Beam Single Spin Asymmetries in Electron-Proton Scattering

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Introduction

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- In general the beam asymmetry, $B$, is

$$B = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}.$$ 

- BSSA disappear for one-photon exchange.
What exchanges can cause BSSA?

Consider the scattering amplitude:

\[ |M|^{2} = |M_{\gamma}|^{2} + |M_{Z}|^{2} + |M_{\gamma\gamma}|^{2} + \cdots \]

At the Born level, the spin orientation has no effect on \( |M_{\gamma}|^{2} \), so the asymmetry vanishes. Higher order effects result in asymmetries dependent on whether the spin is transversely, normally, or longitudinally polarized.
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Consider the scattering amplitude:

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- Higher order effects result in asymmetries dependent on whether the spin is transversely, normally, or longitudinally polarized.
Transverse and Normal Polarization

- Transverse asymmetry $B_t$ behaves like $\cos \phi_s$ and normal asymmetry $B_n$ behaves like $\sin \phi_s$.

![Electron-Proton Scattering Diagram]

**Figure 2.** Electron-Proton Scattering
Motivation: Q-weak Collaboration

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![Graph showing asymmetry as a function of octant number with preliminary results indicated.](image-url)
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Concerned about possible phase offsets due to unconsidered BSSA effects.
Approach

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  - Interference of one- and two-photon exchange.
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- **Elastic Scattering Contribution**
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- **Hadronic Inelastic Intermediate State**
  - Pasquini and Vanderhaeghen (*PRC 70*, 2004) showed that inelastic hadronic intermediate states have a larger contribution to the asymmetry than the elastic case.
  - Consider the case of near forward limit.
Elastic scattering for electron-proton scattering,

\[ e(p_1) + N(p_2) \rightarrow e(p_3) + N(p_4). \]
Kinematic Variables

Elastic scattering process:

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Using the following notation for kinematic variables:

\[ P = \frac{p_2 + p_4}{2}, \quad K = \frac{p_1 + p_3}{2}, \quad q = p_1 - p_3, \quad Q^2 = -q^2; \]

\[ s = (p_1 + p_2)^2, \quad \tau = \frac{Q^2}{4M^2}, \quad \nu = P \cdot K, \quad \epsilon = \frac{\nu^2 - M^4\tau(1 + \tau)}{\nu^2 + M^4\tau(1 + \tau)}; \]

\[ W^2 = (p_2 + q_1)^2, \quad Q_1^2 = -q_1^2, \quad Q_2^2 = -q_2^2, \]

for electron mass \( m_e \) and hadron mass \( M \).
Beam Normal Single Spin Asymmetry: Setup

Using six invariant amplitudes given by Goldberger et al. (Ann. Phys. 2, 1957), we can construct a general elastic lepton-nucleon scattering amplitude:

\[ T = T_{\text{non-flip}} + T_{\text{flip}}, \]
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where

\[ T^{\text{non-flip}} = \frac{e^2}{Q^2} \bar{u}_e(p_3)\gamma_\mu u_e(p_1) \cdot \bar{u}_N(p_4) \left( \tilde{G}_M\gamma^\mu - \tilde{F}_2 \frac{P^\mu}{M} + \tilde{F}_3 \frac{K P^\mu}{M^2} \right) u_N(p_2) \]

and

\[ T^{\text{flip}} = \frac{e^2}{Q^2} \bar{u}_e(p_3)\gamma_\mu u_e(p_1) \cdot \bar{u}_N(p_4) \left( \tilde{G}_M\gamma^\mu - \tilde{F}_2 \frac{P^\mu}{M} + \tilde{F}_3 \frac{K P^\mu}{M^2} \right) u_N(p_2). \]
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and

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T^{\text{flip}} = \frac{e^2}{Q^2} \frac{m_e}{M} \left[ \bar{u}_e(p_3)u_e(p_1) \cdot \bar{u}_N(p_4) \left( \tilde{F}_4 + \tilde{F}_5 \frac{K}{M} \right) u_N(p_2) + \tilde{F}_6 \bar{u}_e(p_3)\gamma_5 u_e(p_1) \cdot \bar{u}_N(p_4)\gamma_5 u_N(p_2) \right].
\]
In the two previous equations,

\[ \tilde{G}_M, \tilde{F}_2, \tilde{F}_3, \tilde{F}_4, \tilde{F}_5, \tilde{F}_6 \]

are complex functions of \( \nu \) and \( Q^2 \). In the Born approximation, these reduce to

