Institute for Theoretical Physics
Eberhard Karls Universität Tübingen

Lepton-Hadron Processes beyond NLO

Newport News, *05.18.2015*

Accardi, Anderle, de Florian, Ringer, Rotstein, Stratmann, Vogelsang

Jefferson Lab

Speaker:
Daniele Paolo Anderle

OUTLINE

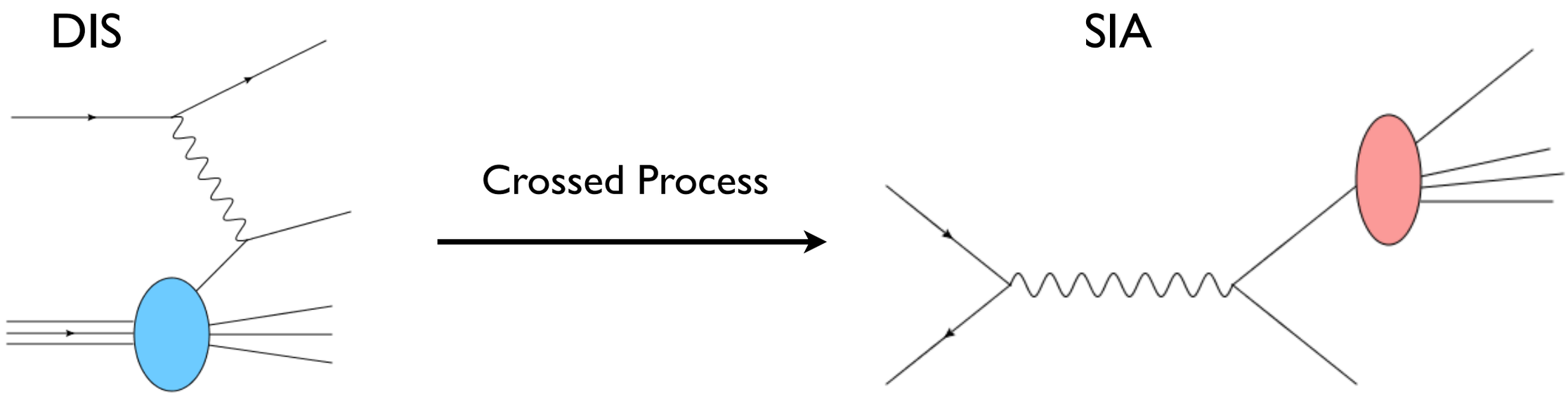
- HMC + THRESHOLD RESUMMATION
- TOWARDS A GLOBAL NNLO FF FIT
- NEW CHANNELS IN SIDIS NNLO F_L
- CONCLUSIONS & OUTLOOK

HMC + THRESHOLD RESUMMATION

Accardi, Anderle, Ringer (Phys. Rev. D 91, 034008 (2015))

We consider two corrections on standard pQCD calculation of SIA and DIS:

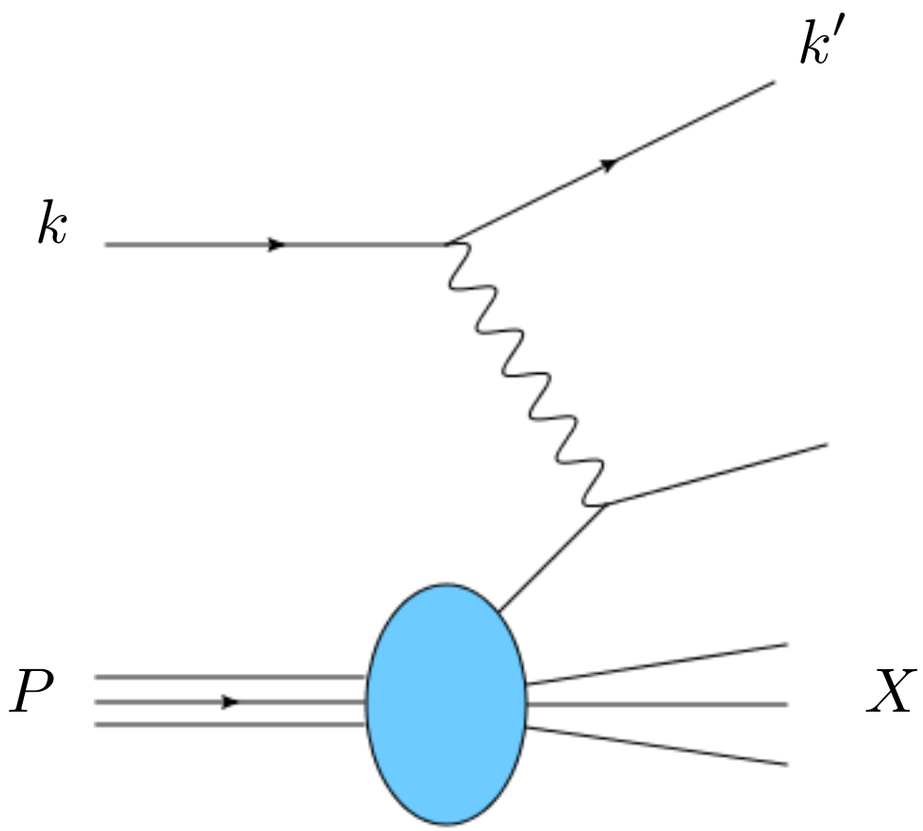
- Threshold resummation
- Hadron Mass Correction



Both corrections become relevant only in some kinematical phase space regions

DEEP INELASTIC SCATTERING

$$l(k)p(P) \to l(k')X$$



Defined kinematic variables:

$$Q^2 \equiv -q^2 = -(k-k')^2$$ Virtual Photon Energy

$$y \equiv \frac{P \cdot q}{P \cdot k}$$ $\propto$ to lepton scattered angle

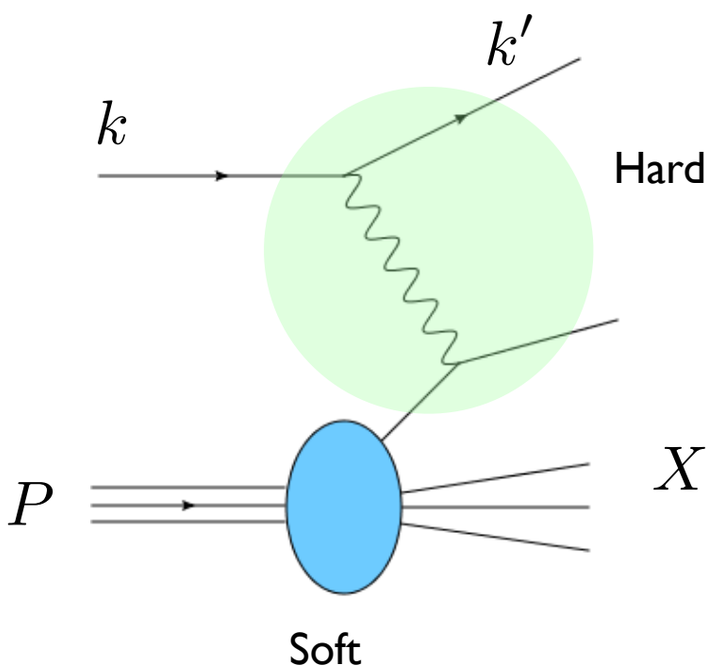
$$x \equiv \frac{Q^2}{2P \cdot q}$$

In a standard pQCD calculation of DIS cross section one is able to write

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}\mathcal{F}_T(x,Q^2) + \frac{1-y}{y}\mathcal{F}_L(x,Q^2)\right]$$

Furmanski, Petronzio; Catani; Kretzer;...

$$\mathcal{F}_i(x,Q^2) = \sum_f \int_x^1 \frac{d\hat{x}}{\hat{x}} f\left(\frac{x}{\hat{x}},\mu^2\right) \mathcal{C}_f^i\left(\hat{x},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)$$



Factorization of long (soft) and short (hard) behavior in the STRUCTURE FUNCTIONS

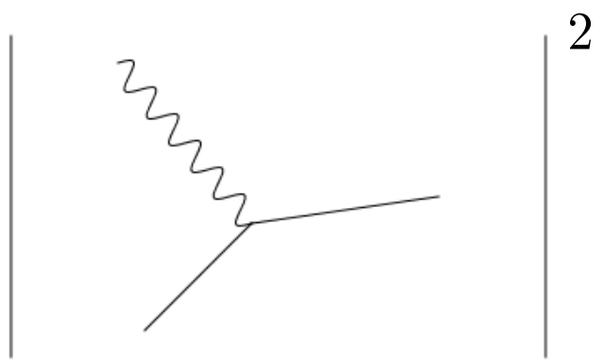
Experimentally-measured
distribution functions PDFs
$f(\xi)$ (fitted)

Theoretically-calculated
coefficient functions

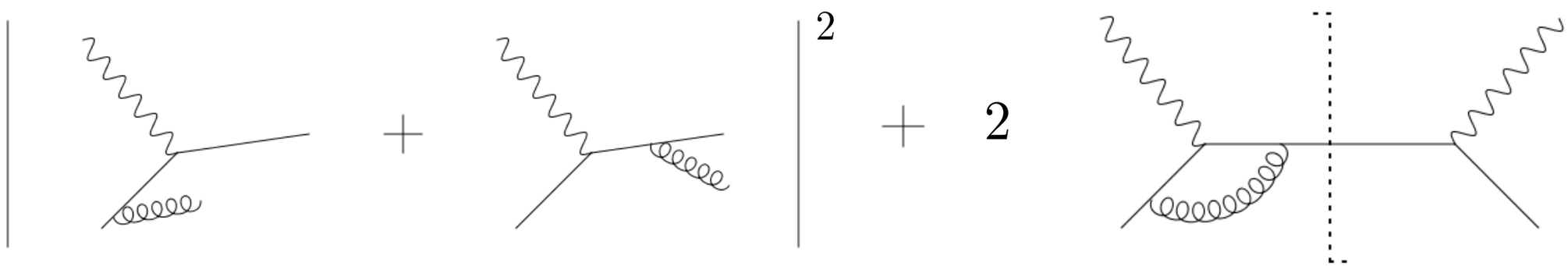
While the PDFs are *UNIVERSAL* do not depend on the specific process, the *coefficient functions* can be calculated *perturbatively* for each process

$$\mathcal{C}_f^i = C_f^{i,(0)} + \frac{\alpha_s(\mu^2)}{2\pi} C_f^{i,(1)} + \mathcal{O}(\alpha_s^2),$$

