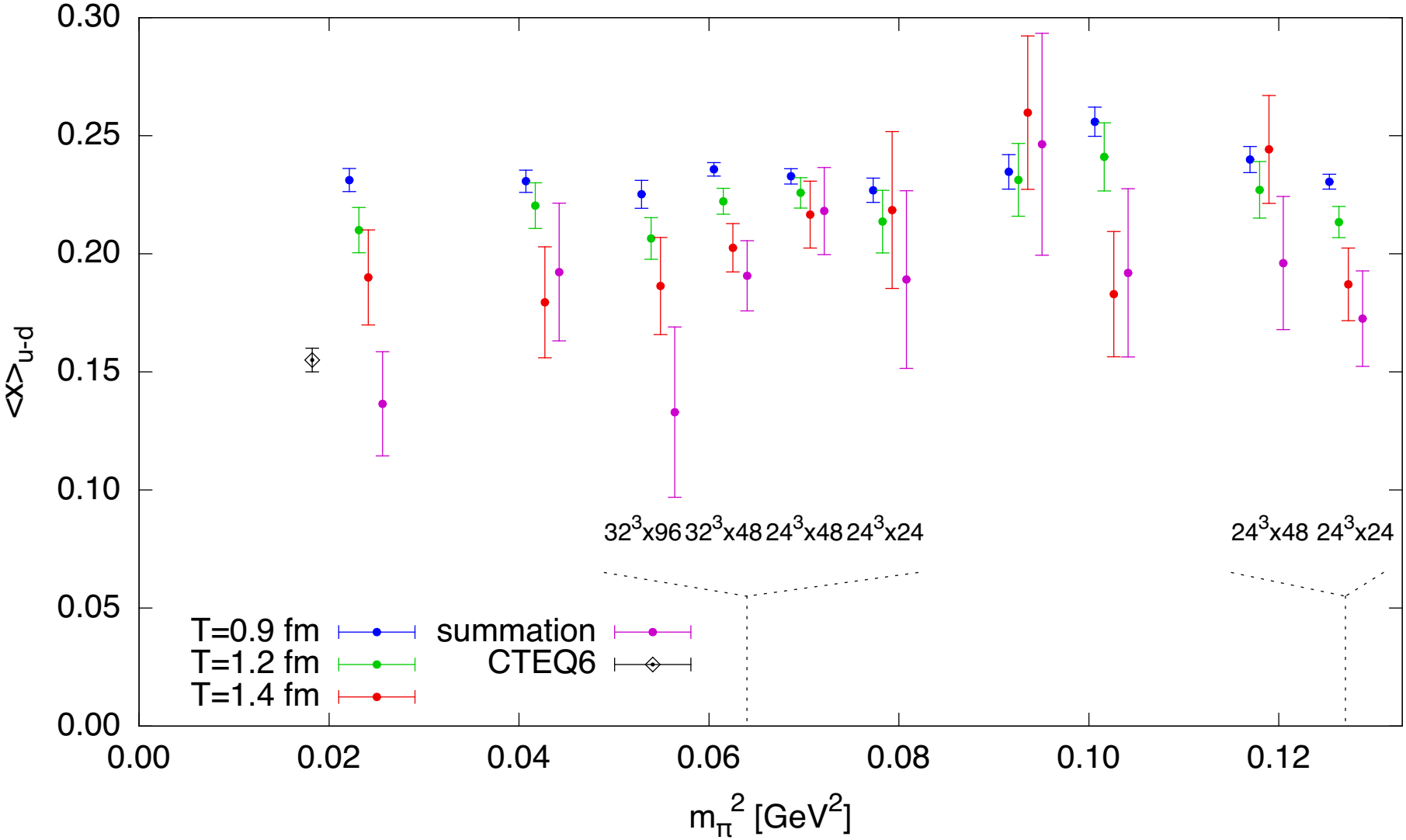
$G_E^v(Q^2)$: lowest pion mass



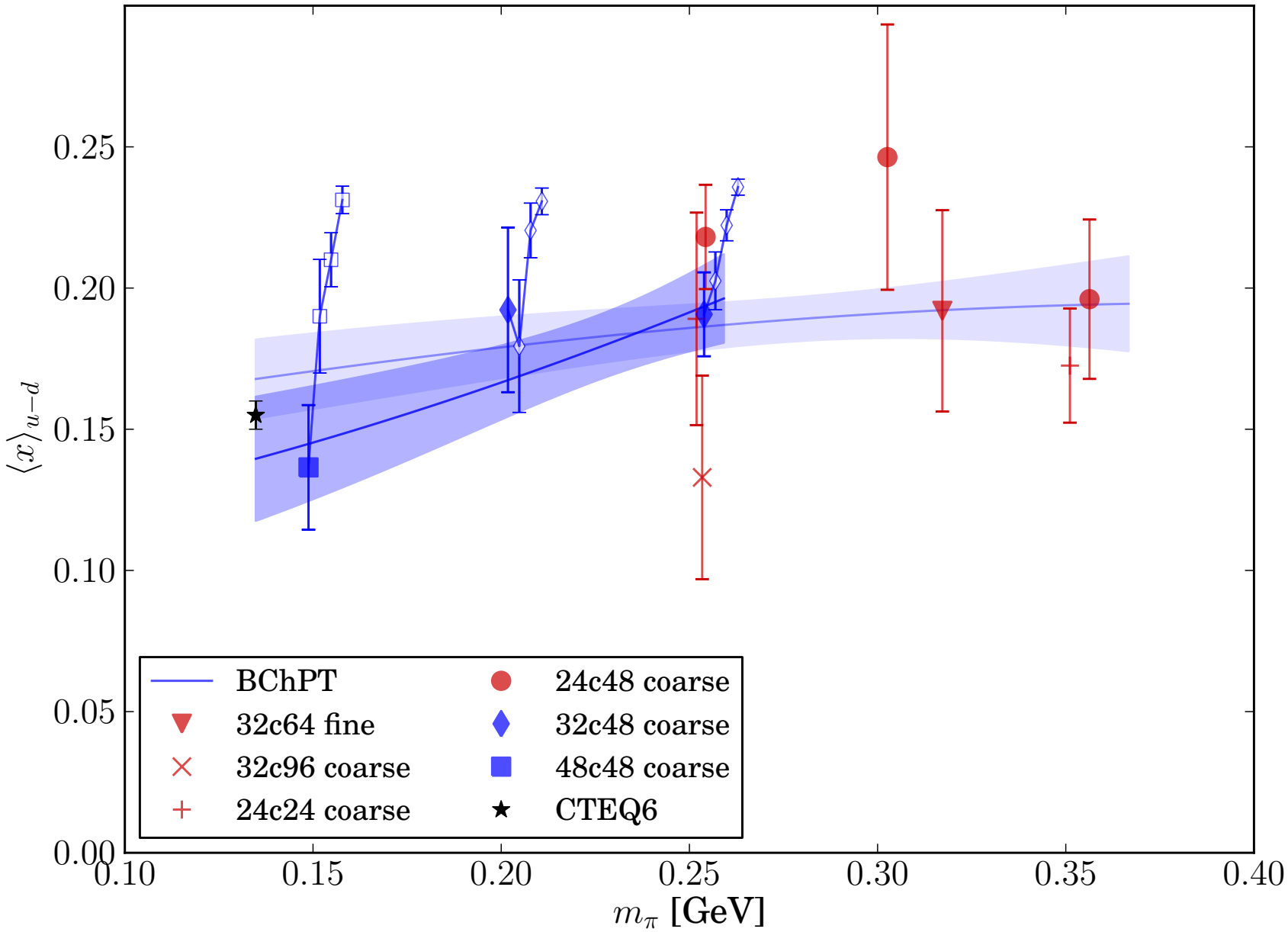
$G_M^v(Q^2)$: lowest pion mass



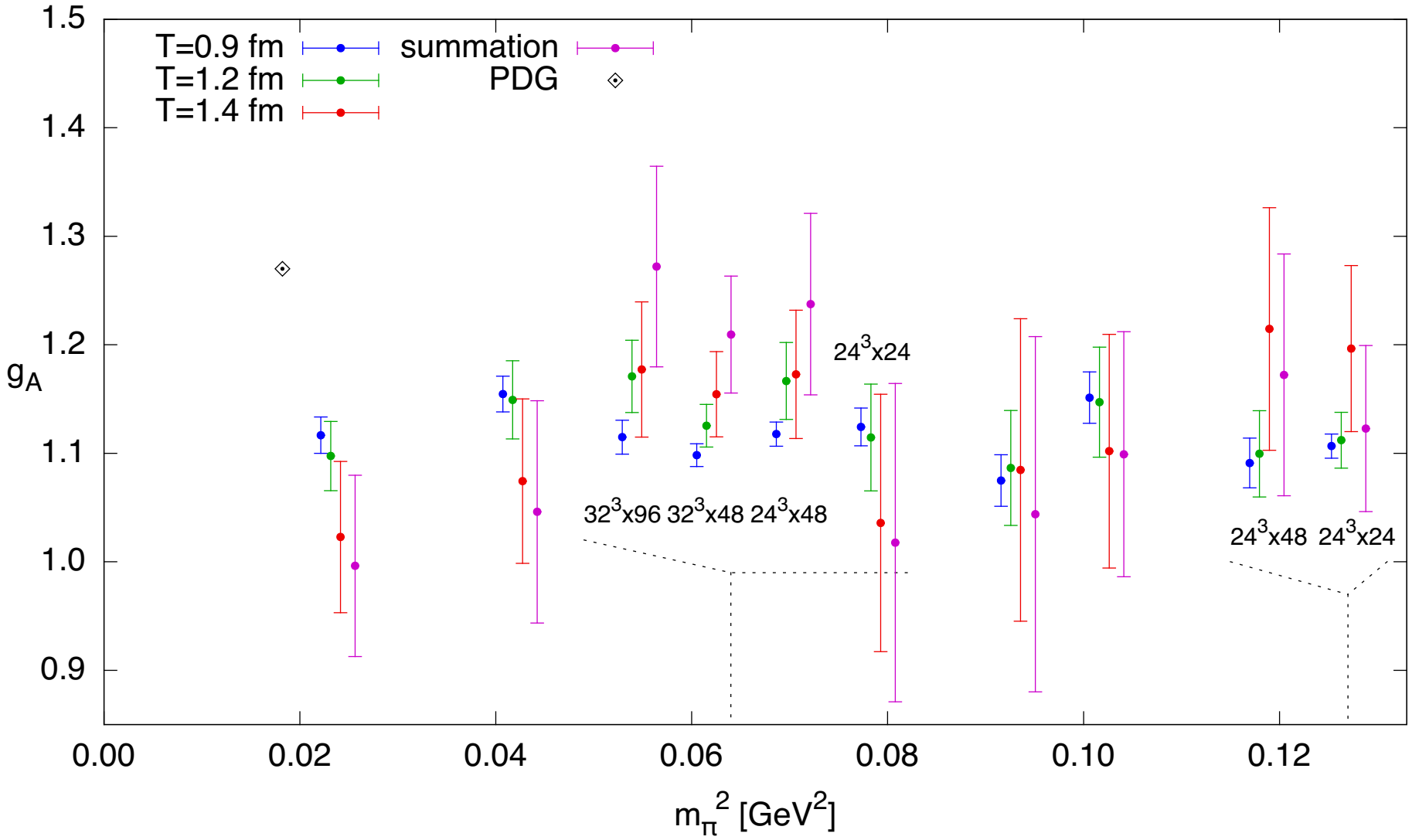
