b_1 Structure Function of the Deuteron

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G. A. Miller, in *Electronuclear Physics with Internal Targets*, ed. R. G. Arnold (World Scientific, Singapore, 1989), p.30.

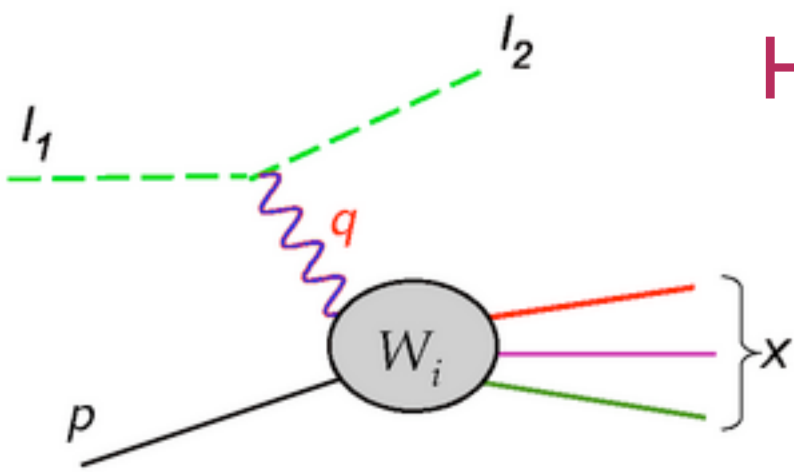
Patricia Solvignon

arXiv:1311.4561v1

Outline

- What is the b_1 Structure Function of the Deuteron?
- Nucleonic contributions negligible
- Pion exchange contributions
- 6 quark, hidden-color contributions
- Double scattering (shadowing) contributions
- Close-Kumano sum rule

b_1 structure function DIS



Hoodbhoy, Jaffe, Manohar NPB312, 571

Direction of photon= spin quantization axis

$$d^2\sigma^{(m)} \propto \ell^{\mu\nu} W^{(m)}_{\mu\nu}$$

unpolarized lepton, polarized target

$$W^{(m)}_{\mu\nu} = \int d^4r \langle T, J=1, J_z=m | [j^\mu(r), j^\nu(0)] | T, J=1, J_z=m \rangle$$

$$F_1(x) = \frac{1}{3} \sum_m W^{(m)}_{11}$$

usual average over target spin directions

$$b_1(x) = W^{(1)}_{11} - W^{(0)}_{11}.$$

Depends on spin direction of target

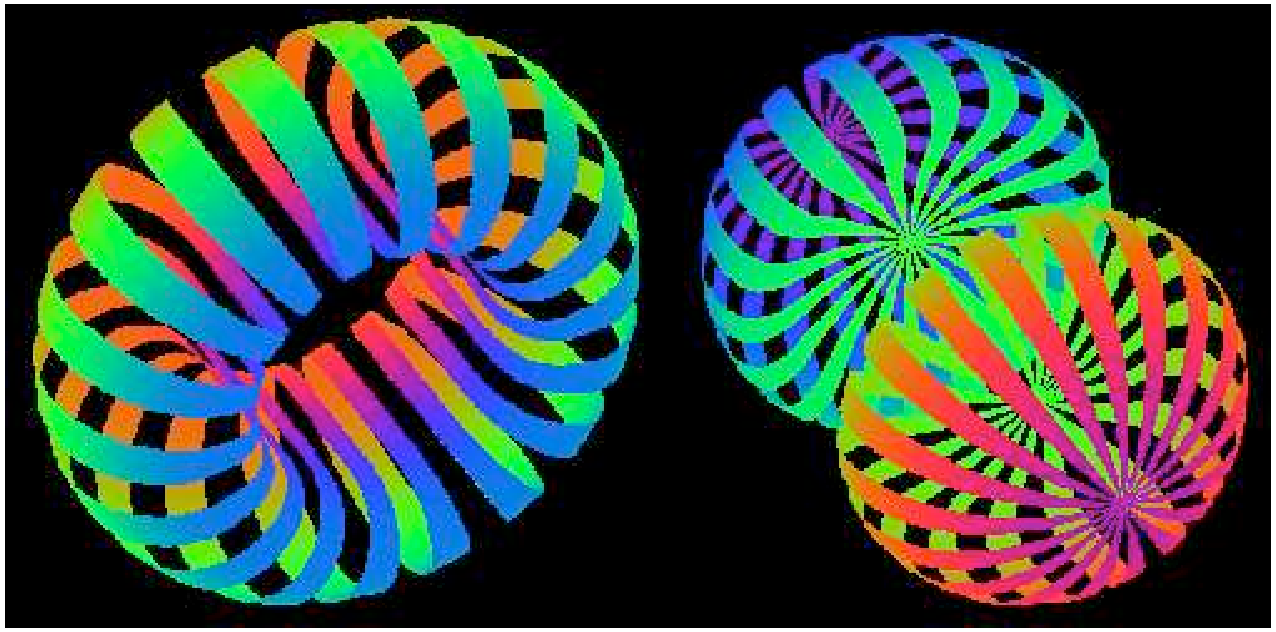
Experimentalist's Definition

$$\sigma_{\rm meas} = \sigma^{\rm U}\left[1 - P_B P_z A_{\parallel} + \frac{1}{2}\, P_{zz} A_{zz}\right].$$

$$P_{zz} = \frac{(n^+ + n^-) - 2n^0}{n^+ + n^- + n^0}, \qquad -2 \leqq P_{zz} < 1.$$

