New Results on QCD Threshold Resummation

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Asmita Mukherjee, George Sterman, Werner Vogelsang

JLab, 12.01.15

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Biography

Jan 2012– present **PhD**, *Institute for Theoretical Physics, University of Tübingen*, Germany.
Thesis title: "Threshold Resummation and Higher Order Effects in Perturbative QCD"
Supervisor: Prof. Werner Vogelsang
Expected to finish in February 2015

Oct 2009– Dec 2011 **Completing diploma studies in physics**, *University of Tübingen*, Germany.
Thesis title: "Contributions of the Weak Gauge Bosons to Spin-Dependent Hard QCD Processes"
Supervisor: Prof. Werner Vogelsang

Oct 2008– Oct 2009 **Certificate of Advanced Studies in Mathematics (CASM)/ Master of Advanced Study (MASt)**, *University of Cambridge*, UK.
St. John's College, Cambridge
Scholarship from DAAD (German Academic Exchange Service)

Oct 2005 **Starting diploma studies in physics**, *University of Tübingen*, Germany.
2005 **Abitur**, *(A-levels)*.

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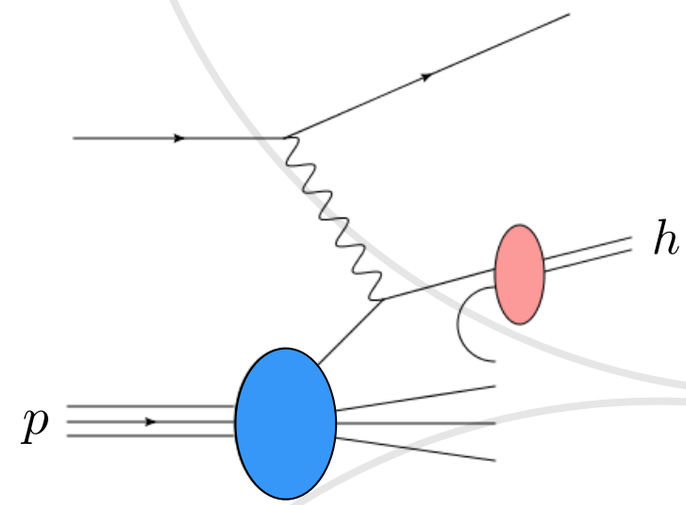
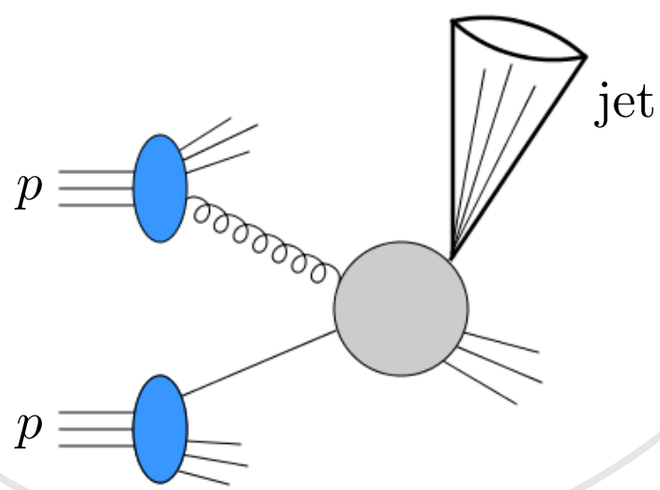
Resummation and Hadron Mass Corrections for Color Singlet Processes

- (Semi-)Inclusive Deep-Inelastic Scattering
- Semi-Inclusive e^+e^- Annihilation
- Double Longitudinal Spin Asymmetries



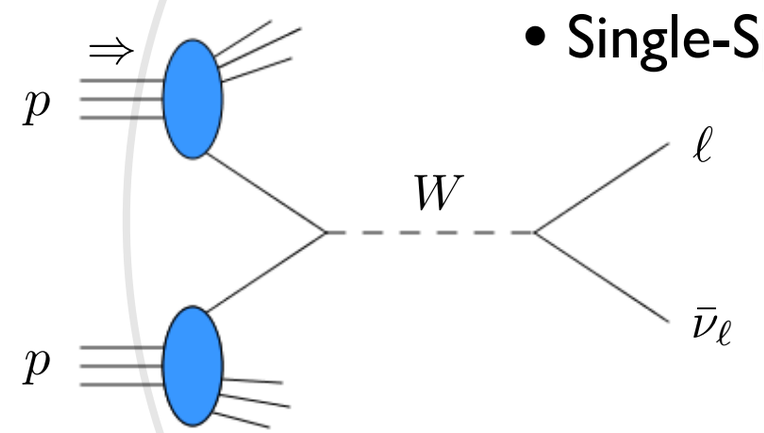
Hadronic QCD Hard Scattering

- Di-Hadron Production
- Single-Inclusive Jet Production



Heavy Gauge Boson Production

- Lepton p_T distribution
- Single-Spin Asymmetries at RHIC



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Outline

- Resummation and HMC for color singlet processes
- QCD Hard Scattering
- Heavy Gauge Boson Production
- Conclusions

Publications:

Anderle, FR, Vogelsang - PRD `13
Anderle, FR, Vogelsang - PRD `13
Accardi, Anderle, FR - PRD `15

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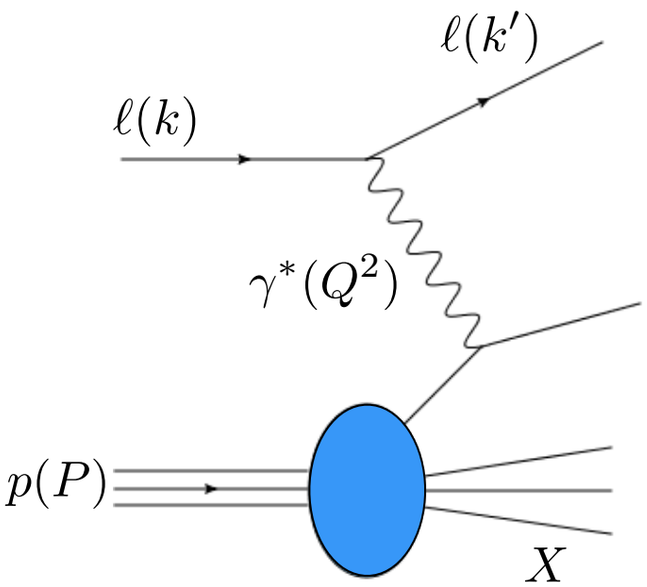
Deep-Inelastic Scattering



Define: $Q^2 \equiv -q^2 = -(k-k')^2$

$$x_B = \frac{Q^2}{2P\cdot q} \qquad y \equiv \frac{P\cdot q}{P\cdot k}$$

Factorized cross section

$$\frac{d^2\sigma}{dx_B dy} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}\mathcal{F}_T(x_B,Q^2) + \frac{1-y}{y}\mathcal{F}_L(x_B,Q^2)\right]$$

universal PDF

with structure functions $i = T,\, L$

$$\mathcal{F}_i(x_B,Q^2) = \sum_f \int_{x_B}^1 \frac{dx}{x} f\left(\frac{x_B}{x},\mu^2\right) \mathcal{C}_f^i\left(x, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

hard scattering part

(up to power corrections $1/Q^2$)

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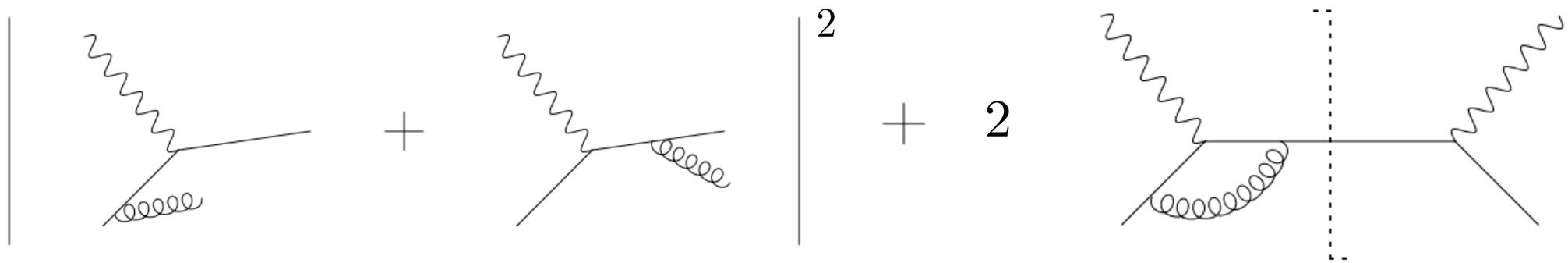
$$\mathcal{F}_i(x_B, Q^2) = \sum_f \int_{x_B}^1 \frac{dx}{x} f\left(\frac{x_B}{x}, \mu^2\right) \mathcal{C}_f^i \left(x, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

$$\mathcal{C}_f^i = C_f^{i,(0)} + \frac{\alpha_s(\mu^2)}{2\pi} C_f^{i,(1)} + \mathcal{O}(\alpha_s^2)$$

at NLO:



large corrections for $x \to 1$

Altarelli et al. `79; Furmanski, Petronzio `82; de Florian et al. `13

$$C_{q,\mathrm{th}}^{1,(1)}(x) = C_F \left[(1+x^2) \left(\frac{\ln(1-x)}{1-x} \right)_+ - \frac{3}{2} \frac{1}{(1-x)_+} - \left(\frac{9}{2} + \frac{\pi^2}{3} \right) \delta(1-x) \right]$$

spoils perturbative convergence for $x \to 1$ even if $\alpha_s \ll 1$

$\overline{\mathrm{MS}}$ scheme

$$\int_0^1 dz\, h(z)\, [g(z)]_+ \equiv \int_0^1 dz\, [h(z) - h(1)]\, g(z)$$

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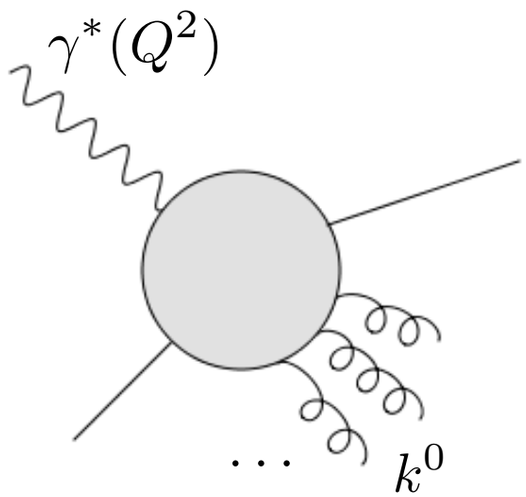
Threshold Logarithms



kth order: $\alpha_s^k \left(\frac{\ln^n(1-x)}{1-x} \right)_+ , \quad \text{with } n \leq 2k-1$

