Strong interactions for the precision frontier in particle and nuclear physics

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Colloquium
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Outline

Introduction: why the strong interactions are difficult

Strong and weak interactions
  • Spectroscopy, CP violation, and Dalitz plots

Strong interactions and electromagnetism
  • The anomalous magnetic moment of the muon

Summary / Outlook
### The Standard Model of particle physics

#### Matter particles:
- **Quarks and leptons**

#### Force carriers:
- **Gauge bosons**

#### How did we find these?
- How to find something beyond?
The Standard Model of particle physics

<table>
<thead>
<tr>
<th>Quark</th>
<th>Mass (MeV/c²)</th>
<th>Charge</th>
<th>Spin</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>2.4</td>
<td>2/3</td>
<td>1/2</td>
<td>up</td>
</tr>
<tr>
<td>d</td>
<td>4.8</td>
<td>2/3</td>
<td>1/2</td>
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</tr>
<tr>
<td>c</td>
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<td>0</td>
<td>1/2</td>
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</tr>
<tr>
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<td>2/3</td>
<td>1/2</td>
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<tr>
<td>t</td>
<td>171.2</td>
<td>0</td>
<td>1/2</td>
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<td>b</td>
<td>4.2</td>
<td>2/3</td>
<td>1/2</td>
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<td>g</td>
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<td>0</td>
<td>1</td>
<td>gluon</td>
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<tr>
<td>H*</td>
<td>~125</td>
<td>0</td>
<td>0</td>
<td>Higgs boson</td>
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<table>
<thead>
<tr>
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<th>Spin</th>
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<tbody>
<tr>
<td>e</td>
<td>0.511</td>
<td>0</td>
<td>1/2</td>
<td>electron</td>
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<tr>
<td>μ</td>
<td>105.7</td>
<td>0</td>
<td>1/2</td>
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<tr>
<td>τ</td>
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<td>0</td>
<td>1/2</td>
<td>tau</td>
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<tr>
<td>νe</td>
<td>~2.2</td>
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<tr>
<td>νμ</td>
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<td>ντ</td>
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<tr>
<th>Gauge boson</th>
<th>Mass (GeV/c²)</th>
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<th>Spin</th>
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<tr>
<td>Z⁰</td>
<td>91.2</td>
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<td>W⁺/W⁻</td>
<td>~86.4</td>
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- matter particle: quarks and leptons
- force carriers: gauge bosons
- how did we find these? how to find something beyond?

- discoveries at high energies: production of new particles
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- **Matter particle:** quarks and leptons
- **Force carriers:** gauge bosons
- **How did we find these?** how to find something beyond?

- Discoveries at high energies: production of new particles
- Discoveries in precision experiments: effects of virtual particles

![Diagram of the Standard Model](image-url)

- mass, charge, spin, name
- e, μ, τ: electron, muon, tau
- u, c, t: up, charm, top
- H*: Higgs boson
- Z°: Z boson
- W±: W boson
- ν: neutrino
- d: down
- s: strange
- b: bottom
- g: gluon
- γ: photon

B. Kubis, Strong interactions for the precision frontier in particle and nuclear physics – p. 3
Why the strong interactions are difficult

Perturbation theory and coupling constants

- expand amplitudes in powers of coupling constants $\alpha$

scattering amplitude $\propto \alpha(\ldots)$
Why the strong interactions are difficult

Perturbation theory and coupling constants

- expand amplitudes in powers of coupling constants $\alpha$

  scattering amplitude $\propto \alpha(\ldots) + \alpha^2(\ldots)$

- works well for electromagnetic and weak interactions: $\alpha \approx 10^{-2}$
Why the strong interactions are difficult

The running coupling constant in Quantum Chromodynamics

- asymptotic freedom at high energies ("weak QCD")
- confinement at low energies ("strong QCD"):

  no quarks + gluons, only (colour-neutral) hadrons
  baryons ($r g b$) + mesons ($r \bar{r}$)
Why the strong interactions are difficult