$$\frac{b_1}{F_1} = -\frac{3}{2} A_{zz}.$$

b_1 structure function: potentially interesting



J. Carlson and R. Schiavilla, Rev. Mod. Phys. **70**, 743 (1998).

J. L. Forest *et al*., Phys. Rev. **C54**, 646 (1996).

constant density surfaces
small np separations

m=0

m=1

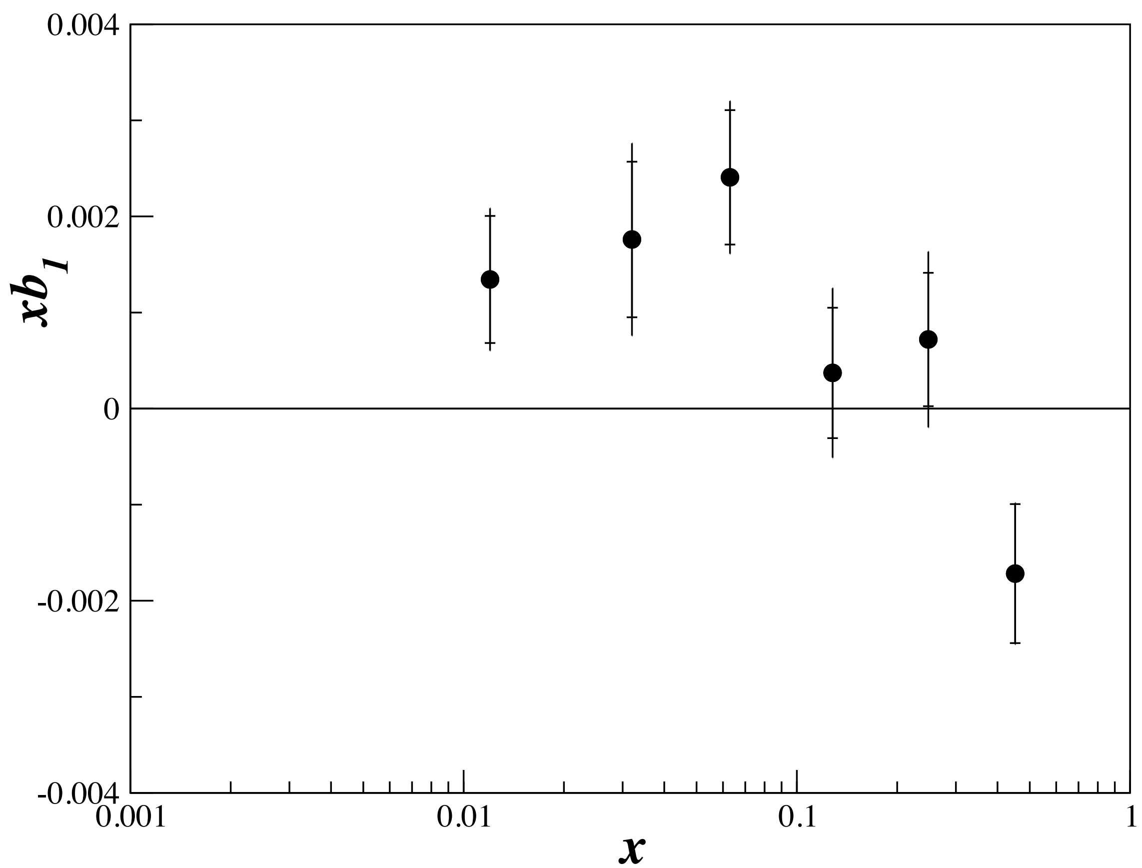
Hoodbhoy et al: b_1 measures the extent to which a target nucleus deviates from a trivial bound state of nucleons

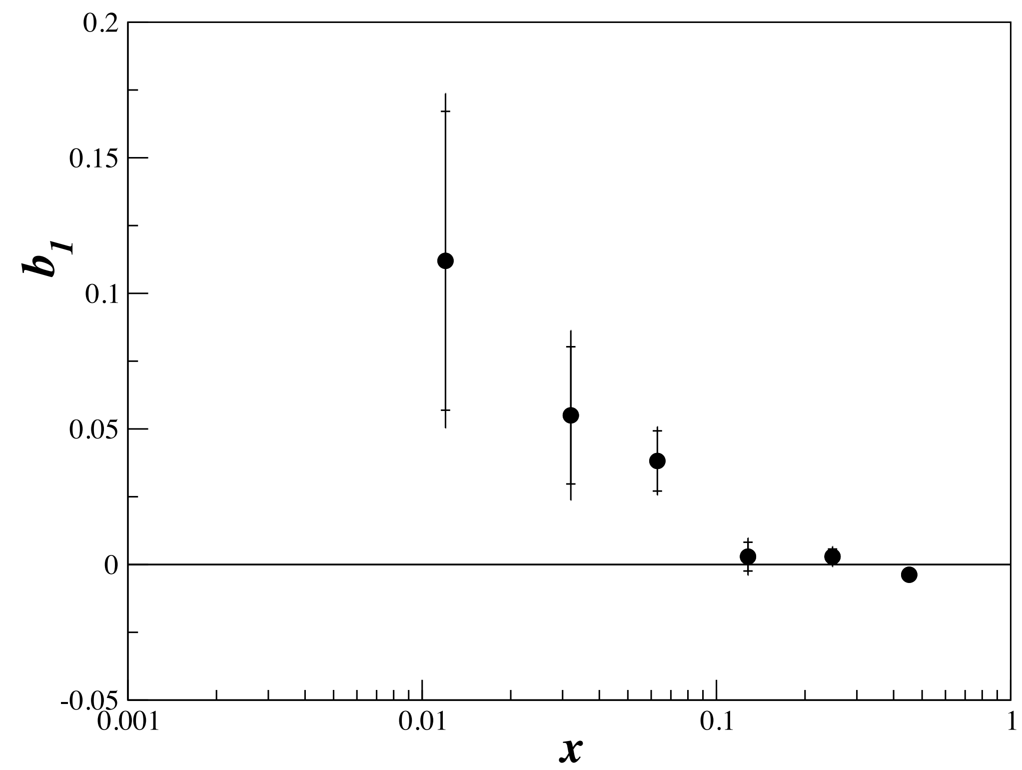
JLab proposal PR12-13-011 K. Silfer et al
HERMES PRL 95,242001

HERMES

C. Riedl, Ph. D thesis, DESY-THESIS-2005-027 (2005).

A. Airapetian *et al*., Phys. Rev. Lett. **95**, 242001 (2005).





General remarks

$$q^{(m)}(x) = \langle T, J=1, m | O | T, J=1, m \rangle$$

Wigner Eckhart: O is tensor of rank 1 or 2

Parity $q^{(1)}=q^{(-1)}$, O can't be rank 1 and give non-zero b_1

O is a rank 2 tensor, so b_1 measures tensor effects

Consequence:

s-wave component of deuteron gives no contribution to b_1 because m is not relevant