- Partonic threshold $x \to 1$: soft gluon radiation from the LO process $\gamma^* q \to q$
- Origin: suppression of real gluon emission while virtual corrections are allowed
- Logarithms may spoil perturbative series, unless taken into account to all orders

$\longrightarrow$ Threshold resummation

Sterman `81; Catani, Trentadue `89

Mellin Transform Space

- Structure function

$$\mathcal{F}_1^N(Q^2) = \int dx_B \, x_B^{N-1} \, \mathcal{F}_1(x_B, Q^2)$$
$$= \left(\int_0^1 dx \, x^{N-1} \mathcal{C}_f^1 \left(x, Q^2/\mu^2, \alpha_s(\mu^2) \right) \right) \left(\int_0^1 dy \, y^{N-1} f(y, \mu^2) \right)$$

- Threshold logarithms

$$\alpha_s^k \left(\frac{\ln^{2k-1}(1-x)}{1-x} \right)_+ \rightarrow \alpha_s^k \ln^{2k} \bar{N}$$

large logarithms in N

Accuracy of Resummation

$$\mathcal{O}(\alpha_s^k): \qquad C_{kn} \times \alpha_s^k \ln^n \bar{N}, \quad \text{where } n \leq 2k$$

Fixed Order

Resummation

LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
...	...	...	...	...	...	
N^{k}LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	...

LL NLL NNLL

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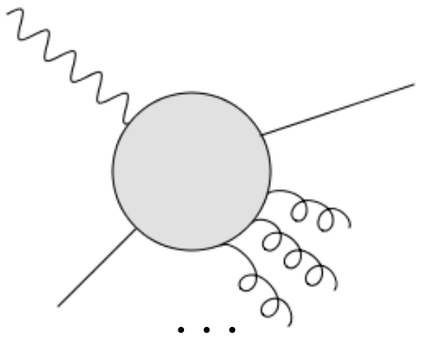
Resummed result



Resummation relies on factorization of

- QCD matrix elements for n-gluon emission in the soft limit
- phase space in Mellin space

in Mellin space: exponentiation of eikonal diagrams

$$\mathcal{C}^{1,N}_{q,\mathrm{res}}(Q^2/\mu^2,\alpha_s(\mu^2)) = e_q^2 H_q\left(Q^2/\mu^2,\alpha_s(\mu^2)\right)\ \Delta^N_q(Q^2/\mu^2,\alpha_s(\mu^2))\, J^N_q(Q^2/\mu^2,\alpha_s(\mu^2))$$

where

$$\log \Delta^N_q \equiv \int_0^1 dx \frac{x^N-1}{1-x}\int_{Q^2}^{(1-x)^2Q^2}\frac{dk_\perp^2}{k_\perp^2}A_q(\alpha_s(k_\perp^2))$$

$$\log J^N_q \equiv \int_0^1 dx \frac{x^N-1}{1-x}\left\{\int_{(1-x)^2Q^2}^{(1-x)Q^2}\frac{dk_\perp^2}{k_\perp^2}A_q(\alpha_s(k_\perp^2)) + \frac{1}{2}B_q(\alpha_s((1-x)Q^2)\right\}$$

calculable perturbatively

Sterman `81; Gatheral `83; Frenkel, Taylor `84

$\overline{\mathrm{MS}}$ scheme

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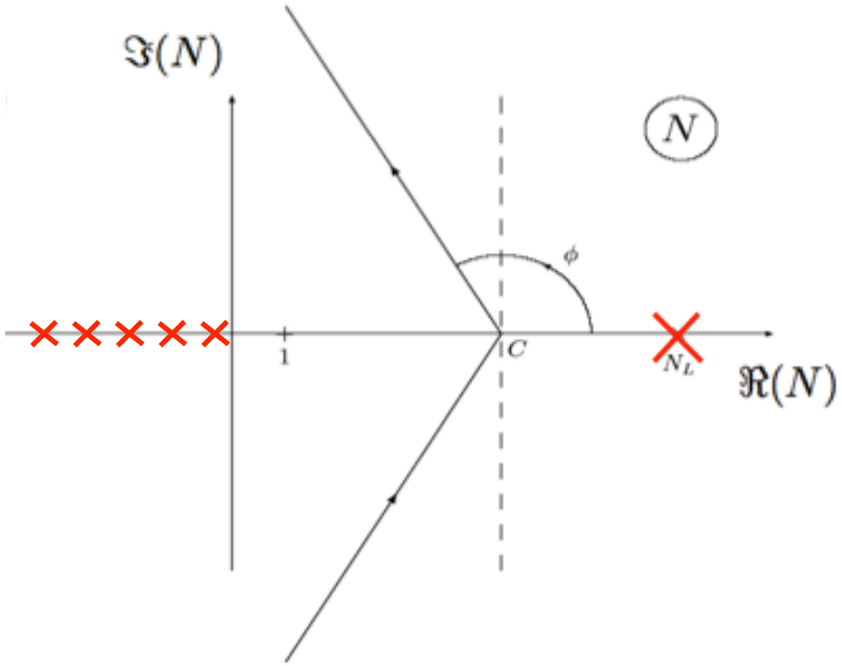
Matching and Minimal Prescription

- Matching procedure (avoiding double counting)

$$d\sigma^{\text{match}} = \left(d\sigma^{\text{resum}} - d\sigma^{\text{resum}}\Big|_{\mathcal{O}(\alpha_s)} \right) + d\sigma^{\text{NLO}}$$

- Inverse Transformation

$$\mathcal{F}_{1,\text{res}}(x_B, Q^2) = \int_{\mathcal{C}_N} \frac{dN}{2\pi i}\, x_B^{-N}\; \mathcal{C}^{1,N}_{q,\text{res}}(Q^2/\mu^2, \alpha_s(\mu^2))\, f^N(\mu^2)$$



Choosing the contour to the left of the Landau pole

Catani, Mangano, Nason, Trentadue `96

Combining Resummation and TMC

- Target Mass Corrections *Accardi, Qiu `09*

$$2F_1^{\mathrm{TMC}}(x_B, Q^2) = \sum_f \int_\xi^{\xi/x_B} \frac{dx}{x} f(x,\mu^2)\, C_f^1\left(\frac{\xi}{x}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

with the Nachtmann variable $\xi = \dfrac{2x_B}{1+\sqrt{1+4x_B^2 m_N^2/Q^2}}$

- Combination with resummation *Accardi, Anderle, FR `15*

$$2F_1^{\mathrm{TMC}}(x_B, Q^2) = \underline{\int_\xi^{\xi_{\mathrm{th}}} \frac{dx}{x} C_f^1\left(\frac{\xi}{x}\right) f(x)} + \int_{\xi_{\mathrm{th}}}^{\xi/x_B} \frac{dx}{x} C_f^1\left(\frac{\xi}{x}\right) f(x) \quad \text{with} \quad \xi_{\mathrm{th}} = \frac{2}{1+\sqrt{1+4m_N^2/Q^2}}$$

Taking moments in ξ : $\left(\int_0^1 dy\, y^{N-1} C_f^1(y)\right)\left(\int_0^{\xi_{\mathrm{th}}} dx\, x^{N-1} f(x)\right)$

as before

truncated moments of the PDF

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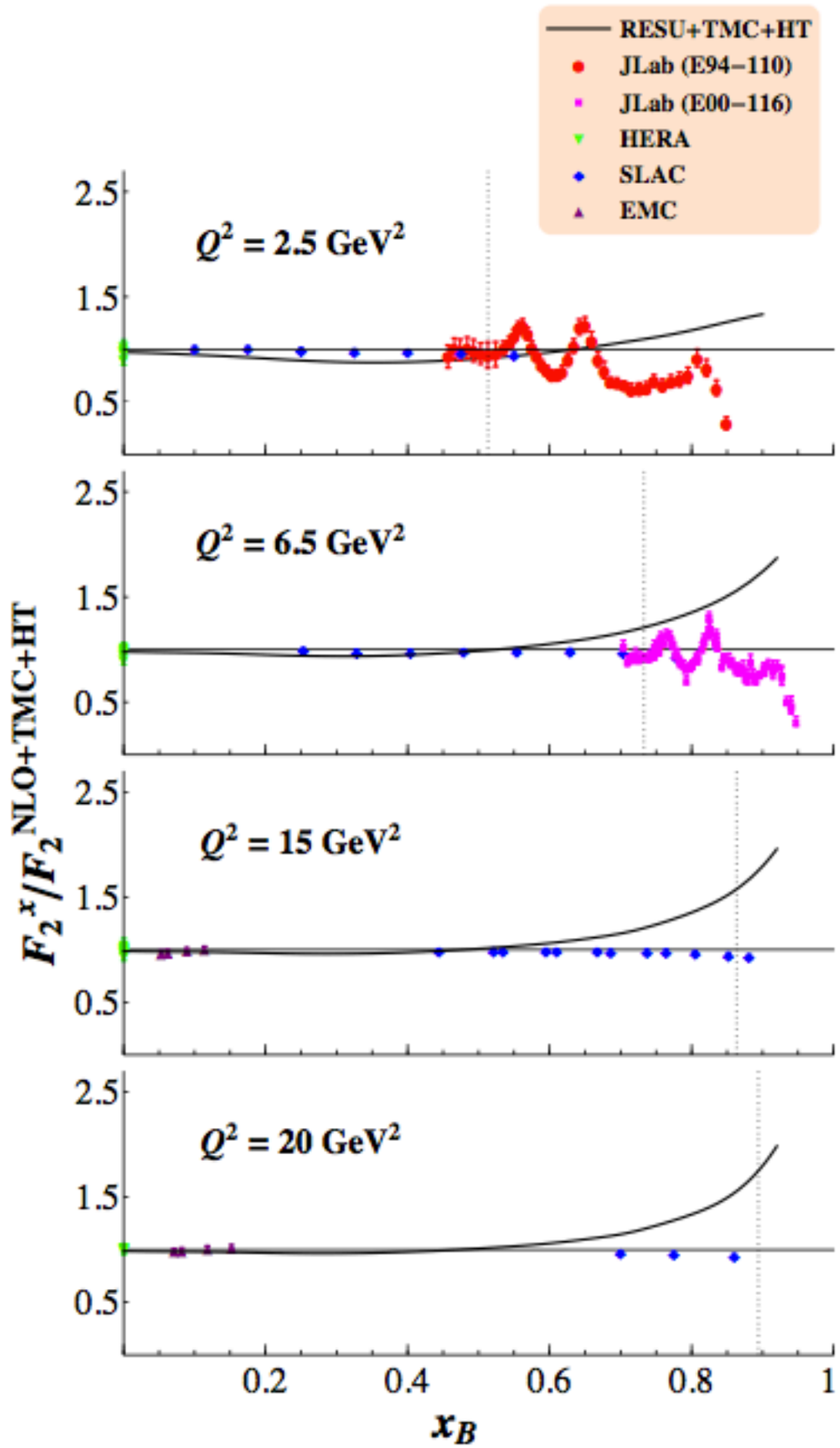
Relevance for the extraction of PDFs

