Exploring Hadron Structure with Transverse Momentum Dependent Factorization

Ted C. Rogers

C.N. Yang Institute for Theoretical Physics, SUNY Stony Brook

• Perturbative QCD and Collinear Factorization.

• Transverse Momentum Dependent (TMD) Factorization.

• TMD project: Implementing TMD-factorization.

Jefferson National Laboratory – February 6, 2013
Example:

- Semi-Inclusive Deep Inelastic Scattering (SIDIS):

\[ -q^2 = -(l_1 - l_2)^2 = Q^2 \gg \Lambda_{QCD}^2 \]
Example 2:

- Drell-Yan:

\[
q^2 = (l_1 + l_2)^2 = Q^2 \gg \Lambda_{QCD}^2
\]

\[
\sim 1/Q
\]

Small Scales
Example 2:

\[ q^2 = (l_1 + l_2)^2 = Q^2 \gg \Lambda_{QCD}^2 \]

\[ \sim 1/Q \]

Small Scales
Collinear (Standard) Case

- Parton Model Picture

\[
\sigma \sim \int \mathcal{H}(Q) \otimes f_q/P(x_1) \otimes f_{\bar{q}}/\bar{P}(x_2)
\]

- **Elementary collision**

- **Number densities**

“Parton Distribution Functions” (PDFs)
Example 2:

- Drell-Yan:

*Collinear case*

Get: \[ \int d\mathbf{q}_T \frac{d\sigma}{d\mathbf{q}_T} \cdots \]

*(Or large \( q_T \))*

\[ q^2 = (l_1 + l_2)^2 = Q^2 \gg \Lambda_{QCD}^2 \]

\[ k_1 \equiv k \]

\[ k_2 \equiv q - k \]

\[ k_{1T} + k_{2T} = q_T \]
Example 2:

- Drell-Yan:

\[ C_0 + C_1 \alpha_s + C_2 \alpha_s^2 + C_3 \alpha_s^3 + \cdots \]

\[
\alpha_s = \text{QCD coupling strength}
\]
QCD Factorization

• Short Distances; Asymptotic Freedom
  – Perturbation Theory

• Large Distance Scales:
  – Hadron Structure  
    *Confinement*

• Physical processes involve both.
  – Need to be separated: *QCD Factorization*
Collinear (Standard) Case

- Parton Model

\[ \sigma \sim \int \mathcal{H}(Q) \otimes f_{q/P}(x_1) \otimes f_{\bar{q}/P}(x_2) \]

- Elementary collision
- Hadron Structure: large distance scales
- Short distance scales

\[ \sim 1/Q \]
Collinear (Standard) Case

- Perturbative QCD factorization theorem:

\[ \sigma \sim \int \mathcal{H}(Q, \mu/Q, \alpha_s(\mu)) \otimes f_{q/P}(x_1; \mu) \otimes f_{\bar{q}/\bar{P}}(x_2; \mu) \]

**Small Coupling: Perturbation Theory**

\[ C_0 + C_1\alpha_s(\mu) + C_2\alpha_s(\mu)^2 + C_3\alpha_s(\mu)^3 + \cdots \]

Error \( \sim \Lambda_{QCD}/Q \)

- **Red**: Perturbatively Calculable
- **Blue**: Non-Perturbative Input Needed
**Collinear (Standard) Case**

- Perturbative QCD factorization theorem:

\[
\sigma \sim \int \mathcal{H}(Q, \mu/Q, \alpha_s(\mu)) \otimes f_{q/P}(x_1; \mu) \otimes f_{\bar{q}/\bar{P}}(x_2; \mu)
\]

Small Coupling: Perturbation Theory

\[C_0 + C_1\alpha_s(\mu) + C_2\alpha_s(\mu)^2 + C_3\alpha_s(\mu)^3 + \cdots\]

Defined in terms of elementary fields

\[\text{Error} \sim \Lambda_{QCD}/Q\]

