Heavy Quarkonium in a Quark-Gluon Plasma

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Charmonium

Eichten, Godfrey, Mahlke, Rosner (2007)
Plan

Non-relativistic EFTs: QED
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- A more realistic setting for Heavy Ion Collisions
QED bound states (hydrogen atom)

\[ \mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi} (i \gamma \cdot \partial - m) \Psi + N^\dagger i D_0 N \]
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- Relevant scales:
  - \( m \) (hard), electron mass
  - \( m\alpha \) (soft), inverse Bohr radius, \( \alpha = e^2 / 4\pi \); \( e \), electron charge
  - \( m\alpha^2 \) (ultrasoft), binding energy
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- We may obtain it by integrating out energies and momenta at the hard and soft scales.
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  - Exploit the hierarchy of energy scales
- The EFT gives equivalent physical results in the region where it holds
  - It may make apparent accidental symmetries in that region, which help constraining the physics.
- Calculations are usually simpler.
Since $\alpha \sim 1/137$, the scales are well separated:

\[ m \gg m\alpha \gg m\alpha^2 \]
EFTs

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$$m \gg m\alpha \gg m\alpha^2$$

EFTs are useful:

- $m$ (hard), QED
- $m\alpha$ (soft), NRQED
- $m\alpha^2$ (ultrasoft), pNRQED
Non-Relativistic QED

\[ \mathcal{L}_{NRQED} = -\frac{1}{4} d_1 F_{\mu \nu} F^{\mu \nu} + \frac{d_2}{m^2} F_{\mu \nu} D^2 F^{\mu \nu} + N^\dagger i D^0 N + \]
\[ + \psi^\dagger (i D^0 + \frac{D^2}{2m} + \frac{D^4}{8m^3} + c_F e \frac{\sigma B}{2m} + c_D e \frac{\nabla E}{8m^2} + \]
\[ + i c_S e \frac{\sigma (D \times E - E \times D)}{8m^2}) \psi + \cdots \]

- \( \psi \) are Pauli spinors
- The matching coefficients \( d_1, d_2, c_F, c_D, c_S, \ldots \) contain all the physics at the hard scale \( m \)

(Caswell, Lepage, 1986)
Potential NRQED

\[ L_{pNRQED} = - \int d^3x \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \int d^3x S^\dagger(t, x) \left( iD_0 + \frac{\nabla^2}{2m} + \frac{Z\alpha}{|x|} + \frac{\nabla^4}{8m^3} + \frac{Ze^2}{m^2} \left( -\frac{c_D}{8} + 4d_2 \right) \delta^3(x) + i cs \frac{Z\alpha}{4m^2} \sigma \cdot \left( \frac{x}{|x|^3} \times \nabla \right) \right) S(t, x) \]

\[ + \int d^3x S^\dagger(t, x) e x \cdot E S(t, x) + \cdots \]

- \( S(t, x) \) is the Hydrogen wave function field
- The potentials encode the physics at the soft scale \( m\alpha \)

(Pineda, Soto, 1997)
Potential NRQED

\[ L_{pNRQED} = - \int d^3x \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \int d^3x S^\dagger(t, x) \left( iD_0 + \frac{\nabla^2}{2m} + \frac{Z\alpha}{|x|} + \right. \]
\[ + \frac{\nabla^4}{8m^3} + \frac{Ze^2}{m^2} \left( -\frac{c_D}{8} + 4d_2 \right) \delta^3(x) + ic_S \frac{Z\alpha}{4m^2} \sigma \cdot \left( \frac{x}{|x|^3} \times \nabla \right) \right) S(t, x) \]
\[ + \int d^3x S^\dagger(t, x) e x \cdot E S(t, x) + \cdots \]

\[ e x \cdot E, \] residual interaction with the e.m. field leading to:

- Non-potential effects \(\text{e.g. Bethe logs in the Lamb shift}\)
- Van der Waals forces
Selected results:

  "Potential NRQED: The Positronium case" $[\alpha^3$ corrections to the spectrum]

  "Order $\alpha^3 \ln(1/\alpha)$ corrections to positronium decays"

  "Order $\alpha^7 \ln(1/\alpha)$ contribution to positronium hyperfine splitting"

  "Renormalization group improvement of the spectrum of hydrogen - like atoms with massless fermions"

  "Hyperfine splitting in positronium to $O(\alpha^7 m_e)$: one-photon annihilation contribution"
Heavy quark-antiquark system:
Heavy Quarkonium