We can also write

\[ \tilde{G}_E = \tilde{G}_M - (1 + \tau) \tilde{F}_2. \]
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\begin{align*}
\tilde{G}_M^{\text{Born}}(\nu, Q^2) &= G_M(Q^2) \\
\tilde{F}_2^{\text{Born}}(\nu, Q^2) &= F_2(Q^2) \\
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Gorchtein et al. found that for a spin parallel (anti-parallel) to the normal polarization vector

\[ S^\mu = (0, \vec{S}_n), \quad \vec{S}_n = (\vec{p}_1 \times \vec{p}_3) / |\vec{p}_1 \times \vec{p}_3| \]

there is a beam normal SSA \( B_n \) due to \( \mathcal{M}_\gamma^* \mathcal{M}_\gamma \). It is equal to
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there is a beam normal SSA \( B_n \) due to \( \mathcal{M}_\gamma^* \mathcal{M}_{\gamma\gamma} \). It is equal to

\[
B_n = \frac{2m_e}{Q} \sqrt{2\epsilon(1 - \epsilon)} \sqrt{1 + \frac{1}{\tau} \left(G_M^2 + G_E^2\right)^{-1}}
\]

\[
\times \left\{ -\tau G_M \text{Im} \left( \tilde{F}_3 + \frac{1}{1 + \tau M^2} \frac{\nu}{\tilde{M}^2} \tilde{F}_5 \right) - G_E \text{Im} \left( \tilde{F}_4 + \frac{1}{1 + \tau M^2} \frac{\nu}{\tilde{M}^2} \tilde{F}_5 \right) \right\}.
\]
Beam Normal SSA

Figure 3. Transverse Spin Polarization
If we consider a general transverse spin

$$\vec{s} = \cos \phi_s \hat{x} + \sin \phi_s \hat{y},$$

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the general beam normal SSA can be written

\[ B_{n, \text{gen}} = B_n \sin \phi_s. \]
Beam Transverse SSA

We found that a beam transverse SSA exists at the Born level due to $M_{\gamma}^*M_Z$. It is equal to

$$B_t = \frac{G_F m^2}{2 \pi \alpha} Q^2 \left( s - M^2 \right) \sqrt{\epsilon (1 - \epsilon)} \tau_2 \left( \tau + 1 \right) \left[ G^2 M + \epsilon \tau G^2 E \right]^{-1} \times \left\{ g_e V G^2 M G \tau \left( \tau + 1 \right) + g_e A \left( G^2 M G \tau (1 + \tau - \nu) + G E G \tau (1 - \tau - \nu) \right) \right\},$$

where $G^Z_M$ and $G^Z_E$ are the weak form factors and $G_F$ is Fermi’s coupling constant.

For a general transverse spin, $B_{t, \text{gen}} = B_t \cos \phi_s$. 
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$$\times \left\{ g^e V G_M G^Z_M \tau (\tau + 1) + g^e A \left( G_M G^Z_A \tau (1 + \tau - \nu) + G_E G^Z_E (1 - \tau - \nu) \right) \right\},$$

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$$\times \left\{ g_V^e G_M G_Z^Z \tau(\tau + 1) + g_A^e \left( G_M G_A^Z \tau(1 + \tau - \nu) + G_E G_E^Z (1 - \tau - \nu) \right) \right\},$$

where $G_M^Z$ and $G_E^Z$ are the weak form factors and $G_F$ is Fermi’s coupling constant.

