The exotic frontiers of lattice QCD

Raúl Briceño
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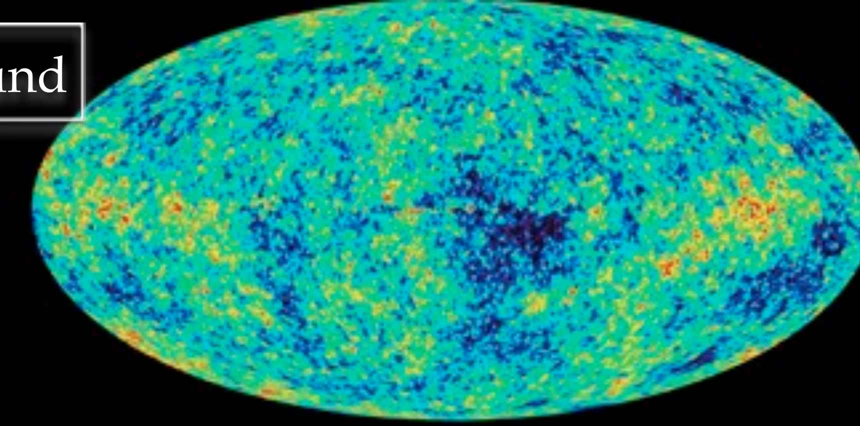
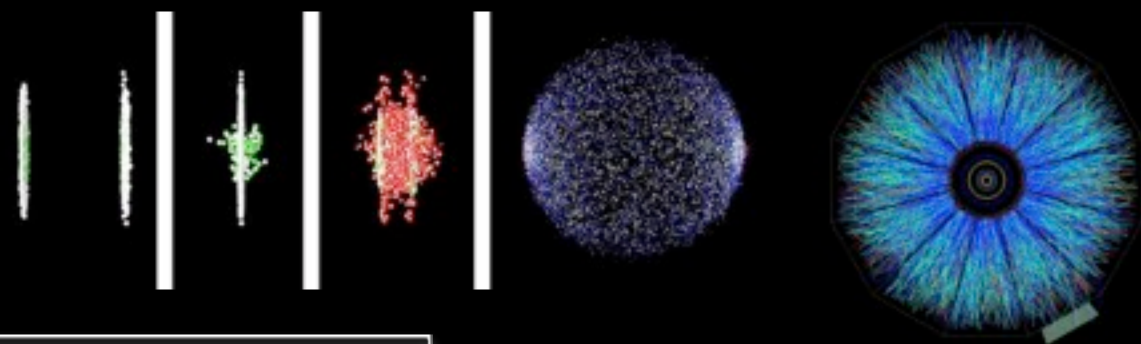
SESAPS, Nov. 2014

QCD

Cosmic microwave background





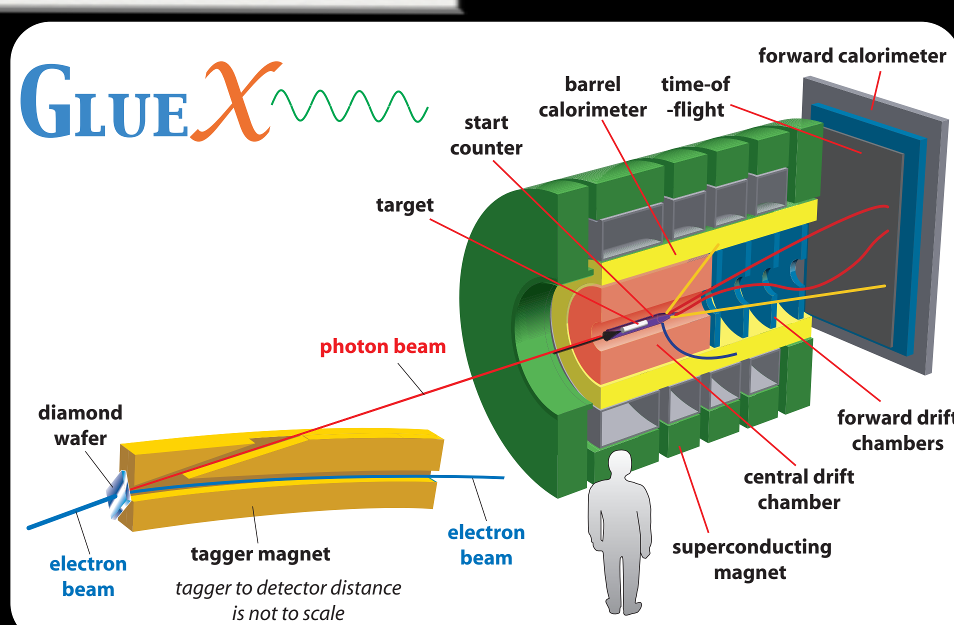
Heavy ion collisions

Stellar evolution

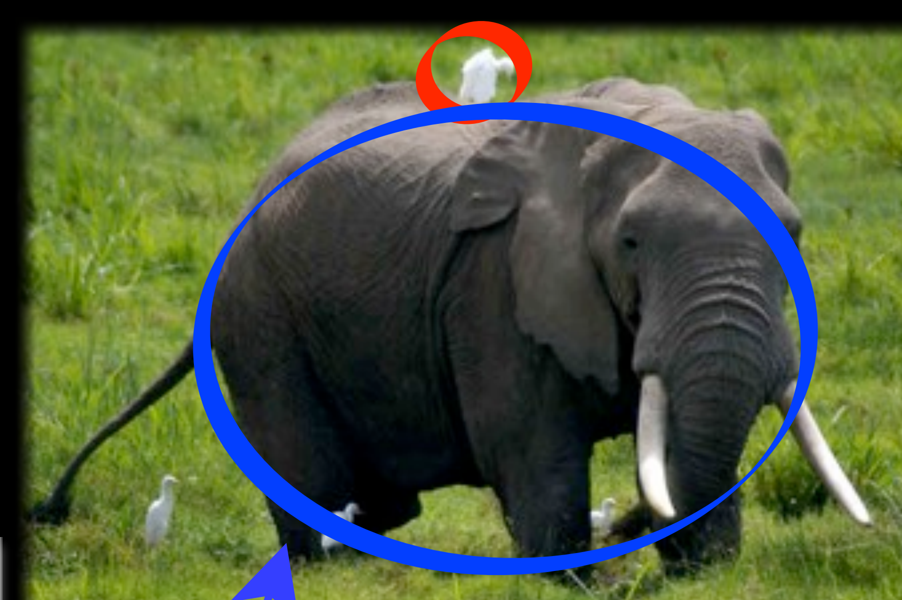
Supernova

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_{\mathbf{f}} \left(\mathbf{i} \not{D} - \mathrm{m_f} \right) \psi_{\mathbf{f}} - \frac{1}{4} \mathrm{tr} \left(\mathrm{GG} \right)$$

Higgs

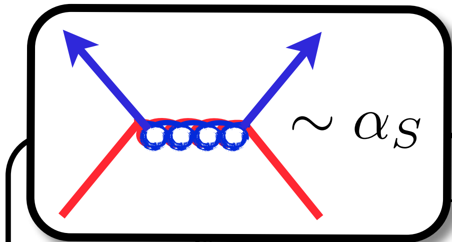


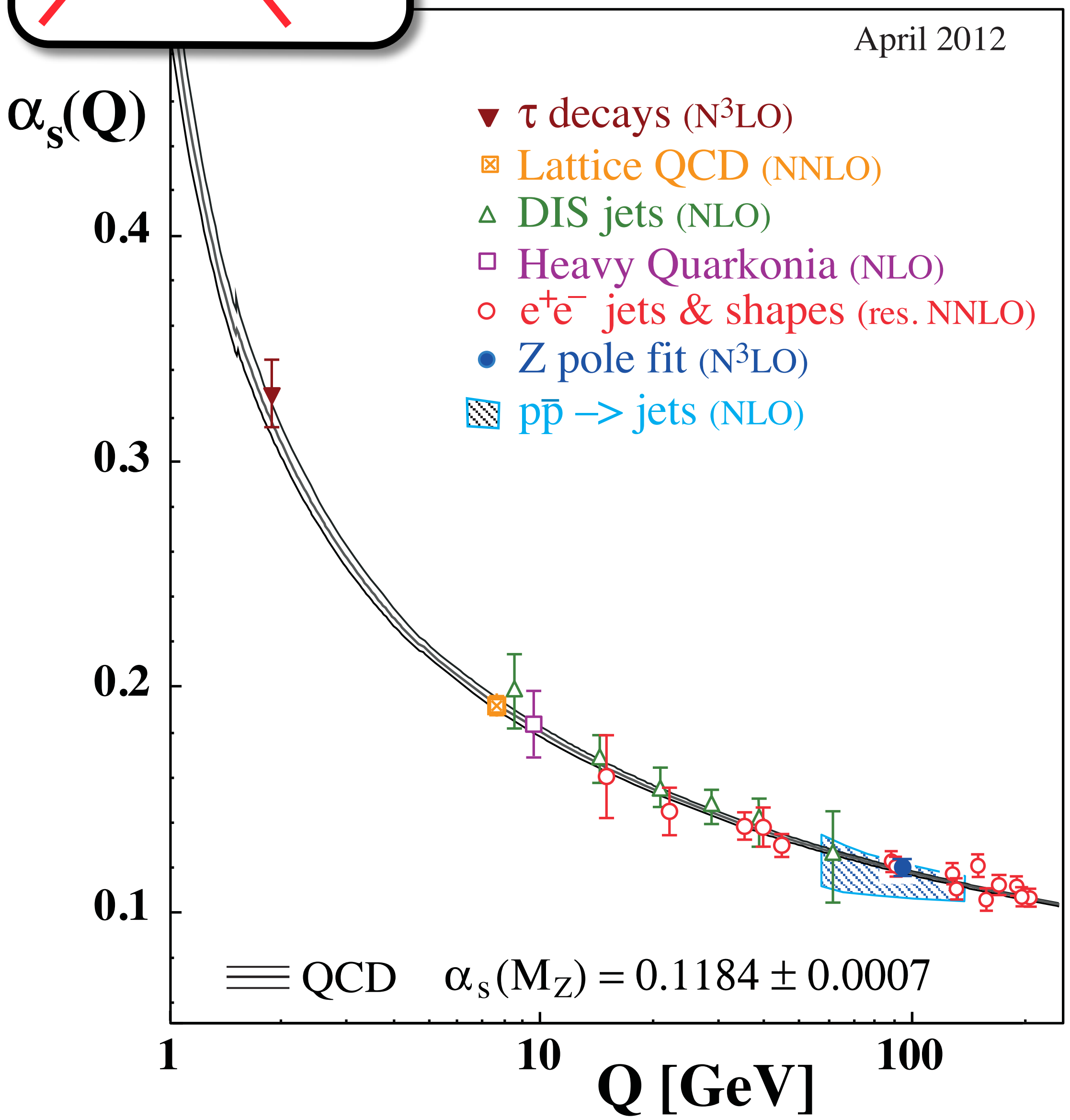
Accelerators



Glue!

Perturbative vs. non-perturbative



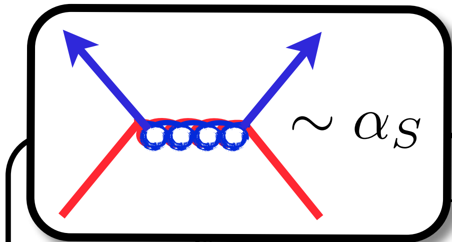


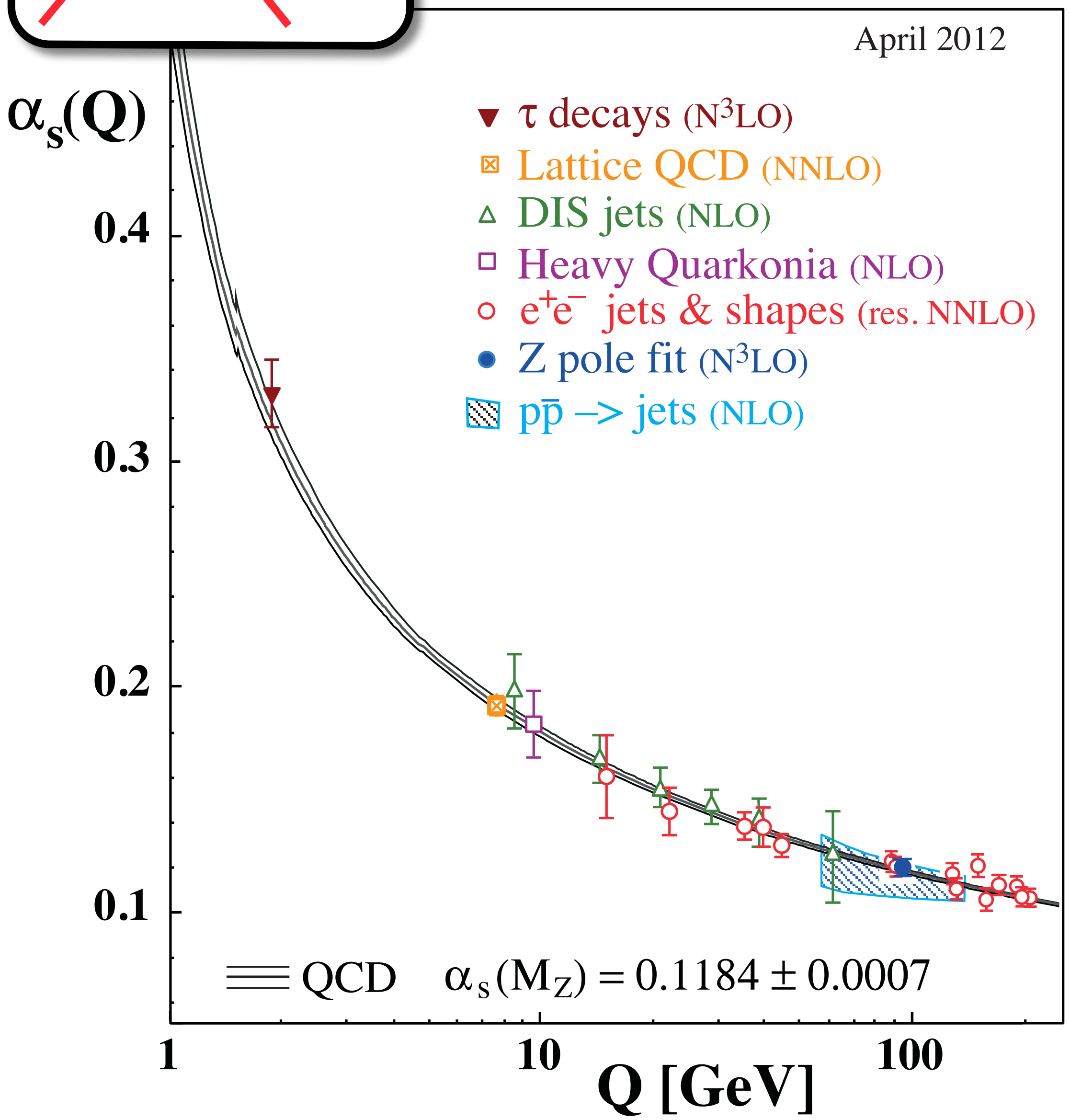
QCD is both perturbative and non-perturbative

Non-perturbative QCD tools:

- Effective field theories
  - low-to-medium energy phenomena
  - parametrizes analytic
  - limited predictive power
- Lattice QCD
  - numerical evaluation of QCD
  - fully predictive
  - no analytic grasp on phenomena
- Dispersive techniques, Dyson-Schwinger, etc.

Perturbative vs. non-perturbative





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nicely complimentary!

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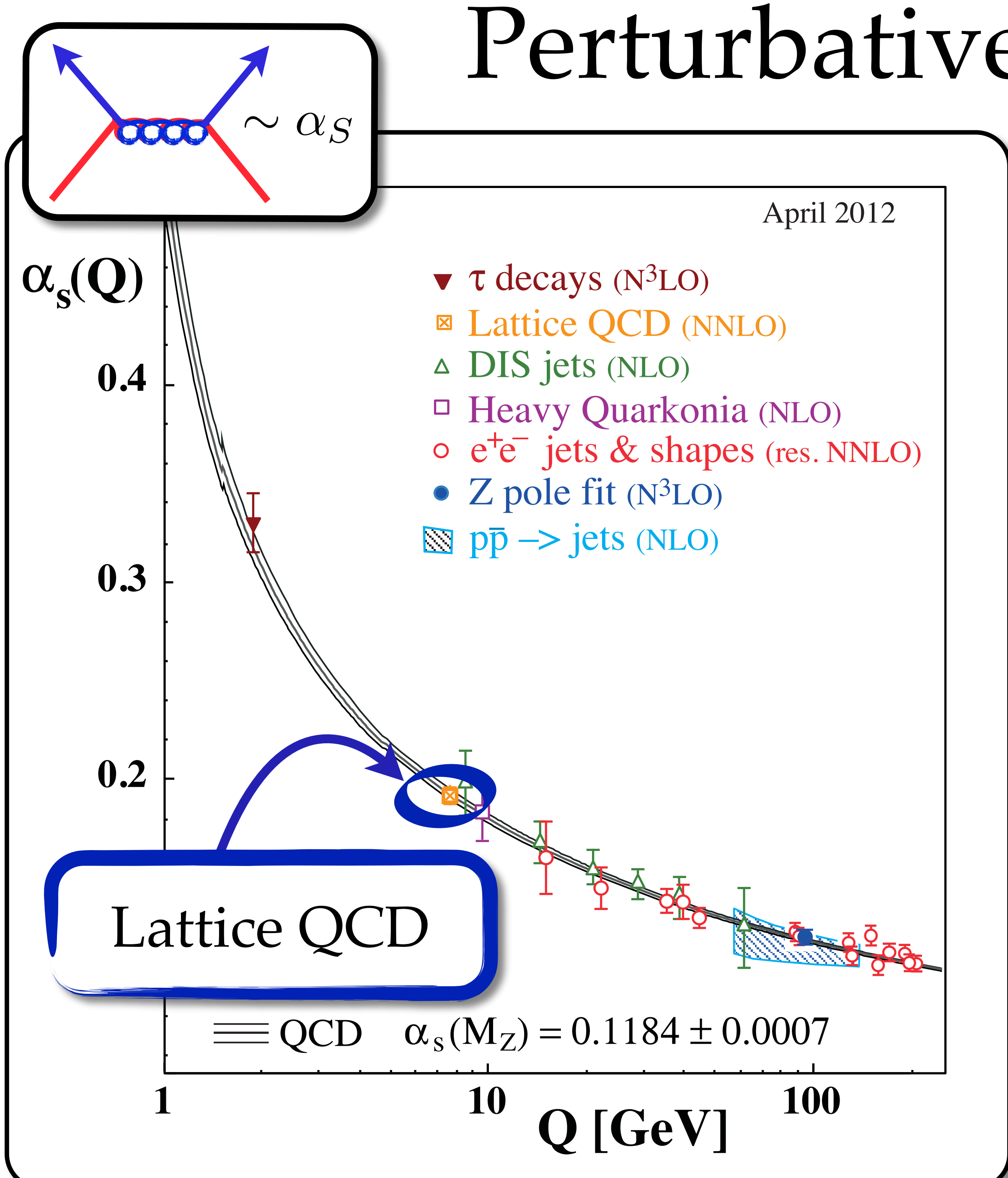

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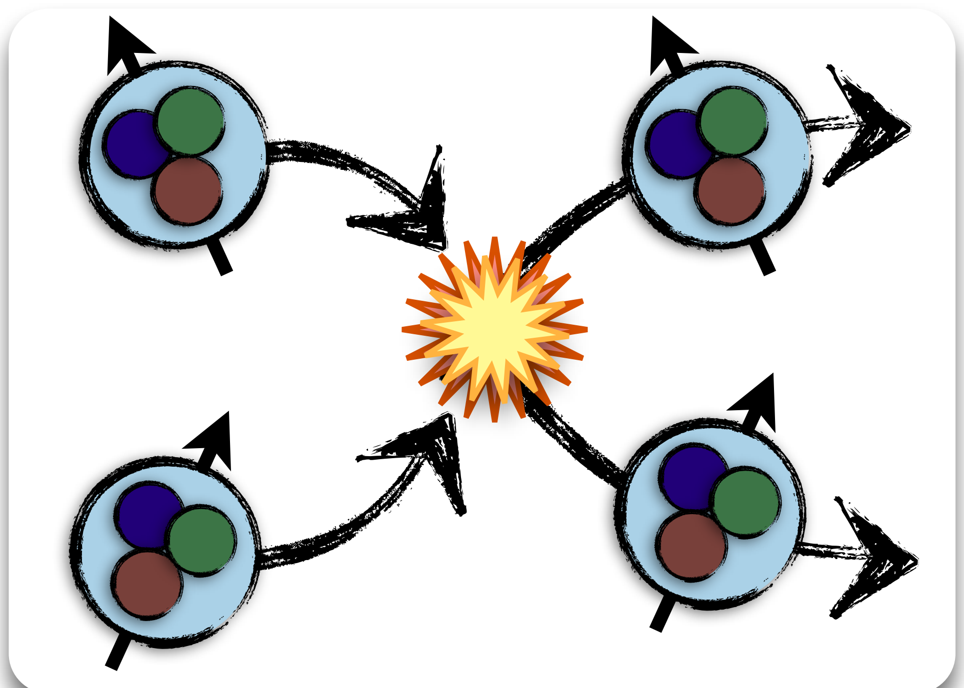
If only life were so easy!



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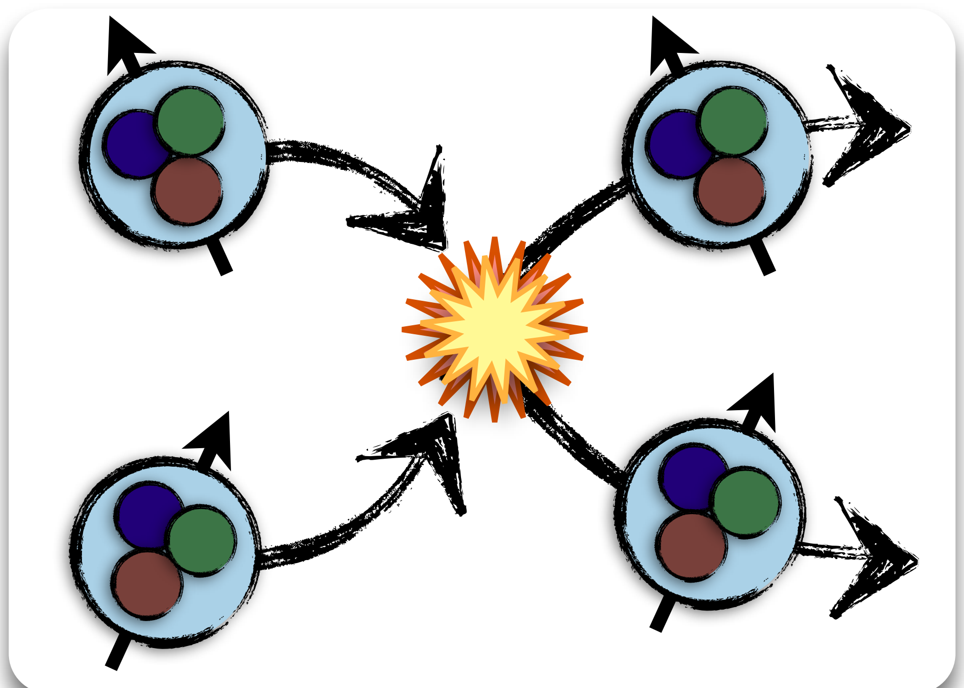
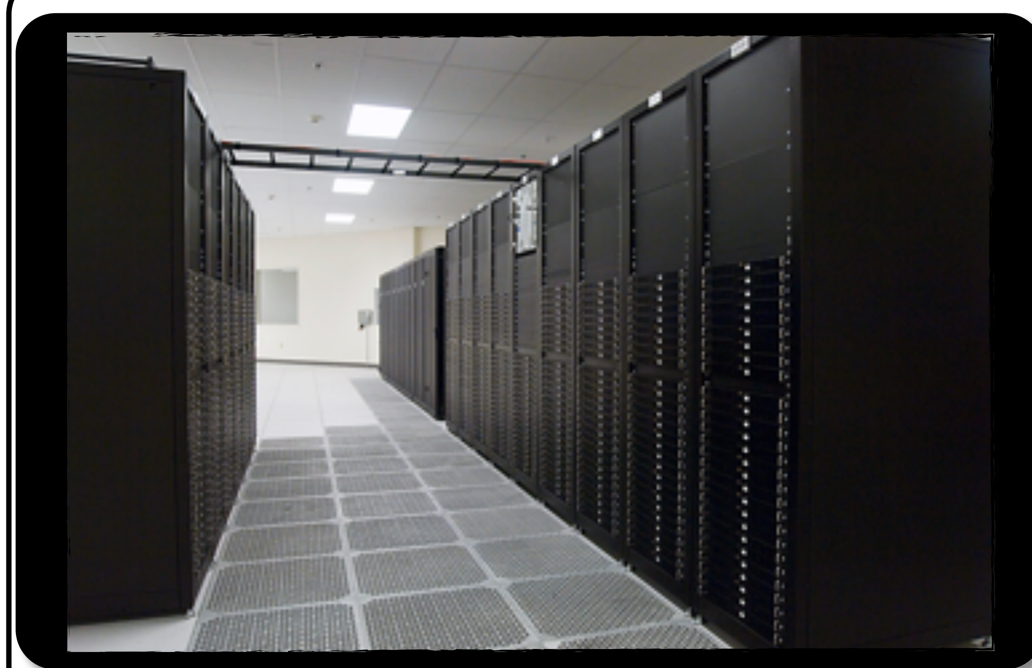
Let the computer do the hard work!

If only life were so easy!

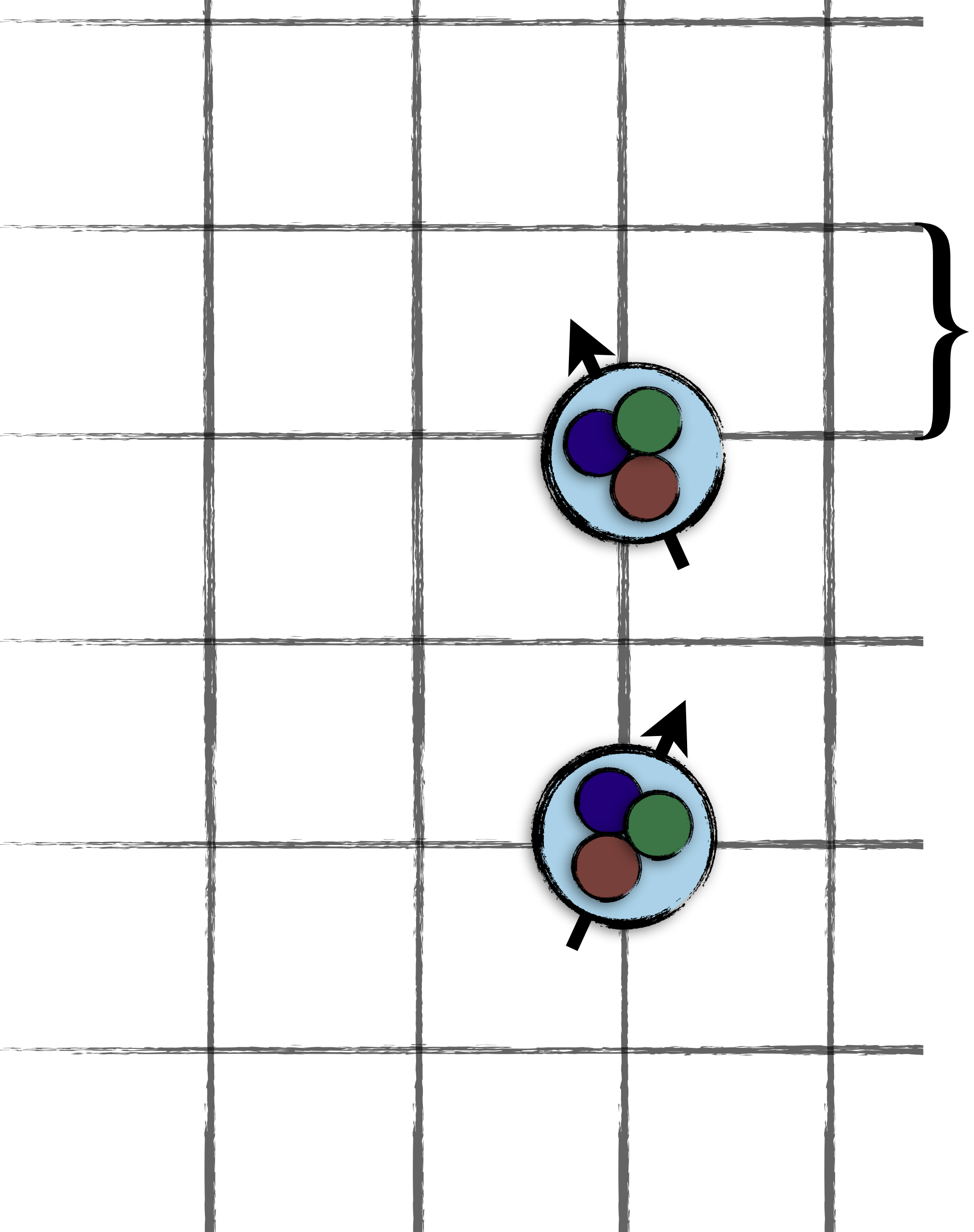




Cluster a JLab
Largest & fastest in Virginia!

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_f \left(i \not{D} - m_f \right) \psi_f - \frac{1}{4} \mathrm{tr}\left(G G \right)$$

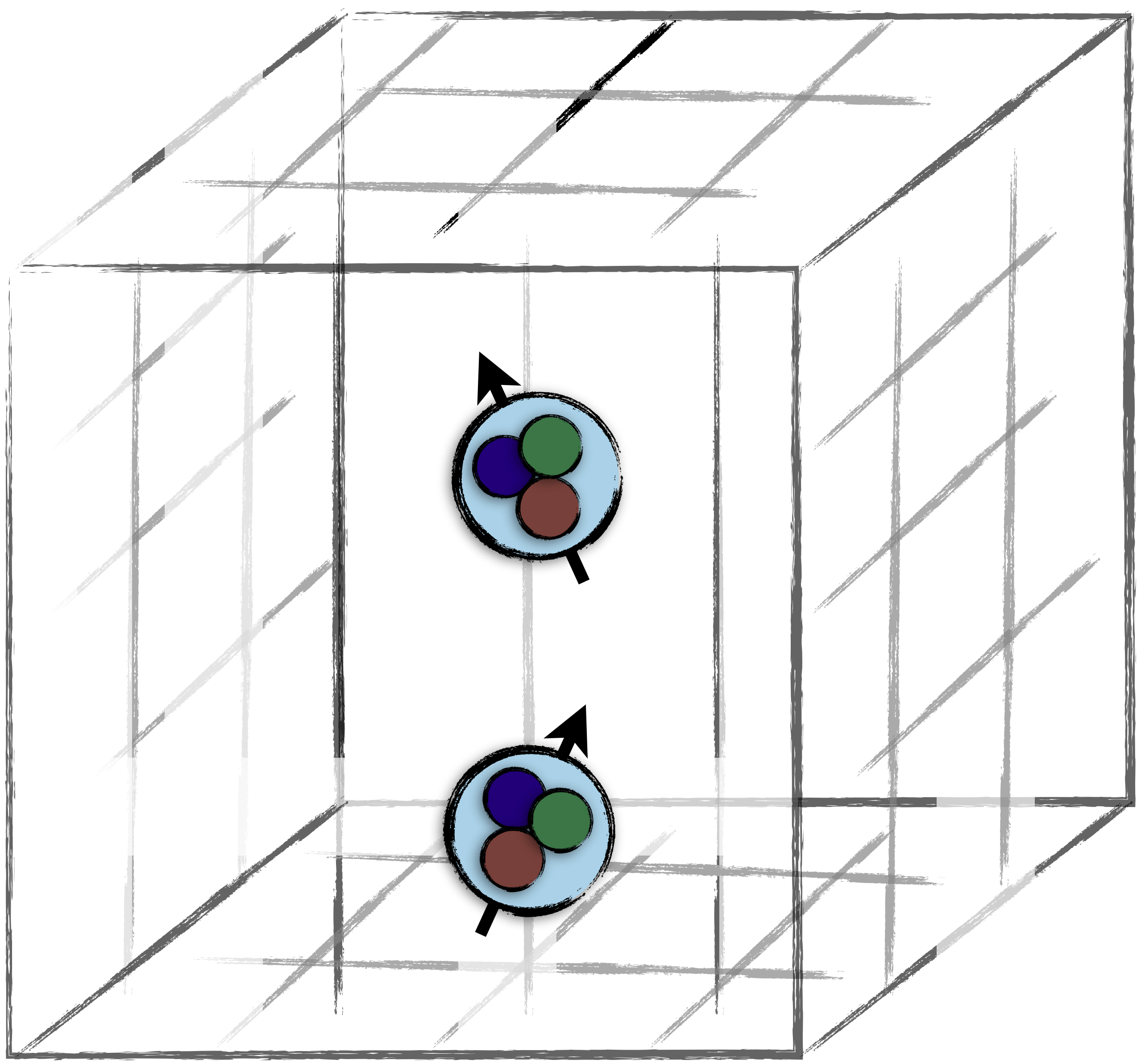
Lattice QCD in four *"easy"* steps



1. Discretize spacetime

- A separation scale between all points in space.
- Regulator
- ~0.1fm = 0.0000000000000001m
- 1million times smaller than hydrogen atom!

$$\mathcal{L}_{\mathbf{QCD}} = \bar{\psi}_{\mathbf{f}} \left(\mathbf{i}\, \not{D} - \mathrm{m_f}\right) \psi_{\mathbf{f}} - \frac{1}{4}\mathrm{tr}\left(\mathrm{GG}\right)$$



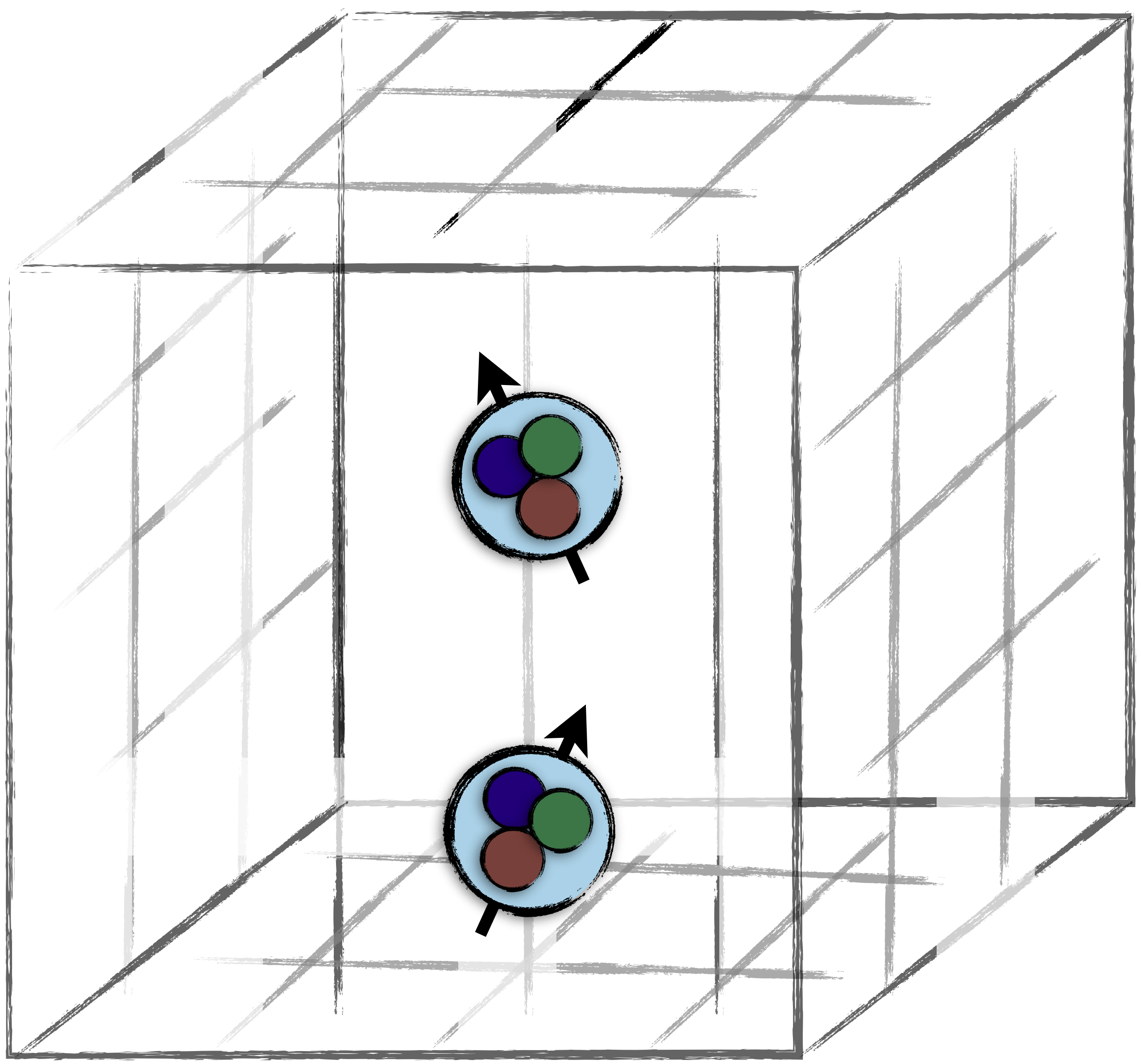
2. Truncate spacetime

- So it fits into a compute!
- ~4-6 fm

- 100,000 times smaller than hydrogen atom!

