Heavy Quarkonium in a Quark-Gluon Plasma

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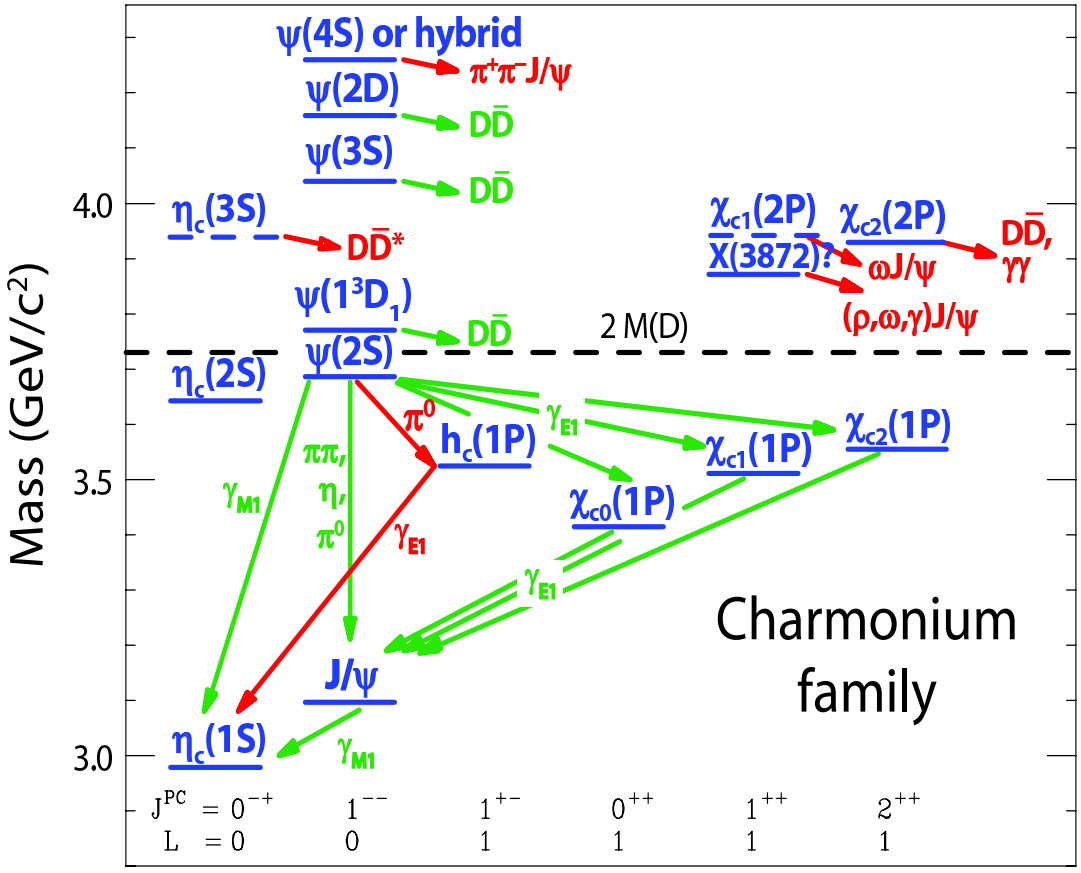
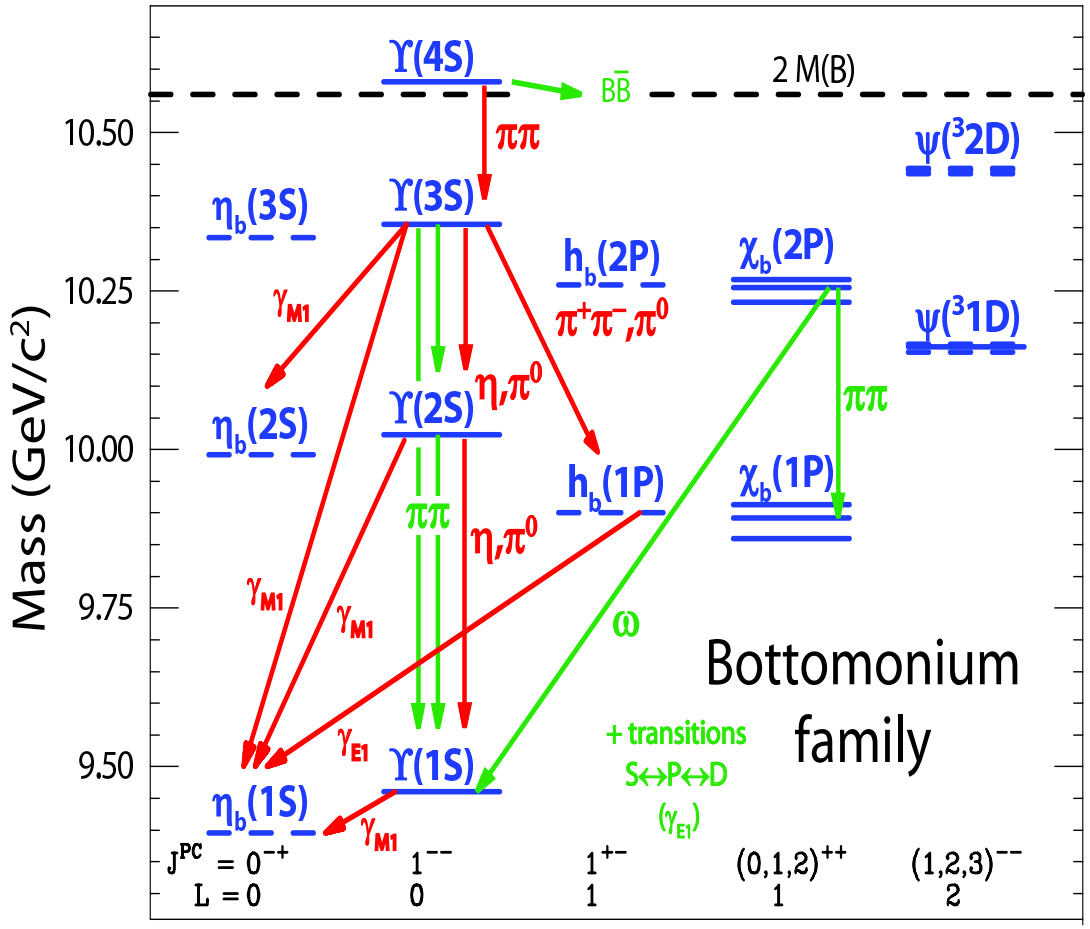
Charmonium



Eichten, Godfrey, Mahlke, Rosner (2007)

Bottomonium



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- A more realistic setting for Heavy Ion Collisions

QED bound states (hydrogen atom)

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi} \left(i \not{D} - m \right) \Psi + N^{\dagger} i D_0 N$$

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- Relevant scales:
  - m (hard), electron mass
  - $m\alpha$ (soft), inverse Bohr radius, $\alpha = e^2/4\pi$; e, electron charge
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- We may obtain it by integrating out energies and momenta at the hard and soft scales.

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- The EFT gives equivalent physical results in the region where it holds
  - It may make apparent accidental symmetries in that region, which help constraining the physics.
  - Calculations are usually simpler.