Nucleon contributions

Miller 1989

G. A. Miller, in *Electronuclear Physics with Internal Targets*, ed. R. G. Arnold (World Scientific, Singapore, 1989), p.30.

$$q^{(m)}(x) = \int_x^\infty dy\, q^N(x/y)\, f^{(m)}(y)$$

$$f^{(m)}(y) = \int d^4p \left[1 + \frac{p^3}{\sqrt{p^2 + M^2}}\right] S^{(m)}_D(p)\, \delta\left(y - \frac{p^0 + p^3}{M_D}\right)$$

$$S_D^{(m)} = \sum_s \langle D, m | b^\dagger_{p,s} \delta(-p_0 + M_D - H) b_{p,s} | D, m\rangle$$

$$b_1(x) = \int_x dy\, (F_1^p(x/y) + F_1^n(x/y))\Delta f_{sd}(y)$$

$$\Delta f_{sd}(y) = \frac{-4\sqrt{2}}{8\pi} \int d^3p\, u(p) w(p)\, (3\cos^2\theta - 1)\ \delta\left(\frac{p\cos\theta + p^0}{M} - y\right)\left[1 + \frac{p\cos\theta}{M}\right].$$

Result: $b_1 = 0$

Remark $\int dx b_1(x) \propto \int dy \Delta f_{sd}(y) \propto \int d^3p\, u(p) w(p)(3\cos^2\theta - 1) = 0$

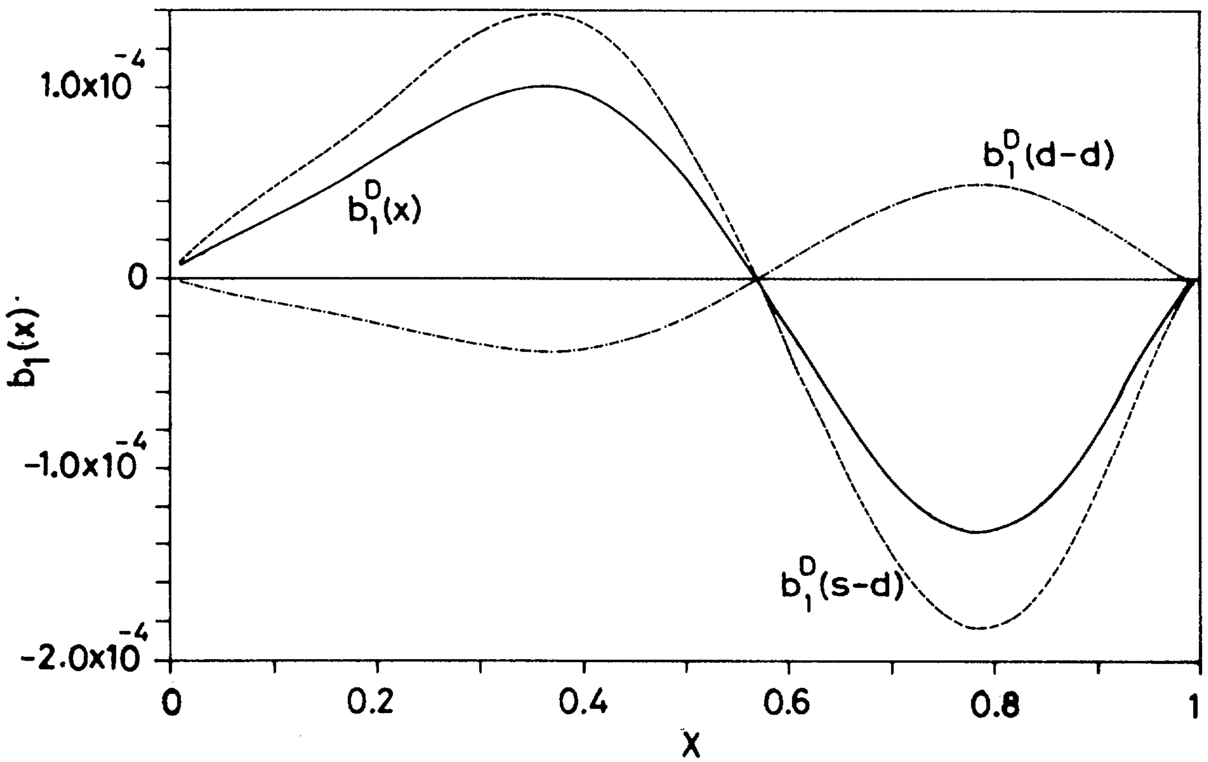
Example of (genesis of) Close-Kumano sum rule

Vanishing integral consistent with b_1 being very small

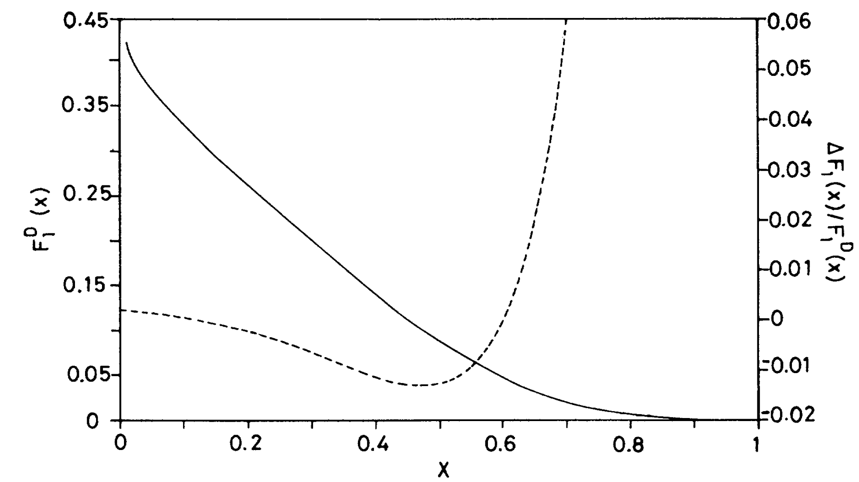
PHYSICAL REVIEW C VOLUME 44, NUMBER 3 SEPTEMBER 1991

Convenient parametrization for deep inelastic structure functions of the deuteron