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Perturbative QCD not applicable for

- hadron spectroscopy
- hadron scattering, decays
- nuclear physics
Why the strong interactions are difficult

Alternative methods

Effective field theories

\[ \rightarrow \text{symmetries, scale separation} \]
Why the strong interactions are difficult

Alternative methods

Effective field theories

→ symmetries, scale separation

Dispersion relations

→ analyticity, unitarity, crossing symmetry
Why the strong interactions are difficult

Alternative methods

Effective field theories

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Dispersion relations

→ analyticity, unitarity, crossing symmetry

Lattice QCD

→ solve discretized version of QCD numerically
Part I:

Strong and weak interactions:
Spectroscopy, CP violation,
and Dalitz plots
Modern spectroscopy

- mesons in the quark model: $q\bar{q}$ states
  $\rightarrow$ certain quantum numbers $J^{PC}$ not allowed:
  $0^{+-}, 0^{--}, 1^{-+}, 2^{+-}, \ldots$ “exotics”
- lattice QCD: $1^{-+}$ state $\approx 1.3$ GeV above the $\rho$

Dudek et al. 2013
Modern spectroscopy

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- experiments:
  COMPASS@CERN
  CLAS, GlueX@JLab
Modern spectroscopy

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- lattice QCD: $1^{--}$ state $\approx 1.3$ GeV above the $\rho$
- experiments:

  \[ \pi \rightarrow \rho ?? \]

  \[ \gamma \rightarrow \rho ?? \]

  COMPASS@CERN  CLAS, GlueX@JLab

Things to control using dispersion relations:
- (exotic) phases interfere with known reference phases
- effects of three-/many-body final-state interactions?
- resonances $\hat{=} \text{poles in the complex scattering-energy plane}$
CP violation in weak interactions

- CP violation: one of the prerequisites for matter–antimatter asymmetry in the universe
**CP violation in weak interactions**

- **CP violation**: one of the prerequisites for matter–antimatter asymmetry in the universe
- in the Standard Model: embedded in the weak interactions, Cabibbo–Kobayashi–Maskawa matrix

\[
V_{\text{CKM}} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

CP-violation given by a complex phase in \( V \)
CP violation in weak interactions

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CP-violation given by a complex phase in \( V \)

- Standard-Model CP violation too small
  \( \rightarrow \) search for new/unconventional sources of CP violation
  - charm \( D \) / beauty \( B \) decays
  - electric dipole moments (proton/neutron/deuteron...)
CP violation in weak interactions

CP violation in partial widths \( \Gamma(P \to f) \neq \Gamma(\bar{P} \to \bar{f}) \)

- at least two interfering decay amplitudes
- different weak (CKM) phases
- different strong (final-state-interaction) phases
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Two-body decays: \( D \rightarrow \pi\pi, K\bar{K} \)

- decay at fixed total energy \( \rightarrow \) fixed strong phase
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two-body decays: \( D \to \pi\pi, K\bar{K} \)
- decay at fixed total energy \( \rightarrow \) fixed strong phase

three-body decays: \( D \to 3\pi, \pi\pi K \)
- Dalitz plot \( \hat{=} \) density distribution in two kinematical variables
- resonances \( \rightarrow \) rapid phase variation enhances CP-violation in parts of the decay region
Amplitude analyses in Dalitz plots

The traditional picture: isobar model / Breit–Wigner resonances

\[ \begin{align*}
B, D & \xrightarrow{\pi} f_0, \rho \ldots \\
\pi & \xrightarrow{\pi} B, D \\
K & \xrightarrow{\kappa, K^* \ldots} K
\end{align*} \]

\[ \begin{align*}
& \text{modulus} \\
& \text{phase [°]}
\end{align*} \]

\[ s [\text{GeV}^2] \]
Amplitude analyses in Dalitz plots

The traditional picture: isobar model / Breit–Wigner resonances

\[ B, D \rightarrow \pi \pi \]

\[ K, K^* \ldots \]

...so what’s not to like?

