Nucleon structure near the physical pion mass

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  ▶ 2004, 2005: knot theory
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  ▶ spatial diquark correlations
  ▶ nucleon structure
Outline

- Introduction
  - Isovector nucleon observables
  - State of past lattice calculations
- Lattice methodology
  - Challenges
  - BMW action; ensembles
  - Systematic errors
- Results
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  - Momentum fraction
  - Axial charge
  - Tensor and scalar charges
- Conclusions
Nucleon observables

Electromagnetic form factors:

\[ \langle p(P', s')|\bar{q} \gamma^\mu q|p(P, s)\rangle = \bar{u}(p', s') \left( \gamma^\mu F_1^q(Q^2) + i\sigma^{\mu\nu} \frac{\Delta^\nu}{2m_p} F_2^q(Q^2) \right) u(p, s), \]

where \( \Delta = P' - P, \ Q^2 = -\Delta^2 \).

Quark momentum fraction:

\[ \langle p(P, s')|\bar{q} \gamma_{\{\mu} D_{\nu\}} q|p(P, s')\rangle = \langle x \rangle_q \bar{u}(P, s')\gamma_{\{\mu} P_{\nu\}} u(P, s) \]

Axial, tensor, and scalar charges:

\[ \langle p(P, s')|\bar{u} \gamma^\mu \gamma_5 d|n(P, s)\rangle = g_A \bar{u}_p(P, s')\gamma^\mu \gamma_5 u_n(P, s) \]
\[ \langle p(P, s')|\bar{u} d|n(P, s)\rangle = g_S \bar{u}_p(P, s')u_n(P, s) \]
\[ \langle p(P, s')|\bar{u} \sigma^{\mu\nu} d|n(P, s)\rangle = g_T \bar{u}_p(P, s')\sigma^{\mu\nu} u_n(P, s) \]
Vector form factors

Dirac and Pauli form factors:

$$\langle p(P', s')|\bar{q}\gamma^\mu q|p(P, s)\rangle = \bar{u}(p', s') \left( \gamma^\mu F_1^q(Q^2) + i\sigma^{\mu\nu} \frac{\Delta_\nu}{2m_p} F_2^q(Q^2) \right) u(p, s),$$

where $\Delta = P' - P$, $Q^2 = -\Delta^2$.

- Isovector combination:

$$F_{1,2}^v = F_{1,2}^u - F_{1,2}^d = F_{1,2}^p - F_{1,2}^n,$$

where $F_{1,2}^{p,n}$ are form factors of the electromagnetic current in a proton and in a neutron.

- Dirac and Pauli radii defined via slope at $Q^2 = 0$:

$$F_{1,2}(Q^2) = F_{1,2}(0) \left( 1 - \frac{1}{6} r_{1,2}^2 Q^2 + O(Q^4) \right);$$

$$F_2(0) = \kappa,$$ the anomalous magnetic moment.

- Proton charge radius, $(r_E^2)^p = (r_1^2)^p + \frac{3\kappa p}{2m_p^2}$, has $7\sigma$ discrepancy between measurements from $e^-p$ scattering and from Lamb shift in muonic hydrogen.
Quark momentum fraction

Measure forward matrix element of quark energy-momentum operator:

\[ \langle p(P, s')|\bar{q}\gamma_{\mu}D_{\nu}q|p(P, s)\rangle = \langle x \rangle_q \bar{u}(P, s')\gamma_{\mu}P_{\nu}u(P, s). \]

\[ \langle x \rangle_q \] is the average momentum fraction carried by quarks \( q \) and \( \bar{q} \); focus on isovector combination \( \langle x \rangle_{u-d} \).

- Phenomenological values come from measurements of parton distribution functions in deep inelastic scattering experiments.
Axial charge

“Benchmark” observable:

\[ \langle p(P, s') | \bar{u} \gamma^\mu \gamma_5 d | n(P, s) \rangle = g_A \bar{u}_p(P, s') \gamma^\mu \gamma_5 u_n(P, s). \]

- Naturally isovector observable; PDG: \( g_A/g_V = 1.2701(25) \) from \( \beta \) decay of polarized neutrons.
- Is also the difference between contributions from the spin of \( u \) and \( d \) quarks to the total proton spin.
Scalar and tensor charges

\[ \langle p(P, s')|\bar{u}d|n(P, s)\rangle = g_S \bar{u}_p(P, s')u_n(P, s) \]
\[ \langle p(P, s')|\bar{u}\sigma^{\mu\nu}d|n(P, s)\rangle = g_T \bar{u}_p(P, s')\sigma^{\mu\nu}u_n(P, s) \]

- Not measured experimentally.
- Recent interest because they are needed to know leading contributions to neutron $\beta$ decay from BSM physics.
- There are plans to measure the tensor charge at JLab using the upcoming 12 GeV upgrade.
Past calculations required large $m_\pi \rightarrow m_\pi^{\text{phys}}$ extrapolations; chiral perturbation theory allows for rapid changes, e.g. log-divergence in Dirac radius as $m_\pi \rightarrow 0$. 