Hafsa Khan and Pervez Hoodbhoy



F_1^D is of order 1

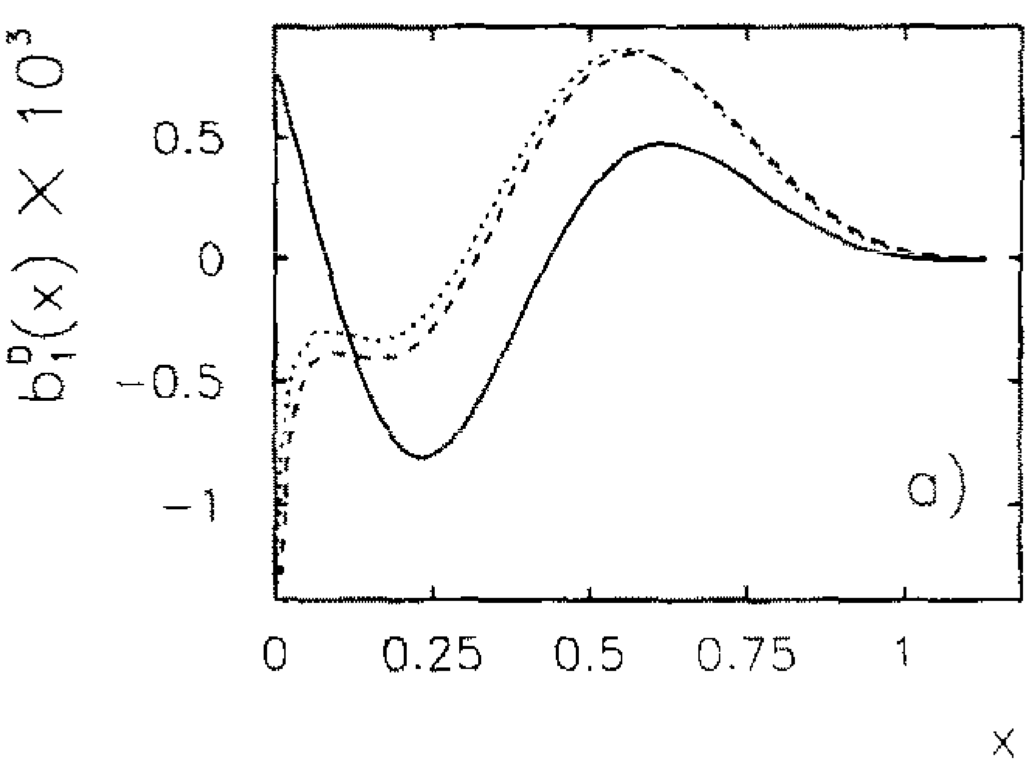


b_1 very small

Physics Letters B 391 (1997) 177–184

Relativistic calculation of structure functions $b_{1,2}(x)$ of the deuteron

A.Yu. Umnikov[1]

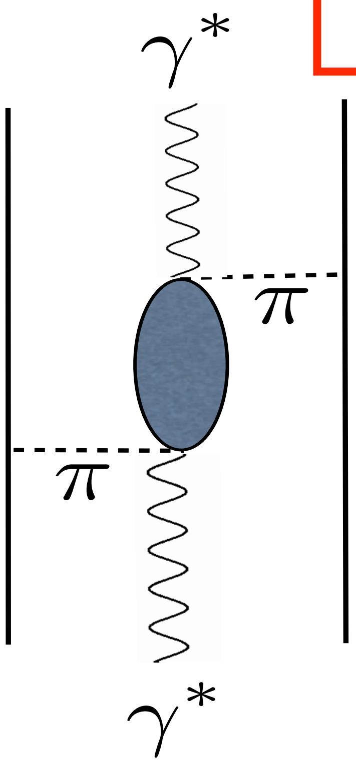


Solid Bethe-Salpeter
Dashed Bonn

b_1 very small

Difference between calcs due to different nucleon sf

Pionic contribution

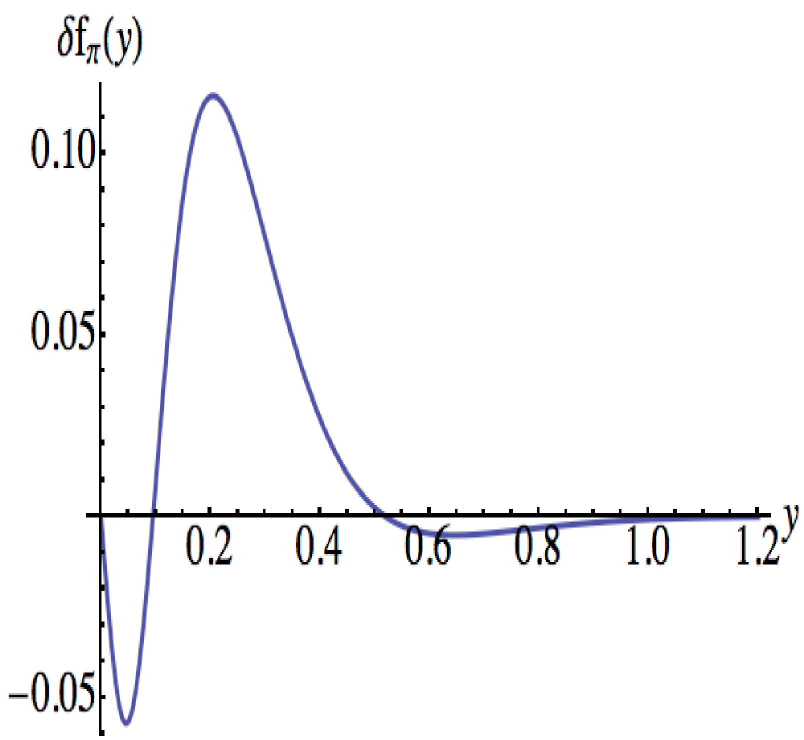


$$\Delta_\pi q^{(m)}(x) = \int_x^\infty \frac{dy}{y} q^\pi(x/y) f_\pi^{(m)}(y),$$

$$f_\pi^{(m)}(y_A) = \int \frac{d\xi^-}{2\pi} e^{-iy_A P_D^+ \xi^-} \langle D, m | \phi_\pi(\xi^-) \phi_\pi(0) | D, m \rangle_c,$$

$$f_\pi^{(m)}(y) = \frac{-3yg^2}{(2\pi)^3} \int \frac{d^3q}{(\mathbf{q}^2 + m_\pi^2)^2} \frac{G_A^2(\mathbf{q}^2)}{G_A^2(0)} \delta(My - q_z) F_m(\mathbf{q}), \qquad F_m(\mathbf{q}) \equiv \int d^3r \langle D, m | e^{-i\mathbf{q}\cdot\mathbf{r}} \boldsymbol{\sigma}_1 \cdot \mathbf{q}\, \boldsymbol{\sigma}_2 \cdot \mathbf{q} | D, m \rangle$$

