Spectral functions and hadron spectroscopy in lattice QCD

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06.01.2014

Outline

Biography

Pre-academia

Academia

Spectral functions in lattice QCD

at finite temperature

- Transport and dissociation in heavy-ion collisions

in the vacuum

- HLO anomalous magnetic moment of the muon

in multi hadron systems

- Bound/Un-bound nature of the H-dibaryon

Overview

Section 1

Biography

Biography: Pre-academia

- Born in Wrexham (Wales/UK) on 26.08.1982 and moved to Lemgo (Germany) in 1985.
- Completion of schooling by obtaining the "Abitur" in 06/2002 in Lemgo.
- Military service as medic with the German navy from 07/2002 until 04/2003.
- From 09/2003 position as Chef-du-Rang aboard "QE2" of Cunard line until 04/2004.
- From 04/2004 "Grundstudium" of physics at Bielefeld University, Germany.
- Successful completion with the "Vordiplom" in 10/2005.

Biography: Academia

- “Hauptstudium” of physics at Bielefeld University, Germany, from 11/2005 to 10/2008. Thesis (12 months) in theoretical particle physics under the supervision of Prof. Dr. Edwin Laermann and Prof. Dr. Frithjof Karsch.
- Thesis project:
  “Improved Staggered Lattice Meson Operators”

- PhD in theoretical particle physics ("magna cum laude") from 01/2009 to 10/2011 under supervision of Prof. Dr. Edwin Laermann, Dr. Olaf Kaczmarek and Prof. Dr. Frithjof Karsch at Bielefeld University, Germany.
- Thesis title:
  "Thermal Dilepton Rates from Quenched Lattice QCD"

Biography: Academia

- Post-Doc in Mainz since 11/2011 (later affiliated to the fairly new "Helmholtz-Institut Mainz").
- Close collaboration with Prof. Dr. Harvey Meyer and Prof. Dr. Hartmut Wittig.

- Further collaborations:
  - Prof. Dr. Damir Becirevic at "Universite Paris Sud XI", France, on Fermilab-type heavy quarks.
  - Prof. Dr. Frithjof Karsch at "Brookhaven National Laboratory", NY, USA, later also co-supervisor.
  - Prof. Dr. Mikko Laine at "Bielefeld University", now "Albert Einstein Institute", Bern, Switzerland on diffusion from HQET.

Section 2

Spectral functions in lattice QCD

Spectral functions in lattice QCD

- Lattice QCD is formulated in Euclidean space-time
- For hadron spectroscopy the central observable on the lattice is the correlation function of two currents J_μ:

$$G_{\mu\nu}(\tau, T) = \int d^3x \langle J_\mu(\tau, \vec{x}) J_\nu(0)^\dagger \rangle \tag{1}$$

  $\Rightarrow$ directly calculable in lattice QCD computations.
- However, there is also a different representation of the correlator as integral over the spectral function $\rho(\omega)$:

$$G_{\mu\nu}(\tau, T) = \int_0^\infty \frac{d\omega}{2\pi}\, \rho_{\mu\nu}(\omega, T)\, \frac{\cosh[\omega(\beta/2 - \tau)]}{\sinh[\omega\beta/2]} \tag{2}$$

  $\Rightarrow$ only indirectly calculable via inverse Laplace-Transform.
- But, $\rho(\omega)$ is common both to Euclidean and Minkowski space-times. $\Rightarrow$ in this sense it is a more "universal" quantity.

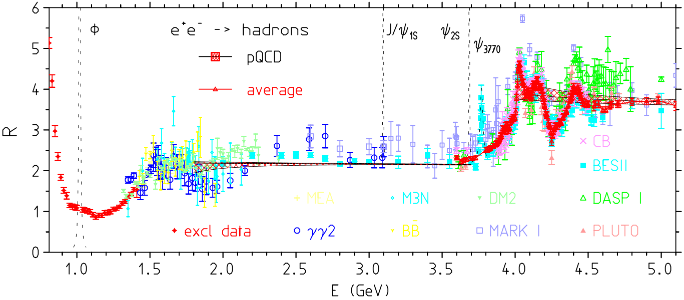
Spectral functions in lattice QCD

- In the case of the electromagnetic current, assuming VMD, the spectral function $\rho(\omega)$ can be linked to

$$R(s) \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{4\pi\alpha(s)^2/(3s)} \tag{3}$$

via the simple relation:

$$\frac{\rho(\omega)}{\omega^2} = \frac{R(\omega^2)}{12\pi^2} \quad \text{where :} \quad s = \omega^2 \tag{4}$$