Isovector quark momentum fraction $\langle x \rangle_{u-d}$



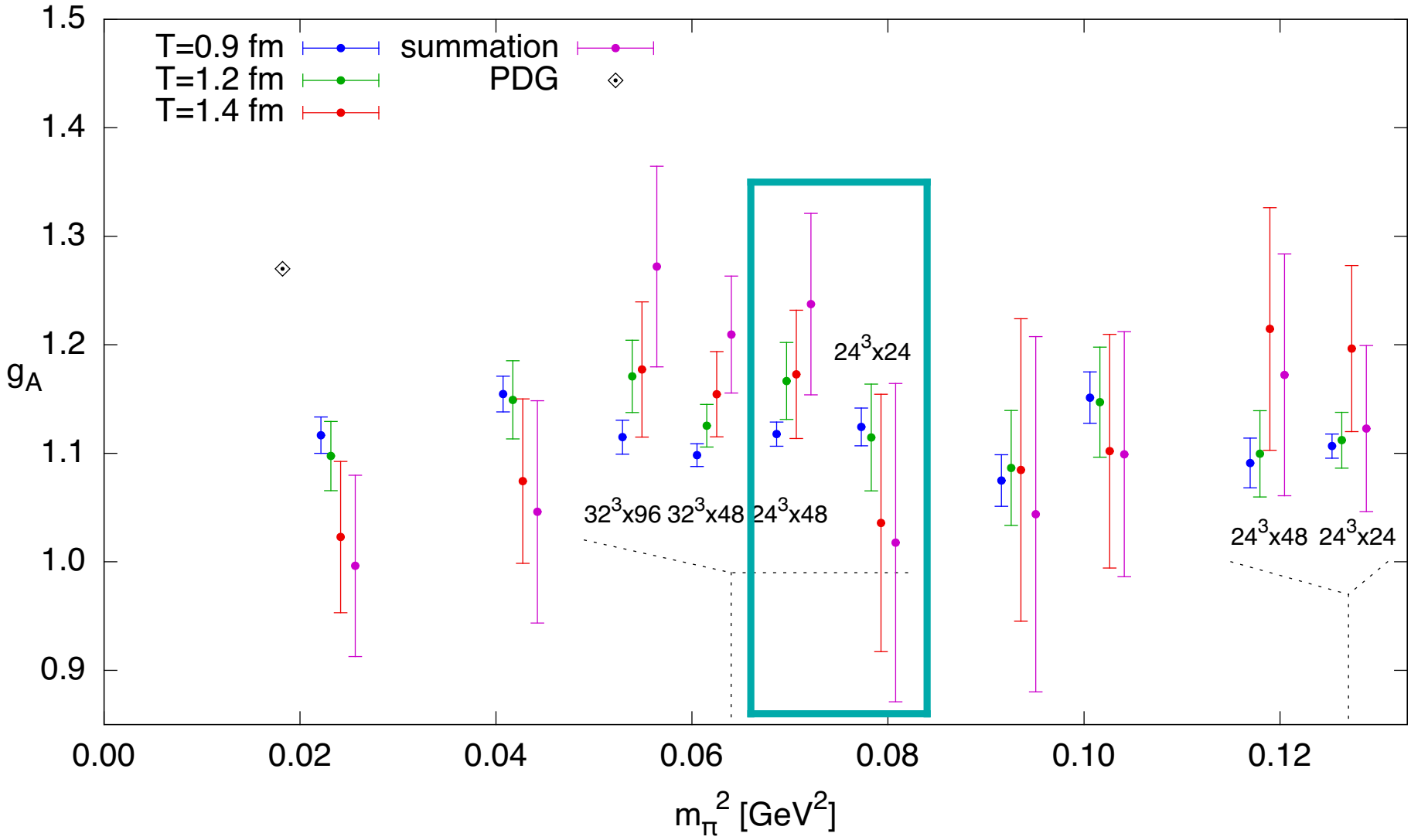
$\langle x \rangle_{u-d}$: chiral extrapolation of summation points



