Outline

- Introduction
- Proton Compton Scattering
- Pion production in $NN \rightarrow NN\pi$
- Few-Body Calculations in Stochastic Variational Method
Effective Field Theory:

- **low-energy** theory of some "fundamental" theory
- external momenta much smaller than some high-energy scale: $Q \ll M_{\text{hep}}$
- the $S$-matrix calculated in an EFT is an *expansion* in the powers of $\chi = Q/M_{\text{hep}}$
- the degrees of freedom (DOFs) $\neq$ those of the underlying theory
- *fundamental symmetries* constrain the dynamics of the EFTs
- renormalizable order by order
- a finite number of parameters (LECs) arises at each order; their values are found by matching with the fundamental theory or from experiment
- *counting rules* tell what order is to be assigned to a particular graph
Effective Field Theories

Chiral EFT

- based on the spontaneously broken (and approximate) chiral symmetry of QCD
- theory of low-energy interactions of baryons and pions (pseudo-Goldstone bosons of QCD)
- the underlying scale is set by the mass of the lightest non-Goldstone boson $\sigma$, $m_\sigma \approx 600$ MeV
- the nucleon mass, $M \approx 940$ MeV, is not a light scale compared with $m_\sigma$
  - treat baryons with heavy-particle formalism — heavy-baryon ChEFT (non-relativistic reduction) (Jenkins, Manohar, 1991)
  - ...or no reduction: on-mass-shell regularization (Gegelia, Japaridze, Wang, 1999, Fuchs, Gegelia, Japaridze, Scherer, 2003)
  - a covariant calculation + an appropriate renormalization
proton Compton scattering in covariant baryon ChEFT
Compton Scattering

Compton Scattering:
- allows to study the structure and the e.m. properties of the nucleon
- the nucleon polarizabilities:

\[ T(\omega, \omega') = \frac{-e^2 Z^2}{4\pi M} + \alpha(\bar{\epsilon}'^* \cdot \bar{\epsilon}')\omega\omega' + \beta(\bar{\epsilon}'^* \times \vec{k}') \cdot (\bar{\epsilon}' \times \vec{k}) + \mathcal{O}(\omega^4) \]

experimental values (proton, Griesshammer et al. (2012)):
\[ \alpha_E^{(p)} = 10.5 \pm 0.9, \quad \beta_M^{(p)} = 2.7 \pm 0.9 \text{ (units: } 10^{-4} \text{ fm}^{-3}) \]

- test ChEFT: just a few loops and no LECs contribute to polarizabilities at the leading order: (Bernard, Kaiser, Meißner, 1992)

\[ \alpha_E^{HBLO} = 12.5, \quad \beta_M^{HBLO} = \frac{1}{48} \frac{e^2 g_A^2}{(4\pi)^2 f^2_\pi m_\pi} = 1.3. \]

\[ \alpha_E^{BLO} = \frac{5}{24} \frac{e^2 g_A^2}{(4\pi)^2 f^2_\pi m_\pi} + \cdots = 6.3, \quad \beta_M^{BLO} = \frac{1}{48} \frac{e^2 g_A^2}{(4\pi)^2 f^2_\pi m_\pi} + \cdots = -1.8. \]

- the \( \Delta \) isobar gives a big contribution to \( \beta \simeq 7 \) (Pascalutsa, Phillips (2003)) — a large counterterm at next-to-leading order in the HBChEFT calculation;

- \( \Delta \) contribution can be more naturally accommodated in covariant BChEFT (V.L., Pascalutsa (2009,2010))
Covariant vs. Heavy Baryon ChEFT

- large $1/M$ corrections to $\alpha_E$ and $\beta_M$
- other sizable effects of HB expansion in Compton scattering?
- adjust polarizabilities to expt. values (V.L., McGovern, Pascalutsa, Phillips (2012))