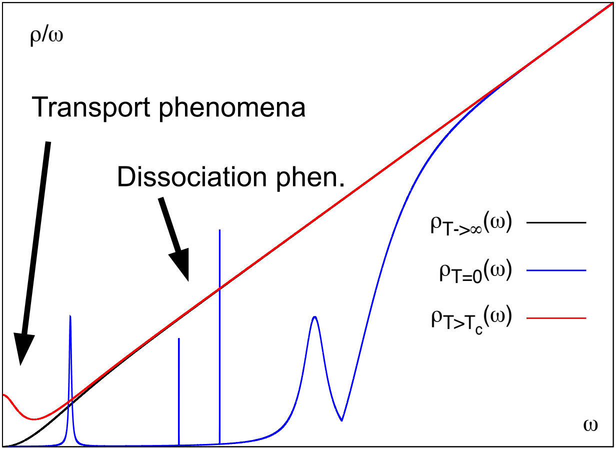
F. Jegerlehner and A. Nyffeler, Phys.Rept. 477 (2009) 1110

Subsection 2

- Transport and dissociation in heavy-ion collisions

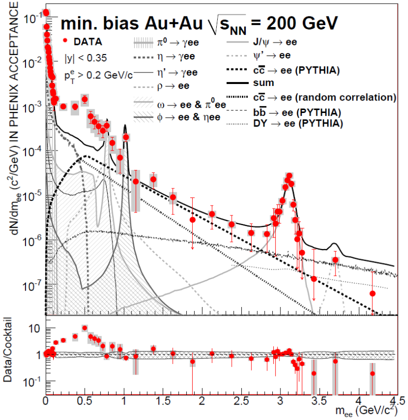
Spectral functions at finite temperature

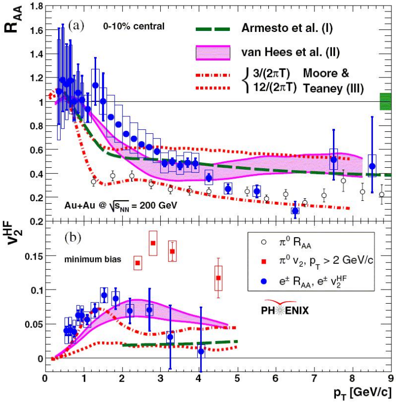
- At finite temperature and especially in the deconfined phase the spf undergoes dramatic changes.
  - Dissociation of bound state particles
  - Emergence of transport phenomena



Spectral functions at finite temperature

- These phenomena have visible effects in heavy-ion collisions
  - Spf of light quarks ⇒ Dilepton rates in the low energy regime
  - Diffusion of heavy quarks ⇒ Elliptic flow
  - Dissociation of heavy quarkonium ⇒ QGP thermometer

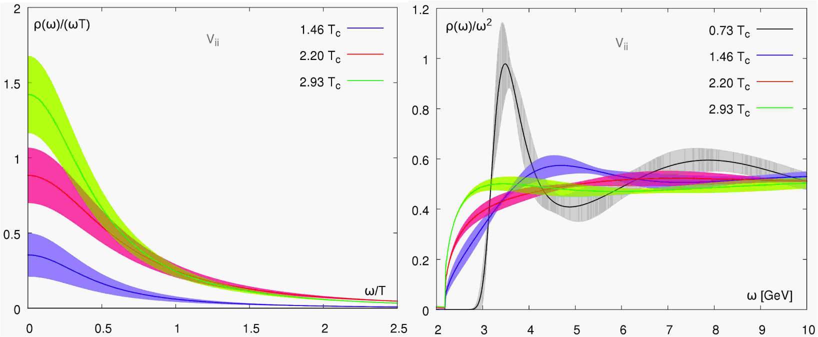




Adare et al.; Phys.Rev. C84 (2011) 044905

Charmonium spf via Maximum Entropy Method (MEM)

- MEM is a Bayesian technique that computes the most probable spectral function given some input model.



- Due to the gap between the transport and particle-peak regions, MEM works well here.
- Clear information for the diffusion coefficient and the dissociation pattern of the shown η_c can be read off the spf.