S. N. Syritsyn et al. (LHPC), Phys. Rev. D 81, 034507 (2010), arXiv:0907.4194
Lattice calculations

Two kinds of quark contractions for $\langle N(t_s)O(t)\bar{N}(0)\rangle$:

Computationally-expensive disconnected diagrams cancel for isovector quantities, so we are focusing on those for now.
Use sequential propagator through the sink: propagators from fixed source and sink allow for measurement at all intermediate times.

Cost grows linearly with number of source-sink separations $T$; past calculations typically used just one.

- Excited-state contamination $\sim \exp(-\Delta ET)$.
- Signal-to-noise $\sim \exp(- (m_N - \frac{3}{2} m_\pi) T)$.
Challenges at low pion mass

Solid curves show expected low-lying $N\left(\frac{1}{2}^+\right)$ excited states in a finite box.
BMW action and ensembles

- $N_f = 2 + 1$ tree-level $O(a)$-improved Wilson fermions coupled to double-HEX-smeared gauge fields.
- Pion mass ranging from 149 MeV to 356 MeV.
- Nine coarse lattices with $a = 0.116$ fm; one fine lattice with $a = 0.09$ fm.
- No disconnected diagrams, so we focus on isovector observables.
- Three source-sink separations for controlling excited-state contributions: $T \in \{0.9, 1.2, 1.4\}$ fm.
Areas of circles scale with number of measurements: largest is 10,000.
Contributions to systematic error

From best-controlled to worst-controlled:

- **Quark masses**: smallest pion mass is 149 MeV, so $m_\pi \rightarrow 135$ MeV chiral extrapolation is under control.

- **Excited states**: use of multiple source-sink separations allows for clear identification of observables where excited states are a problem; the summation method seems to work well for reducing excited-state contamination.

- **Finite volume**: effects expected to be small with $m_\pi L \approx 4$, and two volumes at $m_\pi = 254$ MeV allow for fully-controlled test of finite-size effects.

- **Finite temperature**: at small $m_\pi$, $L_t$ is smaller than the typically used $L_t \approx 2L_x$, but three different time extents at $m_\pi \approx 250$ MeV are useful for identifying possible problems.

- **Discretization**: one finer lattice at $m_\pi = 317$ MeV for consistency check, but no $a \rightarrow 0$ extrapolation.
Systematic error: excited states

Usual approach for extracting matrix elements (forward case):

\[ C_{2\text{pt}}(t) = \langle N(t)N(0) \rangle \quad C_{3\text{pt}}(T, \tau) = \langle N(T)O(\tau)N(0) \rangle \]

- **Ratio-plateau method:** compute ratio

\[
R(T, \tau) = \frac{C_{3\text{pt}}(T, \tau)}{C_{2\text{pt}}(T)} = c_{00} + c_{10}e^{-\Delta E \tau} + c_{01}e^{-\Delta E(T-\tau)} + c_{11}e^{-\Delta ET} + \ldots ,
\]

where \( c_{00} \) is the desired ground-state matrix element. Then average a fixed number of points around \( \tau = T/2 \), yielding asymptotic errors that fall off as \( e^{-\Delta ET/2} \).

- **Summation method:** compute sums

\[
S(T) = \sum_{\tau} R(T, \tau) = b + c_{00}T + dTe^{-\Delta ET} + \ldots,
\]

then find their slope, which gives \( c_{00} \) with errors that fall off as \( Te^{-\Delta ET} \).
Results

Collaborators:
- JRG, J. W. Negele, A. V. Pochinsky (MIT)
- S. N. Syritsyn (LBNL)
- M. Engelhardt (New Mexico State U.)
- S. Krieg (FZ Jülich & U. Wuppertal)

Nucleon structure from Lattice QCD using a nearly physical pion mass
arXiv:1209.1687
- \((r_1^2)^v, \kappa^v, (r_2^2)^v, \langle x \rangle_{u-d}, g_A\)

Nucleon scalar and tensor charges from Lattice QCD with light Wilson quarks
- \(g_S, g_T\)

Nucleon structure with pion mass down to 149 MeV
- \((r_1^2)^v, g_A, g_S, g_T\)
$F_1^v(Q^2)$: $m_\pi = 254$ MeV, $32^3 \times 48$, summation

Dipole fit $F_1(Q^2) \sim \frac{F_1(0)}{(1+Q^2/M_D^2)^2}$ to range $0 \leq Q^2 \leq 0.5$ GeV$^2$. 
Isovector Dirac radius \((r_1^2)^v\)