**Red**: Perturbatively Calculable

**Blue**: Non-Perturbative Input Needed
**Collinear (Standard) Case**

- Perturbative QCD factorization theorem:

\[
\sigma \sim \int \mathcal{H}(Q, \mu/Q, \alpha_s(\mu)) \otimes f_{q/P}(x_1; \mu) \otimes f_{\bar{q}/P}(x_2; \mu)
\]

- **Small Coupling:**
  
  Perturbation Theory

\[
C_0 + C_1\alpha_s(\mu) + C_2\alpha_s(\mu)^2 + C_3\alpha_s(\mu)^3 + \cdots
\]

- **Auxiliary parameter: Arbitrary**

\[
\text{Error} \sim \Lambda_{QCD}/Q
\]

**Red**: Perturbatively Calculable

**Blue**: Non-Perturbative Input Needed
**Evolution (Standard) Case**

- **Perturbative QCD factorization theorem:**
  \[
  \sigma \sim \int \mathcal{H}(Q, \mu/Q, \alpha_s(\mu)) \otimes f_{q/P}(x_1; \mu) \otimes f_{\bar{q}/\bar{P}}(x_2; \mu)
  \]

- **DGLAP evolution**  
  *(Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)*
  \[
  \frac{d}{d \ln \mu} f_{j/P}(x; \mu) = 2 \int P_{jj'}(x') \otimes f_{j'/P}(x/x'; \mu)
  \]

- **Factorization + Evolution:** Universal PDFs  
  "Portable"
Implementing Collinear Factorization

Measurements

Collinear processes
Implementing Collinear Factorization

**Fits**
- CTEQ, MRSTW, etc.

**Measurements**
- Collinear processes

Input For

- Collinear processes
- CTEQ, MRSTW, etc.

Implementing Collinear Factorization

- **Fits**
  - CTEQ, MRSTW, etc..

- **Measurements**
  - Collinear processes

- **Predictions**
  - Used in predictions with small-$\alpha_s$ PT

**Input For**

Flowchart:
- Fits → Predictions
- Measurements → Fits
- Measurements → Predictions
Implementing Collinear Factorization

Fits
CTEQ, MRSTW, etc..

Measurements
Collinear processes

Predictions
with small-$\alpha_s$ PT

Test pQCD theory with

Used In

Input For
Implementing Collinear Factorization

**Fits**
CTEQ, MRSTW, etc..

**Used In**

**Predictions**
with small-$\alpha_s$ PT

**Input For**
Collinear Factorization, Collinear Evolution, Universality

**Measurements**
Collinear processes

Test pQCD theory with
Implementing Collinear Factorization

- **Fits**
  - CTEQ, MRSTW, etc.

- **Input For**
  - New measurements,
  - New processes,
  - Wider range of Q

- **Collinear Factorization, Collinear Evolution, Universality**

- **Used In**
  - Predictions
    - with small-$\alpha_s$ PT

- **Measurements**
  - Collinear processes

- **Test pQCD theory with**
Implementing Collinear Factorization

- **Fits**
  - CTEQ, MRSTW, etc.

- **Used In**
  - New measurements, New processes, Wider range of $Q$

- **Predictions**
  - with small-$\alpha_s$ PT

- **Input For**
  - Collinear Factorization, Collinear Evolution, Universality

- **Measurements**
  - Collinear processes

- **Higher Orders, New Factorization Theorems**

```
Collinear Factorization, Collinear Evolution, Universality
```
Phenomenology

Deep Inelastic Scattering

HERA $F_2$

$F_2^{enl} \log(y)$ vs $Q^2 (GeV^2)$

- ZEUS NLO QCD fit
- H1 PDF 2000 fit

Data sets:
- H1 94-00
- H1 (prelim.) 99/00
- ZEUS 96/97
- BCDMS
- E665
- NMC
Phenomenology

Deep Inelastic Scattering

HERA $F_2$

$F_2^{\text{em}}(x) = F_2(x) \times \log(y)$

$F_2$ vs $x$ for different experiments:

