Spectral functions and hadron spectroscopy in lattice QCD

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Outline

Biography
  Pre-academia
  Academia

Spectral functions in lattice QCD
  at finite temperature
    - Transport and dissociation in heavy-ion collisions in the vacuum
    - HLO anomalous magnetic moment of the muon in multi hadron systems
    - Bound/Un-bound nature of the H-dibaryon

Overview
Section 1

Biography
Biography: Pre-academia

- Born in Wrexham (Wales/UK) on 26.08.1982 and moved to Lemgo (Germany) in 1985.
- Completion of schooling by obtaining the ”Abitur” in 06/2002 in Lemgo.
- Military service as medic with the German navy from 07/2002 until 04/2003.
- From 04/2004 ”Grundstudium” of physics at Bielefeld University, Germany.
- Successful completion with the ”Vordiplom” in 10/2005.
Biography: Academia

- "Hauptstudium" of physics at Bielefeld University, Germany, from 11/2005 to 10/2008. Thesis (12 months) in theoretical particle physics under the supervision of Prof. Dr. Edwin Laermann and Prof. Dr. Frithjof Karsch.

- Thesis project:
  "Improved Staggered Lattice Meson Operators"

- PhD in theoretical particle physics ("magna cum laude") from 01/2009 to 10/2011 under supervision of Prof. Dr. Edwin Laermann, Dr. Olaf Kaczmarek and Prof. Dr. Frithjof Karsch at Bielefeld University, Germany.

- Thesis title:
  "Thermal Dilepton Rates from Quenched Lattice QCD"
Biography: Academia

- Post-Doc in Mainz since 11/2011 (later affiliated to the fairly new "Helmholtz-Institut Mainz").
- Close collaboration with Prof. Dr. Harvey Meyer and Prof. Dr. Hartmut Wittig.

Further collaborations:
- Prof. Dr. Damir Becirevic at "Universite Paris Sud XI", France, on Fermilab-type heavy quarks.
- Prof. Dr. Frithjof Karsch at "Brookhaven National Laboratory", NY, USA, later also co-supervisor.
- Prof. Dr. Mikko Laine at "Bielefeld University", now "Albert Einstein Institute", Bern, Switzerland on diffusion from HQET.
Section 2

Spectral functions in lattice QCD
Spectral functions in lattice QCD

- Lattice QCD is formulated in Euclidean space-time
- For hadron spectroscopy the central observable on the lattice is the correlation function of two currents $J_\mu$:

$$G_{\mu\nu}(\tau, T) = \int d^3x \left\langle J_\mu(\tau, \vec{x}) J_\nu(0) \right\rangle$$  \hspace{1cm} (1)

⇒ directly calculable in lattice QCD computations.

- However, there is also a different representation of the correlator as integral over the spectral function $\rho(\omega)$:

$$G_{\mu\nu}(\tau, T) = \int_0^\infty d\omega \frac{\rho_{\mu\nu}(\omega, T)}{2\pi} \frac{\cosh[\omega(\beta/2 - \tau)]}{\sinh[\omega\beta/2]}$$  \hspace{1cm} (2)

⇒ only indirectly calculable via inverse Laplace-Transform.

- But, $\rho(\omega)$ is common both to Euclidean and Minkowski space-times. ⇒ in this sense it is a more ”universal” quantity.
In the case of the electromagnetic current, assuming VMD, the spectral function $\rho(\omega)$ can be linked to

$$R(s) \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{4\pi\alpha(s)^2/(3s)}$$

via the simple relation:

$$\frac{\rho(\omega)}{\omega^2} = \frac{R(\omega^2)}{12\pi^2} \quad \text{where:} \quad s = \omega^2$$

F. Jegerlehner and A. Nyffeler, Phys.Rept. 477 (2009) 1110
Subsection 2

- Transport and dissociation in heavy-ion collisions
Spectral functions at finite temperature

- At finite temperature and especially in the deconfined phase, the spf undergoes dramatic changes.
  - Dissociation of bound state particles
  - Emergence of transport phenomena

\[
\begin{align*}
\rho/\omega & \\
\text{Transport phenomena} & \\
\text{Dissociation phen.} & \\
\rho_{T>\infty}(\omega) & \\
\rho_{T=0}(\omega) & \\
\rho_{T>T_c}(\omega) & 
\end{align*}
\]
Spectral functions at finite temperature

- These phenomena have visible effects in heavy-ion collisions
  - Spf of light quarks $\Rightarrow$ Dilepton rates in the low energy regime
  - Diffusion of heavy quarks $\Rightarrow$ Elliptic flow
  - Dissociation of heavy quarkonium $\Rightarrow$ QGP thermometer

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Charmonium spf via Maximum Entropy Method (MEM)

- MEM is a Bayesian technique that computes the most probable spectral function given some input model.