Axial charge g_A

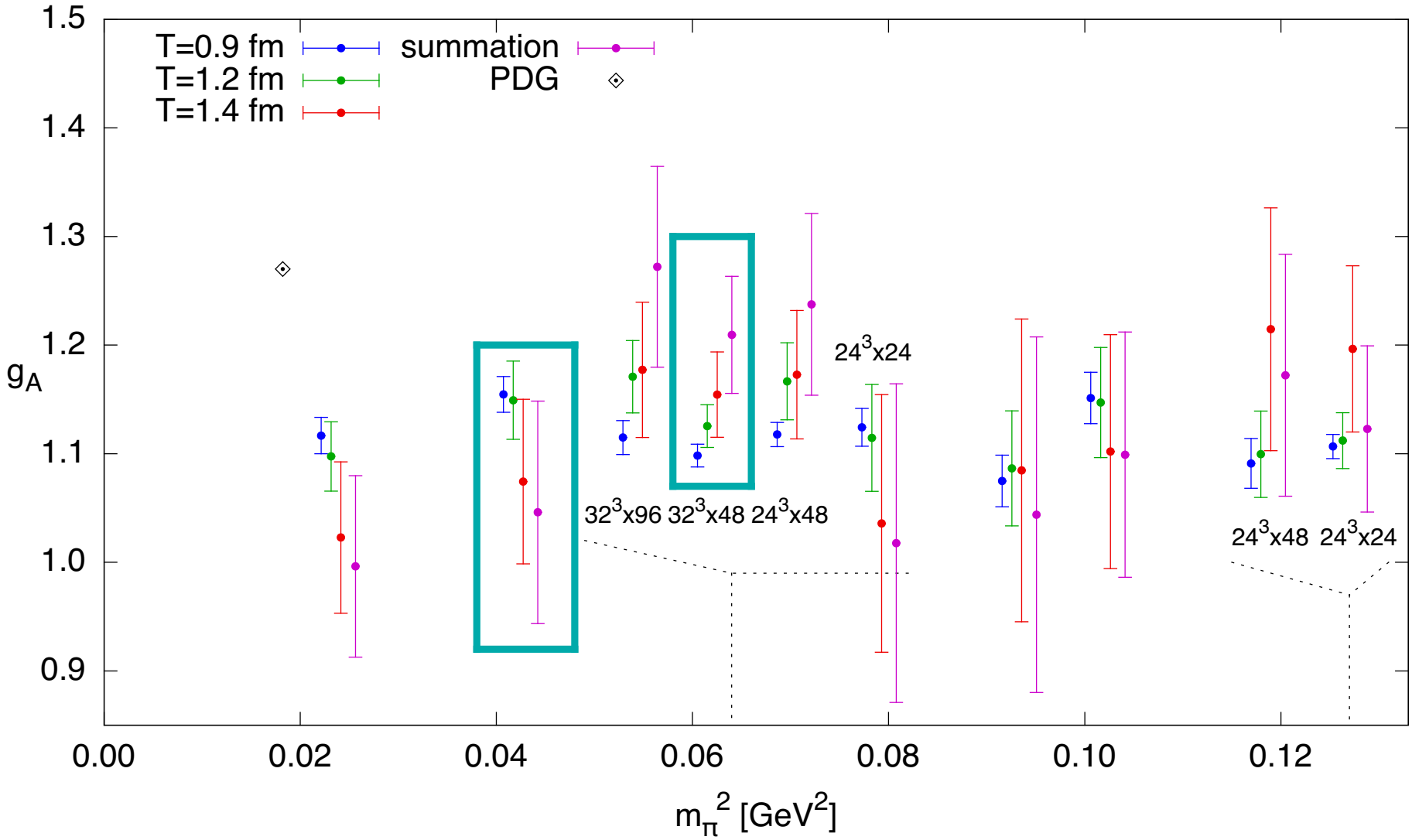


Axial charge g_A



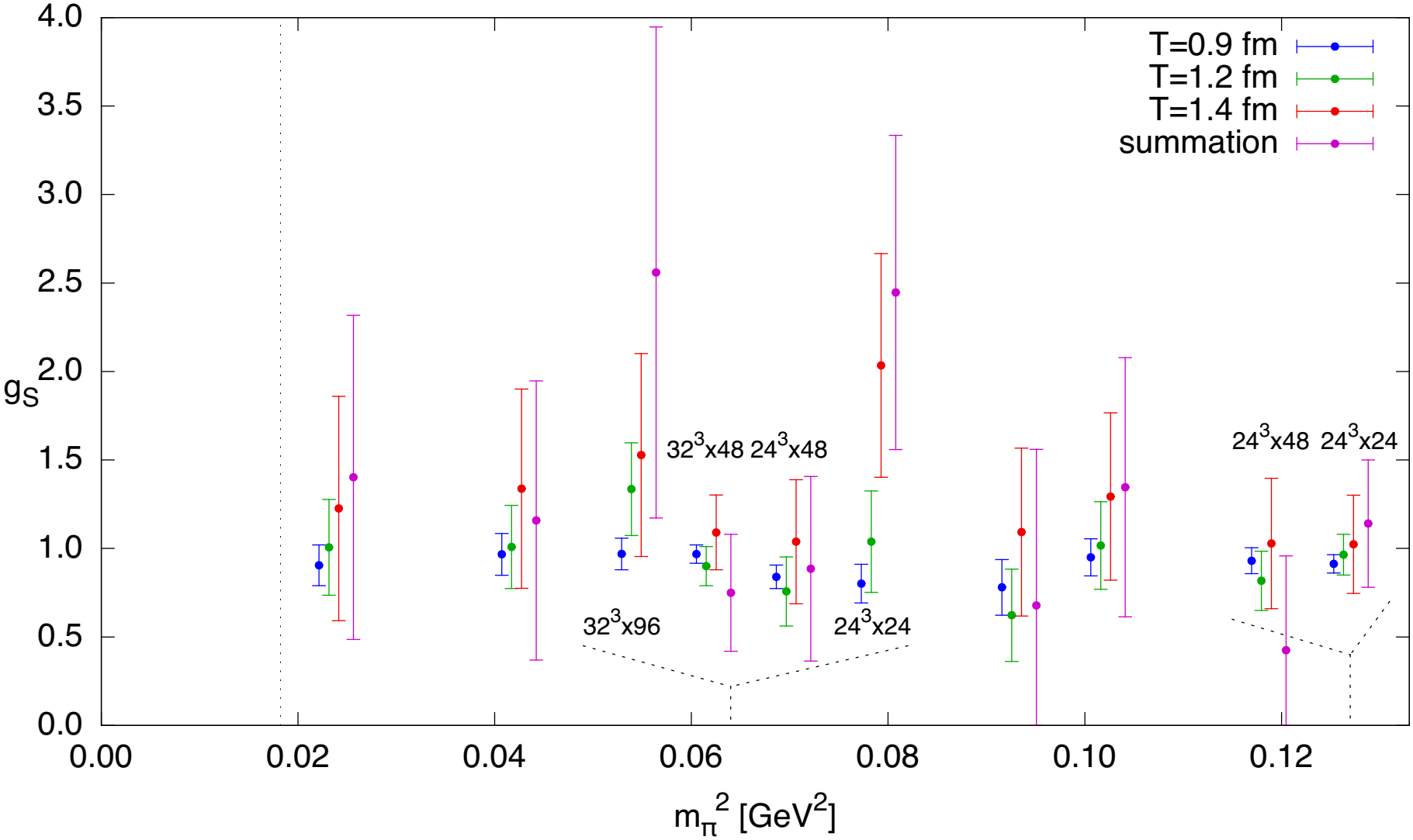
$m_\pi \approx 250$ MeV, $L_x = 24$; $L_t = 48$ versus $L_t = 24$.