- some resonances don’t look like Breit–Wigners at all!

\[ \rightarrow \text{use exact scattering phase shifts instead} \]
The traditional picture: isobar model / Breit–Wigner resonances

\[ B, D \to \pi\pi K \]

\[ f_0, \rho \ldots \]

\[ K \]

\[ \kappa, K^* \ldots \]

\[ K \]

\[ \pi \]

\[ \pi \]

\[ \pi \]

\[ \pi \]

... so what’s not to like?

- some resonances don’t look like Breit–Wigners at all!
  -> use exact scattering phase shifts instead

- 3-particle rescattering
A simple Dalitz plot: $\phi \to 3\pi$

- $2 \times 10^6$ events in 1834 bins

KLOE 2003

- analyzed in terms of:

  sum of 3 Breit–Wigners ($\rho^\pm, \rho^0$)

  + constant background term

Problem:

$\rightarrow$ unitarity fixes Im/Re parts

$\rightarrow$ adding a contact term destroys this relation
A simple Dalitz plot: $\phi \rightarrow 3\pi$

- $2 \times 10^6$ events in 1834 bins
- Analyzed in terms of:
  - Sum of 3 Breit–Wigners ($\rho^\pm$, $\rho^0$)
  - Constant background term

Problem:
- Unitarity fixes Im/Re parts
- Adding a contact term destroys this relation
- Reconcile data with dispersion relations? Niecknig, BK, Schneider 2012
Experimental comparison to $\phi \rightarrow 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012

\[ \chi^2/\text{ndof} = 1.7 \ldots 2.1 \]

→ pairwise interaction only (with correct $\pi\pi$ scattering phase)
Experimental comparison to $\phi \rightarrow 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012

$\chi^2/\text{ndof}$ \quad 1.7 \ldots 2.1 \quad 1.2 \ldots 1.5

$\rightarrow$ full 3-particle rescattering, only overall normalization adjustable
Experimental comparison to $\phi \rightarrow 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012

\[ \chi^2/\text{ndof} \begin{array}{ccc} 1.7 \ldots 2.1 & 1.2 \ldots 1.5 & 1.0 \end{array} \]

→ full 3-particle rescattering, 2 adjustable parameters
  (additional "subtraction constant" to suppress inelastic effects)
Experimental comparison to $\phi \rightarrow 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012

\[ \chi^2 / \text{ndof} \quad 1.7 \ldots 2.1 \quad 1.2 \ldots 1.5 \quad 1.0 \]

- perfect fit respecting analyticity and unitarity possible
- contact term emulates neglected rescattering effects
- no need for "background" — inseparable from "resonance"
Heavier decays: \( D^+ \rightarrow \pi^+ \pi^+ K^- \)
Heavier decays: $D^+ \rightarrow \pi^+ \pi^+ K^-$

- pion–pion: $S_{\pi\pi}^2$, $P_{\pi\pi}^1$
- pion–kaon: $S_{\pi K}^{1/2}$, $P_{\pi K}^{1/2}$, $S_{\pi K}^{3/2}$, $P_{\pi K}^{3/2}$

CLEO 2008
Heavier decays: $D^+ \rightarrow \pi^+ \pi^+ K^-$

CLEO 2008

pion–pion

$S^{2}_{\pi\pi}$  $P^{1}_{\pi\pi}$

pion–kaon

$S^{1/2}_{\pi K}$  $P^{1/2}_{\pi K}$

$S^{3/2}_{\pi K}$  $P^{3/2}_{\pi K}$

$K^*(892)$
Heavier decays: $D^+ \rightarrow \pi^+ \pi^+ K^-$

Fit in the elastic region:

- "improved isobar model" $\chi^2/\text{ndof} \approx 1.4$
- full three-body rescattering $\chi^2/\text{ndof} \approx 1.1$