- It is *a small world after all!*

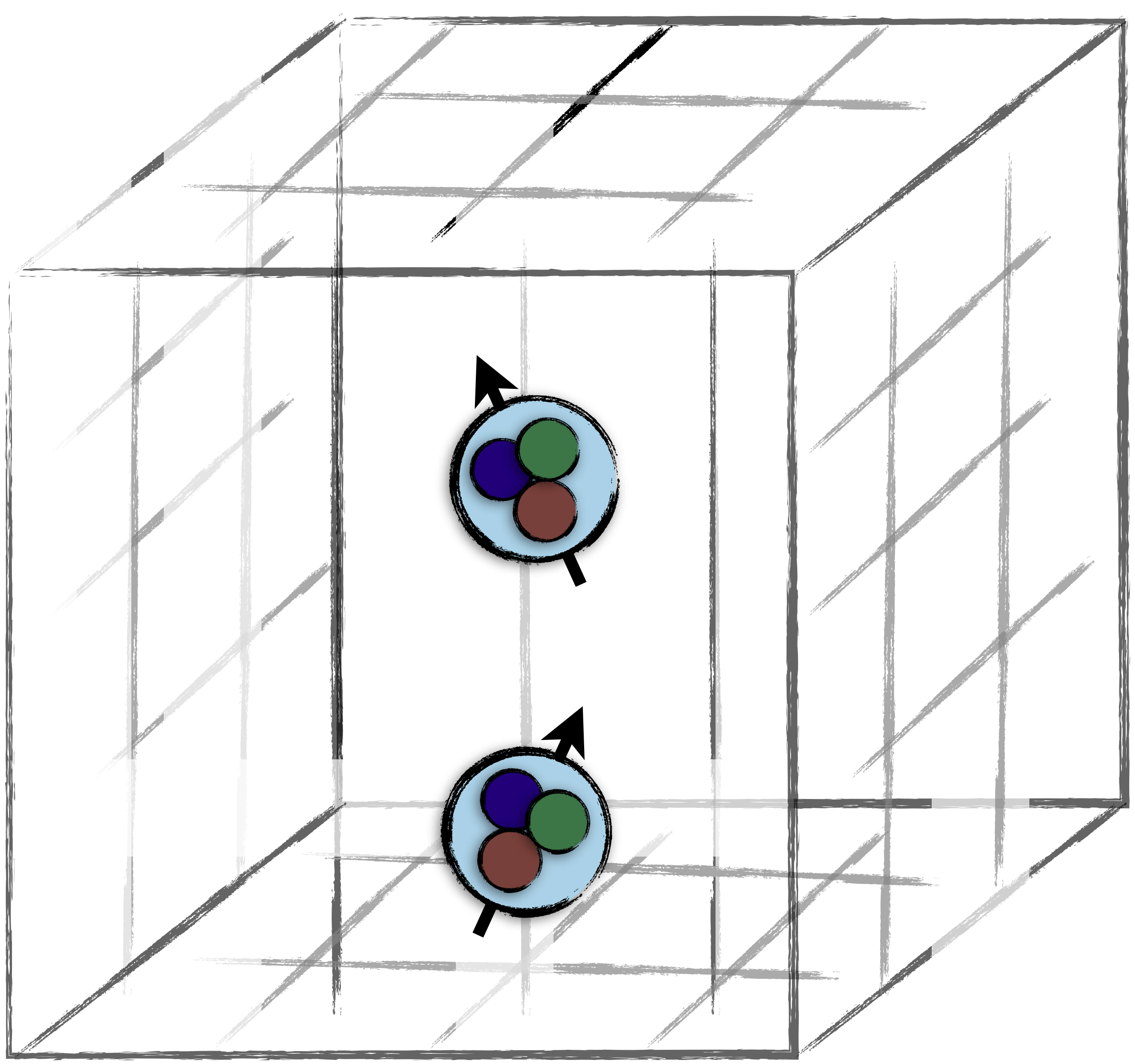
$$\mathcal{L}_{\mathbf{QCD}} = \bar{\psi}_{\mathbf{f}} \left(\mathbf{i} \not{\mathbf{D}} - \mathrm{m_f} \right) \psi_{\mathbf{f}} - \frac{\mathbf{1}}{\mathbf{4}} \mathrm{tr} \left(\mathrm{GG} \right)$$



3. Tune quark masses

- Set them to physical values
- *"Easier said than done!"*
- Dial the quark masses

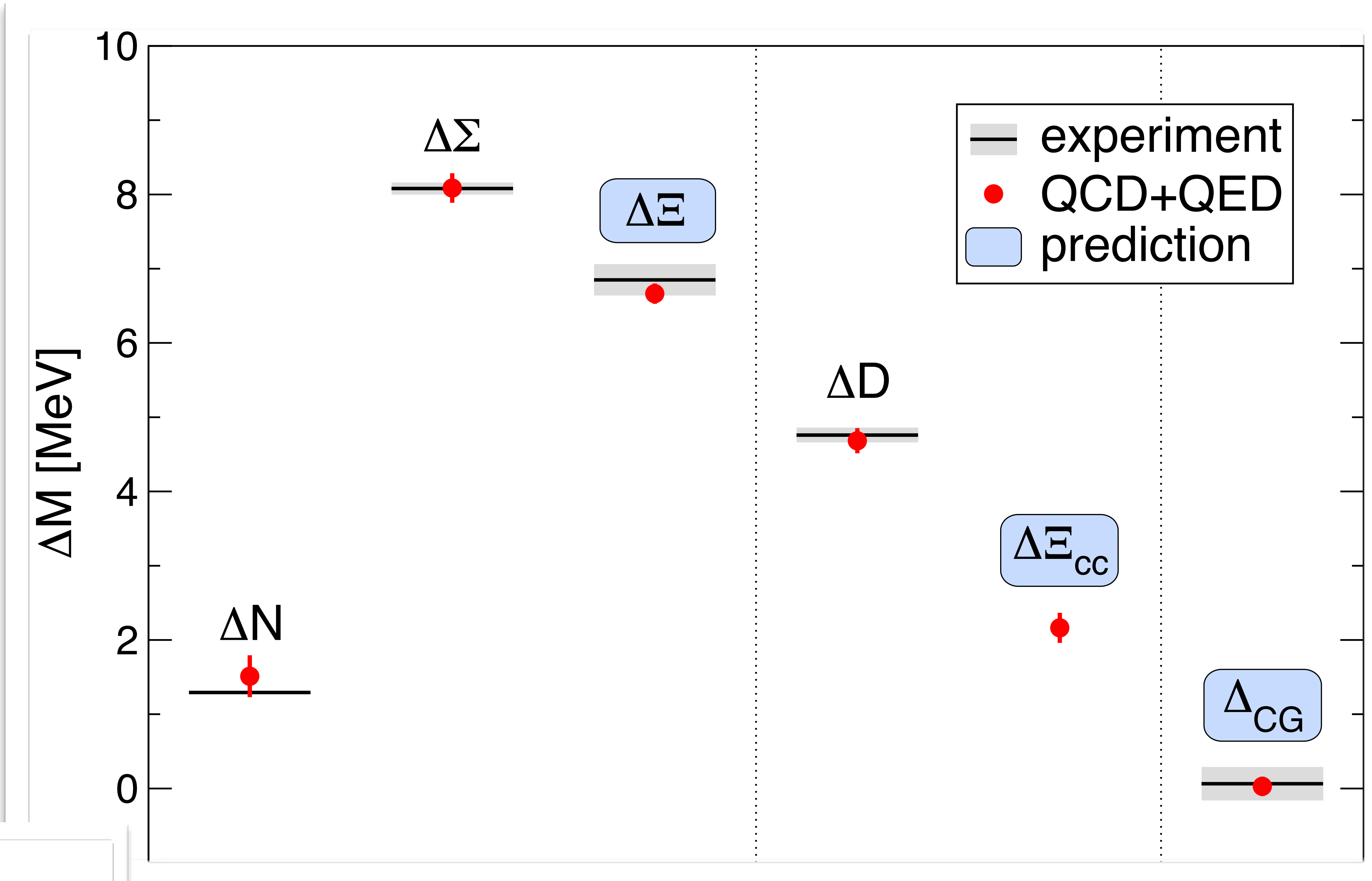
$$\mathcal{L}_{\mathbf{QCD}} = \bar{\psi}_{\mathbf{f}} \left(\mathbf{i} \, \not{D} - \mathrm{m_f} \right) \psi_{\mathbf{f}} - \frac{1}{4} \mathrm{tr} \left(\mathrm{GG} \right)$$



4. Make predictions (or postdiction)

Again, *"Easier said than done!"*

Present day state of the art



$$\Delta N = n - p$$

$$\Delta\Sigma = \Sigma^- - \Sigma^+$$

$$\Delta\Xi = \Xi^- - \Xi^0$$

$$\Delta D = D^\pm - D^0$$

$$\Delta\Xi_{cc} = \Xi_{cc}^{++} - \Xi_{cc}^{+}$$

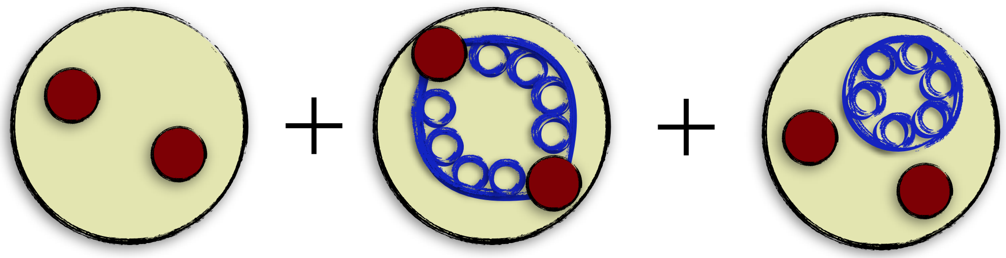
$$\Delta_{\mathrm{CG}} = \Delta N - \Delta\Sigma + \Delta\Xi$$

$$\Delta_{\mathrm{CG}} = \mathrm{Coleman} - \mathrm{Glashow\ difference}$$

BWM Collaboration (2013)

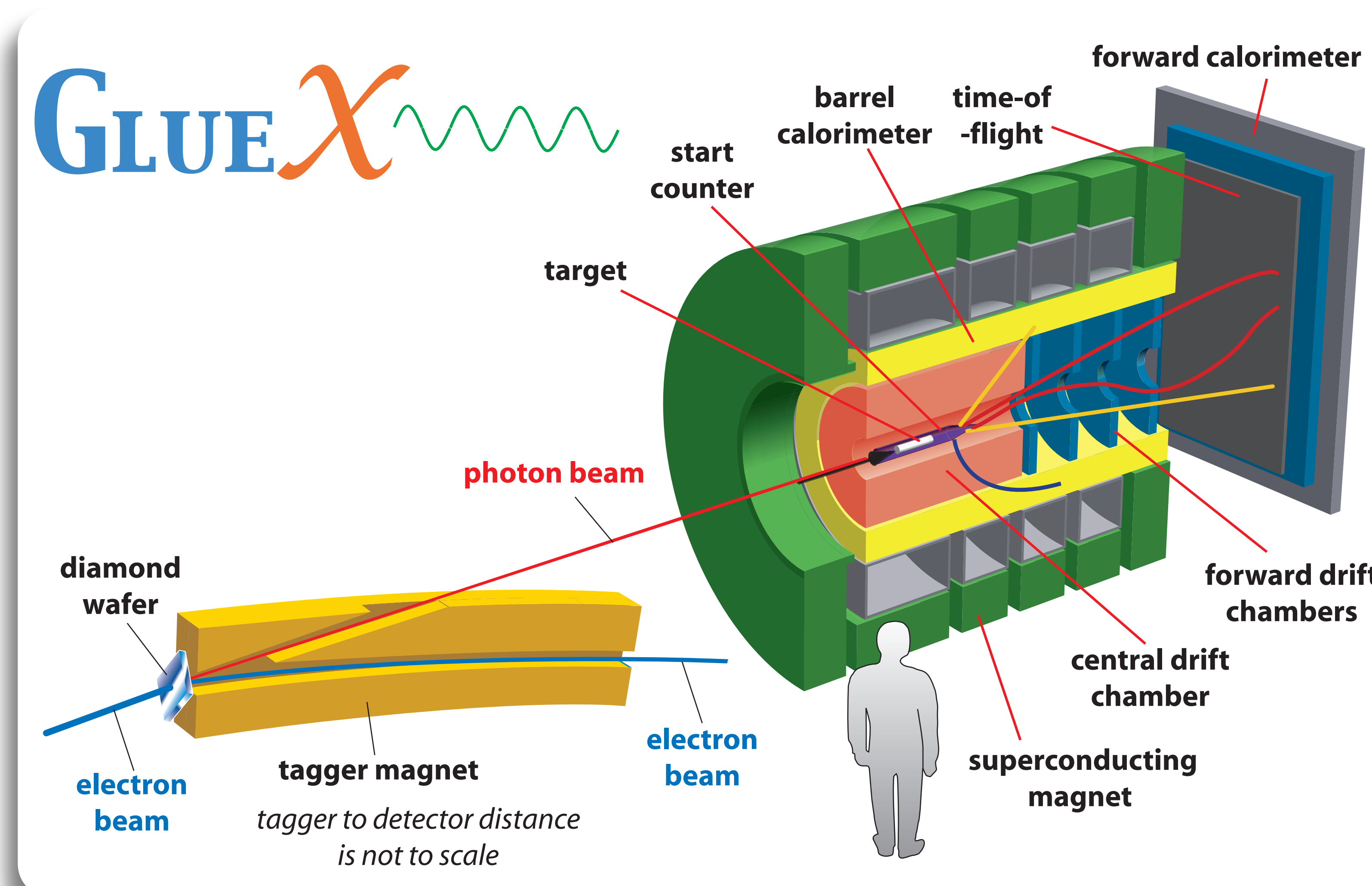
What does the future hold?

$$\pi_1(1800) =$$



$+ \ldots$

- Exotic quantum numbers
- Gluonic excitations
- Hybrid candidate



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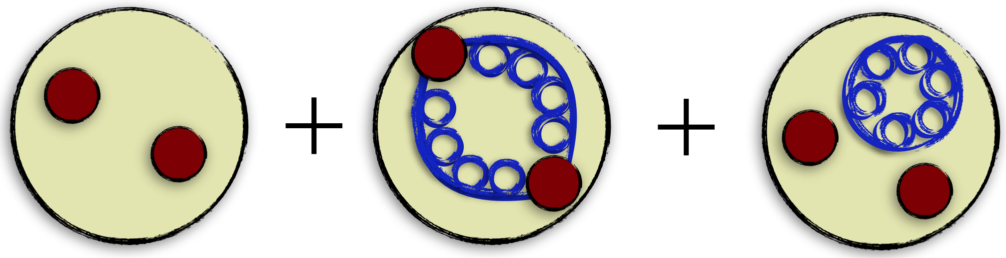
$\pi_1(1800) =$  $+ \ldots$

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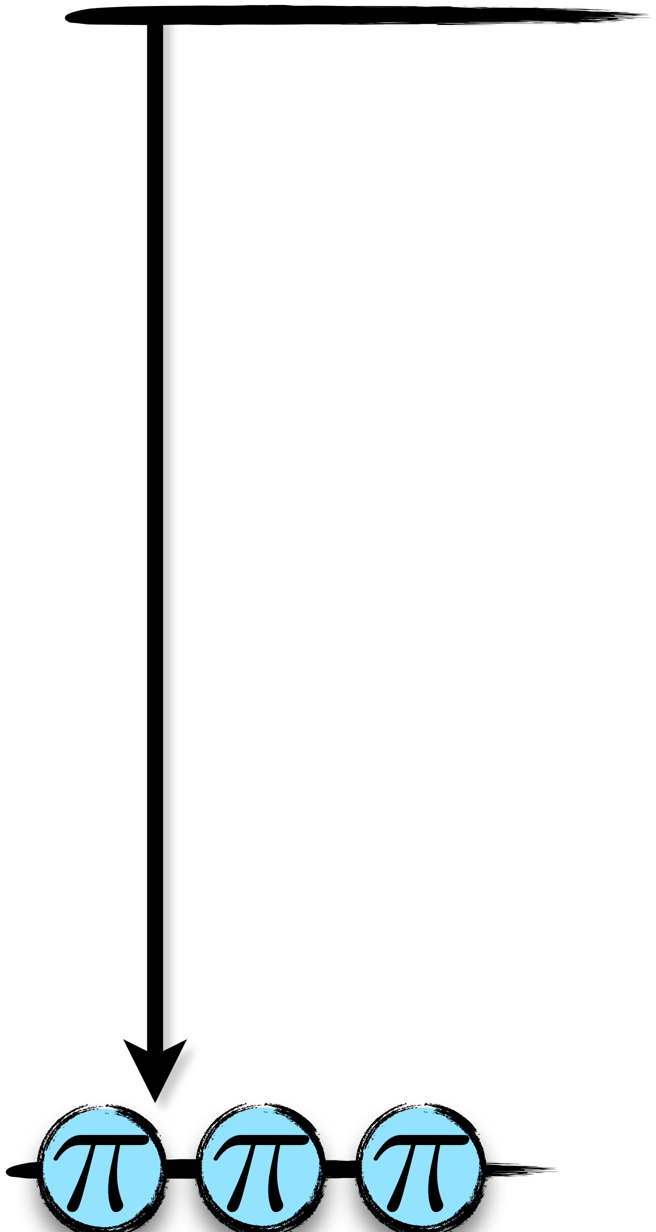

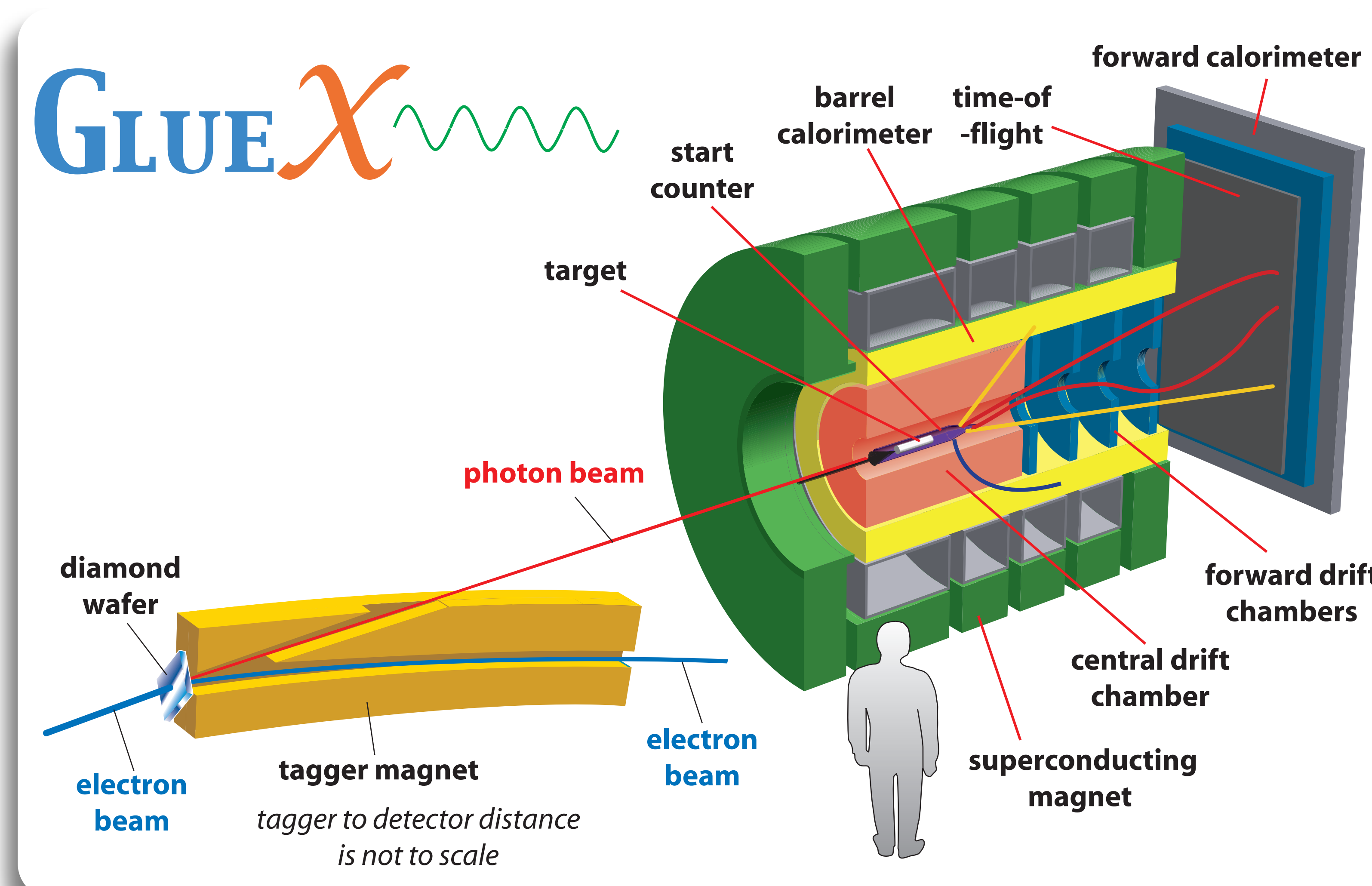

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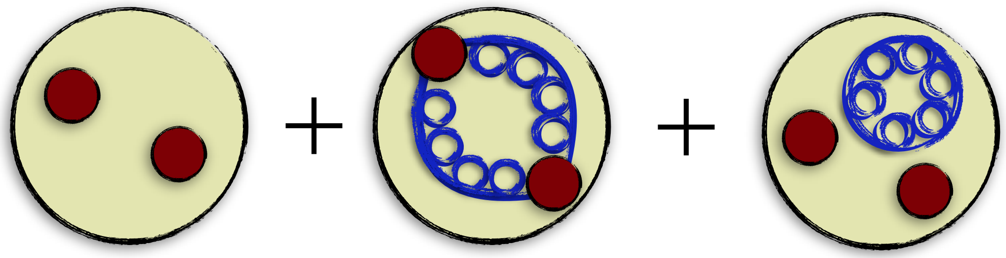
$\pi_1(1800) = $  $+ \ldots$

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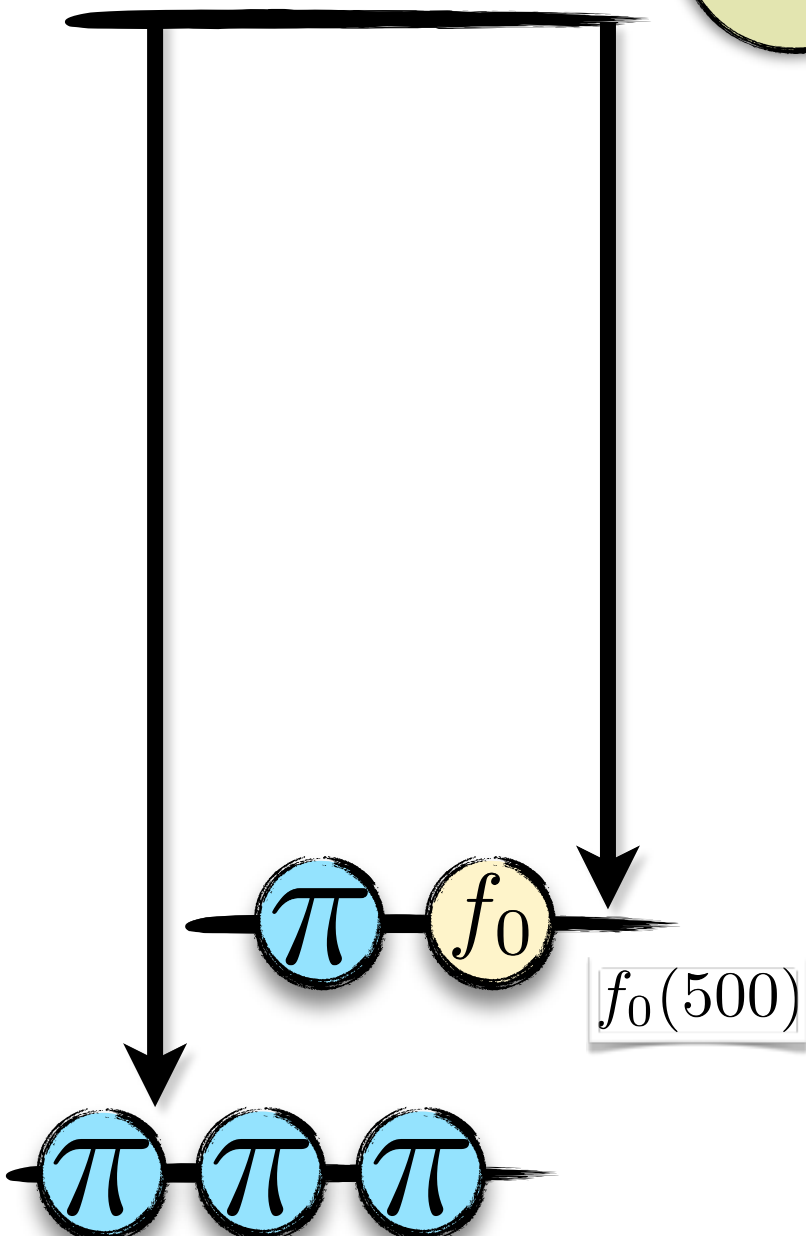

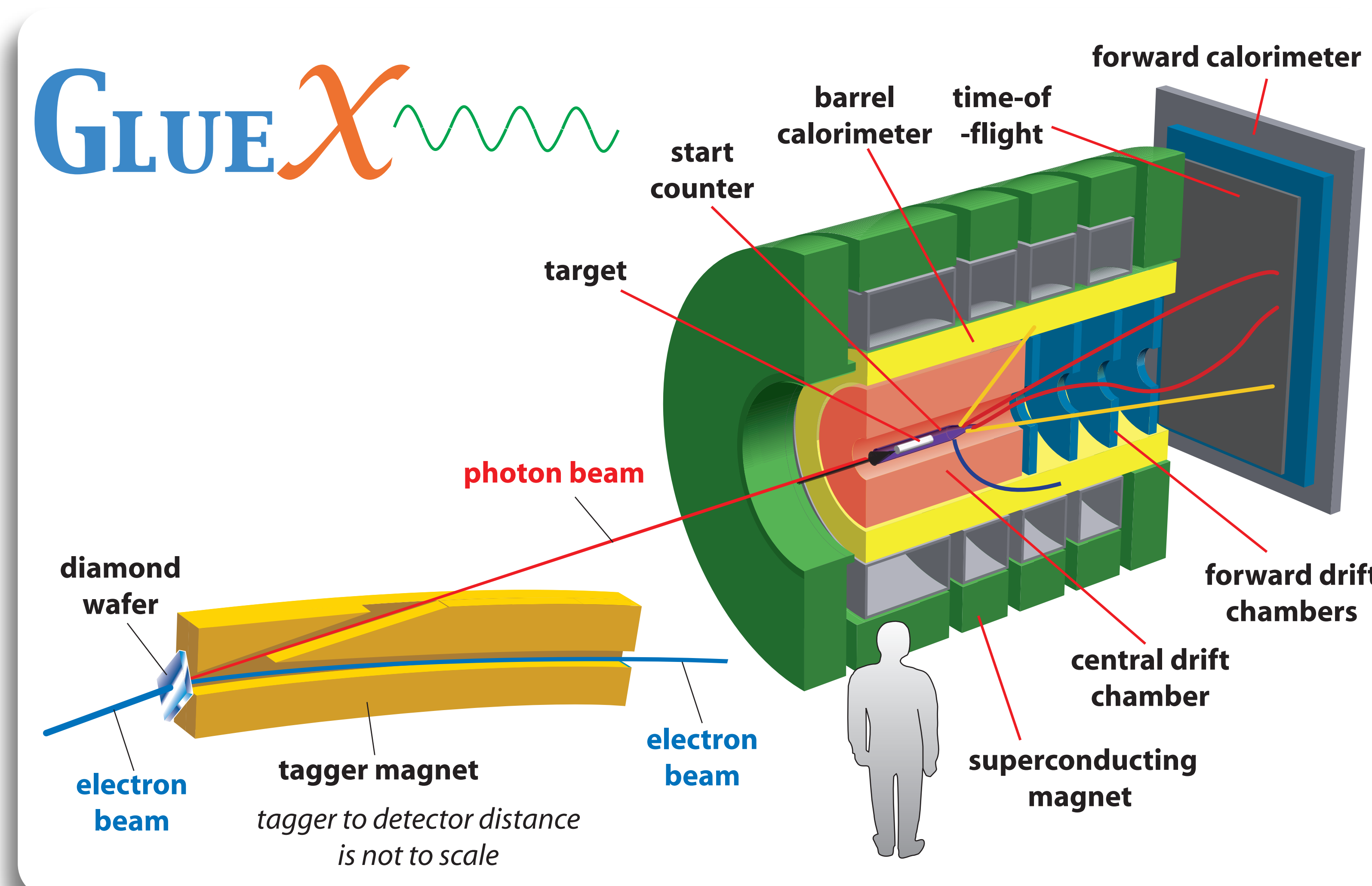

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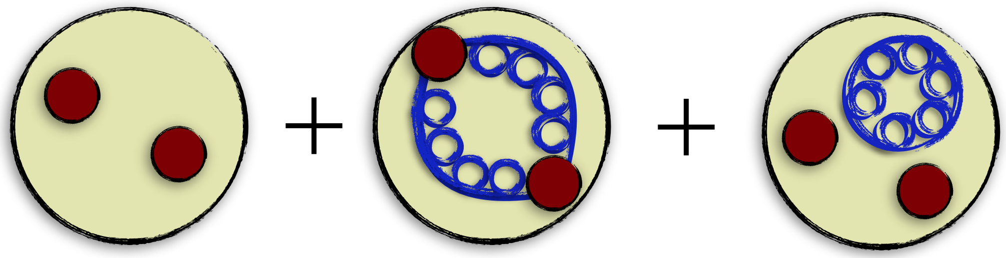
$\pi_1(1800) =$  $+ \ldots$

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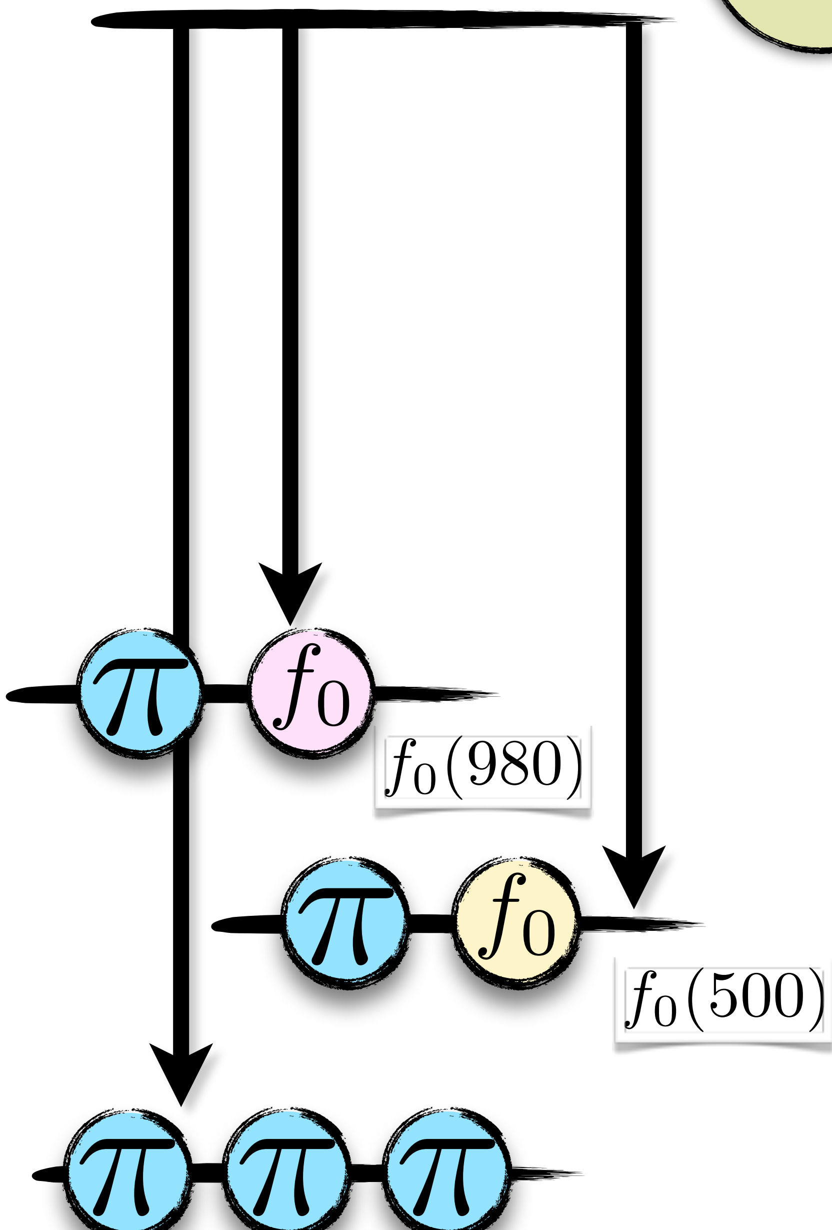

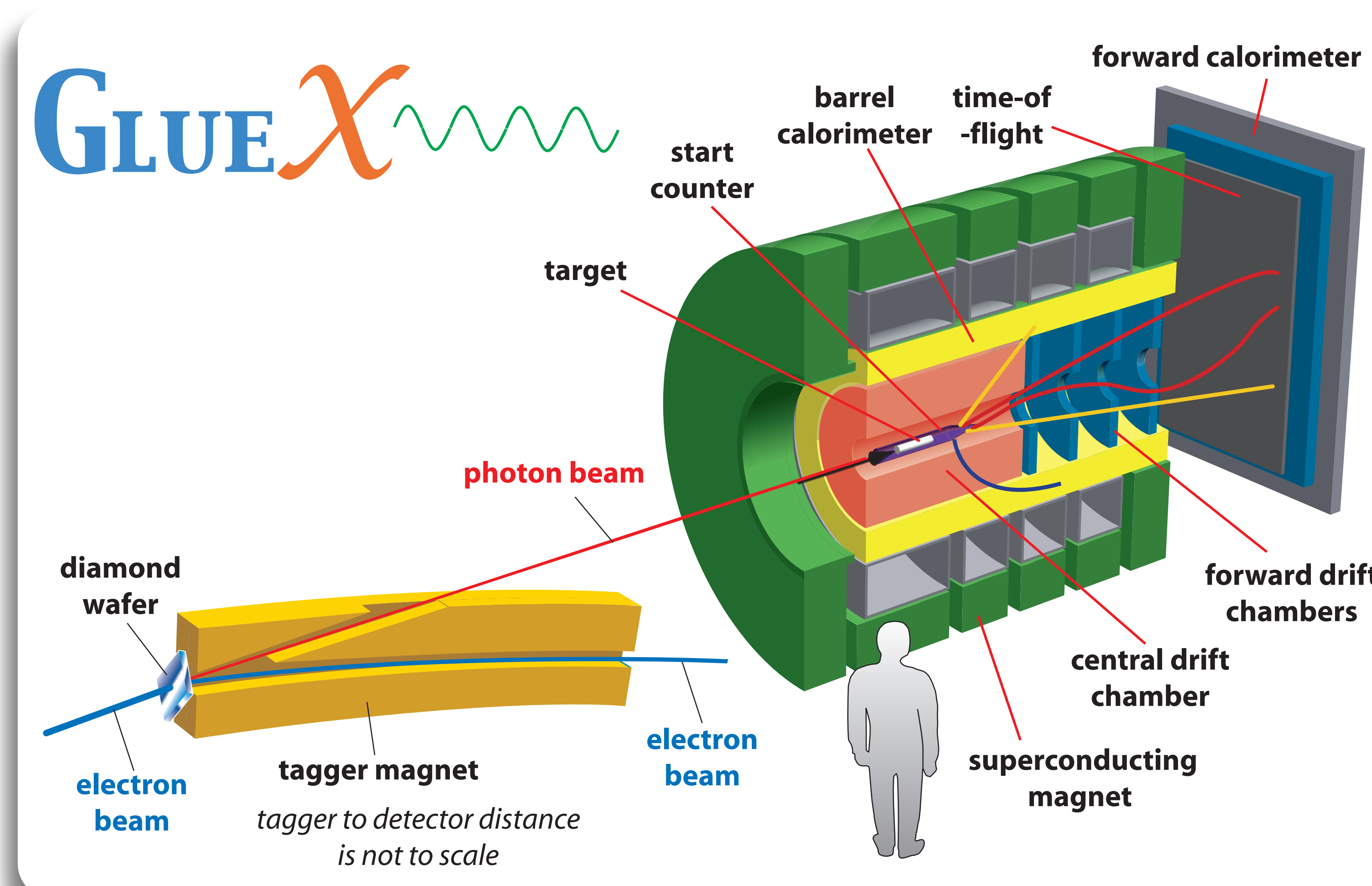

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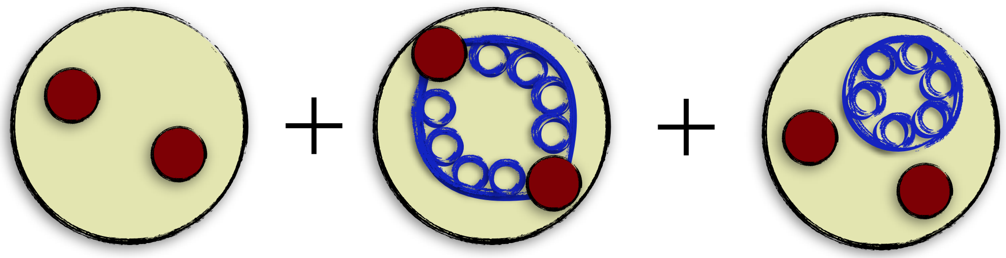
$\pi_1(1800) =$  $+ \ldots$