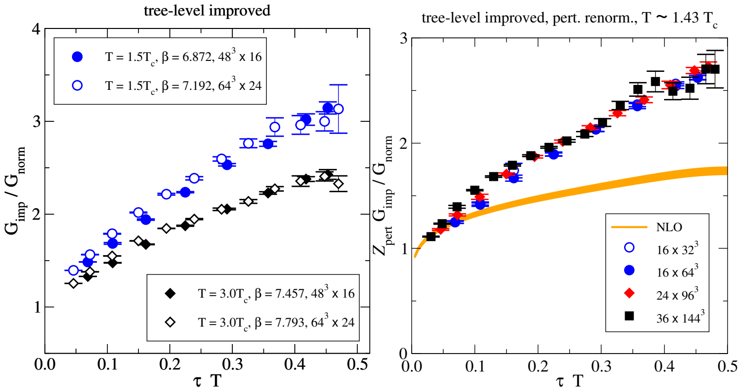
H.T.Ding, A.F., et al.; Phys.Rev. D86 (2012) 014509

Heavy quark diffusion via lattice HQET

- In the quarkonium case the diffusion contribution can be isolated via the HQET correlator of the chromo-electric force:

$$G_E(\tau) \sim \lim_{M\to\infty} \int d^3x \langle J_F(\tau,\vec{x}) J_F(0)^\dagger \rangle \tag{5}$$

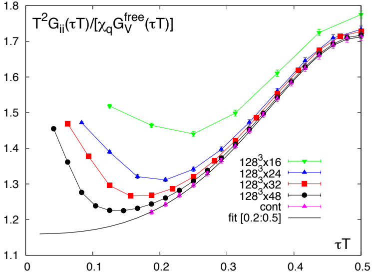
- Note: No systematic extraction of the diffusion constant, yet.

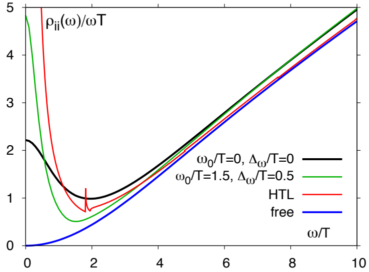


A.F., M. Laine, et al.; PoS LATTICE2011 (2011) 202

σ_{el} in the continuum limit of quenched QCD

- For light quarks here is no gap between the transport region and the continuous spectrum. $\Rightarrow$ MEM is inconclusive.
- Here: Eliminate lattice effects by taking continuum limit.
- Then fit σ_{el} to physics constrained Ansatz.
- The fit result yields $\rho(\omega)$ and its parameters give σ_{el}.





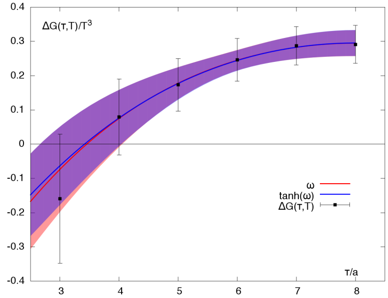
H.T.Ding, A.F., et al.; Phys.Rev. D83 (2011) 034504

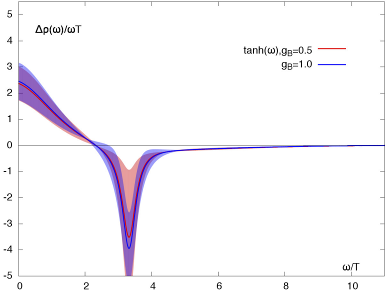
σ_{el} in two-flavour QCD

- Much smaller lattices, continuum limit not feasible.
- But: Both $T > T_c$ and $T \simeq 0$ available.
- Exploit sum rule:

$$0 \equiv \int_{-\infty}^{\infty} \frac{d\omega}{\omega}(\rho_{ii}(\omega, T) - \rho_{ii}(\omega, 0)) = \int_{-\infty}^{\infty} \frac{d\omega}{\omega} \Delta\rho(\omega, T) \quad (6)$$

- Extract σ_{el} from the intercept $\Delta\rho(\omega, T)$.





B.B.Brandt, A.F., et al.; JHEP 1303 (2013) 100

Spectral functions at finite temperature

Achievements so far

- Study of the electrical conductivity of light quarks in the continuum limit of quenched QCD.
- Study of heavy quark diffusion using HQET.
- Charmonium dissociation and diffusion via the Maximum Entropy Method.
- Extending the light quark study to dynamical ensembles and establishing the electrical conductivity also in this regime.

Future goals

- Extend also the charmonium study to the dynamical regime.
- Take the continuum limit of the HQET inspired study.
- Increase the range of available temperatures in the dynamical regime.
- Study the dissociation of the ρ-particle accross the deconfinement phase transition.