Axial charge g_A

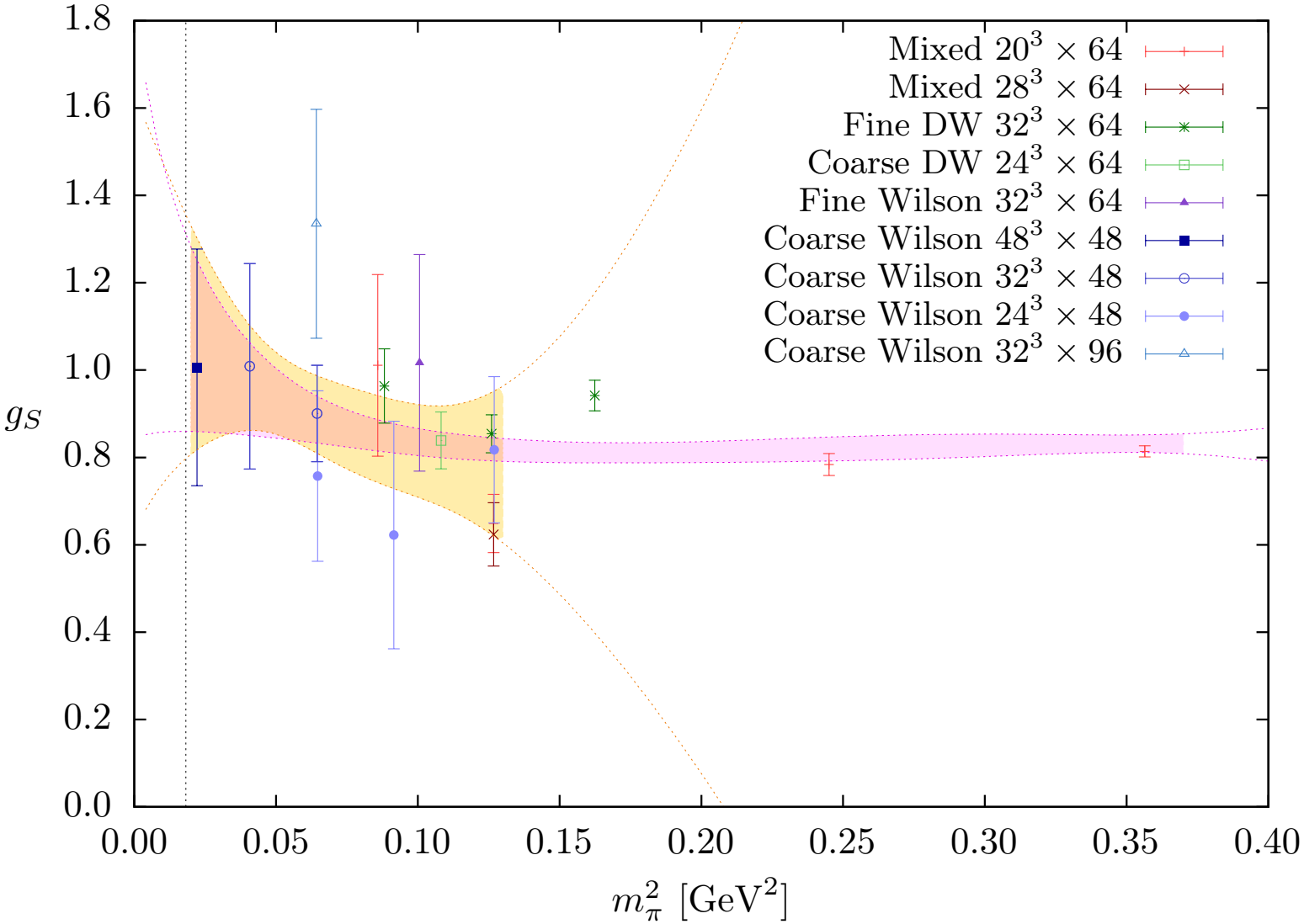


$32^3 \times 48$; $m_\pi = 200$ MeV versus $m_\pi = 250$ MeV.

Scalar charge g_S

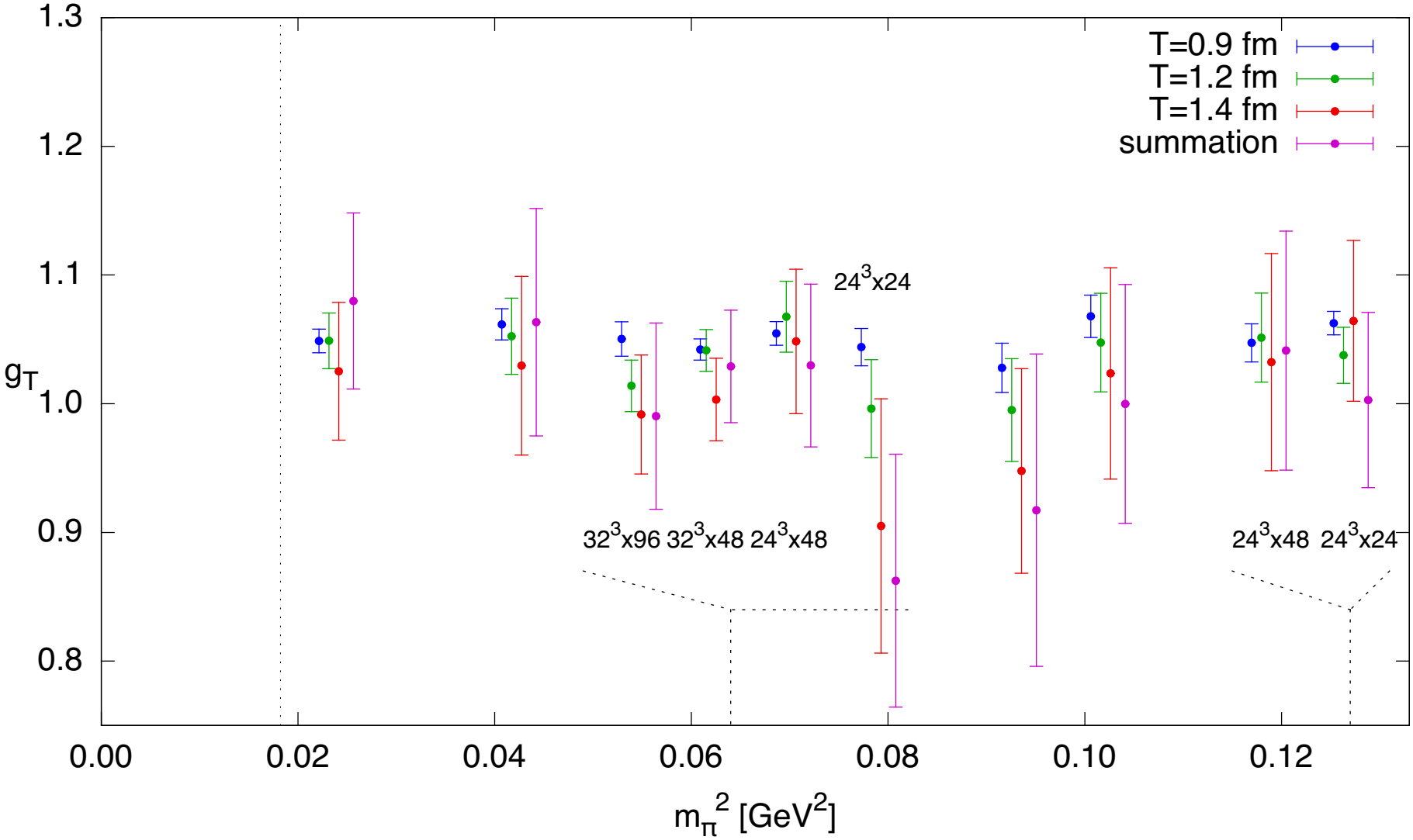


g_S: chiral extrapolation

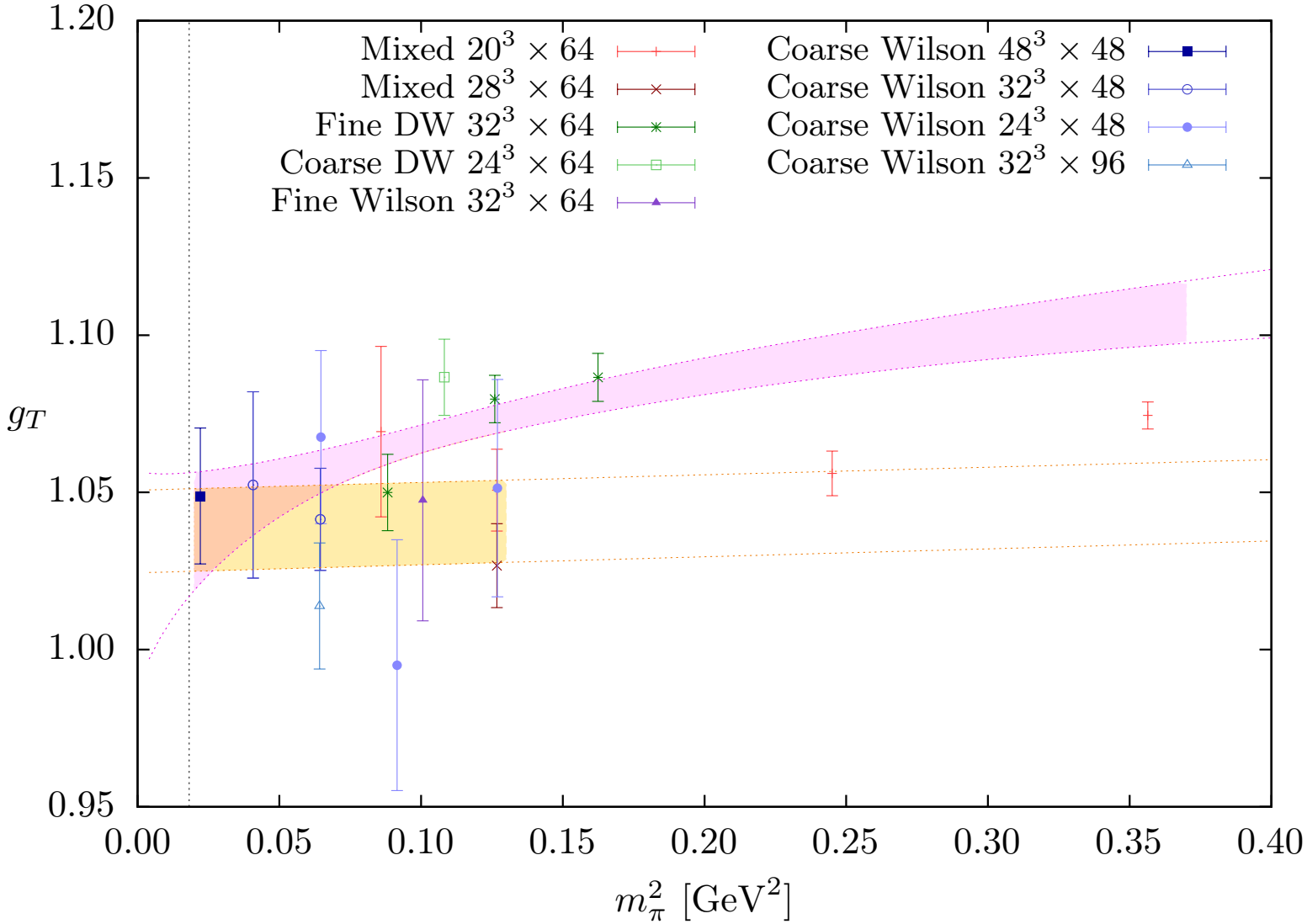


Using middle source-sink separation; also analyzed data from earlier studies.