$\rightarrow$ visible improvement similar to $\phi \rightarrow 3\pi$  

Niecknig, BK in progress
Part II:

Strong interactions and electromagnetism:
The anomalous magnetic moment of the muon
The anomalous magnetic moment of the muon

- Gyromagnetic ratio: magnetic moment ↔ spin
  \[ \vec{\mu} = g \frac{e}{2m} \vec{S} \]

- Dirac theory for spin-1/2 fermions: \( g_\mu = 2 \)
  rad. corr.: \( g_\mu = 2(1 + a_\mu) \), \( a_\mu \) “anomalous magnetic moment”

- One of the most precisely measured quantities in particle physics
  \[ a_\mu = (116592089 \pm 63) \times 10^{-11} \] BNL E821 2006
The anomalous magnetic moment of the muon

- **gyromagnetic ratio**: magnetic moment ↔ spin
  \[ \vec{\mu} = g \frac{e}{2m} \vec{S} \]

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- one of the most precisely measured quantities in particle physics
  \[ a_\mu = (116\,592\,089 \pm 63) \times 10^{-11} \]

  - ... and one with a significant (?) deviation from the Standard Model:
    \[ a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (27.6 \pm 8.6) \times 10^{-11} \]

- new \( g - 2 \) experiment at Fermilab: reduce experimental error by factor 4
The Standard Model prediction for $a_\mu$

<table>
<thead>
<tr>
<th></th>
<th>$a_\mu [10^{-11}]$</th>
<th>$\Delta a_\mu [10^{-11}]$</th>
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<tr>
<td>experiment</td>
<td>116 592 089.</td>
<td>63.</td>
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<tr>
<td>QED $O(\alpha)$</td>
<td>116 140 973.21</td>
<td>0.03</td>
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<td>QED $O(\alpha^2)$</td>
<td>413 217.63</td>
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<td>QED $O(\alpha^3)$</td>
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<td>QED $O(\alpha^5)$</td>
<td>5.09</td>
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<td>QED total</td>
<td>116 584 718.85</td>
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Kinoshita et al. 2012
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B. Kubis, Strong interactions for the precision frontier in particle and nuclear physics – p. 17
### The Standard Model prediction for $a_\mu$

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hadronic part dominates the uncertainty by far!

Jegerlehner, Nyffeler 2009
Davier et al. 2011
and references therein
Hadronic vacuum polarization

- how to control hadronic vacuum polarization?
- characteristic scale set by muon mass → this is not a perturbative QCD problem!
- dispersion relations to the rescue: use the optical theorem!
Hadronic vacuum polarization

- how to control hadronic vacuum polarization?
- characteristic *scale* set by muon mass  
  \[ \rightarrow \text{this is not a perturbative QCD problem!} \]
- dispersion relations to the rescue: use the optical theorem!

\[
\text{Im} \begin{array}{c} \gamma \\ \text{hadrons} \end{array} \Leftrightarrow \left| \begin{array}{c} \gamma \\ \text{hadrons} \end{array} \right|^2 \propto \sigma_{tot}(e^+e^- \rightarrow \text{hadrons})
\]

B. Kubis, Strong interactions for the precision frontier in particle and nuclear physics – p. 18
Hadronic vacuum polarization

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• characteristic scale set by muon mass
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\[ a_{\mu}^{\text{had VP}} \propto \int_{4M_{\pi}^2}^{\infty} K(s) \sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons}) \]

• \( K(s) \): kinematical function, for large \( s \):
  \( K(s) \propto 1/s \),
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- more than 75% of \(a_\mu^{\text{had VP}}\) given by
  energies \(s \leq 1 \text{GeV}^2\) Jegerlehner, Nyffeler 2009
- dominated by \(e^+e^- \rightarrow \pi^+\pi^-\)
  \(\rightarrow\) pion electromagnetic form factor
- well constrained by data KLOE, BABAR…
Hadronic light-by-light scattering

- hadronic light-by-light soon to dominate Standard Model uncertainty
Hadronic light-by-light scattering

- **hadronic light-by-light** soon to dominate Standard Model uncertainty

- different contributions estimated (in $10^{-11}$):

  - $\pi^0, \eta, \eta'$
    - $99\pm16$
  - $\pi^\pm, K^\pm$
    - $-19\pm13$
  - Axials, scalars
    - $15\pm7$
  - Quarks
    - $21\pm3$

→ **hadronic modelling at its worst**...  