Subsection 4

- HLO anomalous magnetic moment of the muon

Spectral functions and $(g-2)_\mu$

- The experimental observation and theoretical predictions of $(g-2)_\mu$ show a dicrepancy of $\sim 3.4\sigma$
- This computation is a precision test of the standard model.
- The leading hadronic order contribution to the anomalous magnetic moment of the muon constitutes one of the major uncertainties (along with light-by-light scattering).

	$a_\mu/10^{-11}$	
Jegerlehner, Nyffeler, PR 477 (2009) *Benayoun, Eur. Phys. J. C (2012) 72:1848*	116591753.7	53.1
QED incl 4-loops+LO 5-loops	116584718.1	0.2
weak 2-loop	153.2	1.8
lead. had. VP (experimentally e^+e^-, τ)	6877.2	46.3
light-by-light (model)	105.0	26.0

Table borrowed from A. Jüttner's presentation at Confinement X, 2012 in Munich, Germany.

Spectral functions, a_μ^{HLO} and $\hat{\Pi}(Q^2)$

- We can write the leading hadronic contribution a_μ^{HLO} as

$$a_\mu^{HLO} = \left(\frac{\alpha}{\pi}\right)^2 \int dQ^2 \, K_{EW}(Q^2, m_\mu) \, \hat{\Pi}(Q^2) \qquad (7)$$

  where $K_{EW}(Q^2, m_\mu)$ is a known electroweak kernel and $\hat{\Pi}(Q^2) = 4\pi^2[\Pi(Q^2) - \Pi(0)]$.

- The key quantity to be calculated is therefore $\hat{\Pi}(Q^2)$, which can be written in terms of $R(s)$ and consequently the spf $\rho(\omega)$

$$\hat{\Pi}(Q^2) = \frac{Q^2}{3} \int_0^\infty ds \frac{R(s)}{s(s+Q^2)} = \int_0^\infty d\omega^2 \frac{4\pi^2 Q^2 \rho(\omega^2)}{\omega^4(\omega^2+Q^2)} \qquad (8)$$

- Idea: Re-write $\rho(\omega^2)$ in terms of the correlator $G(\tau)$!

A new representation of $\hat{\Pi}(Q^2)$ for lattice QCD

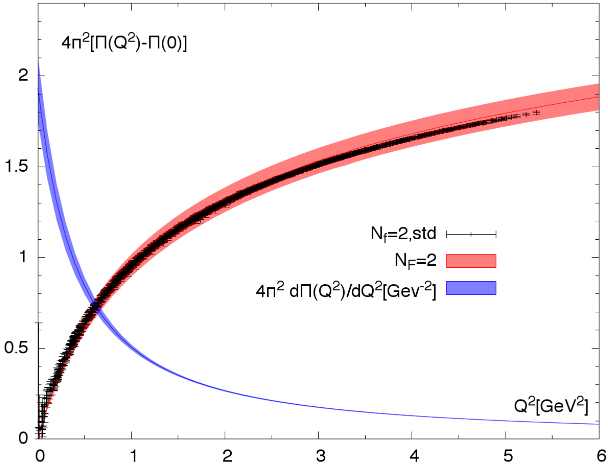
- Replacing $\rho(\omega^2)$ is indeed possible, the result is:

$$\hat{\Pi}(Q^2) = \int_0^\infty d\tau\, G(\tau)\,[\tau^2 - \frac{4}{Q^2}\sin^2(\frac{1}{2}Q\tau)] \qquad (9)$$

- This representation of $\hat{\Pi}(Q^2)$ ...
  - ... is available at any value of the virtuality Q^2, while only the $\vec{p} = 0$ correlator is required.
  - ... does not require an extrapolation of $\hat{\Pi}(Q^2) \to 0$, eliminating one of the largest uncertainties in current lattice results.
  - ... comes at the cost of having to extrapolate the correlator to all times $\tau \to \infty$.
  - ... however, a Lüscher-type analysis and/or highly accurate spectroscopy poses a systematic route to reduce this cost.
  - ... in principle, also a highly accurate determination of the vacuum spf via e.g. MEM could render this issue irrelevant.

First results of the mixed-representation method

- Setup: 96×48^3 lattice with $m_\pi = 324 MeV$ and $m_\pi L = 5.0$.
- Side remark: The mixed-rep. method also enables simply computing derivatives of $\hat{\Pi}(Q^2)$ by change of kernel.