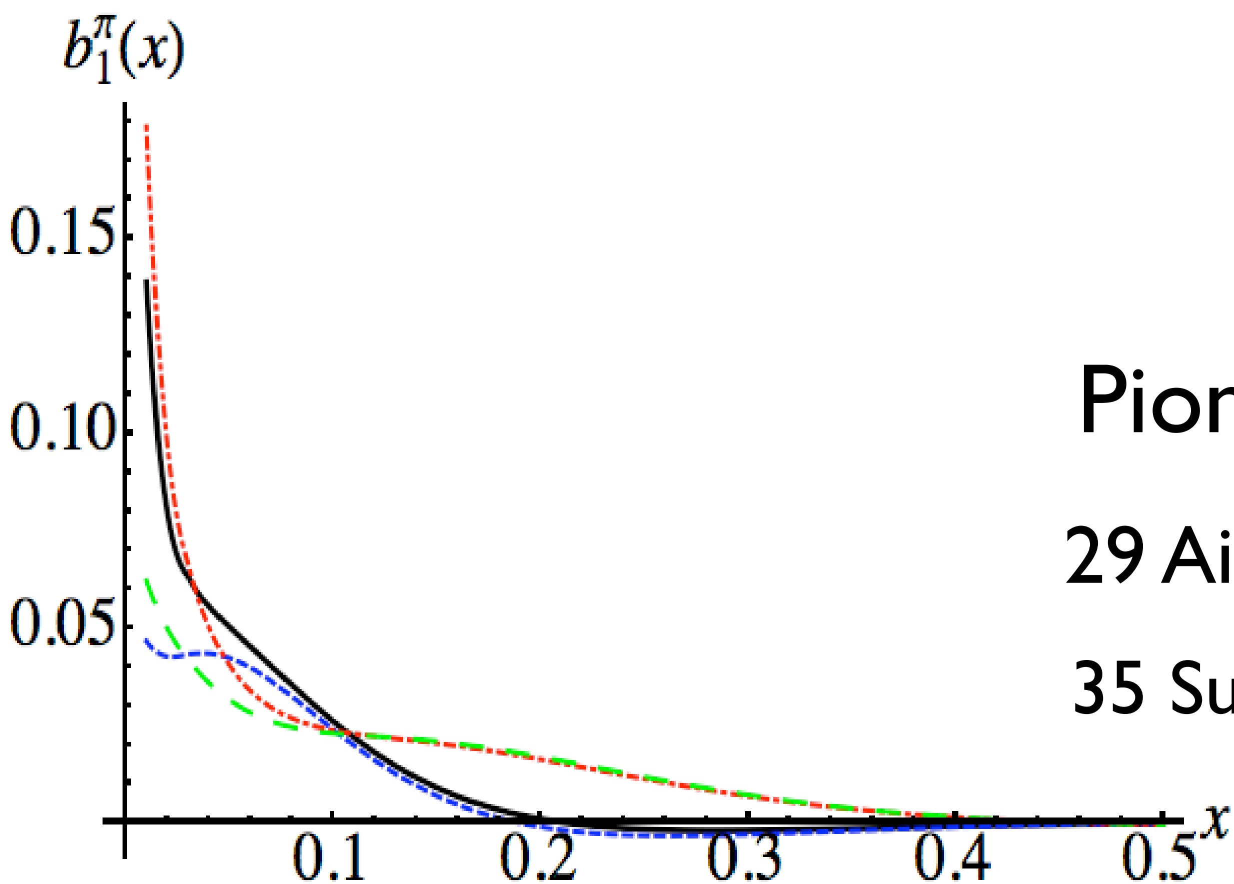
$$\delta f_\pi(y) \equiv f_\pi^{(0)}(y) - f_\pi^{(1)}(y) \quad \sim -y \int \frac{d^3q}{(\mathbf{q}^2 + m_\pi^2)^2} \cdots \delta(My - q_z)(\mathbf{q}^2 - 3q_z^2)$$



$$b_1^\pi(x) = \frac{1}{2} \int_x^\infty \frac{dy}{y} q^\pi(x/y) \delta f_\pi(y).$$

- ~Independent of Deut wave function
- Double node structure -tensor op
- $$\int dy \, \frac{f(y)}{y} = 0$$

Pionic effects



Pion structure function

29 Aicher et al PRL105, 252003

35 Sutton et al PRD45, 2349

FIG. 3: Color online. Computed values of b_1^π, for different pion structure function at $Q^2 = 1.17$ GeV^2. Solid- full structure function [29] short-dashed (blue) valence [29], Dot Dashed (Red) full structure function (mode 3) [35],Long dashed (green) (mode 3) [35]

HERMES

$\langle x \rangle$	$\langle Q^2 \rangle$ [GeV^2]	b_1 [10^{-2}]	$\pm\delta b_1^{\rm stat}$ [10^{-2}]	$\pm\delta b_1^{\rm sys}$ [10^{-2}]	b_1^π[29] [10^{-2}]	b_1^π[35] (1) [10^{-2}]	b_1^π[35] (3) [10^{-2}]
0.012	0.51	11.20	5.51	2.77	10.5	15.5	24.1
0.032	1.06	5.50	2.53	1.84	5.6	6.8	8.9
0.063	1.65	3.82	1.11	0.60	4.2	3.7	4.1
0.128	2.33	0.29	0.53	0.44	1.6	1.3	1.3
0.248	3.11	0.29	0.28	0.24	-0.55	.13	0.12
0.452	4.69	-0.38	0.16	0.03	-0.02	-0.02	-0.022

Non 0 at high x

Hidden color, 6-quark states

- Maybe deuteron has non-nucleon baryonic components
- 6 quark contribution ~orthogonal to two nucleons
- Dominated by hidden color (two color octets form a color singlet Harvey NPA32, 301
- 6 quarks in same s state wave function:
- $$|6q\rangle = \sqrt{1/9}|N^2\rangle + \sqrt{4/45}|\Delta^2\rangle + \sqrt{4/5}|CC\rangle.$$