- ZEUS NLO QCD fit
- H1 PDF 2000 fit

$p\bar{p} \rightarrow \text{Jet} + X$

Tevatron - 1996

$\tilde{F}(x, k_T)$

$\tilde{G}(x, k_T)$

$Q^2(\text{GeV}^2)$

$1/\Delta \eta \int \frac{d^3 p}{dE_d\eta} d\eta$

$n_b(\text{GeV})$
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Jefferson National Laboratory – February 6, 2013
Collinear (Standard) Case

- Perturbative QCD factorization theorem:

\[ \sigma \sim \int \mathcal{H}(Q, \mu/Q, \alpha_s(\mu)) \otimes f_{q/P}(x_1; \mu) \otimes f_{\bar{q}/\bar{P}}(x_2; \mu) \]
Example 2:
Example 2:

\[ q_T = l_{1T} + l_{2T} \]
Example 2:

- Drell-Yan:

**Collinear case**

Get: \[ \int dq_T \frac{d\sigma}{dq_T} \cdots \]

(Or large \( q_T \))

\[ q^2 = (l_1 + l_2)^2 = Q^2 \gg \Lambda_{QCD}^2 \]

\( k_1 \equiv k \)

\( k_2 \equiv q - k \)

\( k_{1T} + k_{2T} = q_T \)
Example 2:

- Drell-Yan:

  \[
  \frac{d\sigma}{dq_T} \ldots
  \]

  For all \(q_T\).

  \[
  q^2 = (l_1 + l_2)^2 = Q^2 \gg \Lambda_{QCD}^2
  \]

  \[
  k_1 \equiv k \\
  k_2 \equiv q - k \\
  k_{1T} + k_{2T} = q_T
  \]
This work was supported by...

\( \sigma = \Lambda_{QCD} \)

\( \frac{d\sigma}{dq_T^2} \)

\( q_T \sim \Lambda_{QCD} \)

\( q_T \)
TMD-Factorization

- Collinear factorization theorem relies on \textit{collinear} approximations.
TMD-Factorization

• Collinear factorization theorem relies on *collinear* approximations.

• Accounting for intrinsic transverse momentum requires *new factorization theorems*.
**Collinear** (Standard) Case

- **Parton Model**

\[ \sigma \sim \int \mathcal{H}(Q) \otimes f_q/P(x_1) \otimes f_{\bar{q}}/\bar{P}(x_2) \]

**Elementary collision**

**Short distance scales**

- **Perturbative QCD factorization theorem**

\[ \sigma \sim \int \mathcal{H}(Q, \mu/Q, \alpha_s(\mu)) \otimes f_q/P(x_1; \mu) \otimes f_{\bar{q}}/\bar{P}(x_2; \mu) \]

**Small Coupling:**

**Perturbation Theory**

**Auxiliary parameter:** 

**Arbitrary Defined in terms of QFT operators**

- **Hadron Structure: large distance scales**

TMD-Factorization

• Parton Model

\[ \sigma \sim \int \mathcal{H}(Q) \otimes F_{q/P}(x_1, k_{1T}) \otimes F_{\bar{q}/\bar{P}}(x_2, q_T - k_{1T}) \]

- Parton Model
- Elementary collision
- Short distance scales
- \( \sim 1/Q \)

Number densities
“Transverse Momentum Dependent Parton Distribution Functions” (TMD PDFs)
TMD-Factorization

• Parton Model

\[ \sigma \sim \int \mathcal{H}(Q) \otimes F_{q/P}(x_1, k_{1T}) \otimes F_{\bar{q}/\bar{P}}(x_2, q_T - k_{1T}) \]

Parton Model

Elementary collision

Short distance scales

\[ \sim 1/Q \]

Number densities

“Transverse Momentum Dependent Parton Distribution Functions” (TMD PDFs)

• Past Approaches:
  – Non-perturbative descriptions:
    • Model
    • Fitting
  – Transverse Momentum Resummation
TMD-Factorization: QCD