- Due to the gap between the transport and particle-peak regions, MEM works well here.

- Clear information for the diffusion coefficient and the dissociation pattern of the shown $\eta_c$ can be read off the spf.

Heavy quark diffusion via lattice HQET

- In the quarkonium case the diffusion contribution can be isolated via the HQET correlator of the chromo-electric force:

\[ G_E(\tau) \sim \lim_{M \to \infty} \int d^3x \langle J_F(\tau, \vec{x}) J_F(0)^\dagger \rangle \]  

(5)

- Note: No systematic extraction of the diffusion constant, yet.
\(\sigma_{el}\) in the continuum limit of quenched QCD

- For light quarks here is no gap between the transport region and the continuous spectrum. \(\Rightarrow\) MEM is inconclusive.
- Here: Eliminate lattice effects by taking continuum limit.
- Then fit \(\sigma_{el}\) to physics constrained Ansatz.
- The fit result yields \(\rho(\omega)\) and its parameters give \(\sigma_{el}\).

\( \sigma_{el} \) in two-flavour QCD

- Much smaller lattices, continuum limit not feasible.
- But: Both \( T > T_c \) and \( T \approx 0 \) available.
- Exploit sum rule:

\[
0 \equiv \int_{-\infty}^{\infty} \frac{d\omega}{\omega} (\rho_{ii}(\omega, T) - \rho_{ii}(\omega, 0)) = \int_{-\infty}^{\infty} \frac{d\omega}{\omega} \Delta \rho(\omega, T) \quad (6)
\]

- Extract \( \sigma_{el} \) from the intercept \( \Delta \rho(\omega, T) \).

Spectral functions at finite temperature

Achievements so far

- Study of the electrical conductivity of light quarks in the continuum limit of quenched QCD.
- Study of heavy quark diffusion using HQET.
- Charmonium dissociation and diffusion via the Maximum Entropy Method.
- Extending the light quark study to dynamical ensembles and establishing the electrical conductivity also in this regime.

Future goals

- Extend also the charmonium study to the dynamical regime.
- Take the continuum limit of the HQET inspired study.
- Increase the range of available temperatures in the dynamical regime.
- Study the dissociation of the $\rho$-particle across the deconfinement phase transition.
Subsection 4

- HLO anomalous magnetic moment of the muon
Spectral functions and \((g - 2)_{\mu}\)

- The experimental observation and theoretical predictions of \((g - 2)_{\mu}\) show a discrepancy of \(\sim 3.4\sigma\).
- This computation is a precision test of the standard model.
- The leading hadronic order contribution to the anomalous magnetic moment of the muon constitutes one of the major uncertainties (along with light-by-light scattering).

<table>
<thead>
<tr>
<th></th>
<th>(a_{\mu}/10^{-11})</th>
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<tbody>
<tr>
<td>Jegerlehner, Nyffeler, PR 477 (2009)</td>
<td>116591753.7 53.1</td>
</tr>
<tr>
<td>QED incl 4-loops+LO 5-loops</td>
<td>116584718.1 0.2</td>
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<tr>
<td>weak 2-loop</td>
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<td>lead. had. VP (experimentally (e^+e^-, \tau))</td>
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<tr>
<td>light-by-light (model)</td>
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</table>

Table borrowed from A. Jüttner’s presentation at Confinement X, 2012 in Munich, Germany.
Spectral functions, $a_{\mu}^{HLO}$ and $\hat{\Pi}(Q^2)$

- We can write the leading hadronic contribution $a_{\mu}^{HLO}$ as

$$a_{\mu}^{HLO} = \left(\frac{\alpha}{\pi}\right)^2 \int dQ^2 K_{EW}(Q^2, m_\mu) \hat{\Pi}(Q^2)$$  \hspace{1cm} (7)

where $K_{EW}(Q^2, m_\mu)$ is a known electroweak kernel and $\hat{\Pi}(Q^2) = 4\pi^2[\Pi(Q^2) - \Pi(0)]$.

