Hadron Mass Corrections in semi-inclusive deep-inelastic scattering

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Hampton University & Jefferson Lab

Theory Center Seminar.
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arXiv: 1505.02739
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Jefferson Lab
Thomas Jefferson National Accelerator Facility

HAMPTON UNIVERSITY
THE STANDARD OF EXCELLENCE

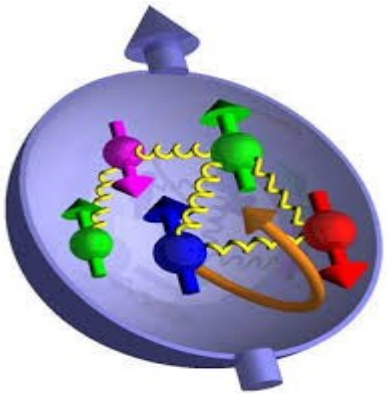
Outline

- Introduction
- Kinematics
- Cross sections at finite Q^2
- Phenomenological implications
- Conclusion and Outlook

Introduction

Hard scattering reactions:

Picture of the nucleon (partons)



Examples:

Deep inelatic scattering (DIS): $e^- p \longrightarrow e^- X$

Semi inclusive Deep inelatic scattering (SIDIS): $e^- p \longrightarrow e^- h X$

Drell Yan (DY): $h_A h_B \longrightarrow e^- e^+ X$

$e^- e+$ annihilation: $e^- e^+ \longrightarrow hX$

Introduction: Light cone vectors

Light cone + direction n^μ

Light cone – direction $\overline{n}^\mu$

$$n^2 = \overline{n}^2 = 0 \qquad n \cdot \overline{n} = 1$$

Components of a vector a^μ

$$a^+ = a \cdot n \qquad a^- = a \cdot \overline{n}$$

Choosing:

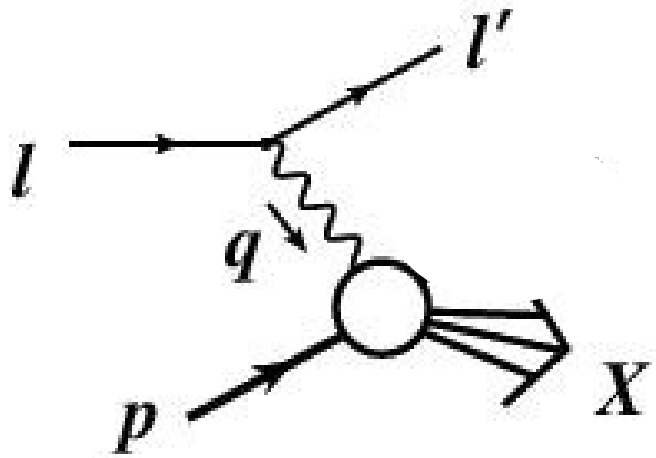
$$n^\mu = (1/\sqrt{2}, \vec{0}_\perp, -1/\sqrt{2})$$
$$\overline{n}^\mu = (1/\sqrt{2}, \vec{0}_\perp, 1/\sqrt{2})$$

We can write

$$a^\pm = (a_0 \pm a_3)/\sqrt{2}$$

Introduction: DIS

DIS Diagram



Theoretical Framework:

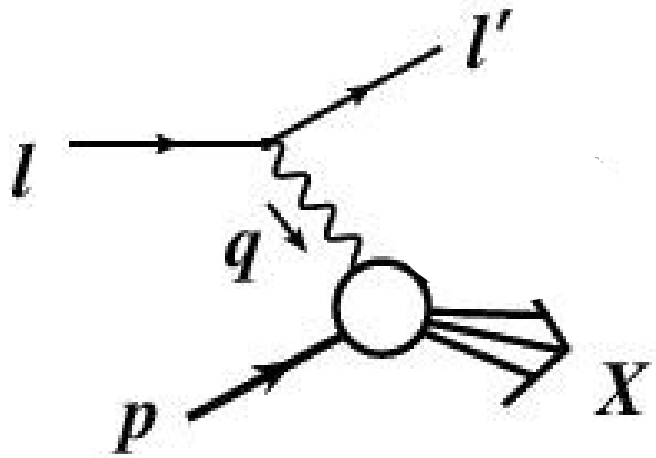
- Operator Product Expasion (OPE).
- Collinear factorization (CF)

OPE ⟺ CF

To be studied:
massive case

Introduction: DIS

DIS Diagram



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To be studied: massive case

DIS Kinematic invariants

Nucleon Mass

$$m_N^2 = p^2$$

Virtuality

$$Q^2 = -q^2$$

Bjorken-x

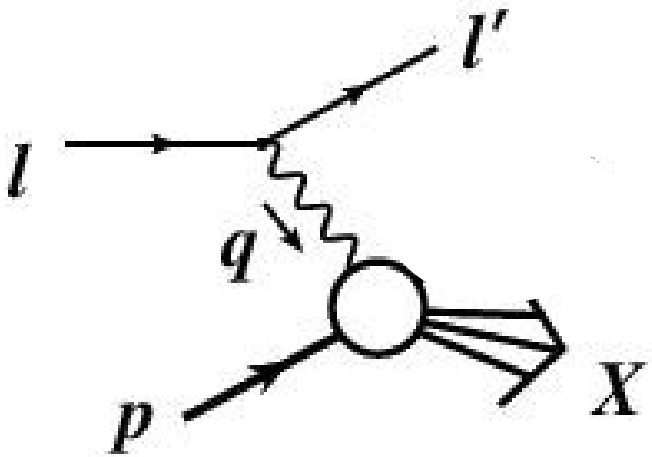
$$x_B = \frac{Q^2}{2p \cdot q}$$

Final state invariant mass

$$W^2 = (p+q)^2$$

Introduction: DIS

DIS Diagram



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DIS Kinematic invariants

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Final state invariant mass

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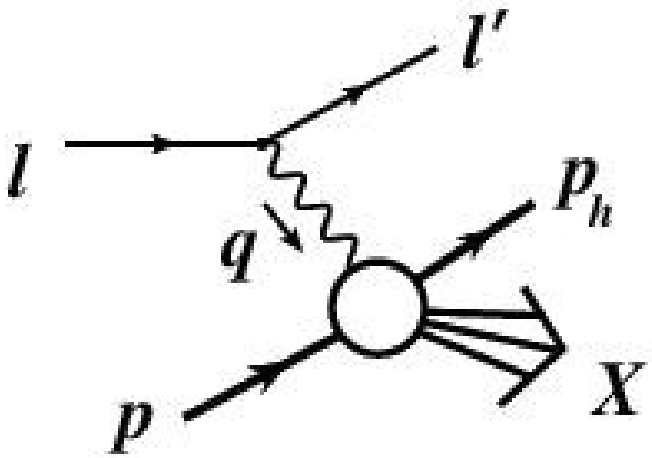
Nachtmann scaling variable

$$\xi = -\frac{q^+}{p^+} = \frac{2x_B}{1+\sqrt{1+4x_B^2 M^2/Q^2}}$$

Bjorken Limit $Q^2 \to \infty \quad \xi \to x_B$

Introduction: SIDIS

SIDIS Diagram



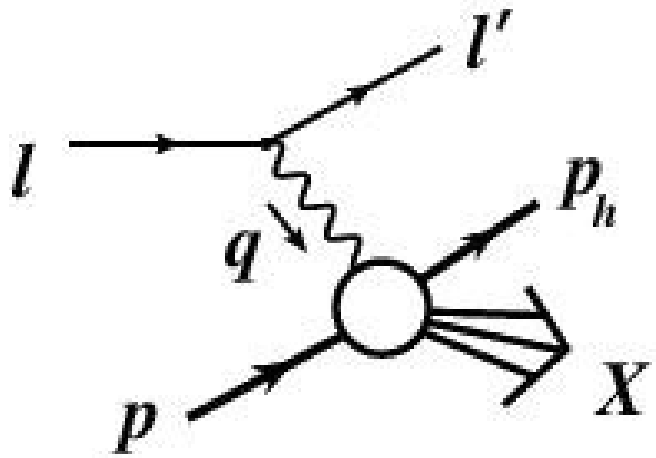
Theoretical Framework:

- Operator Product Expasion (OPE).
- Collinear factorization (CF)

DIS variables

Introduction: SIDIS

SIDIS Diagram



Theoretical Framework:

- Operator Product Expasion (OPE).
- Collinear factorization (CF)

DIS variables + <u>hadronic variables</u>

Fragmentation invariant

$$z_h = \frac{p_h \cdot p}{q \cdot p}$$

Fragmentation variable

$$\zeta_h = \frac{p_h^-}{q^-}$$

Introduction: SIDIS

Advantage vs. DIS:

- Quark flavor decomposition (π vs. K)
- Quark tranverse angular momentum (P_{hT})
- Polarized SIDIS: flavor assymetry?

Disadvantage vs. DIS:

- Needs to understand quark fragmentation
- Experimentally harder:
  - Rates are lower in coincidence experiments
  - Need to identify the hadron.

Introduction: Why Hadron Mass Corrections (HMC's)?

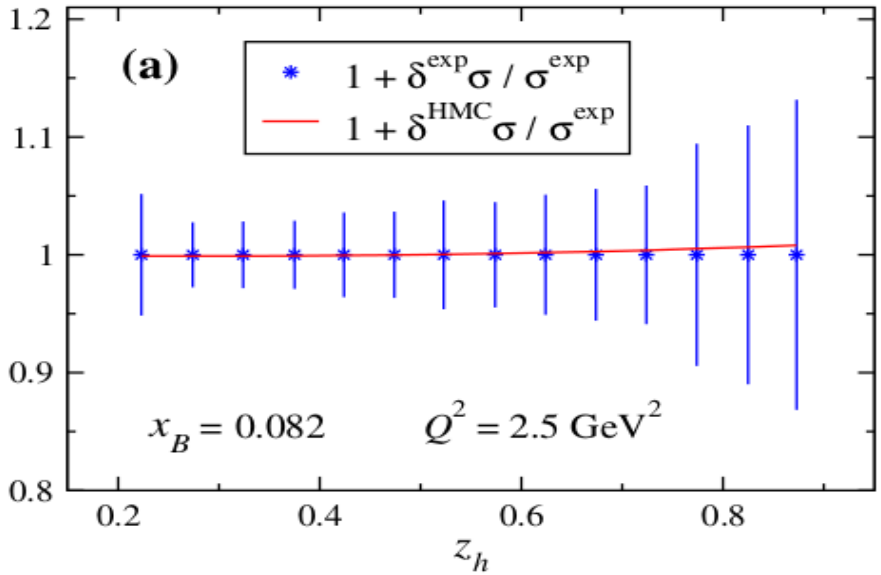
Jlab experiments

- Usually Low Q^2(~1 GeV^2)
- $1/Q^2$ corrections have to be controlled

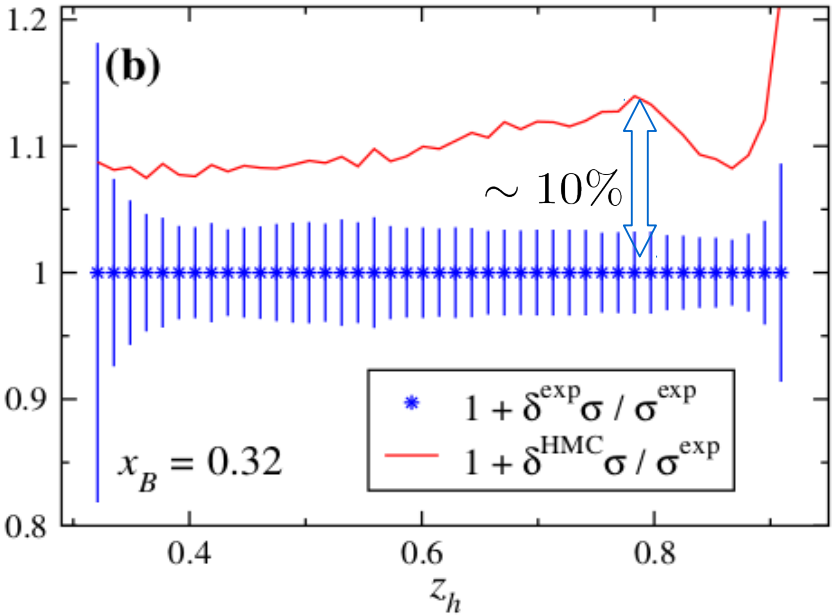
$$O(\frac{M^2}{Q^2}) \equiv \textbf{HMC's!!!}$$

Introduction: HMC's for charged pion production (unpolarized SIDIS)

HERMES experiment (DESY)



Jefferson Lab (E00-108)



Accardi, Hobbs, Melnitchouk
JHEP 0911, 084 (2009)

Introduction: Polarized HMC's (Project)

arXiv: 1505.02739

Hadron mass corrections in semi-inclusive deep-inelastic scattering

J. V. Guerrero[1,2], J. J. Ethier[2,3], A. Accardi[1,2], S. W. Casper[2,4], W. Melnitchouk[2]

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[2] *Jefferson Lab, Newport News, Virginia 23606, USA*

[3] *College of William and Mary, Williamsburg, Virginia 23185, USA*

[4] *Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA*

Starting from the ref. JHEP 0911, 084 (2009) we:

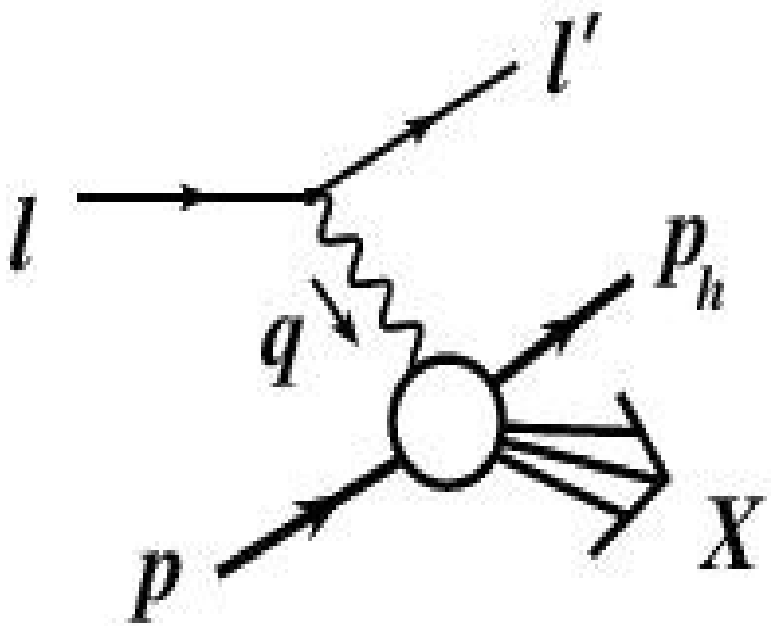
Review all the kinematics

Derive the cross section for the unpolarised case and extend to the polarized case

Estimate the size of the hadron mass corrections at kinematics relevant for current and future experiments

SIDIS: External Kinematics

SIDIS Diagram



(p,q) frame

$$p^\mu = p^+ \, \overline{n}^\mu + \frac{M^2}{2p^+} \, n^\mu$$

$$q^\mu = -\xi p^+ \, \overline{n}^\mu + \frac{Q^2}{2\xi p^+} \, n^\mu$$

Hadron momentum

$$p_h^\mu = \frac{\xi m_{h\perp}^2}{\zeta_h Q^2} \, p^+ \, \overline{n}^\mu + \frac{\zeta_h Q^2}{2\xi p^+} \, n^\mu + p_{h\perp}^\mu$$

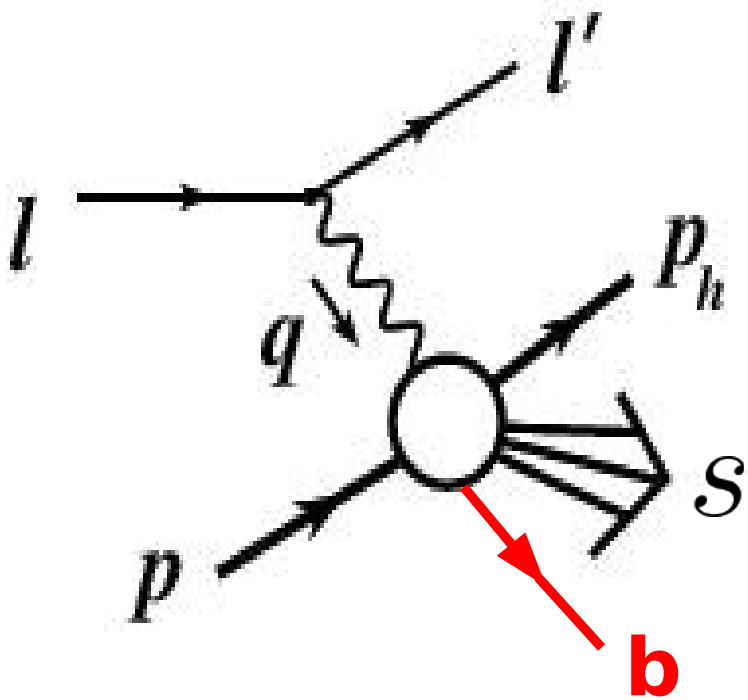
Fragmentation Variable

$$\boxed{\zeta_h = \frac{p_h^-}{q^-} = \frac{z_h}{2} \frac{\xi}{x_B} \left(1 + \sqrt{1 - \frac{4 x_B^2 M^2 m_{h\perp}^2}{z_h^2 \, Q^4}} \right)}$$

with $m_{h\perp}^2 = m_h^2 + p_{h\perp}^2 ;$ $\quad \zeta_h \longrightarrow z_h$, $\; Q^2 \to \infty$

SIDIS: External Kinematics

SIDIS Diagram



Momentum and baryon number conservation

- **Max. x_B**

$$\frac{1}{x_B^{\max}} = 1 + \frac{m_h^2 + 2Mm_h}{Q^2}$$

- **Limits on z_h** $z_h^{\min} \leq z_h \leq z_h^{\max}$

$$z_h^{\min} = 2x_B \frac{Mm_h}{Q^2}$$

$$z_h^{\max} = 1 - 2x_B \frac{M(M_b - M)}{Q^2}$$

SIDIS: External Kinematics

Target rest frame $z_h = \frac{E_h}{\nu}$ $x_B = \frac{Q^2}{2M\nu}$

Energy + Baryon number conservation $E_P + E_\gamma = E_h + E_b + E_s$

$E_s \geq 0$ $E_b \geq M_b$

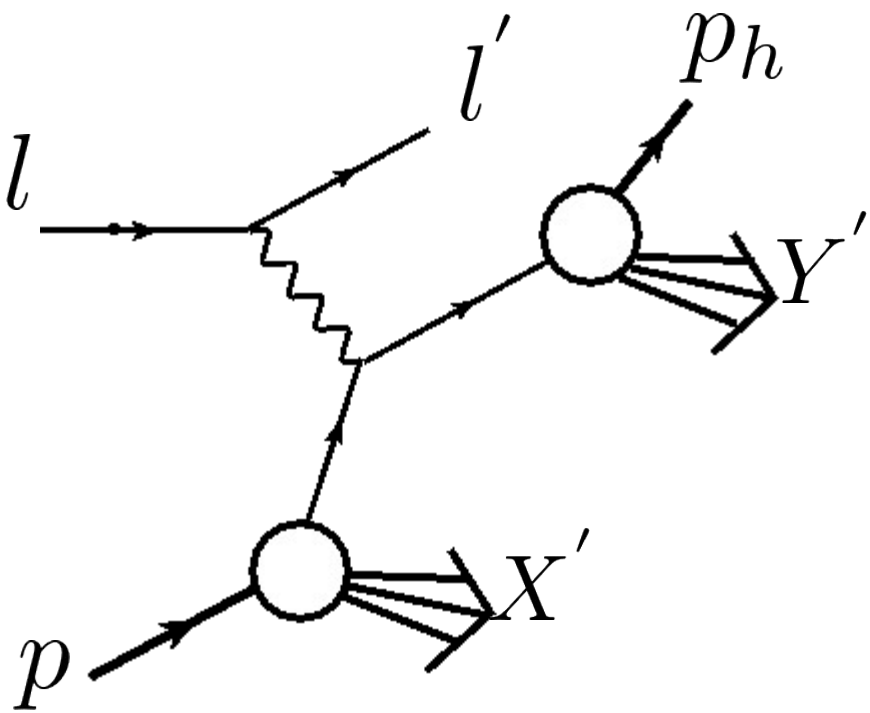
$$\Rightarrow \boxed{z_h^{\max} = 1 - 2x_B \frac{M(M_b - M)}{Q^2}}$$

- **π-production:** $M_b = M$ $\Rightarrow$ $z_h^{\max} = 1$

- **K-production:** $M_b = M_\Lambda$ $\Rightarrow$ $z_h^{\max} = 1 - 2x_B \frac{M(M_\Lambda - M)}{Q^2}$

SIDIS: Current vs. Target Fragmentation

SIDIS Fragmentation (in pQCD)



Alternative Frag. Invariant

$$z_e = \frac{2p_h \cdot q}{q^2}$$

Frag. Variable

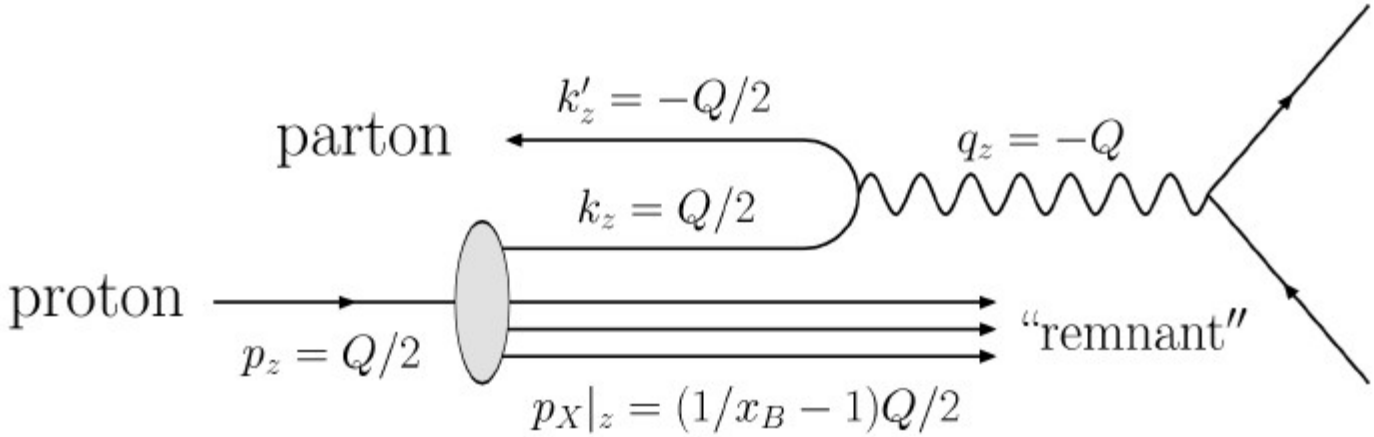
$$\zeta_h = \frac{z_e}{2}\left(1 + \sqrt{1 + \frac{4m_{h\perp}^2}{z_e^2\, Q^2}}\right)$$

SIDIS: Current vs. Target Fragmentation

Alternative Frag. Invariant

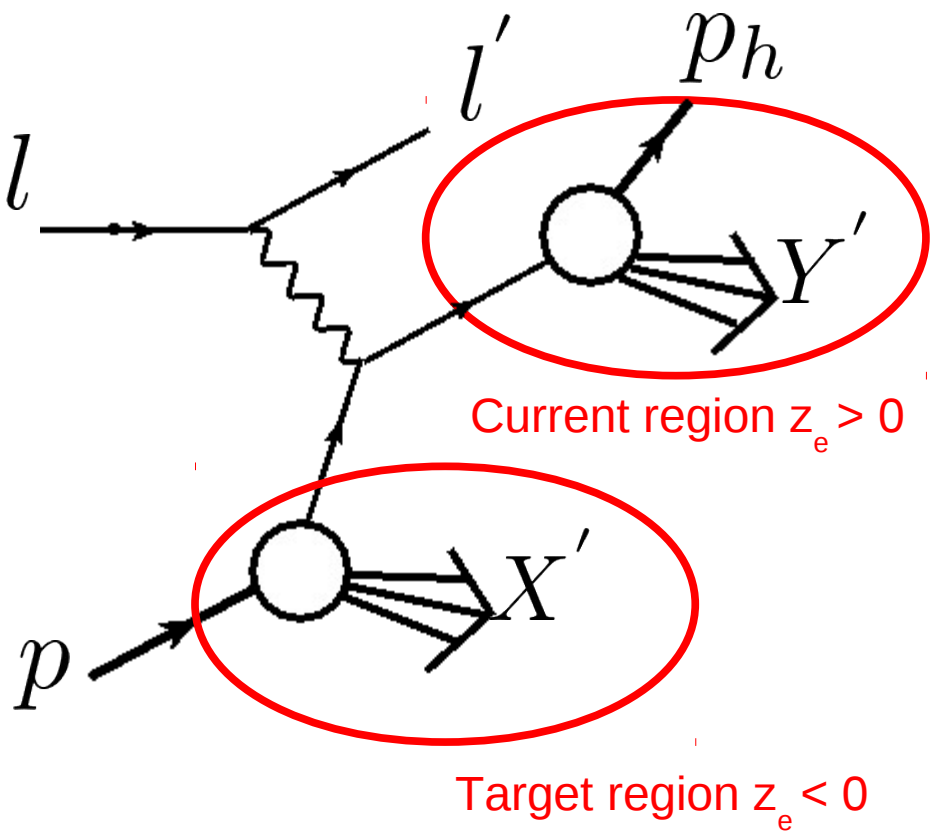
$$z_e = \frac{2p_h \cdot q}{q^2} = \frac{p_h^z}{q^z}$$

Breit frame



SIDIS: Current vs. Target Fragmentation

SIDIS Fragmentation (Frag.)



Alternative Frag. Invariant

$$z_e = \frac{2p_h \cdot q}{q^2} = \frac{p_h^z}{q^z}$$

Breit frame

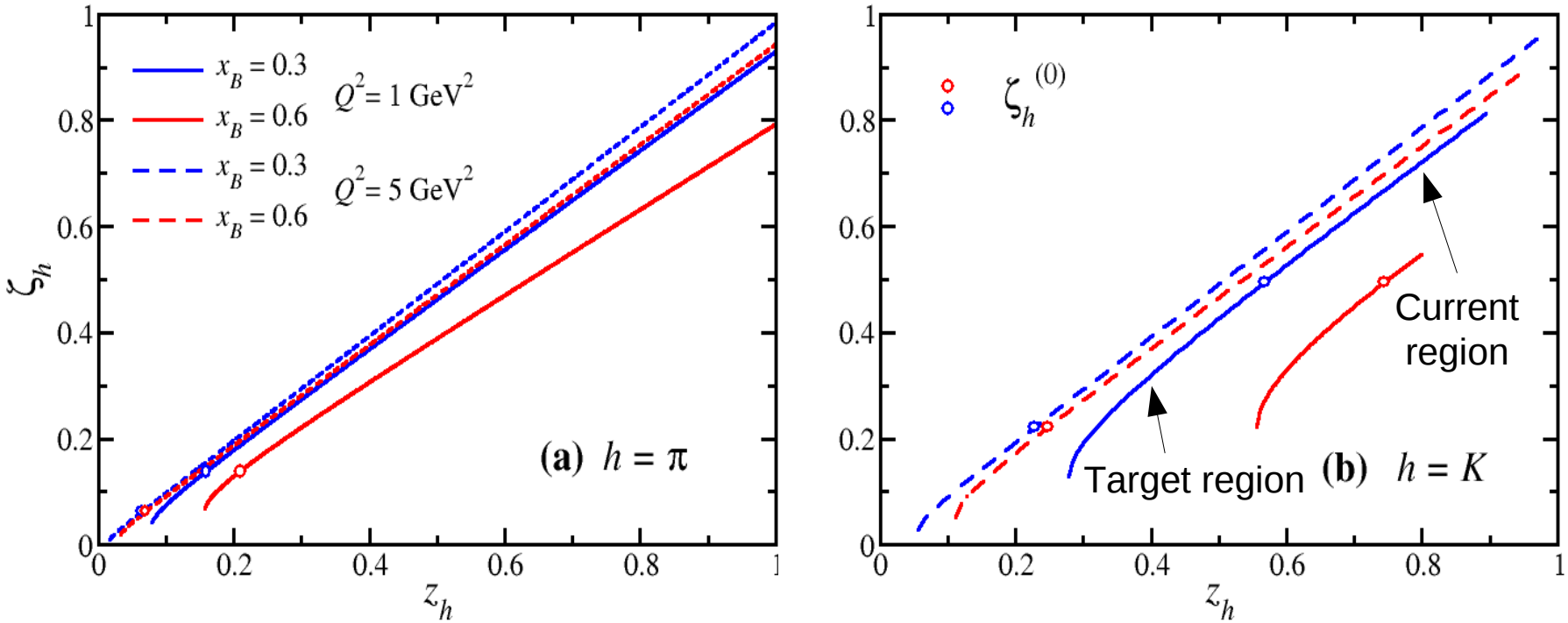
Frag. Variable

$$\zeta_h = \frac{z_e}{2}\left(1 + \sqrt{1 + \frac{4m_{h\perp}^2}{z_e^2\, Q^2}}\right)$$

In the Current region:

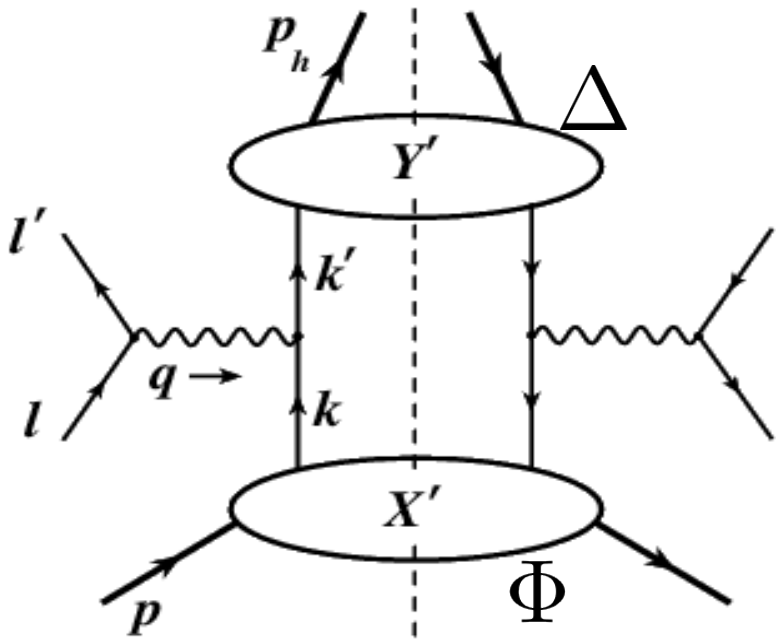
$$\zeta_h > \zeta_h(z_e = 0) = \frac{m_{h\perp}}{Q}$$

SIDIS: External Kinematics



- Bjorken Limit $\zeta_h \to z_h \to z_e$ $\zeta_h^{(0)} \to 0$

SIDIS: Internal Kinematics



Quarks momenta (LO)

$$k^\mu = xp^+\,\bar{n}^\mu + \frac{k^2 + k_\perp^2}{2xp^+}\,n^\mu + k_\perp^\mu$$

$$k'^\mu = \frac{k'^2 + k_\perp'^2}{2p_h^-/z}\,\bar{n}^\mu + \frac{p_h^-}{z}\,n^\mu + k_\perp'^\mu$$

"Collinear" Momenta

$$\widetilde{k}^\mu = xp^+\,\bar{n}^\mu + \frac{\widetilde{k}^2}{2xp^+}\,n^\mu$$

$$\widetilde{k}'^\mu = \frac{\widetilde{k}'^2 + p_{h\perp}^2/z^2}{2p_h^-/z}\,\bar{n}^\mu + \frac{p_h^-}{z}\,n^\mu + \frac{p_{h\perp}^\mu}{z}$$

Initial and scattered quarks are off-shell

Virtualities $\widetilde{k}^2$ and $\widetilde{k}'^2$ will be fixed later

SIDIS: Internal Kinematics

Partonic analog to Bjorken variable

$$\hat{x} = -\frac{q^2}{2\widetilde{k}\cdot q} = \frac{\xi}{x}\frac{1}{1-\xi^2\widetilde{k}^2/x^2Q^2}$$

Four-momentum and baryon number conservation: $\hat{x}_{\min} \leq \hat{x} \leq \hat{x}_{\max}$

$$\frac{1}{\hat{x}_{\min}} = \frac{1}{x_B} - \frac{2Mm_h + \widetilde{k}^2}{Q^2}$$

collinear spectators minimun mass

$$\frac{1}{\hat{x}_{\max}} = 1 + \frac{m_h^2}{\zeta_h Q^2} - \frac{\widetilde{k}^2}{Q^2}\left(1 - \frac{\xi m_h^2}{x\zeta_h Q^2}\right)$$

Mininum current jet invariant mass

SIDIS: Internal Kinematics

Combining:

- These two limits
- $x_B \leq x_B^{max}$

$$\widetilde{k}^2 \geq x(\zeta_h - 1)Q^2/\xi$$

Next step: make a choice of initial and scattered parton virtualities.

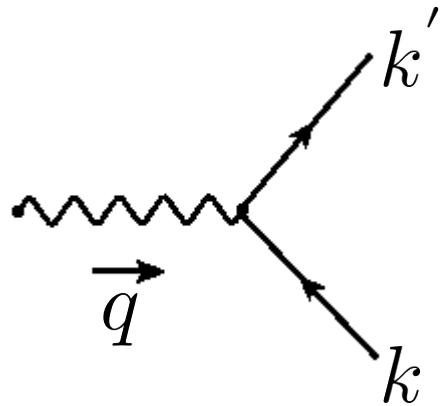
Initial parton:

- Virtuality independent of x $\quad \widetilde{k}^2 \geq 0$
- For light, bound state initial partons $\quad \widetilde{k}^2 \leq 0$

$$\boxed{\widetilde{k}^2 = 0}$$

SIDIS: Internal Kinematics

Final parton: four-momentun conservation



$$x = \xi_h$$

$$\xi_h = \xi\left(1 + \frac{\widetilde{k}'^2}{Q^2}\right)$$

Combining this with $\dfrac{1}{\hat{x}_{\max}}$ $\Rightarrow$ $\widetilde{k}'^2 \geq \dfrac{m_h^2}{\zeta_h}$

Minimal choice of virtuality: $\boxed{\widetilde{k}'^2 = \dfrac{m_h^2}{\zeta_h}}$ $\Rightarrow$ $\boxed{\xi_h = \xi(1 + \dfrac{m_h^2}{\zeta_h Q^2})}$

Cross section at finite Q^2

Spin averaged Cross section

$$\sigma_h \equiv \frac{1}{2}\frac{d\sigma_h^{\uparrow\uparrow+\downarrow\uparrow}}{dx_B\, dQ^2\, dz_h} = \frac{2\pi\alpha^2}{Q^4}\frac{y^2}{1-\varepsilon} J_h \sum_q e_q^2 q(\xi_h, Q^2) D_q^h(\zeta_h, Q^2)$$

Jacobian $J_h = \dfrac{d\zeta_h}{dz_h}$

Unpolarised PDF

Fragmentation Function

Cross section at finite Q^2

Spin averaged Cross section

$$\sigma_h \equiv \frac{1}{2}\frac{d\sigma_h^{\uparrow\uparrow+\downarrow\uparrow}}{dx_B\, dQ^2\, dz_h} = \frac{2\pi\alpha^2}{Q^4}\frac{y^2}{1-\varepsilon} J_h \sum_q e_q^2 q(\xi_h, Q^2) D_q^h(\zeta_h, Q^2)$$

Jacobian $J_h = \frac{d\zeta_h}{dz_h}$

Unpolarised PDF

Fragmentation Function

New scaling variables

$$\xi_h = \xi(1 + \frac{m_h^2}{\zeta_h Q^2})$$

$$\zeta_h = \frac{z_h}{2}\frac{\xi}{x_B}\left(1 + \sqrt{1 - \frac{4x_B^2 M^2 m_{h\perp}^2}{z_h^2\, Q^4}}\right)$$

Ratio longitudinal to transverse photon flux

$$\varepsilon = \frac{1 - y - y^2\gamma^2/4}{1 - y + y^2(1 + \frac{1}{2}\gamma^2)/2}$$

Cross section at finite Q^2

Spin dependent Cross section

$$\Delta\sigma_h \equiv \frac{d\sigma_h^{\uparrow\uparrow-\downarrow\uparrow}}{dx_B\, dQ^2\, dz_h} = \frac{4\pi\alpha^2}{Q^4}\frac{y^2\sqrt{1-\varepsilon^2}}{1-\varepsilon} J_h \sum_q e_q^2 \Delta q(\xi_h, Q^2) D_q^h(\zeta_h, Q^2)$$

Polarised PDF

Cross section at finite Q^2

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Polarised PDF

Bjorken Limit: $Q^2 \to \infty$

$\xi_h \to x_B \qquad \zeta_h \to z_h \qquad J_h \to 1$

Massless cross sections

$$\sigma_h^{(0)} \equiv \sigma_h(\xi_h \to x_B, \zeta_h \to z_h)$$
$$\Delta\sigma_h^{(0)} \equiv \Delta\sigma_h(\xi_h \to x_B, \zeta_h \to z_h)$$

Cross section at finite Q^2

Unpolarised PDF's

$$q(x) = \frac{1}{2}\int dk^- d^2k_\perp \; \left[\gamma^+ \Phi_q(p,S,k)\right]_{k^+=xp^+}$$
$$= \frac{1}{4\pi}\int dw^- \, e^{ixp^+w^-} \langle N|\overline{\psi}_q(0)\,\gamma^+\,\psi_q(w^-n)|N\rangle$$

Polarized PDF's

$$\Delta q(x) = \frac{1}{2}\int dk^- d^2k_\perp \; \left[\gamma^5\gamma^+ \Phi_q(p,S,k)\right]_{k^+=xp^+}$$
$$= \frac{1}{4\pi}\int dw^- \, e^{ixp^+w^-} \langle N|\overline{\psi}_q(0)\,\gamma^5\,\gamma^+\,\psi_q(w^-n)|N\rangle$$

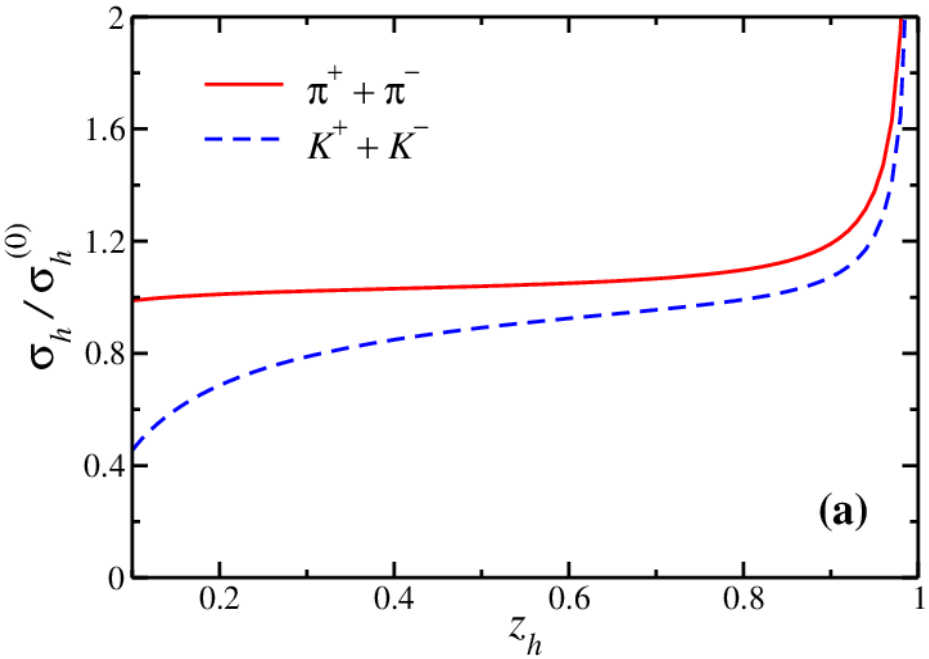
Fragmentation Functions

$$D_q^h(z) = \frac{z}{4}\int dk'^+ d^2k'_\perp \; \left[\gamma^- \Delta_q^h(k',p_h)\right]_{k'^-=p_h^-/z}$$
$$= \frac{z}{8\pi}\sum_{Y'}\int dw^+ \, e^{i(p_h^-/z)w^+} \langle 0|\psi_q(w^+)|h,Y'\rangle\langle h,Y'|\overline{\psi}_q(0)\gamma^-|0\rangle$$

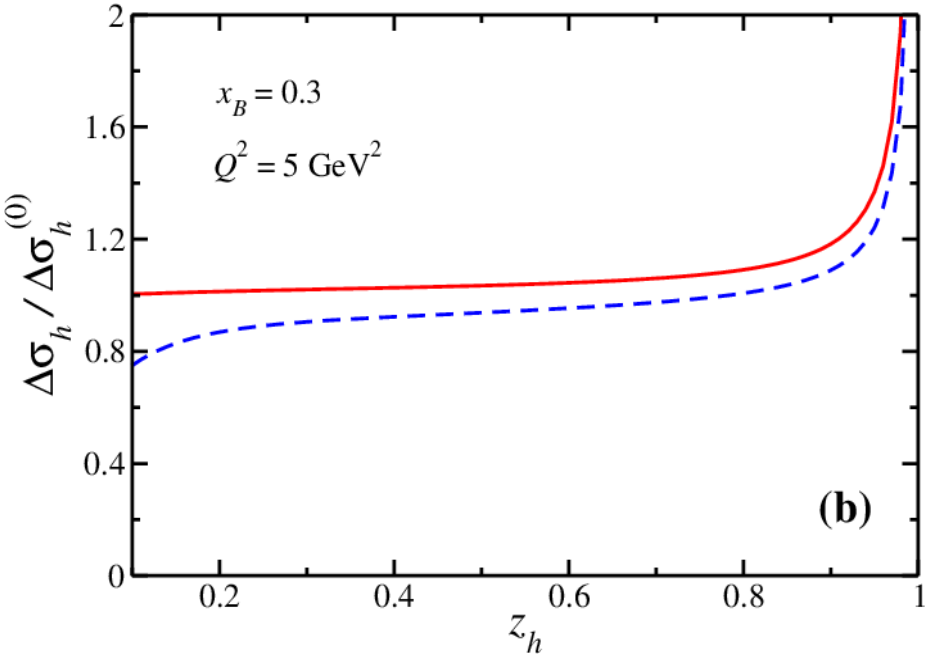
Phenomenological implications

Pions vs. Kaons

Unpolarised



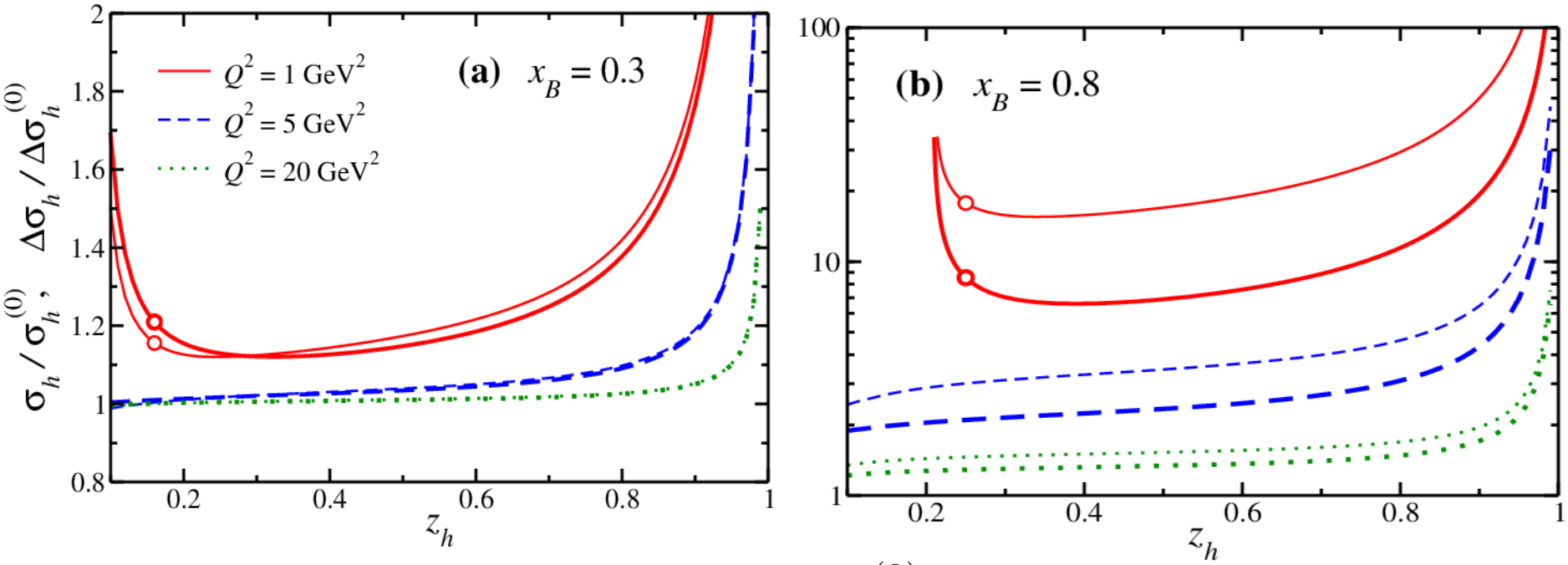
Polarised



Kaons mass effects much larger

Phenomenological implications

Q^2 dependence of the HMC's for $(\pi^+ + \pi^-)$

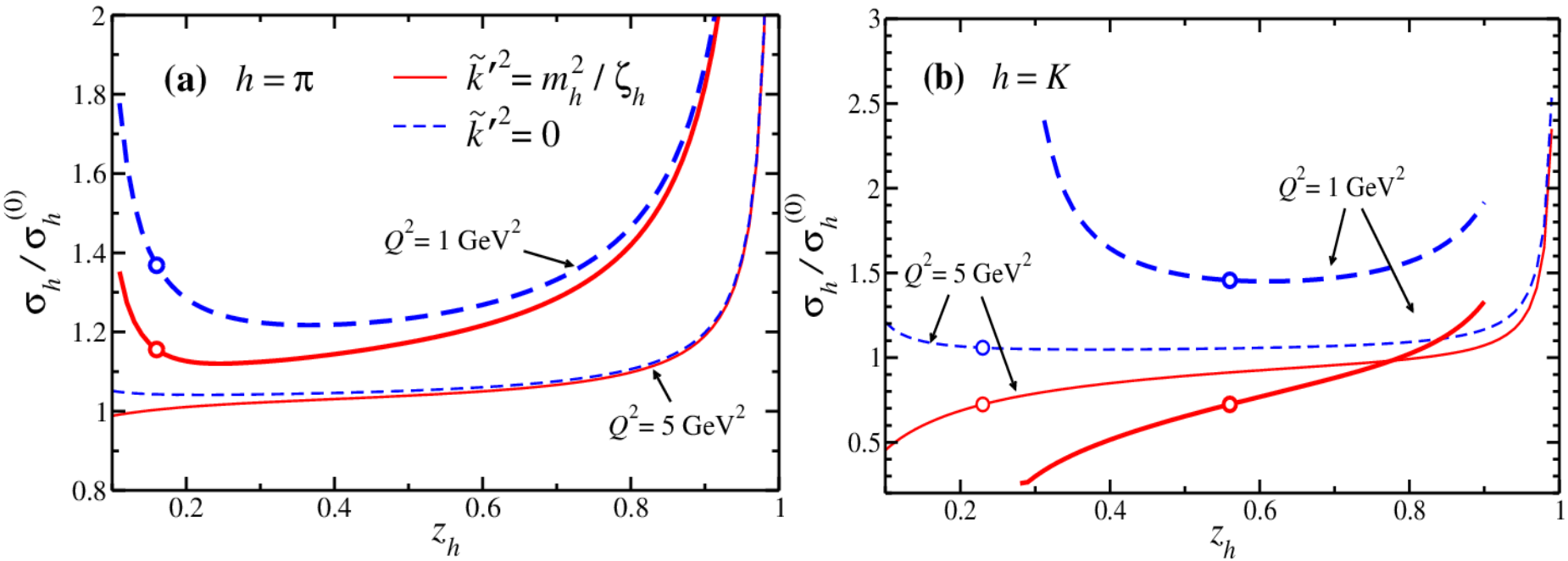


Thin lines: Unpolarized $\sigma_h / \sigma_h^{(0)}$

Thick lines: Polarized $\Delta\sigma_h / \Delta\sigma_h^{(0)}$

Phenomenological implications

Dependence on the parton virtuality



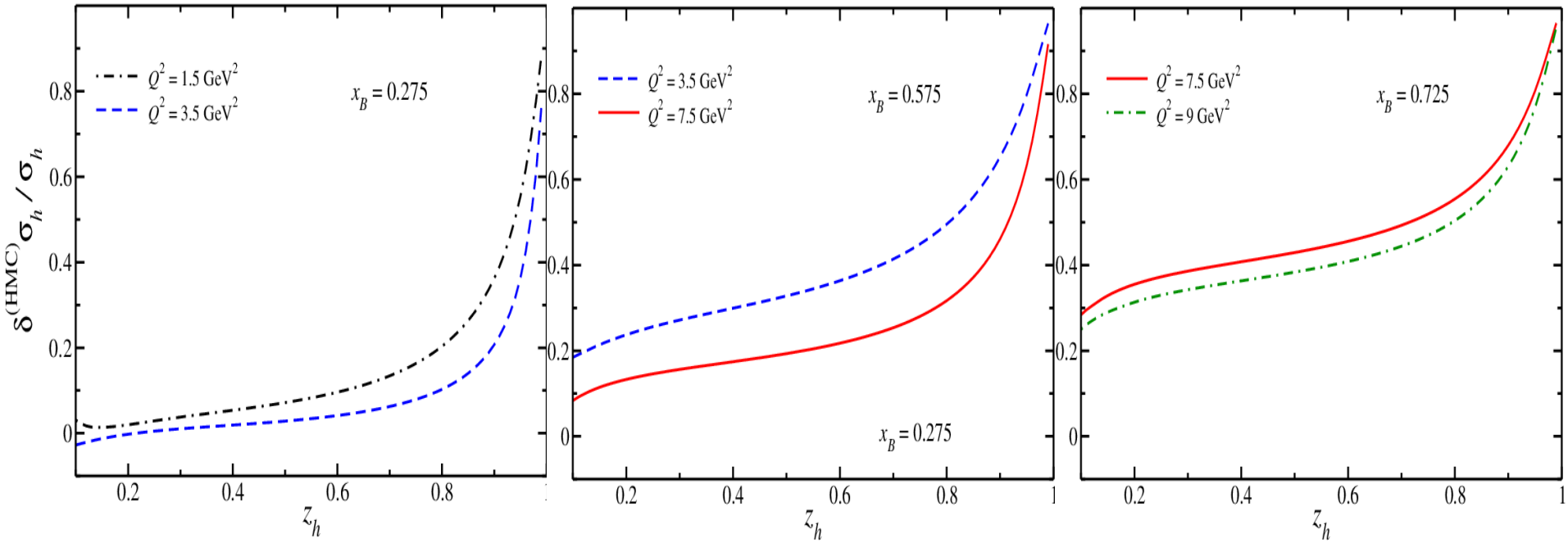
Red lines: Minimal choice $\tilde{k}'^2 = m_h^2/\zeta_h$

Thin lines: Albino et all. Nucl.Phys B803, 42 (2008) $\tilde{k}'^2 = 0$

Mass corrections for specific experiments

Jlab 11 GeV

(Experiments: E12-09-007, E12-13-007, E12-06-109)



$$\frac{\delta^{(\mathrm{HMC})}\sigma_h}{\sigma_h} = \frac{\sigma_h - \sigma_h^{(0)}}{\sigma_h}$$

Mass corrections for specific experiments

Polarization assymetry $A_1^h = \frac{A_\parallel^h}{D}$

Parallel assymetry $A_\parallel^h = \frac{\Delta\sigma_h}{2\sigma_h}$ γ–depolarization factor $D = \frac{1-(1-y)\varepsilon}{1+\varepsilon R}$

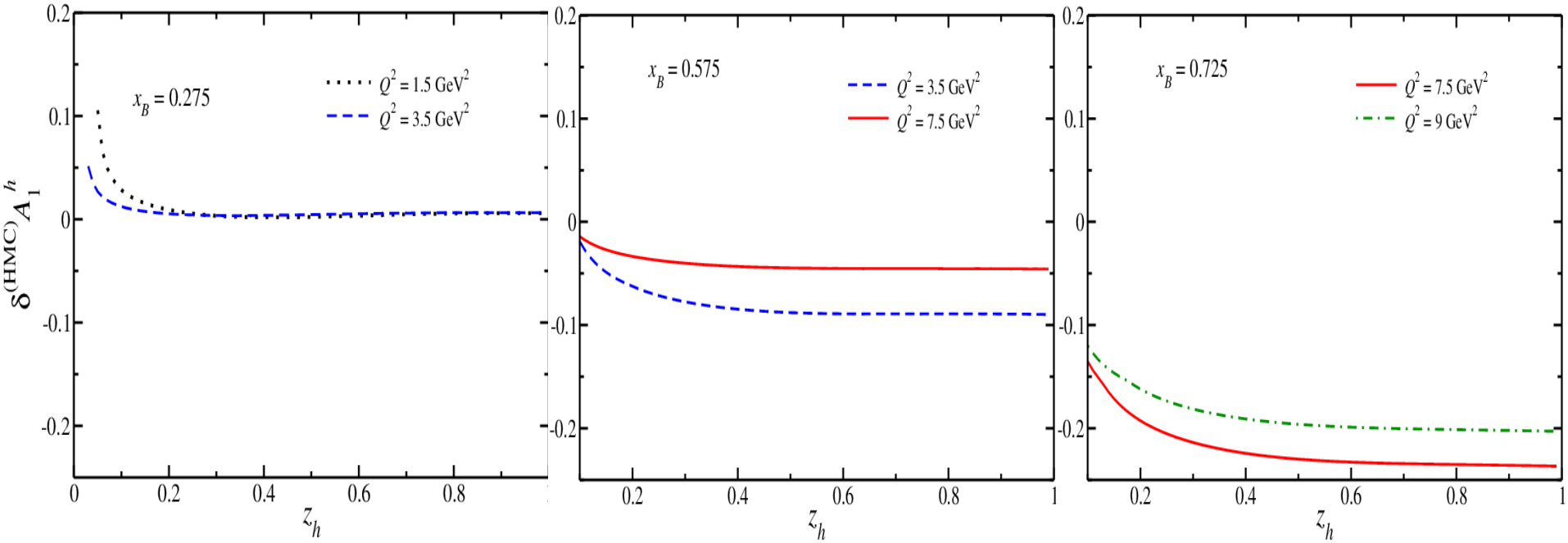
At LO:

$$A_1^h = \frac{\sqrt{1-\varepsilon^2}}{D} \frac{\sum_q e_q^2\, \Delta q(\xi_h, Q^2)\, D_q^h(\zeta_h, Q^2)}{\sum_q e_q^2\, q(\xi_h, Q^2)\, D_q^h(\zeta_h, Q^2)}$$

Mass corrections for specific experiments

Jlab 11 GeV

(Experiment: E12-06-109)



$$\delta^{(\mathrm{HMC})} A_1^h = A_1^h - A_1^{h(0)}$$

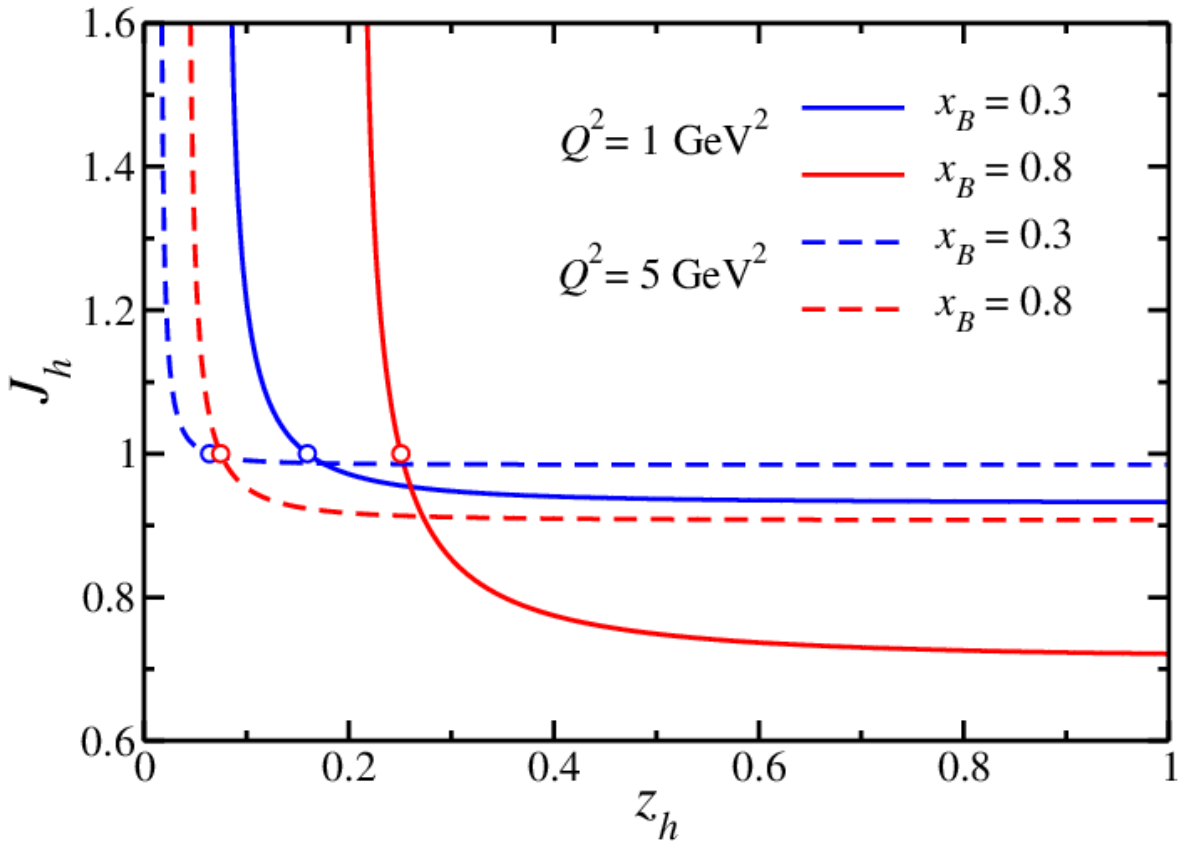
Conclusion and outlook

- HMC's at LO are captured by new scalling variables ξ_h and ζ_h.
  - Matching "internal" and external kinematics requires a massive fragmenting parton with $\widetilde{k}'^2 \geq m_h^2/\zeta_h$
  - x_B and z_h mixed in the new scaling variables ξ_h and ζ_h
- We have quantified HMC's effects numerically: stronger for large as well very small values of z_h, stronger effects at large x_B and Low Q^2 (as DIS).
- More dramatic effects for Kaons compared to pions.
- Jefferson Lab experiments: corrections up to ~40 %- 50 % even at moderately large Q^2.
- Future development: prove factorization at NLO in the prescence of massive fragmenting partons with non-zero virtuality.

Thank you!

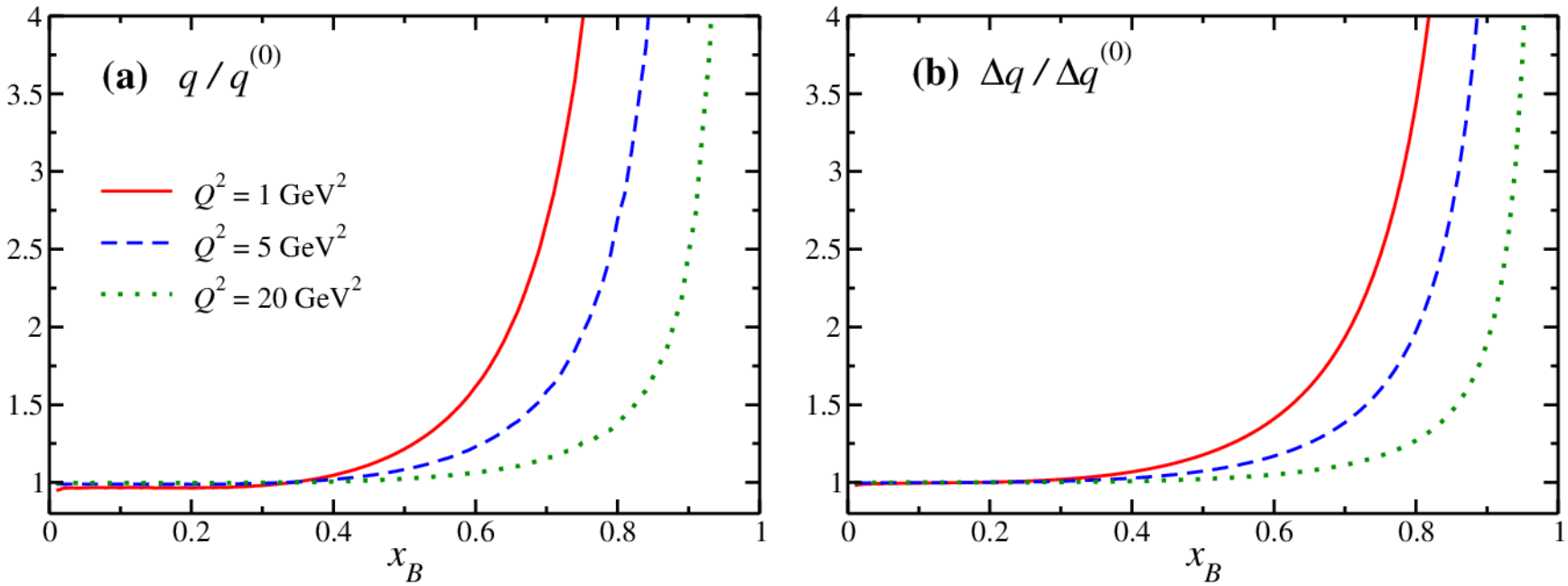
Backup slides

Jacobian



Backup slides

PDF's



Backup slides

Fragmentation function