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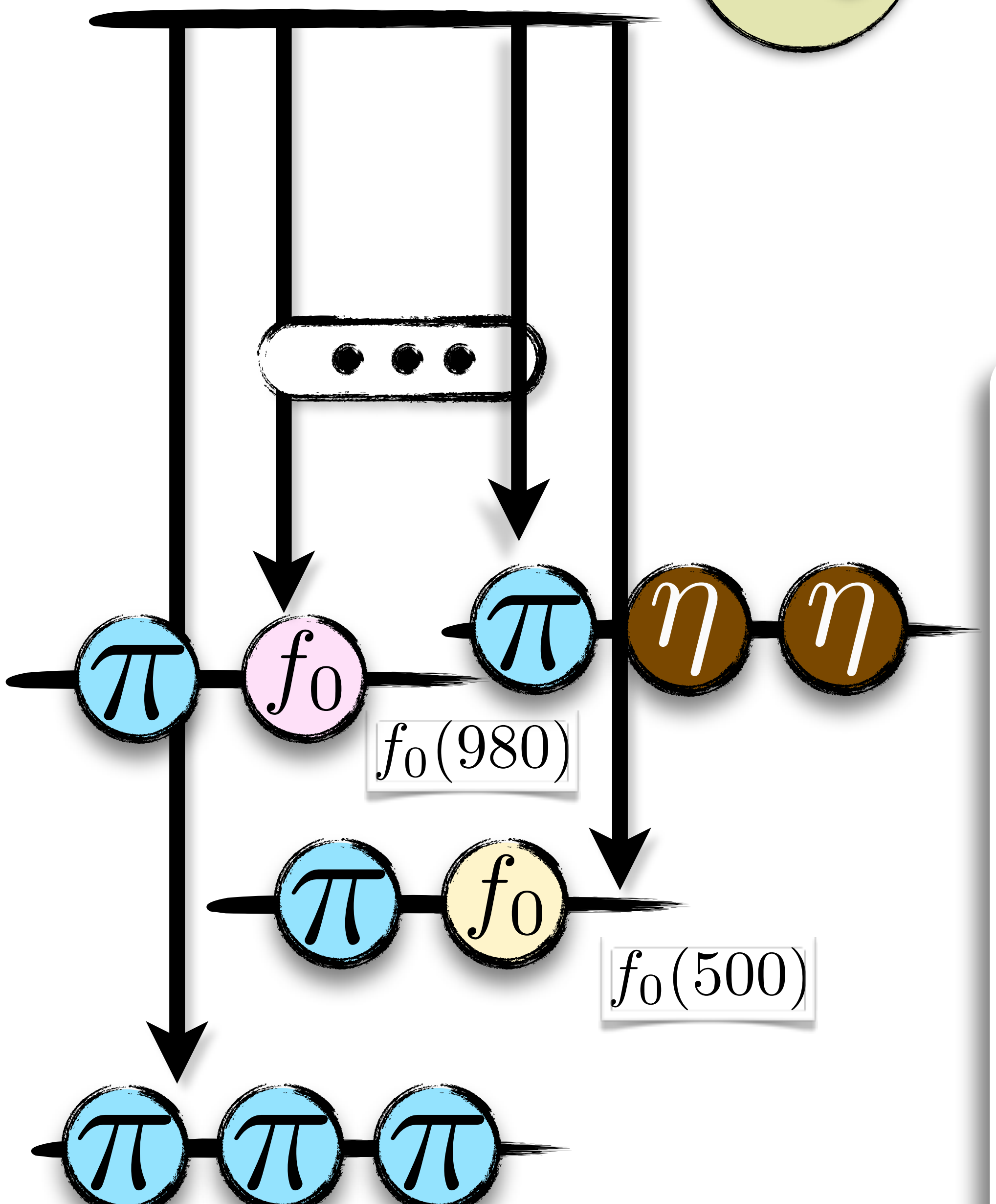
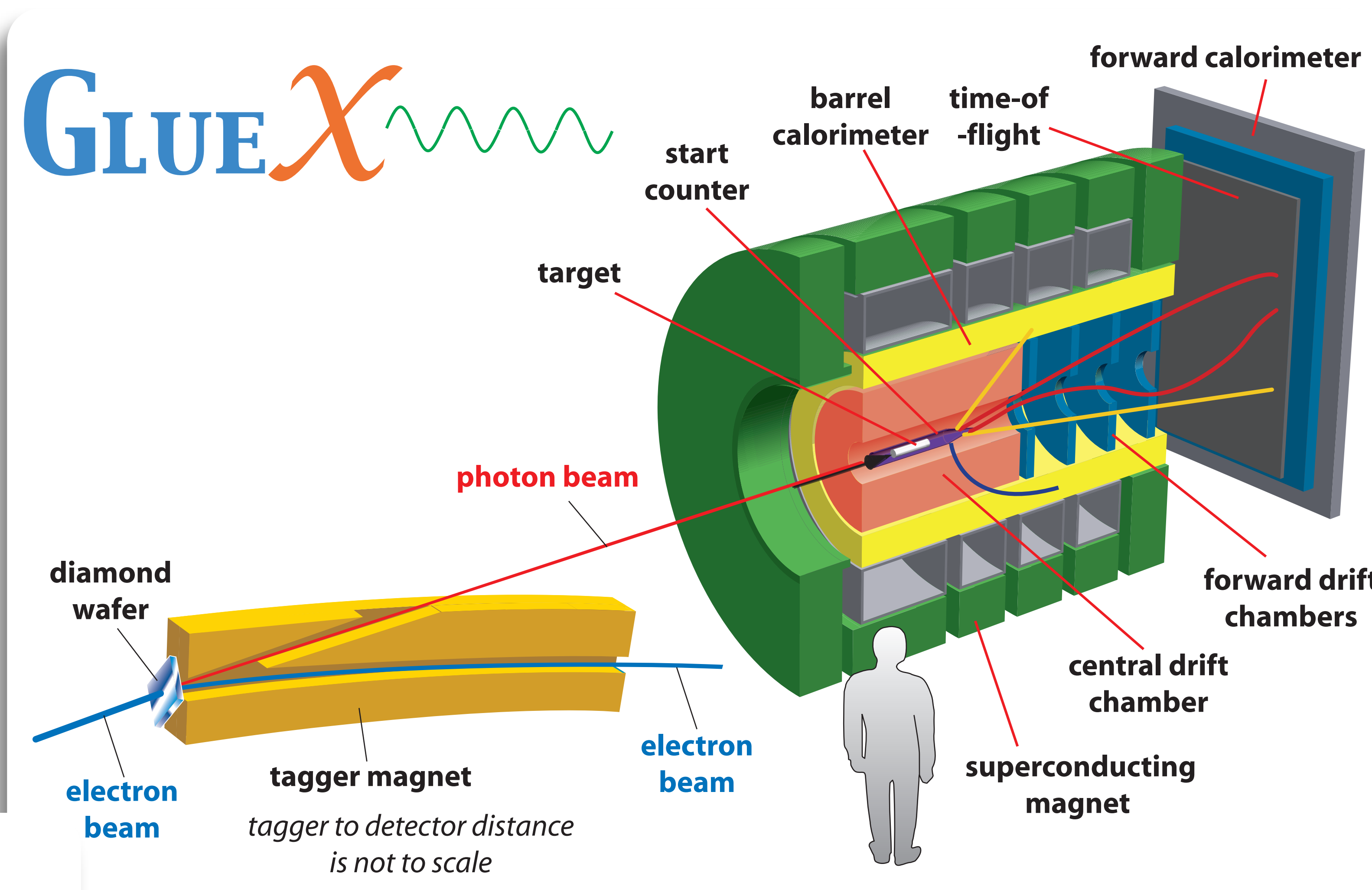


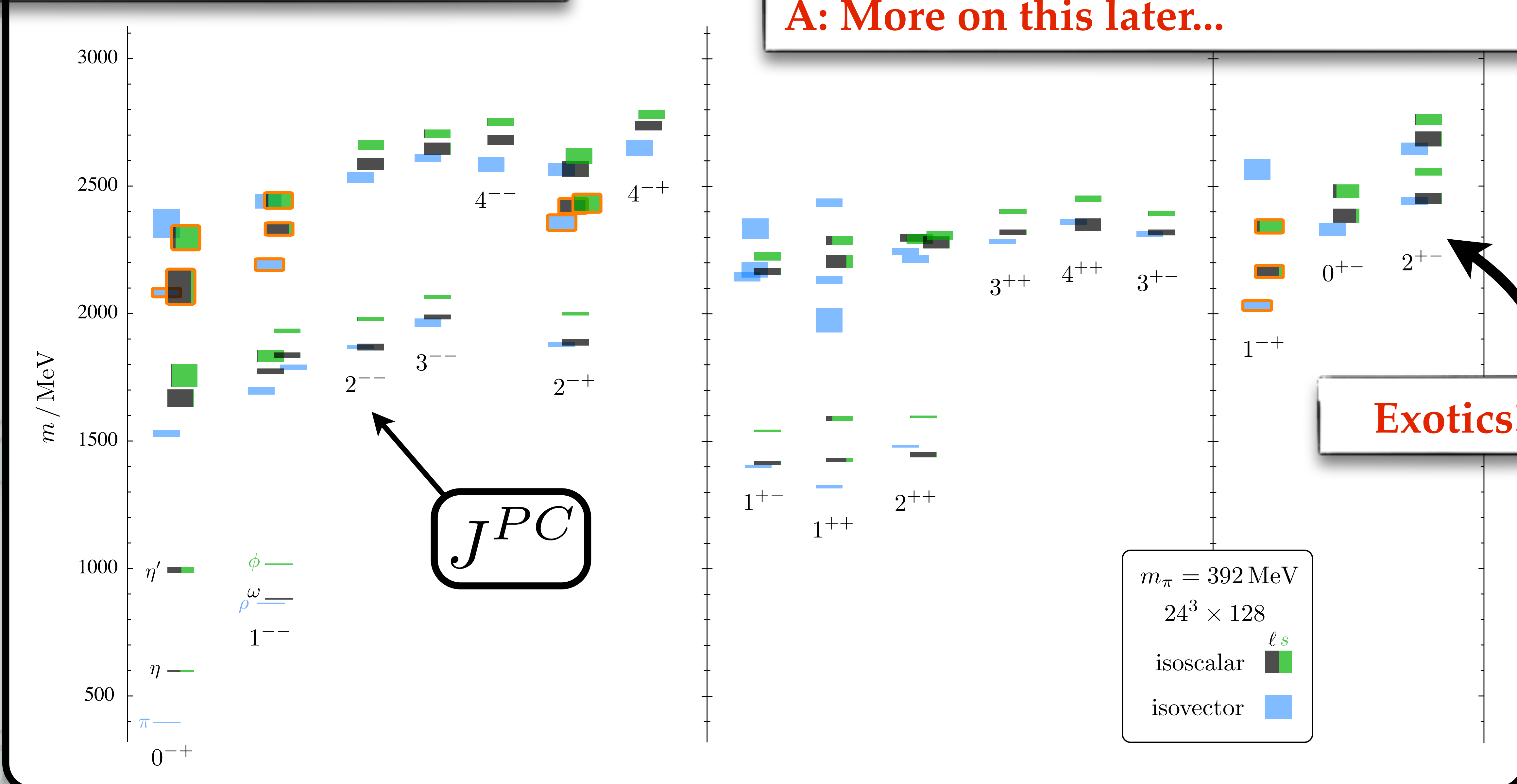


Few-body, multi coupled-channel system!
Not for the faint of heart!

What does the future hold?

State of the art lattice QCD

Q: How to interpret spectrum?
A: More on this later...

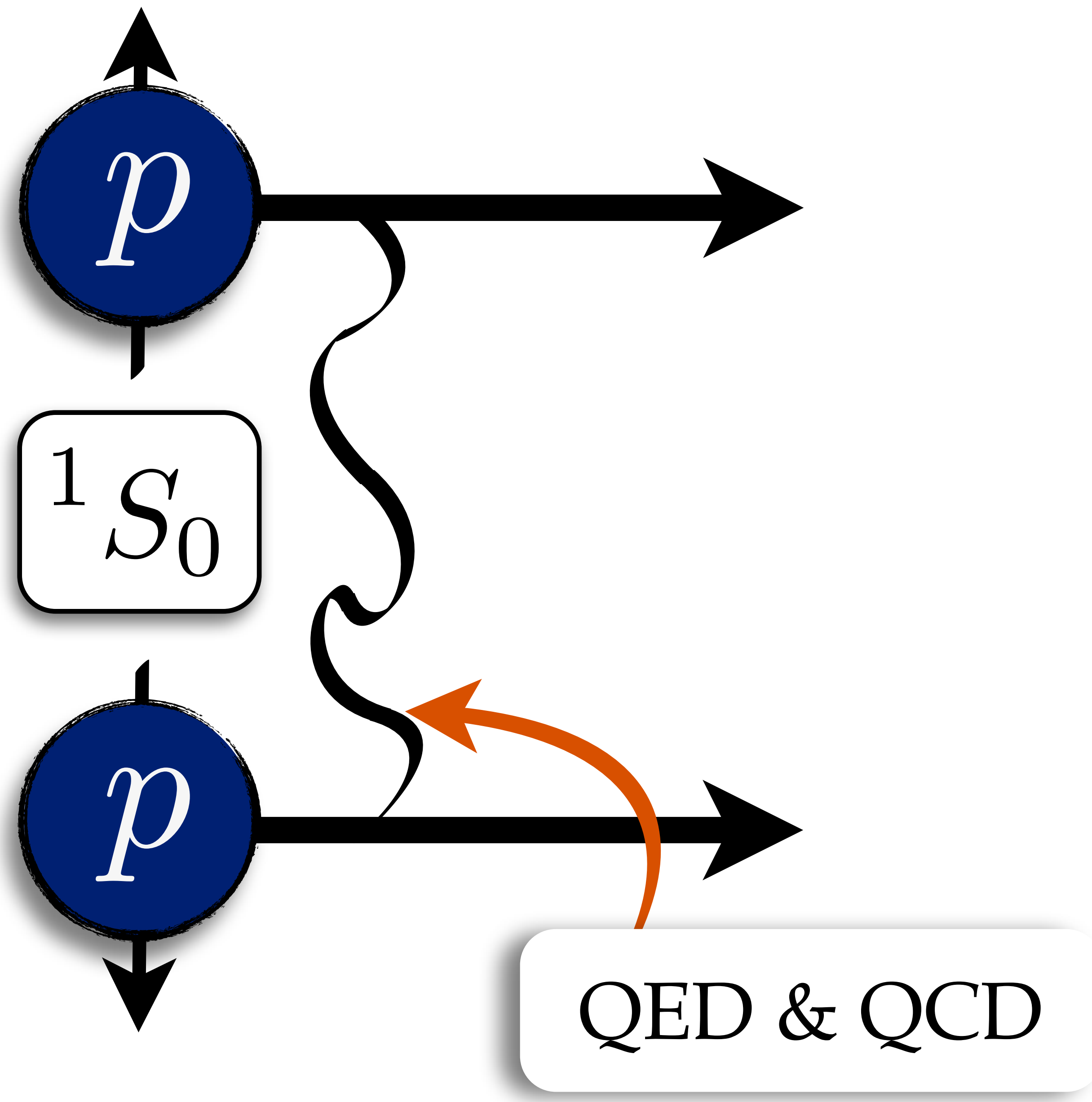


Exotics!

Hadron Spectrum Collaboration:
J. Dudek, R. Edwards, P. Guo & C. Thomas (2013)

What does the future hold?

Parity violation in the NN



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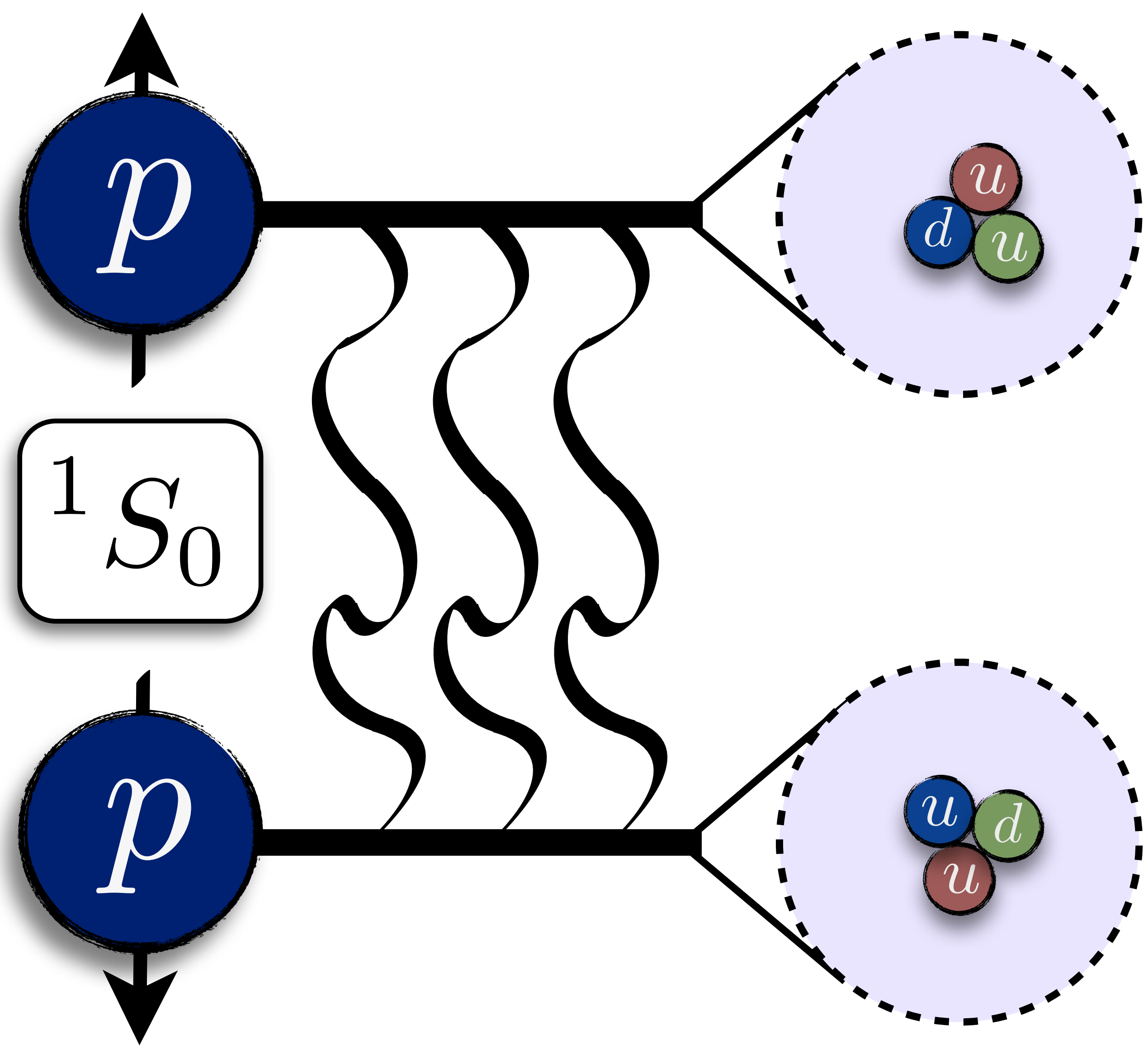
Parity violation in the NN



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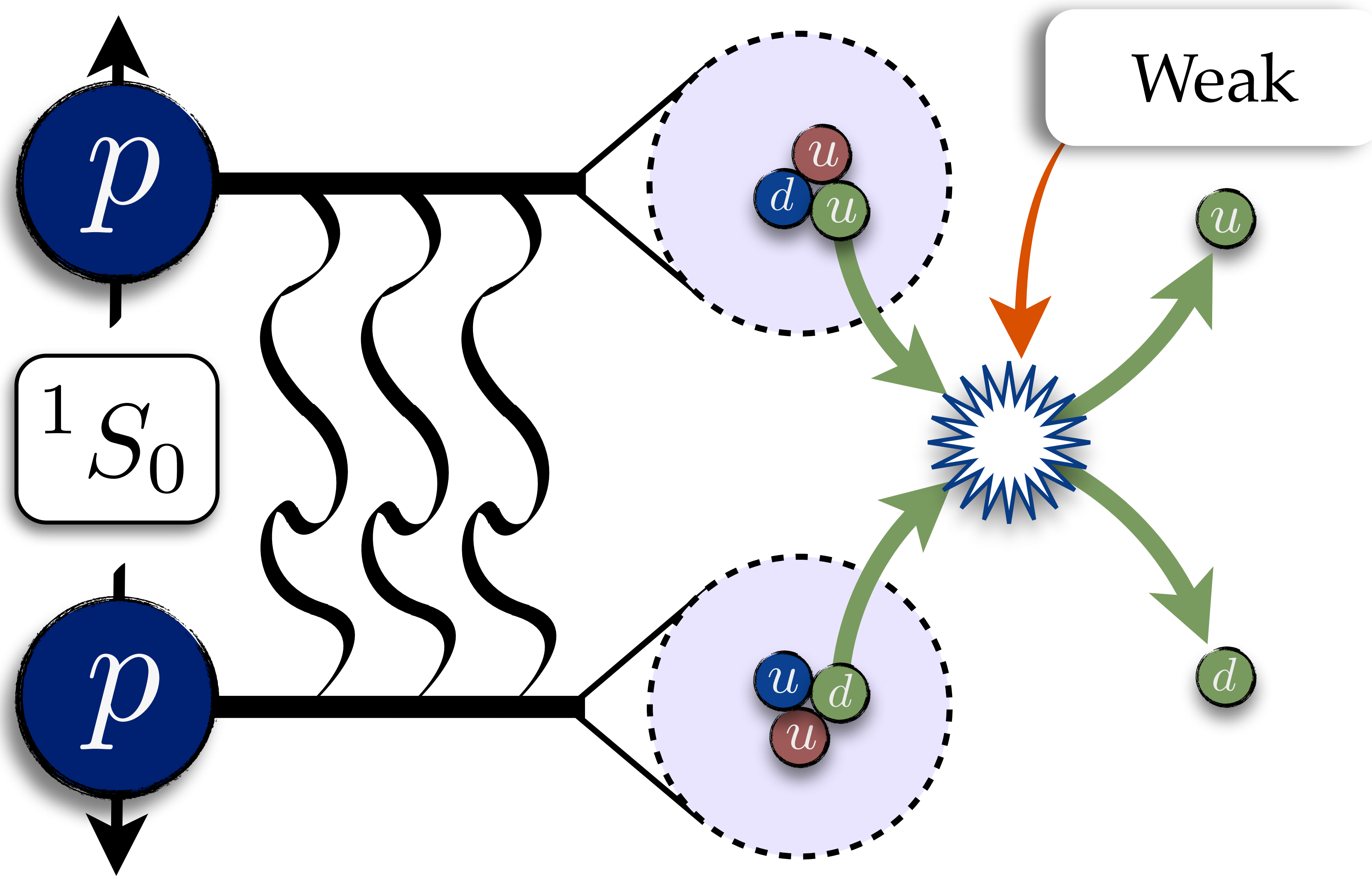

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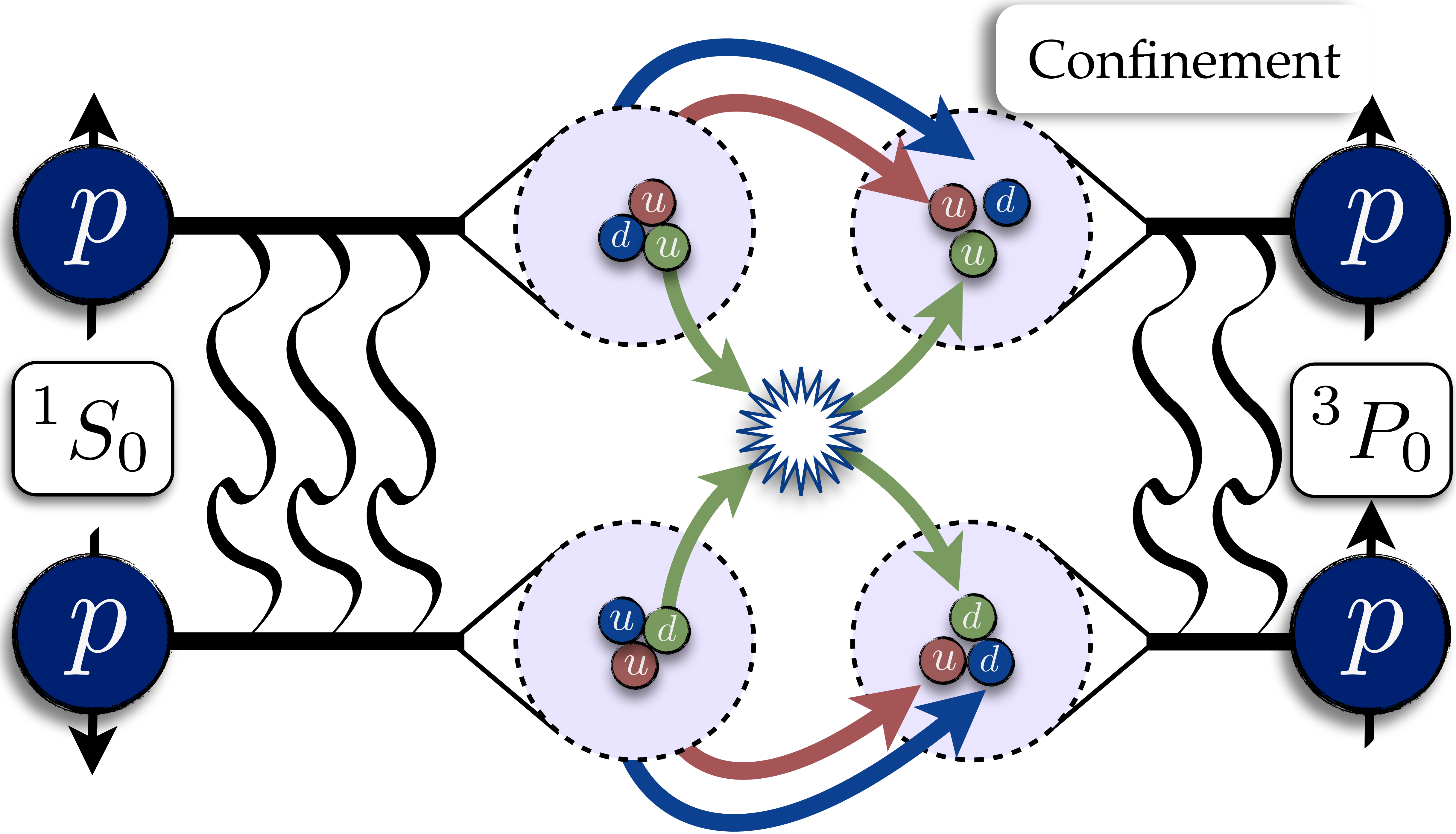

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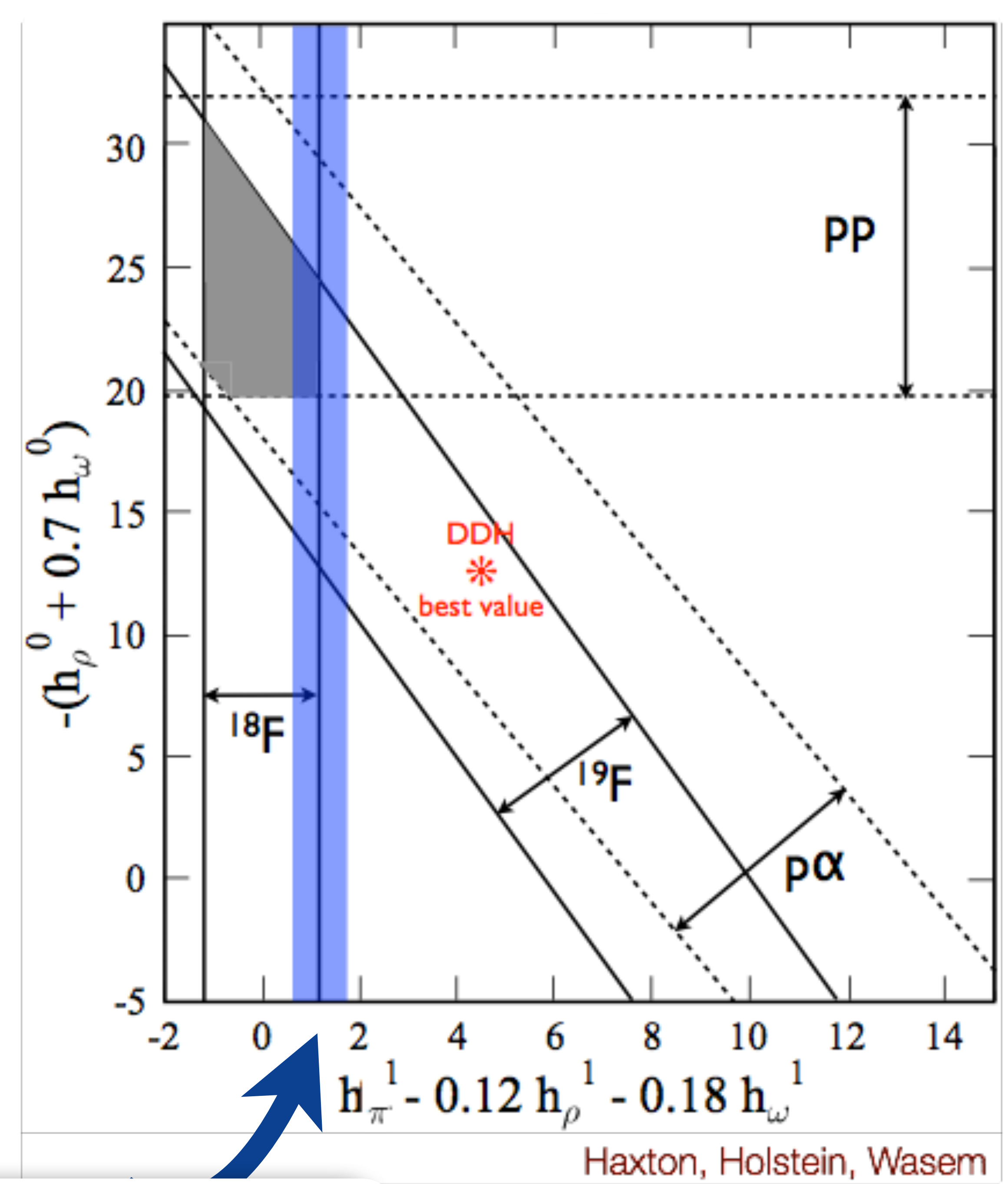

First parity violation calculation from lattice QCD by Joseph Wasem (2012)

Parametrize hadronic parity violation low-energy

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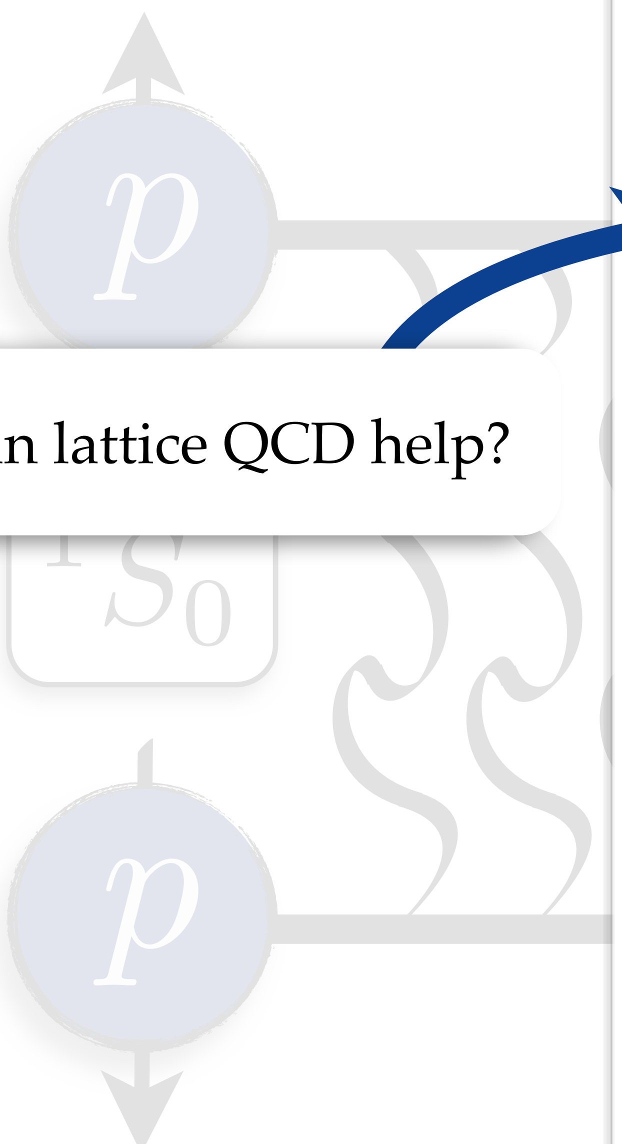
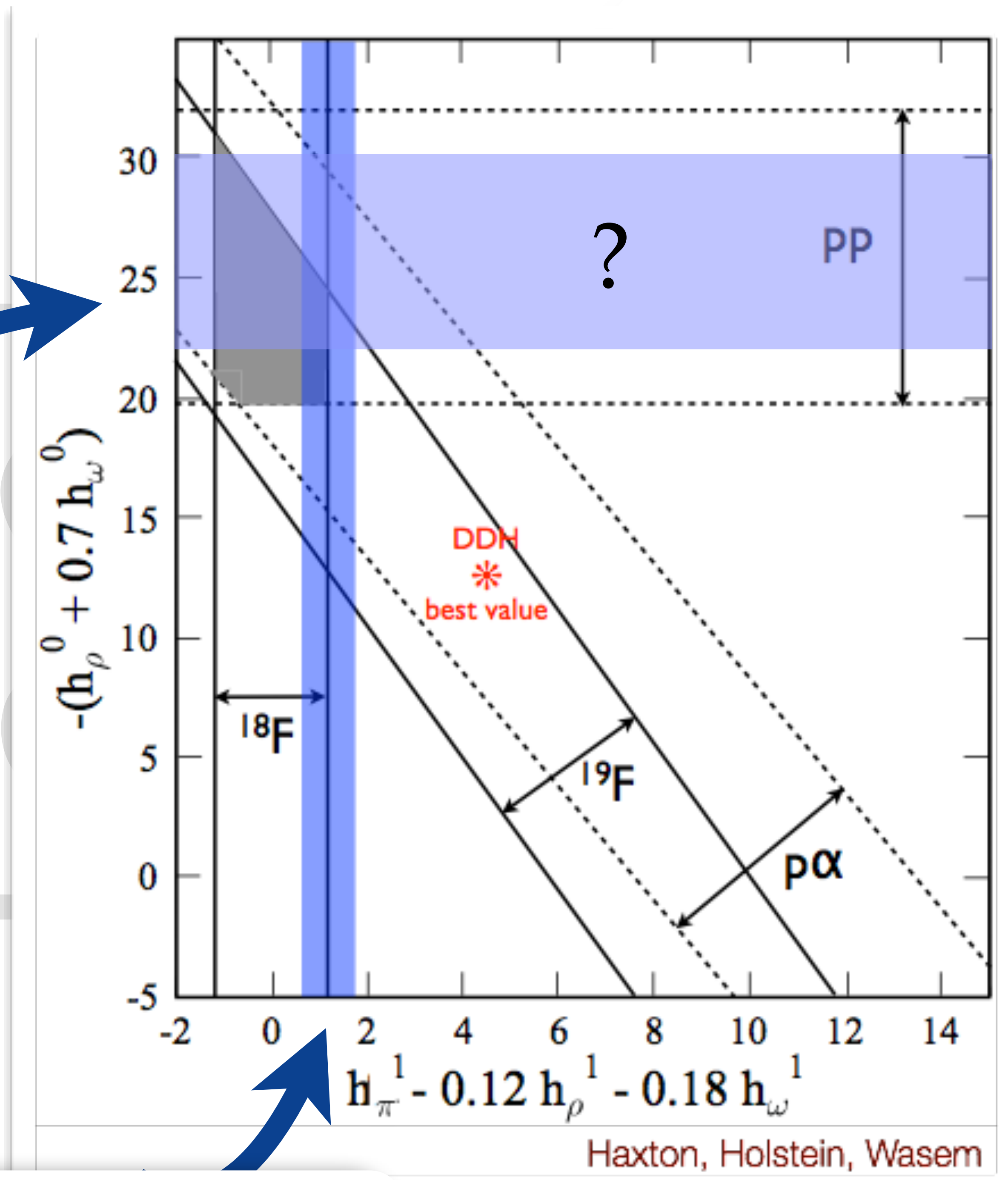


Can lattice QCD help?