Just call this state 6q (mainly hidden color)

Hidden color model-simplest possible

- S-state of deuteron has component with 6 quarks in s -state- S=1,T=0
- D-state has 6-quark component with any one quark in $d_{3/2}$ state

$\mathcal{Y}_{jlm_j}$ is a spinor spherical harmonic.

$$\psi_{j,l,H}(\mathbf{p}) = \sqrt{N_l} f_l(p) \sum_{m_s,m_j} \mathcal{Y}_{jlm_j} \langle jm_j, \frac{1}{2} m_s | 1H \rangle, \qquad J_z = H$$

$$l, j \quad = \quad s_{1/2} \quad \text{or} \quad d_{3/2},$$

$$F_H(x_{6q}) = \frac{1}{2} \int d^3p \bar{\psi}_{1/2,0,H}(\mathbf{p}) \gamma^+ \psi_{3/2,2,H}(\mathbf{p}) \delta \left(\frac{p \cos\theta + E(p)}{M_{6q}} - x_{6q} \right),$$

Harmonic oscillator wave functions

$$f_0(p) = e^{-p^2R^2/2}, \; f_2(p) = -p^2R^2 e^{-p^2R^2/2}$$
$$E(p) = \sqrt{p^2 + m^2}, \; m = 338 \text{ MeV quark mass}, \; R = 1.2 \text{ fm from bag}$$

$$b_1^{6q}(x) = -\sqrt{\frac{N_0 N_2}{2}} \frac{3}{4\pi} \int d^3p f_0 f_2 (3\cos^2\theta - 1) \delta \left(\frac{p\cos\theta + E(p)}{M} - x \right) P_{6q}.$$

P_{6q}= 0.0015 to reproduce Hermes x=0.452 (very small P_{6q})

6 quark model

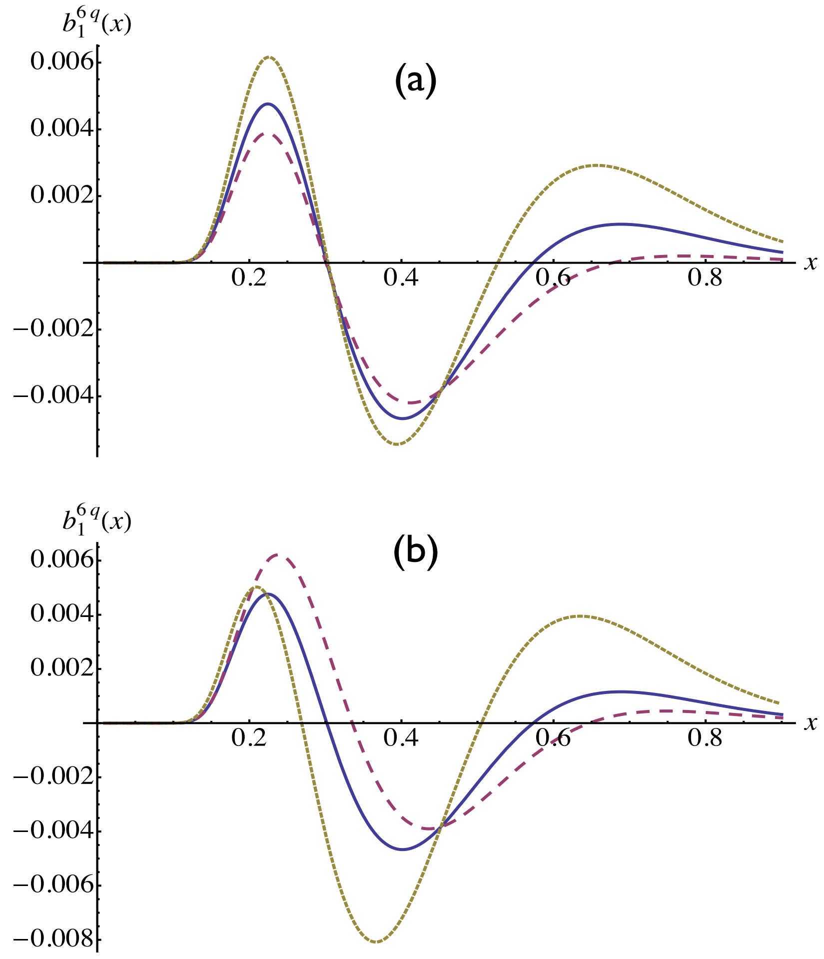


FIG. 4: (Color online) Computed values of b_1^{6q} from Eq. (26). Sensitivity to parameters is displayed. (a) Solid (blue) uses $R = 1.2$ fm, m=338 MeV, long dashed (Red) R is decreased by 10%, dotted(green) R is increased by 10%. (b) Solid (blue) uses $R = 1.2$ fm, m=338 MeV, long dashed (Red) m isincreased by 10%, dotted(green), m is decreased by 10%.