• Unified Formalism

\[ \sigma \sim \int \mathcal{H}(Q, \mu/Q, \alpha_s(\mu)) \otimes F_{q/P}(x_1, k_{1T}, \mu, \zeta_1) \otimes F_{\bar{q}/P}(x_2, q_T - k_{1T}, \mu, \zeta_2) \]

Small Coupling: Perturbation Theory

2 Auxiliary parameters: Arbitrary

\[ \zeta_1 \zeta_2 \sim Q^4 \]

(Collins, Soper, Sterman (CSS) formalism (1982, 1983))

(Collins Extension: (2011), Chaps. 10, 13, 14)
TMD-Evolution

• Recall Collinear / DGLAP:

\[
\frac{d}{d \ln \mu} f_{j/P}(x; \mu) = 2 \int P_{jj'}(x') \otimes f_{j'/P}(x/x'; \mu)
\]
TMD-Evolution

• Recall Collinear / DGLAP:

\[
\frac{d}{d \ln \mu} f_{j/P}(x; \mu) = 2 \int P_{jj'}(x') \otimes f_{j'/P}(x/x'; \mu)
\]

• TMD Case:

\[
\frac{\partial \ln \tilde{F}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)
\]

\[
\frac{d \tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_K(g(\mu))
\]

\[
\frac{d \ln \tilde{F}(x, b_T; \mu, \zeta)}{d \ln \mu} = \gamma_F(g(\mu); \zeta/\mu^2)
\]

((CSS) formalism (1982,1983))

(Collins Extension: (2011), Chaps. 10,13,14)
TMD PDF Definitions

- Defined in terms of elementary field operators.
TMD PDF Definitions

• Defined in terms of elementary field operators.

• Needed to address questions of hadronic structure.
  – Lattice QCD.

(M. Engelhardt et al., (2012))
(B. Musch et al., (2011))
TMD PDF Definitions

• Defined in terms of elementary field operators.

• Needed to address questions of hadronic structure.
  – Lattice QCD.
  – Models and Effective Theories.
TMD PDF Definitions

• Defined in terms of elementary field operators.

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  – Models and Effective Theories.

• Universality / Modified Universality.
TMD PDF Definitions

• Defined in terms of elementary field operators.

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  – Models and Effective Theories.

• Universality / Modified Universality.

• Constrained by factorization derivation.
TMD PDF Definitions

• Defined in terms of elementary field operators.

• Needed to address questions of hadronic structure.
  – Lattice QCD.
  – Models and Effective Theories.

• Universality / Modified Universality.

• Constrained by factorization derivation.

• TMD evolution is independent of method of regulation.
  
  *(Collins, TCR (2012))*
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• Perturbative QCD and Collinear Factorization.

✓

• Transverse Momentum Dependent (TMD) Factorization.

✓

• TMD project: Implementing TMD-factorization.

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TMD vs. Collinear

• TMDs: Rich source of information about hadron structure.
  
  – *TMD Zoo*
TMD vs. Collinear
TMD vs. Collinear
TMD vs. Collinear

• TMDs: Rich source of information about hadron structure.
  – *TMD Zoo*

• More complicated fitting:

\[
f_{f/P}(x) \rightarrow F_{f/P}(x, k_T)
\]

*Need non-perturbative descriptions*
TMD vs. Collinear

- TMDs: Rich source of information about hadron structure.

  - TMD Zoo

- More complicated fitting:

  \[ f_{f/P}(x) \longrightarrow F_{f/P}(x, k_T) \]

  Need non-perturbative descriptions

- Cases of non-universality / TMD-factorization breaking.
Implementing Collinear Factorization

Fits
CTEQ, MRSTW, etc..