Jegerlehner, Nyffeler 2009
Hadronic light-by-light scattering

- hadronic light-by-light soon to dominate Standard Model uncertainty
- different contributions estimated (in $10^{-11}$):

\[ \begin{align*}
\pi^0, \eta, \eta' & \quad 99 \pm 16 \\
\pi^\pm, K^\pm & \quad -19 \pm 13 \\
axials, scalars & \quad 15 \pm 7 \\
quarks & \quad 21 \pm 3
\end{align*} \]

\[ \rightarrow \text{hadronic modelling at its worst...} \quad \text{Jegerlehner, Nyffeler 2009} \]

- largest (and unambiguous!) individual contribution: $\pi^0$ pole term
  \[ \rightarrow \text{depends on } \pi^0 \rightarrow \gamma^*\gamma^* \text{ form factor (doubly virtual in general!)} \]
On the road to a dispersive analysis of $\pi^0 \rightarrow \gamma^* \gamma^*$

The Wess–Zumino–Witten (chiral) anomaly

- $\pi^0 \rightarrow \gamma\gamma$ fixed by the chiral anomaly: $F_{\pi^0\gamma\gamma} = \frac{e^2}{4\pi^2 F_{\pi}}$

$F_{\pi}$: pion decay constant $\rightarrow$ measured at % level

PrimEx 2011
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- analyze the leading hadronic intermediate states:

  see also Gorchtein, Guo, Szczepaniak 2012
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  PrimEx 2011

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\[
\begin{align*}
\gamma_u^* & \quad \ldots & \quad \gamma_s^* \\
\pi^+ & \quad \ldots & \quad \pi^- \\
\pi^0 & \quad \ldots & \quad \pi^0 \\
\omega, \phi & \quad \ldots & \quad \gamma_v^{(*)}
\end{align*}
\]
On the road to a dispersive analysis of $\pi^0 \rightarrow \gamma^* \gamma^*$

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  $F_\pi$: pion decay constant → measured at % level

- analyze the leading hadronic intermediate states:

  \[ \gamma^*(\pi) \rightarrow \pi^+ \pi^- \]

- important ingredients:
  - anomalous process $\gamma \pi \rightarrow \pi \pi$
  - vector-meson radiative decays $\omega/\phi \rightarrow \pi^0 \gamma$
  - transition form factors $\omega/\phi \rightarrow \pi^0 \ell^+ \ell^-$

  → analyze these in turn, using dispersion relations

PrimEx 2011

see also Gorchtein, Guo, Szczepaniak 2012
The anomalous process $\gamma \pi \rightarrow \pi \pi$

- $\gamma \pi \rightarrow \pi \pi$ at zero energy: $F_{3\pi} = \frac{e}{4\pi^2 F_3^\pi} = (9.78 \pm 0.05) \text{ GeV}^{-3}$

how well can we test this low-energy theorem?
The anomalous process $\gamma\pi \rightarrow \pi\pi$

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how well can we test this low-energy theorem?