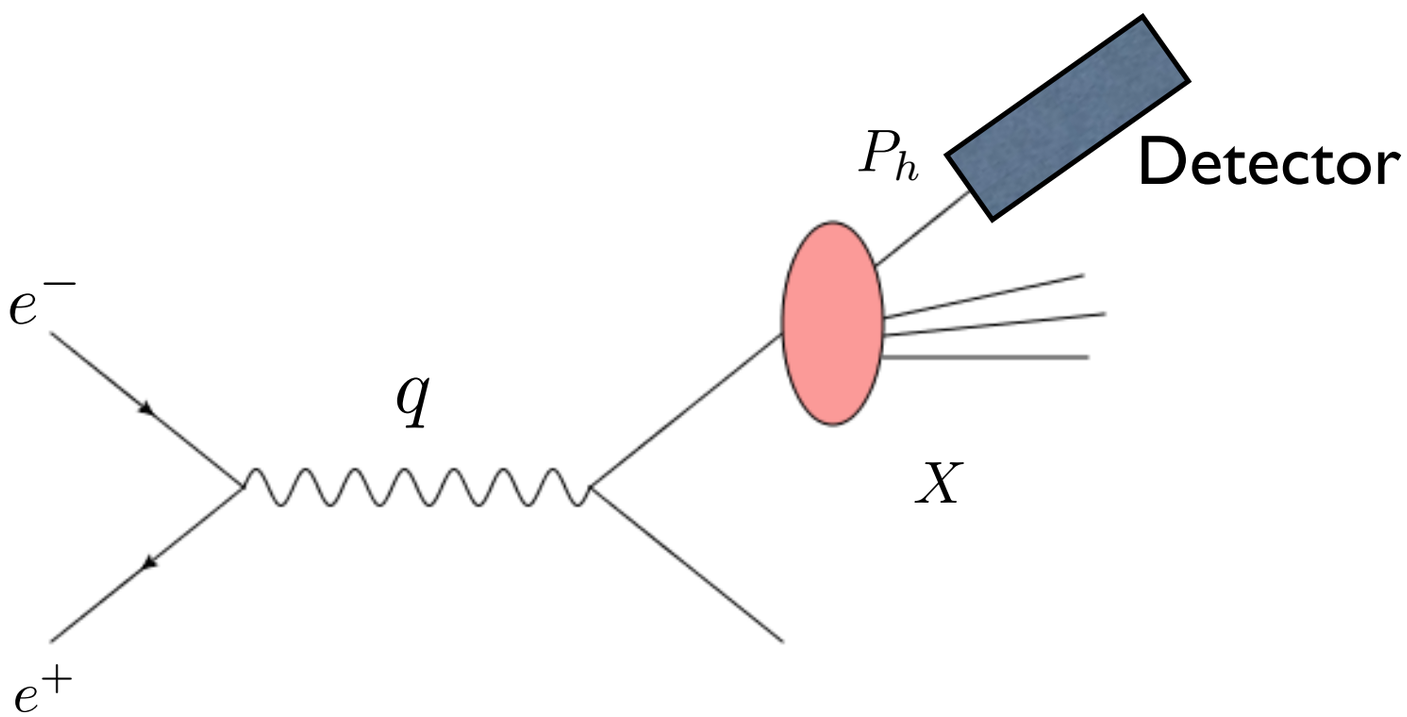
LO:



NLO:



ELECTRON-POSITRON ANNHILATION



Defined kinematic varibles variables:

$q^2 = Q^2$ Virtual Photon Energy

$$x_E \equiv \frac{2P_h \cdot q}{Q^2} = \frac{2E_h}{\sqrt{s}}$$ (c.m.s)

SIA cross section analogous to DIS case.
We treat FFs (parton to hadron) *analogously* to PDFs (hadron to parton):

$$\frac{d^2\sigma^h}{dx_E d\cos\theta} = \frac{\pi\alpha^2}{Q^2} N_C \left[\frac{1+\cos^2\theta}{2} \hat{\mathcal{F}}_T^h(x_E, Q^2) + \sin^2\theta\, \hat{\mathcal{F}}_L^h(x_E, Q^2) \right]$$

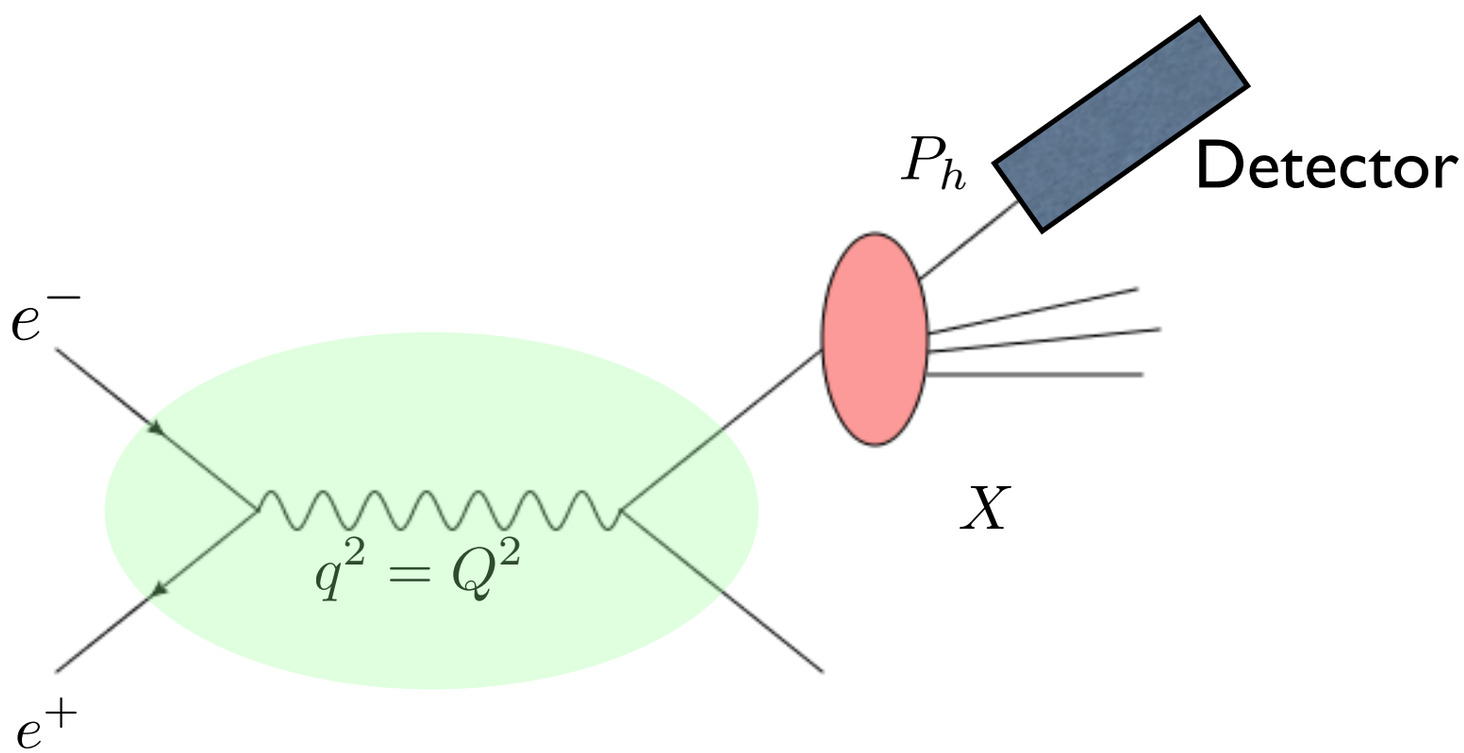
Nason, Webber; Furmanski, Petronzio

where

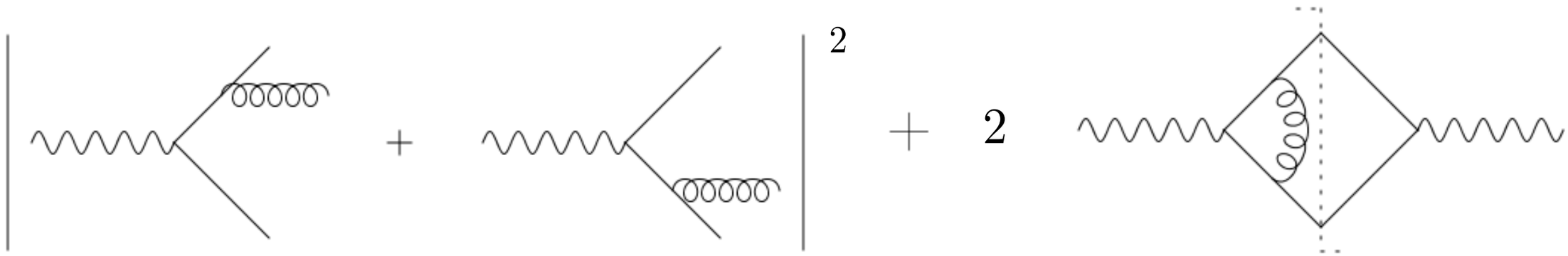
$$\hat{\mathcal{F}}_i^h(x_E, Q^2) = \sum_f \int_{x_E}^1 \frac{d\hat{z}}{\hat{z}} D_f^h\left(\frac{x_E}{\hat{z}}, \mu^2\right) \hat{\mathcal{C}}_f^i\left(\hat{z}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

Theoretically-calculated coefficient functions

Experimentally-measured Fragmentation Functions FFs (fitted)



NLO COEFFICIENT FUNCTION (SIA)



large corrections near threshold $\hat{z} \to 1$

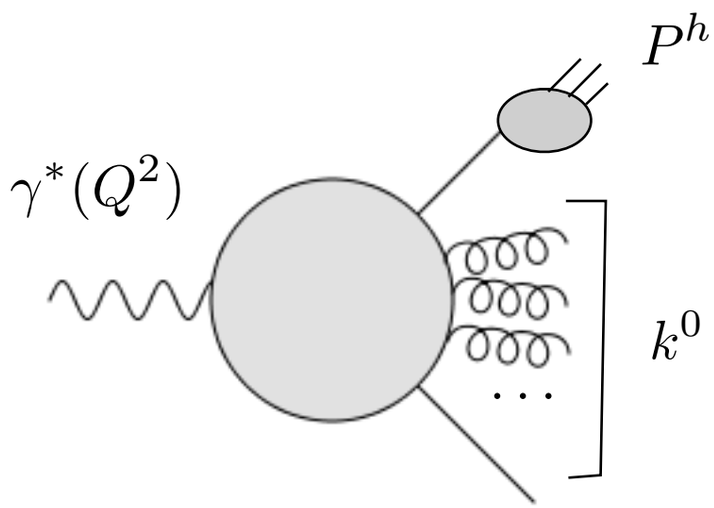
$$\hat{C}_q^{T,(1)} \sim e_q^2 C_F \left[2\left(\frac{\log(1-\hat{z})}{1-\hat{z}}\right)_+ - \frac{3}{2}\frac{1}{(1-\hat{z})_+} + \left(\frac{2\pi^2}{3} - \frac{9}{2}\right)\delta(1-\hat{z}) \right]$$

$\overline{\mathrm{MS}}$ scheme

Altarelli et al.; Furmanski, Petronzio; Nason, Webber...

$$\int_0^1 dz\, f(z) \left(\frac{\ln(1-z)}{1-z}\right)_+ \equiv \int_0^1 dz\, (f(z) - f(1)) \frac{\ln(1-z)}{1-z}$$

THRESHOLD LOGARITHMS



N^kLO Threshold Logarithms

coming from emission of k soft gluon

$$\alpha_s^k \left(\frac{\ln^{2k-1}(1-x)}{1-x} \right)_+$$

spoils perturbative convergence even for $\alpha_s \ll 1$

$x \to 1$ partonic threshold: final state gluon radiation from the basic process $\gamma^* \to q\bar{q}$

