New Results on QCD Threshold Resummation

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JLab, 12.01.15
Biography

Jan 2012–present  PhD, Institute for Theoretical Physics, University of Tübingen, Germany.
Thesis title: “Threshold Resummation and Higher Order Effects in Perturbative QCD”
Supervisor: Prof. Werner Vogelsang
Expected to finish in February 2015

Oct 2009–Dec 2011  Completing diploma studies in physics, University of Tübingen, Germany.
Thesis title: “Contributions of the Weak Gauge Bosons to Spin-Dependent Hard QCD Processes”
Supervisor: Prof. Werner Vogelsang

St. John’s College, Cambridge
Scholarship from DAAD (German Academic Exchange Service)

Oct 2005  Starting diploma studies in physics, University of Tübingen, Germany.
Hadronic QCD Hard Scattering

- Di-Hadron Production
- Single-Inclusive Jet Production

Heavy Gauge Boson Production

- Lepton $p_T$ distribution
- Single-Spin Asymmetries at RHIC

Resummation and Hadron Mass Corrections for Color Singlet Processes

- (Semi-)Inclusive Deep-Inelastic Scattering
- Semi-Inclusive $e^+e^-$ Annihilation
- Double Longitudinal Spin Asymmetries
Outline

• Resummation and HMC for color singlet processes
  • QCD Hard Scattering
  • Heavy Gauge Boson Production
  • Conclusions

Publications:

Anderle, FR, Vogelsang - PRD `13
Anderle, FR, Vogelsang - PRD `13
Accardi, Anderle, FR - PRD `15
Define: \[ Q^2 \equiv -q^2 = -(k - k')^2 \]

\[ x_B = \frac{Q^2}{2P \cdot q} \quad \text{and} \quad y \equiv \frac{P \cdot q}{P \cdot k} \]

Factorized cross section:
\[
\frac{d^2\sigma}{dx_B dy} = \frac{4\pi\alpha^2}{Q^2} \left[ \frac{1 + (1 - y)^2}{2y} \mathcal{F}_T(x_B, Q^2) + \frac{1 - y}{y} \mathcal{F}_L(x_B, Q^2) \right]
\]

universal PDF
\[
\mathcal{F}_i(x_B, Q^2) = \sum_f \int_{x_B}^1 \frac{dx}{x} f \left( \frac{x_B}{x}, \mu^2 \right) C^i_f \left( x, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right)
\]

with structure functions \( i = T, L \)

(up to power corrections \( 1/Q^2 \))
\[ \mathcal{F}_i(x_B, Q^2) = \sum_f \int_{x_B}^1 \frac{dx}{x} f \left( \frac{x_B}{x}, \mu^2 \right) C^i_f \left( x, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) \]

\[ C^i_f = C^i_f(0) + \frac{\alpha_s(\mu^2)}{2\pi} C^i_f(1) + \mathcal{O}(\alpha_s^2) \]

at NLO:

large corrections for \( x \to 1 \)

\[ C^{1, (1)}_{q, \text{th}}(x) = C_F \left[ \left( 1 + x^2 \right) \left( \frac{\ln(1-x)}{1-x} \right) + \frac{3}{2} \frac{1}{(1-x)} + \left( \frac{9}{2} + \frac{\pi^2}{3} \right) \delta(1-x) \right] \]

spoils perturbative convergence for \( x \to 1 \)
even if \( \alpha_s \ll 1 \)

\[ \int_0^1 dz \ h(z) \ [g(z)]_+ = \int_0^1 dz \ [h(z) - h(1)] \ g(z) \]
Threshold Logarithms

\[ \gamma^*(Q^2) \]

\[ k^{th \ order:} \]

\[ \alpha_s^k \left( \frac{\ln^n(1-x)}{1-x} \right)_+, \quad \text{with } n \leq 2k - 1 \]

- Partonic threshold \( x \to 1 \) : soft gluon radiation from the LO process \( \gamma^* q \to q \)
- Origin: suppression of real gluon emission while virtual corrections are allowed
- Logarithms may spoil perturbative series, unless taken into account to all orders