TMC
Higher Twist

+ Resummation



data/ theory
using CJ12 PDFs

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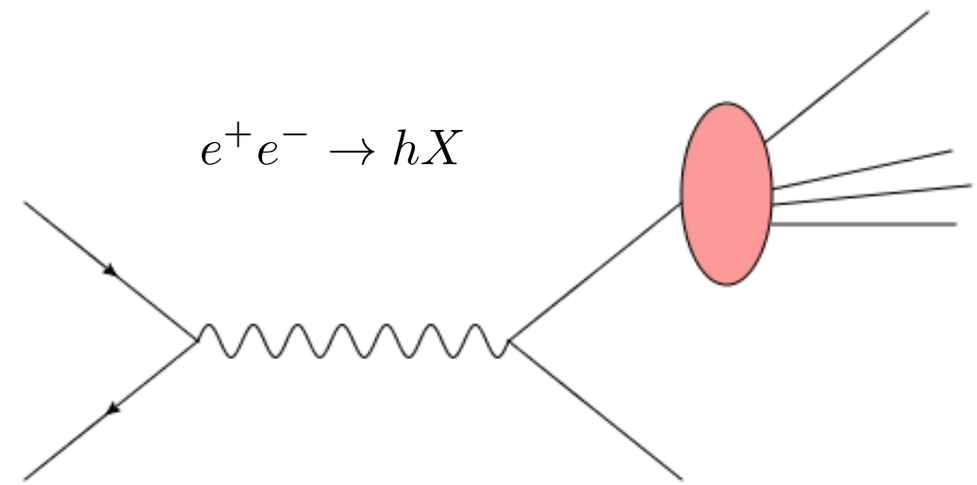
Semi-Inclusive e^+e^- Annihilation

Cross section including HMC

$$\frac{d^2\sigma^h}{dx_E d\cos\theta} = \frac{\pi\alpha^2}{Q^2} N_c \frac{1}{1-\frac{m_h^2}{\xi_E^2 Q^2}} \left[\frac{1+\cos^2\theta}{2} \hat{\mathcal{F}}_1^h(x_E, Q^2) + \sin^2\theta\, \hat{\mathcal{F}}_L^h(x_E, Q^2) \right]$$



Fragmentation Function

$$\hat{\mathcal{F}}_i^h(x_E, Q^2) = \sum_f \int_{x_E}^1 \frac{d\hat{z}}{\hat{z}} D_f^h\left(\frac{x_E}{\hat{z}}, \mu^2\right) \hat{\mathcal{C}}_f^i\left(\hat{z}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

Similar threshold logarithms as for DIS

(``crossed resummation´´ Sterman, Vogelsang `06)

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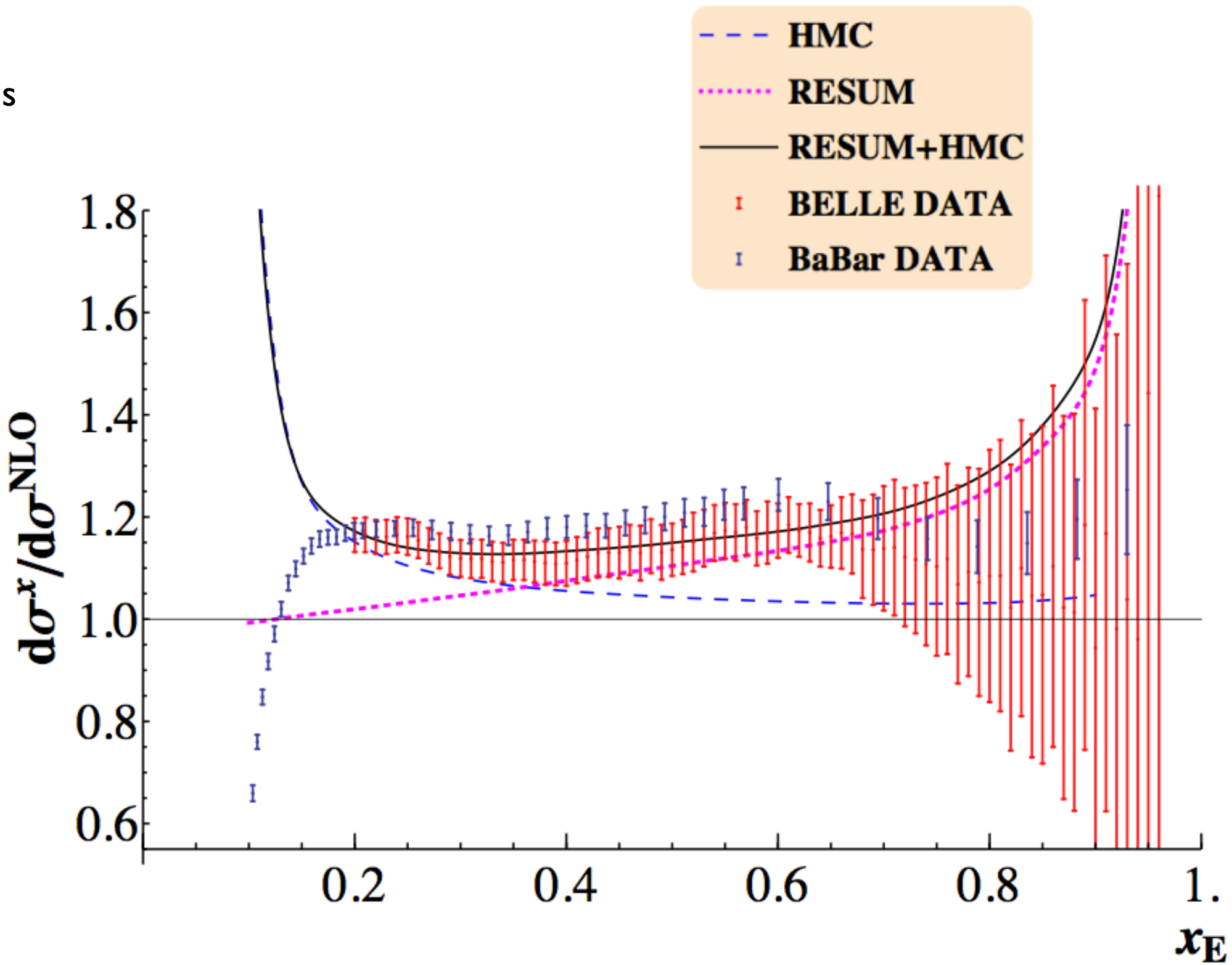
Kaons



BELLE and BaBar data/ theory using DSS `08 FFs

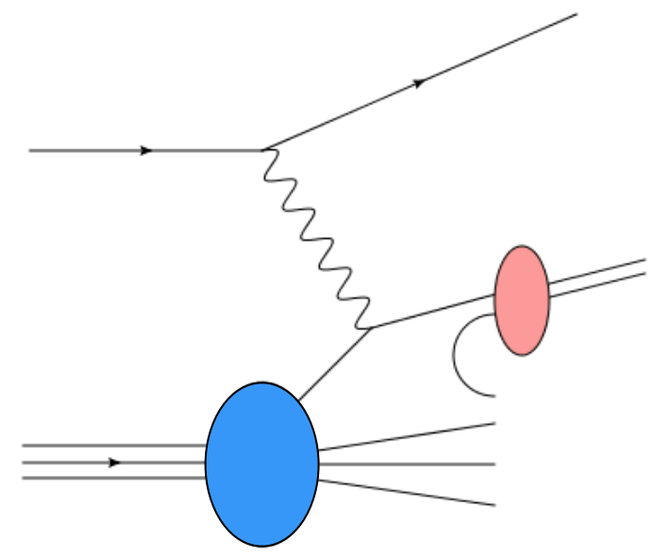
Pions similarly

Semi-Inclusive DIS

- Coefficient function

$$\mathcal{F}_i^h(x,z,Q^2) = \sum_{f,f'} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} f\left(\frac{x}{\hat{x}},\mu^2\right) D_{f'}^h\left(\frac{z}{\hat{z}},\mu^2\right) \mathcal{C}_{f'f}^i\left(\hat{x},\hat{z},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)$$

$$\alpha_s^k \left(\frac{\ln^m(1-\hat{x})}{1-\hat{x}}\right)_+ \left(\frac{\ln^n(1-\hat{z})}{1-\hat{z}}\right)_+$$