![Diagrams]

counterterm contributions to $\alpha_E$ and $\beta_M$

<table>
<thead>
<tr>
<th></th>
<th>$\delta \alpha_E$ ($10^{-4}$ fm$^3$)</th>
<th>$\delta \beta_{M1}$ ($10^{-4}$ fm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tree</td>
<td>10.5</td>
<td>2.7</td>
</tr>
<tr>
<td>$+\Delta$</td>
<td>10.6</td>
<td>-4.4</td>
</tr>
<tr>
<td>$+\pi$, HB, $\Delta$</td>
<td>-9.8</td>
<td>-7.1</td>
</tr>
<tr>
<td>$+\pi$, $\Delta$</td>
<td>-0.8</td>
<td>-1.2</td>
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- appears to remove most of the effects of HB expansion
Covariant vs. Heavy Baryon ChEFT

- adjust polarizabilities to expt. values

\[ \frac{d\sigma}{d\theta_{\text{lab}}} [\text{nb/sr}], \text{unpolarised, vs. lab energy [MeV]} \]

\[ \frac{\Theta_{\text{lab}}}{E_{\text{Lab}}} \]

\[ \Theta_{\text{lab}} = 60^\circ \]

\[ \Theta_{\text{lab}} = 90^\circ \]

\[ \Theta_{\text{lab}} = 110^\circ \]

\[ \Theta_{\text{lab}} = 135^\circ \]

\[ \rightarrow \] HBChEFT and covariant BChEFT give consistent results at one-loop level (difference is of higher order in the expansion);

important in connection with experiments ongoing at MAMI and HIGS

Pion production in $NN$ collisions
Motivation/Results

\( \pi \) production in \( NN \) collisions

- test of ChEFT — the lowest-lying hadron inelasticity of \( NN \)
- \( \pi \) production at threshold (\( s \)-wave) — key to absorptive corrections to \( \pi d \) scattering
  - V.L. et al. (2005, 2006); Baru et al. (2007)
  - \( \Rightarrow \) needed for an extraction of \( \pi N \) scattering lengths Baru et al. (2011)
- necessary for studying CSB and isospin violation
  - Kolck, Miller, Niskanen (2000); Gårdestig et al. (2004); Filin et al. (2009); Miller, Bolton (2009)
  - experiment: \( A_{fb}(pn \rightarrow d\pi^0) \) — Opper et al. (2003); \( dd \rightarrow \alpha\pi^0 \) — Stephenson et al. (2003)
- immediate links to many other reactions (\( \gamma d \rightarrow \pi NN, \pi d \rightarrow \gamma NN \), weak reactions, e.g. \( pp \rightarrow de^+\nu, NNN \) forces)
  - Gårdestig, Phillips (2006); Nakamura (2008); Park et al. (2003); Epelbaum (2002); Nogga et al. (2006); Gazit et al. (2007); Baru et al. (2009)
Modified Power Counting

\[ \text{NN} \rightarrow \text{NN}_\pi \]

Large momentum transfer already at threshold: \( |\vec{p}_{\text{thr}}| = \sqrt{m_\pi (M + m_\pi /4)} \)

— modification of power counting scheme is needed


The expansion parameter in this case is

\[ |\vec{p}_{\text{thr}}| / M \sim \sqrt{m_\pi / M} = \sqrt{\chi} \]

— \( \Delta(1232) \) should be explicitly included: \( (M_\Delta - M) / M \sim |\vec{p}_{\text{thr}}| \)
s-wave pion production in $NN \rightarrow NN\pi$
The Irreducible Diagrams up to NLO – Complete Set

\[ \mathcal{A}_{d\pi^+}^{a+b+c+d1(\text{irr})+d2(\text{irr})} = \frac{g_A^3 |\vec{q}|}{256 f_\pi^5} (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \frac{\vec{q}}{2} \left( -2 + 3 + 0 - \frac{1}{4} - \frac{3}{4} \right) = 0 \]

Leading order pion rescattering vertex goes on shell

\[ (0,0) \quad (0,0) \]

\[ \ell_0 \quad \ell_0 + k_0 \]

\[ (m_\pi, 0) \]

\[ (m_\pi, 0) \]

\[ \propto (\ell_0 + k_0) = \frac{3}{2} m_\pi \rightarrow 2 m_\pi \]
The Irreducible Diagrams up to NLO – Complete Set

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Leading order pion rescattering vertex goes on shell