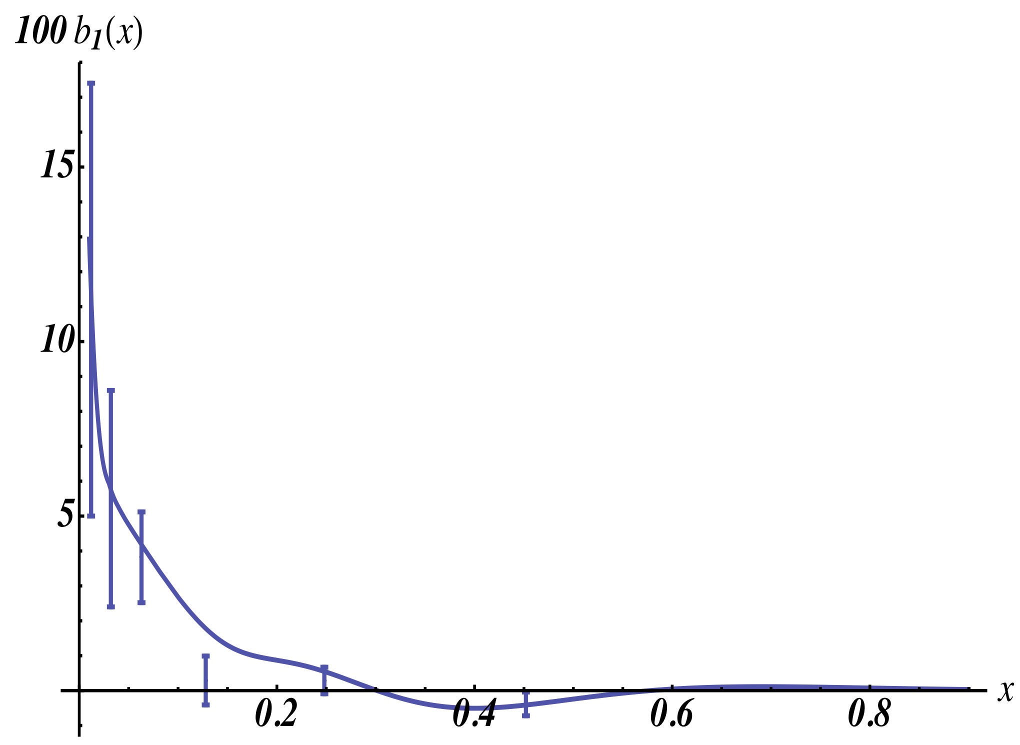
Vary R

Vary m

Small at low x, where pionic effect is relevant

Valence quarks carry higher momentum

pionic and 6q contributions

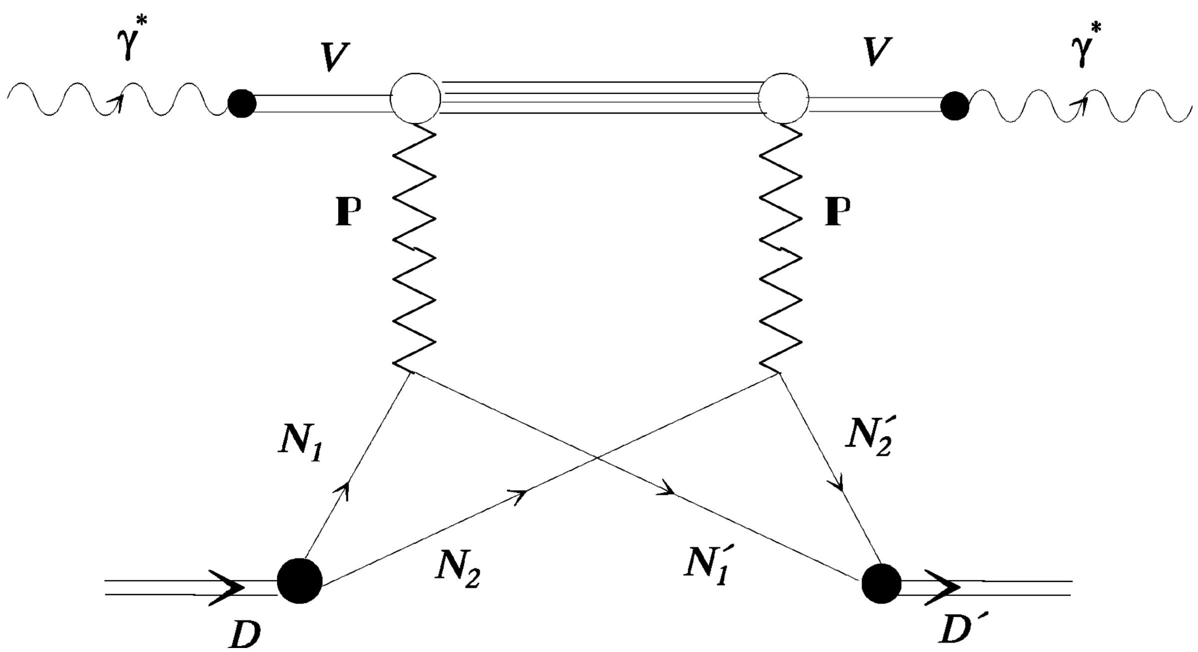


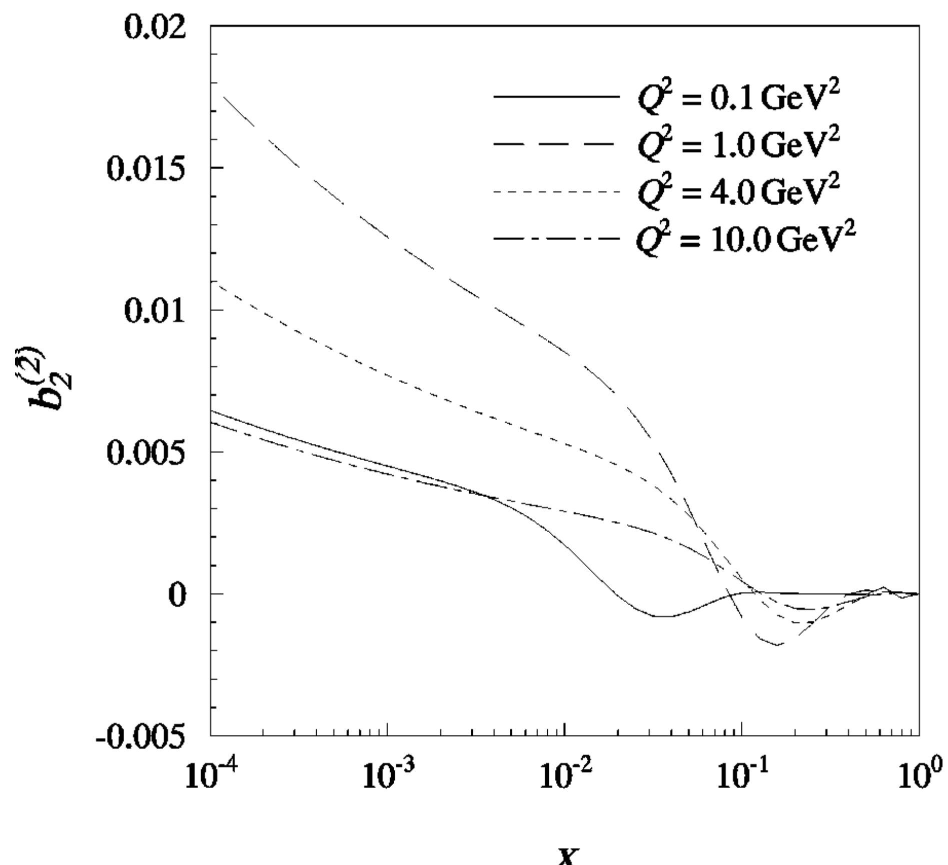
$\langle x \rangle$	$\langle Q^2 \rangle$ [GeV^2]	b_1 [10^{-2}]	$\pm\delta b_1^{\mathrm{stat}}$ [10^{-2}]	$\pm\delta b_1^{\mathrm{sys}}$ [10^{-2}]	b_1^{π}[29] [10^{-2}]	b_1^{π}[35] (1) [10^{-2}]	b_1^{π}[35] (3) [10^{-2}]	b_1^{6q} [10^{-2}]
0.012	0.51	11.20	5.51	2.77	10.5	15.5	24.1	0.00
0.032	1.06	5.50	2.53	1.84	5.6	6.8	8.9	0.00
0.063	1.65	3.82	1.11	0.60	4.2	3.7	4.1	0.00
0.128	2.33	0.29	0.53	0.44	1.6	1.3	1.3	0.01
0.248	3.11	0.29	0.28	0.24	-0.55	.13	0.12	0.41
0.452	4.69	-0.38	0.16	0.03	-0.02	-0.02	-0.022	-0.38