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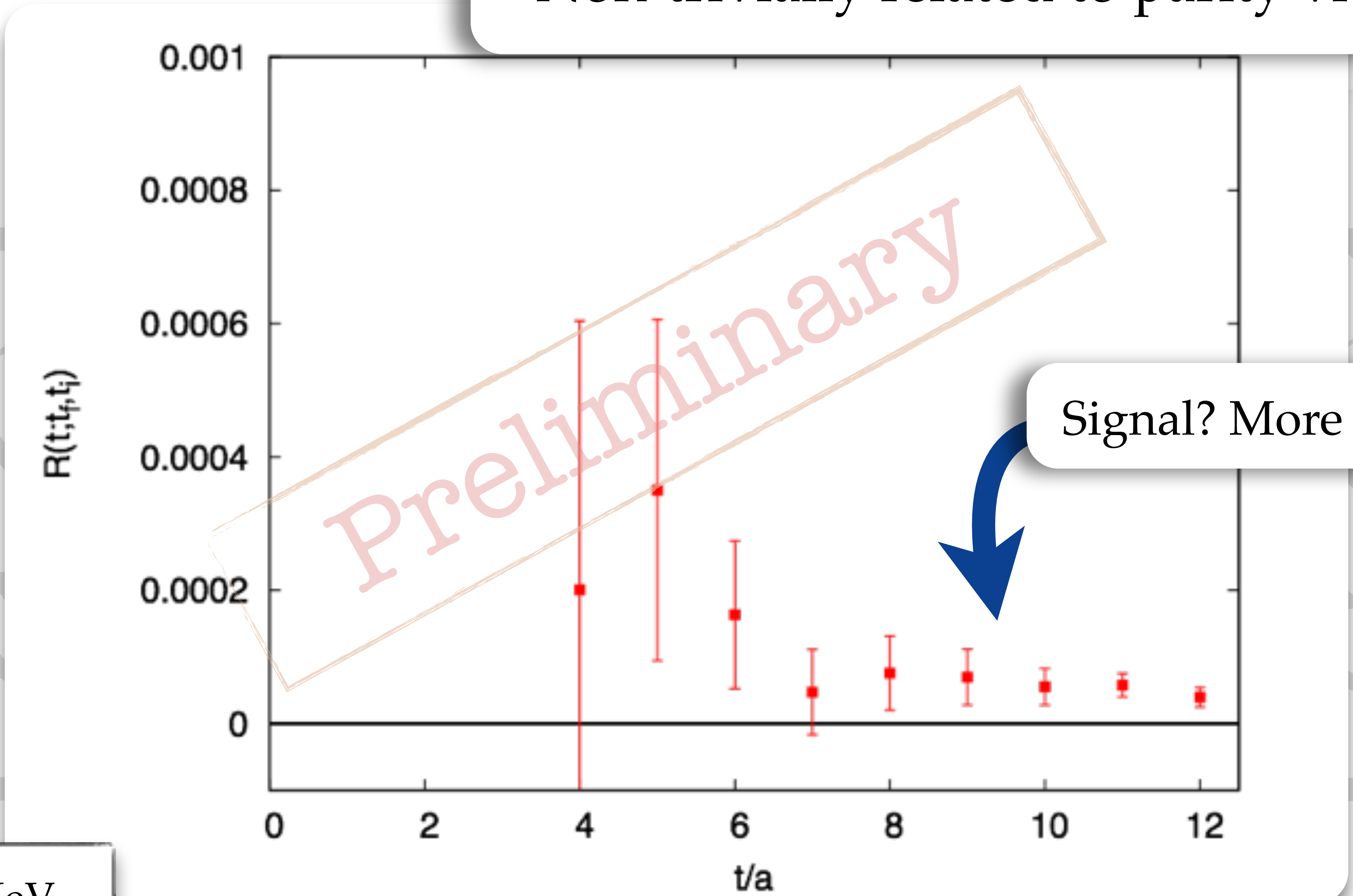
State of the art lattice QCD

Non-trivially related to parity violation!

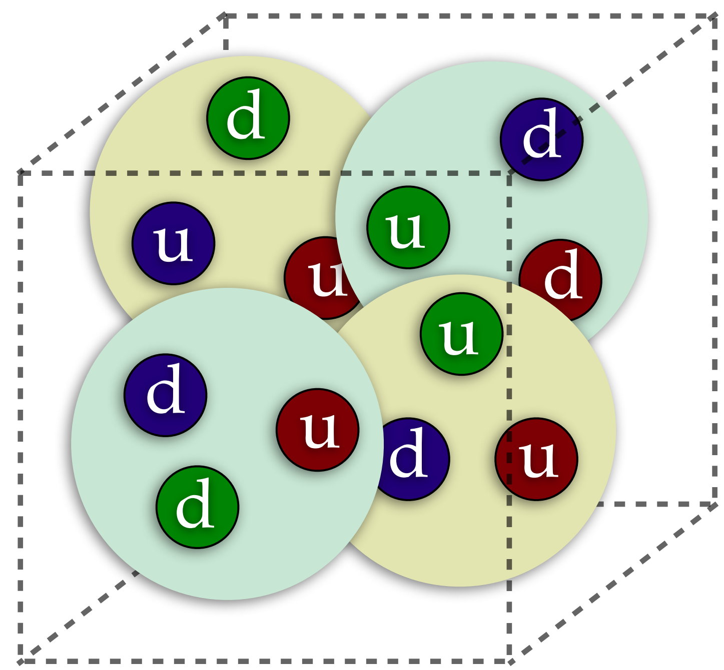


m_π~700MeV

Thorsten Kurth, Amy Nicholson
et al. [CalLat Collaboration]

Challenges with few-body systems

Correlation functions:



- The building block of lattice QCD calculations
- Evaluated numerically using Monte Carlo techniques
- Many challenges for few-body systems:
  - Large number of Wick contractions
  - Poor signal/noise
  - Interpretation of observables

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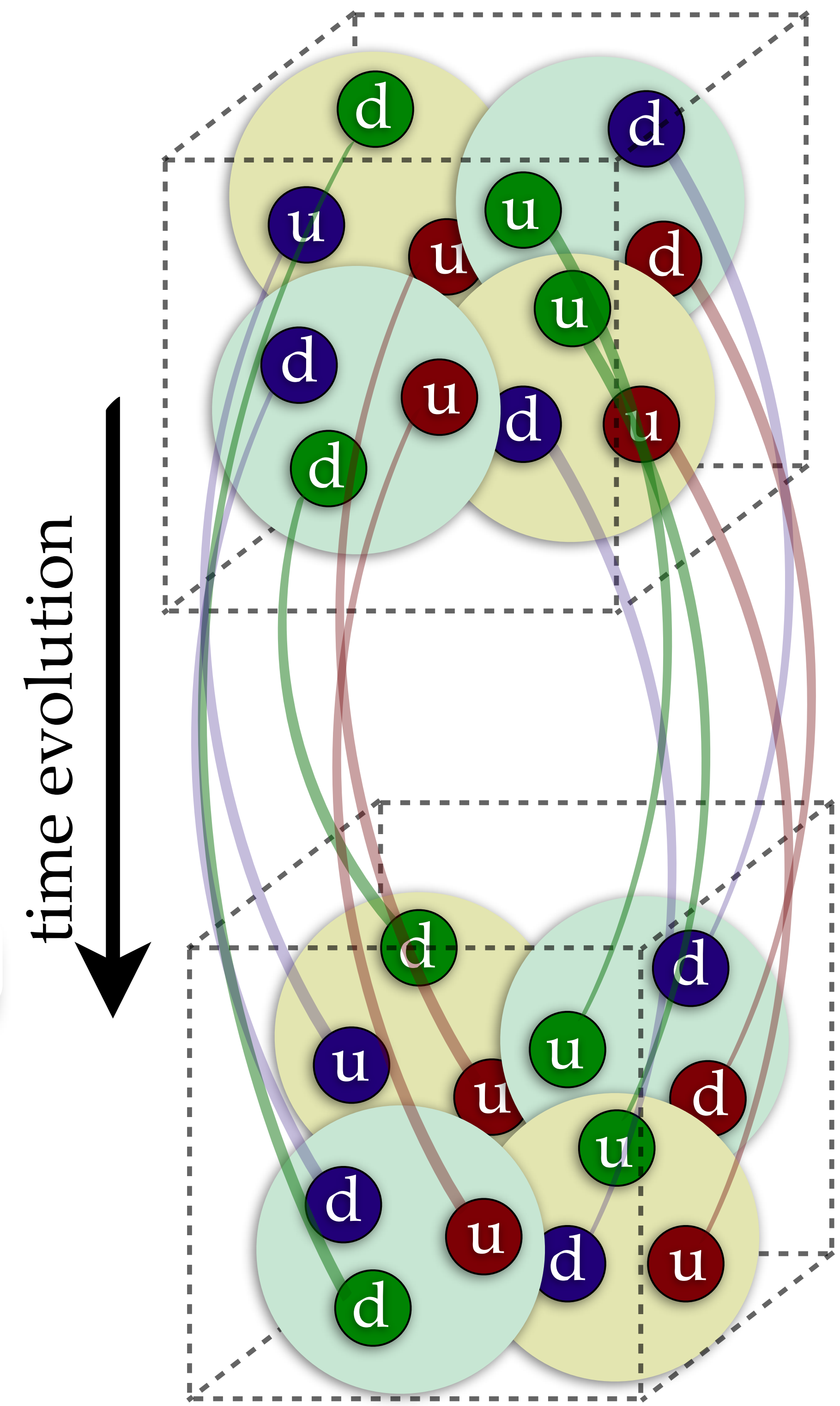
    e.g., naïvely ^{4}He has 6! x 6! = 518,400 contractions!
  - Poor signal/noise
  - Interpretation of observables



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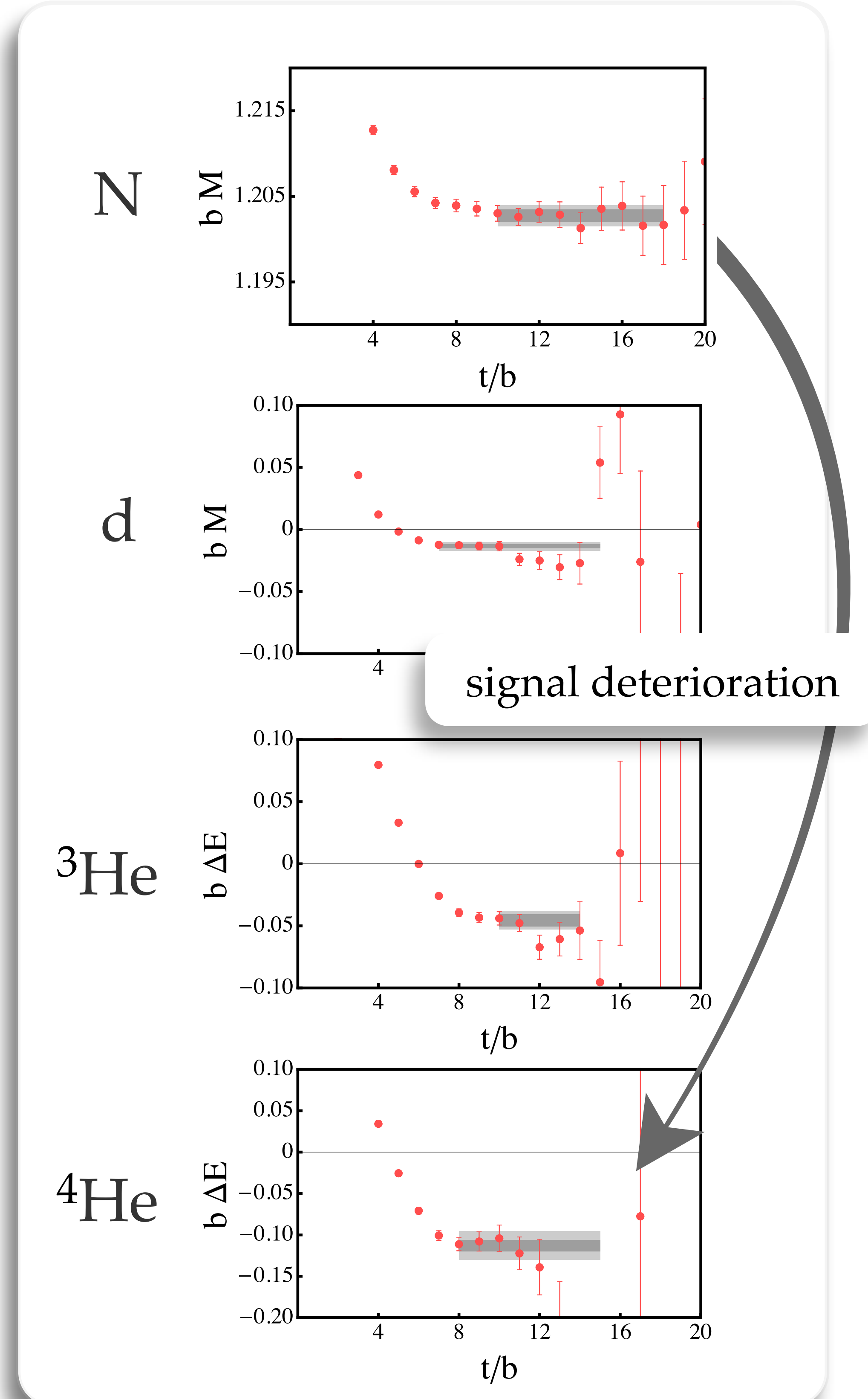
[NPLQCD Coll.] Beane, Chang, Cohen, Detmold, Lin, Luu, Orginos, Parreno, Savage, Walker-Loud (2012)



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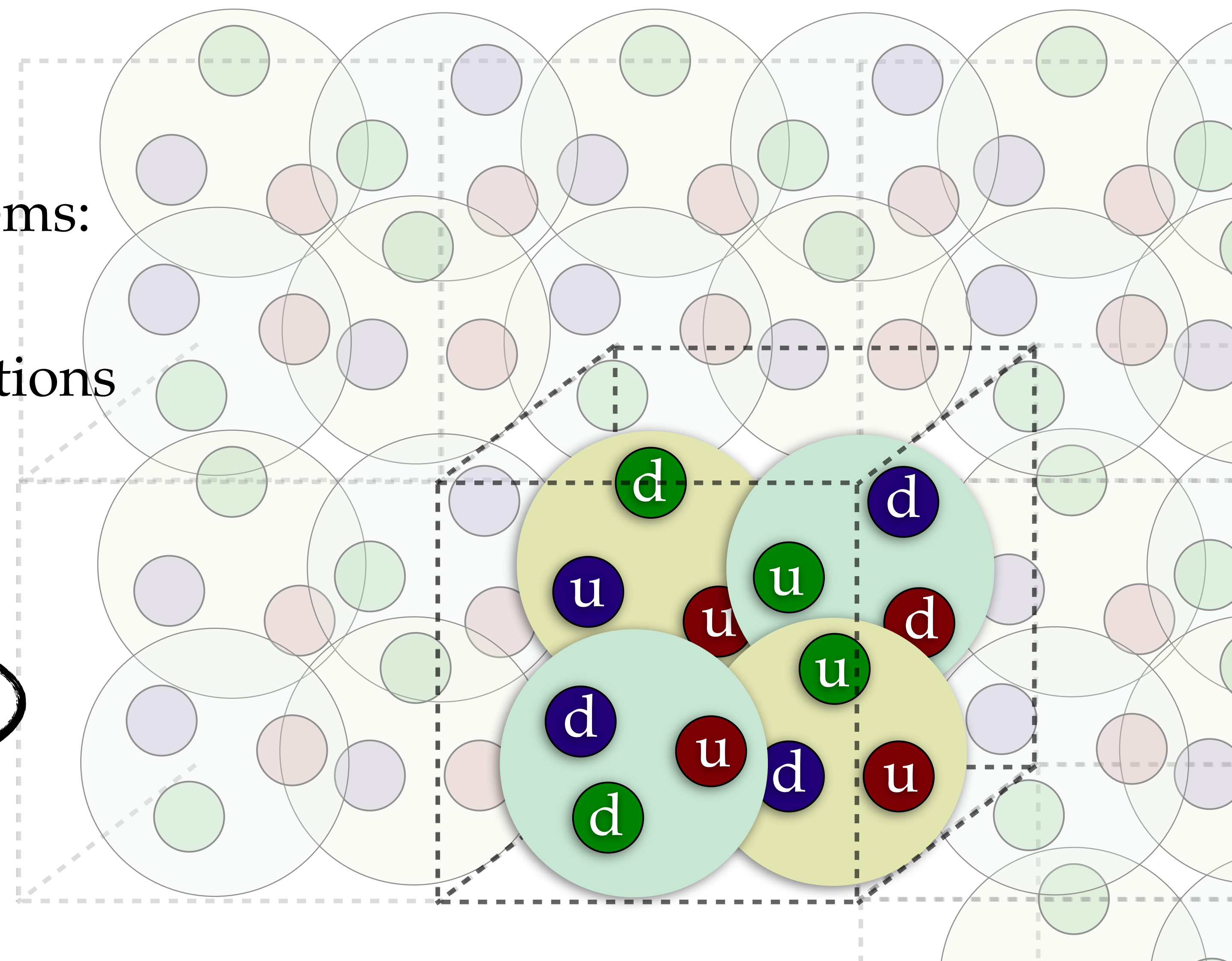
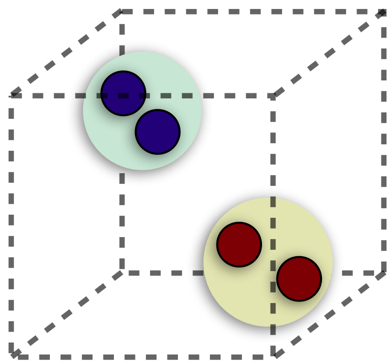


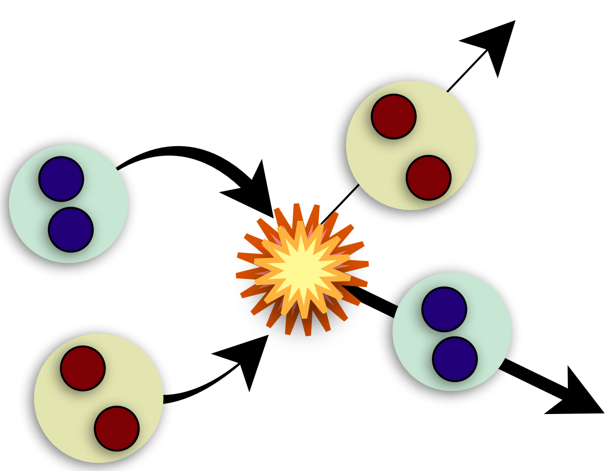
Paving the road towards physics

1 Calculate finite volume spectrum



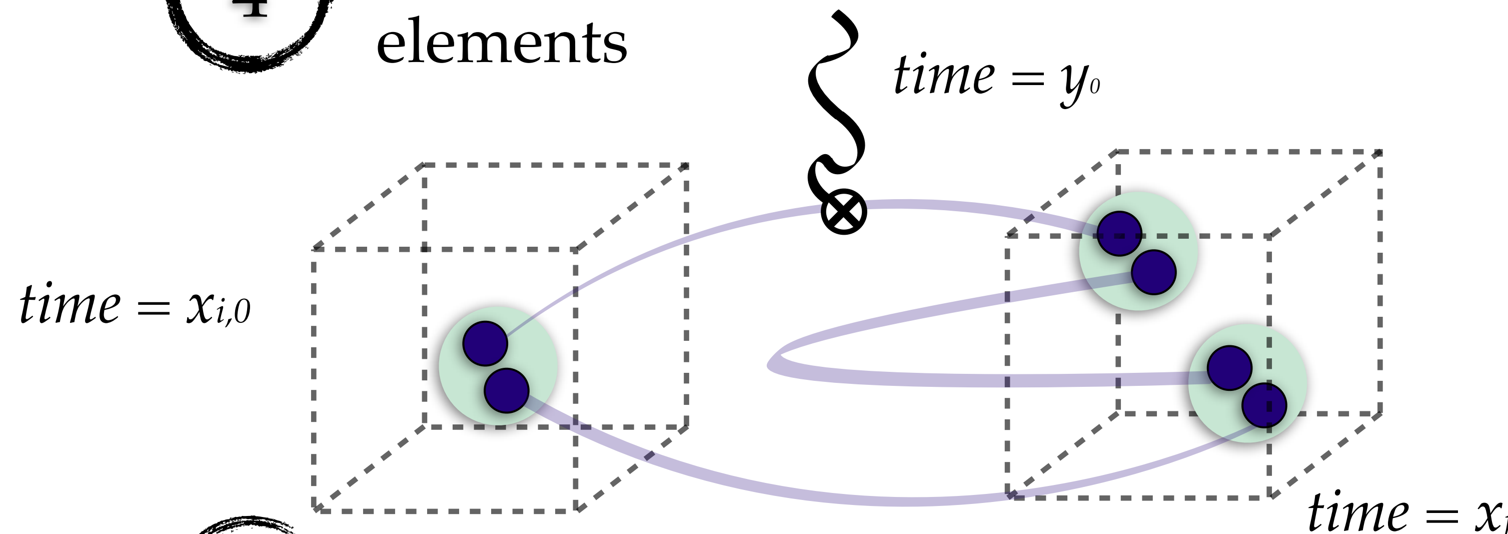
2 Plug into formalism

3 Out goes elastic & inelastic QCD scattering amplitudes



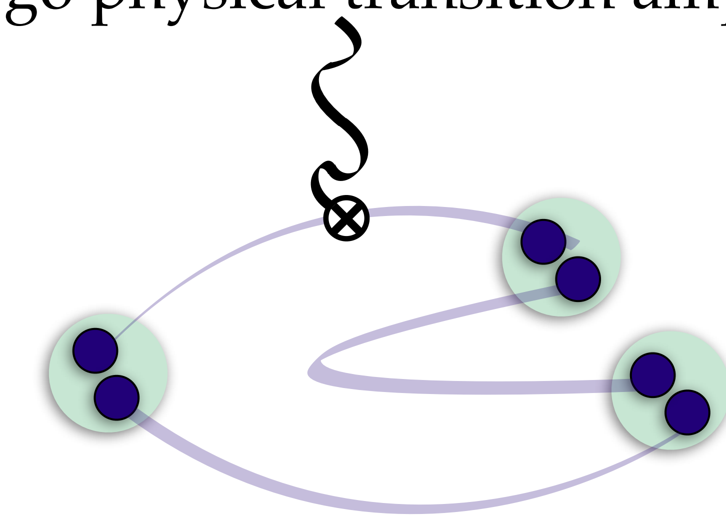
à la mode de **Lüscher (1986)**

4 Calculate finite volume matrix elements



5 Plug spectrum, scattering parameters and finite volume form factor into formalism

6 Out go physical transition amplitude



à la mode de **Lellouch & Lüscher (2000)**

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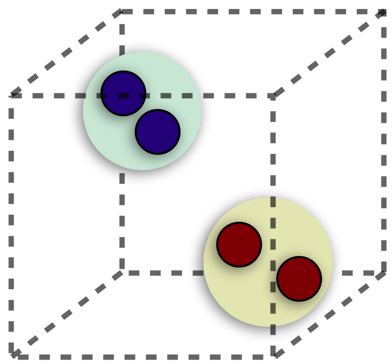

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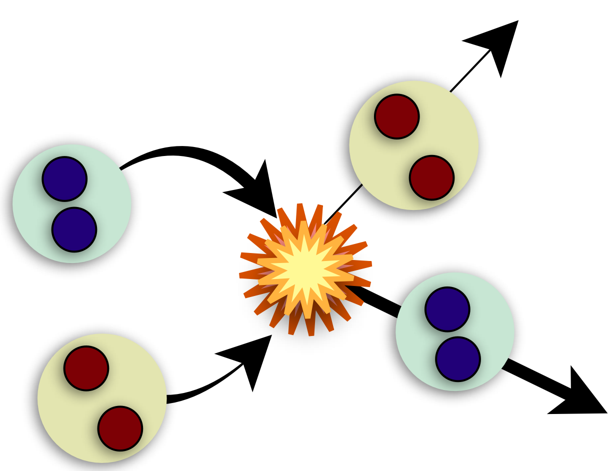

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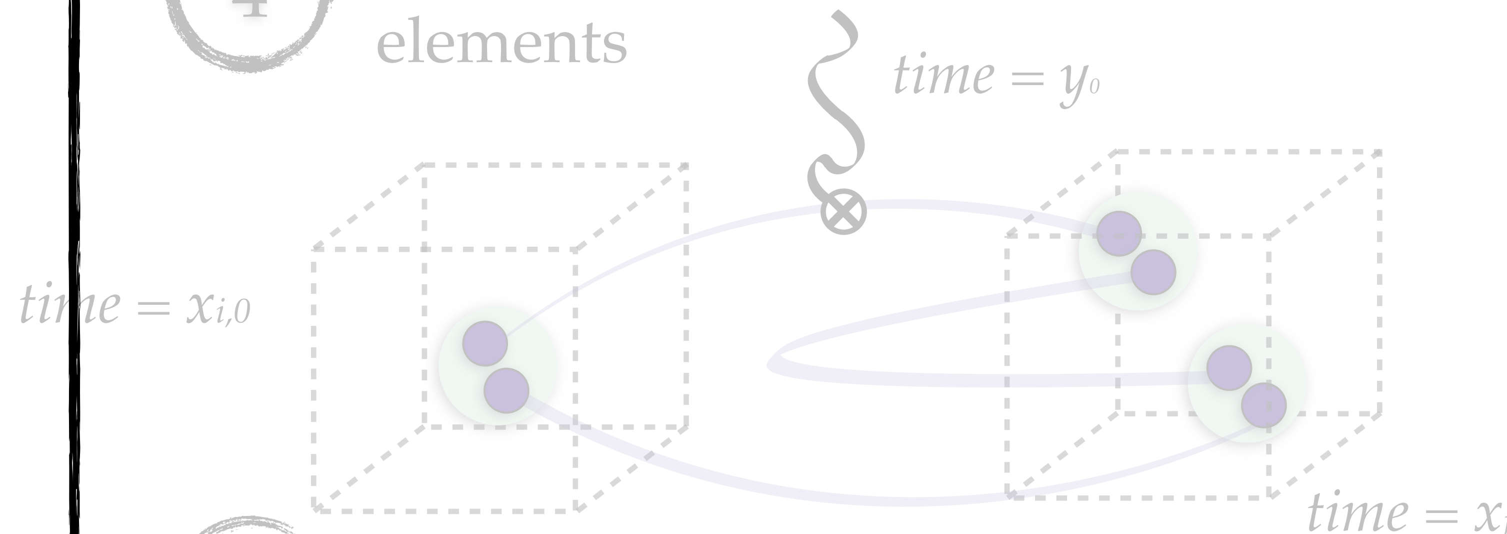

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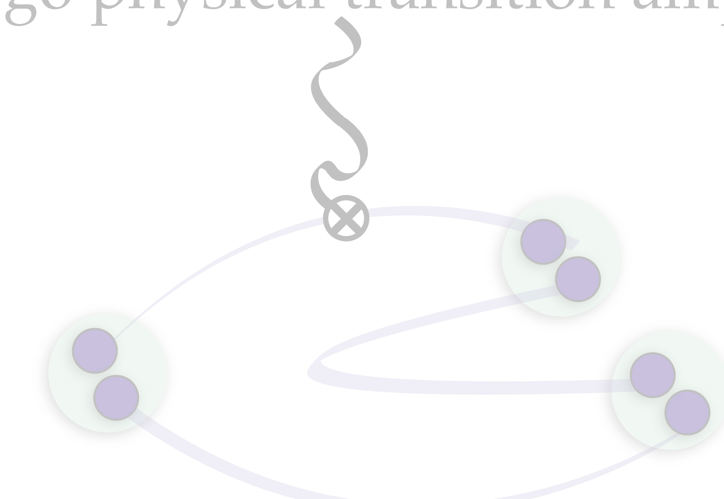


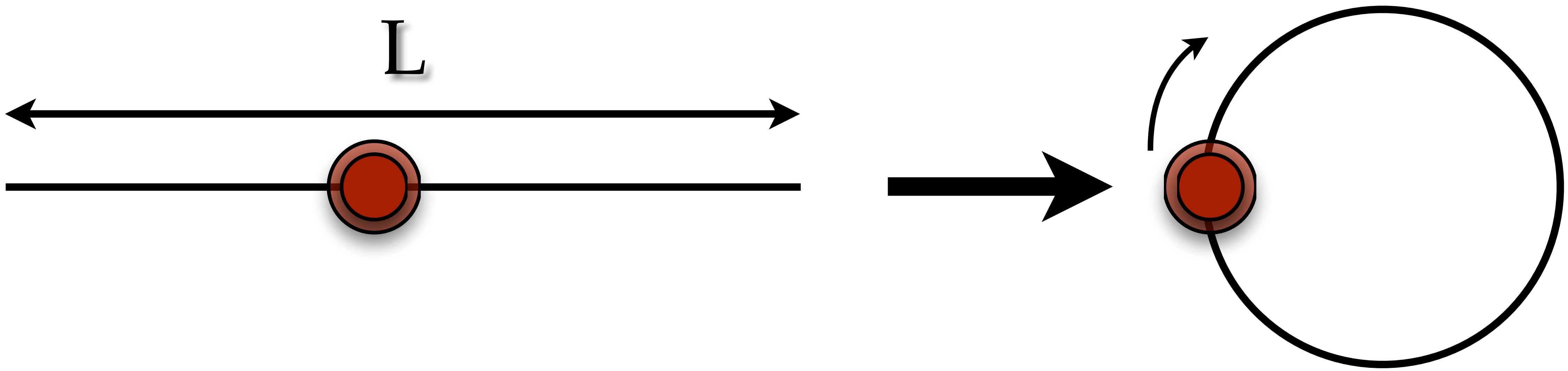
à la mode de **Lellouch & Lüscher (2000)**

A long list of extensions of the Lüscher formalism

- **Lüscher (1986), (1991)** *("Lüscher Formalism")*
- Maiani and Testa (1990)
- Rummukainen and Gottlieb (1995)
- Beane, Bedaque, Parreno, and Savage (2004), (2005)
- Bedaque (2004)
- Li and Liu (2004)
- Detmold and Savage (2004)
- Feng, Li, and Liu (2004)
- Christ, Kim, and Yamazaki (2005)
- Kim, Sachrajda, and. Sharpe (2005)
- Bernard, Lage, Meissner, and Rusetsky (2008)
- Ishizuka (2009)
- Bour, Koenig, Lee, Hammer, and Meissner (2011)
- Davoudi and Savage (2011) (2014)
- Leskovec and Prelovsek (2012)
- Gockeler, Horsley, Lage, Meissner, Rakow (2012)
- Polejaeva and Rusetsky (2012)
- Hansen and Sharpe (2012), (2013)
- RB and Davoudi (2012), (2013)
- Li and Liu (2013)
- Guo, Dudek, Edwards, and Szczepaniak (2013)
- RB, Davoudi, and Luu (2013)
- RB, Davoudi, Luu and Savage (2013)
- Bernard, Lage,Meissner, and Rusetsky (2011)
- RB (2014)
- Li, Li, Liu (2014)
- ...

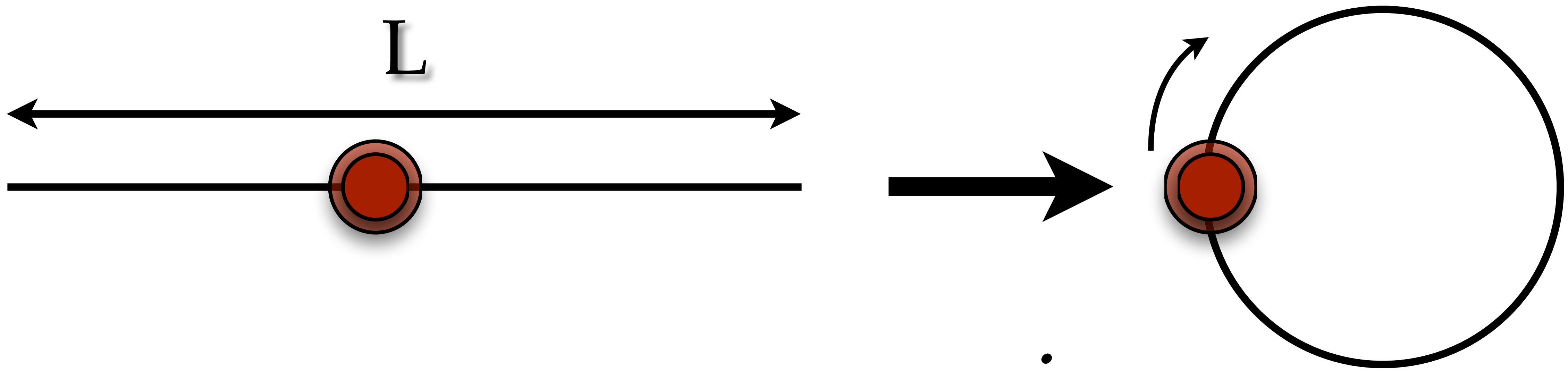
Reinventing the *quantum-mechanical* wheel

(in 1+1 dimensions)



Reinventing the *quantum-mechanical* wheel

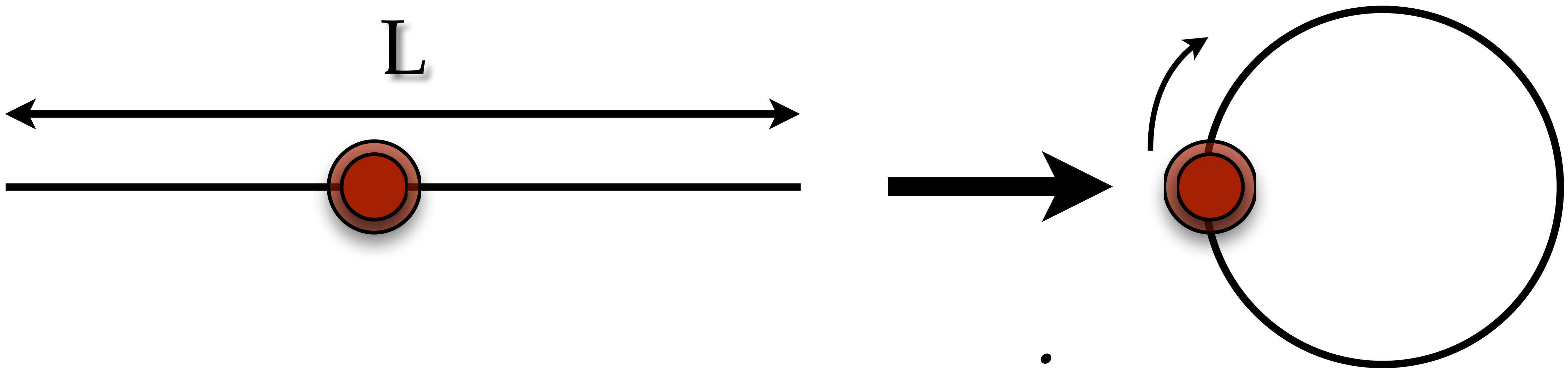
(in 1+1 dimensions)



$$\phi(x) \sim e^{ipx}$$

Reinventing the *quantum-mechanical* wheel

(in 1+1 dimensions)



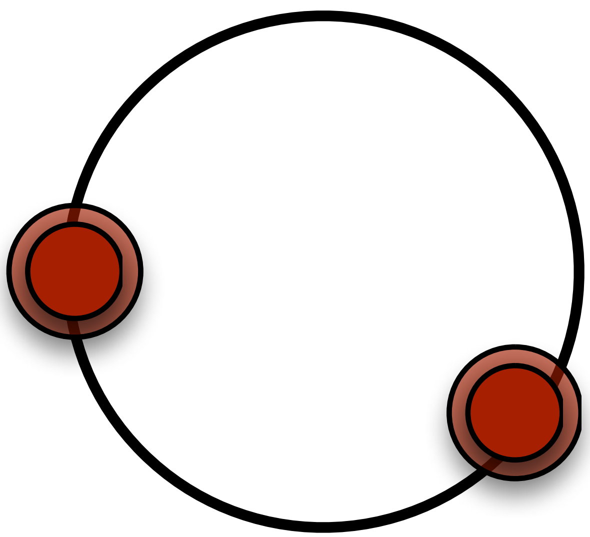
$$\phi(x) \sim e^{ipx}$$

Periodicity: $\phi(L) = \phi(0)$

Quantization condition: $\boxed{L\ p_n = 2\pi n}$

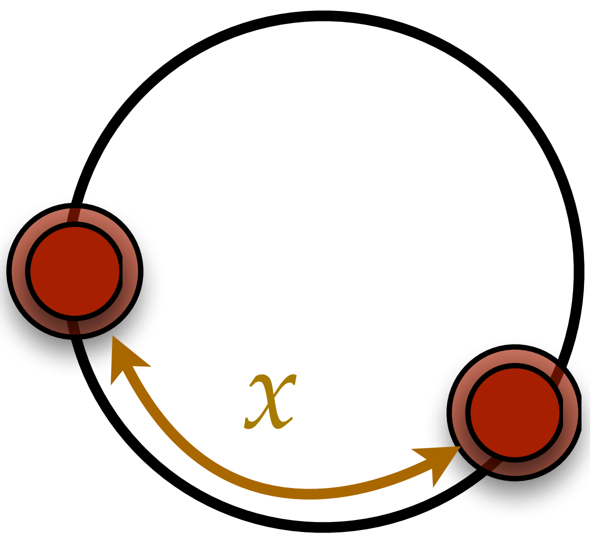
Reinventing the *quantum-mechanical* wheel

Two particles:



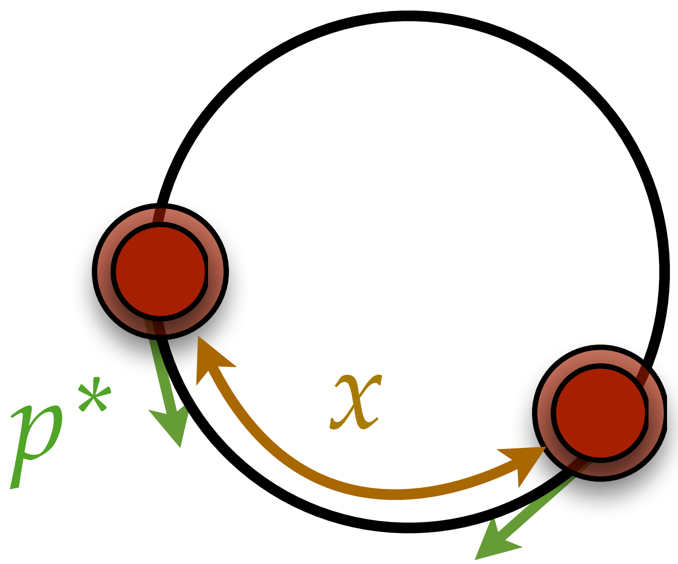
Reinventing the *quantum-mechanical* wheel

Two particles:



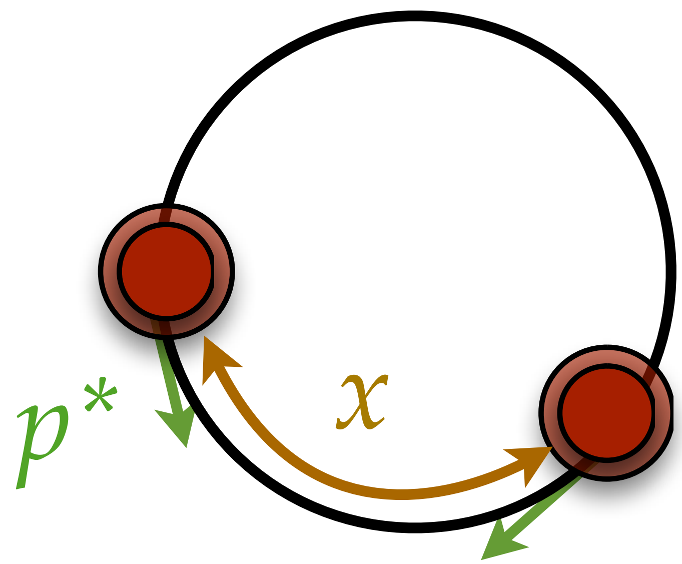
Reinventing the *quantum-mechanical* wheel

Two particles:



Reinventing the *quantum-mechanical* wheel

Two particles:



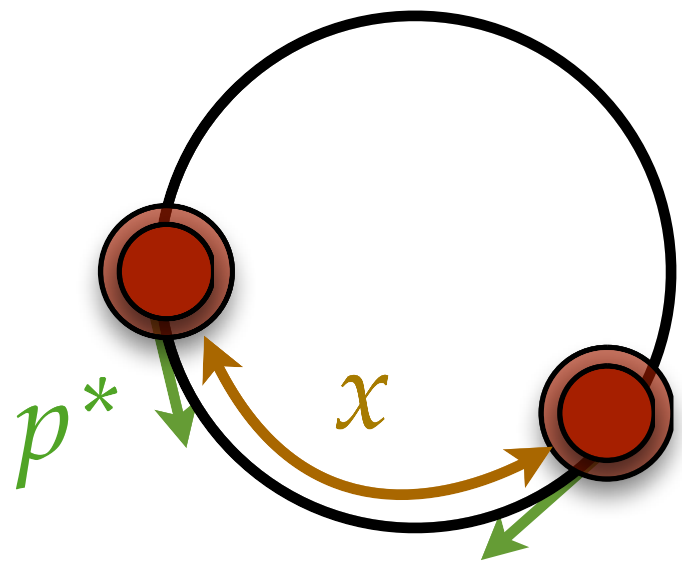
infinite volume scattering phase shift

$$\psi(x) \sim e^{ip^*x + i2\delta(p^*)}$$

Asymptotic wavefunction

Reinventing the *quantum-mechanical* wheel

Two particles:



Asymptotic wavefunction

$$\psi(x) \sim e^{ip^* x + i2\delta(p^*)}$$

infinite volume scattering phase shift

Periodicity:

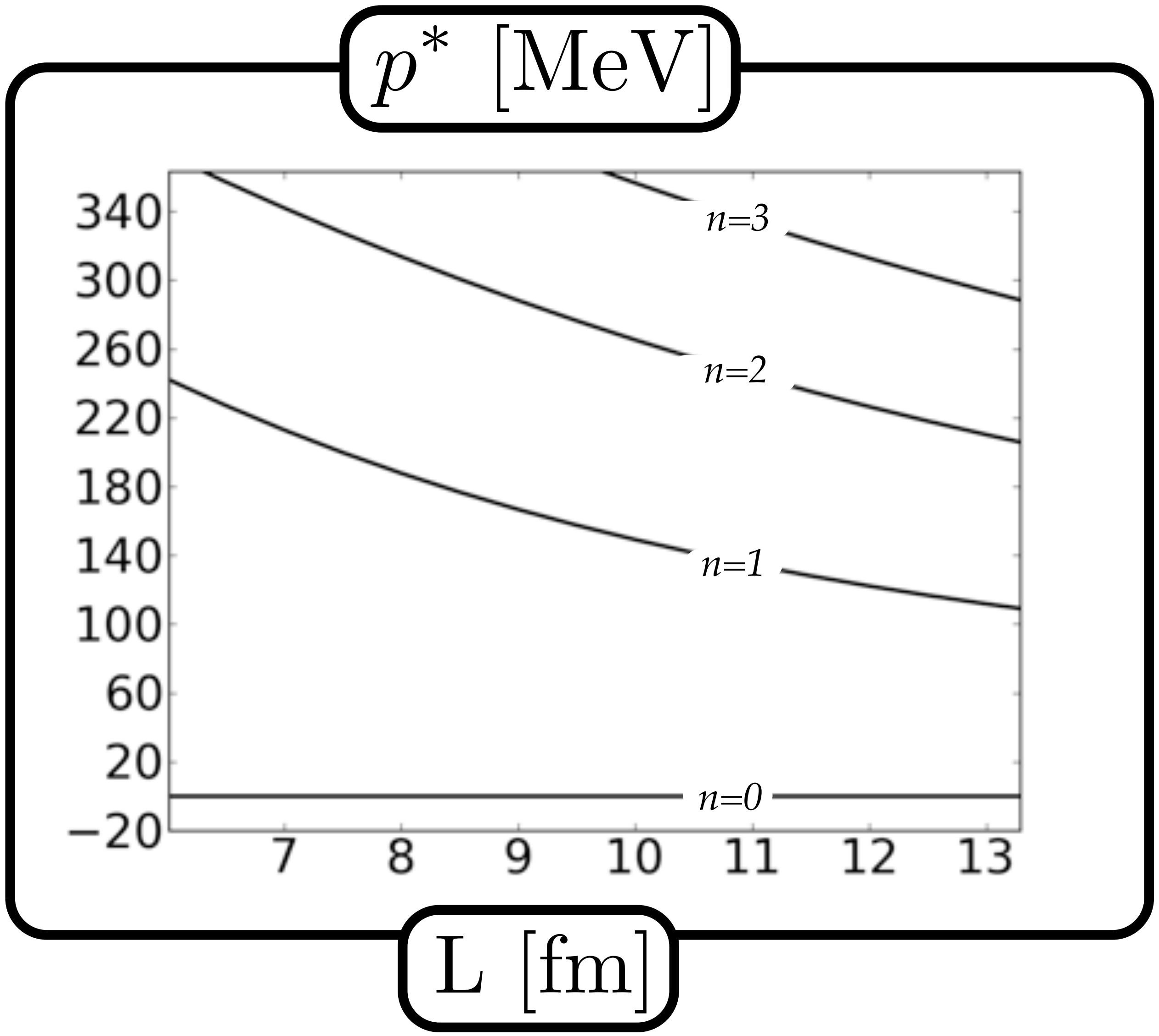
$$\psi(L) = \psi(0)$$

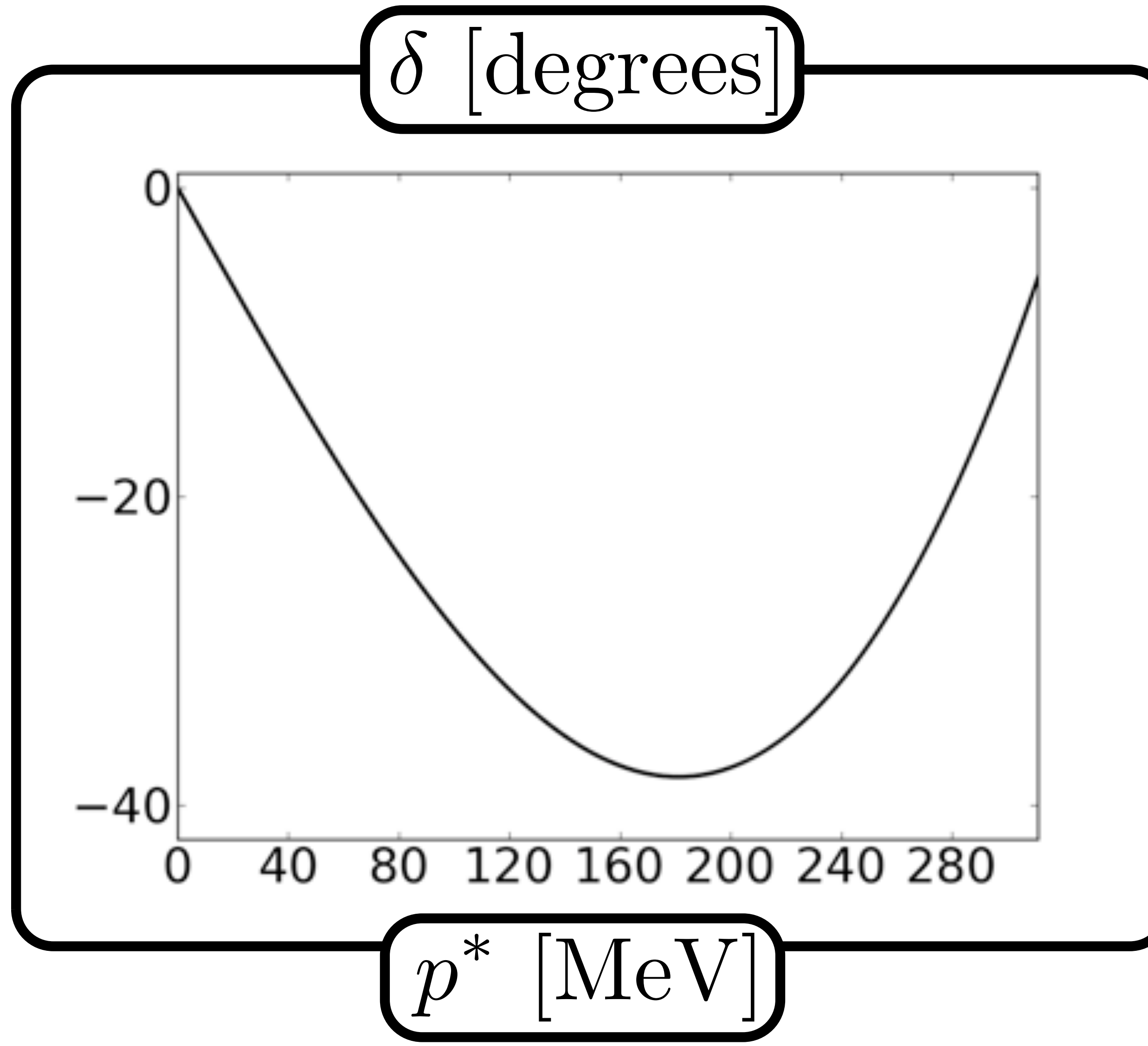
Quantization condition:

$$\boxed{L\ p_n^* + 2\delta(p_n^*) = 2\pi n}$$

Reinventing the *quantum-mechanical* wheel

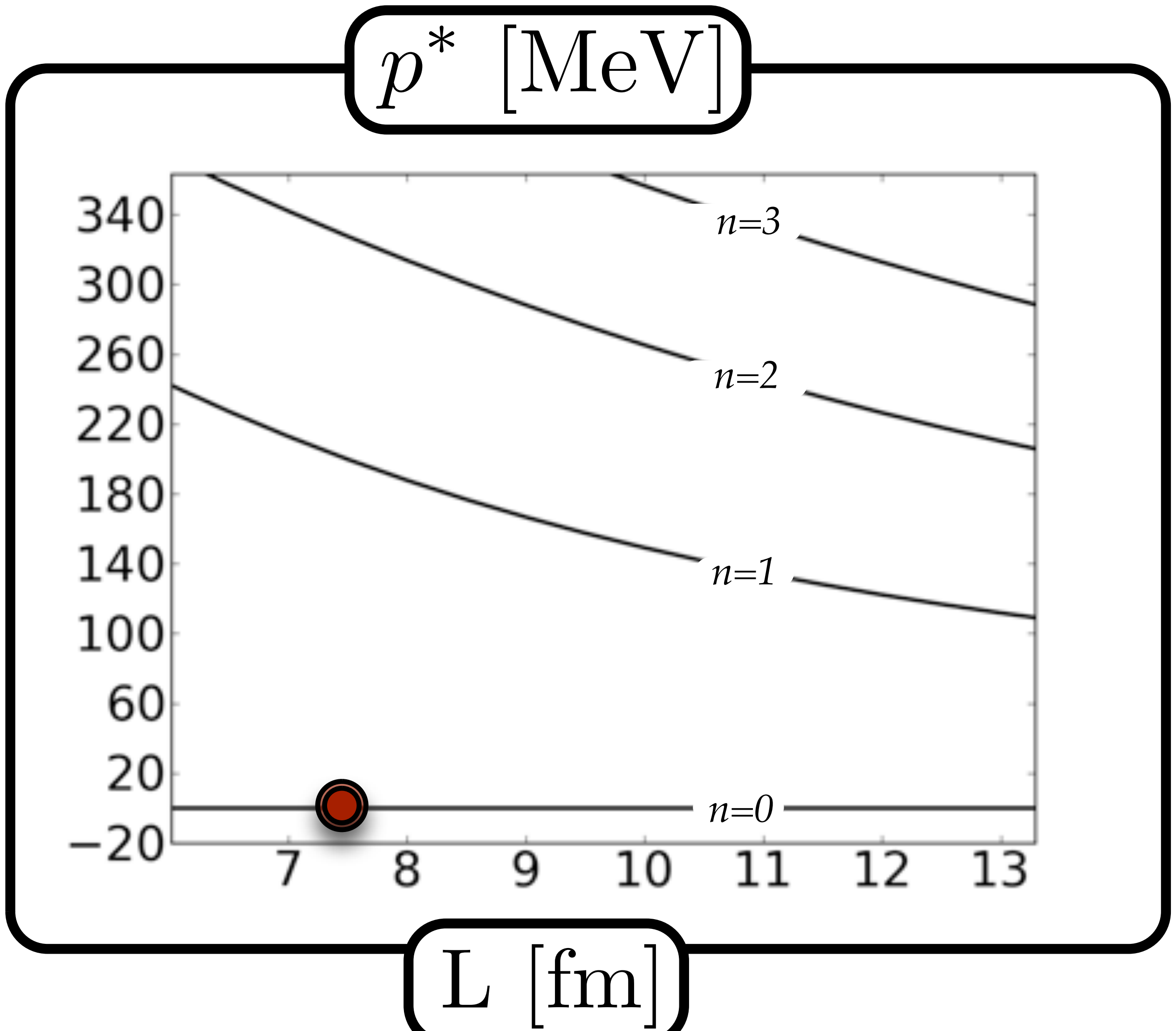
$$\boxed{L\ p_n^* + 2\delta(p_n^*) = 2\pi n}$$

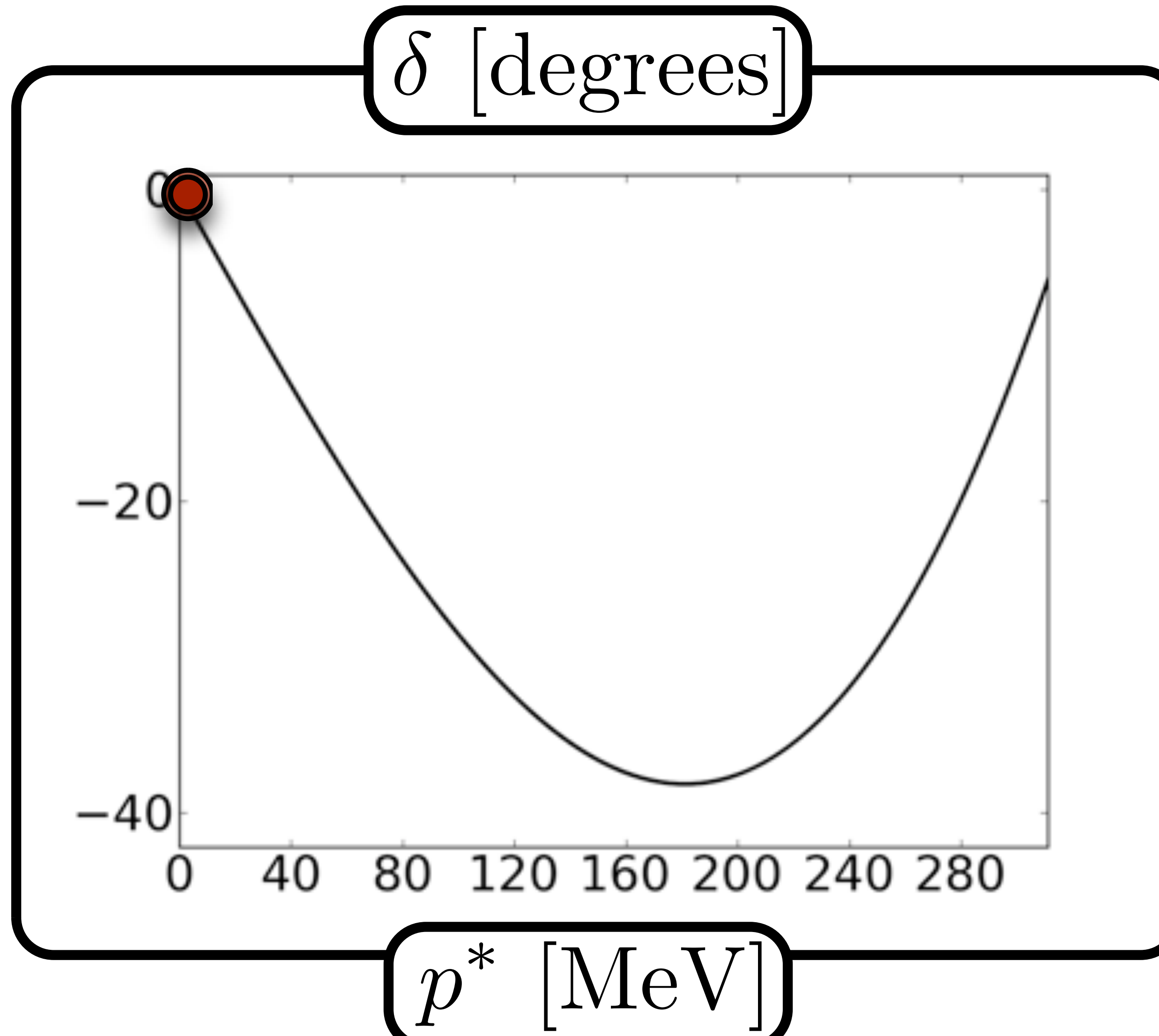




Reinventing the *quantum-mechanical* wheel

$$\boxed{L\ p_n^* + 2\delta(p_n^*) = 2\pi n}$$





Reinventing the *quantum-mechanical* wheel

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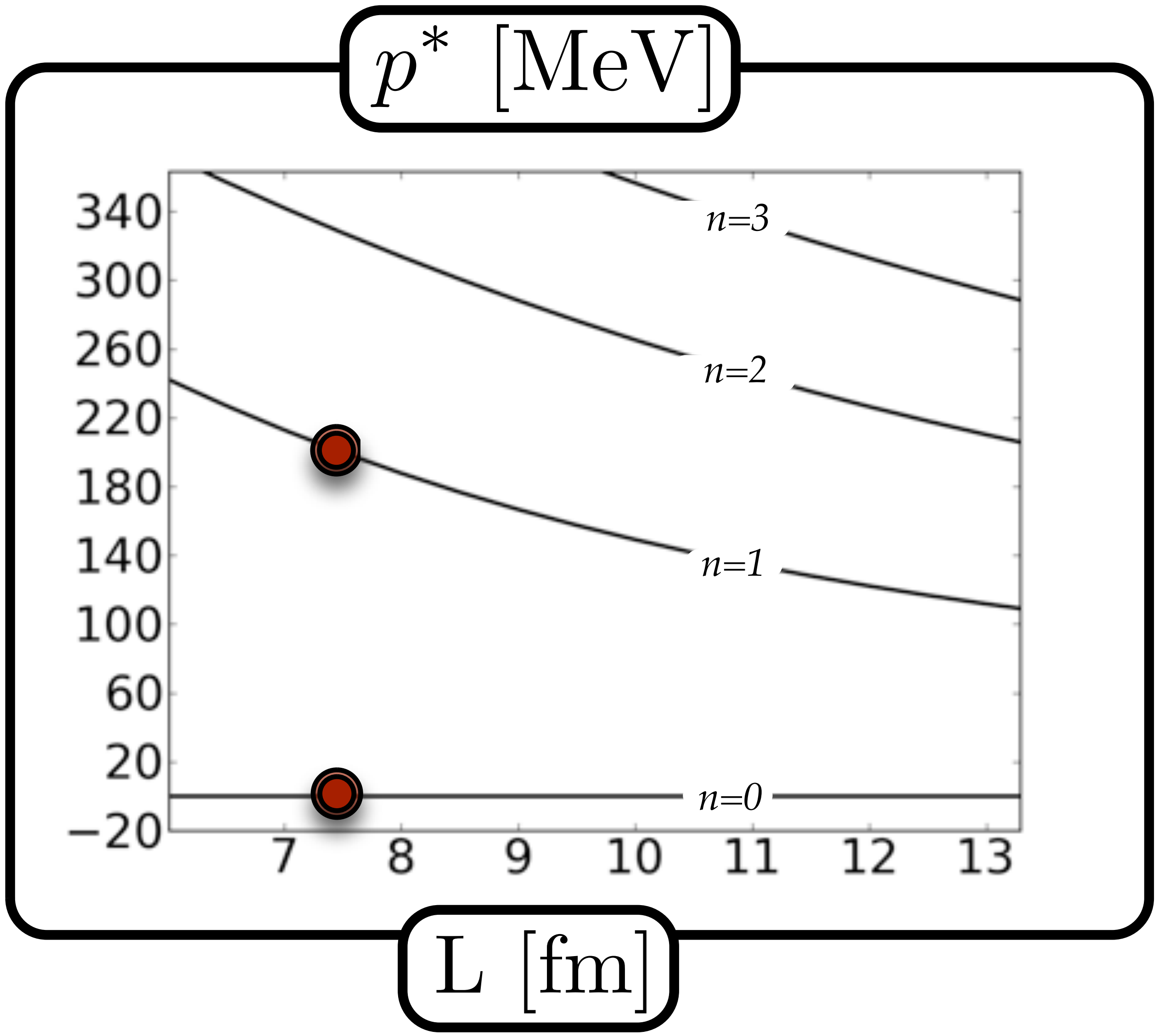

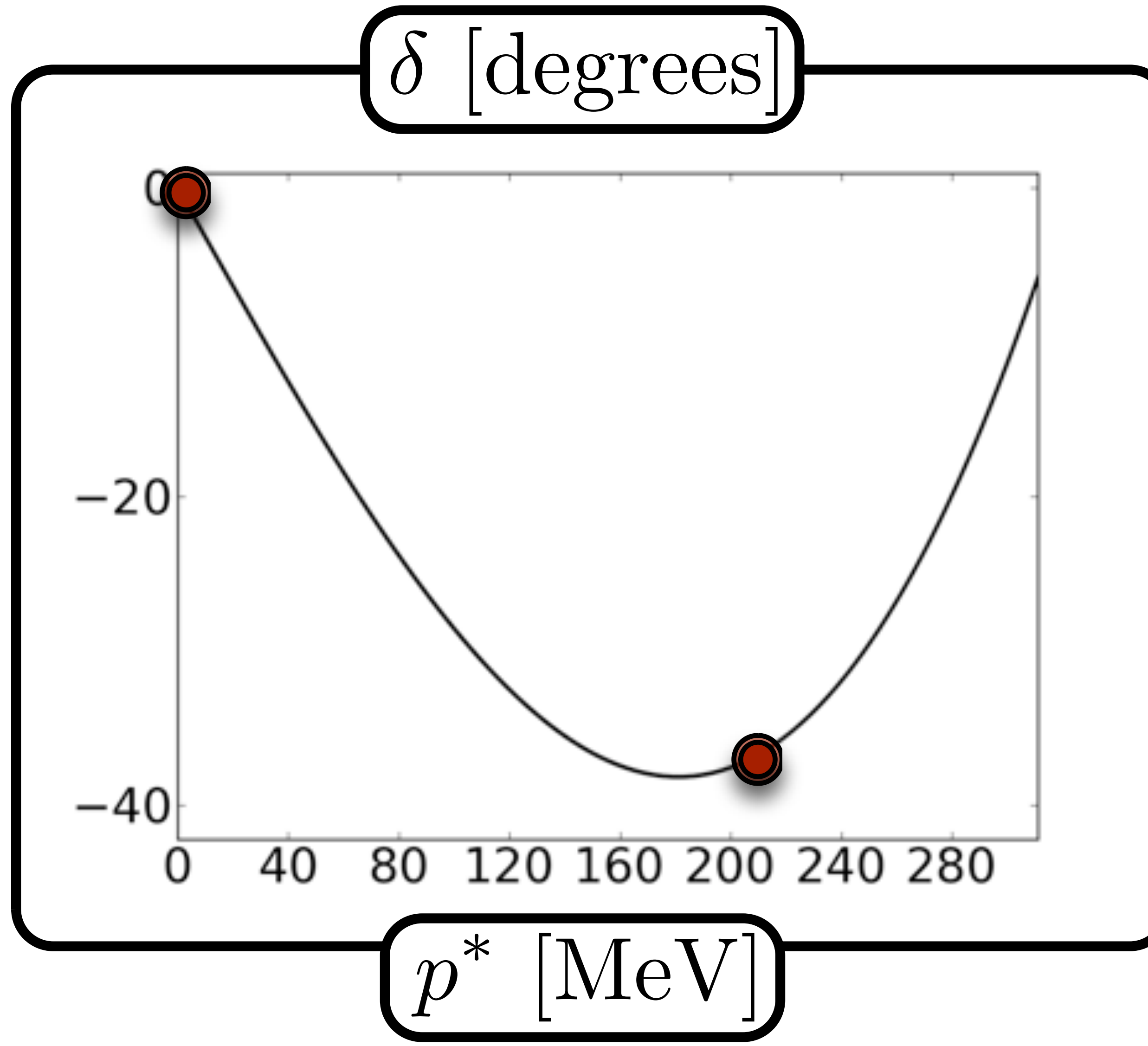

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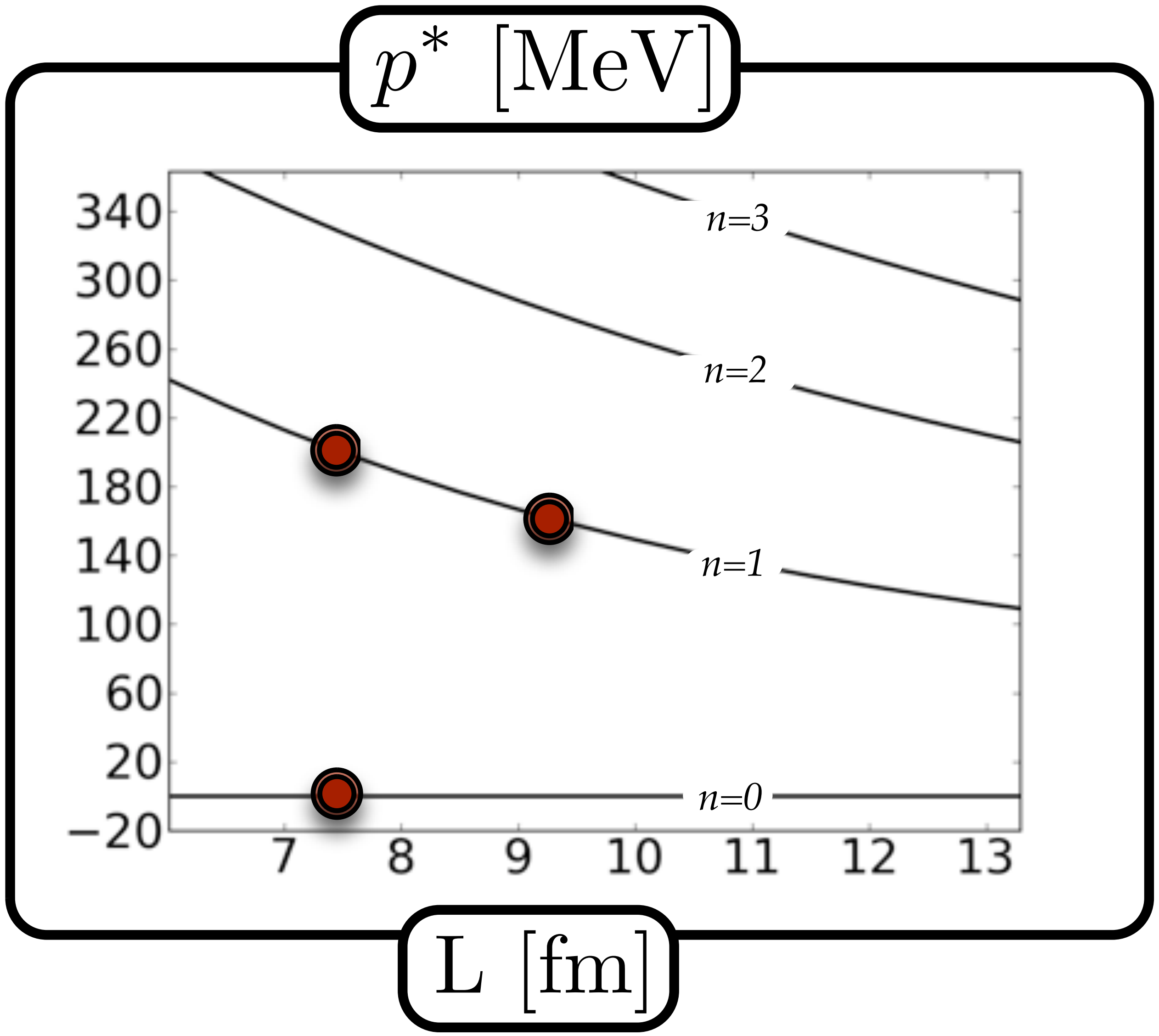
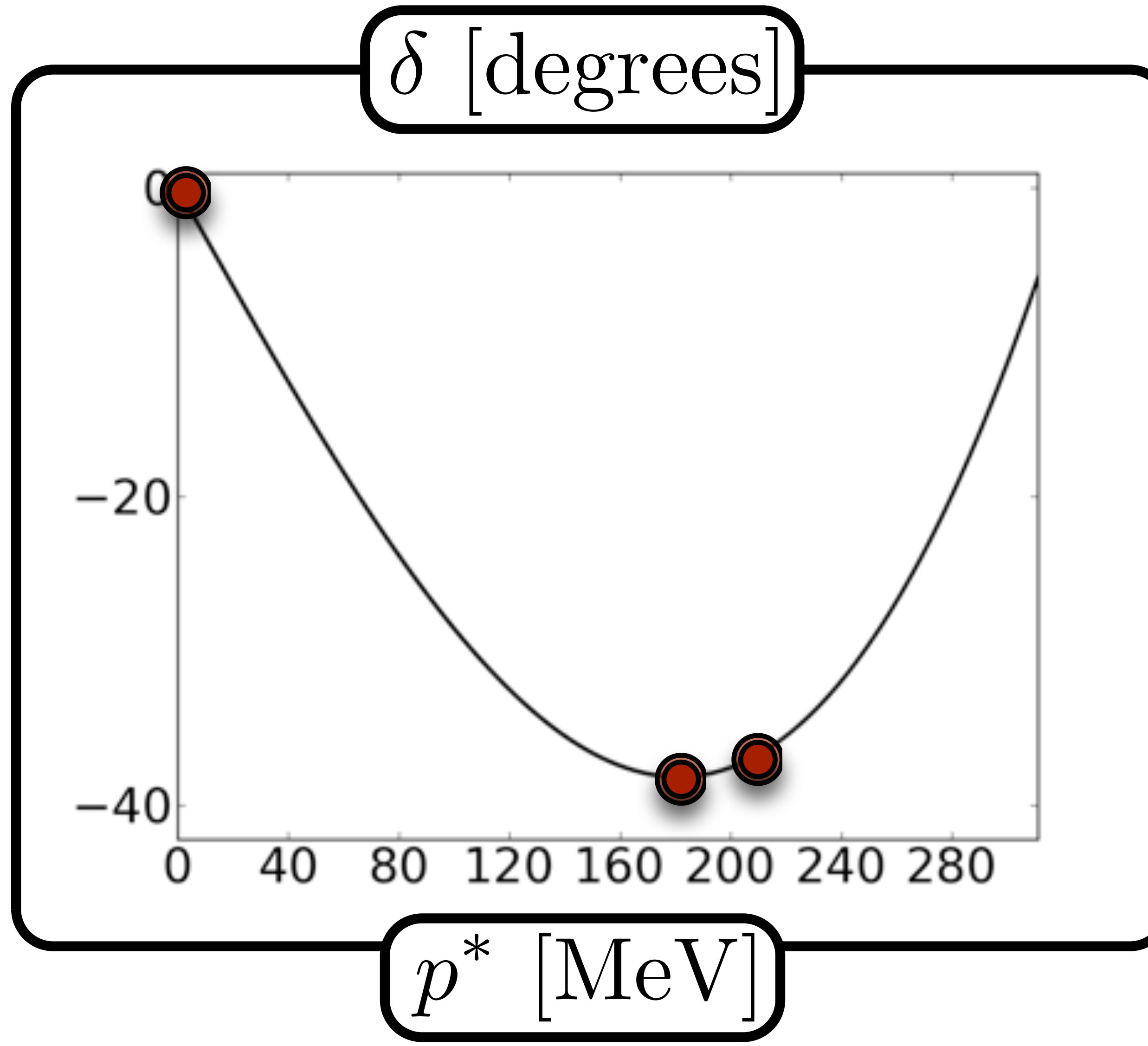
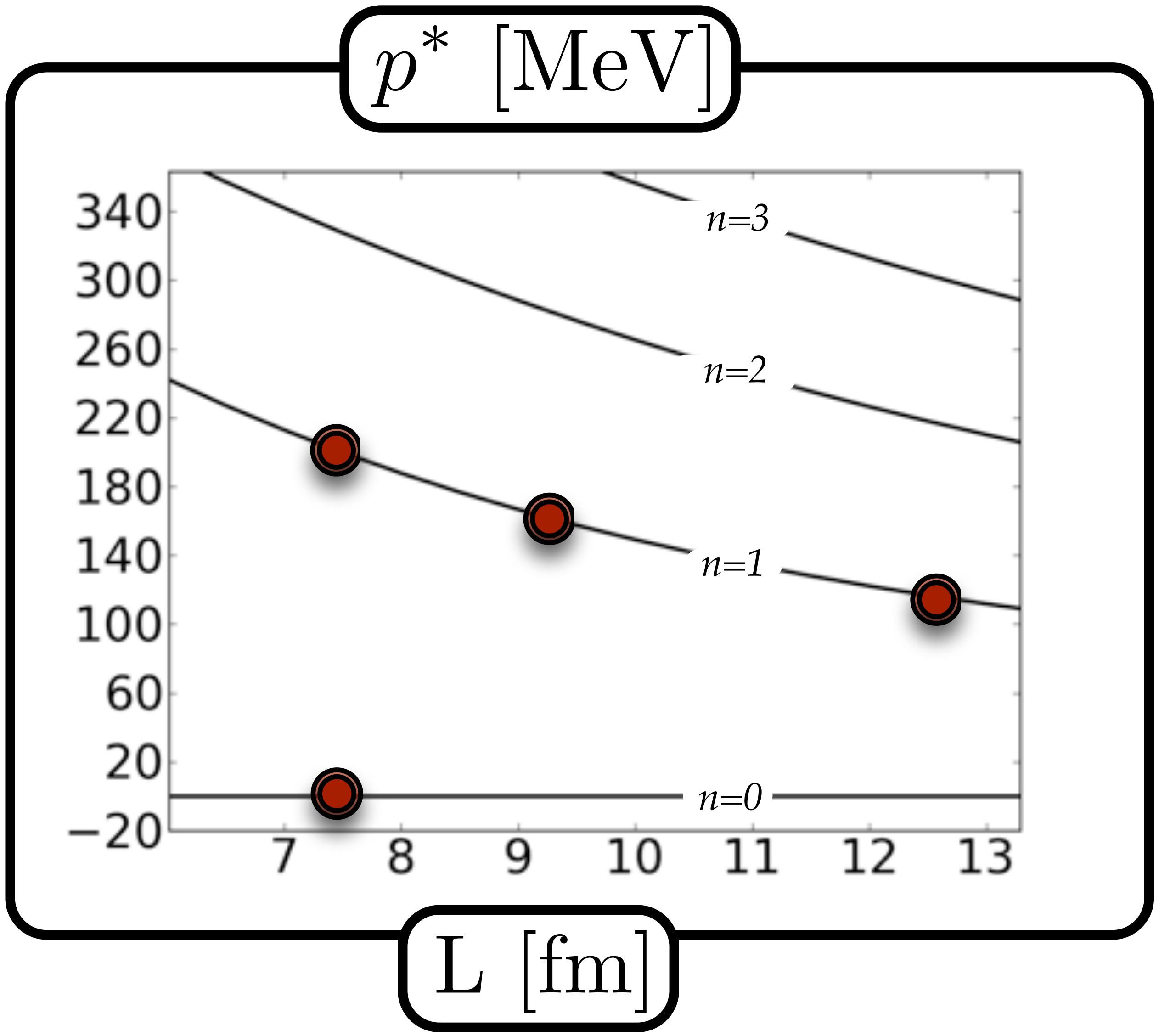
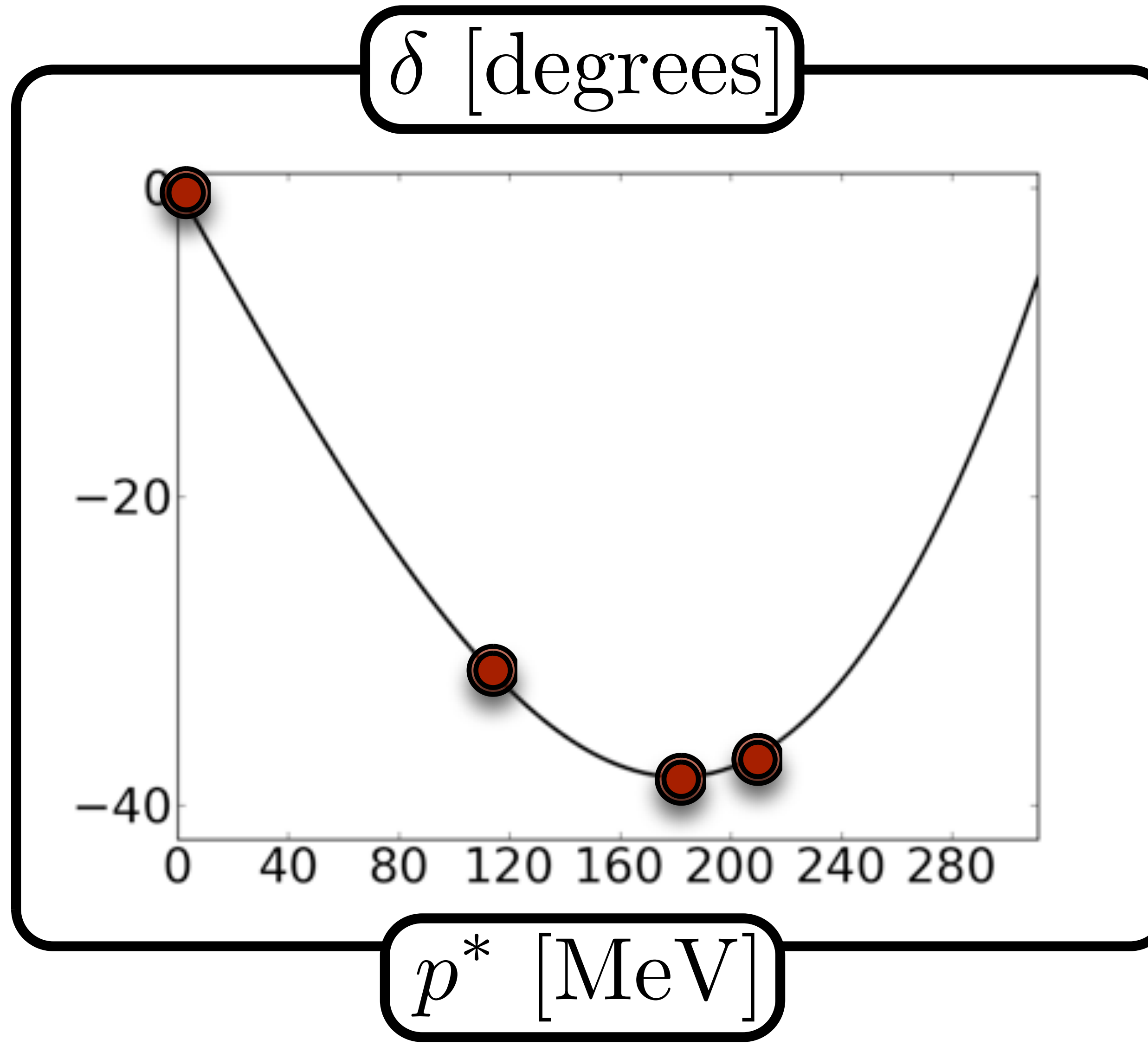