Collinear Factorization,
Collinear Evolution,
Universality

Input For

New measurements,
New processes,
Wider range of Q

Measurements
Collinear processes

Predictions
with small-$\alpha_s$ PT

Used In

Higher Orders,
New Factorization Theorems

Test pQCD theory with
Implementing **TMD-Factorization**

*(Current Work)*

- **Fits**
  - Few compared with collinear case

- **Used In**
  - **Non-perturbative Theory of TMDs**
  - **TMD-Factorization**, **TMD-Evolution**, *(Modified)* **Universality**

- **Predictions**
  - with small-$\alpha_s$ PT

- **Input For**
  - New measurements, New processes, Wider range of $Q$

- **Test pQCD theory with**

- **Measurements**
  - **TMD processes**

- **Higher Orders, New Factorization Theorems**
Implementing **TMD-Factorization**  
(Current Work)

- **Non-perturbative Theory of TMDs**
  - Used In
  - Predictions with small-$\alpha_s$ PT
  - Higher Orders, New Factorization Theorems
- **Fits**  
  Few compared with collinear case
- **Input For**  
  - TMD-Factorization, TMD-Evolution, (Modified) Universality
  - Test pQCD theory with
- **Measurements**  
  - TMD processes
- **Improvements**  
  In non-perturbative theory
- **New measurements, New processes, Wider range of Q**
Implementing TMD-Factorization (Current Work)

- Improvements in non-perturbative theory
- \textbf{Non-perturbative Theory of TMDs}
  - 
  - \textbf{Fits}
    - Few compared with collinear case
  - 
  - \textbf{Predictions}
    - with small-$\alpha_s$ PT

- Lattice QCD, Eff. Theories, Relation to GPDs
- \textbf{TMD-Factorization, TMD-Evolution, (Modified) Universality}

- Input For
  - Measurements
    - TMD processes

- Test pQCD theory with

- Higher Orders, New Factorization Theorems

- New measurements, New processes, Wider range of $Q$
Implementing **TMD-Factorization**

*Current Work*

- **Improvements** in non-perturbative theory
- **Fits**
  - Few compared with collinear case
- **Used In**
  - Non-perturbative Theory of TMDs
- **Predictions**
  - with small-$\alpha_s$ PT
- **TMD-Factorization, TMD-Evolution, (Modified) Universality**
- **Measurements**
  - **TMD processes**
  - New measurements, New processes, Wider range of $Q$
  - Lattice QCD, Eff. Theories, Relation to GPDs
  - Jlab@12GeV, EIC, LHC, Belle...
- **Test pQCD theory with**
- **Input For**
- **Higher Orders, New Factorization Theorems**
- **Jlab@12GeV UPGRADE**
  - double cryo capacity, upgrade magnets and power supplies
  - add arc, upgrade existing Halls
  - add new hall, 5 new cryomodules
Implementing TMD-Factorization (Current Work)

- **Improvements in non-perturbative theory**
- **Non-perturbative Theory of TMDs**
  - Used in New QCD Methods
  - Test pQCD theory with TMD-Factorization, TMD-Evolution, (Modified) Universality
  - New QCD Methods
  - Non-Universality, Factorization Breaking, Color Entanglement

- **Predictions with small-$\alpha_s$ PT**
- **Fits** Few compared with collinear case
- **Measurements** TMD processes
  - New measurements, New processes, Wider range of $Q$
  - Jlab@12GeV, EIC, LHC, Belle, ...
  - Lattice QCD, Eff. Theories, Relation to GPDs

- **Input For**
  - Higher Orders, New Factorization Theorems
  - TMD-Factorization, TMD-Evolution, (Modified) Universality
  - Input For (Current Work)
Specific Fits: Two Approaches

Fixed Scale

Single-Inclusive DIS

Resummation in CSS

Drell-Yan

(Schweitzer, Teckentrup, Metz (2010))

(Landry, et al. (2003))
Unpolarized TMD PDFs

Fixed Scale
(Schweitzer, Teckentrup, Metz (2010))

Resummation in CSS
(Landry, et al. (2003))

(Aybat, TCR (2011))

Up Quark TMD PDF, x = 0.09

\[ F_{u_T}(x=0.09, k_T) (\text{GeV}^{-2}) \]

- \( b_{T,max} = 0.5 \text{ GeV}^{-1} \)
- \( b_{T,max} = 1.5 \text{ GeV}^{-1} \)

- \( Q = \sqrt{2.4} \text{ GeV} \)
- \( Q = 5.0 \text{ GeV} \)
- \( Q = 91.19 \text{ GeV} \)
Sivers Function

FIG. 1: (Color online.) The (negative of the) up quark Sivers function at $x = 0.1$.