- The key quantity to be calculated is therefore $\hat{\Pi}(Q^2)$, which can be written in terms of $R(s)$ and consequently the spf $\rho(\omega)$

$$\hat{\Pi}(Q^2) = \frac{Q^2}{3} \int_0^\infty ds \frac{R(s)}{s(s + Q^2)} = \int_0^\infty d\omega^2 \frac{4\pi^2 Q^2 \rho(\omega^2)}{\omega^4(\omega^2 + Q^2)}$$  \hspace{1cm} (8)

- Idea: Re-write $\rho(\omega^2)$ in terms of the correlator $G(\tau)$!
A new representation of $\hat{\Pi}(Q^2)$ for lattice QCD

- Replacing $\rho(\omega^2)$ is indeed possible, the result is:

$$\hat{\Pi}(Q^2) = \int_0^\infty d\tau \ G(\tau) \left[ \tau^2 - \frac{4}{Q^2} \sin^2\left(\frac{1}{2}Q\tau\right) \right] \quad (9)$$

- This representation of $\hat{\Pi}(Q^2)$ ...
  - ... is available at any value of the virtuality $Q^2$, while only the $\vec{p} = 0$ correlator is required.
  - ... does not require an extrapolation of $\hat{\Pi}(Q^2) \to 0$, eliminating one of the largest uncertainties in current lattice results.
  - ... comes at the cost of having to extrapolate the correlator to all times $\tau \to \infty$.
  - ... however, a Lüscher-type analysis and/or highly accurate spectroscopy poses a systematic route to reduce this cost.
  - ... in principle, also a highly accurate determination of the vacuum spf via e.g. MEM could render this issue irrelevant.
First results of the mixed-representation method

- Setup: $96 \times 48^3$ lattice with $m_\pi = 324 \text{MeV}$ and $m_\pi L = 5.0$.
- Side remark: The mixed-rep. method also enables simply computing derivatives of $\hat{\Pi}(Q^2)$ by change of kernel.
Spectral functions and \((g - 2)_\mu\)

**Achievements so far**

- Development of a new representation for \(\hat{\Pi}(Q^2)\) in lattice QCD.
- Implementation and test of the new method.
- First results achieve a very good agreement with the standard method,…
- … without however having to extrapolate \(\Pi(0)\) or to use twisted-boundary conditions to boost the number of available virtualities.

**Future goals**

- Repeat the analysis on all available CLS ensembles.
- Compute \(a^{HLO}_{\mu}\) in the chiral and continuum limits.
- Develop strategy to fully control systematic uncertainties.
- Combine the two available representations to boost precision.
Subsection 6

- Bound/Un-bound nature of the H-dibaryon
Bound states in multi-baryon systems

- The study of multi-baryon systems poses a difficult challenge to lattice QCD and many interesting questions in this field are unanswered.

- One of these is the quark model prediction of a possibly stable six-quark state, the H-dibaryon (quark composition udsuds).

- Embarking on a study of the H-dibaryon, as the simplest multi-baryon system, some issues to be tackled are:
  - The signal-to-noise ratio is expected to scale as the product of those of the individual baryons.
  - The factorial growth of necessary quark contractions to form the desired system.

- We could handle part of these issues by using the newly developed, sophisticated algorithms put forward by the NPLQCD and HALQCD collaborations.
Bound states in multi-baryon systems

To this extent we implemented a "blocking"-algorithm to carry out the necessary contractions

\[ \sum_{\sigma} u \bar{u} - d \bar{d} - s \bar{s} = u \bar{u} + d \bar{d} + s \bar{s} + \cdots \]

In the next step we coded six different six-quark operators that all have overlap with the H-dibaryon in order to be able to set up a GEVP to compute the dibaryon-operator masses.

\[ X_1 Y_1 - X_2 Y_2 = \langle O_{X_1 Y_1} (t) O_{X_2 Y_2} (0) \rangle \quad (10) \]

where \( X_i Y_i \in \Lambda \Lambda; \Sigma \Sigma; \bar{N} \Xi; \)
First results of H-dibaryon masses

- Setup: 64 × 32³ lattice with $m_\pi = 451\text{MeV}$ and $m_\pi L = 4.7$.
- The GEVP results are promising,
- However, they are not yet precise enough to decide on a bound or unbound nature of the H-dibaryon in our study.
Bound states in multi-baryon systems

Achievements so far

▶ Implementation of a blocking procedure to handle multi-baryon contractions.
▶ Set-up of the necessary code for studying the H-dibaryon.
▶ Spectrum analysis via GEVP.
▶ First results look promising
▶ Still large statistical errors.

Future goals

▶ Increase statistics and reduce errors.
▶ Implement also non-relativistic operators.
▶ Go to larger, more chiral ensembles.
▶ In the farer future, go beyond the H-dibaryon and six-quark states.
Section 3

Overview
Overview

- el. conductivity
- (g-2)
- block-algo.
- H-dibaryon
- HQ diffusion
- scale setting
- Adler function
- multilevel-algo
- charmonium
- MEM
- MPI programming
- screening phen.
- ...etc
- ...and much
- ...more to come
Overview

- Oh. Conductivity
- \( g_2 \)
- Block algo.
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Thank you for your attention!