For a general transverse spin,

$$B_{t, \text{gen}} = B_t \cos \phi_s.$$
Combination of Asymmetries

The interference between one- and two-photon exchange only produces a normal BSSA. The interference between one-photon and $Z$ exchange only produces a transverse BSSA. The combination of these two asymmetries gives

$$B = B_t \cos \phi_s + B_n \sin \phi_s = \sqrt{B_t^2 + B_n^2} \sin (\phi_s + \delta)$$

where $\delta = \tan^{-1} \left( \frac{B_t}{B_n} \right)$. 
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Using Q-weak kinematics, $Q^2 = 0.025 \text{ GeV}$, $E_1 = 1.125 \text{ GeV}$ (electron beam energy), we find

$$\delta \approx \tan^{-1}(B_t / B_n) \approx 1.116 \times 10^{-11} \times 5.350 \times 10^{-6} = 2.086 \times 10^{-11}.$$  

This is too small to affect the Q-weak measurements.
Combination of Asymmetries: Q-weak Kinematics

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- What factors contribute to \( B_n \)?
Inelastic Hadronic Intermediate State

Scattering process:

\[ e(p_1) + N(p_2) \rightarrow e(p_3) + N(p_4) \]

with intermediate leptonic momentum \( k \) and intermediate hadronic momentum \( W = p_2 + q_1 \).
One- and Two-Photon Interference

- We consider the contribution to the normal BSSA from $\mathcal{M}_\gamma \mathcal{M}_{\gamma \gamma}$. 
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- For polarized beam, spinor relation becomes

$$\bar{u}_e(k)u_e(k) = (1 + \gamma_5 \not{s})(\not{k} + m_e).$$
One- and Two-Photon Interference

- We consider the contribution to the normal BSSA from $M_\gamma^* M_\gamma$. 
- For polarized beam, spinor relation becomes 
  $$\bar{u}_e(k) u_e(k) = (1 + \gamma_5 \gamma)(k + m_e).$$ 
- The BSSA comes from the absorptive part of the two-photon exchange amplitude (Pasquini & Vanderhaeghen PRC 70, 2004):
  $$B_n = \frac{2 \text{Im} \left( \sum_{\text{spins}} M_\gamma^* \cdot \text{Abs} M_\gamma \right)}{\sum_{\text{spins}} |M_\gamma|^2}.$$
Leptonic and Hadronic Tensors

We can write this as

\[ B_n = \frac{e^6}{(2\pi)^3 Q^2} \left( \sum_{\text{spins}} |M_\gamma|^2 \right)^{-1} \int \frac{d^3 \vec{k}}{E_k} \frac{1}{Q_1^2 Q_2^2} \text{Im} \{ L_{\alpha\mu\nu} H^{\alpha\mu\nu} \}, \]
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where

\[ L_{\alpha\mu\nu} = \text{Tr} \left[ \frac{1}{2} (1 + \gamma_5 \gamma^5) (p_1^\prime + m_e) \gamma_\alpha (p_1^\prime - q_1 + m_e) \gamma_\mu (p_1^\prime - q_1^\prime + m_e) \gamma_\nu \right]. \]
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where

\[ L_{\alpha\mu\nu} = \text{Tr} \left[ \frac{1}{2} \left( 1 + \gamma_5 \frac{s}{p} \right) (p_1 + m_e) \gamma_\alpha (p_1 - q + m_e) \gamma_\mu (p_1 - q_1 + m_e) \gamma_\nu \right]. \]

and we take the forward limit on the hadronic tensor:

\[ H^{\alpha\mu\nu} = 2p_2^\alpha W^{\mu\nu} \]
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where

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\[ H_{\alpha\mu\nu} = 2 p_2^\alpha W^{\mu\nu} \]


\[ W^{\mu\nu} = \left( -g^{\mu\nu} + \frac{q_1^\mu q_1^\nu}{q_1^2} \right) W_1 + \frac{1}{M^2} \left( p_2^\mu - \frac{p_2 \cdot q_1}{q_1^2} q_1^\mu \right) \left( p_2^\nu - \frac{p_2 \cdot q_1}{q_1^2} q_1^\nu \right) W_2. \]
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\[ B_n = -\frac{1}{(2\pi)^3} \frac{e^2 Q^2}{D(s, Q^2)} \frac{1}{8E_1 E_3 M} \int_0^{2\pi} \int_0^s \int_0^{Q_{1,\text{max}}} d\phi_k dW^2 dQ_1^2 \]