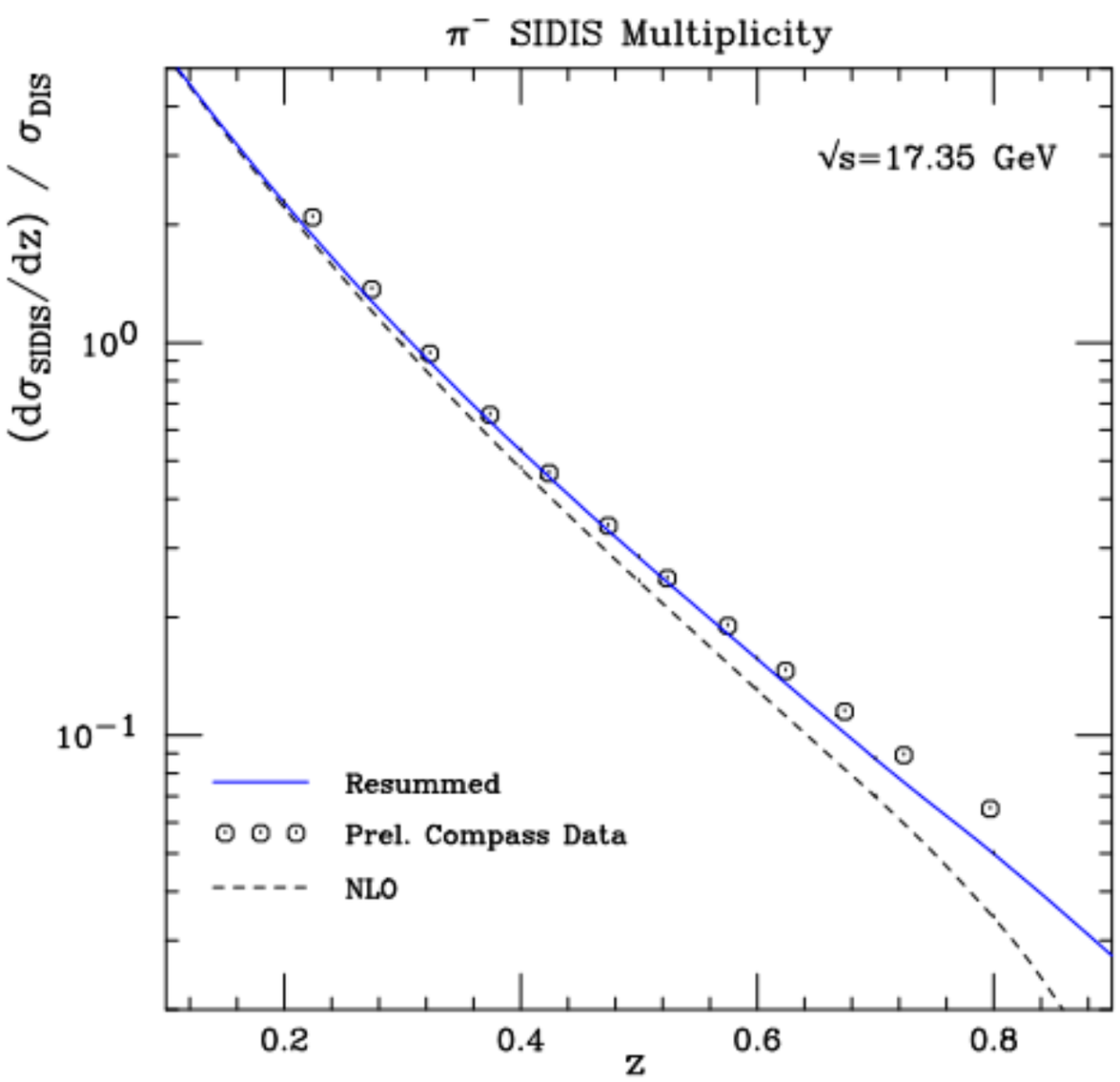
- Double Mellin moments

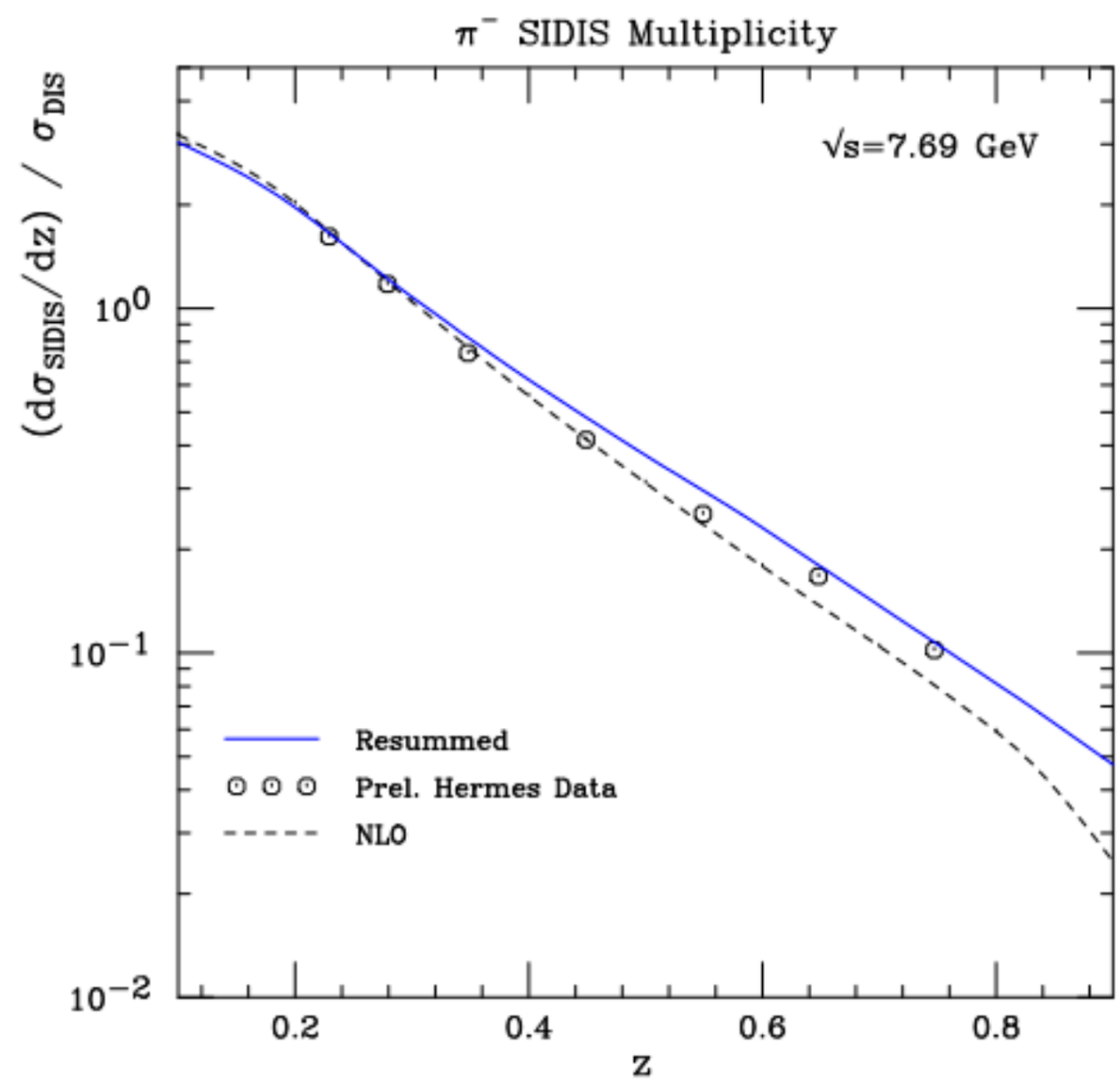
$$\mathcal{C}_{qq}^{T,\mathrm{res}}(N,M,\alpha_s(Q^2))$$

$$\propto \exp\left[\int_0^{Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q\left(\alpha_s(k_\perp^2)\right) \left\{ \int_{\frac{k_\perp^2}{Q^2}}^1 \frac{d\xi}{\xi} \left[\mathrm{e}^{-N\xi - M\frac{k_\perp^2}{\xi Q^2}} - 1\right] + \ln\bar{N} + \ln\bar{M} \right\}\right]$$

(no HMCs yet)