(Collins et al., 2010): Fixed Scale

Bochum Fits

- $Q = \sqrt{2.4}$ GeV
- $Q = 5$ GeV
- $Q = 91.19$ GeV

Torino Fits

(Mixed Scale)

(Anselmino et al., 2010): Fixed Scale

The evolution of the Sivers function is obtained by multiplying the up quark Sivers function by a Gaussian shape, even after evolution to large $k_T$. The Gaussian fits of the Torino group are just the negative of the evolved Sivers function. For the Torino fits, the down quark Sivers function is obtained by multiplying the $-F_{TT}$ at large $Q$, with the 1-2 ratio of the Gaussian fits to the original Sivers functions.

The ratio of the Gaussian fits for a range of values is shown in Table I.

TABLE I. (Fortran, C++, and Wolfram resulting values for the Gaussian parameters is shown in the dashed curves in Fig. 2. A table of the Gaussian fits to the Sivers function at intermediate and large $Q$ can aid in providing meaningful parametrizations of the nonperturbative input over lattice QCD calculations [48].

The Gaussian fits of the Torino group are about as good as those of the Bochum group, with the lower plot found by evolving the Bochum fits to large $Q$. The upper plot is found by evolving the Torino fits to large $Q$. Figure 1 suggests that, apart from the tail at large $Q$, the Sivers function continues to be well described by a Gaussian shape, even after evolution to large $Q_T$. The evolution to large $Q$ is factorized, requiring only a specification of the scale dependence of the Gaussian parameters. This saves having to directly calculate the contribution to evolution results in a substantial modification of the shape and normalization of the TMD PDF, however, it should be emphasized that the perturbative calculation of sign if used in Drell-Yan.

The evolution of the Sivers function is obtained by multiplying the up quark Sivers function by a Gaussian shape, even after evolution to large $k_T$. The Gaussian fits of the Torino group are just the negative of the evolved Sivers function. For the Torino fits, the down quark Sivers function is obtained by multiplying the up quark one. For the Torino fits, the down quark Sivers function is just the negative of the evolved Sivers function.

The evolution to large $Q$ is factorized, requiring only a specification of the scale dependence of the Gaussian parameters. This saves having to directly calculate the contribution to evolution results. The Gaussian fits of the Torino group are about as good as those of the Bochum group.
**Sivers Function**

**Up Quark Sivers Function**

\[ x = 0.1 \]  

(Aybat, Collins, Qiu TCR (2012))

\[
\begin{align*}
A_{UT} & = \sin (\phi_n - \phi_s) \\
A_{UT} & = \sin (\phi_n - \phi_s)
\end{align*}
\]

(Hermes, Compass)

(Aybat, Prokudin, TCR (2012))

(Compass, (2011))
New fits: To do

• Incorporate data from all types of processes.
  – SIDIS, DY, $e^+e^-$, different targets....