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Nucleon structure near the physical \(m_\pi\)

JLab, January 4, 2013
$(r_1^2)^v$: chiral extrapolation of summation points
$\kappa^V$: chiral extrapolation of summation points

\[ \kappa^V = \kappa^{p} - \kappa^{n} \]

HBChPT + Δ

32c64 fine

32c96 coarse

24c24 coarse

48c48 coarse

PDG 2012

$J\text{Lab, January }4, 2013$
$\kappa^v (r_2^2)^v$: chiral extrapolation of summation points
Sachs form factors

\[ G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{2m_N} F_2(Q^2) \]
\[ G_M(Q^2) = F_1(Q^2) + F_2(Q^2) \]

- Slopes at \( Q^2 = 0 \) give rms charge and magnetic radii.
- Compare:
  1. Isovector \( m_\pi = 149 \text{ MeV} \), summation data from lattice calculation.
  2. Parameterization of experimental data:
     4 parameters for each of \( G_{Ep}, G_{Mp}, G_{Mn} \); 2 parameters for \( G_{En} \); determined from fit to experiment.
$G^v_E(Q^2)$: lowest pion mass
$G_M^V(Q^2)$: lowest pion mass

![Graph showing the dependence of $G_M^V$ on $Q^2$.](image-url)
Isovector quark momentum fraction $\langle x \rangle_{u-d}$
\langle x \rangle_{u-d}: \text{chiral extrapolation of summation points}
Axial charge $g_A$

![Graph showing the axial charge $g_A$ vs. $m_{\pi}^2$ for different values of $T$. The graph includes data points for summation and PDG, with various lattice sizes and configurations.](image)
Axial charge $g_A$

$m_\pi \approx 250$ MeV, $L_x = 24; L_t = 48$ versus $L_t = 24$. 
Axial charge $g_A$

$32^3 \times 48; m_\pi = 200 \text{ MeV versus } m_\pi = 250 \text{ MeV}.$
Scalar charge $g_S$
$g_s$: chiral extrapolation

Using middle source-sink separation; also analyzed data from earlier studies.
Tensor charge $g_T$
$g_T$: chiral extrapolation

Using middle source-sink separation; also analyzed data from earlier studies.
Conclusions

- Major new advance for calculations of nucleon structure observables: combination of both
  1. \( m_\pi \) close to physical
  2. serious effort toward control over excited-state contamination

- Calculated isovector Dirac and Pauli radii, anomalous magnetic moment, and momentum fraction are consistent with experiment.

- “Benchmark” axial charge still inconsistent with experiment, but the influence of thermal states has been identified as a serious candidate for being the culprit.
  - We are planning to do a controlled study of finite volume and time effects using \( 32^3 \times 48 \), \( 24^3 \times 48 \), \( 32^3 \times 24 \), and \( 24^3 \times 24 \) ensembles.

- Agreement with experiment will increase confidence in predictions such as
  - \( g_S = 1.08 \pm 0.28 \) (stat) \( \pm 0.16 \) (syst)
  - \( g_T = 1.038 \pm 0.011 \) (stat) \( \pm 0.012 \) (syst)
$g_A \text{ vs. } m_\pi L_x$: summation data
$g_A$ vs. $m_\pi L_t$: summation data

Thermal effects?

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Nucleon structure near the physical $m_\pi$

JLab, January 4, 2013
$F_2^v(Q^2)$: $m_\pi = 254$ MeV, $32^3 \times 48$, summation

Dipole fit $F_2(Q^2) \sim \frac{F_2(0)}{(1+Q^2/M_D^2)^2}$ to range $0 < Q^2 \leq 0.5$ GeV$^2$. 
Isovector anomalous magnetic moment $\kappa^v$

Normalized relative to the physical magneton: $\kappa_{\text{norm}} = \frac{m^\text{phys}_N}{m^\text{lat}_N} F^\text{lat}_2(0)$. 

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Isovector Pauli radius \((r_2^2)^v\)

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Nucleon structure near the physical \(m_\pi\)

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$g_S$ renormalization: using $Z_S$

plotted: $Z_S g_S^{\text{bare}}$
$g_S$ renormalization: using $m_s - m_{ud}$

plotted: $\frac{(m_s - m_{ud})g_S}{m_s^{\text{phys}} - m_{ud}^{\text{phys}}} \times \frac{m_{K,\text{phys}}^2 - m_{\pi,\text{phys}}^2}{m_K^2 - m_\pi^2}$
$g_S$ renormalization: using $m_{ud} + m_{res}$

Plotted:

$$\frac{(m_{ud} + m_{res})g_S}{m_{ud}^{phys}} \times \frac{m^2_{\pi,phys}}{m^2_{\pi}}$$

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Nucleon structure near the physical $m_{\pi}$