Hadron Multiplicities for Pions



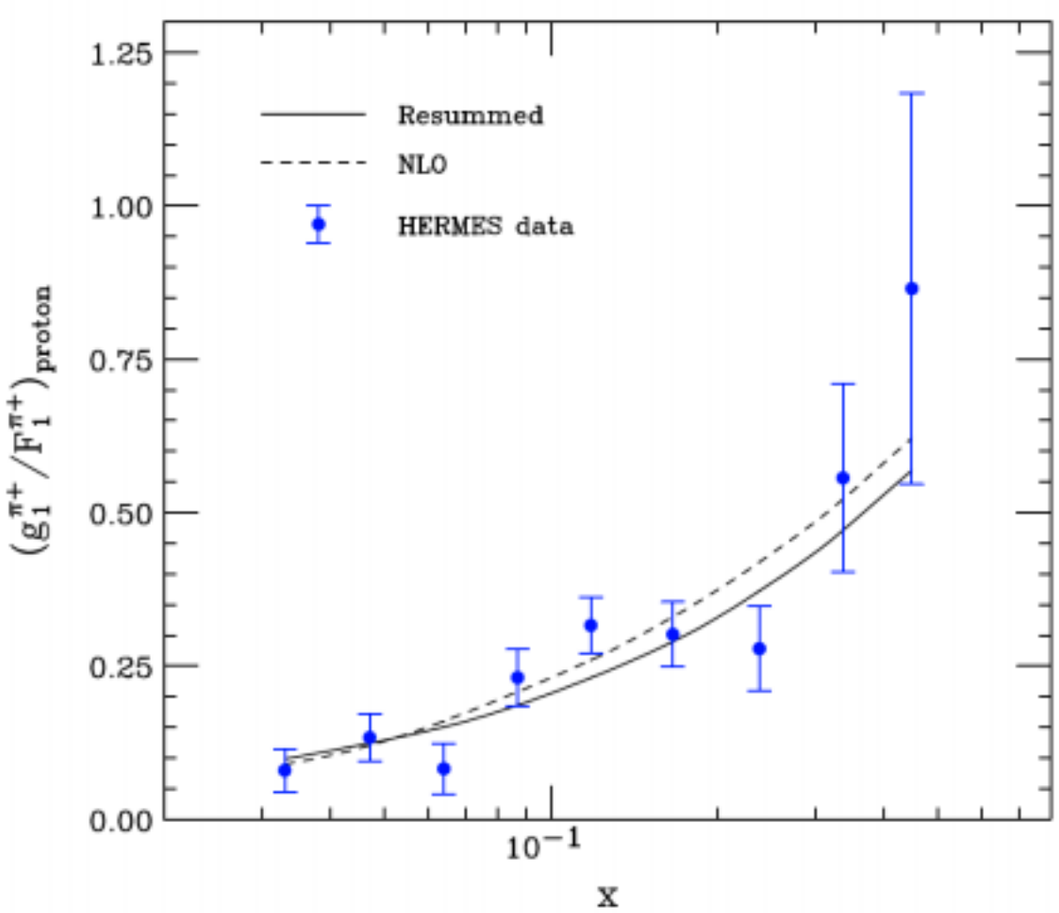


using MSTW PDFs and DSS `08 FFs

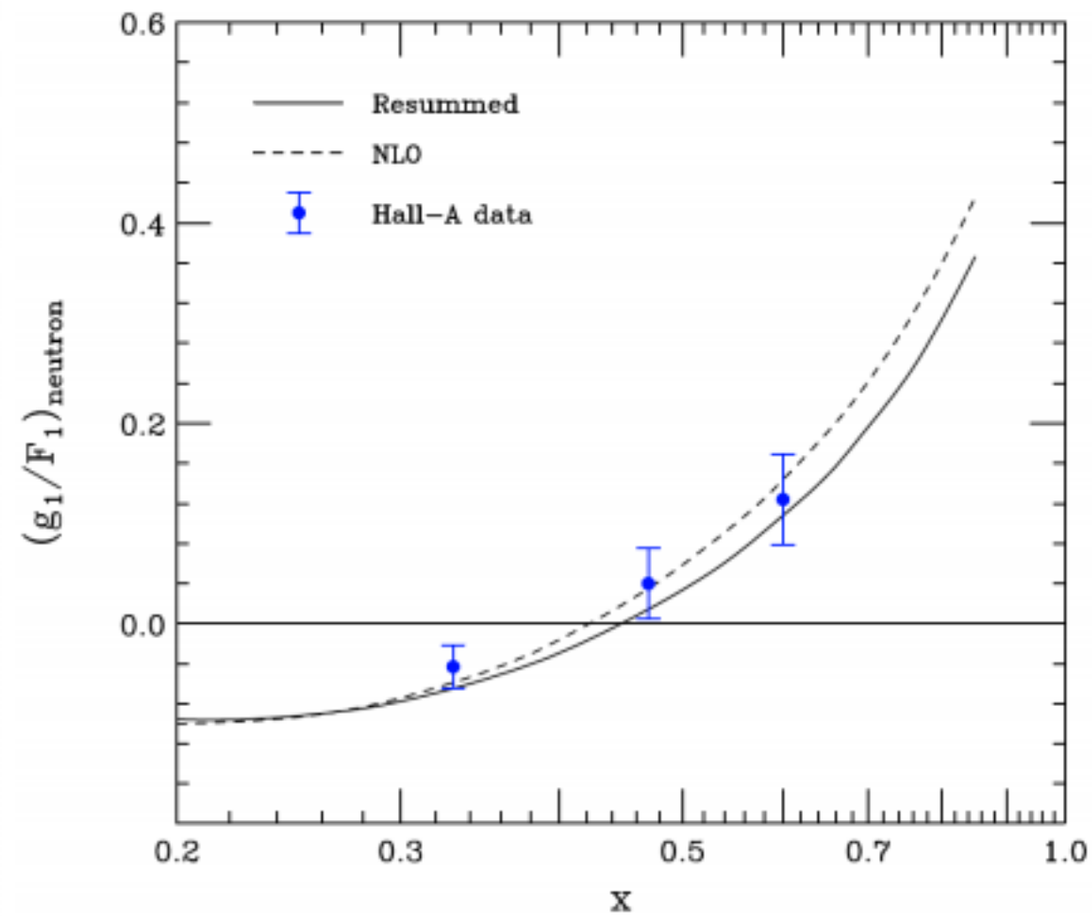
Spin Asymmetries for (SI)DIS

SIDIS: $$A_1^h(x,z,Q^2) \approx \frac{g_1^h(x,z,Q^2)}{F_1^h(x,z,Q^2)}$$

$\vec{\ell}\vec{p} \to \ell h X$ SIDIS, proton target



$\vec{\ell}\vec{p} \to \ell X$ DIS, neutron target



⟶ JLab, 12 GeV upgrade ...

using DSSV, MSTW PDFs and DSS `08 FFs

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Outline

- Resummation and HMC for color singlet processes
- QCD Hard Scattering
- Heavy Gauge Boson Production
- Conclusions

Publications: *de Florian, Hinderer, Mukherjee, FR, Vogelsang - PRL '14*
Hinderer, FR, Sterman, Vogelsang - PRD '15

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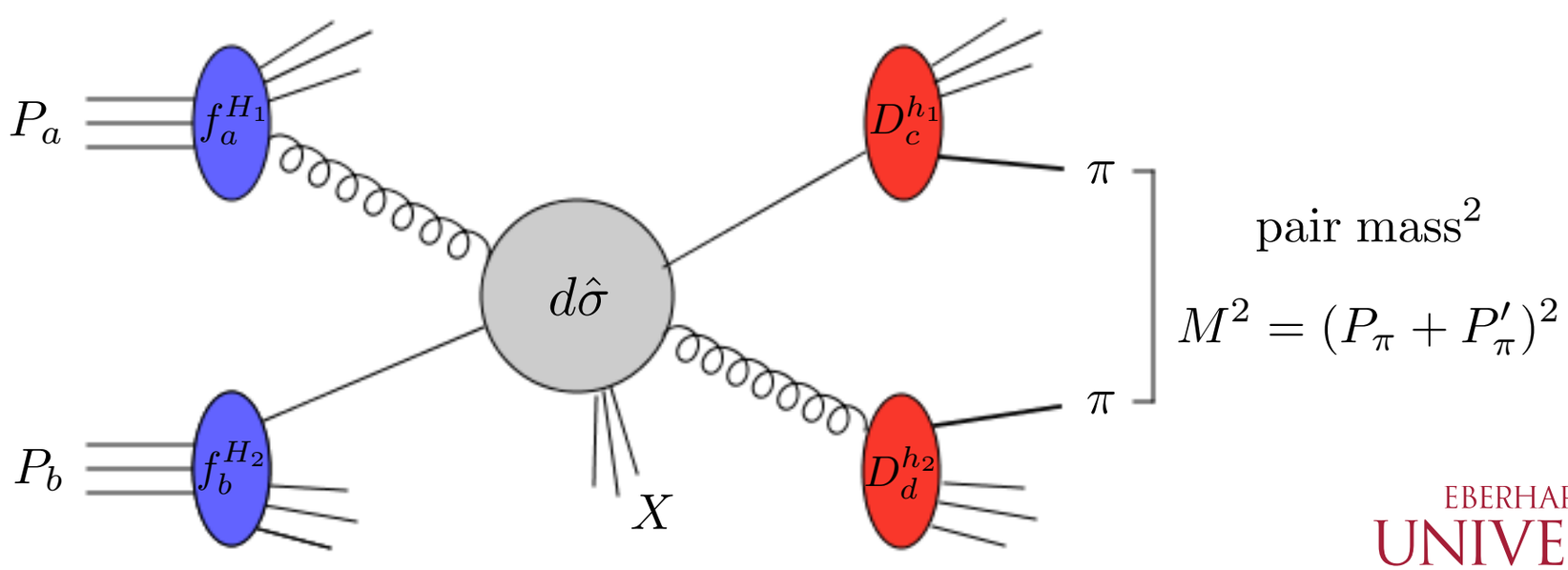
Di-Hadron Production

- ``Full QCD extension´´ of Drell-Yan
- Exisiting data from various experiments not very well described by NLO *Owens `02*
  data from fixed target: NA24, E711, E706
  collider regime: CCOR, RHIC (planned)
- Previous NLL results from *Almeida, Sterman, Vogelsang `09* ⟶ Extension toward NNLL
- Starting point for extension toward NNLL for other processes



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Threshold Resummation toward NNLL

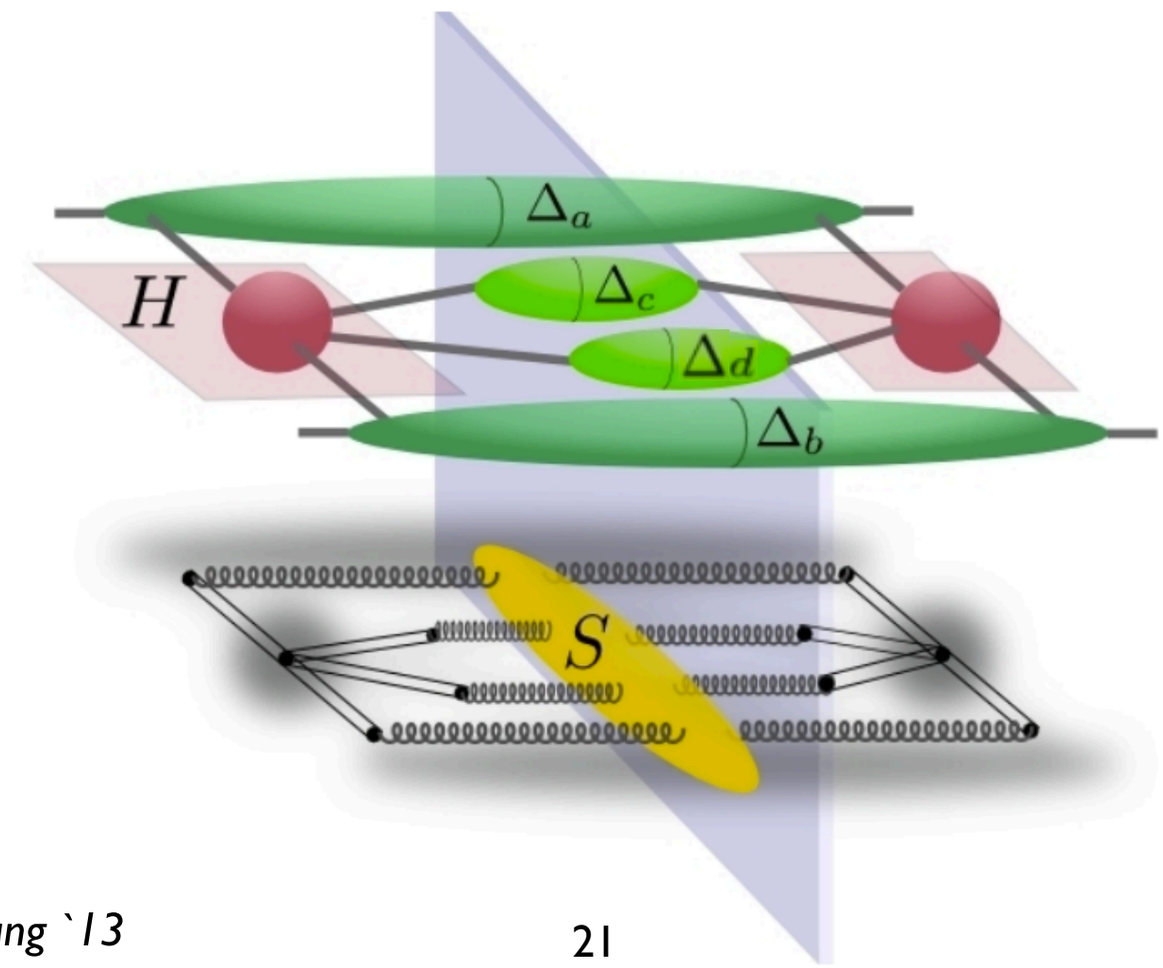
After taking Mellin and Fourier double moments

$$\tilde{\omega}^{\text{resum}}_{ab\to cd}\left(N,\Delta\eta,\alpha_s(\mu_R),\frac{\mu_R}{\hat{m}},\frac{\mu_F}{\hat{m}}\right) = \xi_R\left(\alpha_s(\mu_R),\frac{\mu_R}{\hat{m}}\right)\xi_F^{abcd}\left(\alpha_s(\mu_R),\frac{\mu_F}{\hat{m}}\right)$$

$$\times\,\Delta_a^{N+1}\left(\alpha_s(\mu_R),\frac{\mu_R}{\hat{m}},\frac{\mu_F}{\hat{m}}\right)\Delta_b^{N+1}\left(\alpha_s(\mu_R),\frac{\mu_R}{\hat{m}},\frac{\mu_F}{\hat{m}}\right)\Delta_c^{N+2}\left(\alpha_s(\mu_R),\frac{\mu_R}{\hat{m}},\frac{\mu_F}{\hat{m}}\right)\Delta_d^{N+2}\left(\alpha_s(\mu_R),\frac{\mu_R}{\hat{m}},\frac{\mu_F}{\hat{m}}\right)$$

$$\times\,\mathrm{Tr}\left\{H\left(\Delta\eta,\alpha_s(\mu_R)\right)\mathcal{S}_N^\dagger\left(\Delta\eta,\alpha_s(\mu_R),\frac{\mu_R}{\hat{m}}\right)S\left(\alpha_s(\hat{m}/\bar{N}),\Delta\eta\right)\mathcal{S}_N\left(\Delta\eta,\alpha_s(\mu_R),\frac{\mu_R}{\hat{m}}\right)\right\}_{ab\to cd}$$