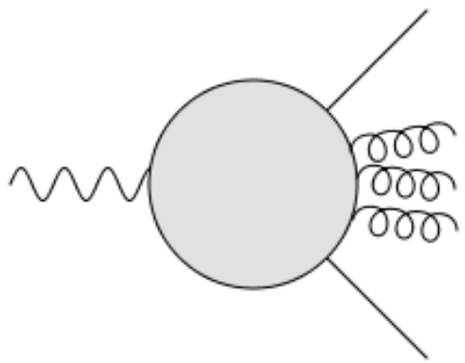
-soft $\frac{k_0}{P_0^h} \equiv 1 - x$

-collinear $k_T \sim k_0\theta \ll (1-x)Q$

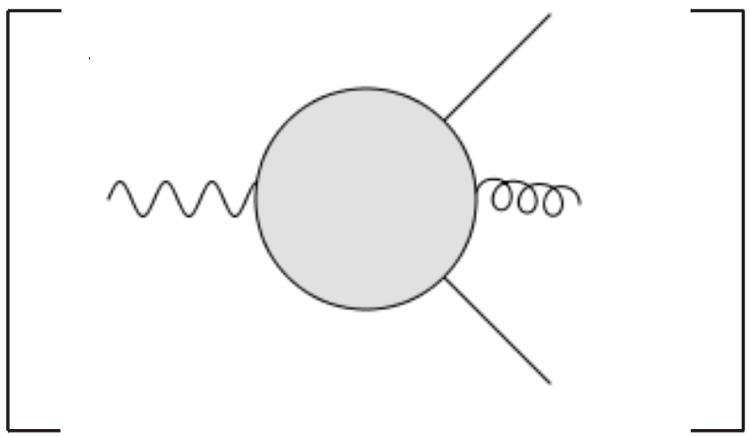
THE EXPONENTIATION

The Resummation of the *Threshold Logs* occurs via the *exponentiation of the "single emission"*

$$\sum_{\text{soft gluons}}$$



$$= \exp$$



Perturbatively calculable (NLO-Calculation)

Resummation relies on the factorisation of

- the matrix element for n-gluon emission in the eikonal approximation (soft gluon approx.)
- the phase space when the Mellin transform is taken

$$\delta\left(1 - k_0 - \sum_{i=1}^{n} k_i\right) = \frac{1}{2\pi i}\int_C dN e^{N(1-k_0-\sum_{i=1}^{n} k_i)}$$

→ Mellin Space

Resummation can be derived in Mellin space

$$\tilde{\mathcal{F}}_i^h(N,Q^2) = \int_0^1 dx_E\, x_E^{N-1}\, \mathcal{F}_i^h(x,Q^2)$$
$$= \sum_f \tilde{D}_f^{h,N} \times \tilde{\mathcal{C}}_f^i(N,Q^2)$$

where for $N \to \infty$ (corresponds to $x \to 1$)

$$\tilde{C}_q^{T,(1)} \sim e_q^2 C_F \left[\boxed{\log \bar{N}^2 + \frac{3}{2}\log \bar{N}} + \left(\frac{5}{6}\pi^2 - \frac{9}{2}\right)\right]$$

$$\bar{N} = N e^{\gamma_E}$$

ACCURACY OF RESUMMATION

$$\mathcal{O}(\alpha_s^k): \qquad C_{kn} \times \alpha_s^k \ln^n \bar{N}, \quad \text{where } n \leq 2k \qquad L = \ln(\bar{N})$$

Fixed Order

LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
...	...	...	...	...	...	
N^{k}LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	...

In Mellin Space

ACCURACY OF RESUMMATION

$$\mathcal{O}(\alpha_s^k): \qquad C_{kn} \times \alpha_s^k \ln^n \bar{N}, \quad \text{where } n \leq 2k \qquad L = \ln(\bar{N})$$

Fixed Order

Resummation

LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
...	...	...	...	...	...	
N^{k}LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	...

LL NLL NNLL

THRESHOLD RESUMMATION

For both DIS and SIA

in Mellin space: exponentiation of the one-loop results

$$\mathcal{C}_q^{T,res} \propto \exp\left[\int_0^1 d\xi \frac{\xi^N - 1}{1-\xi} \times \left\{ \int_{Q^2}^{(1-\xi)Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q(\alpha_s(k_\perp^2)) + \frac{1}{2} B_q\left(\alpha_s((1-\xi)Q^2)\right) \right\}\right]$$

where $A^{(1)} = C_F \ , \quad A^{(2)} = \frac{1}{2} C_F \, K = \frac{1}{2} C_F \left[C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} N_f \right]$

$B^{(1)} = -\frac{3}{2} C_F \ .$

Catani, Trentadue; Stermann

Threshold Resummation acts for DIS and SIA in the same exact way and is relevant for the same Phase Space region:

DIS: $x \to 1$
SIA: $x_E \to 1$

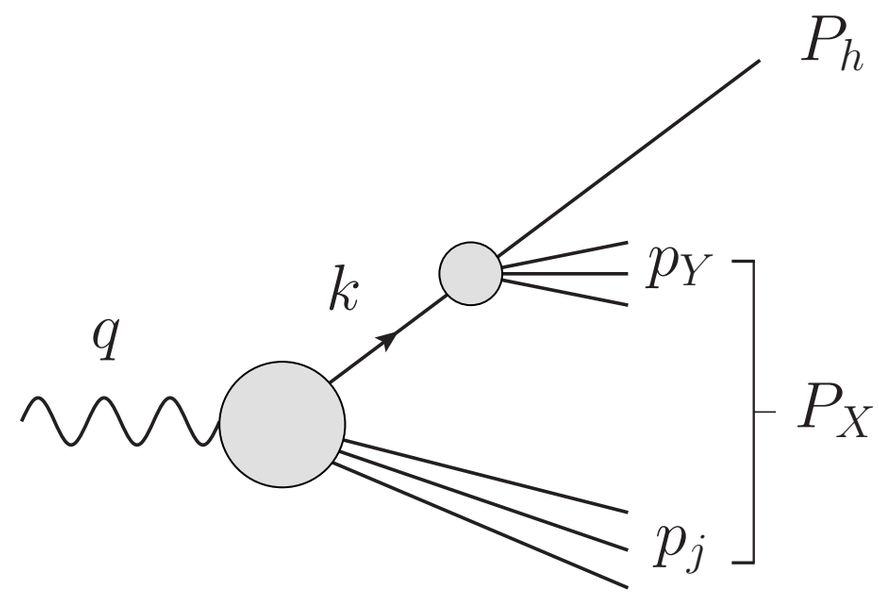
STUDYING THE KINEMATICS (SIA)

we study the kinematics in the $\gamma - h$ frame

$$q = q^+ \bar{n} + \frac{Q^2}{2q^+} n$$

$$P_h = P_h^+ \bar{n} + \frac{m_h^2}{2p_h^+} n$$

$$k = k^+ \bar{n} + \frac{k^2 + k_T^2}{2k^+} n + \mathbf{k}_T$$



we work in collinear factorization

$$z = \frac{P_h^+}{k^+}, \qquad \mathbf{k}_T = 0$$

where the light-cone vectors

$$n^\mu = \frac{1}{\sqrt{2}}(1, 0, 0, -1)$$

$$\bar{n}^\mu = \frac{1}{\sqrt{2}}(1, 0, 0, 1)$$

$$n^2 = \bar{n}^2 = 0 \quad n \cdot \bar{n} = 1$$

$$a^+ = a \cdot n \quad a^- = a \cdot \bar{n}$$

The boson fractional momentum in respect to the hadron is not anymore

$$\cancel{x_E = \frac{2q \cdot P_h}{q^2}}$$

but

$$P_h^+/q^+ = \xi_E = \frac{1}{2} x_E \left(1 + \sqrt{1 - \frac{4}{x_E^2}\frac{m_h^2}{Q^2}}\right)$$

and analogously for DIS

$$\xi = \frac{2x_B}{1 + \sqrt{1 + 4x_B^2 m_N^2/Q^2}}$$

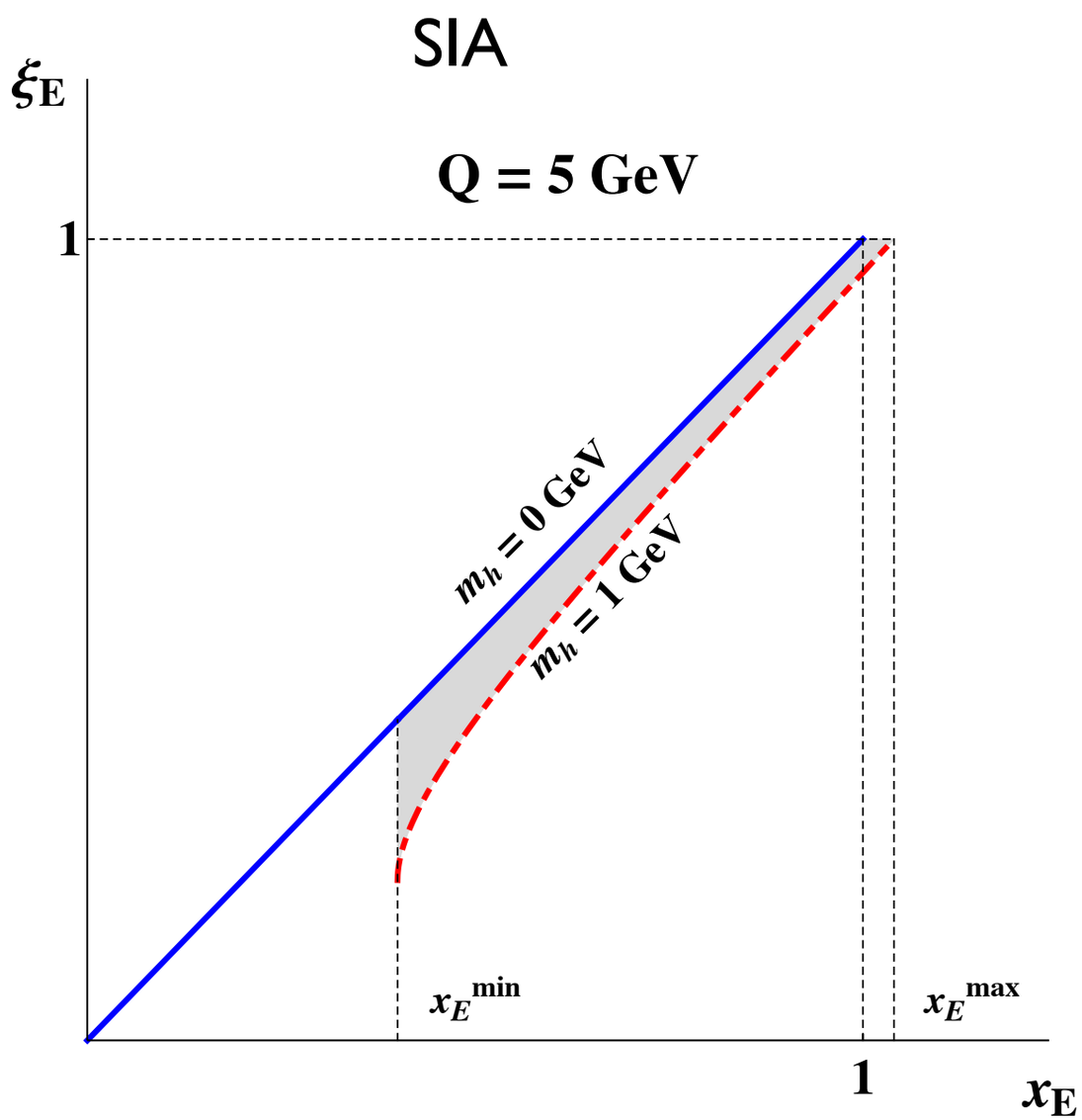
$$\cancel{x_B = \frac{Q^2}{2q \cdot P_h}}$$