Reinventing the *quantum-mechanical* wheel

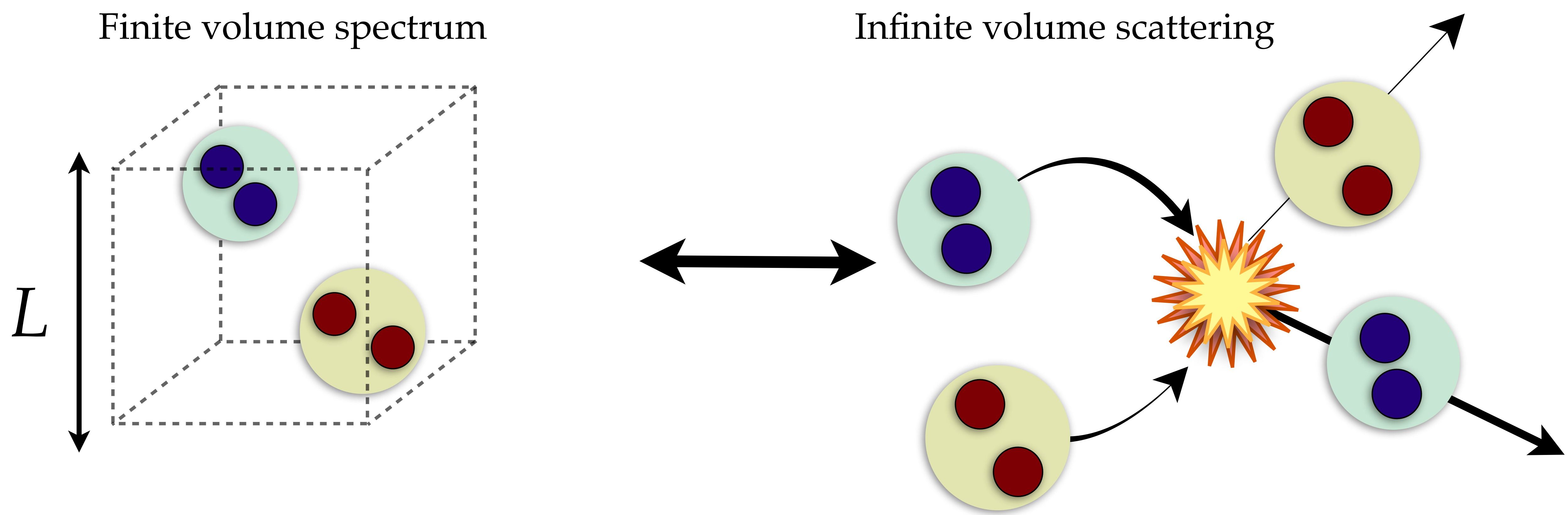
$$\boxed{L\ p_n^* + 2\delta(p_n^*) = 2\pi n}$$





Two-body spectrum

$$\det\left[\mathcal{M}^{-1} + \delta\mathcal{G}^{V}\right] = 0$$

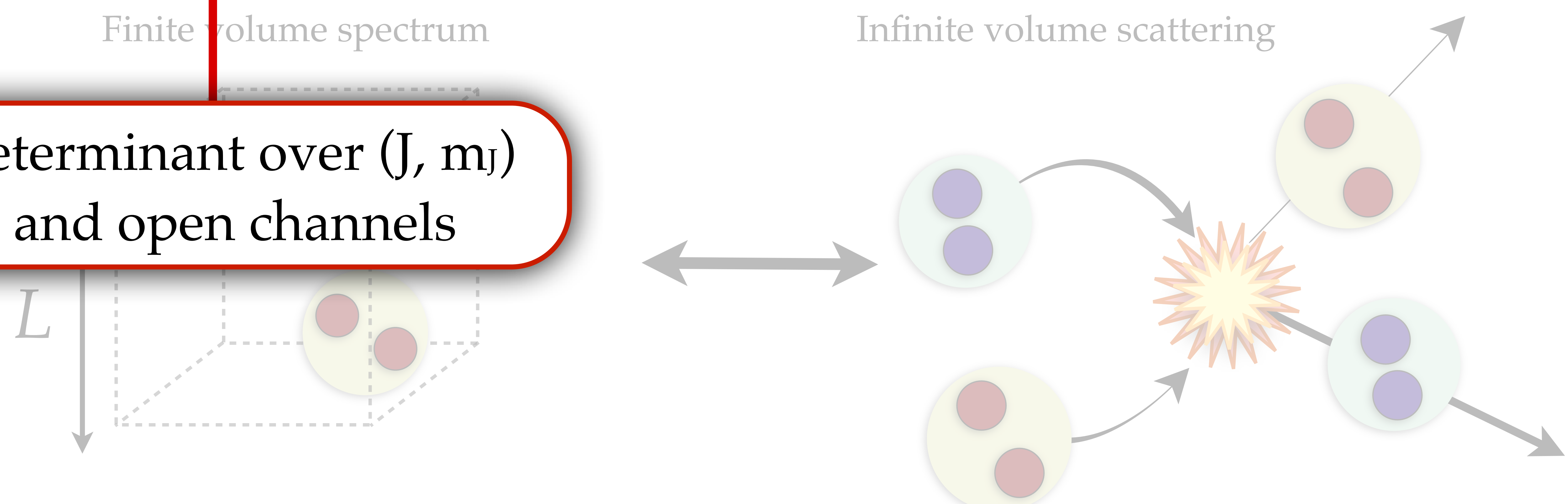


Most general two-body result is found in RB [PRD] (2014)

Two-body spectrum

$$\det \left[\mathcal{M}^{-1} + \delta\mathcal{G}^{V}\right] = 0$$

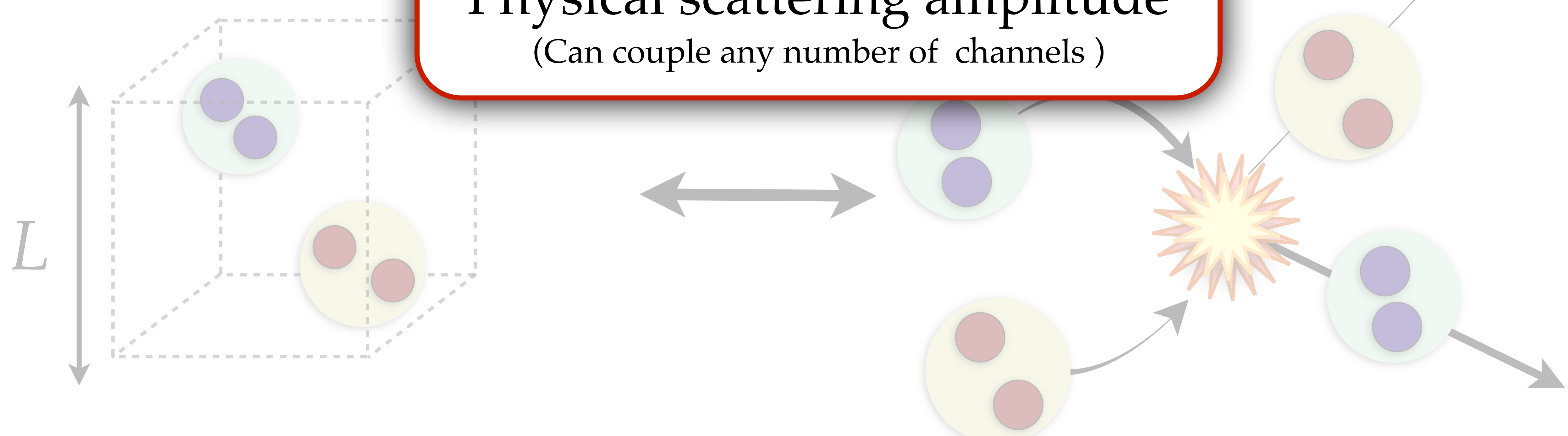
determinant over (J, m_J)
and open channels



Two-body spectrum

$$\det\left[\mathcal{M}^{-1} + \delta\mathcal{G}^{V}\right] = 0$$

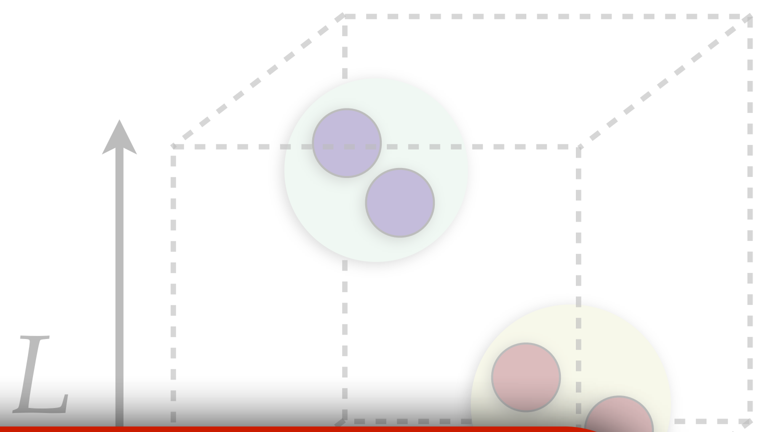
Physical scattering amplitude
(Can couple any number of channels)



Two-body spectrum

$$\det\ \left[\mathcal{M}^{-1} + \delta\mathcal{G}^{V}\right] = 0$$

Finite volume spectrum



Infinite volume scattering

$$\begin{pmatrix} \delta G^V_{00} & \delta G^V_{01} & \delta G^V_{02} & \\ \delta G^V_{10} & \delta G^V_{11} & \delta G^V_{12} & \\ \delta G^V_{20} & \delta G^V_{21} & \delta G^V_{22} & \\ & & & \ddots \end{pmatrix}$$

Typically a sparse matrix, but in general partial waves do mix (as they should!)

e.g. S-wave at rest

$$k^* \cot \delta_S = \frac{1}{\pi L} \sum_{\mathbf{n}} \frac{1}{\mathbf{n}^2 - (k^* L/2\pi)^2}$$

Two-body spectrum

$$\det\left[\mathcal{M}^{-1} + \delta\mathcal{G}^{V}\right] = 0$$

- Model independent & non-perturbative
- Universal: lattice QCD, lattice EFT, cold atoms, etc.
- Arbitrary quantum numbers for two particles
- General volumes with any boundary conditions: periodic, anti-periodic , or any linear combination on any rectangular prism

Two-body spectrum

$$\det\left[\mathcal{M}^{-1}+\delta\mathcal{G}^{V}\right]=0$$

Finite volume spectrum

Infinite volume scattering

Compactly summarizes & generalizes all that has been written on the two-body sector in the literature

A long list of reference

- **Lüscher (1986), (1991)** *("Lüscher Formalism")*
- Maiani and Testa (1990)
- Rummukainen and Gottlieb (1995)
- Beane, Bedaque, Parreno, and Savage (2004), (2005)
- Bedaque (2004)
- Li and Liu (2004)
- Detmold and Savage (2004)
- Feng, Li, and Liu (2004)
- Christ, Kim, and Yamazaki (2005)
- Kim, Sachrajda, and. Sharpe (2005)
- Bernard, Lage, Meissner, and Rusetsky (2008)
- Ishizuka (2009)
- Bour, Koenig, Lee, Hammer, and Meissner (2011)
- Davoudi and Savage (2011) (2014)
- Leskovec and Prelovsek (2012)
- Gockeler, Horsley, Lage, Meissner, Rakow (2012)
- Hansen and Sharpe (2012)
- RB and Davoudi (2012)
- Li and Liu (2013)
- Guo, Dudek, Edwards, and Szczepaniak (2013)
- RB, Davoudi, and Luu (2013)
- RB, Davoudi, Luu and Savage (2013)
- Bernard, Lage,Meissner, and Rusetsky (2011)
- RB (2014)
- Li, Li, Liu (2014)
- ...

Friday, October 10, 14

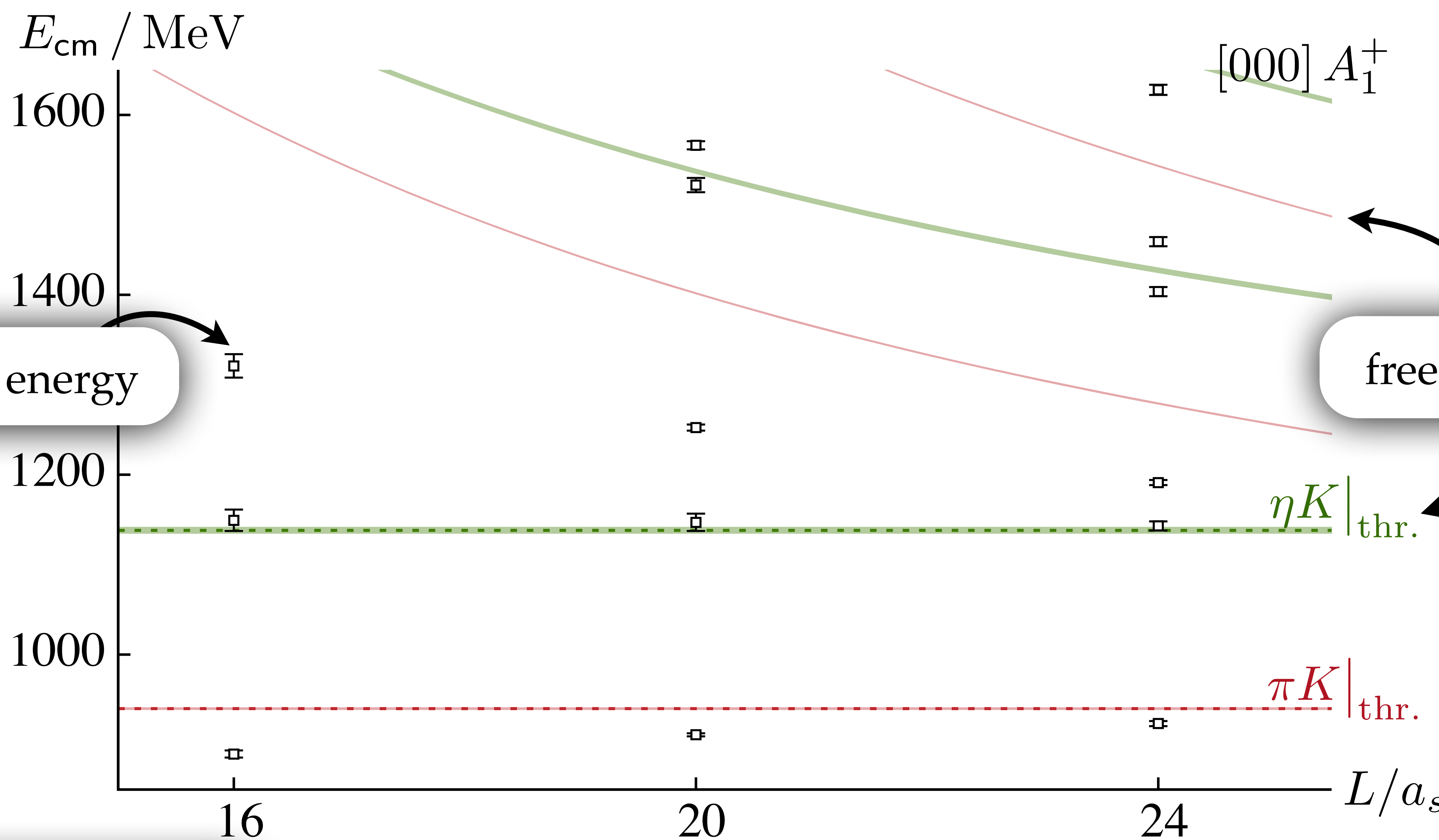
RB [PRD] (2014)

One example: Kπ-Kη

1

Determine finite volume spectra, e.g., Kπ-Kη spectrum using m_π~390MeV

by David Wilson, Dudek, Edwards & Thomas (2014) [Hadron Spectrum Coll]



Lattice QCD energy

free energy level

unboosted
d=**P**L/2π=[000]

Over 100 energy levels determined using **3** different volumes and 5 different types of boosts, **d={[000],[001],[011],[111],[002]}** and allowed cubic rotations.

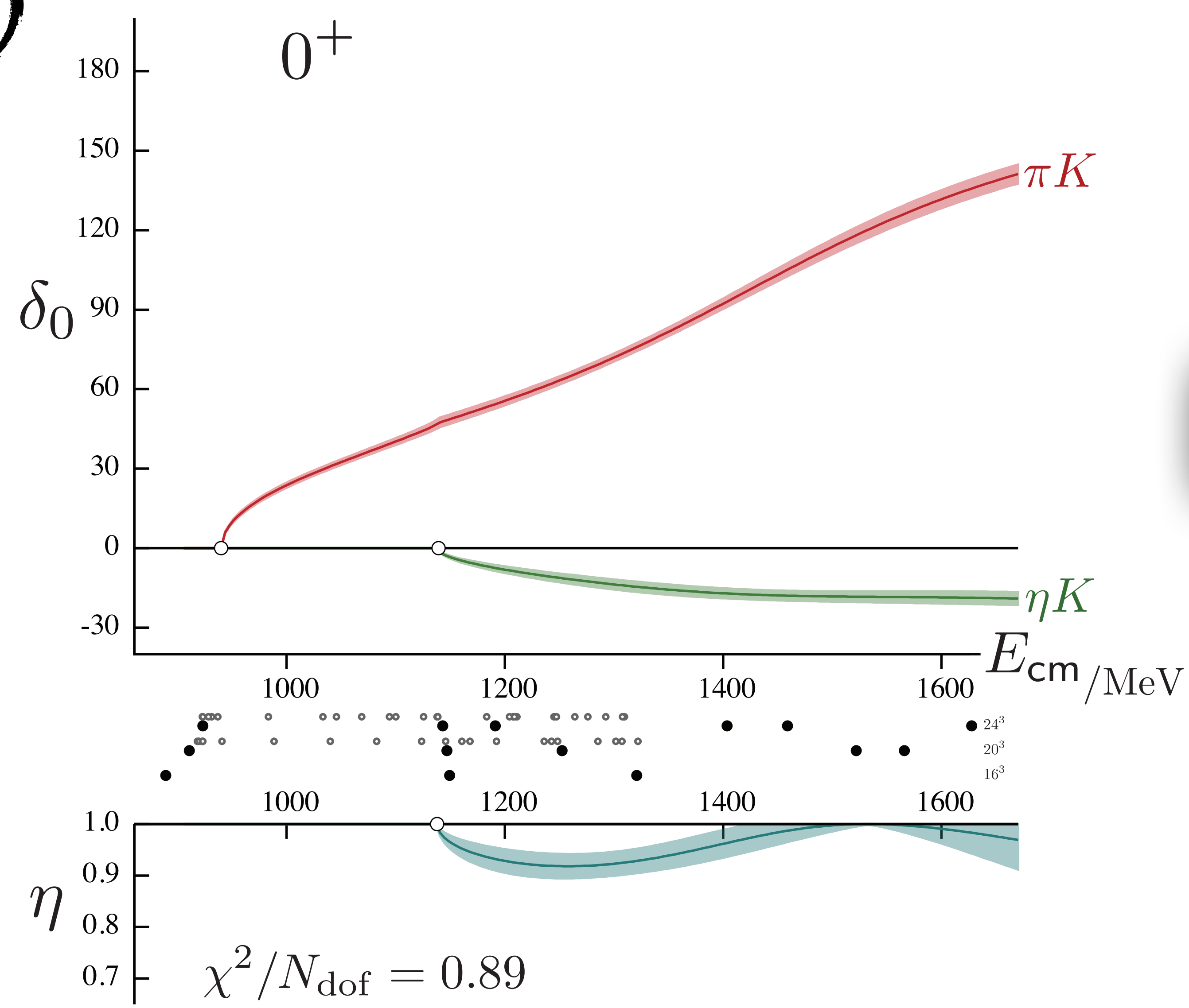
One example: Kπ-Kη

1. Determine finite volume spectra, e.g., Kπ-Kη spectrum using m_π~390MeV

by David Wilson, Dudek, Edwards & Thomas (2014) [Hadron Spectrum Coll]

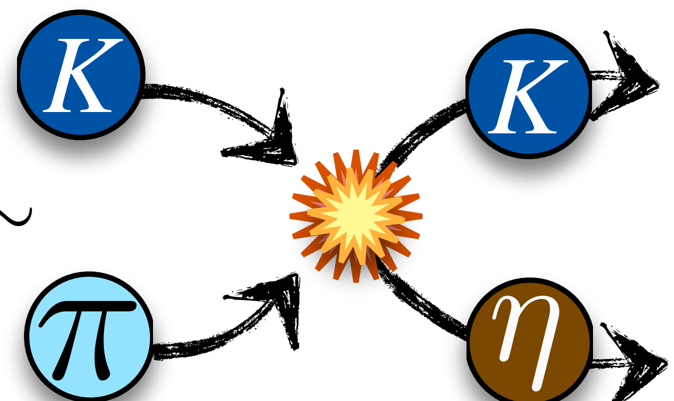
2. $$\det\left[\mathcal{M}^{-1}+\delta\mathcal{G}^{V}\right]=0$$

3.



S-wave phase shifts

$$\sqrt{1-\eta^2}\sim$$



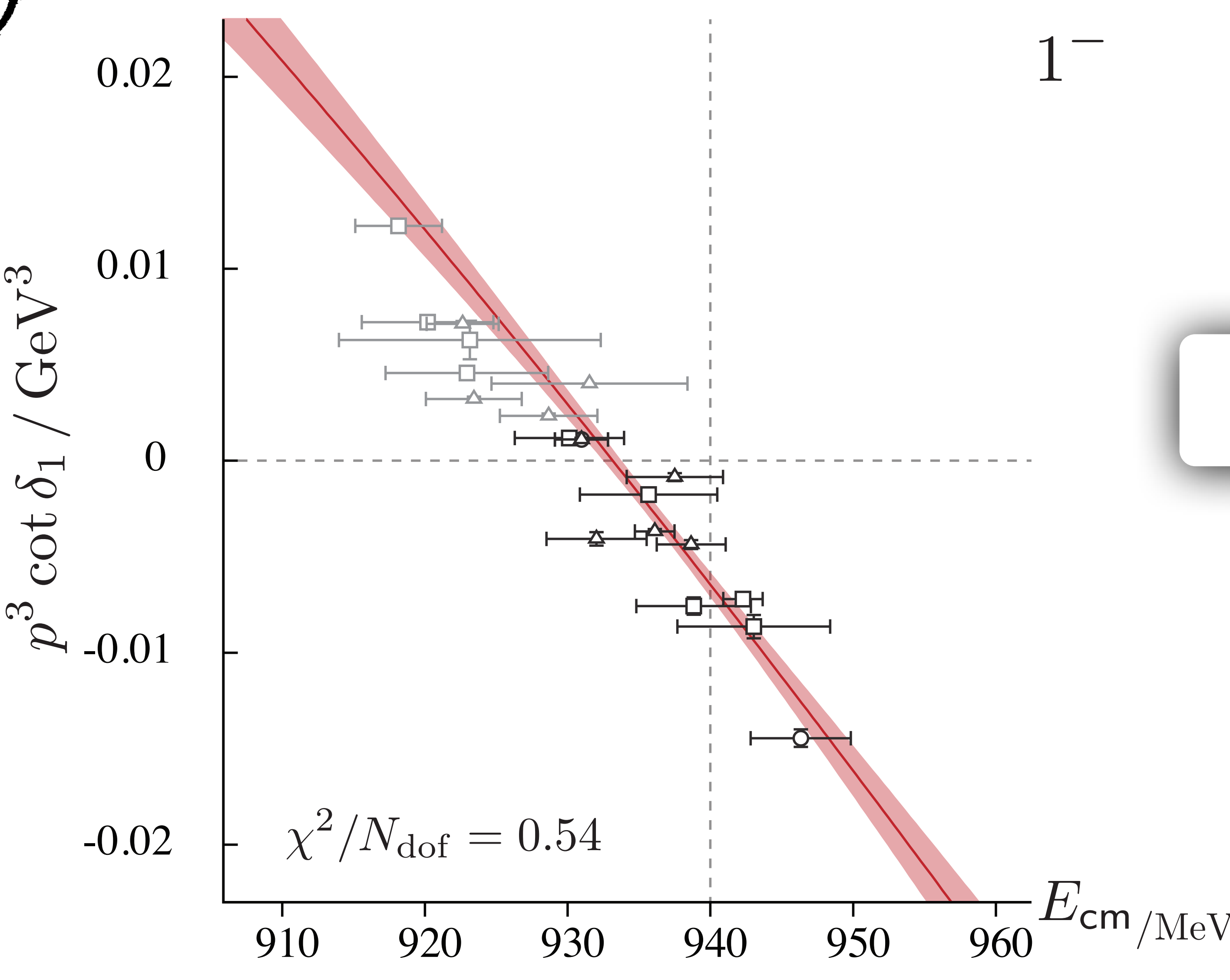
One example: Kπ-Kη

1 Determine finite volume spectra, e.g., Kπ-Kη spectrum using m_π~390MeV

by David Wilson, Dudek, Edwards & Thomas (2014) [Hadron Spectrum Coll]

2 $$\det\left[\mathcal{M}^{-1}+\delta\mathcal{G}^{V}\right]=0$$

3



P-wave phase shifts

One example: Kπ-Kη

1

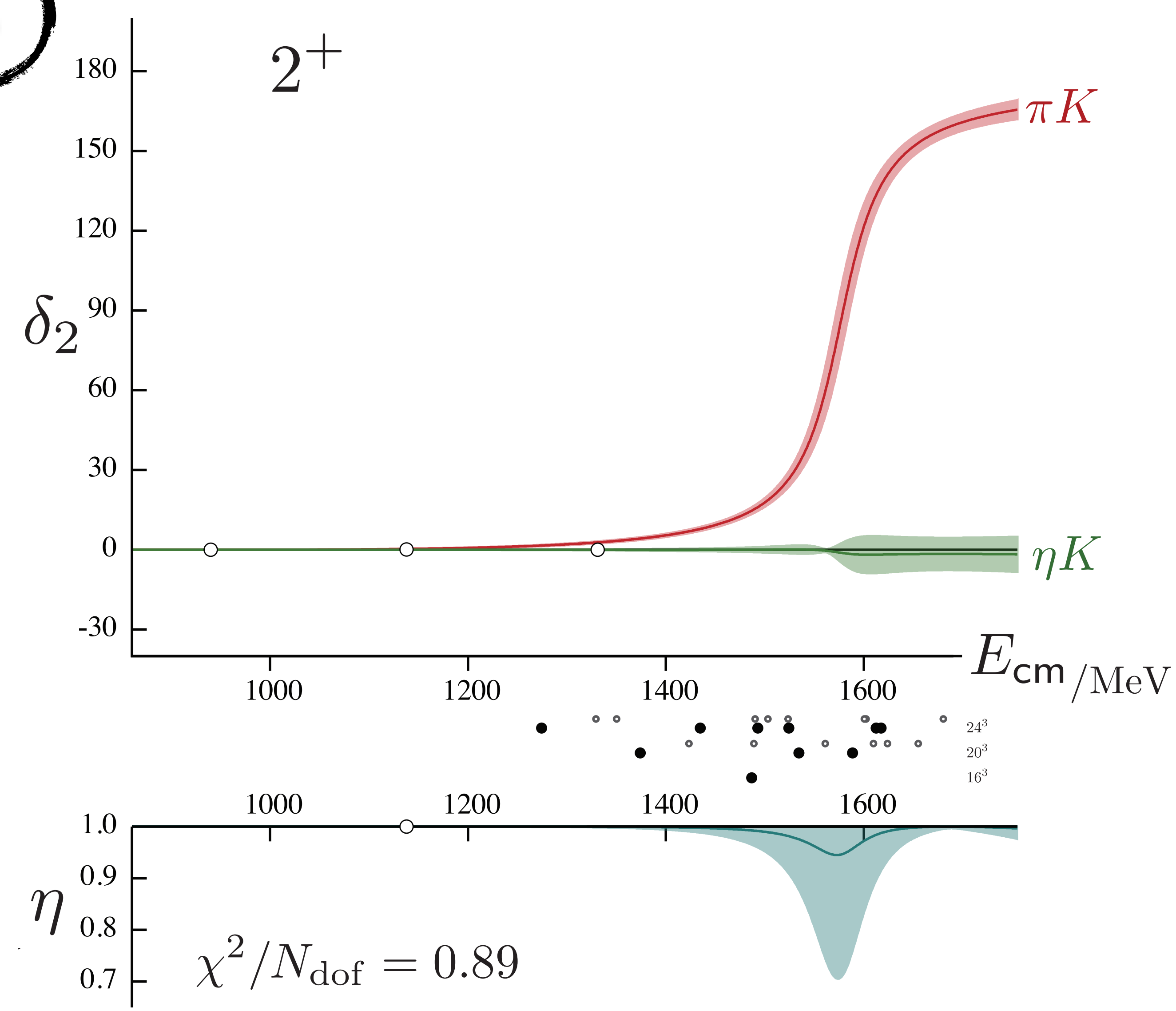
Determine finite volume spectra, e.g., Kπ-Kη spectrum using m_π~390MeV

by David Wilson, Dudek, Edwards & Thomas (2014) [Hadron Spectrum Coll]

2

$$\det\ [\mathcal{M}^{-1} + \delta\mathcal{G}^V] = 0$$

3



D-wave phase shifts

inelasticity

One example: Kπ-Kη

1

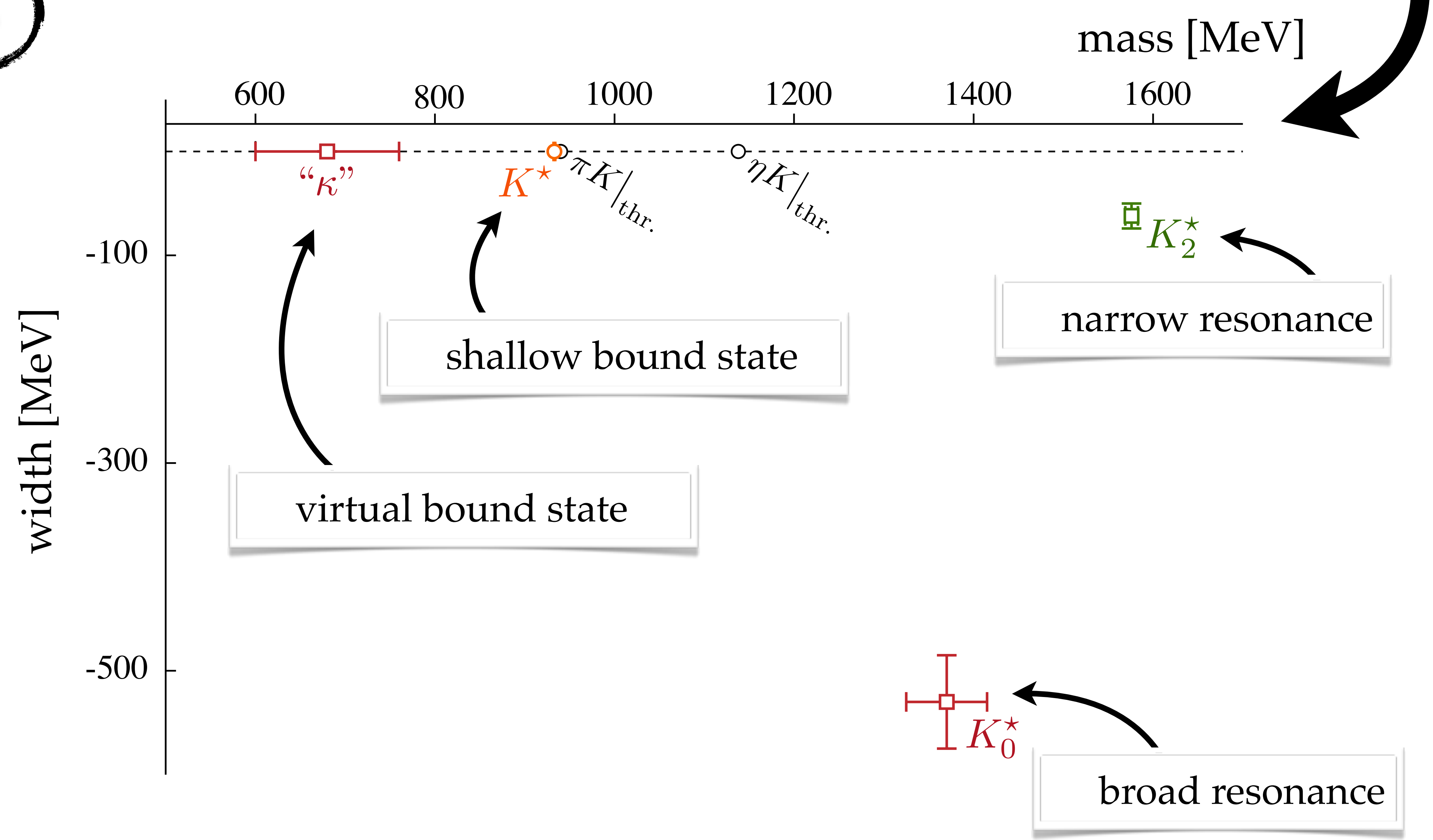
Determine finite volume spectra, e.g., Kπ-Kη spectrum using m_π~390MeV

by David Wilson, Dudek, Edwards & Thomas (2014) [Hadron Spectrum Coll]

2

$$\det\left[\mathcal{M}^{-1} + \delta\mathcal{G}^V\right] = 0$$

3



One example: Kπ-Kη

1

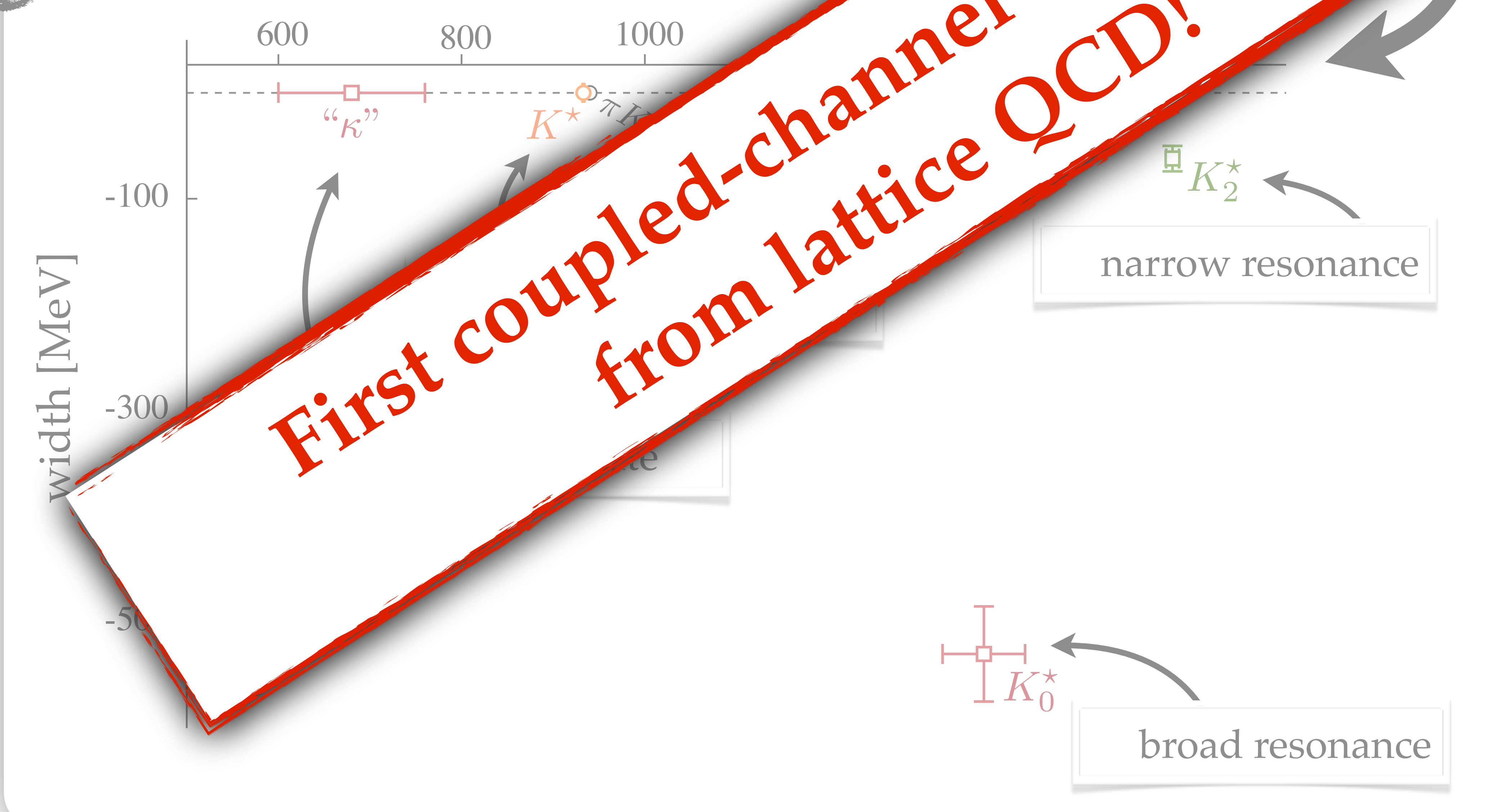
Determine finite volume spectra, e.g., Kπ-Kη spectrum using m_π~390MeV

by David Wilson, Dudek, Ed[...]rds & Thomas (2014) [Hadron Spectru[...]