• Incorporate all types of observables.
  – Unpolarized cross sections, spin asymmetries...
Constraining Non-Perturbative Parts

**TMD PDF:** quark in hadron

\[ F_f/P(x, k_T) \]
\[ F_{\bar{f}}/\bar{P}(x, k_T) \]

**TMD PDF:** antiquark in hadron

\[ F_{\bar{f}}/P(x, k_T) \]
\[ F_f/P(x, k_T) \]

**TMD Fragmentation Function**

\[ D_{h/f}(z, k_T) \]

**More TMDs**

**Drell-Yan**

\[ pp \rightarrow \gamma^*(Z, W) + X \]
\[ p\bar{p} \rightarrow \gamma^*(Z, W) + X \]

**SIDIS**

\[ lp \rightarrow h + X \]

**More Processes Different Targets**

\[ l^+l^- \rightarrow h_1 + h_2 + X \]
Unpolarized Proton vs. Anti-proton

TMD PDF: quark in hadron
\[ F_{f/P}(x, k_T) \]
\[ (F_{\bar{f}/P}(x, k_T)) \]

TMD PDF: antiquark in hadron
\[ F_{\bar{f}/P}(x, k_T) \]
\[ (F_{f/P}(x, k_T)) \]

TMD Fragmentation Function
\[ D_{h/f}(z, k_T) \]  

More TMDs

pp → γ*(Z, W) + X  
p\bar{p} → γ*(Z, W) + X  
Drell-Yan

SIDIS

lp → h + X

l^+l^- → h_1 + h_2 + X

More Processes  
Different Targets
Unpolarized Proton vs. Anti-proton

*Current Project with C. Aidala and B. Field*

*(In Preparation)*

- Focus only on pp and p-pbar Drell-Yan.
- Focus on small $q_T$.
- Use all available data.
- Vary non-perturbative functional forms.
- Test correlation of $x$ and $b_T$.
- Extract any difference between valence and sea.

*(See, e.g., Schweitzer, Strikman, Weiss (2012))"
New fits: *To do*

- Incorporate data from *all* types of processes.
  - SIDIS, DY, $e^+e^-$, different targets....

- Incorporate *all* types of observables.
  - Unpolarized cross sections, spin asymmetries...

- Test/Compare non-perturbative functional forms

- Treatment of large transverse momentum

- Higher orders.
  - Much already exists, some translating to TMD formalism needed

- Test non-universality
  - Sivers sign
  - Factorization breaking
TMD-Factorization Breaking

• Need tests of TMD-factorization.

• Polarizations in final state:
  – Lambda Hyperon: Transverse Polarization
    • Seen in hadron-hadron collision
    • Not seen in SIDIS, e+e−

  *Factorization Breaking*
  (TCR, Mulders (2010))
  (TCR, in preparation)
Summary

• Theoretical advances.

• Implementations needed.

• Progress being made.

• Next:
  – Include Higher Orders
  – Improve treatment of large $q_T$ region
  – Unpolarized SIDIS data.

Please see updates:

https://projects.hepforge.org/tmd/
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- Transverse Momentum Dependent (TMD) Factorization: ✓
- TMD project: Implementing TMD factorization: ✓

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Thank You!

Jefferson National Laboratory – February 6, 2013
Solutions

- After evolution and \( \mu = \sqrt{\zeta} = Q \)

\[
\tilde{F}_{f/P}(x, b_T; Q, Q^2) =
\]

\[ A \left\{ \sum_j \int_{x}^{1} \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \times \right. \]

\[ B \left\{ \times \exp \left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[ \gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\} \right. \times \]

\[ C \left\{ \times \exp \left\{ g_{f/P}(x, b_T) + g_K(b_T) \ln \frac{Q}{Q_0} \right\} \right. \]

**Example Matching Prescription:**

\[ b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{T\text{max}}^2}} \]

\[ \mu_*(b_T) = C_1/b_* \]

**Collinear PDFs**

**Nonperturbative large \( b_T \) behavior**
Constraining Non-Perturbative Parts

\[ g_K(b_T) \ln \frac{Q}{Q_0} \]
Constraining Non-Perturbative Parts

\[ g_K(b_T) \ln \frac{Q}{Q_0} \]

- Drell-Yan
- Z/W production
- SIDIS
- Collins
- Boer-Mulders
- Sivers
- Unpolarized Cross Sections
- Fragmentation Functions
- PDFs
- \( l^+ l^- \) to back-to-back hadrons