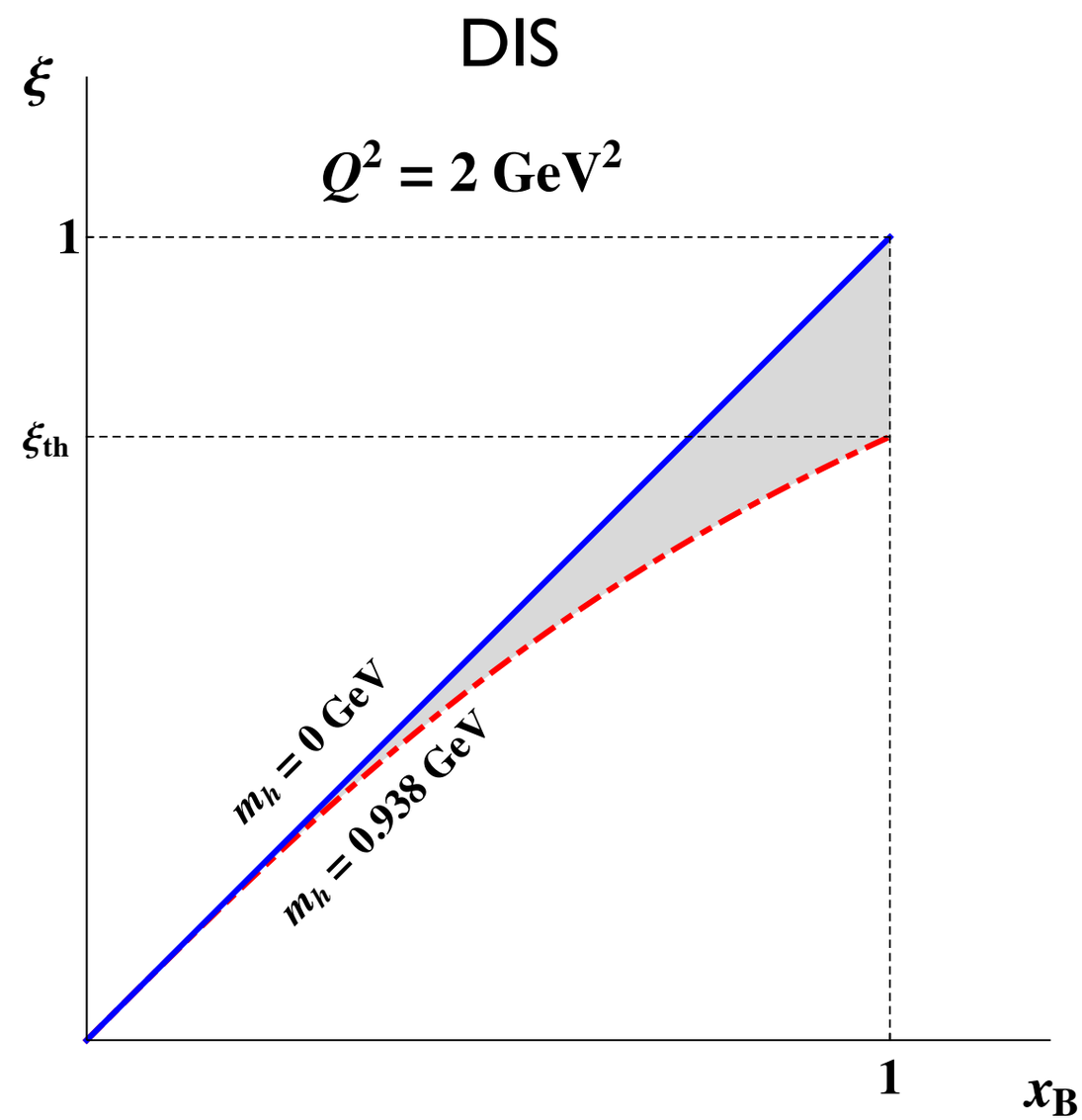
One should use those variables when calculating structure functions, since they represent the right physical fractional momentum variables

$$\mathcal{F}_i(x_E, Q^2) \to \mathcal{F}_i(\xi_E, Q^2)$$
$$\mathcal{F}_i(x_B, Q^2) \to \mathcal{F}_i(\xi, Q^2)$$

Albino et al.

The hadron mass acts kinematically on the two processes in a very different way





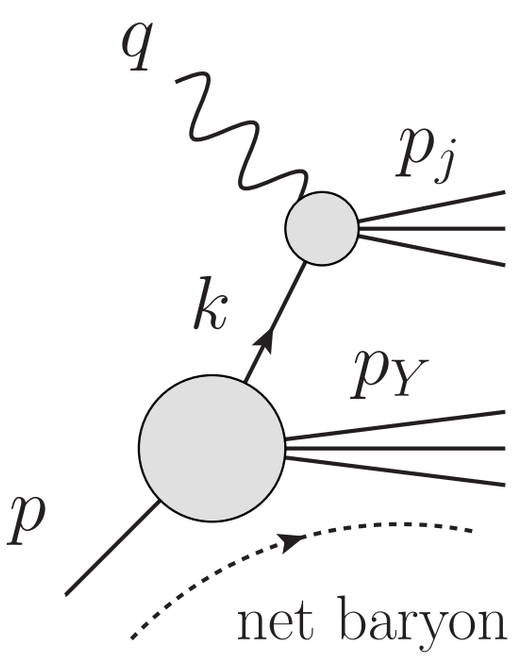
Low x_E effect

High x_B effect

AS RESUMMATION!!!

RESUMMATION AND HMC INTERPLAY (DIS)

Taking into account momentum conservation law and some simple algebra



$$p_j^2 \geq 0 \quad \rightarrow \quad 0 \leq p_j^2 = (q+k)^2 = \left(1 - \frac{\xi}{\hat{x}}\right)\frac{Q^2\hat{x}}{\xi}$$

$$p_Y^2 \geq m_N^2 \quad \rightarrow \quad (q+p)^2 = (p_j + p_Y)^2 \geq (q+k)^2 + m_N^2$$

$$\rightarrow \quad 1 - \frac{1}{x_B} \leq \left(1 - \frac{\hat{x}}{\xi}\right)$$

we find that the partonic momentum fraction $\hat{x}$ is limited as

$$\xi \leq \hat{x} = \frac{k^+}{P_h^+} \leq \xi / x_B$$

In the definition of the structure functions the integration limits need to be modified

$$\mathcal{F}_i(\xi, Q^2) = \sum_f \int_{\xi}^{\xi/x_B} \frac{d\hat{x}}{\hat{x}} f(\hat{x})\, C_f^i\left(\frac{\xi}{\hat{x}}, Q^2\right)$$

Accardi and Qiu(JHEP 0807:090,2008)

This effects also Threshold Resummation correction

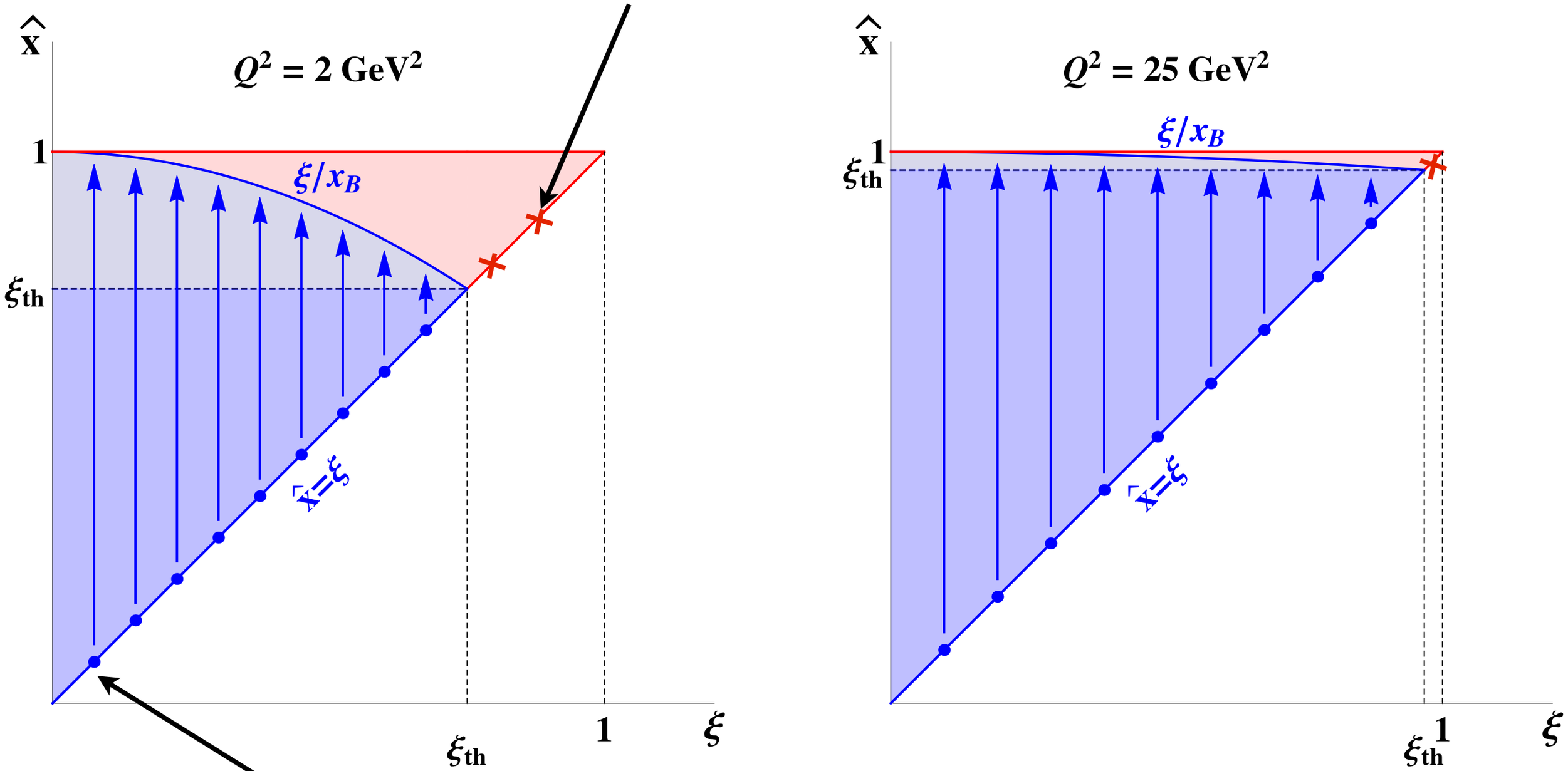
In order to be able to perform the Mellin Transform properly be able to use the resumption formula, we have to define

$$
\begin{aligned}
\mathcal{F}_1^{\mathrm{TMC},N} &= \int_0^1 d\xi\, \xi^{N-1} \int_\xi^{\xi_{\mathrm{th}}} \frac{dx}{x} \mathcal{C}_f^1\left(\frac{\xi}{x}\right) f(x) \\
&= \int_0^1 d\xi\, \xi^{N-1} \int_0^1 dy \int_0^{\xi_{\mathrm{th}}} dx\, \mathcal{C}_f^1(y)\, f(x)\, \delta(xy-\xi) \\
&= \left(\int_0^1 dy\, y^{N-1} \mathcal{C}_f^1(y)\right) \left(\int_0^{\xi_{\mathrm{th}}} dx\, x^{N-1} f(x)\right) \\
&= \mathcal{C}_f^{1,N}\, f_{\xi_{\mathrm{th}}}^N
\end{aligned}
$$