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enhancement factor 4/3 in the amplitude
Results for $pp \rightarrow d\pi^+$

Theoretical uncertainty is \( \mathcal{O}(\frac{m_\pi}{M_N}) \sim 30\% \) (conservative estimate)—hatched bar

Experimental Data:

- these findings allow for a calculation of dispersive and absorptive corrections to $\pi d$ scattering length (V.L. et al. (2006); Baru et al. (2007))
  - serves for an extraction of $\pi N$ scattering lengths (Baru et al. (2011))
- an NNLO $NN \rightarrow NN\pi$ s-wave calculation is called for (Baru et al. (2012))
  - reduce the uncertainty in $pp \rightarrow d\pi^+$
  - necessary for a description of $pp \rightarrow pp\pi^0$
$p$-wave pion production in $NN \rightarrow NN\pi$
NN → NNπ and other reactions: 4Nπ LEC

▶ a single 4Nπ contact term with a LEC \( d \) enters all the reactions
  ▶ momentum transfer \( \sim \sqrt{m_\pi M} \): NN → NNπ
  ▶ momentum transfer \( \sim m_\pi \): all others

▶ we want to describe them all with one LEC \( d \) (with reasonable accuracy at NNLO)
  a very nontrivial test of ChEFT

▶ Gazit et al (2000): works for 3N binding energies
▶ does it work for different channels in NN → NNπ?
\textbf{p-wave $NN \rightarrow NN\pi$: production mechanism}

Baru et al (2009)

- we fit $d$ to experimental data — cross sections and asymmetries
  - $A_y = \frac{d\sigma_{↑↑} - d\sigma_{↑↓}}{d\sigma_{↑↑} + d\sigma_{↑↓}}$ — target analysing power
  - $\implies ^1S_0 \rightarrow ^3S_1 p$ in $pp \rightarrow pn\pi^+$, $pp \rightarrow d\pi^+$
  - $\implies ^3S_1 \rightarrow ^1S_0 p$ in $pn \rightarrow pp\pi^-$
- $d = d(\Lambda)$ depends on the regularization/NN interaction used
- $d$ absorbs the short-range part of the production operator
  \[
  A_{s.r.} \propto \frac{(\vec{p} - \vec{p}')^2}{(\vec{p} - \vec{p}')^2 + m^2_\pi} = \text{const} + \mathcal{O}(N^4\text{LO})
  \]
Results: $pp \rightarrow d\pi^+$, $np \rightarrow pp\pi^-$

$np \rightarrow pp\pi^-$

$\eta = 0.6$
energy cut-off: $E_{pp} \leq 1.5$ MeV
data: TRIUMF, PSI

$pp \rightarrow d\pi^+$

$\eta = 0.14$
data: TRIUMF, IUCF

- Positive $d \sim 3$ is preferred in both reactions
- New data on $np \rightarrow pp\pi^-$ at low $\eta$ (expt: ANKE@COSY (2011))
New Data: $pp \to pp\pi^0$, $np \to pp\pi^-$

ANKE@COSY (2011)

$np \to pp\pi^-$

$\eta = 0.6$

energy cut-off:

$E_{pp} \leq 1.5$ MeV

$pp \to pp\pi^0$

Complete amplitude analysis: $s$-, $p$-, $d$-wave $\pi$ production amplitudes

ChEFT/hybrid helps justify fixing the phases of $d$-waves

$d$-wave is significant!

ChEFT calculation taking into account $d$-waves is in progress
few-body variational calculations in pionless EFT
Pionless Nuclear EFT

- High momentum scale $\sim m_\pi$: $Q \lesssim m_\pi$, $E \lesssim 20$ MeV for NN
- Contact interactions (with derivatives) $\Rightarrow$ delta-functions

\[ V = C_0 N^\dagger N + C_2 N^\dagger \nabla^2 N + C_4 (\nabla^2 N)^\dagger \nabla^2 N + \cdots \]

Weinberg (1990), Kaplan, Savage, Wise (1998), Kong, Ravndal (1999), ...
Beane, Bertulani, Cohen, Hammer, Higa, Gelman, van Kolck, Phillips, Rupak, ...