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- Charmonium ($c\bar{c}$, $J/\psi$ family)
- Bottomonium ($b\bar{b}$, $\Upsilon$ family)
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Also:

- \(B_c (b\bar{c})\)
- \(t\bar{t}\)
- \(\tilde{q}\bar{q}, \tilde{g}\bar{g}, \cdots\)
- Double-heavy baryons \((ccq, q = u, d, s)\)
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$Q\bar{Q}$ bound state ,  $m_Q >> \Lambda_{QCD}$ ,  $\alpha_s(m_Q) << 1$
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\( Q\bar{Q} \) bound state, \( m_Q \gg \Lambda_{QCD} \), \( \alpha_s(m_Q) \ll 1 \)

- Heavy quarks move slowly \( v \ll 1 \)
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  - \( m_Q >> m_Qv >> m_Qv^2 \)
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- Non-relativistic system $\rightarrow$ multiscale problem
  - $m_Q >> m_Q v >> m_Q v^2$
  - $m_Q >> \Lambda_{QCD}$
- EFTs are useful (N. Brambilla, A. Pineda, JS and A. Vairo, Rev. Mod. Phys. 77, 1423 (2005))
NRQCD


\[ m_Q \gg m_Q v, \quad m_Q v^2, \quad \Lambda_{QCD} \]

\[ \mathcal{L}_\psi = \psi^\dagger \left\{ iD_0 + \frac{1}{2m_Q} D^2 + \frac{1}{8m_Q^3} D^4 + \frac{c_F}{2m_Q} \sigma \cdot gB + \right. \]

\[ + \left. \frac{c_D}{8m_Q^2} (D \cdot gE - gE \cdot D) + i \frac{c_S}{8m_Q^2} \sigma \cdot (D \times gE - gE \times D) + \ldots \right\} \psi \]

\( c_F, c_D \) and \( c_S \) are short distance matching coefficients which depend on \( m_Q \) and \( \mu \) (factorization scale)
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NRQCD (Cont.)

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  - Integrate out energy scales $\sim m_Q v \rightarrow pNRQCD$, A. Pineda and JS, Nucl. Phys. Proc. Suppl. 64, 428 (1998)
  - $\mathcal{L}_{pNRQCD}$ depends on the relative size between $\Lambda_{QCD}$ and $m_Q v^2$. 
NRQCD (Cont.)

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- Proposals:
  - Integrate out energy scales \( \sim m_Qv \) \( \rightarrow \) \( pNRQCD \), A. Pineda and JS, Nucl. Phys. Proc. Suppl. 64, 428 (1998)
  - \( \mathcal{L}_{pNRQCD} \) depends on the relative size between \( \Lambda_{QCD} \) and \( m_Qv^2 \)

- Mode decomposition
The hierarchy $m_Q v \gg m_Q v^2$ is not taken advantage of.

Proposals:

- $\mathcal{L}_{pNRQCD}$ depends on the relative size between $\Lambda_{QCD}$ and $m_Q v^2$.

- $\Lambda_{QCD}$ is neglected.
\[ pNRQCD \]

\[ \Lambda_{QCD} \lesssim m_Q v^2 \quad : \text{weak coupling regime} \]

\[ \mathcal{L}_{pNRQCD} = \int d^3 r \quad \text{Tr} \left\{ S^\dagger \left( i \partial_0 - h_s(r, p, P_R, S_1, S_2, \mu) \right) S + \right. \]

\[ + O^\dagger \left( i D_0 - h_o(r, p, P_R, S_1, S_2, \mu) \right) O \}

\[ + V_A(r, \mu) \text{Tr} \left\{ O^\dagger r \cdot gE S + S^\dagger r \cdot gE O \right\} \]

\[ + \frac{V_B(r, \mu)}{2} \text{Tr} \left\{ O^\dagger r \cdot gE O + O^\dagger Or \cdot gE \right\} + \ldots \]

\[ h_s, h_o = \text{quantum mechanical Hamiltonians with scale dependent potentials calculable in perturbation theory in } \alpha_s(m_Q v) \]
pNRQCD (Cont.)