Can reproduce data, so far
JLab experiment needed to test
no other known mechanism contributes at the higher values of x

Shadowing -double scattering

Bora, Jaffe PRD57,6906





$b_2 = 2xb_1$

Small at JLab x

Prediction for JLab

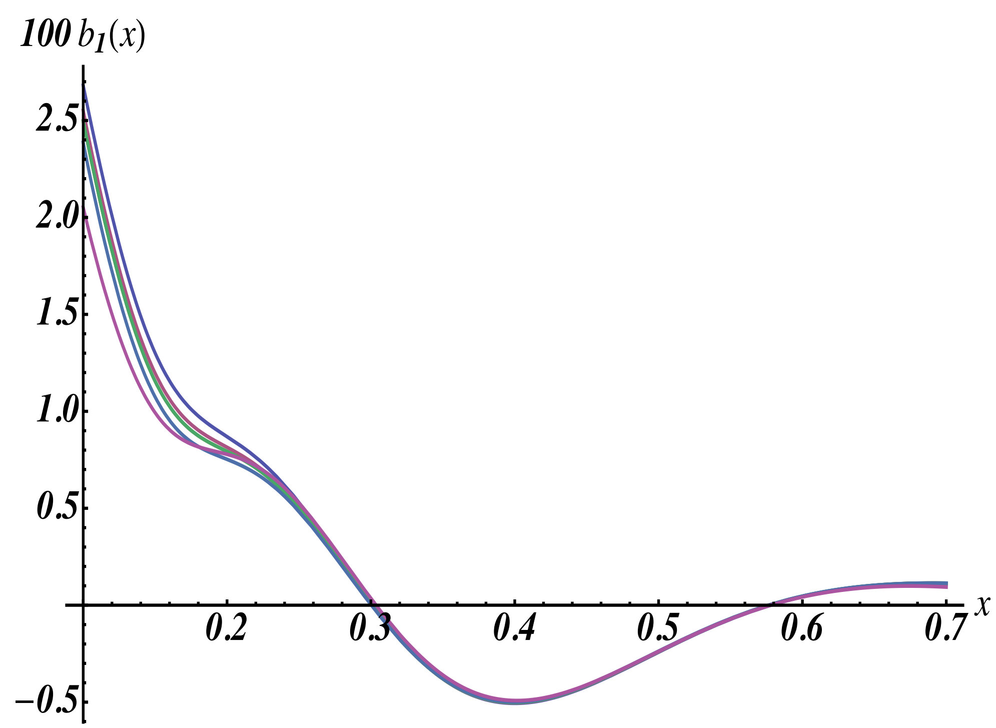


FIG. 6: (Color online) Computed values of $100\,(b_1^{\pi}+b_1^{6q})$, for values of $Q^2\ =\ 1.17$, 1.76, 2.12 and 3.25 GeV^2 [29] distributions and for [35] (lowest curve at $x=0.15$). For the other curves, b_1^{π} increases as Q^2 increases for small values of x.

Close Kumano PR D42, 2377 Sum Rule CKSR

$$\int dx b_1(x) = 0$$

- Derived assuming b_1 is carried by valence quarks
- Analogous to Gottfried sum rule for the integral of F_{2p}-F_{2n} which assumed $\bar{u} = \bar{d}$
- various effects of the sea violate CKSR
- violations may be more interesting than the sum rule

CKSR-pion effect

$$\int_0^1 dx b_1^\pi(x) = \frac{1}{2}\int_0^1 dx \int_x^\infty \frac{dy}{y} q^\pi(x/y)\delta f_\pi(y)$$
$$= \frac{1}{2}\int_0^2 dy \delta f_\pi(y) \int_0^1 du q^\pi(u). \qquad \neq 0, \quad = \infty$$

CKSR- nucleon

$$\int b_1^N(x) = \int_0^2 dy \int d^3p F_d(p)(3\cos^2\theta - 1)\delta(y - \frac{p\cos\theta + E}{M}) \int_0^1 F_{1N}(u)du$$

If integrate over **all** y, get $0 \times \infty$
Can't, so get ∞

CKSR - shadowing - CKSR is not 0

CKSR- six quark effect

$$b_1^{6q}(x) = -\sqrt{\frac{N_0 N_2}{2}} \frac{3}{4\pi} \int d^3p f_0 f_2 (3\cos^2\theta - 1)\delta\left(\frac{p\cos\theta + E(p)}{M} - x\right) P_{6q}.$$

Integral over all x vanishes

Summary of b_1 results

- Pionic effects sizable for $x<0.2$
- Reproduces HERMES data there
- 6-quark hidden color effects can enter at larger values of x
- Combination reproduces HERMES data
- Predictions made for future JLAB data
- CKSR does not hold except for 6-quark effects
- If CKSR holds, b_1 must be both positive and negative
- Observing such would provide evidence for 6-quark hidden-color components of the deuteron