\[ \times \frac{\text{Im} \{ L_{\alpha\mu\nu} H^{\alpha\mu\nu} \}}{Q_1^2 | \vec{k} | (1 - \cos \theta \cos \theta_k - \sin \theta \sin \theta_k \cos \phi_k)} \]

where \( E_1 \) and \( E_3 \) are the energies of the incoming and outgoing leptons, \( \theta_k \) is the angle between \( \vec{p}_1 \) and \( \vec{k} \), and \( \phi_k \) is the azimuthal angle.
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\[ D(s, Q^2) = \frac{8Q^4}{1 - \epsilon} \left\{ G_M^2 + \frac{\epsilon}{\tau} G_E^2 \right\}, \]
Once again, the beam single spin asymmetry is given by:

\[
B_n = -\frac{1}{(2\pi)^3} \frac{e^2 Q^2}{D(s, Q^2)} \frac{1}{8E_1 E_3 M} \int_0^{2\pi} d\phi_k \int_M^s dW^2 \int_0^{Q_{1,\text{max}}} dQ_1^2 \times \frac{\text{Im} \{L_{\alpha\mu\nu} H^{\alpha\mu\nu}\}}{Q_1^2 |\vec{k}| (1 - \cos \theta \cos \theta_k - \sin \theta \sin \theta_k \cos \phi_k)}
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Once again,

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\[ \times \frac{\text{Im} \{ L_{\alpha\mu\nu} H^{\alpha\mu\nu} \}}{Q_1^2 |\vec{k}| (1 - \cos \theta \cos \theta_k - \sin \theta \sin \theta_k \cos \phi_k)} \]

The tensor contraction is

\[ \text{Im} L_{\alpha\mu\nu} H^{\alpha\mu\nu} = \frac{8m_e}{M^2 Q_1^2} \left[ 2W_1 M^2 Q_1^2 (\epsilon^{p_1 p_2 q s} + \epsilon^{p_2 q q_1 s}) \right. \]

\[ + W_2 \epsilon^{p_2 q q_1 s} \left( (M^2 - Q_1^2 - W^2)(p_1 \cdot p_2) - M^2 Q_1^2 \right) \right]. \]
Once again,

\[
B_n = -\frac{1}{(2\pi)^3} \frac{e^2 Q^2}{D(s, Q^2)} \frac{1}{8E_1 E_3 M} \int_0^{2\pi} d\phi_k \int_0^s dW^2 \int_0^{Q_{1,\text{max}}} dQ_1^2 \int \frac{\text{Im} \{L_{\alpha\mu\nu} H^{\alpha\mu\nu}\}}{Q_1^2 |\vec{k}| (1 - \cos \theta \cos \theta_k - \sin \theta \sin \theta_k \cos \phi_k)} \]

The tensor contraction is

\[
\text{Im} L_{\alpha\mu\nu} H^{\alpha\mu\nu} = \frac{8m_e}{M^2 Q_1^2} \left[ 2W_1 M^2 Q_1^2 (\epsilon^{p_1 p_2 q s} + \epsilon^{p_2 q q_1 s}) + W_2 \epsilon^{p_2 q q_1 s} \left( (M^2 - Q_1^2 - W^2)(p_1 \cdot p_2) - M^2 Q_1^2 \right) \right].
\]

→ Currently evaluating for Q-weak kinematics.
Beam single spin asymmetries result from the difference in cross sections produced by the beam polarization in electron-proton scattering experiments.
Conclusion

1. Beam single spin asymmetries result from the difference in cross sections produced by the beam polarization in electron-proton scattering experiments.

2. The interference between one- and two-photon exchange amplitudes produces a beam normal single spin asymmetry.

3. The interference between one-photon and $Z$ exchange amplitudes produces a beam transverse single spin asymmetry.

4. The combination of these two asymmetries produces a small phase shift that may be detectable in the future.

5. The inelastic hadronic intermediate state produces an important contribution to the BSSA (soon to be quantified).

6. The framework developed here could potentially be used to compute other beam or target SSA at near-forward angles.
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References


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Questions?