Truncated-Moments of PDF

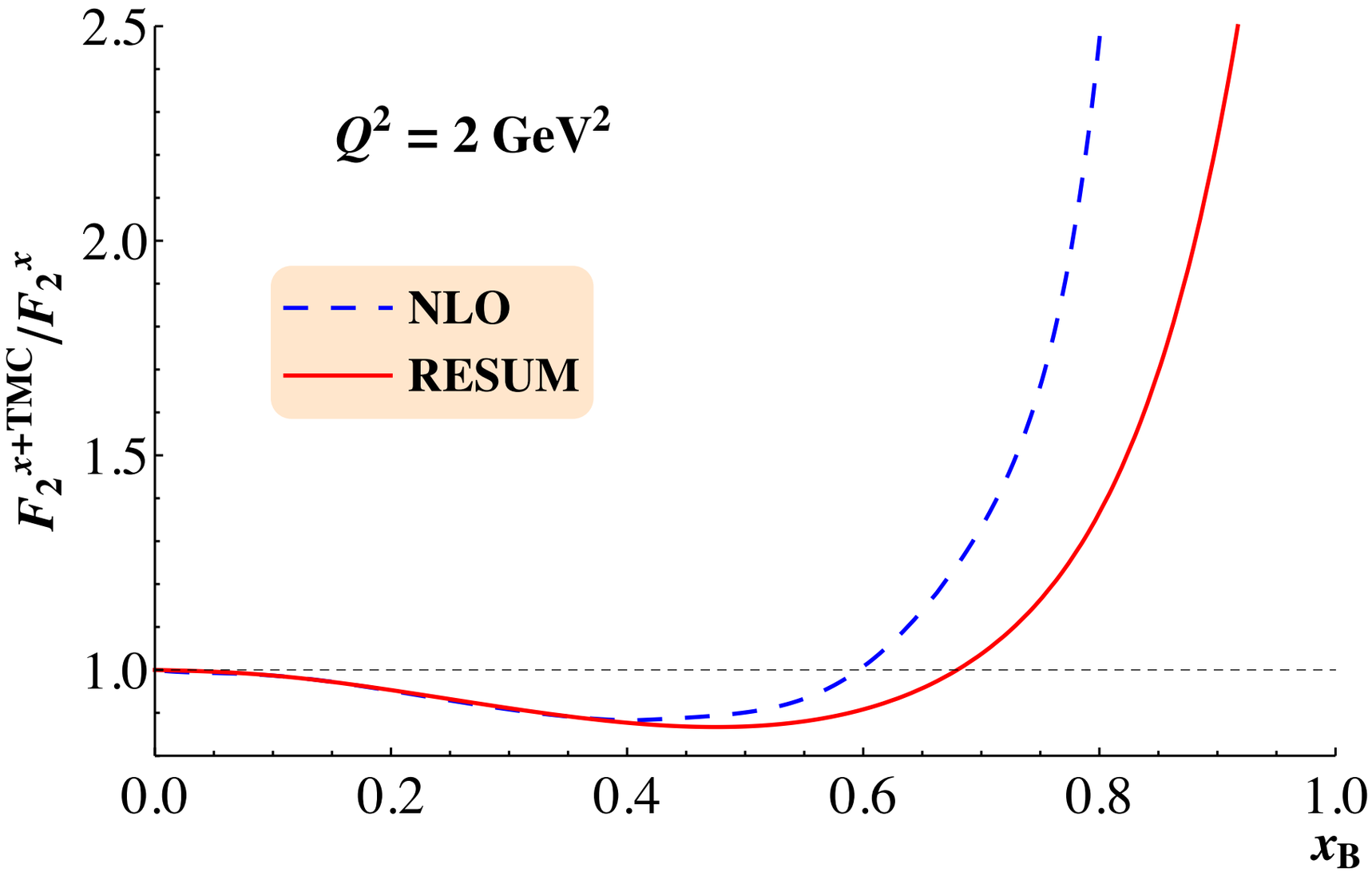
Integration support

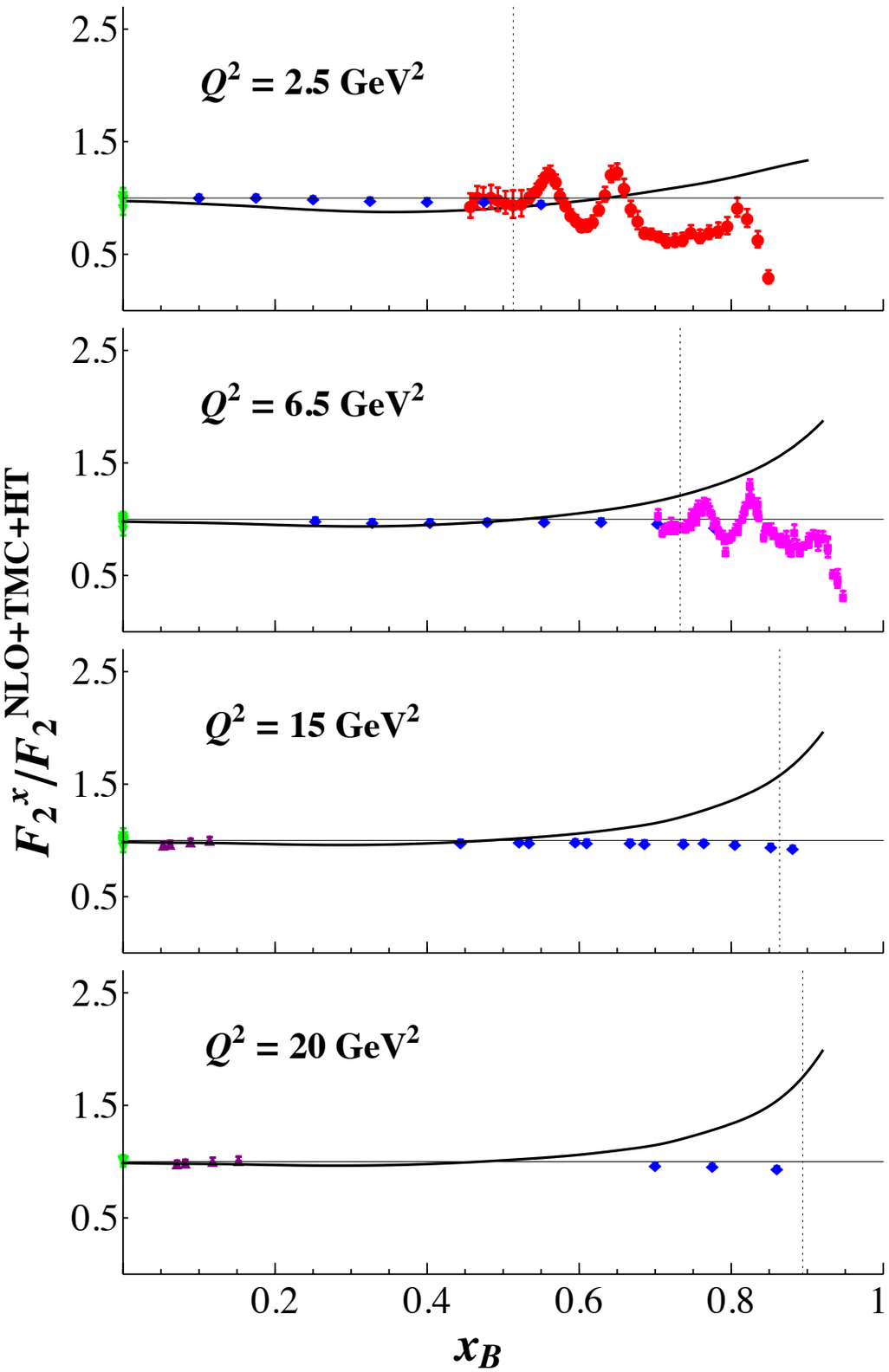
threshold logs excluded from integration