Refactorization
near threshold



Kidonakis, Oderda, Sterman `98
Bonciani, Catani, Mangano, Nason `98
Banfi, Salam, Zanderighi
Dokshitzer, Marchesini
Almeida, Sterman, Vogelsang `09

Figure adapted from
de Florian, Pfeuffer, Schäfer, Vogelsang `13

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- Hard-Scattering Matrix: derived from virtual corrections

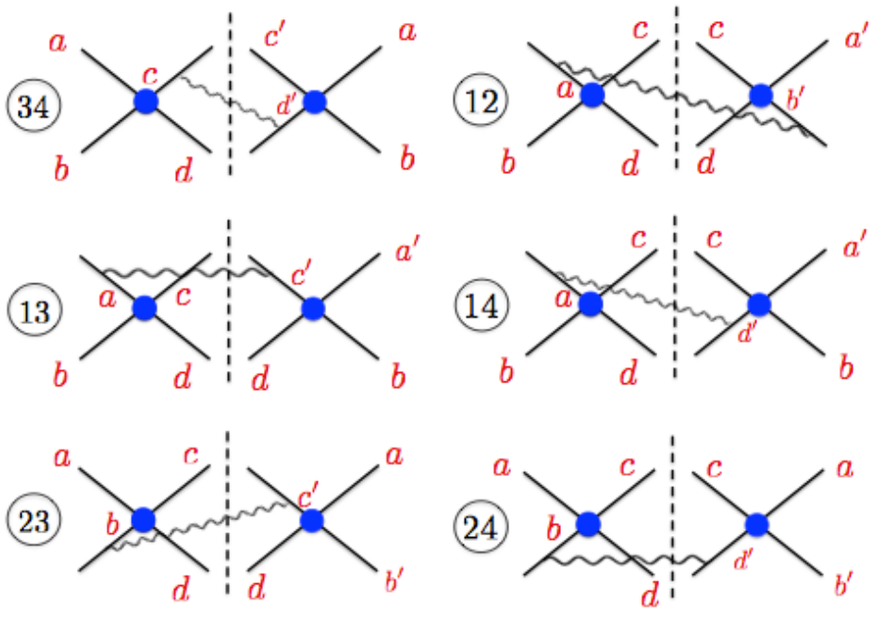
$qq' \to qq'$

$$H^{(1)} = \begin{pmatrix} \mathcal{R}e\left\{\frac{1}{2N_c^2}\left[\frac{s^2+u^2}{t^2}\left(-4C_F L(t)^2 + 2X_1(s,t,u)L(t) + 2Y\right) + \frac{s^2}{t^2}(C_A - 4C_F)Z(s,t,u) - \frac{u^2}{t^2}(2C_A - 4C_F)Z(u,t,s)\right]\right\} & \left(H^{(1)}\right)_{12} = \left(H^{(1)}\right)^*_{21} \\ \frac{1}{2N_c^2}\left[\frac{s^2+u^2}{t^2}X_2(s,t,u)L(t) - \frac{s^2}{t^2}\frac{C_F}{2C_A}Z(s,t,u) + \frac{u^2}{t^2}\frac{C_F}{2C_A}Z(u,t,s)\right] & 0 \end{pmatrix}_{2\times 2}$$

with e.g. $X_1(s,t,u) = 6C_F - 4\pi b_0 + 8C_F[L(s) - L(u)] - 2C_A[2L(s) - L(t) - L(u)]$ ...

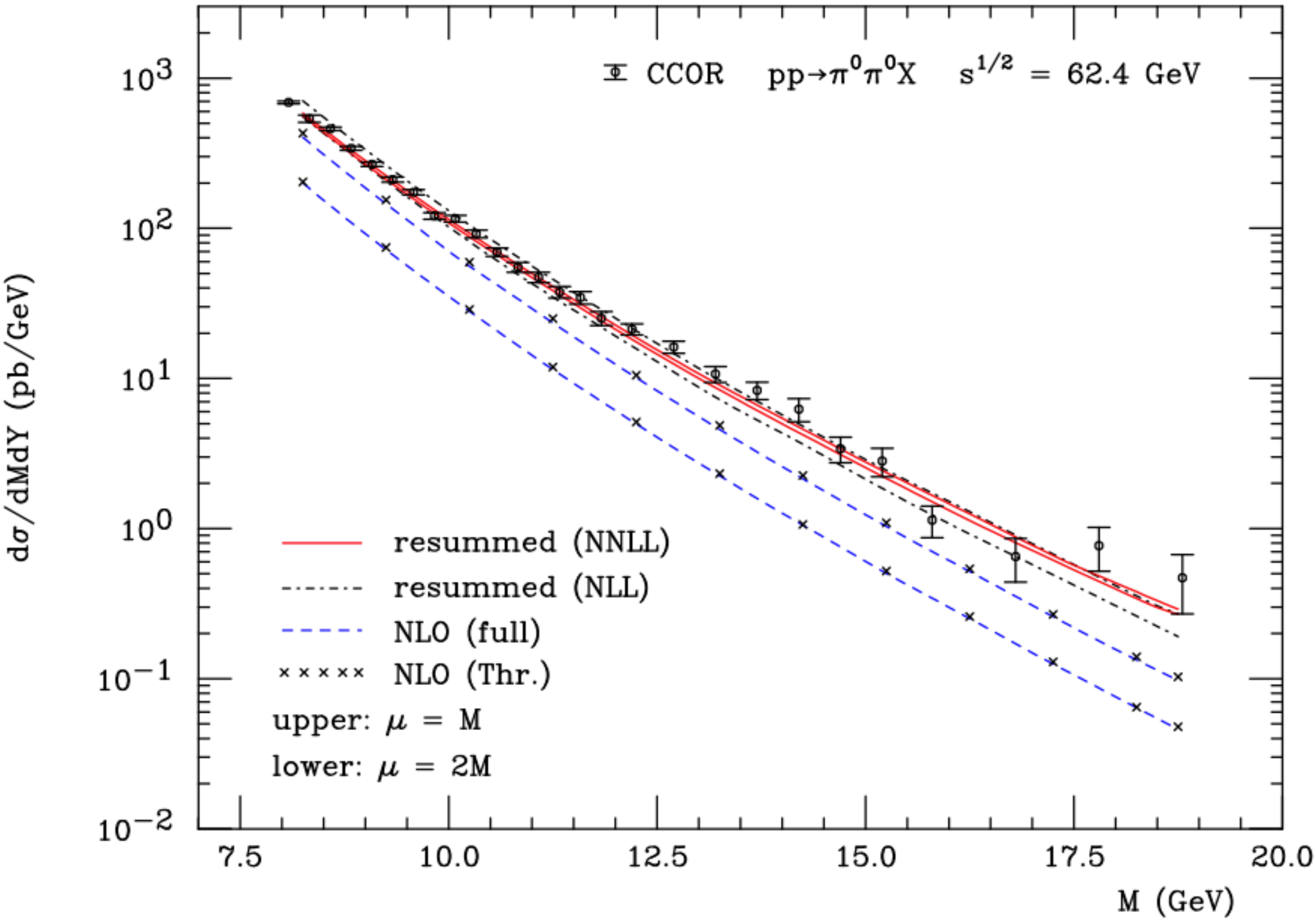
and $L(t) = \log\frac{-t}{s}, \quad L(u) = \log\frac{-u}{s}, \quad L(s) = -i\pi$