A.F. et al., Phys.Rev. D88 (2013) 054502

Spectral functions and $(g-2)_\mu$

Achievements so far

- Development of a new representation for $\hat{\Pi}(Q^2)$ in lattice QCD.
- Implementation and test of the new method.
- First results achieve a very good agreement with the standard method,...
- ... without however having to extrapolate $\Pi(0)$ or to use twisted-boundary conditions to boost the number of available virtualities.

Future goals

- Repeat the analysis on all available CLS ensembles.
- Compute a_μ^{HLO} in the chiral and continuum limits.
- Develop strategy to fully control systematic uncertainties.
- Combine the two available representations to boost precision.

Subsection 6

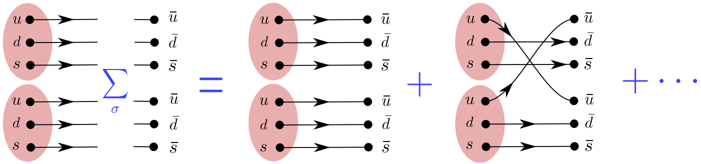
- Bound/Un-bound nature of the H-dibaryon

Bound states in multi-baryon systems

- The study of multi-baryon systems poses a difficult challenge to lattice QCD and many interesting questions in this field are unanswered.
- One of these is the quark model prediction of a possibly stable six-quark state, the H-dibaryon (quark composition udsuds).
- Embarking on a study of the H-dibaryon, as the simplest multi-baryon system, some issues to be tackled are:
  - The signal-to-noise ratio is expected to scale as the product of those of the individual baryons.
  - The factorial growth of necessary quark contractions to form the desired system.
- We could handle part of these issues by using the newly developed, sophisticated algorithms put forward by the NPLQCD and HALQCD collaborations.

Bound states in multi-baryon systems

- To this extent we implemented a "blocking"-algorithm to carry out the necessary contractions



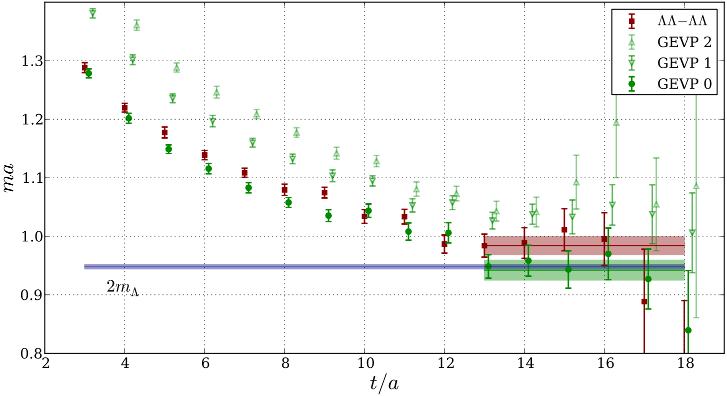
- In the next step we coded six different six-quark operators that all have overlap with the H-dibaryon in order to be able to set up a GEVP to compute the dibaryon-operator masses.

$$X_1 Y_1 - X_2 Y_2 = \langle O_{X_1 Y_1}(t) O_{X_2 Y_2}(0) \rangle \qquad (10)$$

where $$X_i Y_i \in \Lambda\Lambda;\ \Sigma\Sigma;\ N\Xi;$$

First results of H-dibaryon masses

- Setup: 64×32^3 lattice with $m_\pi = 451 MeV$ and $m_\pi L = 4.7$.
- The GEVP results are promising,
- However, they are not yet precise enough to decide on a bound or unbound nature of the H-dibaryon in our study.



A. F., C. Miao, T. D. Rae, H. Wittig; PoS LATTICE2013 (2013) 440

Bound states in multi-baryon systems

Achievements so far

- Implementation of a blocking procedure to handle multi-baryon contractions.
- Set-up of the necessary code for studying the H-dibaryon.
- Spectrum analysis via GEVP.
- First results look promising
- Still large statistical errors.

Future goals

- Increase statistics and reduce errors.
- Implement also non-relativistic operators.
- Go to larger, more chiral ensembles.
- In the farer future, go beyond the H-dibaryon and six-quark states.

Section 3

Overview

Overview

el. conductivity	(g-2)	block-algo.
H-dibaryon	HQ diffusion	scale setting
Adler function	multilevel-algo	charmonium
MEM	MPI programming	screening phen.
...etc	...and much	...more to come

Overview

el. conductivity	(g-2)	block-algo.
H-dibaryon	HQ diffusion	scale setting
Adler functio…	[illegible]	…harmonium
MEM	MPI programming	screening phen.
...etc	...and much	...more to come

Thank you for your attention!