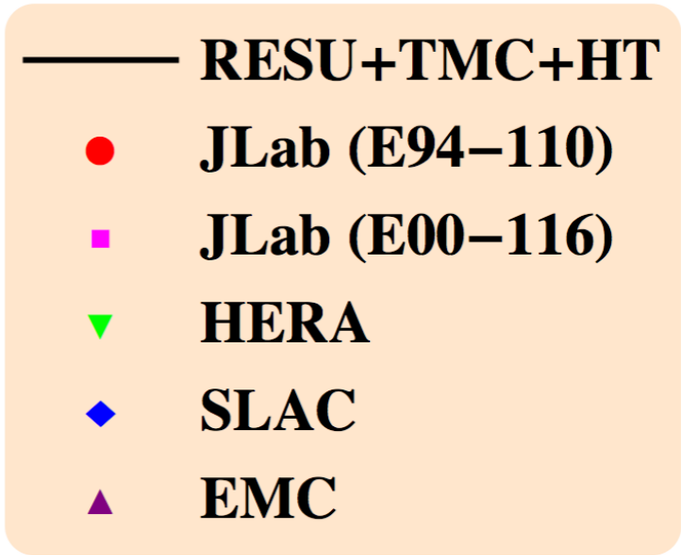


While integrating near the blue dots, the big threshold logs are encountered

For DIS the TMC and Threshold Resummation do not act independently





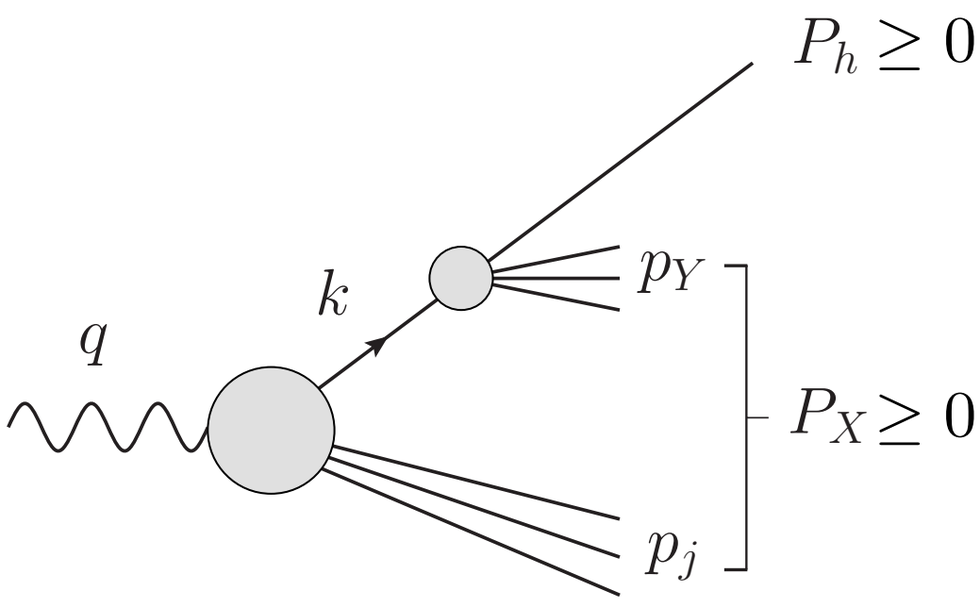


F. Aaron et al. (H1 and ZEUS Collaboration), JHEP 1001, 109 (2010), hep-ex/0911.0884.
L. Whitlow, E. Riordan, S. Dasu, S. Rock, and A. Bodek, Phys.Lett. B282, 475 (1992).
J. Aubert et al. (European Muon Collaboration), Nucl.Phys. B259, 189 (1985)
Y. Liang et al. (Jefferson Lab Hall C E94-110 Collaboration) (2004), nucl-ex/0410027.
S. Malace et al. (Jefferson Lab E00-115 Collaboration),Phys.Rev. C80, 035207 (2009),nucl-ex/0905.2374

with CJ PDF Owens, Accardi, Melnitchouk (Phys.Rev. D87, 094012 (2013))

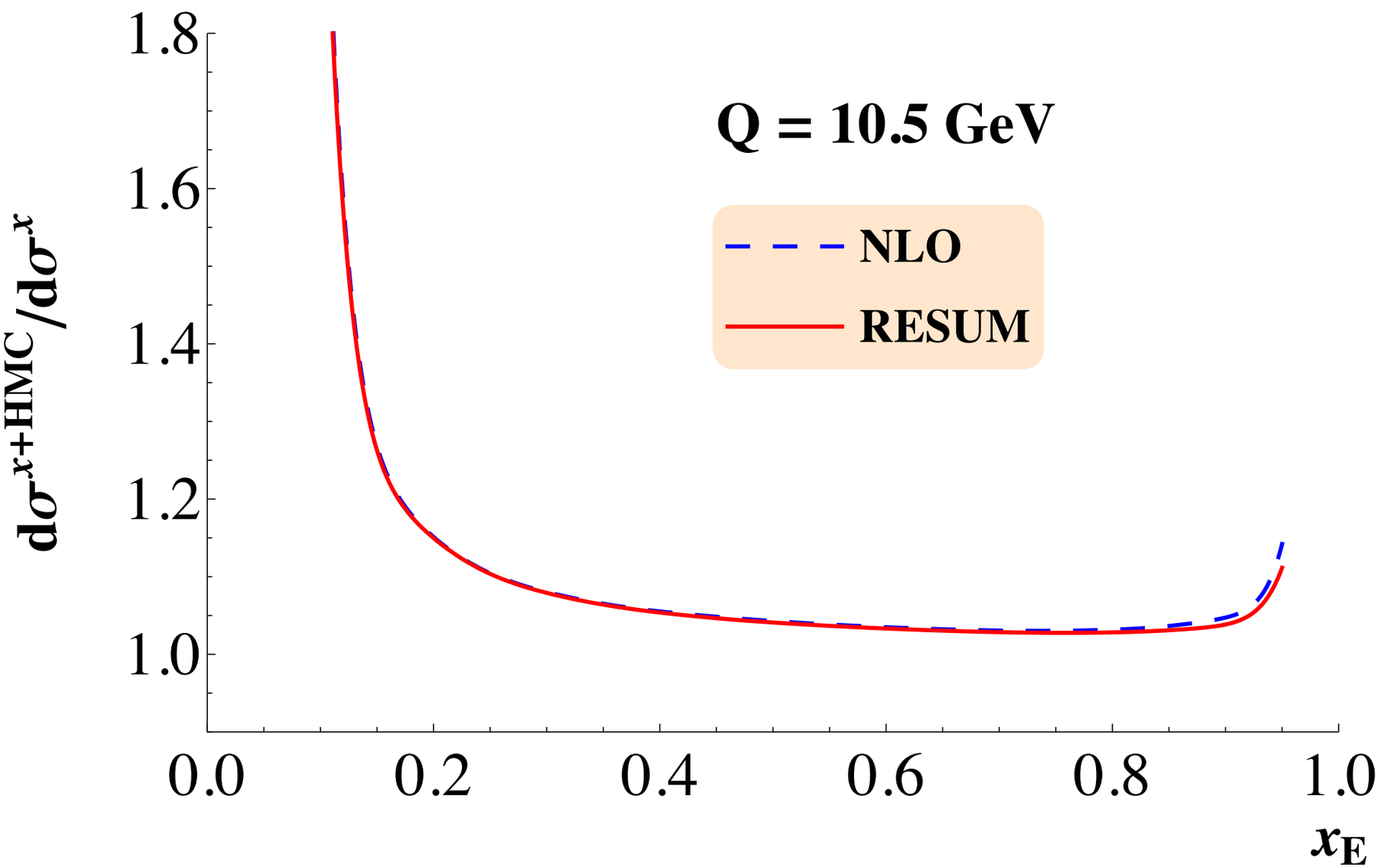
RESUMMATION AND HMC INTERPLAY (SIA)

Following the same type of reasoning, we end up with no modification of the integration limits where the Threshold Logs become important



$$\xi_E \leq \hat{z} \leq 1$$

No interplay between the two effects is found since they act independently on two different kinematical regions