see also Kelley, Schwartz `11; Broggio et al. `14



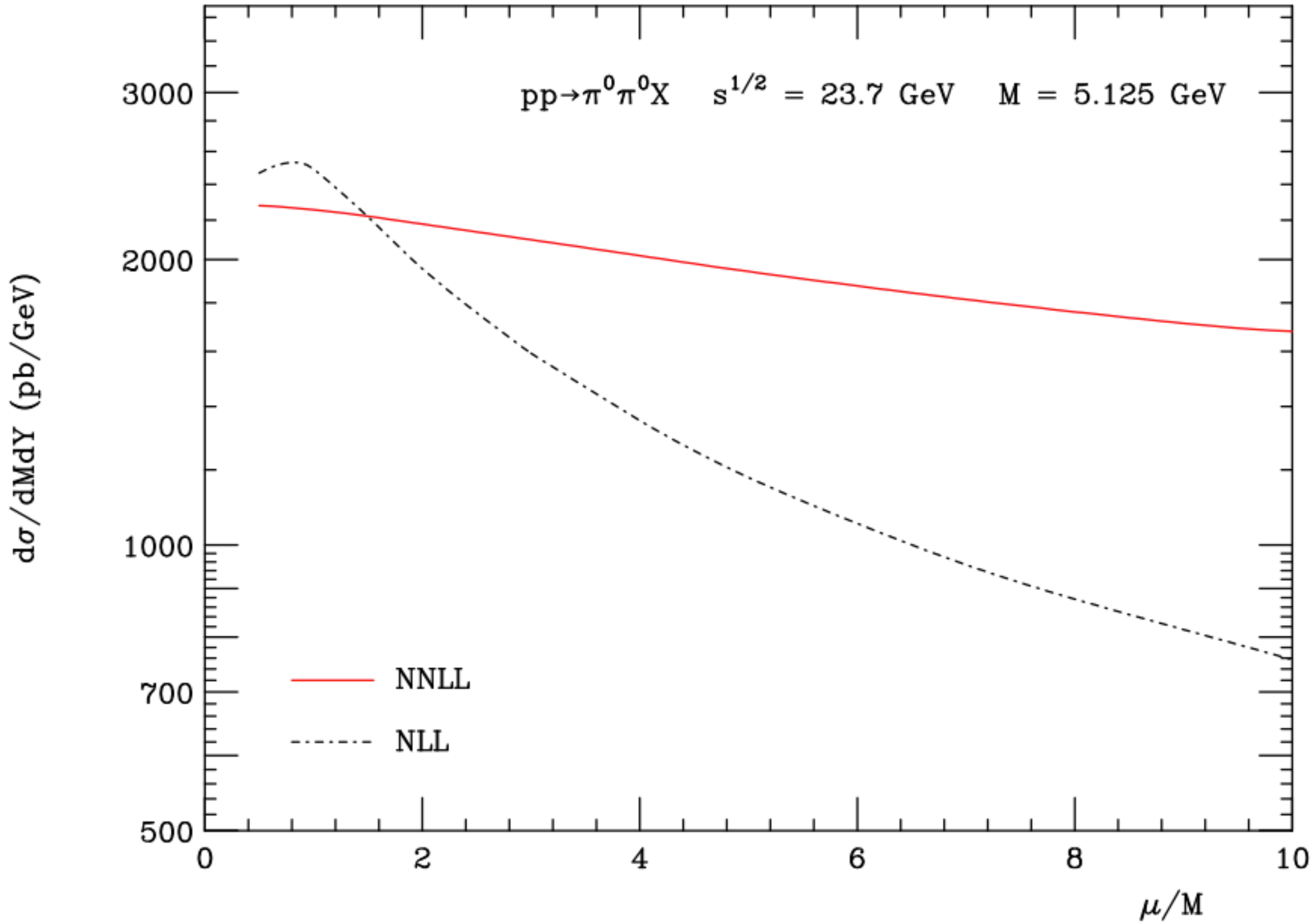
- Soft Matrix: derived from real emission diagrams in the eikonal approximation

$$S^{(1)} = \frac{C_F}{2}\begin{pmatrix} \mathrm{Li}_2\left(-\frac{u}{t}\right) + (2 - N_c^2)\,\mathrm{Li}_2\left(-\frac{t}{u}\right) & -2N_c\,\mathrm{Li}_2\left(-\frac{t}{u}\right) \\ -2N_c\,\mathrm{Li}_2\left(-\frac{t}{u}\right) & -4N_c^2\,\mathrm{Li}_2\left(-\frac{u}{t}\right) \end{pmatrix}$$



CCOR

CTEQ6 NLO PDFs, DSS `14 FFs



Scale dependence

CTEQ6 NLO PDFs, DSS `14 FFs

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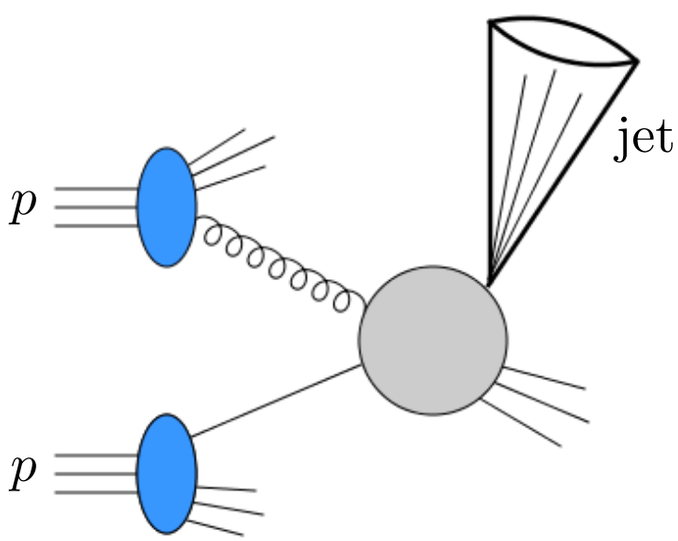
Single-Inclusive Jet Production

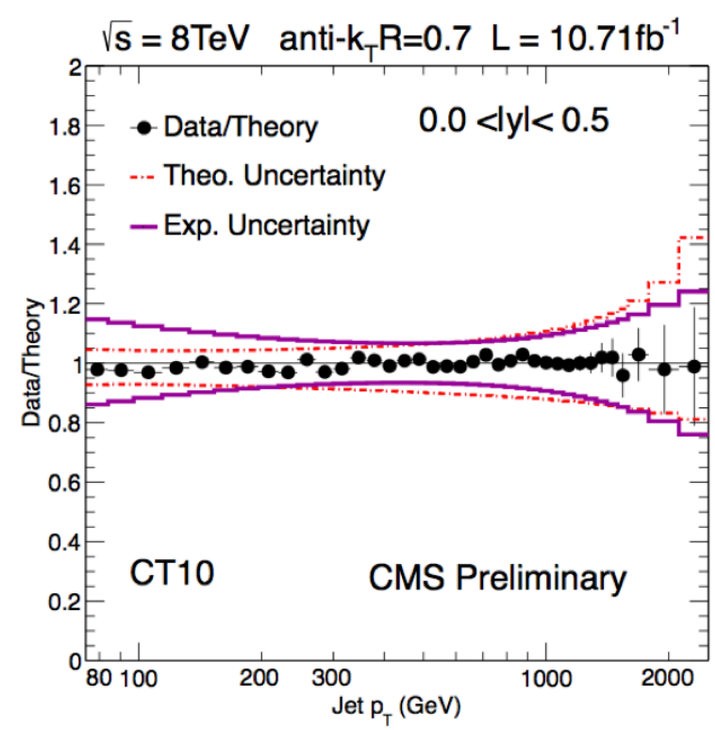
- Large theoretical uncertainties especially at high p_T
- PDFs and α_s are constrained by collider jet data
- High p_T jets are a promising observable for the search of BSM physics at the LHC
- Analytical results obtained in the ``Narrow Jet Approximation´´

Jäger, Stratmann, Vogelsang '04; Mukherjee, Vogelsang '13

$$\mathcal{A}\log R + \mathcal{B} + \mathcal{O}(R^2)$$

- First results based on NLL; Preliminary results toward NNLL





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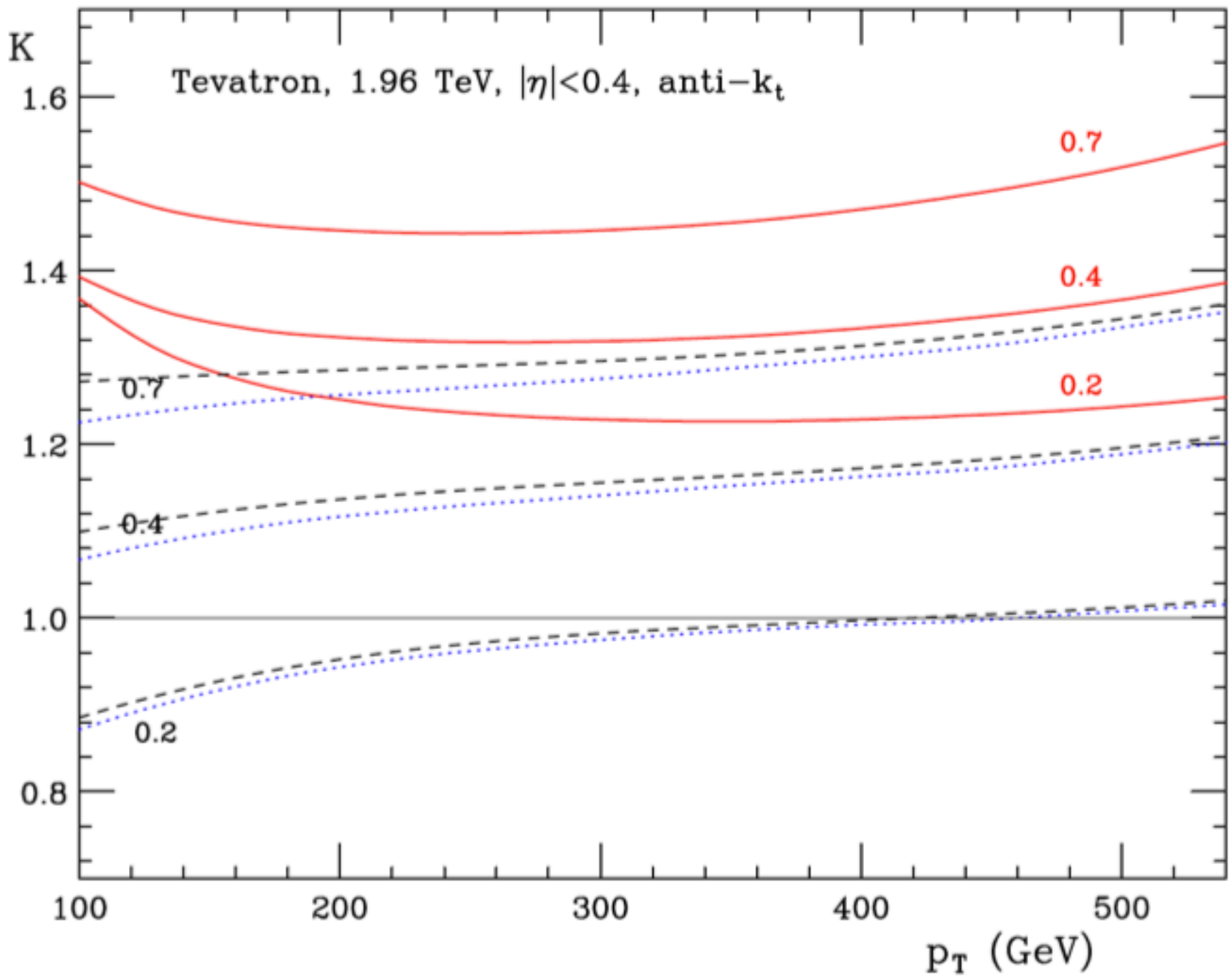
Approximate NNLO-NLL results