$\Lambda_{QCD} \lesssim m v$ : strong coupling regime

$L_{pNRQCD} = \int d^3 x_1 \int d^3 x_2 \ S^\dagger (i \partial_0 - h_s(x_1, x_2, p_1, p_2, S_1, S_2)) S,$

$h_s(x_1, x_2, p_1, p_2, S_1, S_2) = \frac{p_1^2}{2m_Q} + \frac{p_2^2}{2m_Q} + V_s(x_1, x_2, p_1, p_2, S_1, S_2),$ 

$V_s = V_s^{(0)} + \frac{V_s^{(1)}}{m_Q} + \frac{V_s^{(2)}}{m_Q^2} + \cdots ,$

All $V_s$s can be, and most of them have been, calculated on the lattice
Weak coupling: recent applications

- Spectrum and decays of lowest lying states in bottomonium, charmonium and $B_c$
  - Bottomonium spectrum (Kiyo, Sumino, 2013)
  - Magnetic transitions (Pineda, Segovia, 2013)

- Precision determination of Standard Model parameters
  - Heavy quark masses
    - $\tilde{m}_b(\bar{m}_b) = 4201(43) \text{ MeV}$, from NNNLO spectrum (Ayala, Cvetic, Pineda, 2014)

- Strong coupling constant $\alpha_s$
  - $\alpha_s(M_Z) = 0.1166^{+0.0012}_{-0.0008}$, from comparing NNNLO static energy with lattice data (Bazavov, Brambilla, Garcia i Tormo, Petreczky, JS, Vairo, 2014)
Heavy Quarkonium in a QGP

Quark-Gluon Plasma (QGP)

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It is expected to be produced in Heavy Ion Collision (HIC) experiments (RHIC, LHC)
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- Quark-Gluon Plasma (QGP)
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  - It is expected to be produced in Heavy Ion Collision (HIC) experiments (RHIC, LHC)
  - Properties of the QGP are extracted indirectly by comparing
    - $AA \rightarrow X + \text{anything}$
    - $pp \rightarrow X + \text{anything}$
  - $X =$ Hard Probe (jets, heavy quarks, heavy quarkonia, ...) is particularly useful
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  - Early proposal (Matsui, Satz, 86):

    $$ V(r) \rightarrow V(r, T) = -\frac{C_f\alpha_s}{r} e^{-m_D r}, \quad m_D \sim gT $$
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- Can we quantify these arguments from QCD?
QED as a toy model for QCD

- Muonic Hydrogen ~ Heavy Quarkonium (Eiras, JS, 00)
  - Muon, Proton ~ Heavy Quarks
  - Photon ~ Gluons
  - Electron, Positron ~ Light Quarks
- Electron-positron plasma (EPP) ~ Quark-gluon plasma
  - Photon ~ Gluons
  - Electron, Positron ~ Light Quarks
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- The $m_e = 0$ case
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The Muonic Hydrogen Atom in a EPP

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  - Closer to the heavy quarkonium case
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- The $m_e \neq 0$ case
  - Technically more involved
  - Relevant for actual muonic hydrogen in an electron-positron plasma

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\[ m \neq 0, \ T = 0 \] case:

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\( m = 0, T \neq 0 \) case:
- \( T \) (hard), temperature
- \( eT \) (soft), Debye mass
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\( m \neq 0, T \neq 0 \) case: what is the interplay among the scales above?
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- $m = 0, \ T \neq 0$ case:
  - $T$ (hard), QED
  - $eT$ (soft), HTL (Hard Thermal Loops)
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- **m \neq 0, T \neq 0** case: contributions of energies above **T** are exponentially suppressed by Boltzmann factors
Hard Thermal Loops EFT ($m=0$)

$$\delta \mathcal{L}_{HTL} = \frac{1}{2} m_D^2 F^{\mu\alpha} \int \frac{d\Omega}{4\pi} \frac{k_\alpha k_\beta}{-(k.\partial)^2} F^{\mu\beta} + m_f^2 \bar{\psi} \gamma^\mu \psi \int \frac{d\Omega}{4\pi} \frac{k_\mu}{k.D} \psi$$

$$k = (1, \hat{k}), \quad m_D^2 = e^2 T^2 / 3, \quad m_f^2 = e^2 T^2 / 8$$

(Braaten, Pisarsky, 1992)
The $m_e = 0, T \ll m_\mu \alpha/n$ case

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    Analytic results possible for:
    - $T \gg m_\mu \alpha^2/n^2 \gg eT$
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    Analytic results possible for:
    - $T \gg m_\mu \alpha^2 / n^2 \gg eT$. Results available for QCD
      $O(m_Q \alpha_5^5)$ (Brambilla, Escobedo, Ghiglieri, JS, Vairo (10))
    - $T \gg eT \gg m_\mu \alpha^2 / n^2$
The $T \ll m\alpha/n$ case

- For $T = \beta^{-1} \ll m\alpha^2/n^2$:

\[
\delta E_n = -\frac{4\pi^3\alpha}{45\beta^4} \langle n | x \frac{\bar{P}_n}{(H_0 - E_n)} x | n \rangle \left(1 + \mathcal{O}\left(\left(\frac{n^2}{\beta m\alpha}\right)^2\right)\right)
\]

\[
\delta \Gamma_n = 0
\]

- For $T = \beta^{-1} \gg m\alpha^2/n^2$:

\[
\delta E_n = \frac{\alpha\pi}{3m\beta^2} + \frac{2\alpha}{3\pi} \sum_r |\langle n | v | r \rangle|^2 (E_n - E_r) (\ln\left(\frac{2\pi}{\beta |E_n - E_r|}\right) - \gamma)
\]

\[
\times \left(1 + \mathcal{O}\left(\left(\frac{\beta m\alpha}{n^2}\right)^2\right)\right)
\]

\[
\delta \Gamma_n = \frac{4Z^2\alpha^3}{3\beta n^2} \left(1 + \mathcal{O}\left(\frac{\beta m\alpha}{n^2}\right)\right)
\]
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  - For $T \sim m_\mu \alpha / n \Rightarrow eT \gg m_\mu \alpha^2 / n^2 \Rightarrow$ further contributions to the potential from the scale $eT$ (HTLs necessary)
  - For $T \gg m_\mu \alpha / n$, and $eT \sim m_\mu \alpha / n \Rightarrow$ all contributions to the potential from HTLs
The \( m_e = 0, \; m_\mu \alpha/n \ll T \ll m_\mu \) case

- The results also hold for heavy quarkonium [ \( \alpha \leftrightarrow C_f \alpha_s, \]
  \[ m_\mu \leftrightarrow m_Q/2, \; m_D^2 = e^2 T^2 / 3 \leftrightarrow m_D^2 = g^2 T^2 (N_c + N_f / 2) \] ]
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- One can start from NRQED, and integrate out the largest scale, \( T \)
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The $m_e = 0$, $m_\mu \alpha/n \ll T \ll m_\mu$ case

$$V(r, T) = -\frac{\alpha e^{-m_D r}}{r} - \alpha m_D + i\alpha T \phi(m_D r) + O\left(\frac{\alpha T^2}{m_\mu}\right)$$

$$\phi(x) = 2 \int_0^\infty \frac{dzz}{(z^2 + 1)^2} \left[\frac{\sin(zx)}{zx} - 1\right]$$

- It has an imaginary part! (Laine, Philipsen, Romatschke, Tassler, 06; Escobedo, JS, 08; Brambilla, Ghiglieri, Vairo, Petreczky, 08)

- The dissociation temperature becomes

$$T_d \sim m_\mu \alpha^{2/3} / \ln^{1/3} \alpha < m_\mu \alpha^{1/2}$$

where $m_\mu \alpha^{1/2} \sim m_D \sim eT$ is the scale of the disociation temperature for the screening mechanism (Matsui, Satz, 86)
The $m_q = 0$, $m_Q \alpha_s \ll T \ll m_Q$ case

- The melting temperature $T_d$ can be parametrically estimated to be $T_d \sim -m_Q \alpha_s^{2/3} / \ln^{1/3} \alpha_s$
- $\Upsilon(1S) \rightarrow T_d \sim 500 \text{MeV}$
- $J/\psi \rightarrow T_d \sim 200 \text{MeV}$

- The $\Upsilon(1S)$ spectral function

![Graph showing the spectral function at different temperatures](image-url)
Moving through the QGP

- Bound state at rest, the medium moves at velocity $v$ (Weldom, 82)

$$f(\beta k^0) \rightarrow f(\beta^\mu k_\mu) = \frac{1}{e^{\beta^\mu k_\mu} \pm 1}, \quad \beta^\mu = \frac{\gamma}{T}(1, v)$$

$$v = |v|, \quad \gamma = \frac{1}{\sqrt{1 - v^2}}$$

- $O(3)$ rotational symmetry is reduced to $O(2)$

- In light cone coordinates $k_+ = k_0 + k_3, \quad k_- = k_0 - k_3$

$$\beta^\mu k_\mu = \frac{1}{2} \left( \frac{k_+}{T_+} + \frac{k_-}{T_-} \right), \quad T_+ = T \sqrt{\frac{1 + v}{1 - v}}, \quad T_- = T \sqrt{\frac{1 - v}{1 + v}}$$

- For $v \sim 1$ (moderate velocities), $T_+ \sim T \sim T_-$
- For $v \sim 1$ (relativistic velocities), $T_+ \gg T \gg T_-$

- Collinear region, $k_+ \sim T_+, \quad k_- \sim T_-$
- Soft (ultrasoft) region, $k_+ \sim k_- \sim T_-$
Moving through the QGP

Results (Escobedo, Mannarelli, JS, 11; + Giannuzzi, 13):

- For $T \lesssim m_{\alpha}/n$ the thermal decay width ($\Gamma$) decreases with $v$
- For $eT \sim m_{\alpha}/n \ll T$:
  - At moderate velocities $\Gamma$ increases with $v$
  - At ultrarelativistic velocities $\Gamma$ decreases with $v$
- The $\Upsilon(1S)$ spectral function at $v \neq 0$
  
  $$T = 250 \text{ MeV} \quad T = 400 \text{ MeV}$$
Motivation for $\nu \neq 0$

(H. Pereira Da Costa, for the PHENIX collaboration, arXiv:1007.3688)
Motivation for $\nu \neq 0$

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- The $J/\psi$ suppression depends on the rapidity
Motivation for $\nu \neq 0$

(ALICE collaboration, arXiv:1202.1383)
Motivation for $\nu \neq 0$

The $J/\psi$ suppression may depend on the transverse momentum

(ALICE collaboration, arXiv:1202.1383)
Motivation for $\nu \neq 0$

The $J/\psi$ suppression does depend on the transverse momentum

(ALICE collaboration, arXiv:1311.0214)
Motivation for $\nu \neq 0$

The $J/\psi$ suppression does depend on the transverse momentum.

Is this due, at least in part, to the fact that the in-medium properties of $J/\psi$ depend on the velocity?
Embeding it in Heavy Ion Collisions

M.A. Escobedo, proceedings Confinement 2014
N. Brambilla, M.A. Escobedo, JS, A. Vairo, in preparation

- Not really an equilibrium QGP
  - $t_0$, formation time
  - Expansion ($T$ decreases)
  - $t_F$, hadronization time ($T \sim T_c$)

- Open quantum system approach:
  - At $t = 0$, $\rho = \rho_{HQ} \otimes \rho_{ldf}$
  - From $0 < t < t_0$, $\rho_{HQ}$ and $\rho_{ldf}$ evolve independently, $\rho_{HQ}$ as in the vacuum, and $\rho_{ldf}$ to $\rho_{QGP}$
  - From $t_0 < t < t_F$, $\rho_{QGP}$ evolves according to the Bjorken model, and $\rho_{HQ}$ according to pNRQCD ($rT \ll 1$)
Dilepton emission

\[ d\mathcal{R} = -\frac{e^2 L_{\mu\nu}(k_1, k_2)}{|k_1||k_2|(k_1 + k_2)^4} \int d^4 \lambda_1 \, d^4 \lambda_2 e^{-i(k_1 + k_2)(\lambda_1 - \lambda_2)} \text{Tr} (\rho J_\mu(\lambda_1) J^\nu(\lambda_2)) \]

McLerran, Toimela (85)

- \( k_1, k_2 \), lepton momenta
- \( J_\mu(\lambda_1) \) QCD electromagnetic current

The formula is adapted to include:

- Heavy quarks in \( J_\mu(\lambda_1) \)
- Non thermalized heavy quarks in \( \rho = \rho_{HQ} \otimes \rho_{ldf} \)
Evolution of $\rho_{HQ}$

\[ \rho_{HQ} = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix} \]

For $\rho_s$ we have:

\[ \frac{d\rho_s(t)}{dt} = -i h_{s,\text{eff}}(t)\rho_s(t) + i\rho_s(t)h_{s,\text{eff}}^\dagger(t) + F(\rho_o(t), t) \]
Evolution of $\rho_{HQ}$

For $\rho_o$ we have:

$$\frac{d\rho_o(t)}{dt} = -i h_{o,eff}(t) \rho_o(t) + i \rho_o(t) h_{o,eff}^\dagger(t) + \mathcal{F}_1(\rho_s(t), t) + \mathcal{F}_2(\rho_o(t), t)$$

For quasistatic evolution of $\rho_{QGP}$ ($dT/dt \ll TE$), $h_{s,eff}(t)$, $h_{o,eff}(t)$, $\mathcal{F}(\rho_o(t), t)$, $\mathcal{F}_1(\rho_s(t), t)$, $\mathcal{F}_2(\rho_o(t), t)$, can be obtained from equilibrium distributions.

If in addition $t$ is large ($t \gg 1/E$), $h_{s,eff}(t)$, $h_{o,eff}(t)$ become slowly varying in time and can be obtained directly from previous calculations.

We have further assumed that $\rho_o$ is diagonal in color space.
Solution of $\rho_{HQ}(t)$

- Initial conditions:
  - Scaling of NRQCD LO production: $\rho_o = b \rho_s / \alpha_s(m)$, $b = \mathcal{O}(1)$
  - $Tr(\rho_s) + Tr(\rho_o) = 1$

- Rewrite the evolution equation in the Lindblad form

  \[
  \frac{d\rho_{HQ}}{dt} = -i[H, \rho_{HQ}] + \sum_i (C_i \rho_{HQ} C_i^\dagger - \frac{1}{2} \{C_i^\dagger C_i, \rho_{HQ}\})
  \]

- Decompose in partial waves and truncate beyond a given angular momentum ($\int_{t_0}^{t} dt' r^2 T(t')^3 \ll 1$)

- Use standard libraries to numerically solve the Lindblad equation (Johansson, Nation, Nori, 12)
Solution of $\rho_{HQ}(t)$

- The case $1/r \gg T \gg E \gg m_D$

\[
H = \frac{1}{2} \begin{pmatrix}
    h_{s,eff} + h_{s,eff}^\dagger & 0 \\
    0 & h_{o,eff} + h_{o,eff}^\dagger
\end{pmatrix}.
\]

\[
C^0_i = \sqrt{\frac{4T_F\alpha_s(\mu_E)T}{3}} \left( \frac{2ip_i}{M_b} + \frac{N_c\alpha_s(1/a_o)r_i}{2r} \right) \begin{pmatrix}
    0 & 1 \\
    0 & 0
\end{pmatrix}
\]

\[
C^1_i = \sqrt{\frac{4C_F\alpha_s(\mu_E)T}{3}} \left( -\frac{2ip_i}{M_b} + \frac{N_c\alpha_s(1/a_o)r_i}{2r} \right) \begin{pmatrix}
    0 & 0 \\
    1 & 0
\end{pmatrix}
\]

\[
C^2_i = \frac{2}{M_b} \sqrt{\frac{(N_c^2 - 4)\alpha_s(\mu_E)T}{N_c}} p_i \begin{pmatrix}
    0 & 0 \\
    0 & 1
\end{pmatrix}
\]
$R_{AA}$ for bottomonium

LO NRQCD production scaling $(b = 1), \mu_E = 2\pi T$
$R_{AA}$ for bottomonium

More singlet than LO NRQCD production scaling ($b = 0.1$), $\mu_E = 2\pi T$
More octet than LO NRQCD production scaling ($b = 10$), $\mu_E = 2\pi T$
Solution of $\rho_{HQ}(t)$

- The case $1/r \gg T \sim m_D \gg E$
- $g(T)$ large, more realistic case, but non-perturbative inputs needed:

$$\int d\lambda_0 \langle T E^{A,i}(t + \frac{\lambda_0}{2}, 0) E^{A,i}(t - \frac{\lambda_0}{2}, 0) \rangle$$

- Two real functions, one of them related to the heavy quark energy loss, available on the lattice (Kaczmarek, 14)
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More on this in the future...
Conclusions

Non-Relativistic EFTs have been combined to Thermal EFTs in order to elucidate the behavior of Non-relativistic bound states in a thermal bath under well controlled approximations.
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- The role of the relative motion of the bound state with respect to the thermal bath has been also analyzed and turns out to produce non-trivial modifications in the decay width.
Conclusions

- Non-Relativistic EFTs have been combined to Thermal EFTs in order to elucidate the behavior of Non-relativistic bound states in a thermal bath under well controlled approximations.

- The role of the relative motion of the bound state with respect to the thermal bath has been also analyzed and turns out to produce non-trivial modifications in the decay width.

- The above results are being incorporated in a more realistic framework suitable for Heavy Ion Collisions.