Tensor charge g_T



g_T: chiral extrapolation

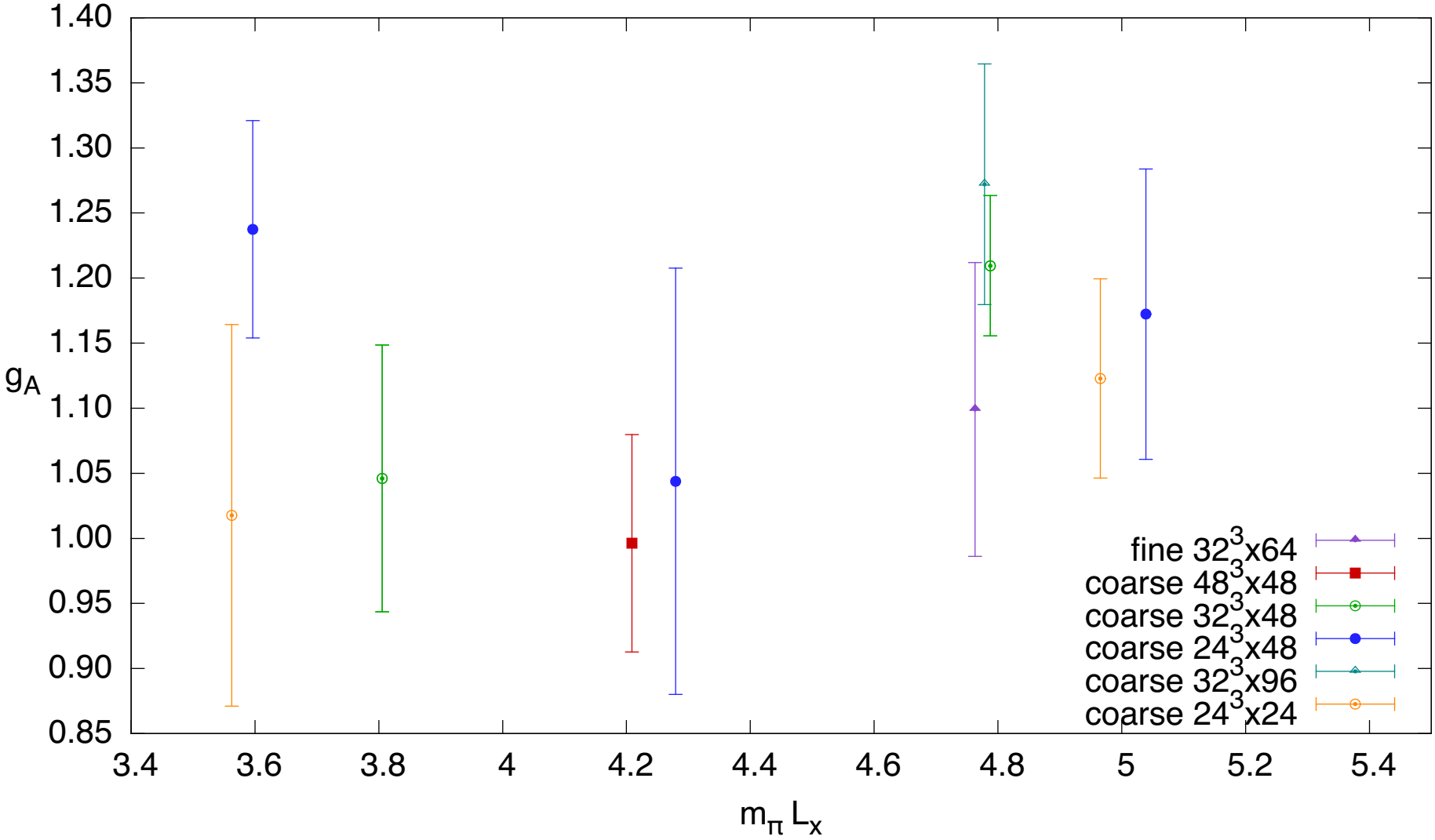


Using middle source-sink separation; also analyzed data from earlier studies.

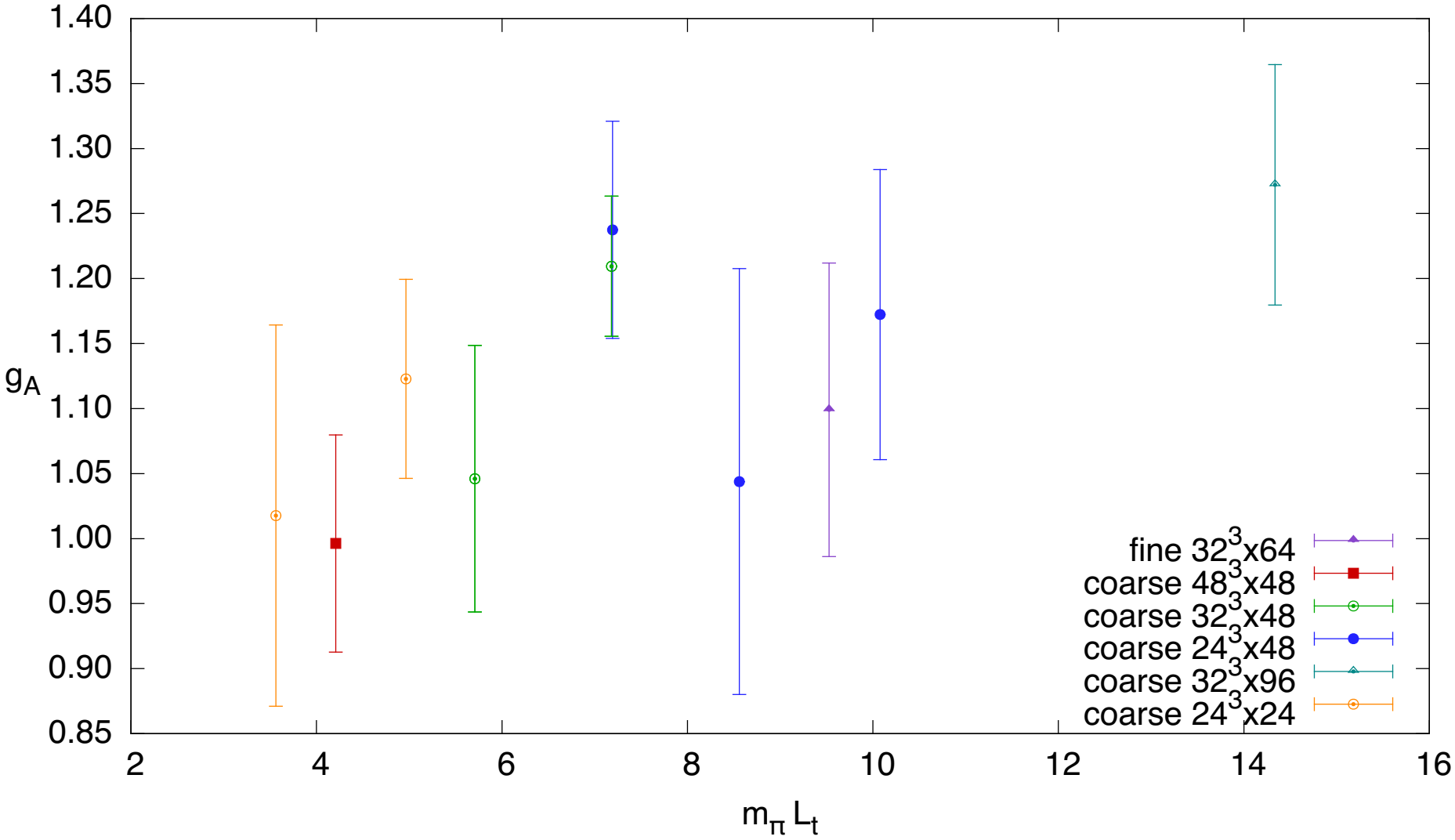
Conclusions

- Major new advance for calculations of nucleon structure observables: combination of both
  1. m_π close to physical
  2. serious effort toward control over excited-state contamination
- Calculated isovector Dirac and Pauli radii, anomalous magnetic moment, and momentum fraction are consistent with experiment.
- "Benchmark" axial charge still inconsistent with experiment, but the influence of thermal states has been identified as a serious candidate for being the culprit.
  - We are planning to do a controlled study of finite volume and time effects using $32^3 \times 48$, $24^3 \times 48$, $32^3 \times 24$, and $24^3 \times 24$ ensembles.
- Agreement with experiment will increase confidence in predictions such as
  - $g_S = 1.08 \pm 0.28\,(\text{stat}) \pm 0.16\,(\text{syst})$
  - $g_T = 1.038 \pm 0.011\,(\text{stat}) \pm 0.012\,(\text{syst})$