Approximate NNLO-NLL results (planned to be) used by PDF groups, ``fastNLO´´... For example, the latest NNPDF3.0 set *Ball et al. `14*

using MSTW-nnlo PDFs

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Outline

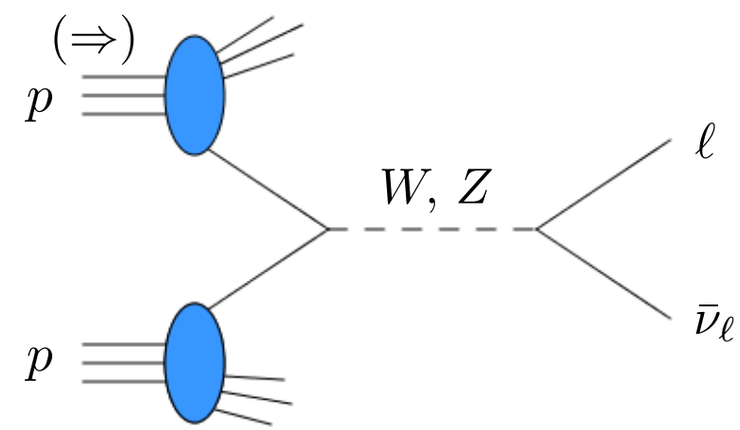
- Resummation and HMC for color singlet processes
- QCD Hard Scattering
- Heavy Gauge Boson Production
- Conclusions

Publications: *FR, Vogelsang - to be submitted*

Heavy Gauge Boson Production

- New analytical results at NLO

$$q\bar{q}' \to \ell^{\pm} X \ , \ qg \to \ell^{\pm} X$$



- Structure $\quad 1 - v = \frac{p_T^\ell}{\sqrt{s}} \mathrm{e}^{-\hat{\eta}} \qquad vw = \frac{p_T^\ell}{\sqrt{s}} \mathrm{e}^{+\hat{\eta}}$

$$\frac{d\hat{\sigma}^{\mathrm{NLO}}}{dvdw} = \frac{\alpha_s}{2\pi} \left\{ \frac{f_{\mathrm{LO}}(v)}{(s - M_W^2)^2 + \Gamma^2 M_W^2} \left[A \left(\frac{\ln(1-w)}{1-w} \right)_+ + B(v) \frac{1}{(1-w)_+} \right. \right.$$
$$\left. + C(v)\delta(1-w) \right]$$
$$\left. + \ldots + \frac{\ln\left(\frac{(ws - M_W^2)^2 + \Gamma^2 M_W^2}{M_W^4 + \Gamma^2 M_W^2} \right)}{(ws - M_W^2)^2 + \Gamma^2 M_W^2} \quad + \ldots \right\}$$

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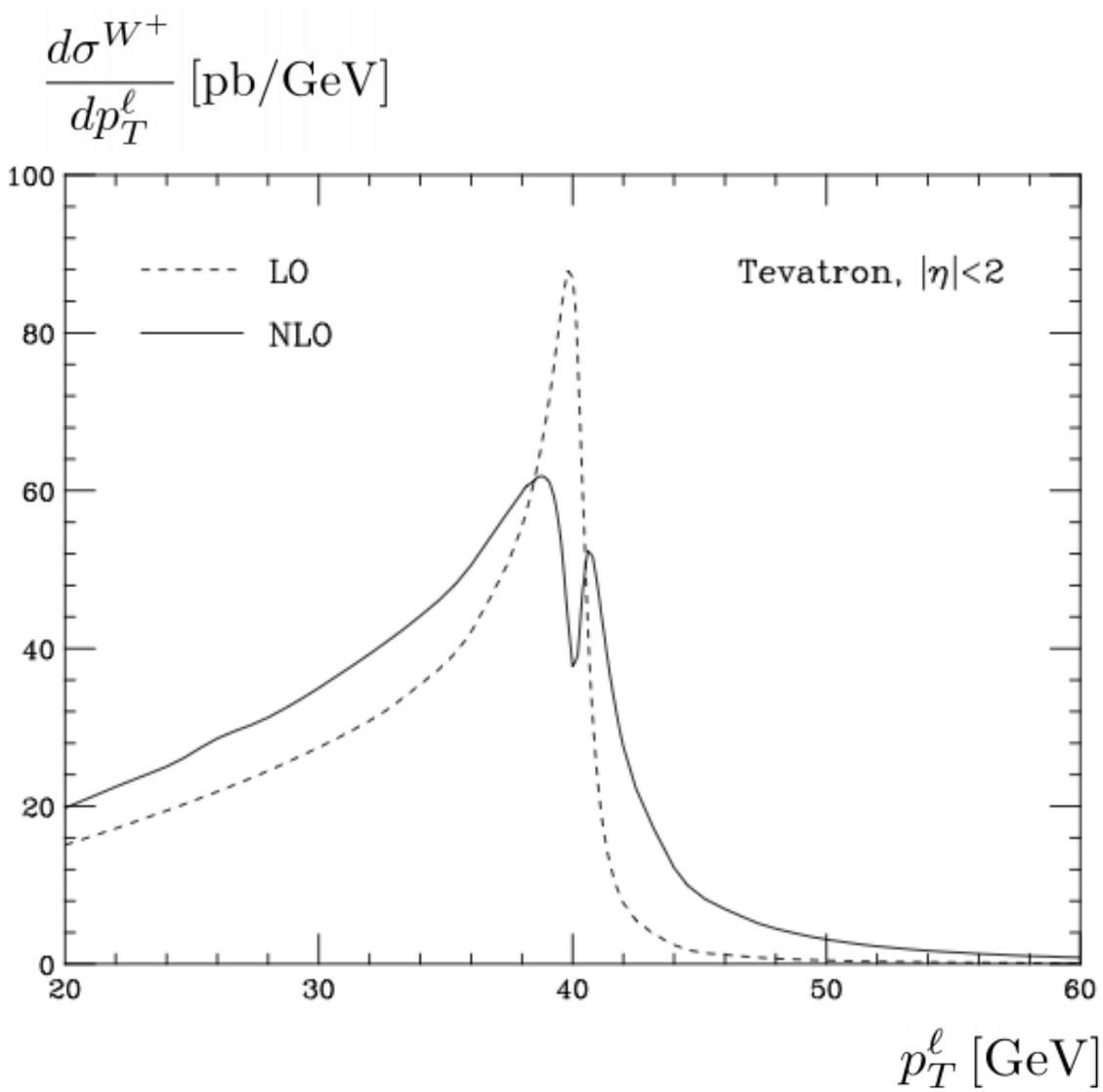
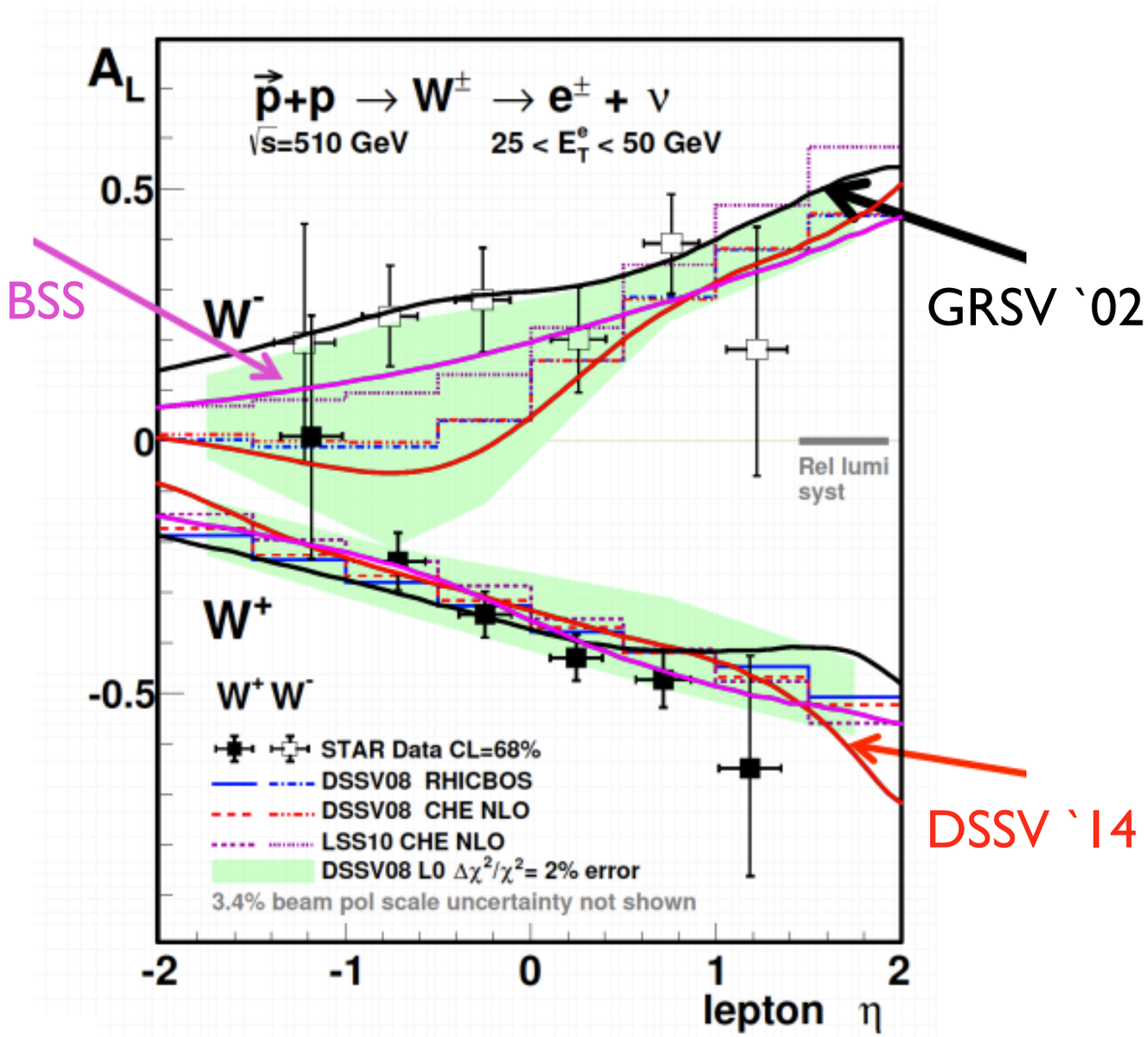
- Analytical results help to understand the structure around $p_T^\ell \sim M_W/2$
  $\longleftrightarrow$ interplay with resummation

- Polarized results are being used for including results on SSA from RHIC into the DSSV global analysis of polarized PDFs





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Conclusions

- Resummation and HMC for color-singlet processes. Future extension to NNLO, NNLL for data coming from JLab at 12 GeV
- More work necessary concerning HMCs
- Threshold resummation toward NNLL for di-hadron and Single-Inclusive Jet Production
- Possible extensions to processes such as di-jet, single-inclusive hadron production, photons etc.
- W, Z boson production for the (un-)polarized cross section
- RHIC data will be included in the DSSV PDF set