2

$$\det \left[\quad \right] = 0$$

3



First coupled-channel calculation from lattice QCD!

Three-body spectrum

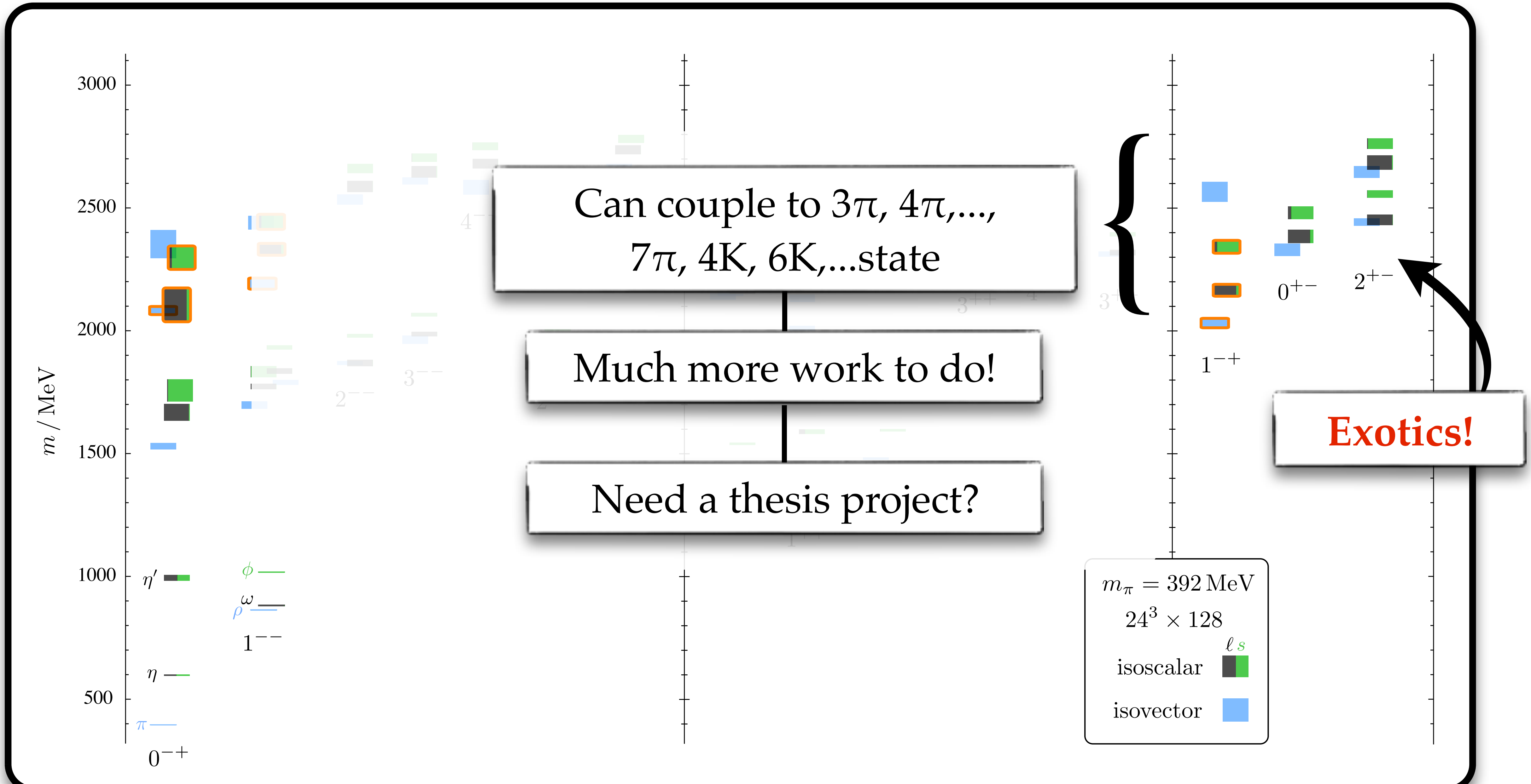
Fresh off the press

$$\det \left[\mathcal{K}_{3,\mathrm{df}}^{-1} + F_3^V\right] = 0$$

- Model independent & non-perturbative
- Universal: lattice QCD, lattice EFT, cold atoms, etc.
- Only holds for three, identical boson
- Only holds for periodic, cubic volumes
- No N-body result yet!

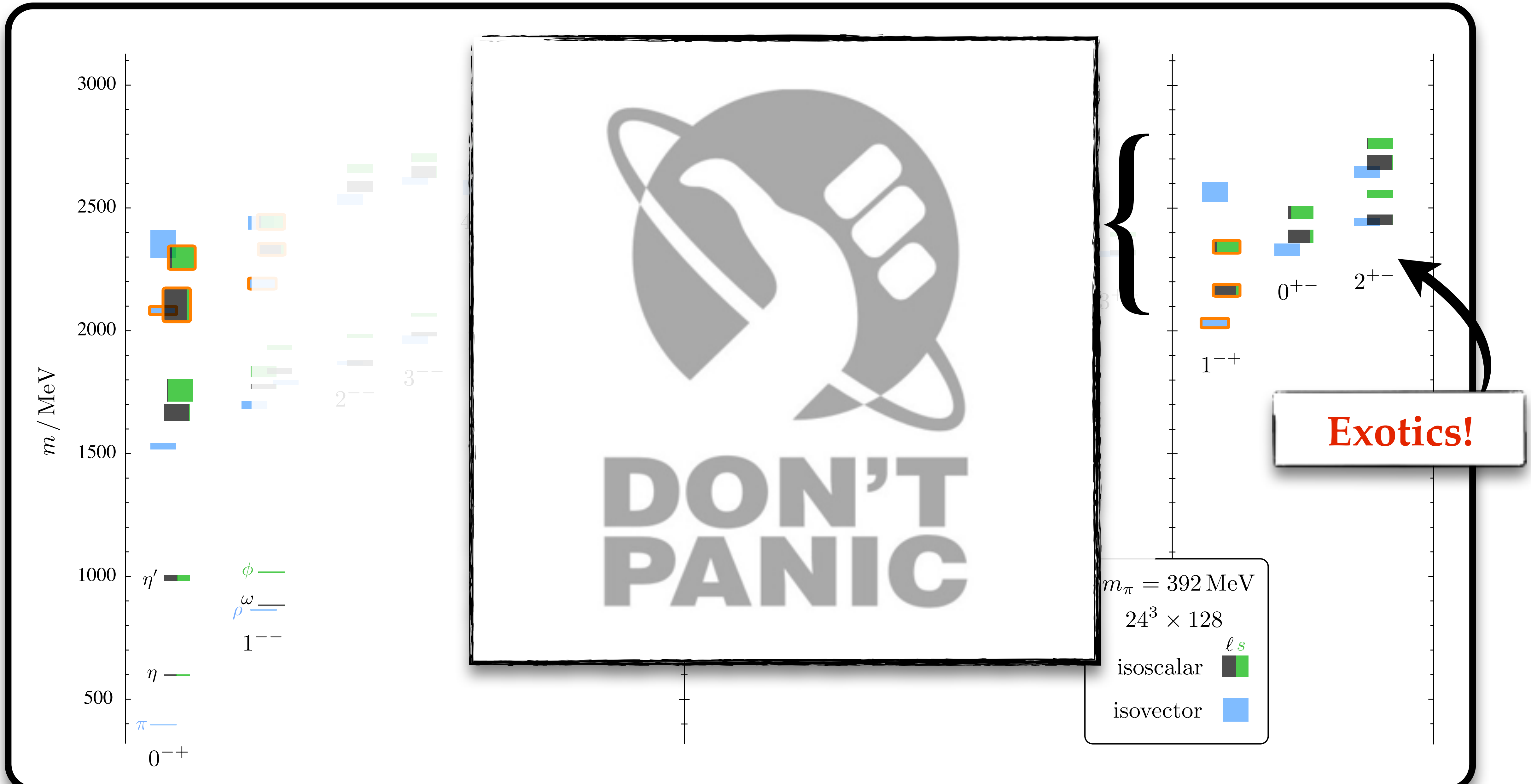
Hansen and Sharpe (2014)

N-body spectrum



Hadron Spectrum Collaboration:
J. Dudek, R. Edwards, P. Guo & C. Thomas (2013)

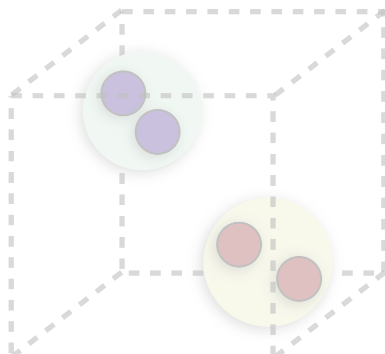
N-body spectrum



Hadron Spectrum Collaboration:
J. Dudek, R. Edwards, P. Guo & C. Thomas (2013)

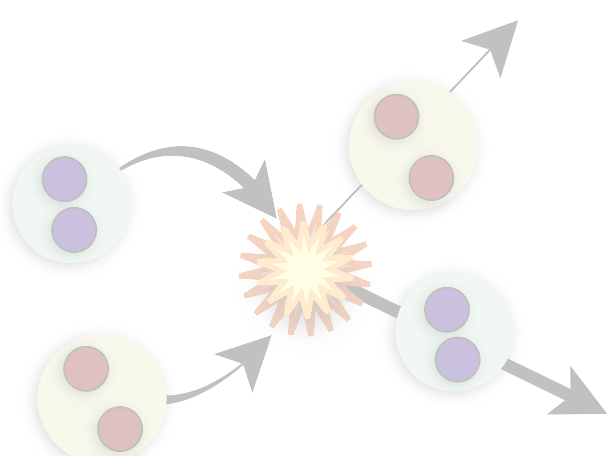
Paving the road towards physics

1 Calculate finite volume spectrum



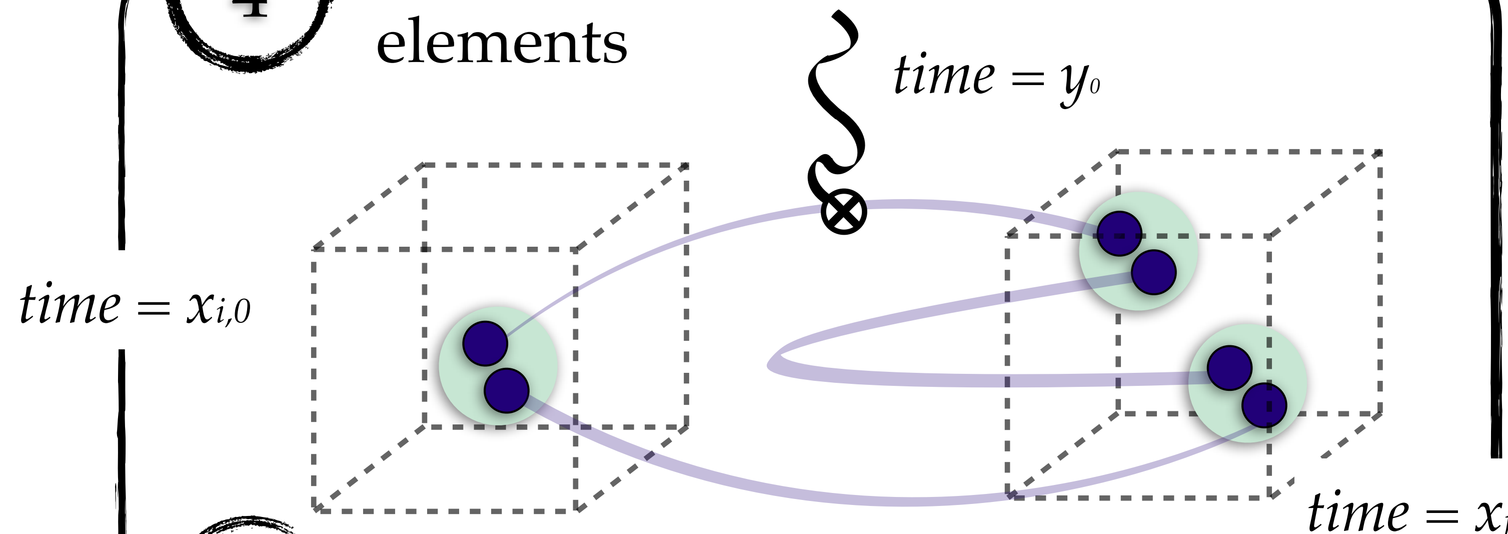
2 Plug into formalism

3 Out goes elastic & inelastic QCD scattering amplitudes



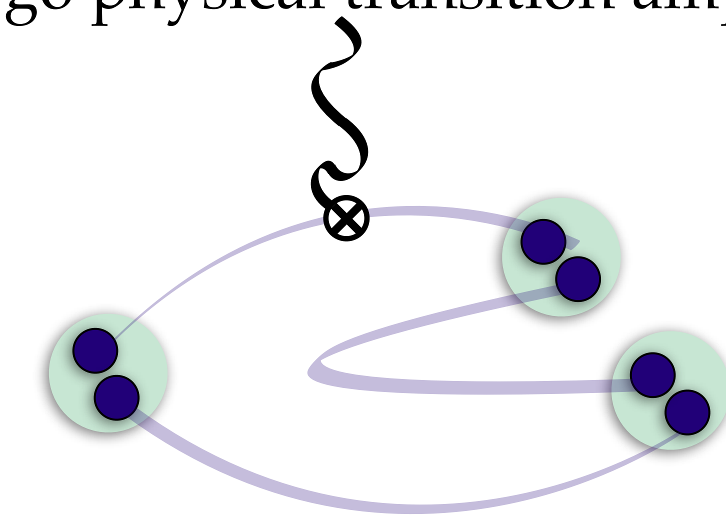
à la mode de Lüscher (1986)

4 Calculate finite volume matrix elements



5 Plug spectrum, scattering parameters and finite volume form factor into formalism

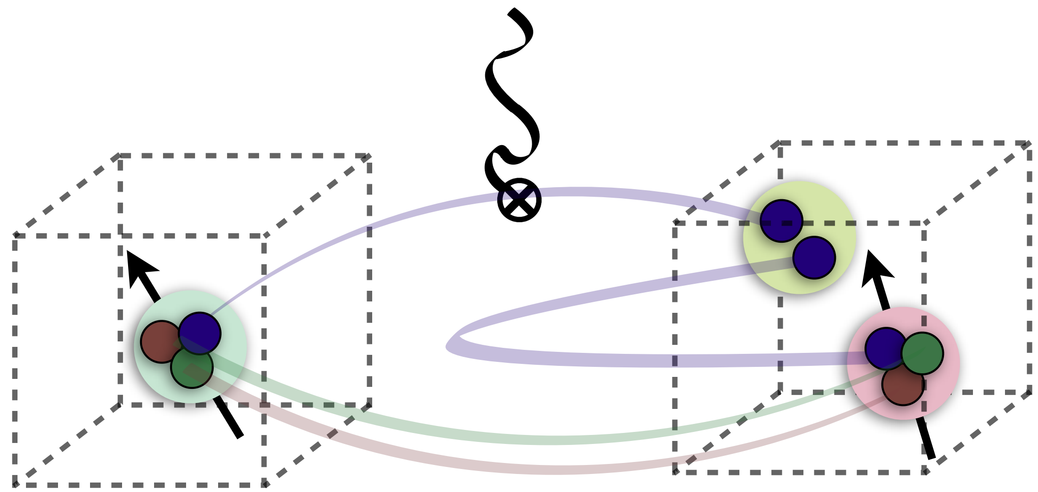
6 Out go physical transition amplitude



***à la mode de* Lellouch & Lüscher (2000)**

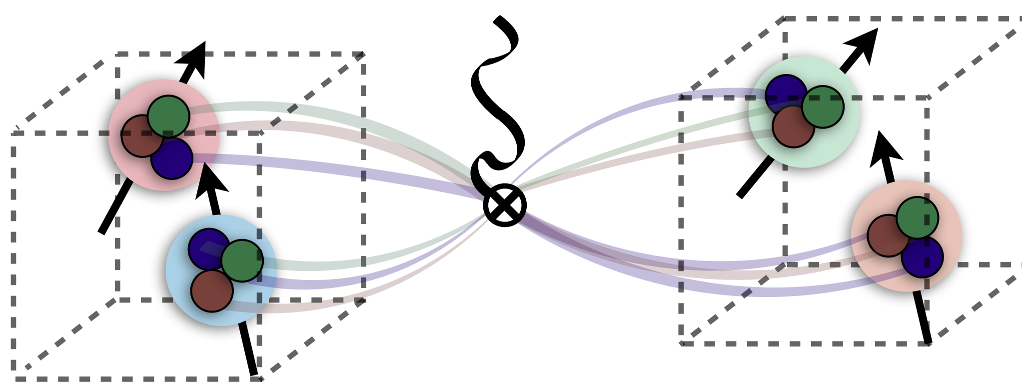
Transition amplitudes

- 1-to-2 with/without intrinsic spin:

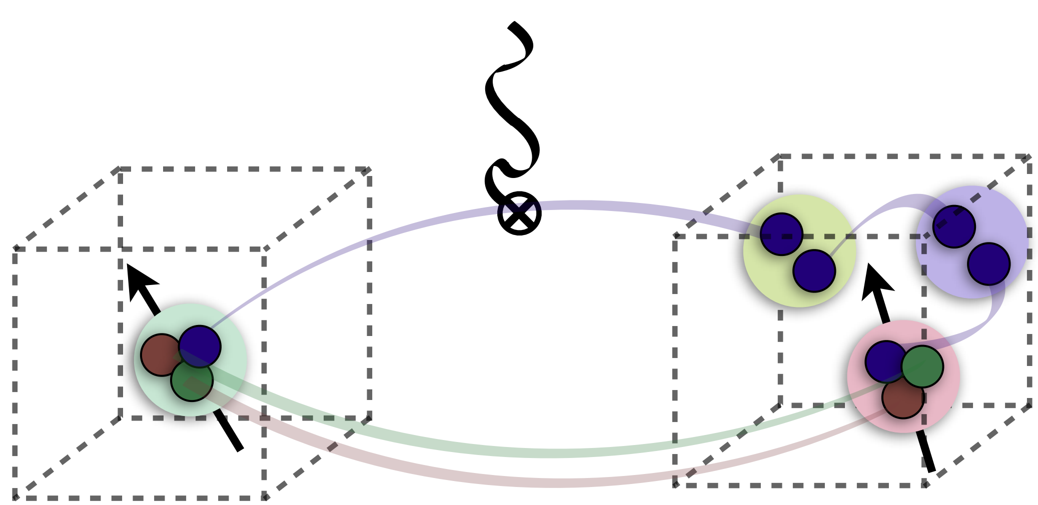


RB, Hansen & Walker-Loud (2014) / RB & Hansen [in preparation]

- 2-to-2 with intrinsic spin:

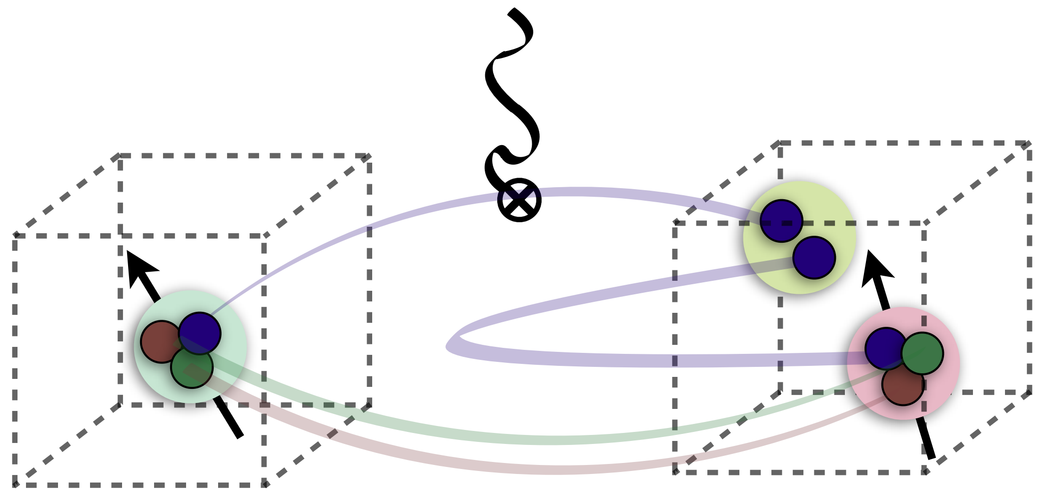


- 1-to-3 with intrinsic spin:



Transition amplitudes

- 1-to-2 with/without intrinsic spin:



RB, Hansen & Walker-Loud (2014) / RB & Hansen [in preparation]

- 2-to-2 with intrinsic spin:

- 1-to-3 with intrinsic spin:

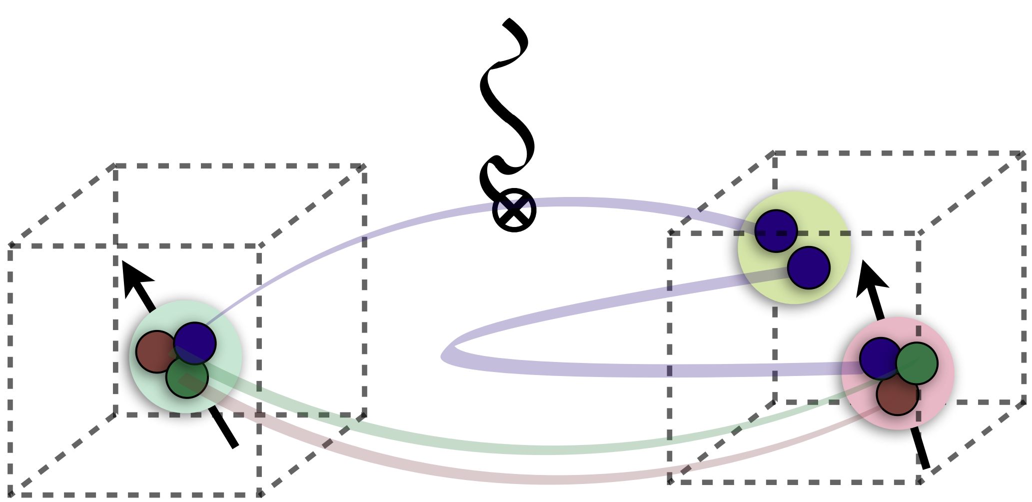
Transition Amplitudes

$$|\langle \underline{f; L}|\underline{j_\Lambda}|\underline{i; L}\rangle| = \frac{1}{\sqrt{2E_i}}\sqrt{\left[\mathcal{A}^\dagger_{\Lambda,i\to f}\ \mathcal{R}_{\Lambda,f}\ \mathcal{A}_{\Lambda,i\to f}\right]}$$

final state

initial state

external current



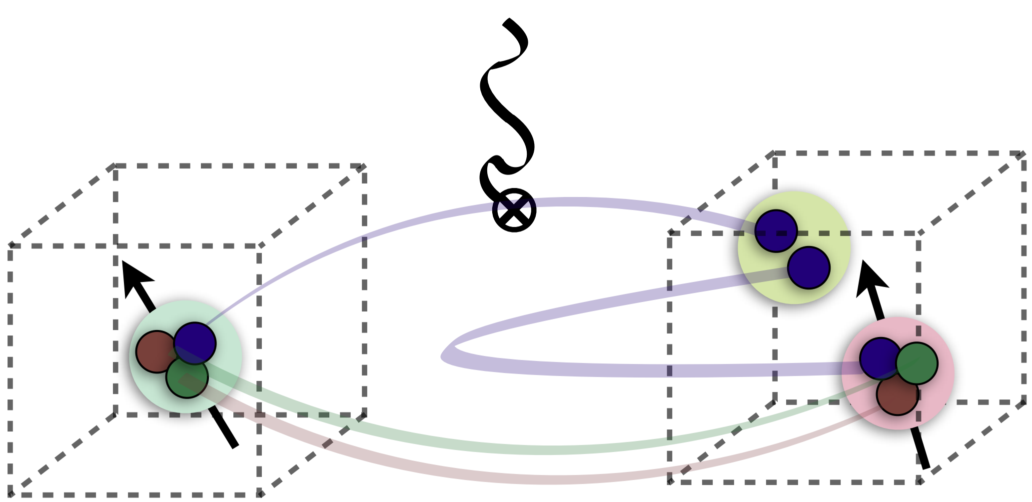
RB, Hansen & Walker-Loud (2014)

RB & Hansen [in prepation]

Transition Amplitudes

$$|\langle f;L|j_\Lambda|i;L\rangle| = \frac{1}{\sqrt{\underline{2E_i}}}\sqrt{\left[\mathcal{A}^\dagger_{\Lambda,i\to f}\ \mathcal{R}_{\Lambda,f}\ \mathcal{A}_{\Lambda,i\to f}\right]}$$

energy of initial particle



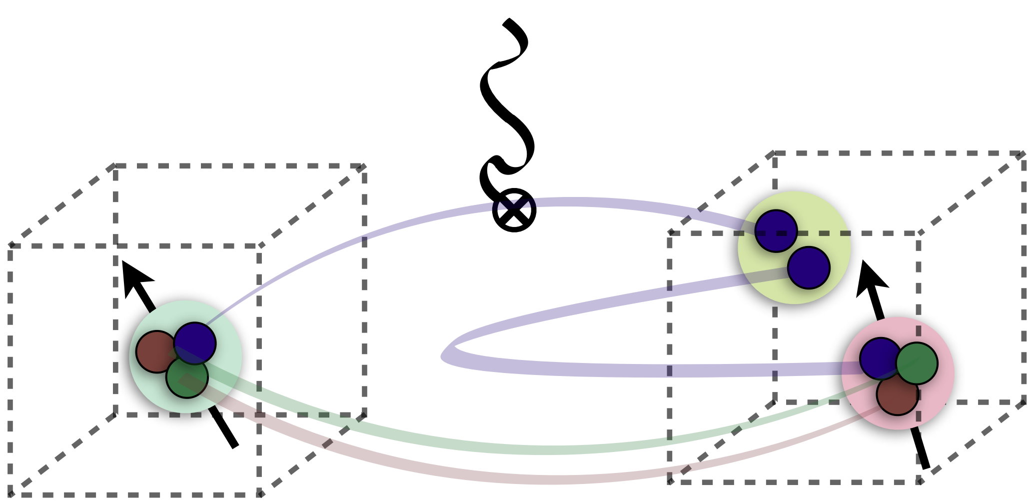
RB, Hansen & Walker-Loud (2014)

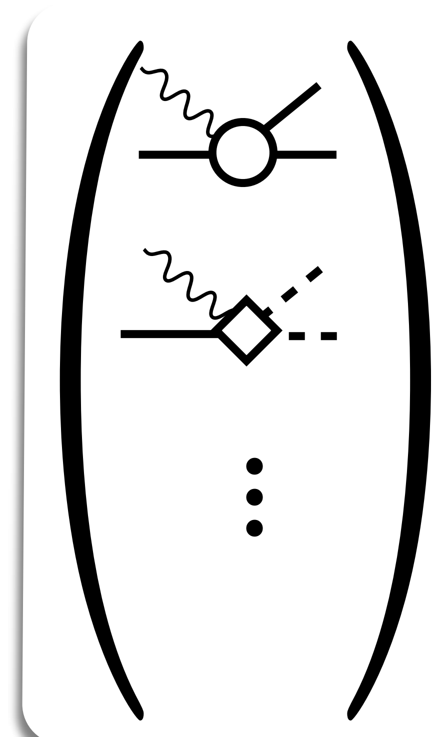
RB & Hansen [in prepation]

Transition Amplitudes

$$|\langle f; L | j_\Lambda | i; L\rangle| = \frac{1}{\sqrt{2E_i}} \sqrt{\left[\underline{\mathcal{A}^\dagger_{\Lambda,i\to f}}\ \mathcal{R}_{\Lambda,f}\ \underline{\mathcal{A}_{\Lambda,i\to f}}\right]}$$

fully dressed, on-shell infinite volume transition amplitude!