g_A vs. $m_\pi L_x$: summation data

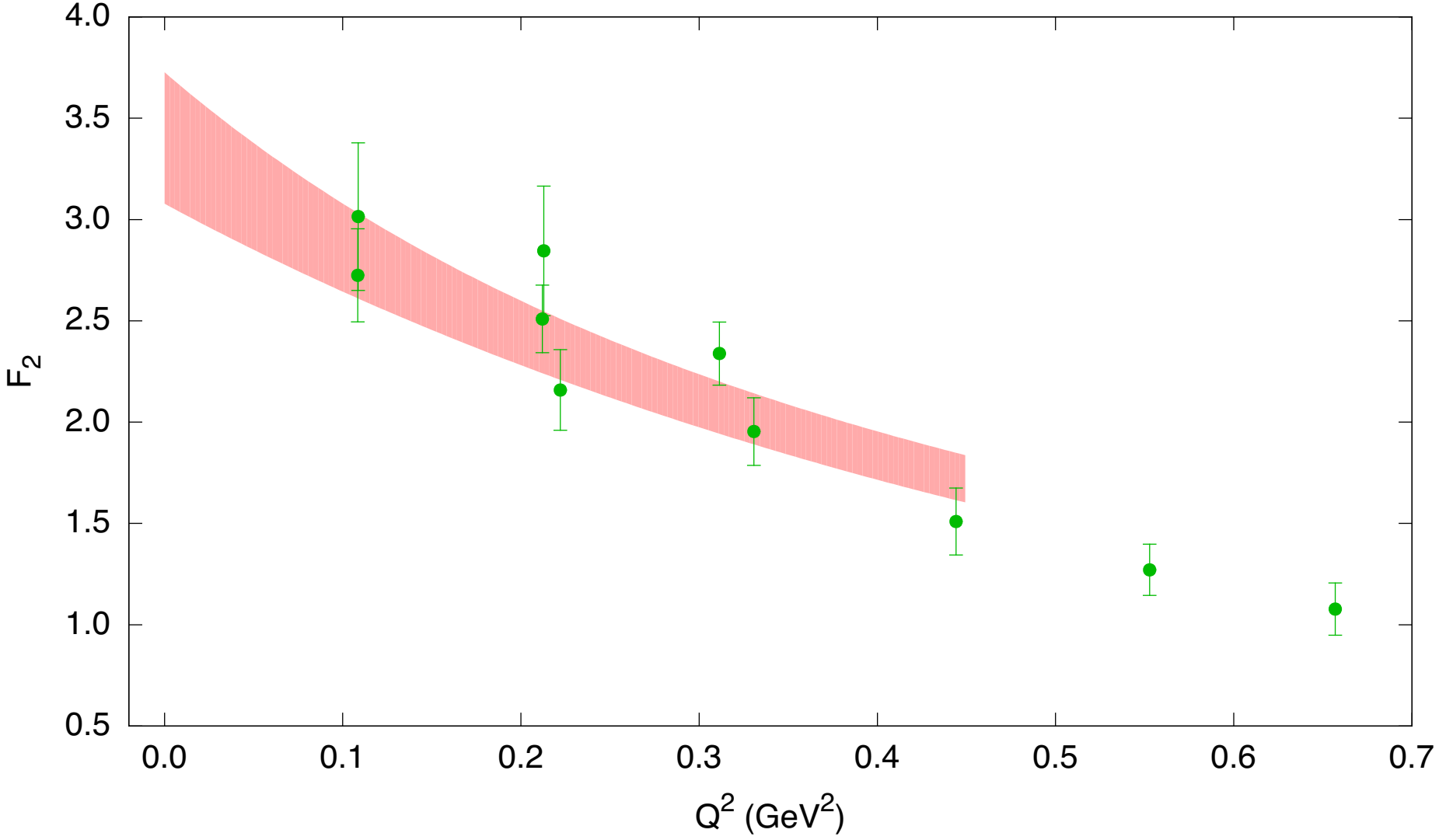


g_A vs. $m_\pi L_t$: summation data



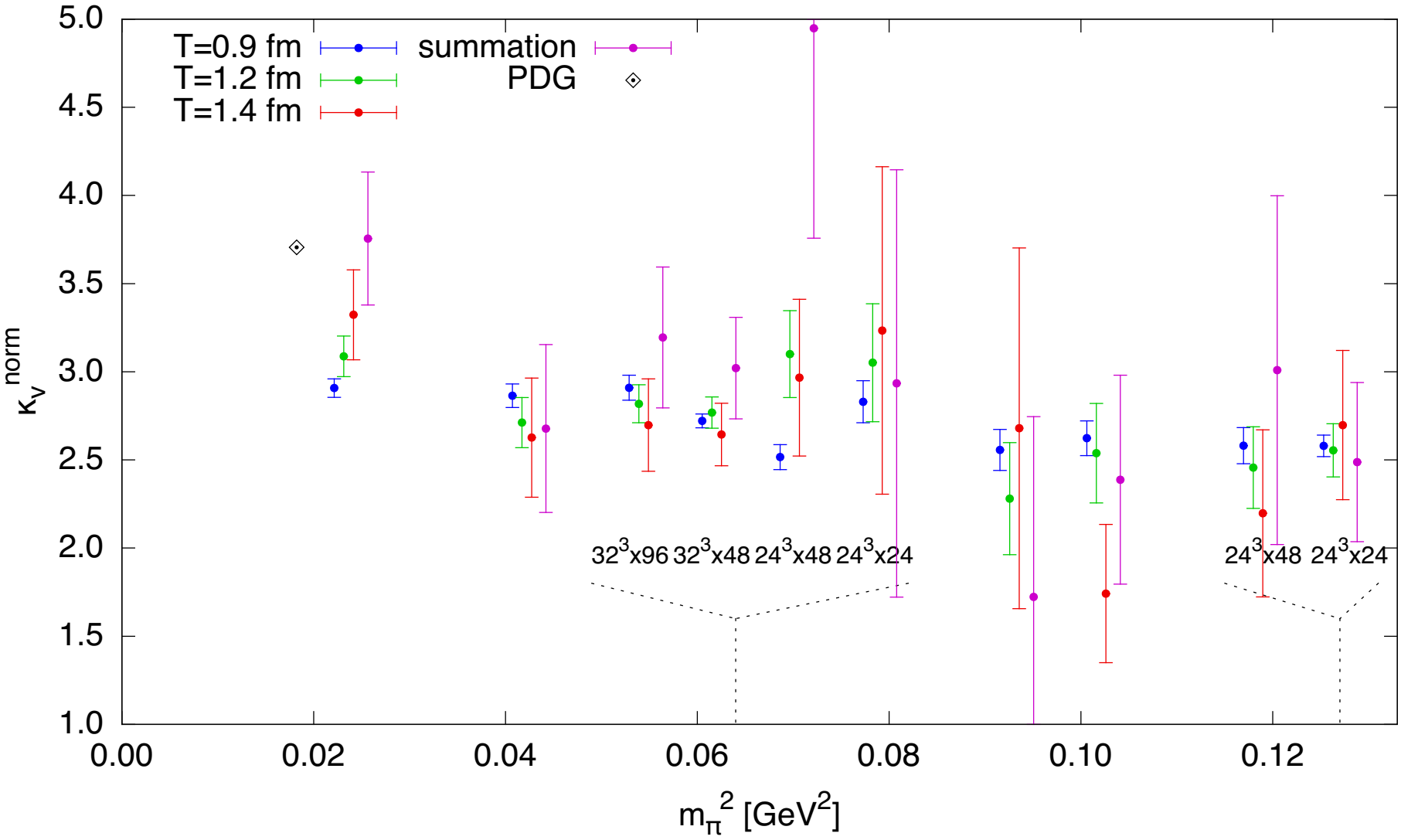
Thermal effects?