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- EFTs are useful:
  - m (hard), QED
  - $m\alpha$ (soft), NRQED
  - $m\alpha^2$ (ultrasoft), pNRQED

Non-Relativistic QED

$$\mathcal{L}_{NRQED} = -\frac{1}{4} d_1 F_{\mu\nu} F^{\mu\nu} + \frac{d_2}{m^2} F_{\mu\nu} D^2 F^{\mu\nu} + N^\dagger i D^0 N +$$
$$+\psi^\dagger (i D^0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} + c_F e \frac{\boldsymbol{\sigma}\mathbf{B}}{2m} + c_D e \frac{\boldsymbol{\nabla}\mathbf{E}}{8m^2} +$$
$$+ i c_S e \frac{\boldsymbol{\sigma}(\mathbf{D}\times\mathbf{E} - \mathbf{E}\times\mathbf{D})}{8m^2}) \psi + \cdots$$

- ψ are Pauli spinors
- The matching coefficients $d_1, d_2, c_F, c_D, c_S, \ldots$ contain all the physics at the hard scale m

(Caswell, Lepage, 1986)

Potential NRQED

$$L_{pNRQED} = -\int d^3\mathbf{x}\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \int d^3\mathbf{x}S^\dagger(t,\mathbf{x})\Bigg(iD_0 + \frac{\boldsymbol{\nabla}^2}{2m} + \frac{Z\alpha}{|\mathbf{x}|} +$$

$$+\frac{\boldsymbol{\nabla}^4}{8m^3} + \frac{Ze^2}{m^2}\left(-\frac{c_D}{8} + 4d_2\right)\delta^3(\mathbf{x}) + ic_S\frac{Z\alpha}{4m^2}\boldsymbol{\sigma}\cdot\left(\frac{\mathbf{x}}{|\mathbf{x}|^3}\times\boldsymbol{\nabla}\right)\Bigg)S(t,\mathbf{x})$$

$$+\int d^3\mathbf{x}S^\dagger(t,\mathbf{x})e\mathbf{x}\cdot\mathbf{E}S(t,\mathbf{x}) + \cdots$$

- $S(t,\mathbf{x})$ is the Hydrogen wave function field
- The potentials encode the physics at the soft scale $m\alpha$

(Pineda, Soto, 1997)

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- $e\mathbf{x}\cdot\mathbf{E}$, residual interaction with the e.m. field leading to:
  - Non-potential effects (e.g. Bethe logs in the Lamb shift)
  - Van der Waals forces

pNRQED

- Selected results:

  - A. Pineda, JS, Phys.Rev.D59:016005,1999,
    " Potential NRQED: The Positronium case" [α^3 corrections to the spectrum]
  - B. A. Kniehl, A.A. Penin, Phys.Rev.Lett.85:1210,2000 ,
    " Order $\alpha^3 \ln(1/\alpha)$ corrections to positronium decays"
  - B. A. Kniehl, A.A. Penin, Phys.Rev.Lett.85:5094,2000,
    "Order $\alpha^7 \ln(1/\alpha)$ contribution to positronium hyperfine splitting"
  - A. Pineda, Phys.Rev.A66:062108,2002 ,
    " Renormalization group improvement of the spectrum of hydrogen - like atoms with massless fermions"
  - M. Baker, P. Marquard, A. Penin, J. Piclum, M. Steinhauser, Phys. Rev. Lett. 112, 120407 (2014),
    "Hyperfine splitting in positronium to $O(\alpha^7 m_e)$: one-photon annihilation contribution"

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  - t-$\bar{t}$
  - $\tilde{q}\bar{\tilde{q}}$, $\tilde{g}\bar{\tilde{g}}$, $\cdots$
  - Double-heavy baryons (ccq, $q = u, d, s$)

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  - $m_Q >> m_Q v >> m_Q v^2$
  - $m_Q >> \Lambda_{QCD}$
- EFTs are useful (N. Brambilla, A. Pineda, JS and A. Vairo, Rev. Mod. Phys. 77, 1423 (2005))

NRQCD

W.E. Caswell and G.P. Lepage, Phys. Lett. **167B**, 437 (1986)
G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51** (1995) 1125

$$m_Q \quad >> \quad m_Q v \quad , \quad m_Q v^2 \quad , \quad \Lambda_{QCD}$$

$$\begin{aligned}\mathcal{L}_\psi &= \psi^\dagger \Bigg\{ iD_0 + \frac{1}{2m_Q}\mathbf{D}^2 + \frac{1}{8m_Q^3}\mathbf{D}^4 + \frac{c_F}{2m_Q}\boldsymbol{\sigma}.g\mathbf{B} + \\ &+ \frac{c_D}{8m_Q^2}\left(\mathbf{D}.g\mathbf{E} - g\mathbf{E}.\mathbf{D}\right) + i\,\frac{c_S}{8m_Q^2}\boldsymbol{\sigma}.\left(\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times\mathbf{D}\right) + \ldots \Bigg\}\psi\end{aligned}$$

c_F, c_D and c_S are short distance matching coefficients which depend on m_Q and μ (factorization scale)

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    - Mode decomposition

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      - $\mathcal{L}_{pNRQCD}$ depends on the relative size between Λ_{QCD} and $m_Q v^2$
    - Mode decomposition $\longrightarrow$ vNRQCD, M. E. Luke, A. V. Manohar and I. Z. Rothstein, Phys. Rev. D **61**, 074025 (2000)
      - Λ_{QCD} is neglected

pNRQCD

$\Lambda_{QCD} \lesssim m_Q v^2$: weak coupling regime

$$\mathcal{L}_{\rm pNRQCD} = \int d^3\mathbf{r}\, {\rm Tr} \Big\{ {\rm S}^\dagger \left(i\partial_0 - h_s(\mathbf{r}, \mathbf{p}, \mathbf{P_R}, \mathbf{S}_1, \mathbf{S}_2, \mu) \right) {\rm S} +$$

$$+ {\rm O}^\dagger \left(iD_0 - h_o(\mathbf{r}, \mathbf{p}, \mathbf{P_R}, \mathbf{S}_1, \mathbf{S}_2, \mu) \right) {\rm O} \Big\}$$

$$+ V_A(r,\mu) {\rm Tr} \left\{ {\rm O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, {\rm S} + {\rm S}^\dagger \mathbf{r} \cdot g\mathbf{E}\, {\rm O} \right\}$$

$$+ \frac{V_B(r,\mu)}{2} {\rm Tr} \left\{ {\rm O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, {\rm O} + {\rm O}^\dagger {\rm O} \mathbf{r} \cdot g\mathbf{E} \right\} + \ldots$$

h_s, $h_o =$ quantum mechanical Hamiltonians with scale dependent potentials calculable in perturbation theory in $\alpha_{\rm s}(m_Q v)$

pNRQCD (Cont.)

$\Lambda_{QCD} \lesssim mv$: strong coupling regime

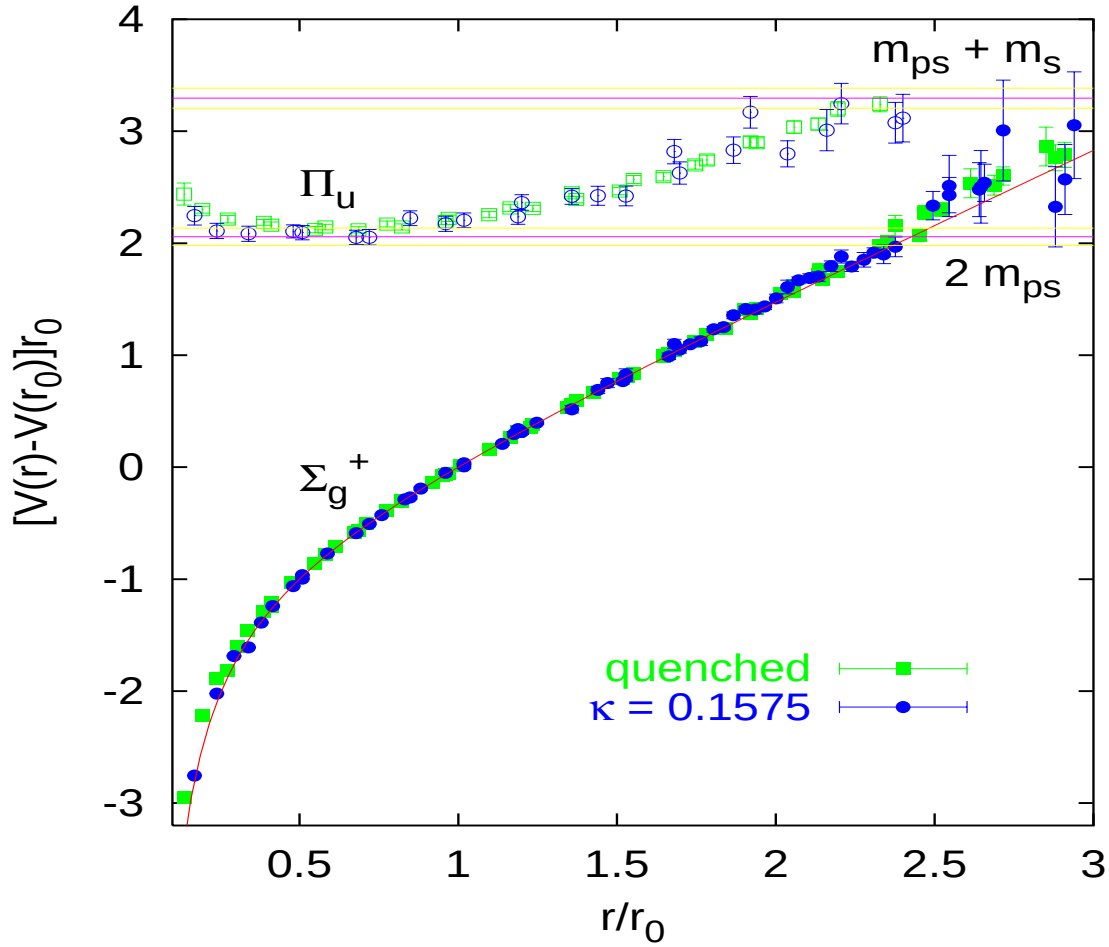
$$L_{\text{pNRQCD}} = \int d^3\mathbf{x}_1 \int d^3\mathbf{x}_2 \; S^\dagger \big(i\partial_0 - h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2)\big) S,$$

$$h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2) = \frac{\mathbf{p}_1^2}{2m_Q} + \frac{\mathbf{p}_2^2}{2m_Q} + V_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2),$$

$$V_s = V_s^{(0)} + \frac{V_s^{(1)}}{m_Q} + \frac{V_s^{(2)}}{m_Q^2} + \cdots ,$$

All V_ss can be, and most of them have been, calculated on the lattice

pNRQCD (Cont.)



G.S. Bali at al. (TXL Collaboration), Phys. Rev. **D62**,(2000):054503

Weak coupling: recent applications

- Spectrum and decays of lowest lying states in bottomonium, charmonium and B_c
  - Bottomonium spectrum (Kiyo, Sumino, 2013)
  - Magnetic transitions (Pineda, Segovia, 2013)
- Precision determination of Standard Model parameters
  - Heavy quark masses
    - $\bar{m}_b(\bar{m}_b) = 4201(43)$ MeV , from NNNLO spectrum (Ayala, Cvetic, Pineda, 2014)
  - Strong coupling constant α_s
    - $\alpha_s(M_Z) = 0.1166^{+0.0012}_{-0.0008}$, from comparing NNNLO static energy with lattice data (Bazavov, Brambilla, Garcia i Tormo, Petreczky, JS, Vairo, 2014)

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- Quark-Gluon Plasma (QGP)
  - If $T \gg \Lambda_{QCD}$ then $\alpha_{\rm s}(T) \ll 1 \Rightarrow$ weakly coupled quarks and gluons
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  - It is expected to be produced in Heavy Ion Collision (HIC) experiments (RHIC, LHC)
    - Properties of the QGP are extracted indirectly by comparing
      - $A\,A \longrightarrow X +$ anything
      - $p\,p \longrightarrow X +$ anything
    - $X =$ Hard Probe (jets, heavy quarks, heavy quarkonia,. . .) is particularly useful

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$$V(r) \longrightarrow V(r,T) = -\frac{C_f \alpha_{\rm s}}{r} e^{-m_D r} \ , \quad m_D \sim gT$$

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    - Can we quantify these arguments from QCD?

QED as a toy model for QCD

- Muonic Hydrogen $\sim$ Heavy Quarkonium (Eiras, JS, 00)
  - Muon, Proton $\sim$ Heavy Quarks
  - Photon $\sim$ Gluons
  - Electron, Positron $\sim$ Light Quarks
- Electron-positron plasma (EPP)$\sim$ Quark-gluon plasma
  - Photon $\sim$ Gluons
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M.A. Escobedo, JS, Phys. Rev. A 78, 032520 (2008); A 82, 042506 (2010)

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    - Technically more involved
    - Relevant for actual muonic hydrogen in an electron-positron plasma

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- $m \neq 0$, $T \neq 0$ case: what is the interplay among the scales above?

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- $m \neq 0$, $T \neq 0$ case: contributions of energies above T are exponentially suppressed by Boltzmann factors

Hard Thermal Loops EFT (m=0)

$$\delta\mathcal{L}_{HTL} = \frac{1}{2}m_D^2 F^{\mu\alpha}\int\frac{d\Omega}{4\pi}\frac{k_\alpha k_\beta}{-(k.\partial)^2}F^{\mu\beta} + m_f^2\bar{\psi}\gamma^\mu\int\frac{d\Omega}{4\pi}\frac{k_\mu}{k.D}\psi$$

$$k = (1, \hat{\mathbf{k}}), \qquad m_D^2 = e^2T^2/3, \qquad m_f^2 = e^2T^2/8$$

(Braaten, Pisarsky, 1992)

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  - For $T = \beta^{-1} \gg m_\mu \alpha^2 / n^2$, HTL resummations necessary

The $m_e = 0$, $T \ll m_\mu \alpha / n$ case

pNRQED can be used as a starting point

- The potentials remain the same as in the $T = 0$ case
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    - $T \gg eT \gg m_\mu \alpha^2 / n^2$

The $T \ll m\alpha/n$ case

- For $T = \beta^{-1} \ll m\alpha^2/n^2$:

$$\begin{aligned}\delta E_n &= -\frac{4\pi^3\alpha}{45\beta^4}\langle n|\mathbf{x}\frac{\bar{P}_n}{(H_0 - E_n)}\mathbf{x}|n\rangle\left(1 + \mathcal{O}\left((\frac{n^2}{\beta m\alpha})^2\right)\right)\\ \delta\Gamma_n &= 0\end{aligned}$$

- For $T = \beta^{-1} \gg m\alpha^2/n^2$:

$$\begin{aligned}\delta E_n &= \frac{\alpha\pi}{3m\beta^2} + \frac{2\alpha}{3\pi}\sum_r |\langle n|\mathbf{v}|r\rangle|^2(E_n - E_r)(\ln(\frac{2\pi}{\beta|E_n - E_r|}) - \gamma)\\ &\quad \times(1 + \mathcal{O}\left((\frac{\beta m\alpha}{n^2})^2\right))\\ \delta\Gamma_n &= \frac{4Z^2\alpha^3}{3\beta n^2}\left(1 + \mathcal{O}\left(\frac{\beta m\alpha}{n^2}\right)\right)\end{aligned}$$

The $m_e = 0$, $T \ll m_\mu$ case

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  - For $T \gg m_\mu \alpha / n$, and $eT \sim m_\mu \alpha / n \Rightarrow$ all contributions to the potential from HTLs

The $m_e = 0$, $m_\mu \alpha / n \ll T \ll m_\mu$ case

- The results also hold for heavy quarkonium [$\alpha \leftrightarrow C_f \alpha_s$, $m_\mu \leftrightarrow m_Q/2$, $m_D^2 = e^2 T^2/3 \leftrightarrow m_D^2 = g^2 T^2 (N_c + N_f/2)$]

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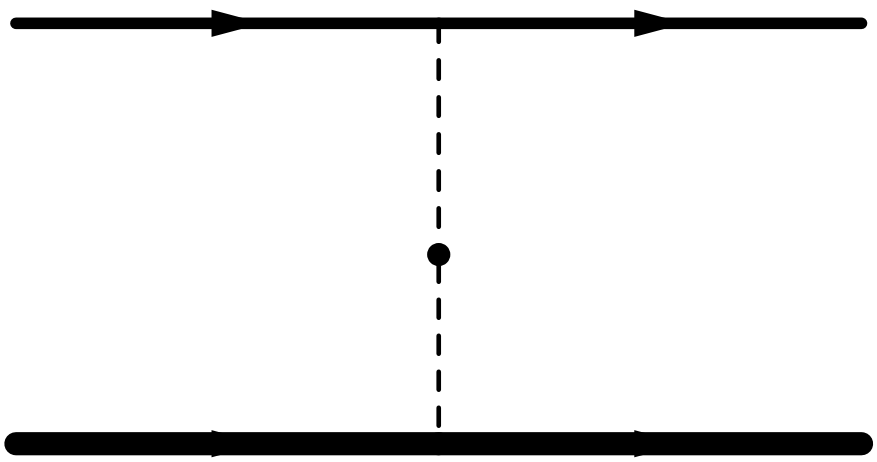
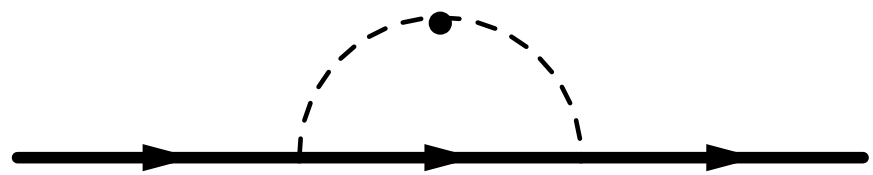




The $m_e = 0$, $m_\mu \alpha / n \ll T \ll m_\mu$ case

$$V(r,T) = -\frac{\alpha e^{-m_D r}}{r} - \alpha m_D + i\alpha T \phi(m_D r) + \mathcal{O}\left(\frac{\alpha T^2}{m_\mu}\right)$$

$$\phi(x) = 2\int_0^\infty \frac{dz z}{(z^2+1)^2}\left[\frac{\sin(zx)}{zx} - 1\right]$$

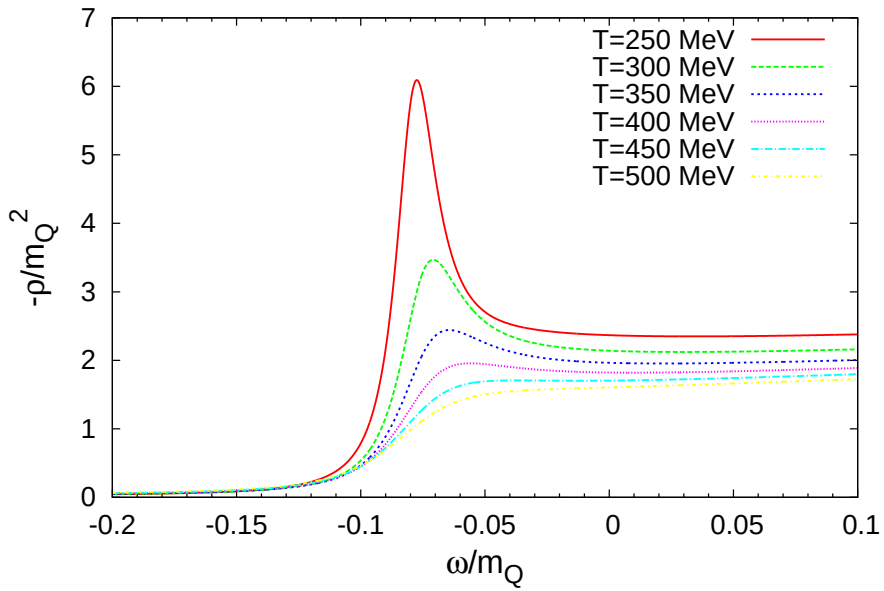
- It has an imaginary part ! (Laine, Philipsen, Romatschke, Tassler, 06; Escobedo, JS, 08; Brambilla, Ghiglieri, Vairo, Petreczky, 08)
- The dissociation temperature becomes

$$T_d \sim m_\mu \alpha^{2/3} / \ln^{1/3} \alpha \; < \; m_\mu \alpha^{1/2}$$

where $m_\mu \alpha^{1/2}$ ($\sim m_D \sim eT$) is the scale of the disociation temperature for the screening mechanism (Matsui, Satz, 86)

The $m_q = 0$, $m_Q \alpha_s \ll T \ll m_Q$ case

- The melting temperature T_d can be parametrically estimated to be $T_d \sim -m_Q \alpha_s^{2/3} / \ln^{1/3} \alpha_s$
  - $\Upsilon(1S) \longrightarrow T_d \sim 500 MeV$
  - $J/\psi \longrightarrow T_d \sim 200 MeV$
- The $\Upsilon(1\mathrm{S})$ spectral function



Moving through the QGP

- Bound state at rest, the medium moves at velocity v (Weldom, 82)

$$f(\beta k^0) \rightarrow f(\beta^\mu k_\mu) = \frac{1}{e^{|\beta^\mu k_\mu|} \pm 1}\,, \;\; \beta^\mu = \frac{\gamma}{T}(1, \mathbf{v})$$

$v = |\mathbf{v}|$, $\gamma = 1/\sqrt{1-v^2}$

- $O(3)$ rotational symmetry is reduced to $O(2)$
- In light cone coordinates $k_+ = k_0 + k_3$, $k_- = k_0 - k_3$

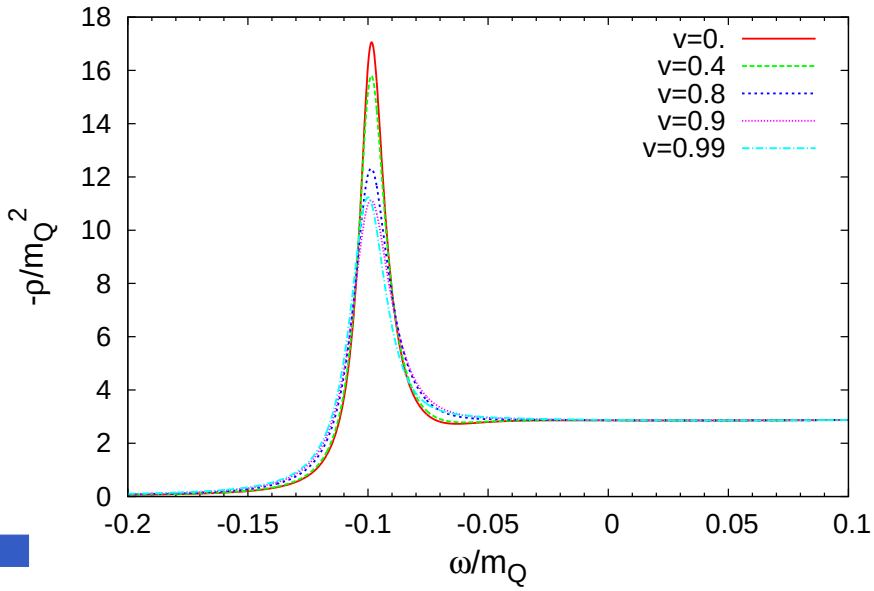
$$\beta^\mu k_\mu = \frac{1}{2}\left(\frac{k_+}{T_+} + \frac{k_-}{T_-}\right)\,, \;\; T_+ = T\sqrt{\frac{1+v}{1-v}}\,, \;\; T_- = T\sqrt{\frac{1-v}{1+v}}$$

  - For $v \nsim 1$ (moderate velocities), $T_+ \sim T \sim T_-$
  - For $v \sim 1$ (relativistic velocities), $T_+ \gg T \gg T_-$
    - Collinear region, $k_+ \sim T_+$, $k_- \sim T_-$
    - Soft (ultrasoft) region, $k_+ \sim k_- \sim T_-$

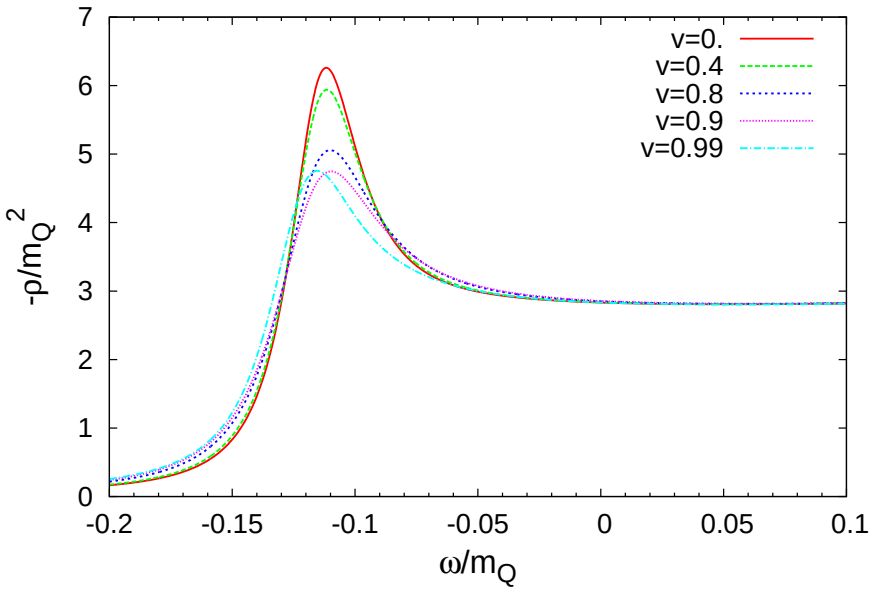
Moving through the QGP

- Results (Escobedo, Mannarelli, JS, 11; + Giannuzzi, 13):
  - For $T \lesssim m\alpha/n$ the thermal decay width (Γ) decreases with v
  - For $eT \sim m\alpha/n \ll T$:
    - At moderate velocities Γ increases with v
    - At ultrarelativistic velocities Γ decreases with v
  - The $\Upsilon(1\mathrm{S})$ spectral function at $v \neq 0$

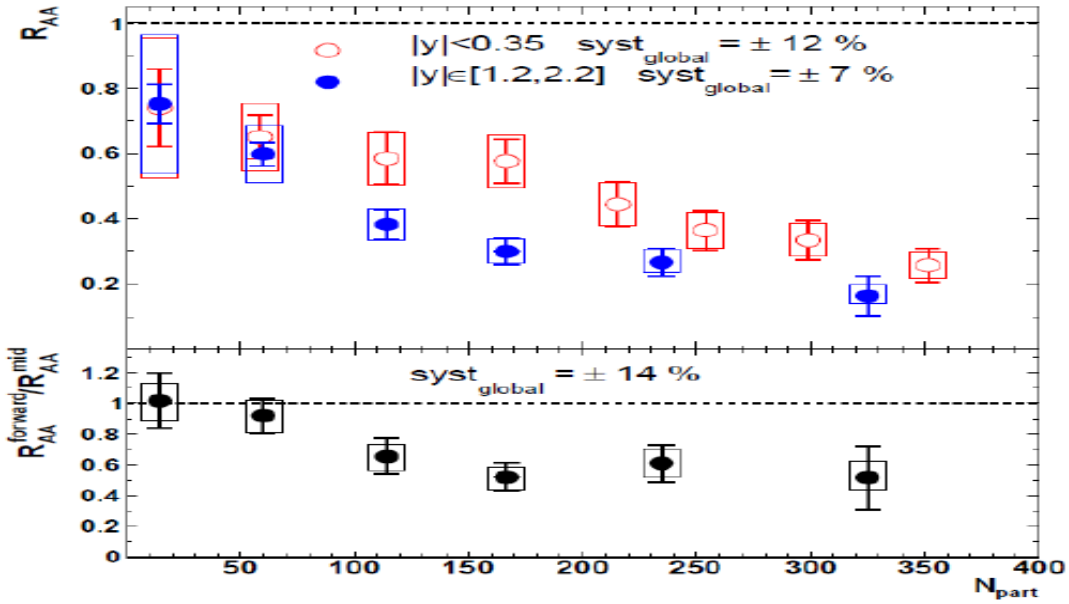
$T = 250$ MeV



$T = 400$ MeV



Motivation for $v \neq 0$



(H. Pereira Da Costa, for the PHENIX collaboration, arXiv:1007.3688)

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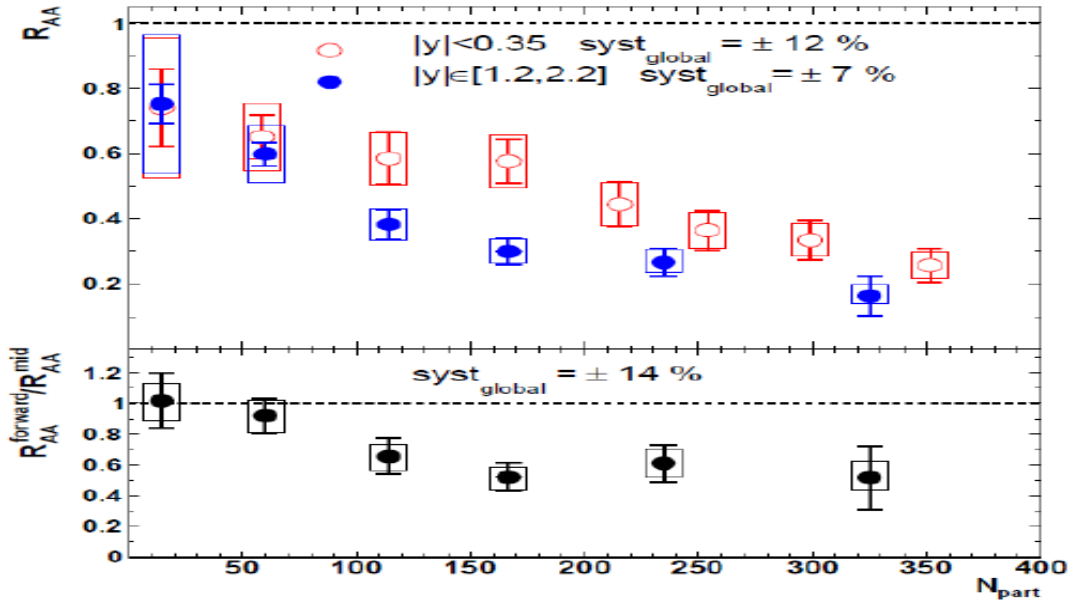

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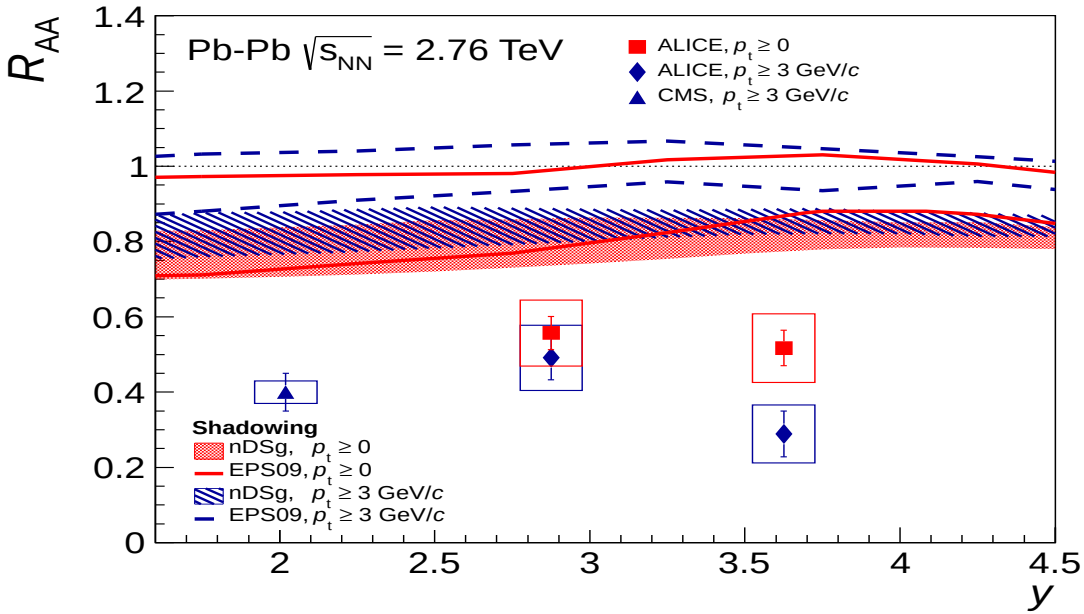
- The J/ψ suppression depends on the rapidity

Motivation for $v \neq 0$



(ALICE collaboration, arXiv:1202.1383)

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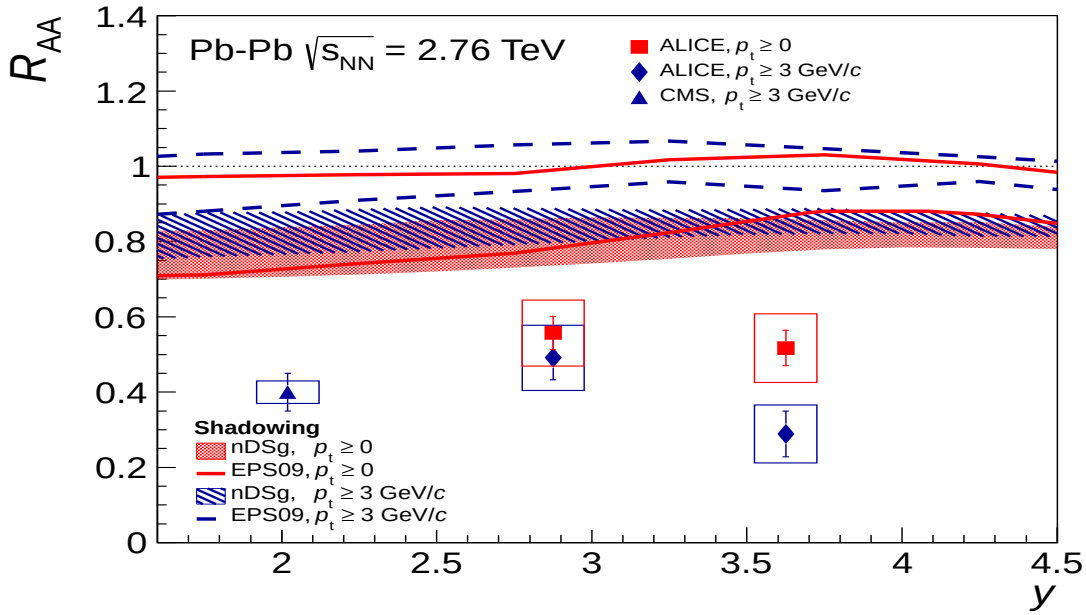
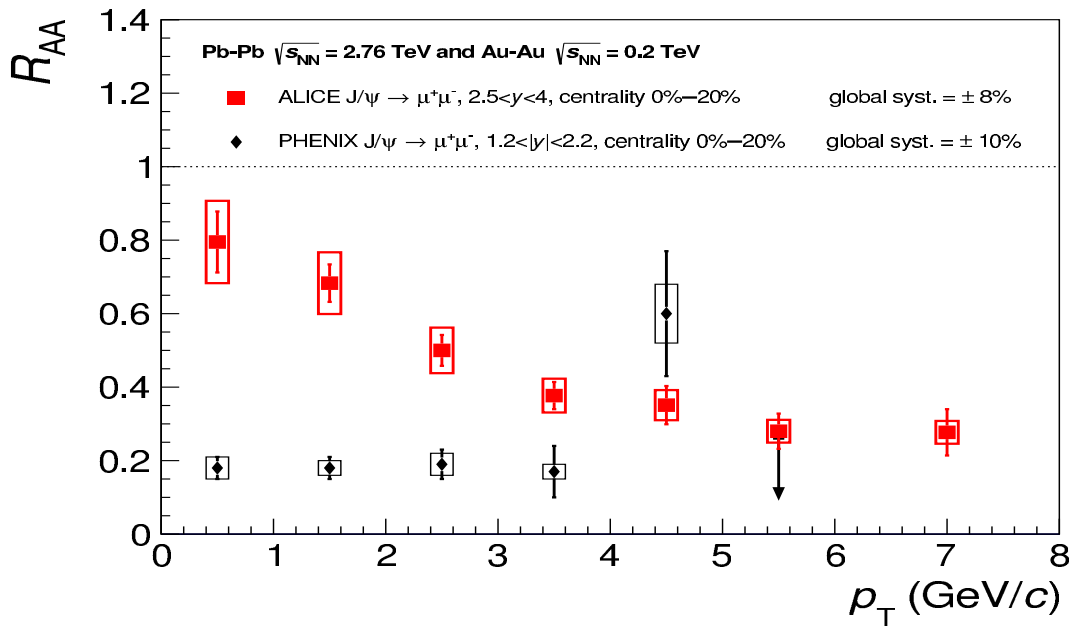


(ALICE collaboration, arXiv:1202.1383)

- The J/ψ suppression may depend on the transverse momentum

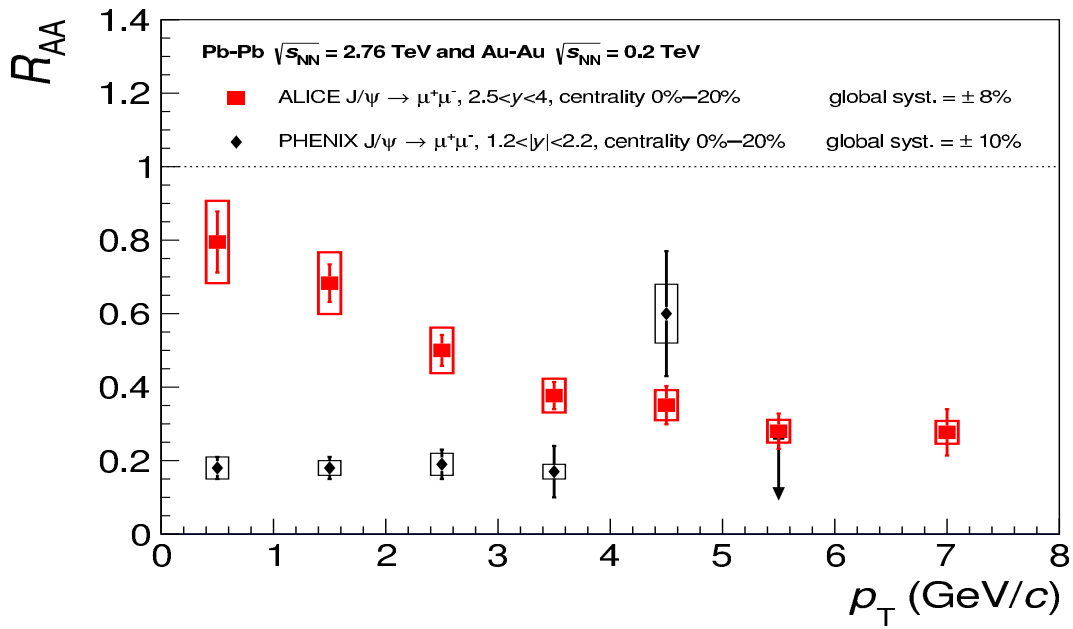
Motivation for $v \neq 0$



(ALICE collaboration, arXiv:1311.0214)

- The J/ψ suppression does depend on the transverse momentum

Motivation for $v \neq 0$



(ALICE collaboration, arXiv:1311.0214)

- The J/ψ suppression does depend on the transverse momentum
- Is this due, at least in part, to the fact that the in-medium properties of J/ψ depend on the velocity ?

Embeding it in Heavy Ion Collisions

M.A. Escobedo, proceedings Confinement 2014

N. Brambilla, M.A. Escobedo, JS, A. Vairo, in preparation

- Not really an equilibirium QGP
  - t_0, formation time
  - Expansion (T decreases)
  - t_F, hadronization time ($T \sim T_c$)
- Open quantum system approach:
  - At $t = 0$, $\rho = \rho_{HQ} \otimes \rho_{ldf}$
  - From $0 < t < t_0$, ρ_{HQ} and ρ_{ldf} evolve independently, ρ_{HQ} as in the vacuum, and ρ_{ldf} to ρ_{QGP}
  - From $t_0 < t < t_F$, ρ_{QGP} evolves according to the Bjorken model, and ρ_{HQ} according to pNRQCD ($rT \ll 1$)

Dilepton emission

$$d\mathcal{R} = -\frac{e^2 L_{\mu\nu}(k_1,k_2)}{|\mathbf{k_1}||\mathbf{k_2}|(k_1+k_2)^4}\int d^4\lambda_1\, d^4\lambda_2 e^{-i(k_1+k_2)(\lambda_1-\lambda_2)} Tr(\rho J^\mu(\lambda_1) J^\nu(\lambda_2))$$

McLerran, Toimela (85)

- k_1, k_2, lepton momenta
- $J^\mu(\lambda_1)$ QCD electromagnetic current

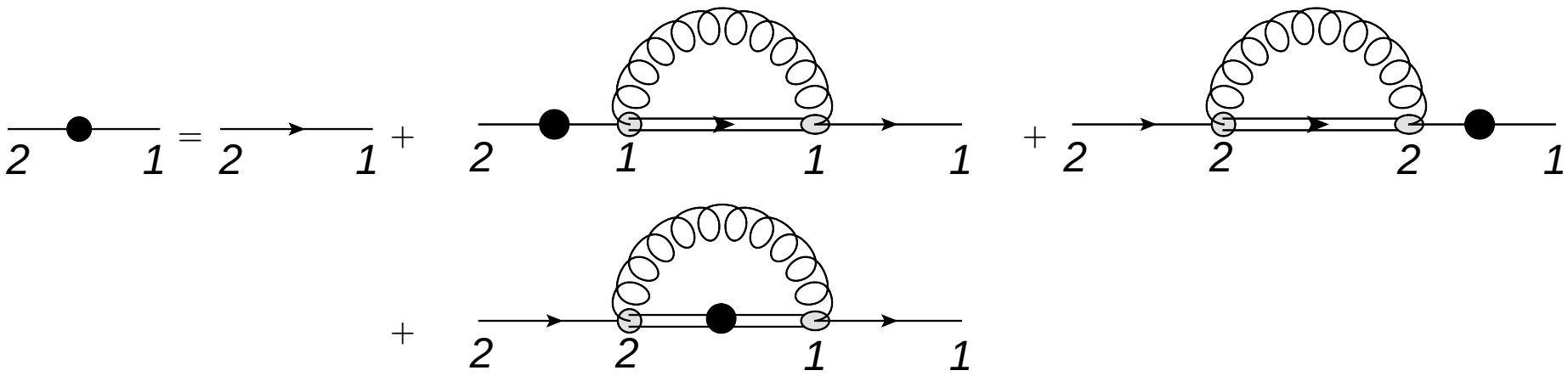
The formula is adapted to include:

- Heavy quarks in $J^\mu(\lambda_1)$
- Non thermalized heavy quarks in $\rho = \rho_{HQ} \otimes \rho_{ldf}$

Evolution of ρ_{HQ}

$$\rho_{HQ} = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

For ρ_s we have:



$$\frac{d\rho_s(t)}{dt} = -ih_{s,eff}(t)\rho_s(t) + i\rho_s(t)h^{\dagger}_{s,eff}(t) + \mathcal{F}(\rho_o(t), t)$$

Evolution of ρ_{HQ}

For ρ_o we have:

$$\frac{d\rho_o(t)}{dt} = -ih_{o,eff}(t)\rho_o(t) + i\rho_o(t)h^{\dagger}_{o,eff}(t) + \mathcal{F}_1(\rho_s(t),t) + \mathcal{F}_2(\rho_o(t),t)$$

- For quasistatic evolution of ρ_{QGP} ($dT/dt \ll TE$) , $h_{s,eff}(t)$, $h_{o,eff}(t)$, $\mathcal{F}(\rho_o(t),t)$, $\mathcal{F}_1(\rho_s(t),t)$, $\mathcal{F}_2(\rho_o(t),t)$, can be obtained from equilibrium distributions.
- If in addition t is large ($t \gg 1/E$), $h_{s,eff}(t)$, $h_{o,eff}(t)$ become slowly varying in time and can be obtained directly from previous calculations.
- We have further assumed that ρ_o is diagonal in color space

Solution of $\rho_{HQ}(t)$

- Initial conditions:
  - Scaling of NRQCD LO production: $\rho_o = b\rho_s/\alpha_{\rm s}(m)$, $b = \mathcal{O}(1)$
  - $Tr(\rho_s) + Tr(\rho_o) = 1$
- Rewrite the evolution equation in the Lindblad form

$$\frac{d\rho_{HQ}}{dt} = -i[H, \rho_{HQ}] + \sum_i (C_i \rho_{HQ} C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho_{HQ}\})$$

- Decompose in partial waves and truncate beyond a given angular momentum ($\int_{t_0}^{t} dt' r^2 T(t')^3 \ll 1$)
- Use standard libraries to numerically solve the Lindblad equation (Johansson, Nation, Nori, 12)

Solution of $\rho_{HQ}(t)$

- The case $1/r \gg T \gg E \gg m_D$

$$H = \frac{1}{2}\begin{pmatrix} h_{s,eff} + h^\dagger_{s,eff} & 0 \\ 0 & h_{o,eff} + h^\dagger_{o,eff} \end{pmatrix}.$$

$$C_i^0 = \sqrt{\frac{4T_F\alpha_s(\mu_E)T}{3}}\left(\frac{2ip_i}{M_b} + \frac{N_c\alpha_s(1/a_o)r_i}{2r}\right)\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

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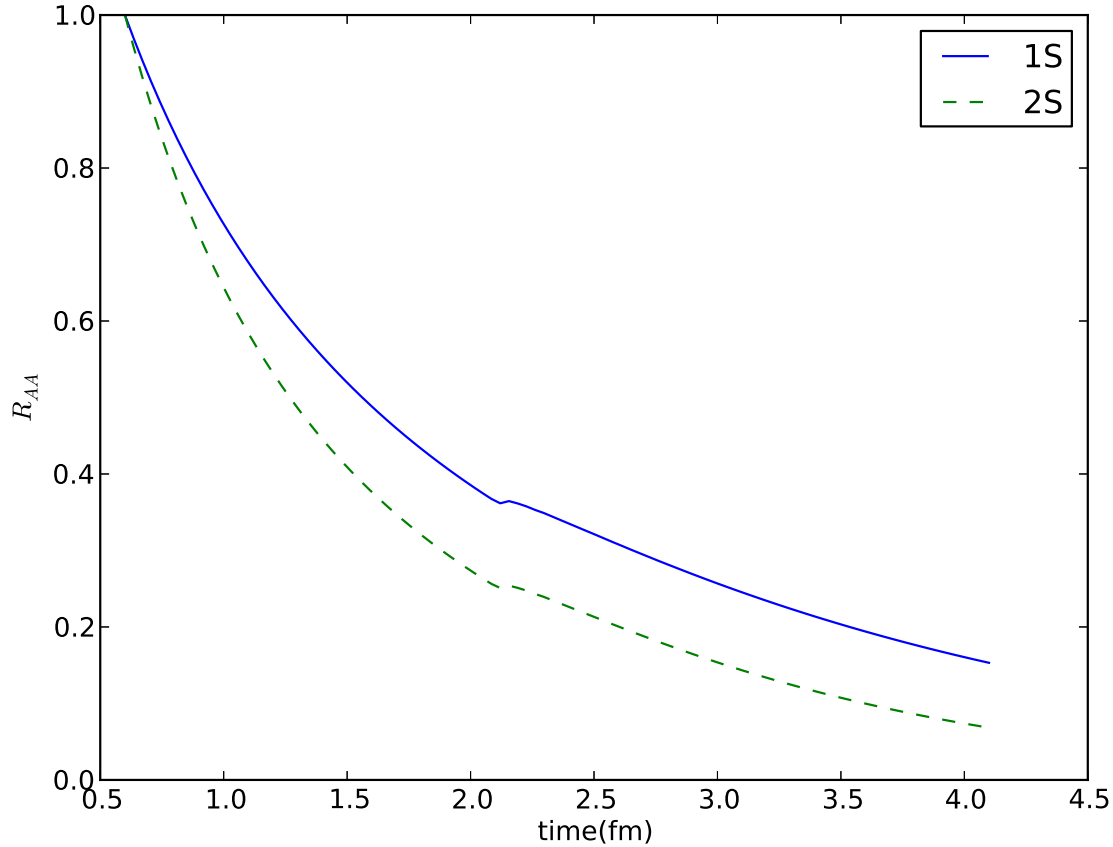
$$C_i^2 = \frac{2}{M_b}\sqrt{\frac{(N_c^2-4)\alpha_s(\mu_E)T}{N_c}}p_i\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

R_{AA} for bottomonium



LO NRQCD production scaling ($b = 1$), $\mu_E = 2\pi T$

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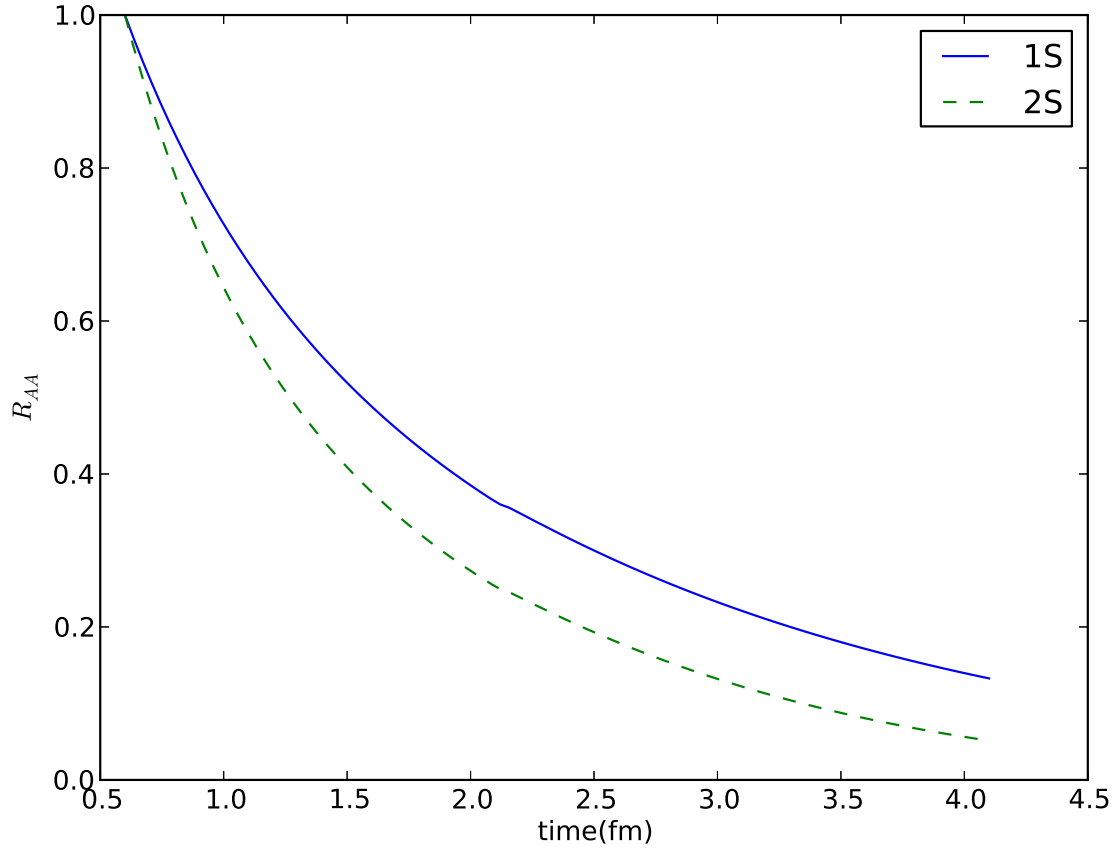

More singlet than LO NRQCD production scaling ($b = 0.1$), $\mu_E = 2\pi T$

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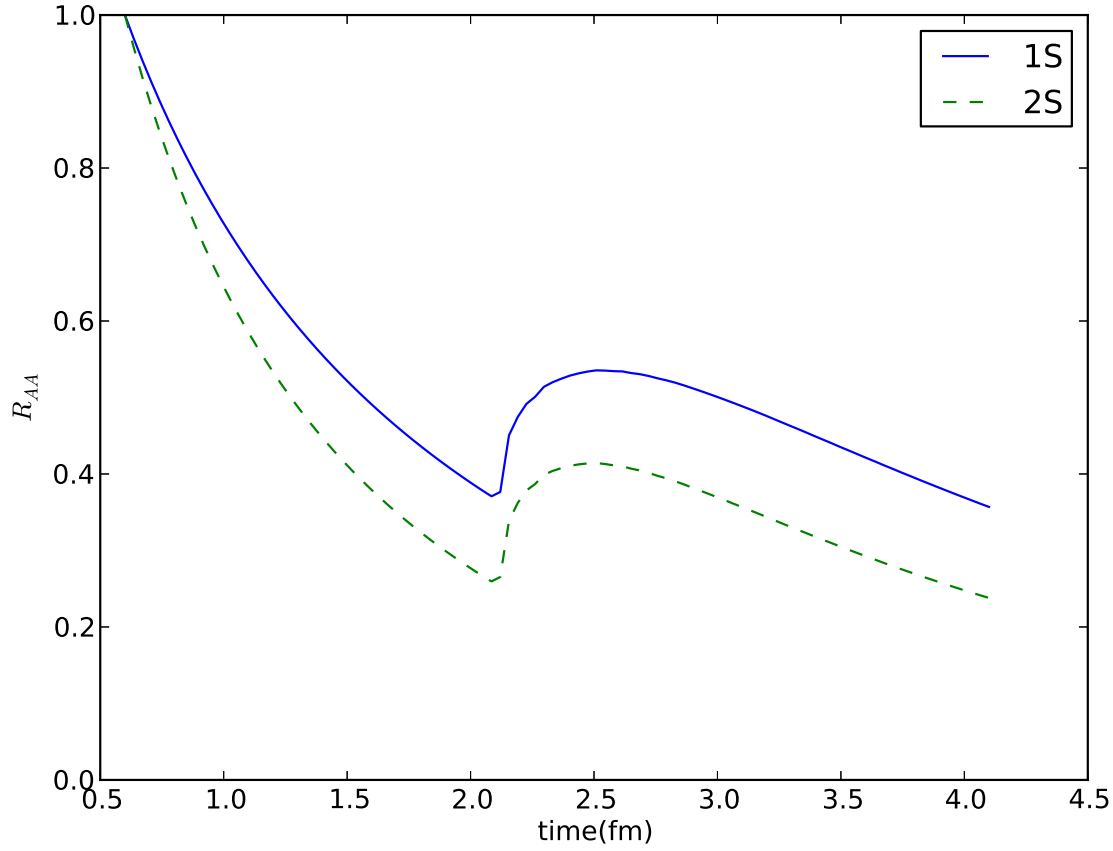

More octet than LO NRQCD production scaling ($b = 10$), $\mu_E = 2\pi T$

Solution of $\rho_{HQ}(t)$

- The case $1/r \gg T \sim m_D \gg E$
  - $g(T)$ large, more realistic case, but non-perturbative inputs needed:

$$\int d\lambda_0 \langle \mathcal{T} E^{A,i}(t + \frac{\lambda_0}{2}, \mathbf{0}) E^{A,i}(t - \frac{\lambda_0}{2}, \mathbf{0}) \rangle$$

  - Two real functions, one of them related to the heavy quark energy loss, available on the lattice (Kaczmarek, 14)

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More on this in the future...

Conclusions

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- Non-Relativistic EFTs have been combined to Thermal EFTs in order to elucidate the behavior of Non-relativistic bound states in a thermal bath under well controlled approximations
- The role of the relative motion of the bound state with respect to the thermal bath has been also analyzed and turns out to produce non-trivial modifications in the decay width
- The above results are being incorporated in a more realistic framework suitable for Heavy Ion Collisions