RB, Hansen & Walker-Loud (2014)

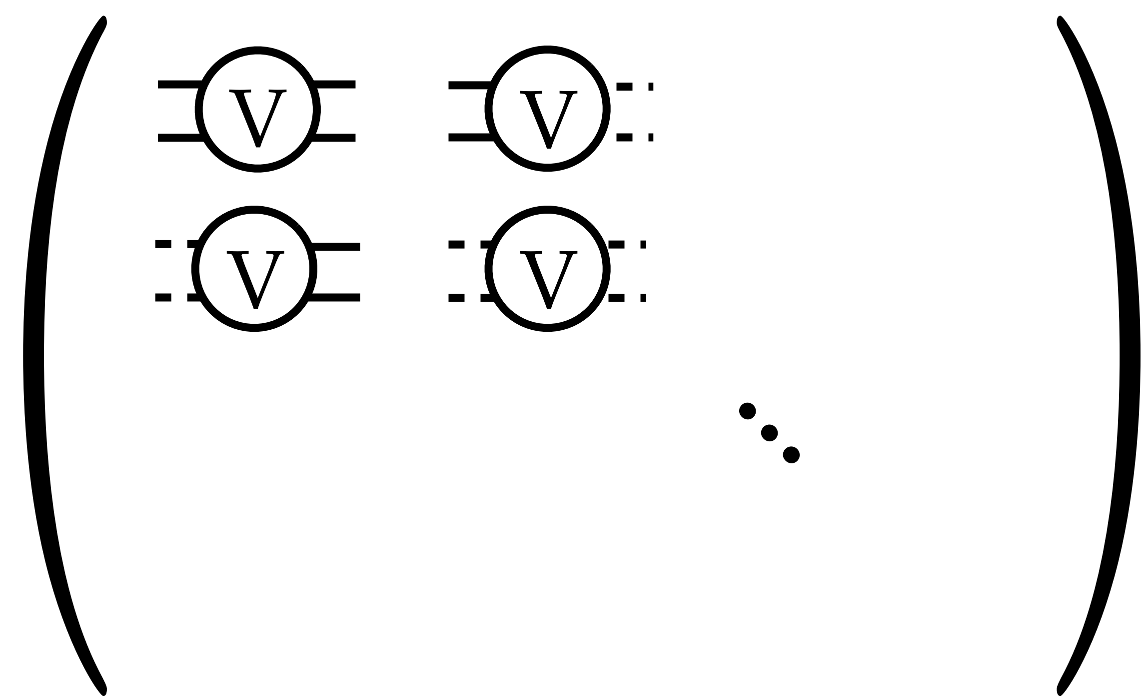
RB & Hansen [in prepation]

a vector in the space of open channels

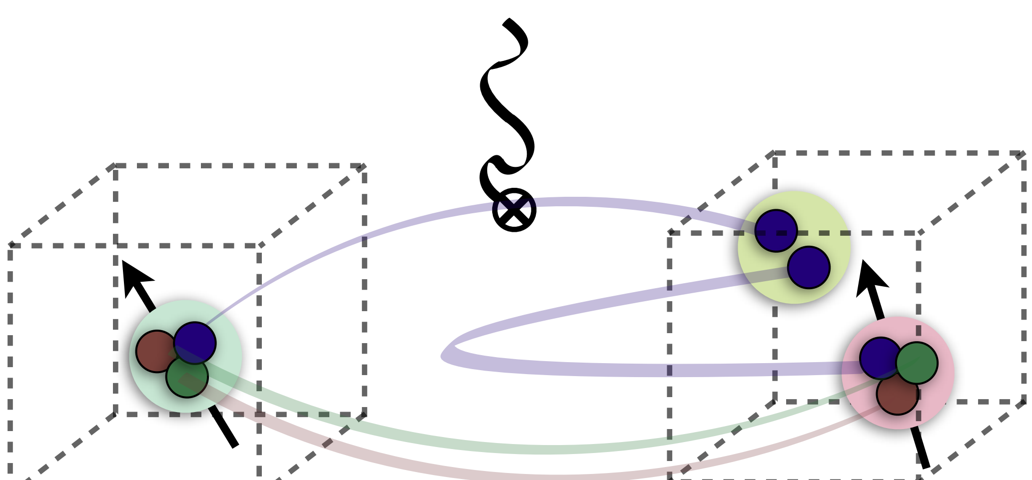
Transition Amplitudes

$$|\langle f; L | j_\Lambda | i; L\rangle| = \frac{1}{\sqrt{2E_i}} \sqrt{\left[\mathcal{A}^\dagger_{\Lambda,i\to f}\ \underline{\mathcal{R}_{\Lambda,f}}\ \mathcal{A}_{\Lambda,i\to f}\right]}$$

two-particle propagator residue, depends on energy, phase shifts and derivative of phase shifts



a matrix in the space of open channels



RB, Hansen & Walker-Loud (2014)

RB & Hansen [in prepation]

Transition Amplitudes

$$|\langle f;L|j_\Lambda|i;L\rangle| = \frac{1}{\sqrt{2E_i}}\sqrt{\left[\mathcal{A}^\dagger_{\Lambda,i\to f}\ \mathcal{R}_{\Lambda,f}\ \mathcal{A}_{\Lambda,i\to f}\right]}$$

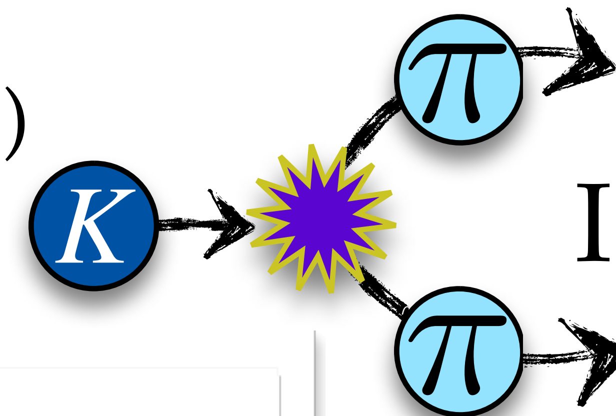
- Model independent & non-perturbative
- Universal: lattice QCD, lattice EFT, cold atoms, etc.
- Arbitrary quantum numbers for two particles
- General volumes with any boundary conditions: periodic, anti-periodic , or any linear combination on any rectangular prism

RB, Hansen & Walker-Loud (2014)

RB & Hansen [in prepation]

Examples: K-to-$\pi\pi$

First consider by Lüscher & Lellouch (2000)



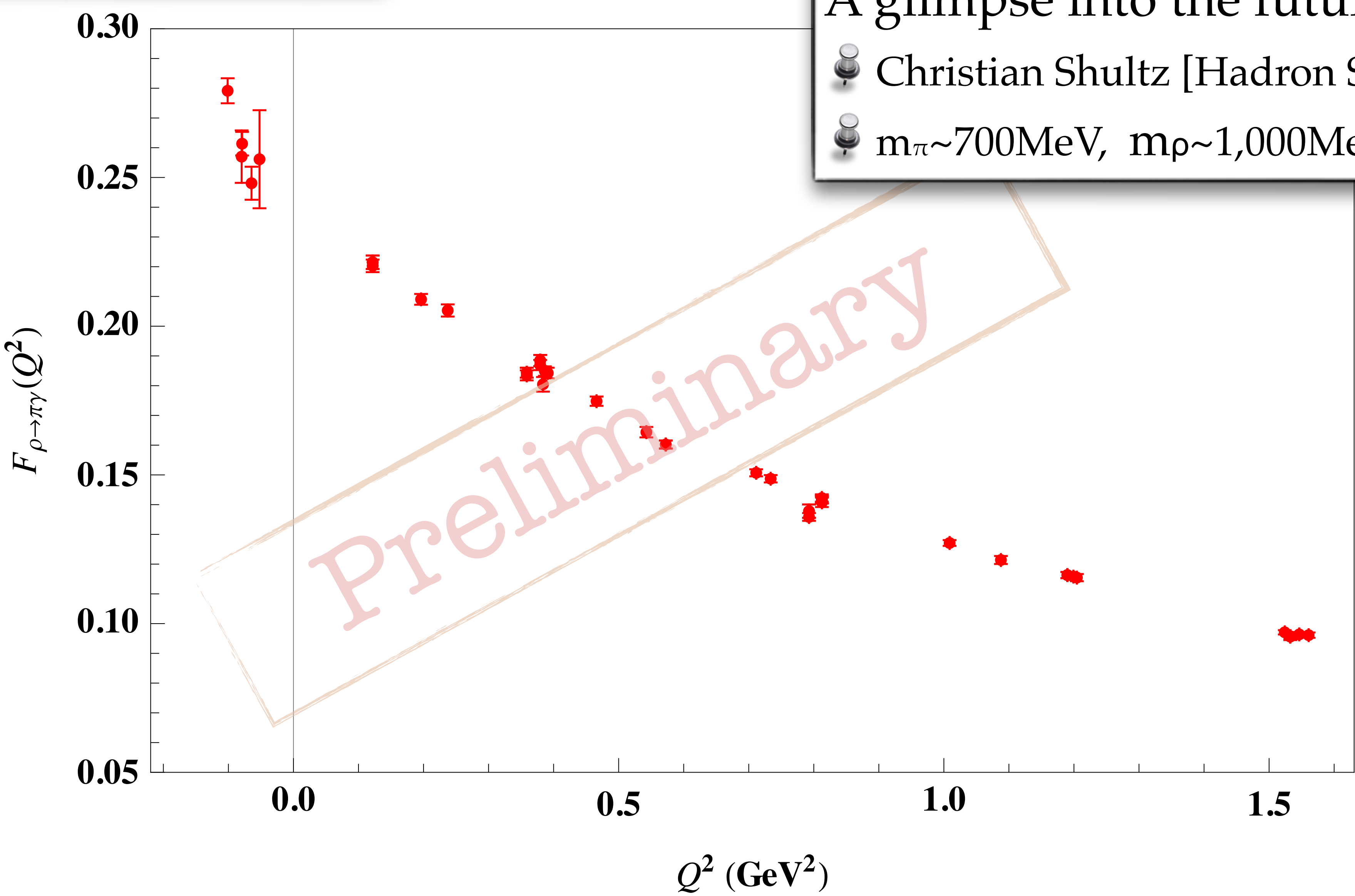
$$\frac{|\mathcal{A}_{K\to\pi\pi}|^2}{|\langle\pi\pi|\mathcal{J}|K\rangle_L|^2} = \frac{16\pi E_i\ E^*_{n_f}}{q^*_{n_f}\xi}\ \frac{\partial(\delta_S+\phi^{\mathbf{d}}_{00})}{\partial P_{0,M}}\Bigg|_{P_{0,M}=E_{n_f}}$$

	m_π[MeV]	m_K[MeV]	Re $\mathcal{A}_{I=2}$ $[10^{-8}\ GeV]$	Re $\mathcal{A}_{I=0}$ $[10^{-8}\ GeV]$	Re $\mathcal{A}_{I=0}$/Re $\mathcal{A}_{I=2}$
Lattice QCD	422(7)	878(15)	4.911(31)	45(10)	9.1(2.1)
Lattice QCD	329(6)	662(11)	2.668(14)	32.1(4.6)	12.0(1.7)
Lattice QCD	142.9(1.1)	511.3(3.9)	1.38(5)(26)	-	-
Experiment	135 - 140	494 - 498	1.479(4)	33.2(2)	22.45(6)

RBC and UKQCD Collaborations (2013)

Examples: $\pi\gamma^*$-to-$\pi\pi$

$\rho \to \pi\gamma^*$



A glimpse into the future:

- Christian Shultz [Hadron Spectrum Coll.]
- m_π~700MeV, m_ρ~1,000MeV, Stable ϱ

Status of formalism

(somewhat bias estimate)

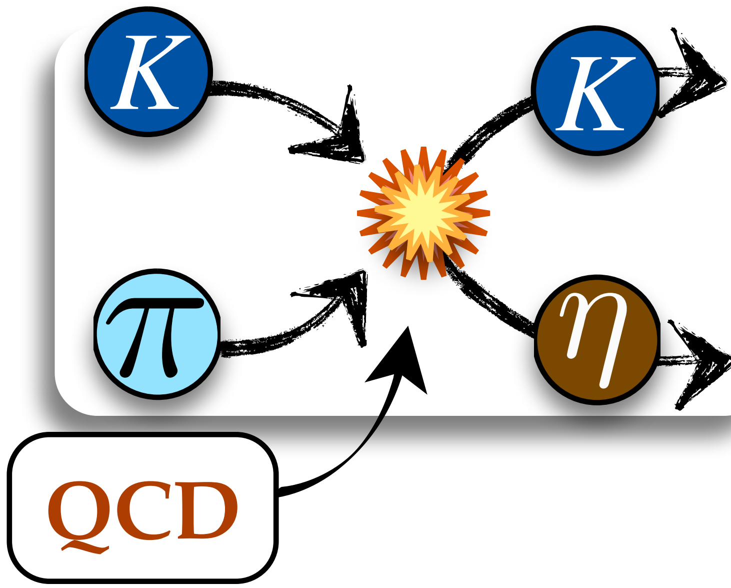
- Spectroscopy / scattering:

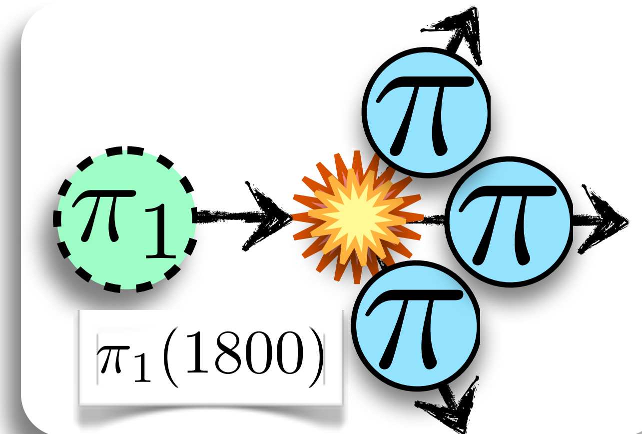
- Electromagnetic form factors:

- Fundamental symmetries:

Status of formalism
(somewhat bias estimate)

- Spectroscopy / scattering:





- Electromagnetic form factors:

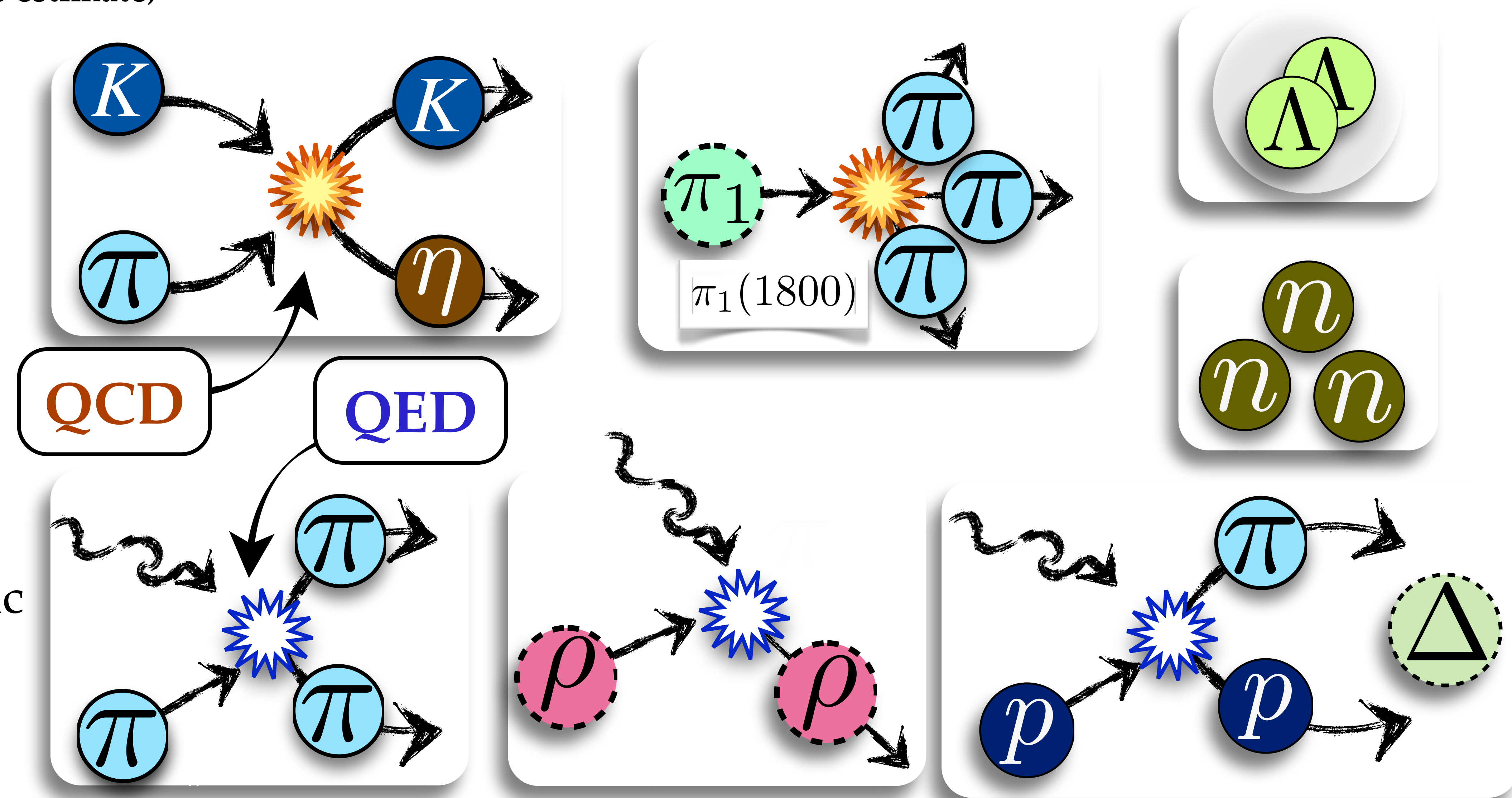
- Fundamental symmetries:

Status of formalism

(somewhat bias estimate)

- Spectroscopy / scattering:

- Electromagnetic form factors:

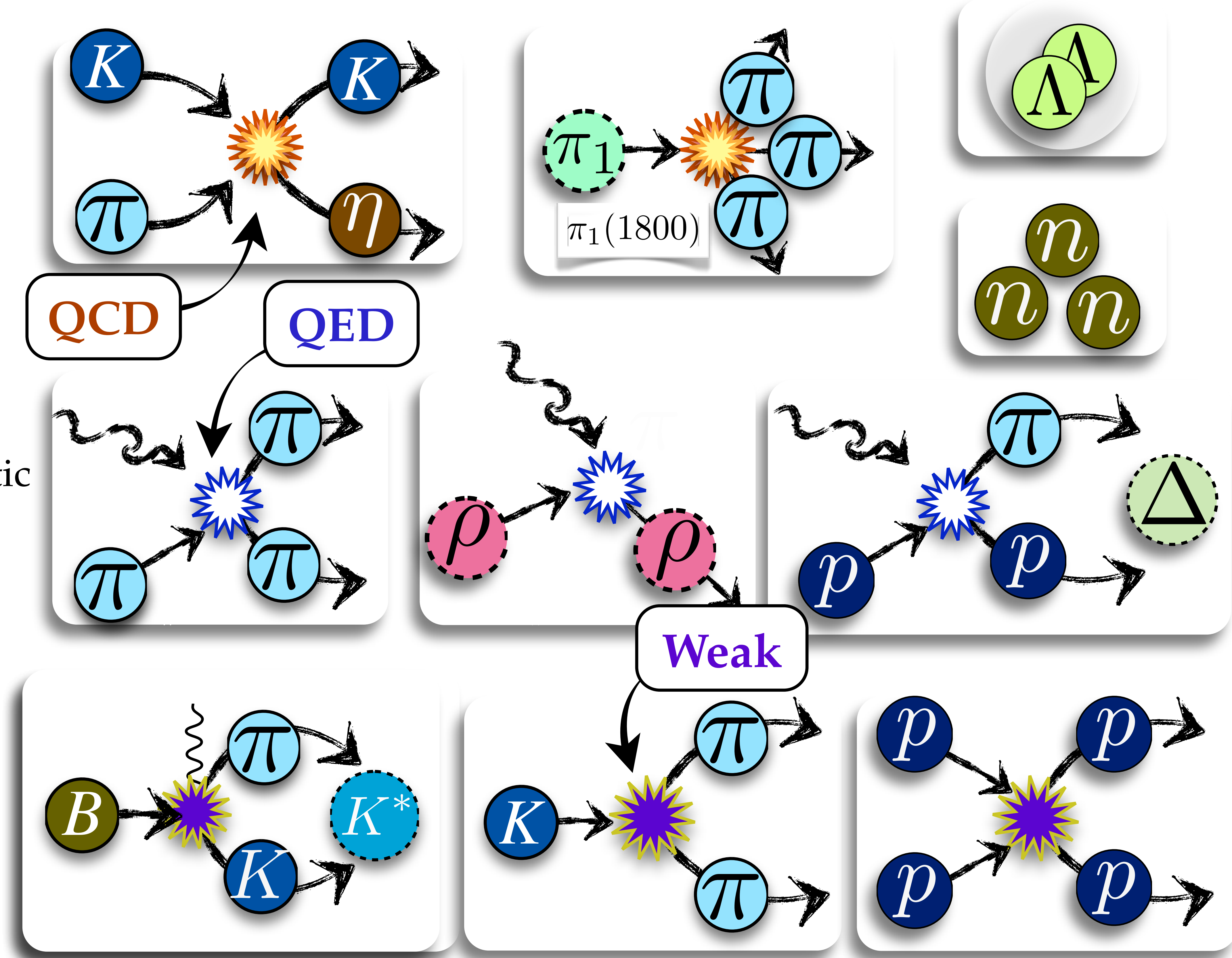


- Fundamental symmetries:

Status of formalism

(somewhat bias estimate)

- Spectroscopy / scattering:
- Electromagnetic form factors:
- Fundamental symmetries:

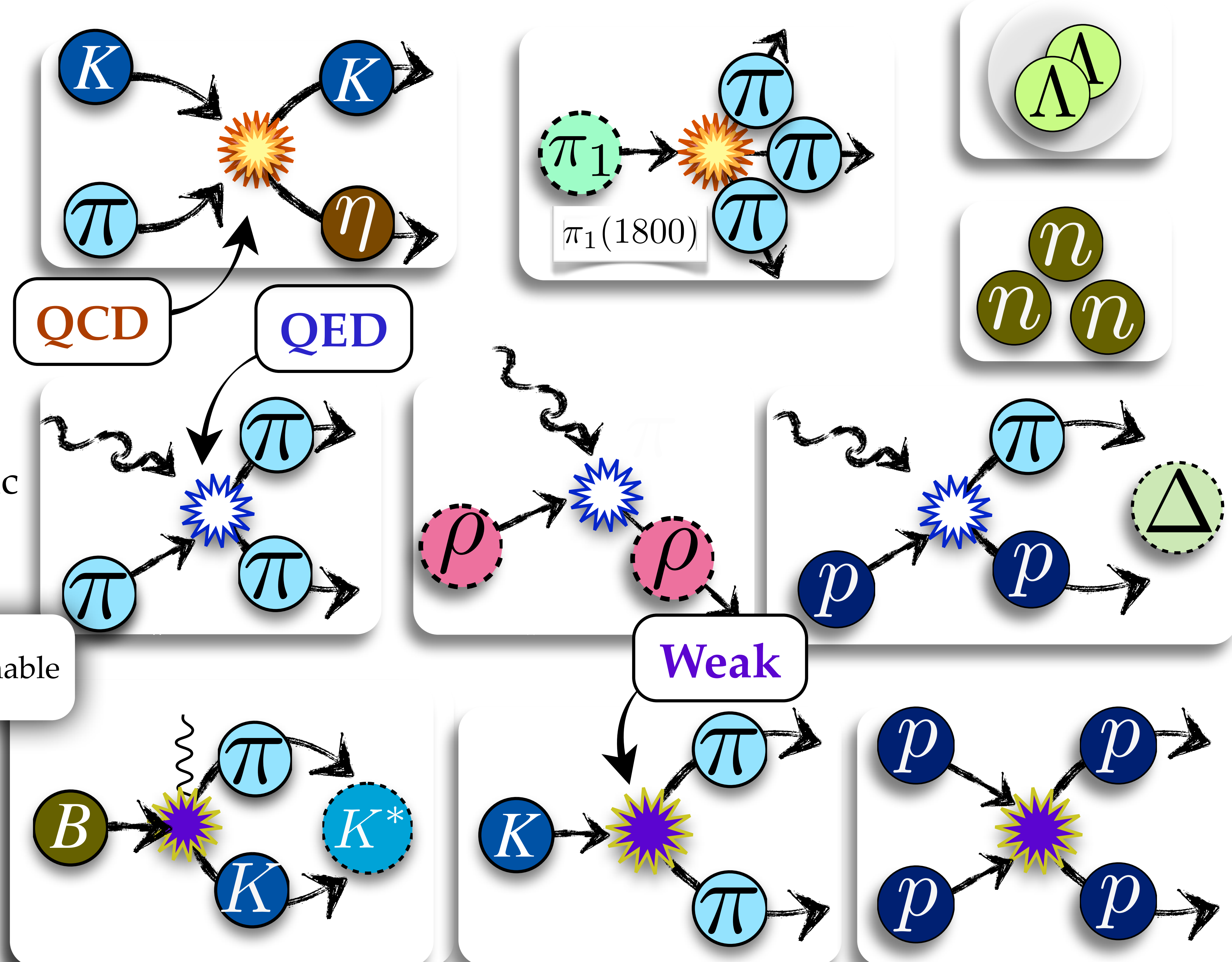


Status of formalism
(somewhat bias estimate)

- Spectroscopy / scattering:
- Electromagnetic form factors:

formally indistinguishable

- Fundamental symmetries:

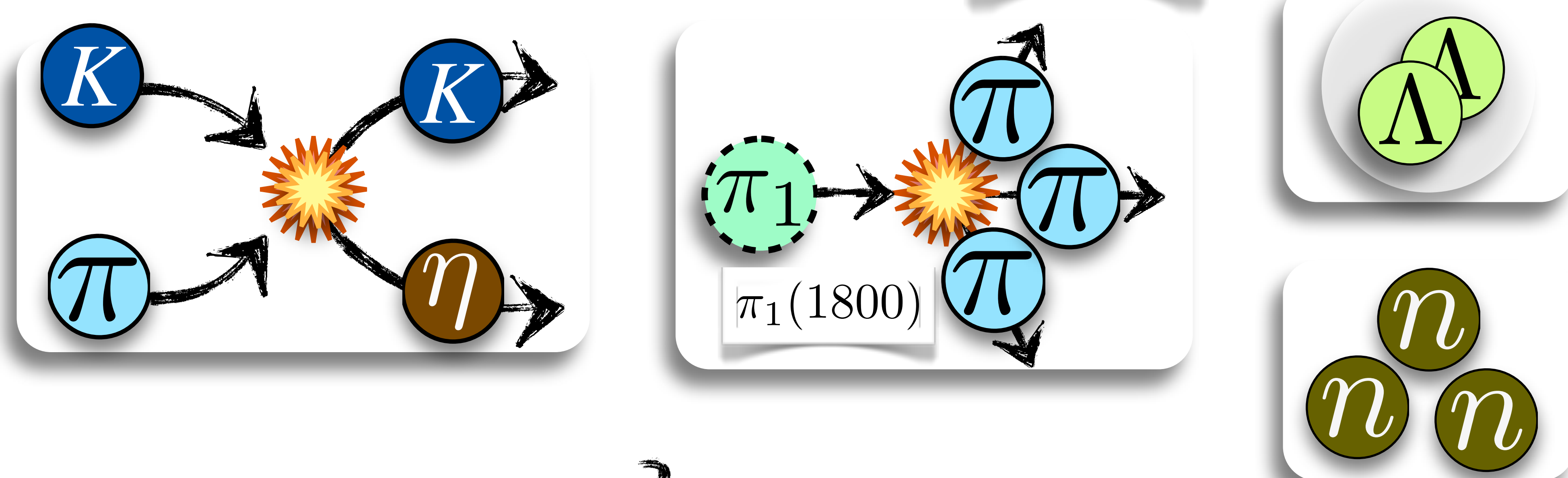


Status of formalism

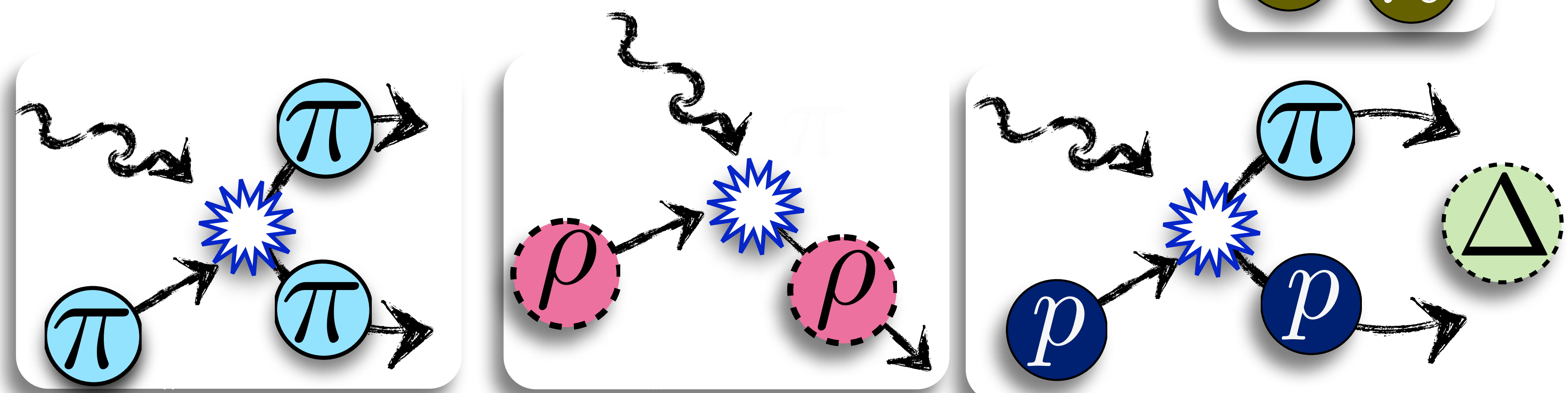
(somewhat bias estimate)

: Under control

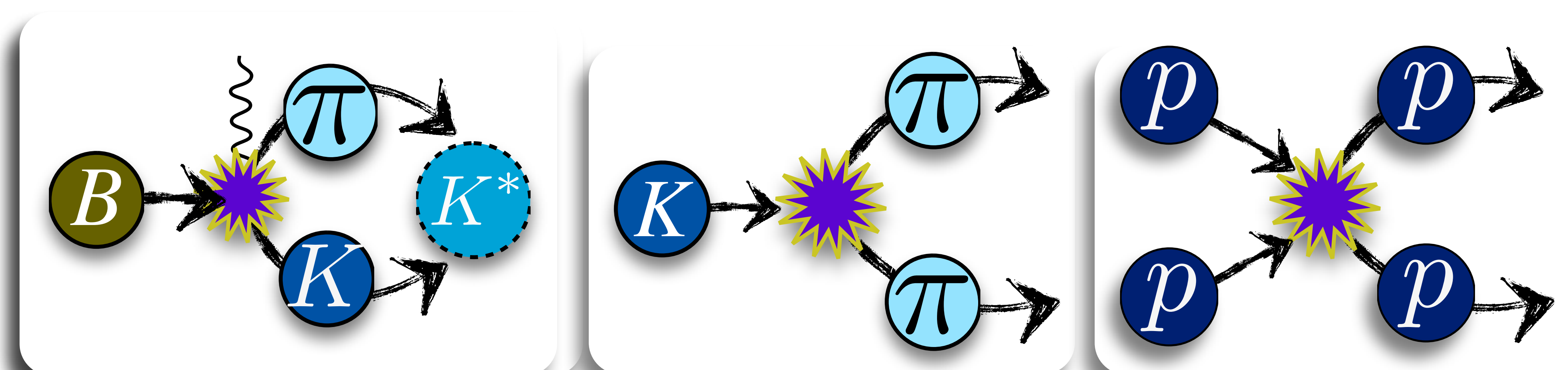
- Spectroscopy / scattering:



- Electromagnetic form factors:



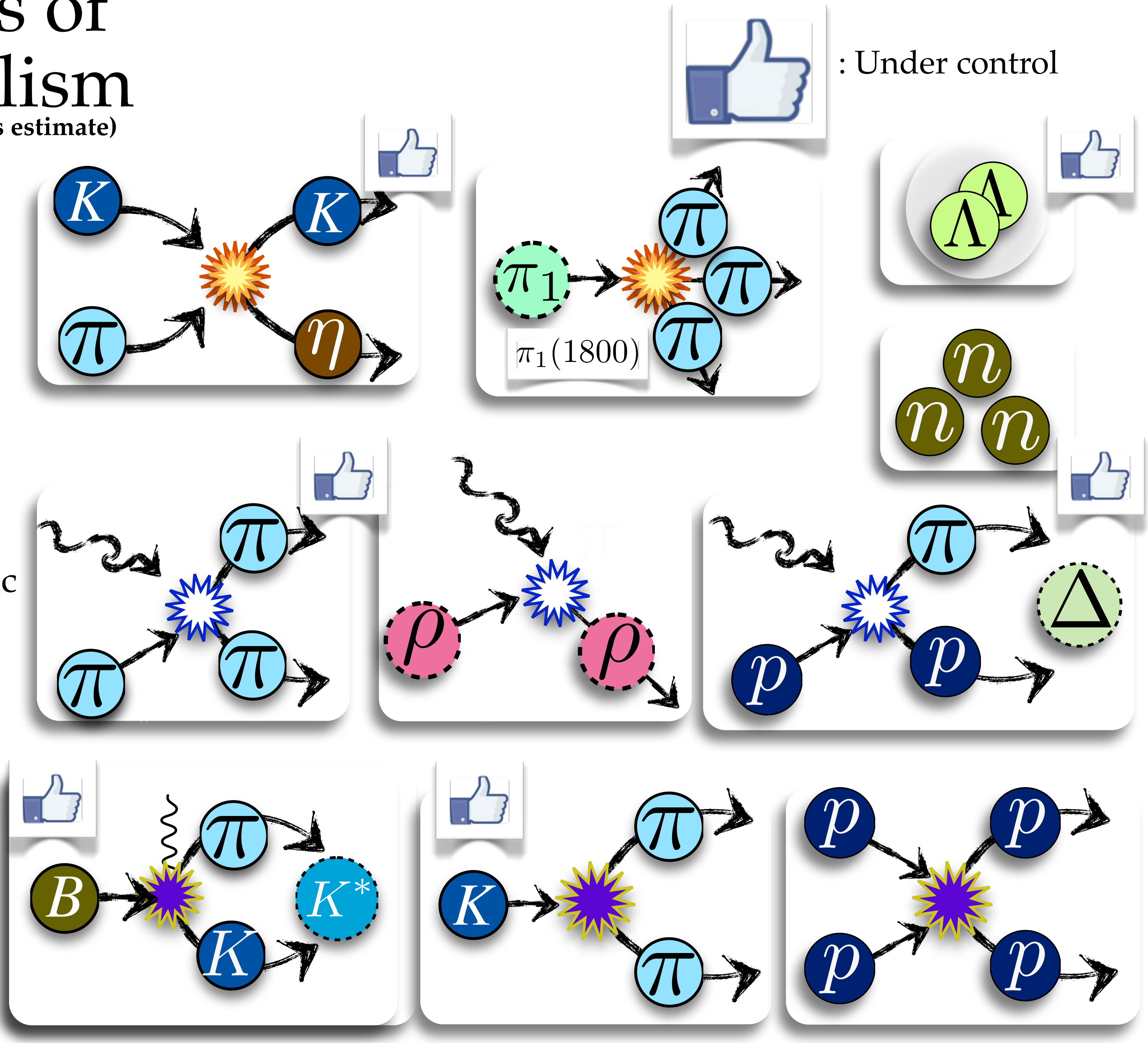
- Fundamental symmetries:



Status of formalism

(somewhat bias estimate)

- Spectroscopy / scattering:
- Electromagnetic form factors:
- Fundamental symmetries:



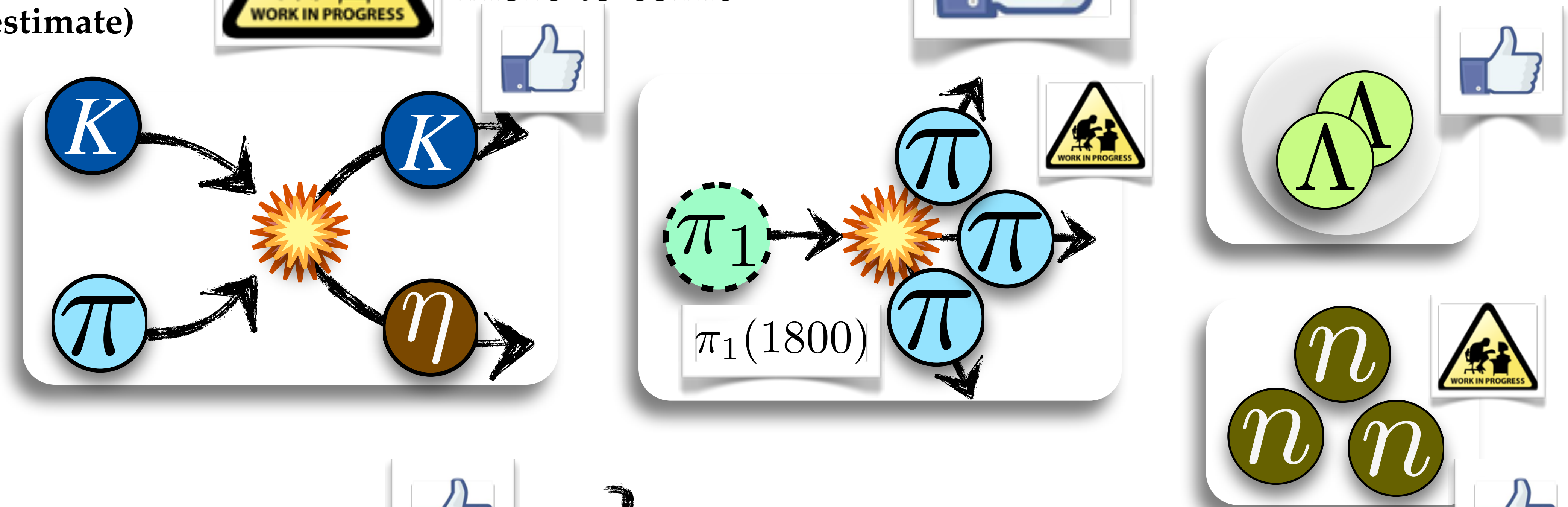
Status of formalism

(somewhat bias estimate)

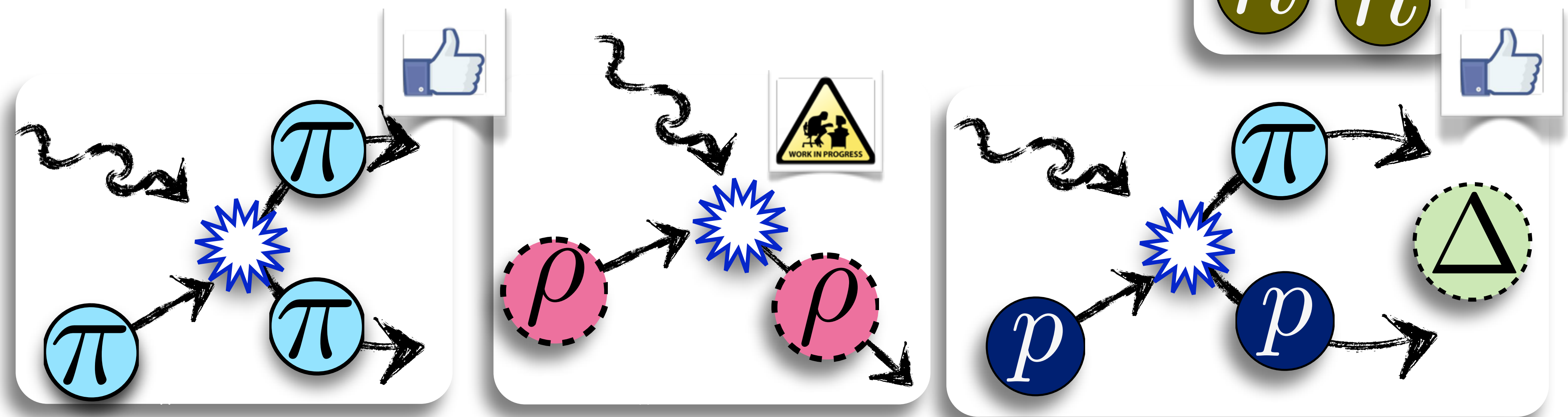
: progress made / more to come

: Under control

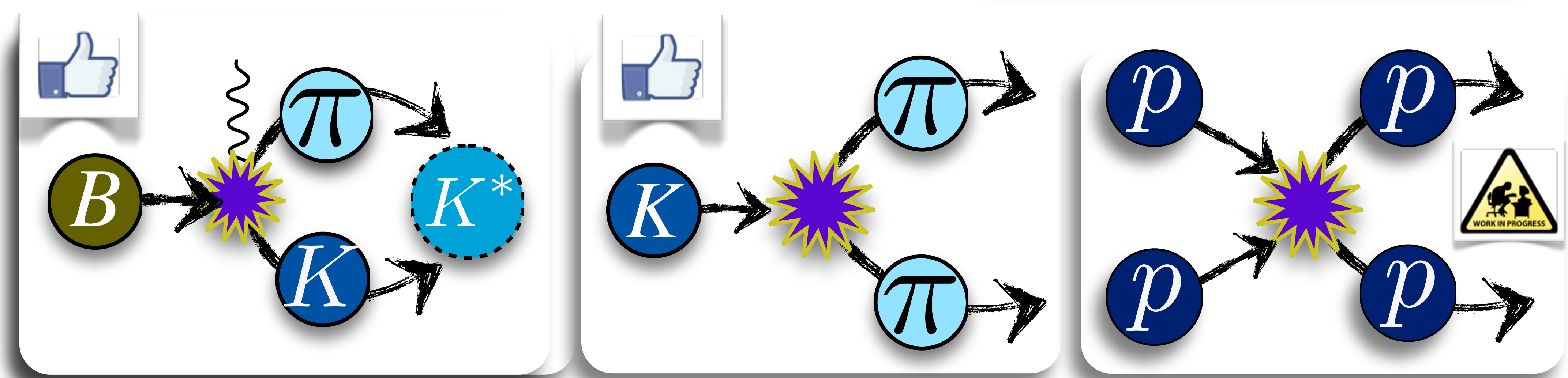
- Spectroscopy / scattering:



- Electromagnetic form factors:



- Fundamental symmetries:



3-5 year goal

- Spectroscopy / scattering:
- Electromagnetic form factors:
- Fundamental symmetries: