Photon Emission in Neutral Current Interactions with Nucleons and Nuclei

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Overview

1. Introduction
   - A brief resume
   - Introduction of NC$\gamma$ theoretical study

2. Theoretical Model
   - NC$\gamma$ on the nucleon
   - Incoherent NC$\gamma$ on nuclei
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3. Comparison with MiniBooNE Estimates

4. Summary
Introduction

A brief resume
Who am I?

- **Name:** En Wang
- **Date of birth:** Dec. 1985
- **Marital status:** Married
- **The place of birth:** Henan, China

**Figure:** Shao-Lin temple, Henan
# Education

<table>
<thead>
<tr>
<th><strong>Ph.D</strong> — 03.2011 – 12.2014</th>
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<tr>
<td>IFIC, University of Valencia &amp; CSIC, Valencia, Spain</td>
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<tr>
<td><strong>Title:</strong> Electroweak processes in nucleons and nuclei at intermediate energies</td>
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<td><strong>Advisors:</strong> Drs. Juan M. Nieves and Luis Alvarez-Ruso</td>
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<th><strong>Master</strong> — 09.2007 – 07.2010</th>
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<td>Department of Physics, Zhengzhou University, Henan, China</td>
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<td><strong>Title:</strong> Study of meson properties in quark models</td>
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<td><strong>Advisor:</strong> Prof. De-Min Li</td>
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<th><strong>Bachelor</strong> — 09.2003 – 07.2007</th>
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<td><strong>Title:</strong> Study of mass spectra of the charmonium</td>
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<tr>
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Research

♣ Neutrino-nucleon and neutrino-nucleus interaction; Photon emission in neutral current interactions at intermediate energies
♣ Effective hadron interactions with chiral symmetry: Resonances and unitary re-summations
♣ Photo-production of the Λ(1520) resonance
Introduction of NC$\gamma$ theoretical study
e-like events in the MiniBooNE experiment

- neutralino-mode excess: $162.0 \pm 47.8$ events
- antineutrino-mode excess: $78.4 \pm 28.5$ events

A. Aguilar-Arevalo et al., PRL 110(2013),161801.
This anomaly of e-like events

- Radiative decay of heavy neutrinos. S. Gninenko, PRL 103(2009), 241802.
e-like events in the MiniBooNE experiment

It could have its origin in poorly understood background and unknown systematics.

- MiniBooNE detector cannot distinguish the signature produced by electrons or photons.
- $\text{NC } \pi^0$ where the $\gamma\gamma$ decay is not identified
- $\Delta \to N\gamma$ decay - one of the largest background

(A. Aguilar-Arevalo et al., PRL 110(2013), 161801)
Theoretical Model
**NC\(\gamma\)** on the nucleons and nuclei

- **NC\(\gamma\)** reaction on nucleons
  \[
  \nu(k) + N(p) \rightarrow \nu(k') + N(p') + \gamma(q_\gamma)
  \]

- **NC\(\gamma\)** on nuclei consists of incoherent and coherent reactions,
  \[
  \nu(k) + A(p) \rightarrow \nu(k') + X(p') + \gamma(q_\gamma)
  \]
  \[
  \nu(k) + A_Z|_{gs}(p_A) \rightarrow \nu(k') + A_Z|_{gs}(p'_A) + \gamma(q_\gamma)
  \]

At the relevant energies for MiniBooNE, the reaction is dominated by the excitation of the \(\Delta(1232)\) resonance but there are also non-resonant contributions that, close to threshold, are fully determined by the effective chiral Lagrangian of strong interactions.
Amplitude for NC$\gamma$ on the nucleon

The reaction is,

$$\nu(k) + N(p) \rightarrow \nu(k') + N(p') + \gamma(q_{\gamma})$$

The amplitude is given by,

$$M_r = \frac{G_F e}{\sqrt{2}} l_\alpha J^\alpha_r$$

$$= \frac{G_F e}{\sqrt{2}} \bar{u}_\nu(k') \gamma_\alpha (1 \mp \gamma_5) u_\nu(k) \times \left( i\epsilon^{*}_\mu(r) \bar{u}(p') \Gamma^{\mu\alpha} u(p) \right),$$

- the $\mp$ is for neutrino and antineutrino, respectively.
- $\epsilon^{*}_\mu(r)$ is the photon polarization vector.
- $\Gamma^{\mu\alpha}$ is the hadronic matrix element.
Model for the hadronic matrix elements

- First row: direct and crossed nucleon pole terms
- Second row: direct and crossed Δ(1232) pole terms
- Third row: direct and crossed heavier resonance [N(1440), N(1520) and N(1535)] pole terms
- Forth row: $t$-channel $\pi$ exchange term
Nucleon pole terms

\[ \Gamma_{N}^{\mu \alpha} = \tilde{J}_{EM}^{\mu}(q\gamma)(\not{p} + \not{q} + M)J_{NC}^{\alpha}(q)D_{N}(p + q) 
+ \tilde{J}_{NC}^{\alpha}(-q)(\not{p}' - \not{q} + M)J_{EM}^{\mu}(-q\gamma)D_{N}(p' - q) \]

where \( \tilde{J} = \gamma_{0}J^{\dagger}\gamma_{0} \), and \( D_{N} \) is the nucleon propagator, given by,

\[ D_{N}(p) = \frac{1}{p - M}. \]

The weak NC and electromagnetic (EM) currents are given by,

\[ J_{NC}^{\alpha}(q) = \gamma^{\alpha}\tilde{F}_{1}(q^{2}) + \frac{i}{2M}\sigma^{\alpha\beta}q_{\beta}\tilde{F}_{2}(q^{2}) - \gamma^{\alpha}\gamma_{5}\tilde{F}_{A}(q^{2}), \]

\[ J_{EM}^{\mu}(q\gamma) = \gamma^{\mu}F_{1}(0) + \frac{i}{2M}\sigma^{\mu\nu}q_{\gamma\nu}F_{2}(0), \]
Nucleon pole terms

The weak NC and EM currents:

\[ J_{NC}^\alpha(q) = \gamma^\alpha \tilde{F}_1(q^2) + \frac{i}{2M} \sigma^{\alpha\beta} q_\beta \tilde{F}_2(q^2) - \gamma^\alpha \gamma_5 \tilde{F}_A(q^2), \]

\[ J_{EM}^\mu(q\gamma) = \gamma^\mu F_1(0) + \frac{i}{2M} \sigma^{\mu\nu} q_\gamma \gamma_\nu F_2(0), \]

The NC vector form factors:

\[ \tilde{F}_{1,2}^{(p)} = (1 - 4 \sin^2 \theta_W) F_{1,2}^{(p)} - F_{1,2}^{(n)} - F_{1,2}^{(s)} \]

\[ \tilde{F}_{1,2}^{(n)} = (1 - 4 \sin^2 \theta_W) F_{1,2}^{(n)} - F_{1,2}^{(p)} - F_{1,2}^{(s)} \]

- \( F_{1,2}^{(p)} \) — (p, n) EM form factors
- \( F_{1,2}^{(s)} \) — strange EM form factors, to be neglected!!!

\[ F_{1}^{(N)} = \frac{G_E^N + \tau G_M^N}{1 + \tau}, \quad F_{2}^{(N)} = \frac{G_M^N - G_E^N}{1 + \tau}, \quad N = p, n \]
The weak neutral current:

\[ J_{\text{NC}}^\alpha(q) = \gamma^\alpha \tilde{F}_1(q^2) + \frac{i}{2M} \sigma^{\alpha\beta} q_\beta \tilde{F}_2(q^2) - \gamma^\alpha \gamma_5 \tilde{F}_A(q^2), \]

The NC axial form factors (dipole parametrization):

\[ \tilde{F}_A^{(p,n)} = \pm F_A - F_A^{(s)}, \quad (+ \rightarrow p, - \rightarrow n) \]

\[ F_A(q^2) = g_A \left(1 - \frac{q^2}{M_A^2}\right)^{-2} \]

- \( g_A = 1.267, \ m_A = 1.016 \ \text{GeV} \).
- \( F_A^{(s)} \) — strange axial form factors, to be neglected!!!
\[ \Gamma_{\mu\alpha} = \tilde{J}_{EM}(p', q) \Lambda_\delta(p + q) J_{NC}^\sigma(p, q) D_\Delta(p + q), \]
\[ + \tilde{J}_{NC}(p', -q) \Lambda_\delta(p' - q) J_{EM}^\sigma(p, -q) D_\Delta(p' - q), \]

where the propagator \( D_\Delta \) is,
\[ D_\Delta(p) = \frac{1}{p^2 - M_\Delta^2 + iM_\Delta \Gamma_\Delta(p^2)}, \]

\( \Lambda^{\mu\nu} \) is the spin 3/2 projection operator, which is given by in the momentum space,
\[ \Lambda^{\mu\nu}(p_\Delta) = -(p_\Delta + M_\Delta) \left[ g^{\mu\nu} - \frac{1}{3} \gamma^{\mu} \gamma^{\nu} - \frac{2}{3} \frac{p_\Delta^{\mu} p_\Delta^{\nu}}{M_\Delta^2} + \frac{1}{3} \frac{p_\Delta^{\mu} \gamma^{\nu} - p_\Delta^{\nu} \gamma^{\mu}}{M_\Delta} \right]. \]

\( \Gamma_\Delta \) is the resonance width.
The weak NC and EM currents are given by,

\[ J^{\beta \mu}_{NC}(p, q) = \left[ \frac{\tilde{C}^V_3}{M} (g^{\beta \mu} q - q^\beta \gamma^\mu) + \frac{\tilde{C}^V_4}{M^2} (g^{\beta \mu} q \cdot p_\Delta - q^\beta p^\mu_\Delta) \right. \]

\[ + \left. \frac{\tilde{C}^V_5}{M^2} (g^{\beta \mu} q \cdot p - q^\beta p^\mu) \right] \gamma_5 + \frac{\tilde{C}^A_3}{M} (g^{\beta \mu} q - q^\beta \gamma^\mu) \]

\[ + \frac{\tilde{C}^A_4}{M^2} (g^{\beta \mu} q \cdot p_\Delta - q^\beta p^\mu_\Delta) + \frac{\tilde{C}^A_5}{M^2} g^{\beta \mu} + \frac{\tilde{C}^A_6}{M^2} q^\beta q^\mu, \]

\[ J^{\beta \mu}_{EM}(p, -q_\gamma) = -\left[ \frac{C^V_3}{M} (g^{\beta \mu} q_\gamma - q^\gamma_\beta \gamma^\mu) + \frac{C^V_4}{M^2} (g^{\beta \mu} q_\gamma \cdot p_{\Delta c} - q^\gamma_\beta p^\mu_{\Delta c}) \right. \]

\[ + \left. \frac{C^V_5}{M^2} (g^{\beta \mu} q_\gamma \cdot p - q^\gamma_\beta p^\mu) \right] \gamma_5, \]

\[ p_\Delta = p + q = p' + q_\gamma \text{ and } p_{\Delta c} = p' - q = p - q_\gamma. \]
$N - \Delta(1232)$ form factors

NC vector form factors and EM transition form factors:
- $\tilde{C}_i^V$ — NC vector form factors
- $C_i^V$ — EM transition form factors

The NC vector form factors are related to the EM ones:

$$\tilde{C}_i^V(q^2) = (1 - 2 \sin^2 \theta_W) C_i^V(q^2) \quad (1)$$

The EM form factors can be related to the helicity amplitudes $A_{1/2}, A_{3/2}$ and $S_{1/2}$. 
The relation between EM form factors and helicity amplitudes

$N - \Delta$ EM form factor $\tilde{C}_i^\nu$ can be obtained from the helicity amplitudes, $A_{1/2}, A_{3/2}$ and $S_{1/2}$, which extracted from pion photo- and electro-production.

\begin{align*}
A_{1/2} &= \sqrt{\frac{2\pi\alpha}{kR}} \left\langle S_z^* = \frac{1}{2} \left| \epsilon^{(+)} J_{EM}^\mu \right| S_z = -\frac{1}{2} \right\rangle \frac{1}{\sqrt{2M} \sqrt{2M_R}}, \\
A_{3/2} &= \sqrt{\frac{2\pi\alpha}{kR}} \left\langle S_z^* = \frac{3}{2} \left| \epsilon^{(+)} J_{EM}^\mu \right| S_z = \frac{1}{2} \right\rangle \frac{1}{\sqrt{2M} \sqrt{2M_R}}, \\
S_{1/2} &= -\sqrt{\frac{2\pi\alpha}{kR}} \left\langle S_z^* = \frac{1}{2} \left| \frac{\vec{k}}{\sqrt{Q^2}} \epsilon^{(0)} J_{EM}^\mu \right| S_z = \frac{1}{2} \right\rangle \frac{1}{\sqrt{2M} \sqrt{2M_R}},
\end{align*}

We adopt the parametrization of the helicity amplitude obtained in the MAID analysis.

(D. Drechsel, et al., EPJA 34(2007), 69 and http://www.kph.uni-mainz.de/MAID)
we assume a standard dipole form for the axial NC form factors

\[
\tilde{C}_5^A(Q^2) = -C_5^A(0) \left(1 + \frac{Q^2}{M_A^2}\right)^{-2},
\]

\[
\tilde{C}_6^A(Q^2) = \frac{M^2}{m_\pi^2 + Q^2} \tilde{C}_5^A(Q^2),
\]

\[
\tilde{C}_4^A(Q^2) = -\frac{\tilde{C}_5^A(Q^2)}{4},
\]

\[
\tilde{C}_3^A(Q^2) = 0,
\]

- the cross section of NCγ strongly depends on \( C_5^A(q^2) \).
Off diagonal Goldberger-Treiman relations

The off diagonal GT relation amounts to

$$C_5^A(0) = -\sqrt{\frac{2}{3}} \frac{f^*}{m_\pi}$$  (2)

The axial coupling $C_5^A(0)$ is now expressed in terms of $f^*/m_\pi$ extracted from the $\Delta \rightarrow \pi N$ decay width.

- $C_5^A(0) = 1.0 \pm 0.11$ and $m_A = 0.93$ GeV, fitted to the $\nu$-induced $\pi$ production ($\nu_\mu p \rightarrow \mu^- p \pi^+$) ANL and BNL bubble chamber data.
- the uncertainties of our model mainly comes from the $C_5^A(0) = 1.0 \pm 0.11$. 

The hadronic matrix element $\pi$ pole term is given by,

$$\Gamma^{\mu\alpha} = -i C_{p,n} \frac{gA}{4\pi^2 f^2_\pi} \left( \frac{1}{2} - 2\sin^2\theta_W \right) \epsilon^{\sigma\delta\mu\alpha} q_{\gamma\sigma} q_{\delta}(p' - p)\gamma_5 D_\pi(p' - p),$$

where,

$$C_{p,n} = \pm 1$$

$$D_\pi(p) = \frac{1}{p^2 - m^2_\pi} \leftarrow \pi \text{ propagator}$$
The hadronic matrix elements of $N(1440), N(1535)$ are similar to that of nucleon pole.

- the hadronic matrix elements of $N(1520)$ is similar to that of $\Delta(1232)$ pole terms.

- $N - N^*$ vector form factors can be obtained from helicity amplitudes.

- We have assumed a standard dipole form for the axial form factors.

- The axial couplings are obtained by the off diagonal Goldberger-Treiman relations.
NC$\gamma$ cross section on the nucleon for neutrino
The cross section of NC\(\gamma\) on nucleon for antineutrino

![Graph showing the cross section of NC\(\gamma\) on nucleon for antineutrino](image-url)
Incoherent NC$\gamma$ on nuclei
It consists of the incoherent and coherent reactions.

\[ \nu(k) + A(p) \rightarrow \nu(k') + X(p') + \gamma(q_{\gamma}) \]

For the incoherent reaction, the final nucleus is either broken or left in some excited state.

\[ \nu(k) + A_Z|_{gs}(p_A) \rightarrow \nu(k') + A_Z|_{gs}(p'_A) + \gamma(q_{\gamma}) \]

For the coherent reaction, the final nucleus is left in its ground state.
## Nuclear effects

### Fermi motion
We adopt the relativistic **local Fermi gas approximation**. The target nucleon moves in a local Fermi sea of momentum $k_F$ defined as a function of the local density of protons and neutrons, independently.

### Pauli blocking
Final nucleons are not allowed to take occupied states.

### In-medium modification of $\Delta$ properties
The $\Delta$ resonance acquires a selfenergy because of several effects such as Pauli blocking of the final nucleon and absorption processes: $\Delta N \rightarrow NN$, $\Delta N \rightarrow NN\pi$ or $\Delta NN \rightarrow NNN$. 
Incoherent reaction

The differential cross section for the incoherent reaction is,

\[
\frac{d\sigma^A}{d\sigma_N} = 2 \int d^3\vec{r} \int \frac{d^3\vec{p}}{(2\pi)^3} n_N(\vec{p}, \vec{r}) \left[ 1 - n_N(\vec{p}', \vec{r}) \right] d\sigma_N,
\]

- Relativistic Local Fermi Gas: \( k_F(\vec{r}) = \left[ 3\pi^2 \rho(\vec{r}) \right]^{1/3} \)
- Fermi motion: \( n(\vec{r}, \vec{p}) = \Theta(k_F(\vec{r}) - |\vec{p}|) \)
- Pauli blocking: \( 1 - n(\vec{r}, \vec{p}) \)

where \( k_F(\vec{r}) \) is the Fermi momentum, given by,

\[
k_F(\vec{r}) = \left[ 3\pi^2 \rho(\vec{r}) \right]^{1/3},
\]
In-medium modification of the $\Delta(1232)$ resonance

For propagator $D_\Delta$ is,

$$D_\Delta(p) = \frac{1}{p^2 - M_\Delta^2 + iM_\Delta \Gamma_\Delta(p^2)},$$

Replace

- $\Gamma_\Delta/2 \rightarrow \Gamma^\text{Pauli}_\Delta/2 - \text{Im}\Sigma_\Delta(\rho)$
- $\Gamma^\text{Pauli}_\Delta$ — free width of $\Delta \rightarrow N\pi$ modified by Pauli blocking
- $\text{Im}\Sigma_\Delta(\rho)$, including many body process: $\Delta N \rightarrow NN$, $\Delta N \rightarrow NN$ and $\Delta NN \rightarrow NNNN$

Incoherent reaction

- Figure (a) shows the cross-section $\sigma$ for $\nu^{12}\text{C}$ as a function of $E_{\nu}$ (GeV) for different cases: 0.89-1.11, All-Free, All-Full, no N*-Free, and no N*-Full.
- Figure (b) shows the cross-section $\bar{\nu}^{12}\text{C}$ for the same cases.

Theoretical Model Comparison with MiniBooNE Estimates

Summary
Coherent NC$\gamma$ on nuclei
Coherent reaction

The amplitude is given by,

\[ \mathcal{M}_r = \frac{G_F}{\sqrt{2}} l_\alpha J_{\text{coh}}^\alpha(r), \]

the hadronic current \( J_{\text{coh}}^\alpha(r) \) is given by,

\[ J_{\text{coh}}^\alpha(r) = i e \epsilon^*_\mu(r) \int d^3 \vec{r} \ e^{i(\vec{q} - \vec{q}'_\gamma) \cdot \vec{r}} \big( \rho_p \Gamma^\mu_\alpha_p + \rho_n \Gamma^\mu_\alpha_n \big) \]

- nuclear correction: \( \Gamma^\Delta_\Delta/2 \rightarrow \Gamma^\text{Pauli}_\Delta/2 - \text{Im} \Sigma_\Delta(\rho) \)
- coherent sum of all nucleons:

\[ \Gamma^\mu_\alpha_N = \frac{1}{2} \sum_i \text{Tr} \left[ \bar{u} \Gamma^\mu_\alpha_{i,N} u \right] \quad (3) \]
Coherent reaction

\[
\sigma (10^{-42} \text{cm}^2) \quad E_\nu \text{(GeV)}
\]

(a) $^{12}\text{C}$

(b) $\bar{\nu}^{12}\text{C}$

- 0.89-1.11
- All
- no N*
- $\Delta$
- N+\Delta (Zhang)
Comparison with MiniBooNE Estimates
CCQE reconstructed (anti)neutrino energy

Assuming that the visible energy \( (E_e) \) is generated by an electron from \( \nu_e + n \rightarrow p + e^- \) in a bound proton/neutron at rest, the reconstructed (anti)neutrino energy is given by,

\[
E_{\nu}^{QE} = \frac{2(M_N - E_B)E_e - \left[ E_B^2 - 2M_N E_B + m_e^2 + \Delta M^2 \right]}{2 \left[ (M_N - E_B) - E_e(1 - \cos \theta_e) \right]},
\]

For a photon which is mis-identified as an electron in the MiniBooNE detector, we have to replace \( E_e \) and \( \theta_e \) by \( E_{\gamma} \) and \( \theta_{\gamma} \). The binding energy \( E_B = 34 \) MeV.

(A. Aguilar-Arevalo et al., PRD 84(2011), 072005)
The events

NC\(\gamma\) events at the MiniBooNE detector is given by,

\[
\frac{dN}{dE\gamma d\cos \theta\gamma} = \varepsilon(E\gamma) \sum_{l=\nu\mu, \bar{\nu}\mu} N_{\text{POT}}^{(l)} \times \\
\sum_{t=p,^{12}\text{C}} N_t \int dE_\nu \phi_l(E_\nu) \frac{d\sigma_{l,t}(E_\nu)}{dE\gamma d\cos \theta\gamma}.
\]

- \(d\sigma_{l,t}(E_\nu) / (dE\gamma d\cos \theta\gamma)\): cross section for NC\(\gamma\) on proton, incoherent and coherent reaction on Carbon
- \(N_{\text{POT}}^{(l)}\): the total number of protons on target (POT)
  \(N_\nu^{\text{POT}} = 6.46 \times 10^{20}\) and \(N_{\bar{\nu}}^{\text{POT}} = 11.27 \times 10^{20}\)
- \(N_t\): the number of protons/carbon nuclei in the target (806 tons CH\(_2\))
- \(\varepsilon(E\gamma)\): energy dependent detection efficiency
- \(\phi_l(E_\nu)\): neutrino/antineutrino fluxes
Beam flux and detection efficiency at MiniBooNE

\[ \phi \times 10^{-12}/\text{cm}^2/\text{POT}/50\text{MeV} \]

\[ E_{\nu} \ (\text{GeV}) \]

\[ \nu - \text{mode} \]
\[ \nu_{\mu} \]
\[ \nu_{\mu} \]
\[ \nu_{e} \]
\[ \bar{\nu}_{e} \]

\[ \bar{\nu} - \text{mode} \]
\[ \nu_{\mu} \]
\[ \nu_{\mu} \]
\[ \nu_{e} \]
\[ \bar{\nu}_{e} \]
Comparison to the MB estimate

![Graph showing comparison to the MB estimate](image-url)

- **ν-mode**
  - Events/ΔQE
  - $E_{ν}^{QE}$ (GeV)
  - Lines: 0.89-1.11, 1.0, no N*, MB

- **ν̄-mode**
  - Events/ΔQE
  - $E_{ν}^{QE}$ (GeV)
  - Lines: 0.89-1.11, 1.0, no N*, MB
$E_\gamma$ distribution of the photon events
$\cos \theta_\gamma$ distribution of the photon events
Summary
Summary

- We have studied NC$\gamma$ on nucleons and nuclei in the intermediate energy region ($E_\nu \sim 1$ Gev).
- Our model consists of nucleon pole, $\Delta$ pole, heavier resonances pole and $t$-channel pion-exchange.
- Large reduction ($\sim 30\%$) on the cross section due to nuclear effects — Fermi motion, Pauli blocking and in-medium modification of $\Delta$ resonance.
- Theoretical error dominated by $N - \Delta$ axial transition properties.

- Our prediction is consistent with MiniBooNE estimate.
- NC$\gamma$: insufficient to explain the excess of e-like events at MiniBooNE.
Thank you!