- Loops divergent (couple to arbitrary high momenta)

\[ T = + + \cdots \]

- Need to regularize and renormalize

$\Rightarrow$ use a (Gaussian) formfactor:
Two Nucleons

\[
V(r) = \left( C_0 + C_0^t \vec{\tau}_1 \cdot \vec{\tau}_2 \right) \exp \left( -\frac{r^2}{2\sigma^2} \right) + \left( C_1 + C_2^t \vec{\tau}_1 \cdot \vec{\tau}_2 \right) r^2 \exp \left( -\frac{r^2}{2\sigma^2} \right) + \cdots
\]

solve the Lippman-Schwinger equation; fit the parameters to the observables:

- works well up to \( T_{\text{lab}} \sim 20\text{MeV}; \)
- deuteron bound state is at the right position too;
- we want few-body bound states: \(^3\text{H}, \(^3\text{He}, \(^4\text{He}, \ldots\)
  \implies\) we need three-body forces (one contact term at leading order), and we are going to use the Stochastic Variational Method (SVM) (Varga, Suzuki, 1998).
Stochastic Variational Method

Hamiltonian

\[ H = T + V = \sum_{i=1}^{N+1} \frac{p_i^2}{2m_i} + \sum_{i<j}^{N+1} V_{ij}(\mathbf{r}_i - \mathbf{r}_j) + \sum_{i<j<k}^{N+1} V_{ijk}(\mathbf{r}_i - \mathbf{r}_j, \mathbf{r}_k - \mathbf{r}_j) \]

Trial function

\[ |\Psi\rangle = \sum_{\alpha} |\psi_{J^z L S}^\alpha \rangle, \]

Basis functions for the system of \( N \) nucleons:

\[ |\psi_{J^z L S}^\alpha \rangle = \sum_{M,S_z} C_{LMSS_z}^{JJZ} |f_{KLM}\rangle |\chi_{SS_z}^\alpha \rangle, \]

\[ |f_{KLM}\rangle = |f_{KLM}(\mathbf{x}, A, u)\rangle = v^{2K} \mathbb{Y}_{LM}(\mathbf{v}) \exp \left( -\frac{1}{2} A^{ij} x_i^T x_j \right), \]

- \( x_i \) is the \( i \)-th Jacobi coordinate,
- \( \alpha = (A, u) \), where \( A \) is an \((N - 1) \times (N - 1)\) matrix of parameters, and \( u \) is an \((N - 1)\)-component vector; \( v = u^i x_i^T \)
- \( A \) and \( u \) are parameters varied randomly and picked such that the lowest (generalized) eigenvalue of \( H \) is minimized \( \Rightarrow \) ground state
Stochastic Variational Method

Features of SVM:

+ (relatively) easy to implement and modify
+ easily scalable
+-/ best for ground states — excited states need extra care
  - not exact (although reasonably accuracy is expected)
  - computationally rather expensive (but still reasonable, e.g, $\sim 20$ hours for $^4\text{He}$)

Ongoing:

▶ $^3\text{H}$, $^3\text{He}$, $^4\text{He}$ wave functions:
  charge radii, magnetic moments;
  use the w.f. to calculate low energy reactions (e.g., Compton scattering on $^3\text{He}$)

▶ study Efimov physics (it is easy to change the potential and see what happens)

▶ heavier nuclei (starting from $^5$, $^6\text{He}$... going to $A = 8$ should be easy (hopefully!)
  but probably a supercomputer will be needed)
Outlook

▶ Compton scattering
  ▶ deuteron Compton scattering around $\pi$ photoproduction energies and beyond
    $\implies$ three-body $\pi NN$ dynamics around threshold
    $\implies$ neutron polarizabilities
▶ $NN \rightarrow NN\pi$
  ▶ get a consistent description of $p$-wave and $d$-wave $\pi$ production
    $\implies$ connection to $NNN$ forces
▶ few-body variational calculations
  ▶ incorporate scattering states
    $\implies$ few-body reactions
  ▶ use a more “realistic“ potential, e.g., that from ChEFTs
    $\implies$ higher energy