Threshold resummation

\[ \text{Sterman '81; Catani, Trentadue '89} \]
Mellin Transform Space

• Structure function

\[ F_1^N(Q^2) = \int dx_B x_B^{N-1} F_1(x_B, Q^2) \]

\[ = \left( \int_0^1 dx x^{N-1} C_f^1(x, Q^2/\mu^2, \alpha_s(\mu^2)) \right) \left( \int_0^1 dy y^{N-1} f(y, \mu^2) \right) \]

• Threshold logarithms

\[ \alpha_s^k \left( \frac{\ln^{2k-1}(1-x)}{1-x} \right) \rightarrow \alpha_s^k \ln^{2k} N \]

large logarithms in \( N \)
### Accuracy of Resummation

\[ \mathcal{O}(\alpha_s^k) : C_{kn} \times \alpha_s^k \ln^n \bar{N}, \quad \text{where } n \leq 2k \]

<table>
<thead>
<tr>
<th>Fixed Order</th>
<th>Resummation</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>1</td>
</tr>
<tr>
<td>NLO</td>
<td>(\alpha_s L^2)</td>
</tr>
<tr>
<td>NNLO</td>
<td>(\alpha_s^2 L^4)</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N^k LO</td>
<td>(\alpha_s^k L^{2k})</td>
</tr>
<tr>
<td>LL</td>
<td>NLL</td>
</tr>
</tbody>
</table>
Resummed result

Resummation relies on factorization of

- QCD matrix elements for n-gluon emission in the soft limit
- phase space in Mellin space

in Mellin space: exponentiation of eikonal diagrams

\[ C_{q,\text{res}}^{1,N}(Q^2/\mu^2, \alpha_s(\mu^2)) = e_q^2 H_q(Q^2/\mu^2, \alpha_s(\mu^2)) \Delta_q^N(Q^2/\mu^2, \alpha_s(\mu^2)) J_q^N(Q^2/\mu^2, \alpha_s(\mu^2)) \]

where

\[ \log \Delta_q^N = \int_0^1 dx \frac{x^N - 1}{1 - x} \int_{Q^2}^{(1-x)^2 Q^2} \frac{dk^2}{k_{\perp}^2} A_q(\alpha_s(k_{\perp}^2)) \]

\[ \log J_q^N = \int_0^1 dx \frac{x^N - 1}{1 - x} \left\{ \int_{(1-x)^2 Q^2}^{(1-x)Q^2} \frac{dk^2}{k_{\perp}^2} A_q(\alpha_s(k_{\perp}^2)) + \frac{1}{2} B_q(\alpha_s((1-x)Q^2)) \right\} \]

calculable perturbatively

\[ \text{Sterman `81; Gatheral `83; Frenkel, Taylor `84} \]
Matching and Minimal Prescription

• Matching procedure (avoiding double counting)

\[ d\sigma^{\text{match}} = \left( d\sigma^\text{resum} - d\sigma^\text{resum}_{\mathcal{O}(\alpha_s)} \right) + d\sigma^\text{NLO} \]

• Inverse Transformation

\[ \mathcal{F}_{1,\text{res}}(x_B, Q^2) = \int_{C_N} \frac{dN}{2\pi i} \ x_B^{-N} C_{q,\text{res}}^{1,N}(Q^2/\mu^2, \alpha_s(\mu^2)) f^N(\mu^2) \]

Choosing the contour to the left of the Landau pole

*Catani, Mangano, Nason, Trentadue `96*
Combining Resummation and TMC

• Target Mass Corrections  Accardi, Qiu ’09

\[ 2F_1^{\text{TMC}}(x_B, Q^2) = \sum_f \int_{\xi}^{\xi/x_B} \frac{dx}{x} f(x, \mu^2) c_f^1 \left( \frac{\xi}{x}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) \]

with the Nachtmann variable  \[ \xi = \frac{2x_B}{1 + \sqrt{1 + 4x_B^2 m_N^2/Q^2}} \]

• Combination with resummation  Accardi, Anderle, FR ’15

\[ 2F_1^{\text{TMC}}(x_B, Q^2) = \int_{\xi}^{\xi_{\text{th}}} \frac{dx}{x} c_f^1 \left( \frac{\xi}{x} \right) f(x) + \int_{\xi_{\text{th}}}^{\xi/x_B} \frac{dx}{x} c_f^1 \left( \frac{\xi}{x} \right) f(x) \]

with  \[ \xi_{\text{th}} = \frac{2}{1 + \sqrt{1 + 4m_N^2/Q^2}} \]

Taking moments in  \( \xi : \)
\[ \left( \int_0^1 dy y^{N-1} c_f^1(y) \right) \left( \int_0^{\xi_{\text{th}}} dx x^{N-1} f(x) \right) \]

as before  truncated moments of the PDF
Relevance for the extraction of PDFs

TMC
Higher Twist
+ Resummation

data/ theory
using CJ12 PDFs
Semi-Inclusive $e^+e^-$ Annihilation

Cross section including HMC

$$\frac{d^2\sigma^h}{dx_E d\cos\theta} = \frac{\pi\alpha^2}{Q^2} N_c \left[ \frac{1 + \cos^2 \theta}{2} \hat{F}_1^h(x_E, Q^2) + \sin^2 \theta \hat{F}_L^h(x_E, Q^2) \right]$$

Fragmentation Function

$$\hat{F}_i^h(x_E, Q^2) = \sum_f \int_{x_E}^1 \frac{d\hat{z}}{\hat{z}} D_f^i \left( \frac{x_E}{\hat{z}}, \mu^2 \right) \hat{C}_f \left( \hat{z}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right)$$

Similar threshold logarithms as for DIS

(``crossed resummation´´ Sterman,Vogelsang `06)
Kaons

BELLE and BaBar data/ theory using DSS '08 FFs
Semi-Inclusive DIS

• Coefficient function

\[ F^h_q(x, z, Q^2) = \sum_{f,f'} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} f \left( \frac{x}{\hat{x}}, \frac{\mu^2}{\hat{\mu}^2} \right) D^h_{f'} \left( \frac{z}{\hat{z}}, \frac{Q^2}{\hat{\mu}^2}, \alpha_s(\hat{\mu}^2) \right) C^{i}_{f,f'} \left( \hat{x}, \hat{z}, \frac{Q^2}{\hat{\mu}^2}, \alpha_s(\hat{\mu}^2) \right) \]

\[ \alpha_s^k \left( \frac{\ln^m(1 - \hat{x})}{1 - \hat{x}} \right) \left( \frac{\ln^n(1 - \hat{z})}{1 - \hat{z}} \right) \]

• Double Mellin moments

\[ C^{T,\text{res}}_{qq}(N, M, \alpha_s(Q^2)) \]

\[ \propto \exp \left[ \int_0^{Q^2} \frac{dk^2_{\perp}}{k^2_{\perp}} A_q(\alpha_s(k^2_{\perp})) \left\{ \int_0^{1} \frac{d\xi}{\xi} \left[ e^{-N \xi - M \frac{k^2_{\perp}}{\xi Q^2}} - 1 \right] + \ln N + \ln M \right\} \right] \]

(no HMCs yet)
Hadron Multiplicities for Pions

\[ \frac{d\sigma_{\text{SIDIS}}}{dz} / \sigma_{\text{DIS}} \]

- Resummed
- Prel. Compass Data
- NLO

\[ \sqrt{s} = 17.35 \text{ GeV} \]

\[ \frac{d\sigma_{\text{SIDIS}}}{dz} / \sigma_{\text{DIS}} \]

- Resummed
- Prel. Hermes Data
- NLO

\[ \sqrt{s} = 7.69 \text{ GeV} \]

using MSTW PDFs and DSS `08 FFs
Spin Asymmetries for (SI)DIS

**SIDIS:**

\[
A_1^h(x, z, Q^2) \approx \frac{g_1^h(x, z, Q^2)}{F_1^h(x, z, Q^2)}
\]

\[\bar{\ell} p \rightarrow \ell h X \quad \text{SIDIS, proton target}\]

\[\bar{\ell} p \rightarrow \ell X \quad \text{DIS, neutron target}\]

JLab, 12 GeV upgrade ...

using DSSV, MSTW PDFs and DSS ’08 FFs
Outline

- Resummation and HMC for color singlet processes
- QCD Hard Scattering
- Heavy Gauge Boson Production
- Conclusions

Publications:
- de Florian, Hinderer, Mukherjee, FR, Vogelsang - PRL ’14
- Hinderer, FR, Sterman, Vogelsang - PRD ’15
Di-Hadron Production

• "Full QCD extension" of Drell-Yan

• Existing data from various experiments not very well described by NLO Owens '02
  data from fixed target: NA24, E711, E706
  collider regime: CCOR, RHIC (planned)

• Previous NLL results from Almeida, Sterman, Vogelsang '09

• Starting point for extension toward NNLL for other processes

\[ M^2 = (P_\pi + P_\pi')^2 \]
Threshold Resummation toward NNLL

After taking Mellin and Fourier double moments

\[
\tilde{\omega}_{ab\to cd}^{\text{resum}} \left( N, \Delta \eta, \alpha_s(\mu_R), \frac{\mu_R}{\tilde{m}}, \frac{\mu_F}{\tilde{m}} \right) = \xi_R \left( \alpha_s(\mu_R), \frac{\mu_R}{\tilde{m}} \right) \xi_F^{abcd} \left( \alpha_s(\mu_R), \frac{\mu_F}{\tilde{m}} \right) \\
\times \Delta_a^{N+1} \left( \alpha_s(\mu_R), \frac{\mu_R}{\tilde{m}}, \frac{\mu_F}{\tilde{m}} \right) \Delta_b^{N+1} \left( \alpha_s(\mu_R), \frac{\mu_R}{\tilde{m}}, \frac{\mu_F}{\tilde{m}} \right) \Delta_c^{N+2} \left( \alpha_s(\mu_R), \frac{\mu_R}{\tilde{m}}, \frac{\mu_F}{\tilde{m}} \right) \Delta_d^{N+2} \left( \alpha_s(\mu_R), \frac{\mu_R}{\tilde{m}}, \frac{\mu_F}{\tilde{m}} \right) \\
\times \text{Tr} \left\{ H (\Delta \eta, \alpha_s(\mu_R)) S_N^\Delta \left( \Delta \eta, \alpha_s(\mu_R), \frac{\mu_R}{\tilde{m}} \right) S \left( \alpha_s(\tilde{m}/\tilde{N}), \Delta \eta \right) S_N \left( \Delta \eta, \alpha_s(\mu_R), \frac{\mu_R}{\tilde{m}} \right) \right\}_{ab\to cd}
\]

Refactorization near threshold

Kidonakis, Oderda, Sterman `98
Bonciani, Catani, Mangano, Nason `98
Banfi, Salam, Zanderighi
Dokshitzer, Marchesini
Almeida, Sterman, Vogelsang `09

Figure adapted from
de Florian, Pfeuffer, Schäfer, Vogelsang `13
• Hard-Scattering Matrix: derived from virtual corrections

\[
H^{(1)} = \begin{pmatrix}
\mathcal{R} \left\{ \frac{1}{2N_c^2} \left[ \frac{s^2 + u^2}{t^2} \left( -4C_F L(t)^2 + 2X_1(s, t, u)L(t) + 2Y \right) + \frac{s^2}{t^2} \left( C_A - 4C_F \right) Z(s, t, u) - \frac{u^2}{t^2} \left( 2C_A - 4C_F \right) Z(u, t, s) \right] \right\} \\
\frac{1}{2N_c^2} \left[ \frac{s^2 + u^2}{t^2} X_2(s, t, u)L(t) - \frac{s^2}{t^2} \frac{C_F}{2C_A} Z(s, t, u) + \frac{u^2}{t^2} \frac{C_F}{2C_A} Z(u, t, s) \right]
\end{pmatrix}
\]

with e.g.

\[
X_1(s, t, u) = 6C_F - 4\pi b_0 + 8C_F[L(s) - L(u)] - 2C_A[2L(s) - L(t) - L(u)]
\]

and

\[
L(t) = \log \frac{-t}{s}, \quad L(u) = \log \frac{-u}{s}, \quad L(s) = -i\pi
\]

• Soft Matrix: derived from real emission diagrams in the eikonal approximation

\[
S^{(1)} = \frac{C_F}{2} \begin{pmatrix}
\text{Li}_2 \left( -\frac{u}{t} \right) + \left( 2 - N_c^2 \right) \text{Li}_2 \left( -\frac{t}{u} \right) - 2N_c \text{Li}_2 \left( -\frac{t}{u} \right) - 2N_c \text{Li}_2 \left( -\frac{t}{u} \right) - 4N_c^2 \text{Li}_2 \left( -\frac{u}{t} \right) & 0
\end{pmatrix}
\]

\[qq' \rightarrow qq'\]

\[
(H^{(1)})_{12} = (H^{(1)})^{*}_{21}
\]

\[
\begin{pmatrix}
0 \\
2 \times 2
\end{pmatrix}
\]

see also Kelley, Schwartz `11; Broggio et al. `14
**Introduction**

Perturbative QCD Resummation

**Conclusions**

**Perturbative QCD**

Threshold Resummation

**Hadronic Production**

**Conclusions**

W±

QCD Resummation for SIDIS

Phenomenological Results

Annihilation

**Conclusions**

e+e-

QCD Resummation for SIDIS

Hadron multiplicities

Longitudinal spin asymmetries

**Conclusions**

QCD Resummation for SIDIS

Annihilation

Longitudinal spin asymmetries

**Conclusions**

Approximate NNLO results

Mellin code and NNLL

**Conclusions**

Color Singlet Processes

QCD Hard Scattering

Heavy Gauge Boson Production

**Conclusions**

CTEQ6 NLO PDFs, DSS `14 FFs

CCOR
Scale dependence

CTEQ6 NLO PDFs, DSS `14 FFs
Single-Inclusive Jet Production

• Large theoretical uncertainties especially at high $p_T$
• PDFs and $\alpha_s$ are constrained by collider jet data
• High $p_T$ jets are a promising observable for the search of BSM physics at the LHC
• Analytical results obtained in the "Narrow Jet Approximation"

\[ A \log R + B + O(R^2) \]

• First results based on NLL; Preliminary results toward NNLL

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Jäger, Stratmann, Vogelsang ’04; Mukherjee, Vogelsang ’13

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$\sqrt{s} = 8\text{TeV}$ anti-$k_T$ $R=0.7$ $L = 10.71\text{fb}^{-1}$

- Data/Theory
- Theo. Uncertainty
- Exp. Uncertainty

CT10 CMS Preliminary
Approximate NNLO-NLL results (planned to be) used by PDF groups, `\texttt{fastNLO}`... For example, the latest NNPDF3.0 set \cite{Ball:2014uwa} using MSTW-nnlo PDFs.
Outline

- Resummation and HMC for color singlet processes
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Publications: FR, Vogelsang - to be submitted
Heavy Gauge Boson Production

• New analytical results at NLO

\[ q\bar{q}^\prime \rightarrow \ell^\pm X, \quad qg \rightarrow \ell^\pm X \]

• Structure

\[ 1 - v = \frac{p_T^\ell}{\sqrt{s}} e^{-\hat{\eta}} \quad \nu w = \frac{p_T^\ell}{\sqrt{s}} e^{+\hat{\eta}} \]

\[
\frac{d\sigma^{\text{NLO}}}{dv dw} = \frac{\alpha_s}{2\pi} \left\{ \frac{f_{\text{LO}}(v)}{(s - M_W^2)^2 + \Gamma^2 M_W^2} \left[ A \left( \frac{\ln(1 - w)}{1 - w} \right)_+ + B(v) \frac{1}{(1 - w)_+} \right. \right.
\]

\[
+ C(v) \delta(1 - w) \left. \right] + \left. \frac{\ln \left( \frac{(ws - M_W^2)^2 + \Gamma^2 M_W^2}{M_W^4 + \Gamma^2 M_W^2} \right)}{(ws - M_W^2)^2 + \Gamma^2 M_W^2} \right. + \ldots \right\} \]
• Analytical results help to understand the structure around $p_T^\ell \sim M_W/2$

• Polarized results are being used for including results on SSA from RHIC into the DSSV global analysis of polarized PDFs
Outline

- Resummation and HMC for color singlet processes
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- Conclusions
Conclusions

- Resummation and HMC for color-singlet processes. Future extension to NNLO, NNLL for data coming from JLab at 12 GeV
- More work necessary concerning HMCs
- Threshold resummation toward NNLL for di-hadron and Single-Inclusive Jet Production
- Possible extensions to processes such as di-jet, single-inclusive hadron production, photons etc.
- W, Z boson production for the (un-)polarized cross section
- RHIC data will be included in the DSSV PDF set