Primakoff reaction:

\[ \pi^- \rightarrow \pi^- \pi^0 \gamma^* \rightarrow \pi^- \pi^- Z \]

COMPASS, work in progress
The anomalous process $\gamma\pi \rightarrow \pi\pi$

- $\gamma\pi \rightarrow \pi\pi$ at zero energy: $F_{3\pi} = \frac{e}{4\pi^2 F_\pi^3} = (9.78 \pm 0.05) \text{ GeV}^{-3}$

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Primakoff reaction:

- same quantum numbers as $\phi \rightarrow 3\pi$:

  Truong 2002, Hoferichter, BK, Sakkas 2012
The anomalous process \( \gamma \pi \rightarrow \pi \pi \)

- \( \gamma \pi \rightarrow \pi \pi \) at zero energy: 
  \[
  F_{3\pi} = \frac{e}{4\pi^2 F^3_\pi} = (9.78 \pm 0.05) \text{GeV}^{-3}
  \]
  how well can we test this low-energy theorem?

Primakoff reaction:

- same quantum numbers as \( \phi \rightarrow 3\pi \):
  \[\rightarrow \text{extract } F_{3\pi} \text{ from the complete spectrum} \]
  \[\text{model-independently, using } \pi \pi \text{ phase shift!} \]

Hoferichter, BK, Sakkas 2012

COMPASS, work in progress

B. Kubis, Strong interactions for the precision frontier in particle and nuclear physics – p. 21
Transition form factors $\omega(\phi) \rightarrow \pi^0 \ell^+ \ell^-$

- $\pi^0 \rightarrow \gamma^* \gamma^*$ form factor linked to $\omega(\phi) \rightarrow \pi^0 \gamma^*$ transition:
Transition form factors $\omega(\phi) \rightarrow \pi^0 \ell^+ \ell^-$

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Transition form factors $\omega(\phi) \rightarrow \pi^0 \ell^+ \ell^-$

- $\pi^0 \rightarrow \gamma^* \gamma^*$ form factor linked to $\omega(\phi) \rightarrow \pi^0 \gamma^*$ transition:

- $\omega$ transition form factor related to pion vector form factor $\times \omega \rightarrow 3\pi$ decay amplitude

- Form factor normalization yields rate $\Gamma(\omega \rightarrow \pi^0 \gamma)$

(2nd most important $\omega$ decay channel)

$\rightarrow$ works at 95% accuracy

Schneider, BK, Niecknig 2012
Numerical results: \( \omega \to \pi^0 \mu^+ \mu^- \)

- Unable to account for steep rise in data (from heavy-ion collisions) \( \text{NA60 2009, 2011} \)
- More "exclusive" data?! \( \text{CLAS?} \)
- \( \omega \to 3\pi \) Dalitz plot? \( \text{CLAS?} \)
Towards a dispersive analysis of $e^+e^- \rightarrow \pi^0\gamma$

- combine isoscalar and isovector contribution to $e^+e^- \rightarrow \pi^0\gamma$:
  - parameterise $e^+e^- \rightarrow 3\pi$
    - e.g. in terms of (dispersively improved) $\omega + \phi$ Breit–Wigner propagators with good analytic properties
      
      Lomon, Pacetti 2012; Moussallam 2013
  - prediction for $e^+e^- \rightarrow \pi^0\gamma$
Fit to $e^+e^- \rightarrow 3\pi$ data

- one subtraction/normalisation at $q^2 = 0$ fixed by $\gamma \rightarrow 3\pi$
- fitted: $\omega$, $\phi$ residues, one additional (linear) subtraction
Comparison to $e^+ e^- \rightarrow \pi^0 \gamma$ data

- "prediction"—no further parameters adjusted
- data well reproduced
Summary

2 pieces of modern strong-interaction physics using dispersion relations:

-Dalitz plot analyses
  - rigorous using modern phase shift input
  - allow to understand ad-hoc "background"
  - methods applicable for many kinds of "amplitude analyses": CP violation, spectroscopy issues
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- Dalitz plot analyses
  - rigorous using modern phase shift input
  - allow to understand ad-hoc "background"
  - methods applicable for many kinds of "amplitude analyses":
    - CP violation, spectroscopy issues

- The muon anomalous magnetic moment
  - hadronic physics needs to be constrained well before claiming "new physics"
  - interrelate as much experimental information as possible