$F_2^v(Q^2)$: $m_\pi = 254$ MeV, $32^3 \times 48$, summation



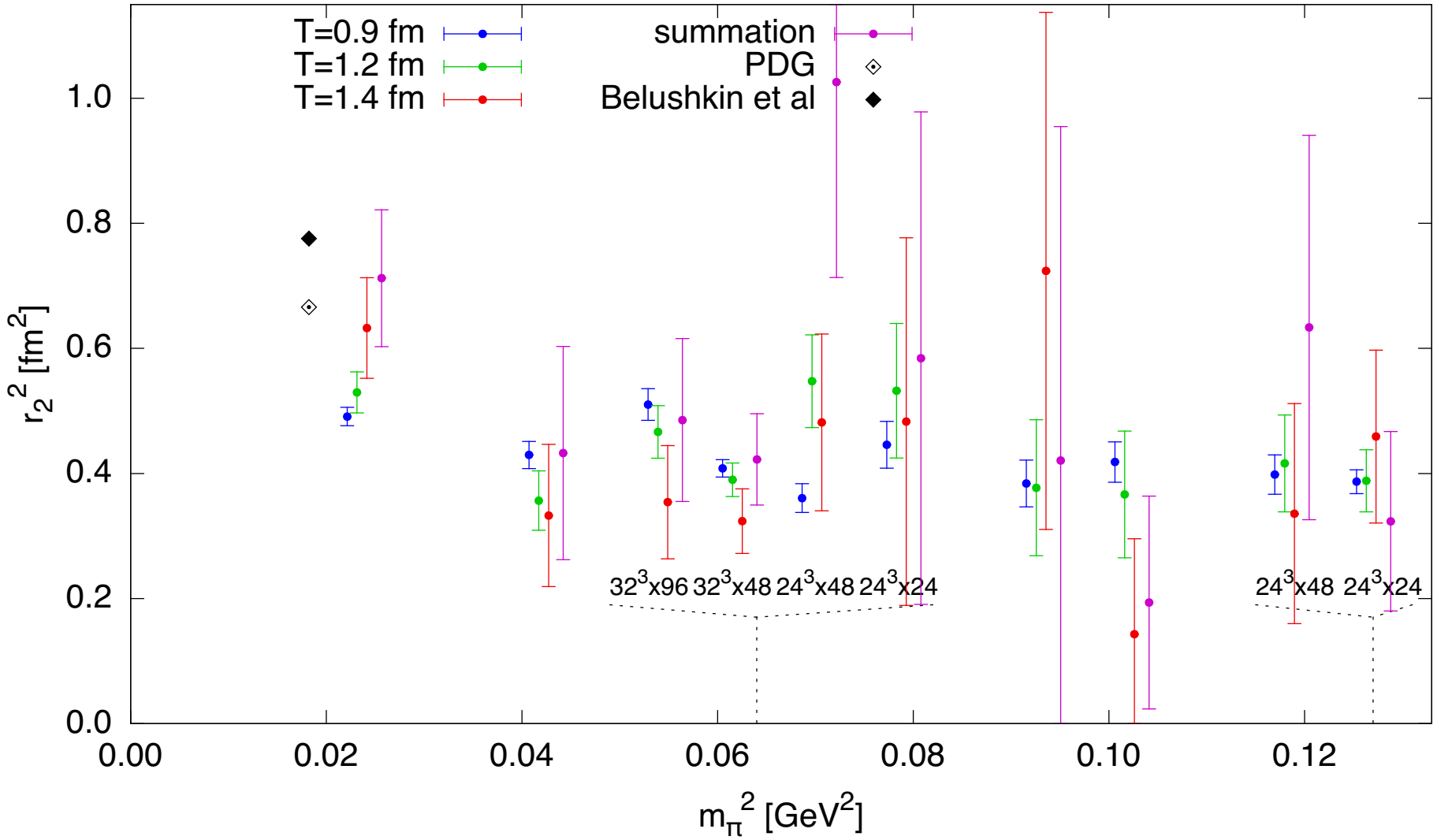
Dipole fit $F_2(Q^2) \sim \frac{F_2(0)}{(1+Q^2/M_D^2)^2}$ to range $0 < Q^2 \leq 0.5$ GeV^2.

Isovector anomalous magnetic moment κ^v

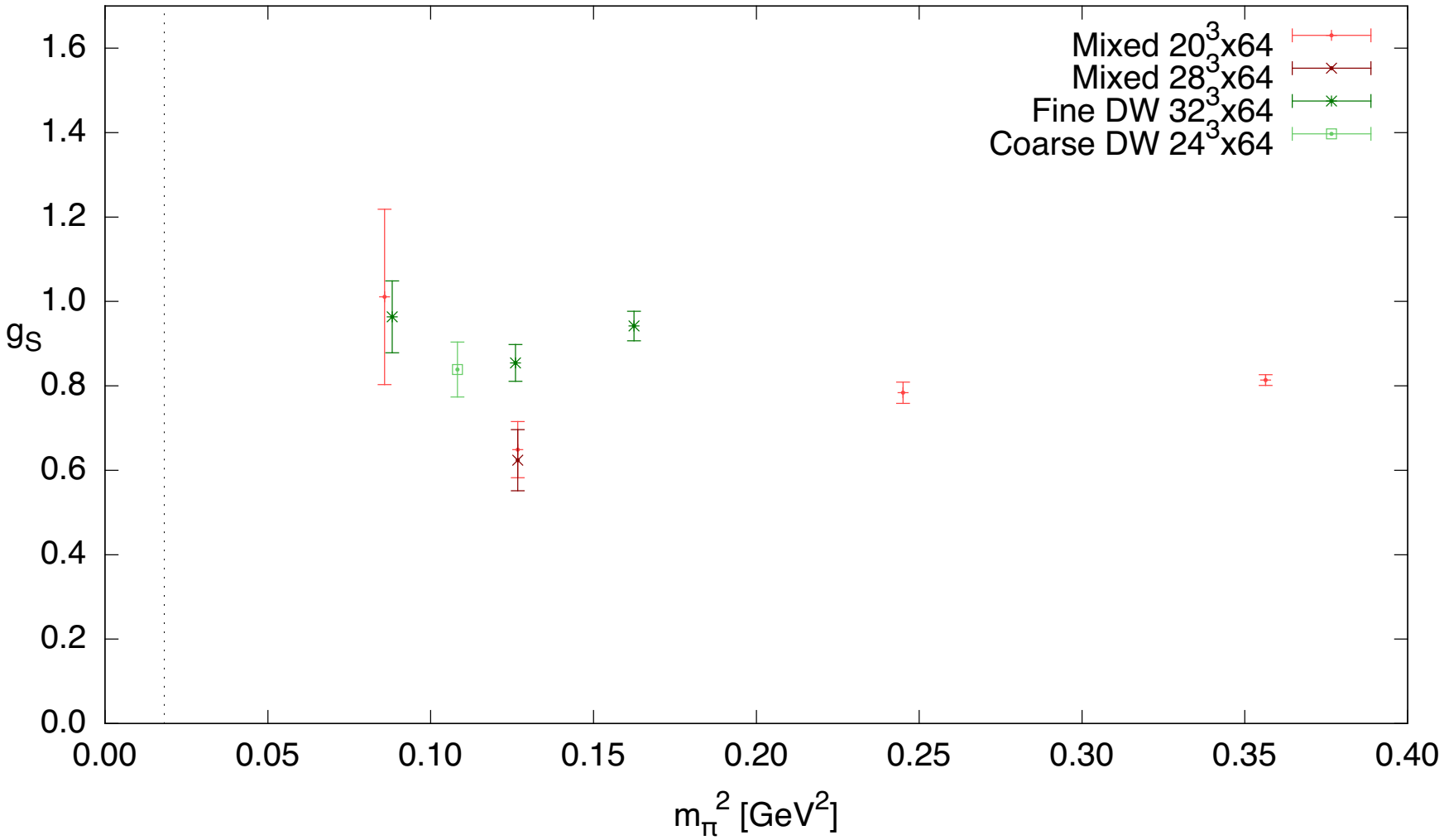


Normalized relative to the physical magneton: $\kappa^{\text{norm}} = \frac{m_N^{\text{phys}}}{m_N^{\text{lat}}} F_2^{\text{lat}}(0)$.

Isovector Pauli radius $(r_2^2)^v$

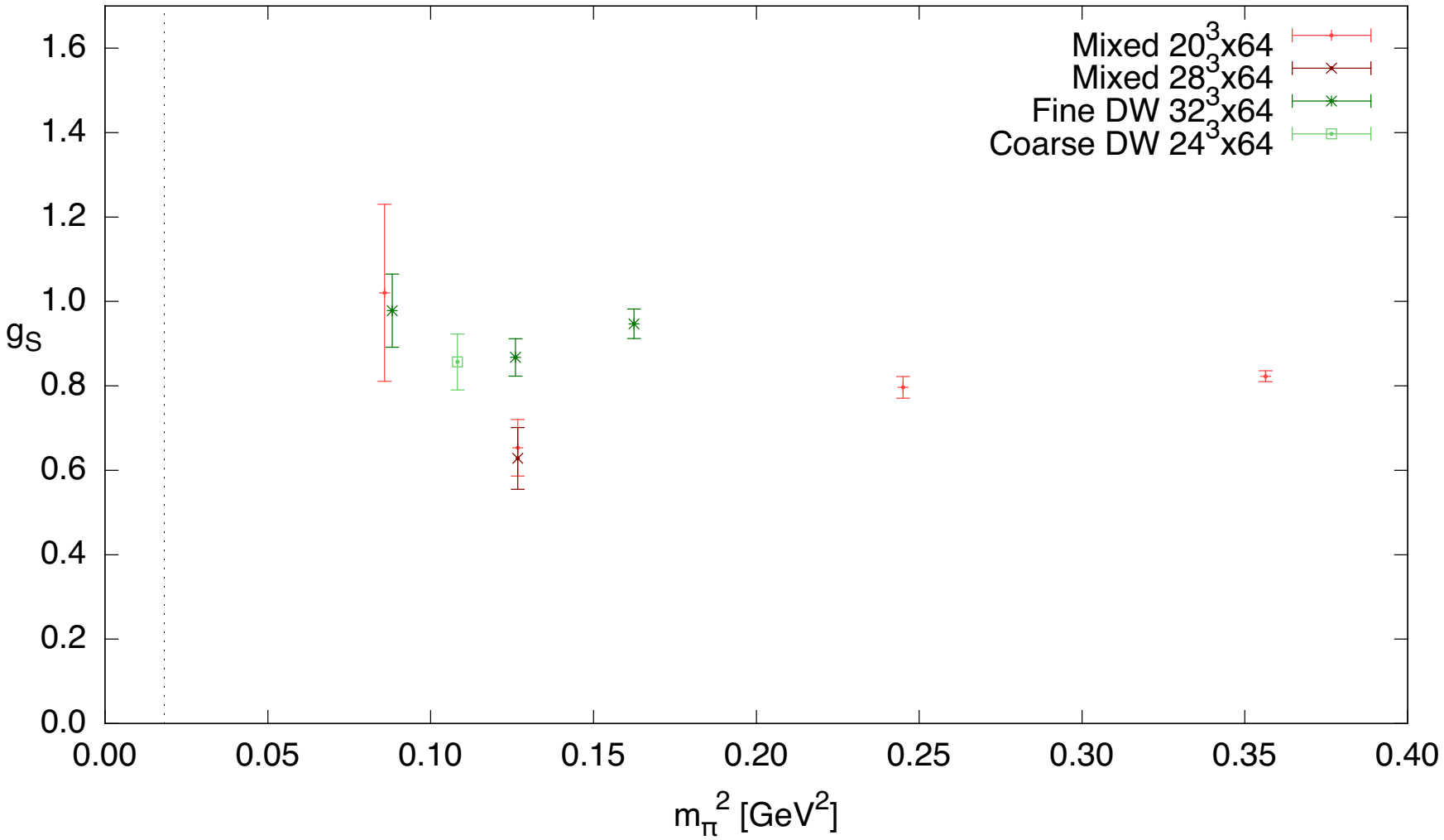


g_S renormalization: using Z_S



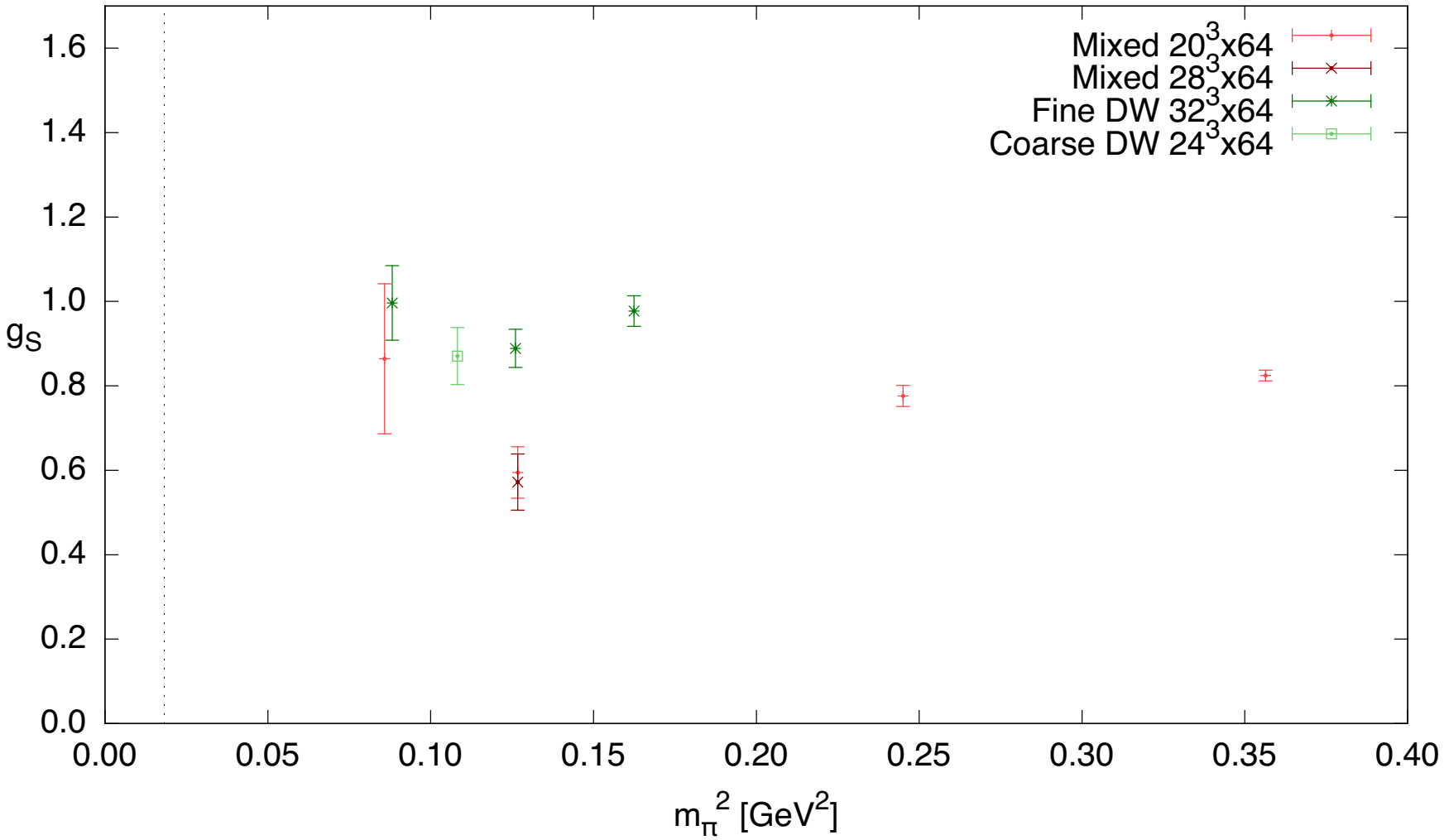
plotted: $Z_S g_S^{\text{bare}}$

g_S renormalization: using $m_s - m_{ud}$



plotted: $\frac{(m_s - m_{ud}) g_S}{m_s^{\text{phys}} - m_{ud}^{\text{phys}}} \times \frac{m_{K,\text{phys}}^2 - m_{\pi,\text{phys}}^2}{m_K^2 - m_\pi^2}$

g_S renormalization: using $m_{ud} + m_{\text{res}}$



plotted: $\frac{(m_{ud}+m_{\text{res}})g_S}{m_{ud}^{\text{phys}}} \times \frac{m^2_{\pi,\text{phys}}}{m^2_\pi}$