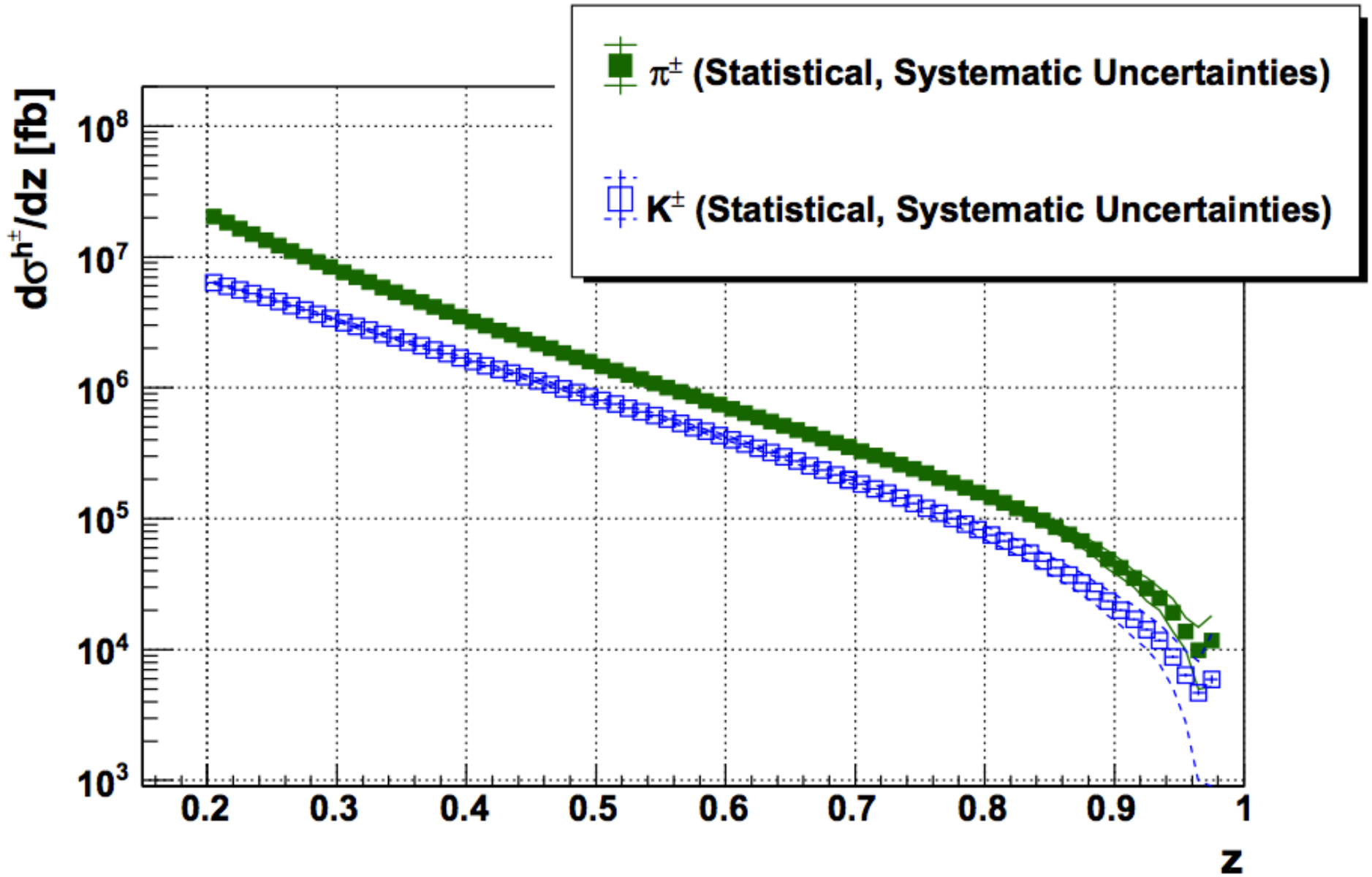


BELLE AND BABAR DATA

BELLE Kinematics

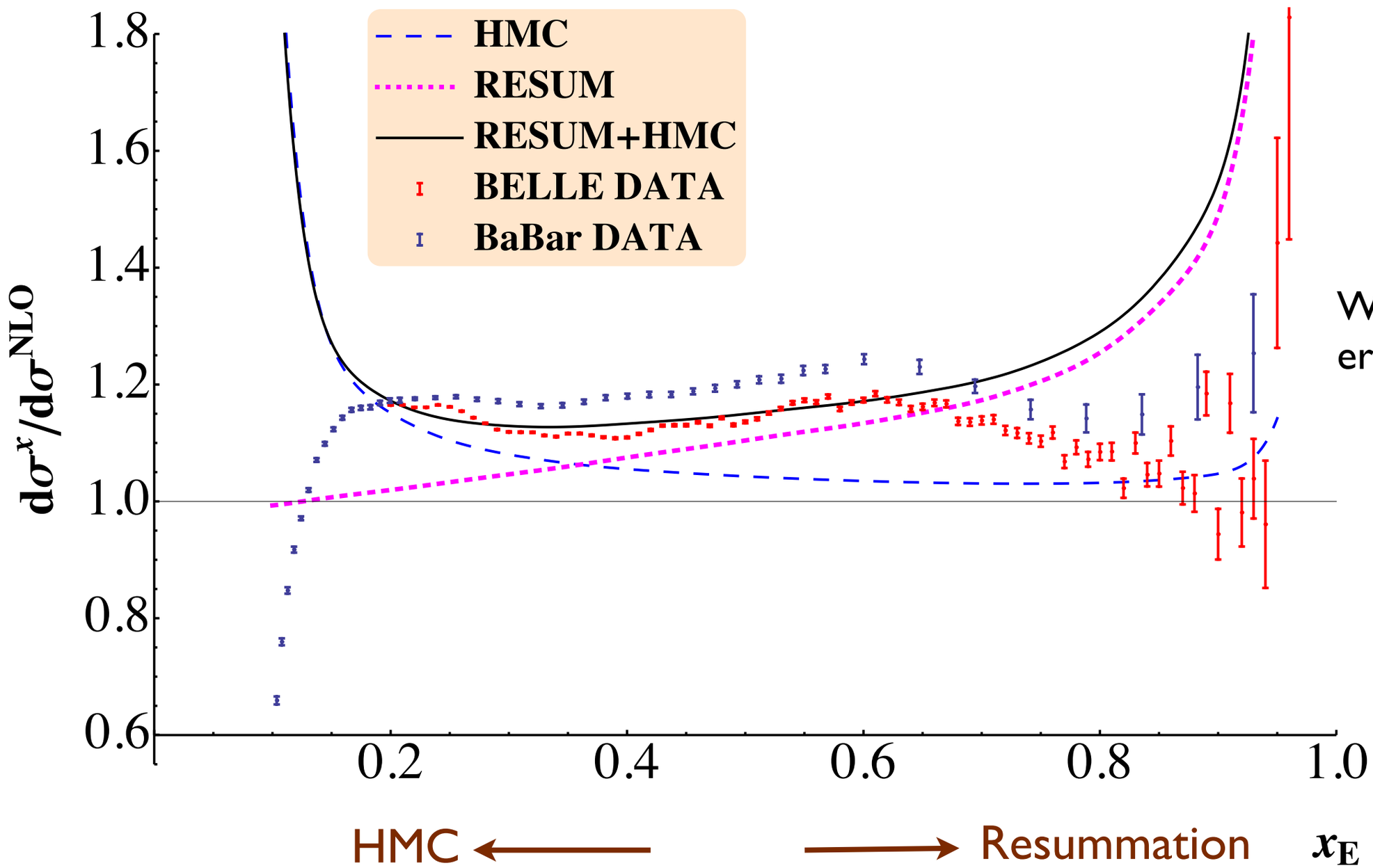
$\sqrt{s} = 10.5\,\mathrm{GeV}$

$-1 < cos\theta < 1$



Belle collaboration arXiv: 1301.6183

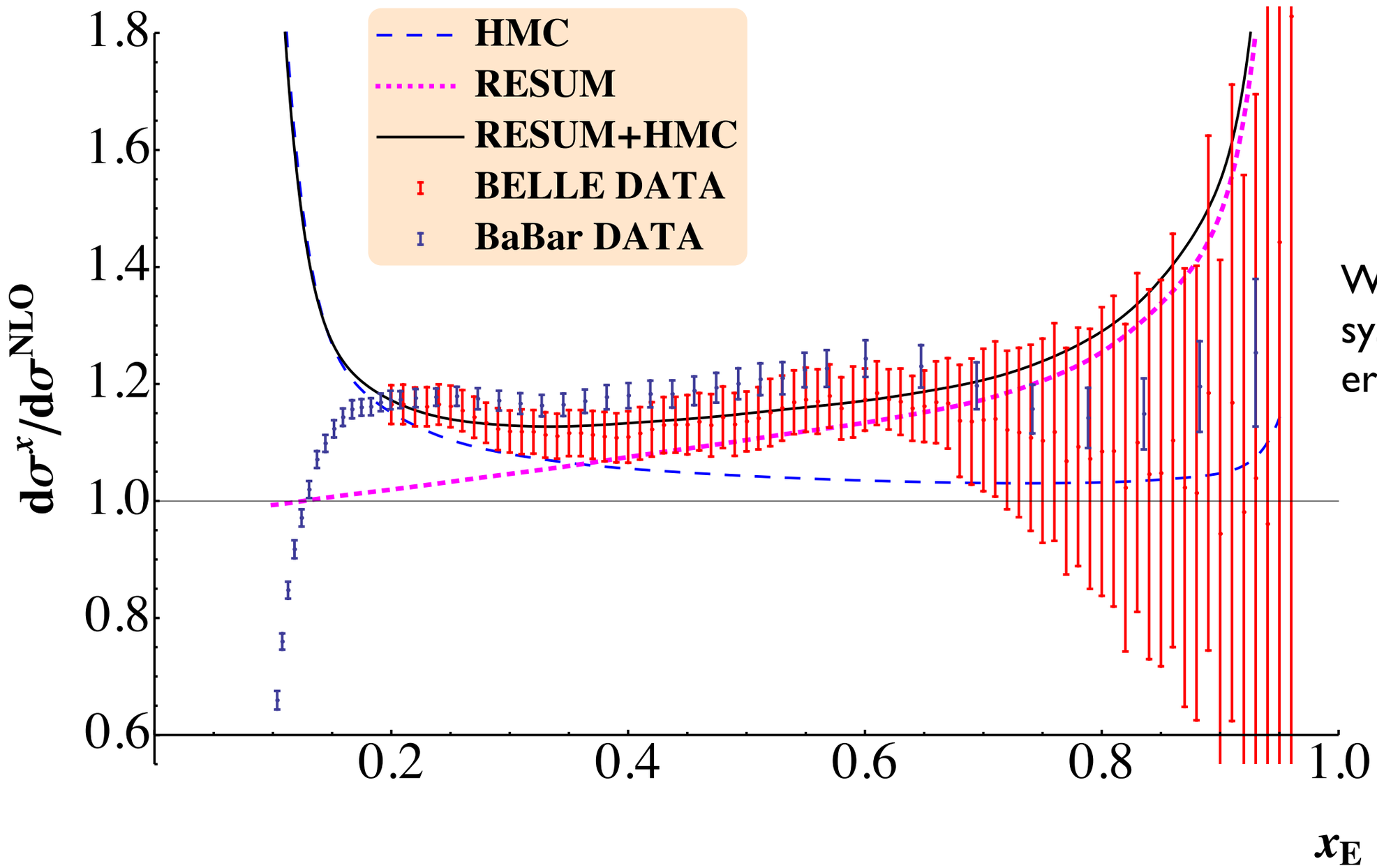
For Kaons one has to take into account HMC



With statistical errors

Belle collaboration arXiv: 1301.6183; BaBar collaboration arXiv: 1306.2895

For Kaons one as to take into account HMC



With systematical errors

Belle collaboration arXiv: 1301.6183; BaBar collaboration arXiv: 1306.2895

OUTLINE

- HMC + THRESHOLD RESUMMATION
- TOWARDS A GLOBAL NNLO FF FIT
- NEW CHANNELS IN SIDIS NNLO F_L
- CONCLUSIONS & OUTLOOK

TOWARDS A GLOBAL NNLO FF FIT

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

DATA SETS:

SI-e^+e^- → old: TPC(Phys. Rev. Lett 61, 1263 (1998)), SLD(Phys. Rev. D59,052001 (1999)), ALEPH(Phys. Lett. B357, 487 (1995)), DELPHI(Eur. Phys. J. C5, 585 (1998),Eur. Phys. J.C6, 19 (1999)) OPAL(Eur. Phys. J. C16, 407 (2000),Eur. Phys. J.C7, 369 (1999)), TASSO(Z. Phys.C42, 189 (1989))

SIDIS → old: EMC(Z. Phys. C52, 361 (1991)), JLAB(Phys. Rev. Lett. 98, 022001)

SI- p(anti-)p → old: CDF(Phys. Rev. Lett. 61,1819 (1988)), UA1(Nucl. Phys. B335,261 (1990)), UA2(Z. Phys. C27, 329 (1985))

TOWARDS A GLOBAL NNLO FF FIT

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

DATA SETS:

SI-e^+e^- → new: BaBar(Phys. Rev. D 88, 032011 (2013)), Belle(Phys. Rev. Lett. 111, 062002 (2013))

SIDIS → new: HERMES(Ph.D. thesis, Erlangen Univ., Germany, September 2005), Compass(PoS DIS 2013, 202 (2013)), JLAB@12GeV

SI- p(anti-)p → new: Phenix(Phys. Rev. D 76,051106 (2007)), Alice(Phys. Lett. B 717, 162 (2012).), Brahms(Phys. Rev. Lett. 98, 252001 (2007)), Star(Phys. Rev. Lett. 97, 152302 (2006))

TOWARDS A GLOBAL NNLO FF FIT

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

NNLO EVOLUTION KERNELS:

Splitting functions → NNLO-Non Singlet: Mitov, Moch, Vogt (Phys.Lett. B638 (2006) 61-67)

NNLO-Singlet: Moch, Vogt (Phys.Lett.B659 (2008) 290-296)

NNLO-Singlet: Almasy, Mitov, Moch, Vogt (Nucl.Phys. B854 (2012)) 133-152)

Both computed in x-Space and in Mellin Space

TOWARDS A GLOBAL NNLO FF FIT

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

NNLO COEFFICINT FUNCTIONS:

SI-e^+e^- → x-Space — Rijken, van Neerven (Phys.Lett.B386(1996)422, Nucl.Phys.B488(1997)233,Phys.Lett.B392(1997)207)

Mellin-Space — Mitov, Moch (Nucl.Phys.B751 (2006) 18-52)
Blümlein, Ravindran (Nucl.Phys.B749 (2006) 1-24)

SIDIS → NOT COMPUTED YET but work in progress

$$\gamma q' \to q\bar{q}q'$$
$$\gamma g \to q\bar{q}q'$$

Anderle, de Florian, Rotstein, Vogelsang

SI- p(anti-)p → NOT COMPUTED YET

TOWARDS A GLOBAL NNLO FF FIT

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

NNLO COEFFICINT FUNCTIONS:

SIDIS → Soft gluon Resummed results (can be expanded @ NNLO)

Anderle, Ringer, Vogelsang (Phys.Rev. D87 (2013) 094021, Phys.Rev. D87 (2013) 3, 034014)

Sl- p(anti-)p → Soft gluon Resummed results (can be expanded @ NNLO)

Work in progress for $\frac{d\sigma}{dp_T d\eta}^{(NNLL)}$ Hinderer, Ringer, Sterman, Vogelsang

TOWARDS A GLOBAL NNLO FF FIT

Anderle, Ringer, Stratmann

Ingredients needed to achieve the goal:

NNLO COEFFICINT FUNCTIONS:

To include the last processes we need a

NNLO Mellin Space Fitting Program

THE TIME-LIKE EVOLUTION

In the factorisation procedure, the absorption of *collinear singularities* by fragmentation functions (FF)(in case of massless partons) leads to scaling violation and the appearance of a factorisation scale μ_F

The scale dependance of FF is governed by the Time-Like DGLAP

$$\frac{\partial}{\partial \ln \mu_F^2} D_i^h(x, \mu_F^2) = \sum_j \int_x^1 \frac{dy}{y} P_{ji}\left(y, \alpha_s(\mu_F^2)\right) D_j^h\left(\frac{x}{y}, \mu_F^2\right)$$

Time-Like Splitting function perturbatively calculable $P_{ji}(y, \alpha_s) = \sum_{k=0} a_s^{k+1} P_{ji}^{(k)}(y)$

Usually rewritten into $2n_f - 1$ equations (charge conjugation and flavour symmetry)

NON-SINGLET

$$D^h_{\mathrm{NS};v} = \sum_{i=1}^{n_f} (D^h_{q_i} - D^h_{\bar{q}_i})$$

$$D^h_{\mathrm{NS};\pm} = (D^h_{q_i} \pm D^h_{\bar{q}_i}) - (D^h_{q_j} \pm D^h_{\bar{q}_j})$$

$$\frac{\partial}{\partial \ln \mu_F^2} D^h_{\mathrm{NS};\pm,v}(x,\mu_F^2) = P^{\pm,\mathrm{v}}(x,\mu_F^2) \otimes D^h_{\mathrm{NS};\pm,v}(x,\mu_F^2)$$

and two coupled

SINGLET

$$D^h_\Sigma = \sum_{i=1}^{n_f} (D^h_{q_i} + D^h_{\bar{q}_i})$$

$$D^h_g$$

$$\frac{\partial}{\partial \ln \mu_F^2} \begin{pmatrix} D^h_\Sigma(x,\mu_F^2) \\ D^h_g(x,\mu_F^2) \end{pmatrix} = \begin{pmatrix} P^{\mathrm{qq}} & 2n_f P^{\mathrm{gq}} \\ \frac{1}{2n_f} P^{\mathrm{qg}} & P^{\mathrm{gg}} \end{pmatrix} \otimes \begin{pmatrix} D^h_\Sigma(x,\mu_F^2) \\ D^h_g(x,\mu_F^2) \end{pmatrix}$$

The splitting functions are accordingly separated in the singlet and non-singlet sectors

NON-SINGLET

$$P_{\rm ns}^{\pm} = P_{\rm qq}^{\rm v} \pm P_{\rm q\bar{q}}^{\rm v}$$

$$P_{\rm ns}^{\rm v} = P_{\rm qq}^{\rm v} - P_{\rm q\bar{q}}^{\rm v} + n_f (P_{\rm qq}^{\rm s} - P_{\rm q\bar{q}}^{\rm s}) \equiv P_{\rm ns}^{-} + P_{\rm ns}^{\rm s}$$

SINGLET

$$P_{\rm qq} = P_{\rm ns}^{+} + n_f (P_{\rm qq}^{\rm s} + P_{\rm \bar{q}q}^{\rm s}) \equiv P_{\rm ns}^{+} + P_{\rm ps}$$

$$P_{\rm gq} \equiv P_{\rm gq_i} = P_{\rm g\bar{q}_i}$$

$$P_{\rm qg} \equiv n_f \, P_{\rm q_i g} = n_f \, P_{\rm \bar{q}_i g}$$

The splitting functions are accordingly separated in the singlet and non-singlet sectors

NON-SINGLET

$$P^{\pm}_{\rm ns} = P^{\rm v}_{\rm qq} \pm \cancel{P^{\rm v}_{\rm q\bar{q}}}$$

$$P^{\rm v}_{\rm ns} = P^{\rm v}_{\rm qq} - \cancel{P^{\rm v}_{\rm q\bar{q}}} + n_f(\cancel{P^{\rm s}_{\rm qq}} - \cancel{P^{\rm s}_{\rm q\bar{q}}}) \equiv P^{-}_{\rm ns} + \cancel{P^{\rm s}_{\rm ns}}$$

@LO

$$P^{\rm v}_{\rm ns} = P^{\pm}_{\rm ns}$$

SINGLET

$$P_{\rm qq} = P^{+}_{\rm ns} + n_f(\cancel{P^{\rm ps}_{\rm qq}} + \cancel{P^{\rm ps}_{\rm qq}}) \equiv P^{+}_{\rm ns} + \cancel{P_{\rm ps}}$$

$$P_{\rm gq} \equiv P_{{\rm g q}_i} = P_{{\rm g}\bar{\rm q}_i}$$

$$P_{\rm qg} \equiv n_f\, P_{{\rm q}_i {\rm g}} = n_f\, P_{\bar{\rm q}_i {\rm g}}$$

The splitting functions are accordingly separated in the singlet and non-singlet sectors

NON-SINGLET

$$P_{\rm ns}^{\pm} = P_{\rm qq}^{\rm v} \pm P_{\rm q\bar{q}}^{\rm v}$$

$$P_{\rm ns}^{\rm v} = P_{\rm qq}^{\rm v} - P_{\rm q\bar{q}}^{\rm v} + n_f \cancel{(P_{\rm qq}^{\rm s} - P_{\rm q\bar{q}}^{\rm s})} \equiv P_{\rm ns}^{-} + \cancel{P_{\rm ns}^{\rm s}}$$

@NLO

$$P_{\rm qq}^{\rm S} = P_{\rm q\bar{q}}^{\rm S}$$

$$P_{\rm ns}^{\rm v} = P_{\rm ns}^{-}$$

SINGLET

$$P_{\rm qq} = P_{\rm ns}^{+} + n_f (P_{\rm qq}^{\rm s} + P_{\rm \bar{q}q}^{\rm s}) \equiv P_{\rm ns}^{+} + P_{\rm ps}$$

$$P_{\rm gq} \equiv P_{{\rm gq}_i} = P_{{\rm g\bar{q}}_i}$$

$$P_{\rm qg} \equiv n_f \, P_{{\rm q}_i {\rm g}} = n_f \, P_{{\rm \bar{q}}_i {\rm g}}$$

The splitting functions are accordingly separated in the singlet and non-singlet sectors

NON-SINGLET

$$P_{\rm ns}^{\pm} = P_{\rm qq}^{\rm v} \pm P_{\rm q\bar{q}}^{\rm v}$$

$$P_{\rm ns}^{\rm v} = P_{\rm qq}^{\rm v} - P_{\rm q\bar{q}}^{\rm v} + n_f (P_{\rm qq}^{\rm s} - P_{\rm q\bar{q}}^{\rm s}) \equiv P_{\rm ns}^{-} + \boxed{P_{\rm ns}^{\rm s}}$$

Responsable for s, $\bar{s}$ asymmetry

@NNLO

$$[s - \bar{s}](x, Q^2) \neq 0$$

German,Catani,
de Florian,Vogelsang
(arXiv:hep-ph/0406338)

SINGLET

$$P_{\rm qq} = P_{\rm ns}^{+} + n_f (P_{\rm qq}^{\rm s} + P_{\rm \bar{q}q}^{\rm s}) \equiv P_{\rm ns}^{+} + P_{\rm ps}$$

$$P_{\rm gq} \equiv P_{{\rm gq}_i} = P_{{\rm g\bar{q}}_i}$$

$$P_{\rm qg} \equiv n_f \, P_{{\rm q}_i {\rm g}} = n_f \, P_{{\rm \bar{q}}_i {\rm g}}$$

THE SOLUTION

We can solve the integro-differential DGLAP equation analytically in Mellin space at each fixed order since it becomes an Ordinary Differential Equation

$$\begin{aligned}\frac{\partial \boldsymbol{q}(N,a_{\rm s})}{\partial a_{\rm s}} &= \{\beta_{\rm N^mLO}(a_{\rm s})\}^{-1}\,\boldsymbol{P}_{\rm N^mLO}(N,a_{\rm s})\,\boldsymbol{q}(N,a_{\rm s})\\ &= -\frac{1}{\beta_0 a_{\rm s}}\Big[\boldsymbol{P}^{(0)}(N)+a_{\rm s}\Big(\boldsymbol{P}^{(1)}(N)-b_1\boldsymbol{P}^{(0)}(N)\Big)\\ &\qquad + a_{\rm s}^2\Big(\boldsymbol{P}^{(2)}(N)-b_1\boldsymbol{P}^{(1)}(N)+(b_1^2-b_2)\boldsymbol{P}^{(0)}(N)\Big)+\ldots\Big]\,\boldsymbol{q}(N,a_{\rm s})\end{aligned}$$

$$f(N,\alpha_s)=\int_0^1 dy\, y^{N-1} f(y,\alpha_s) \qquad N\in\mathbb{C}$$

where here $\mathbf{P}(N,\alpha_S)$ and $\mathbf{q}(N,\alpha_S)$ are the Mellin-Transform of either singlet or non-singlet splitting function and FF respectively

the general solution can be expressed in terms of the evolution matrices *U* (constructed from the splitting functions) as a simple multiplication

$$\begin{aligned}\boldsymbol{q}(N,a_{\rm s}) &= \boldsymbol{U}(N,a_{\rm s})\,\boldsymbol{L}(N,a_{\rm s},a_0)\,\boldsymbol{U}^{-1}(N,a_0)\,\boldsymbol{q}(N,a_0)\\ &= \left[1+\sum_{k=1}^{\infty}a_{\rm s}^k\,\boldsymbol{U}_k(N)\right]\boldsymbol{L}(a_{\rm s},a_0,N)\left[1+\sum_{k=1}^{\infty}a_0^k\,\boldsymbol{U}_k(N)\right]^{-1}\boldsymbol{q}(a_0,N)\end{aligned}$$

where *L* is defined by the LO solution

$$\boldsymbol{q}_{\rm LO}(N,a_{\rm s},N) = \left(\frac{a_{\rm s}}{a_0}\right)^{-\boldsymbol{R}_0(N)}\boldsymbol{q}(N,a_0) \equiv \boldsymbol{L}(N,a_{\rm s},a_0)\,\boldsymbol{q}(N,a_0)$$

$$\boldsymbol{R}_0 \equiv \frac{1}{\beta_0}\boldsymbol{P}^{(0)}$$

TRUNCATED AND ITERATED SOLUTION

Since both $\beta_{\mathrm{N^mLO}}$ and $\boldsymbol{P}_{\mathrm{N^mLO}}$ have an expansion in powers of α_s

there are different ways of defining the N^{m}LO solution

$$
\begin{aligned}
\boldsymbol{q}_{\mathrm{N^3LO}}(a_{\mathrm{s}}) \;=\; \Big[& \boldsymbol{L} + a_{\mathrm{s}}\,\boldsymbol{U}_1\,\boldsymbol{L} - a_0\,\boldsymbol{L}\,\boldsymbol{U}_1 \\
& + a_{\mathrm{s}}^2\,\boldsymbol{U}_2\,\boldsymbol{L} - a_{\mathrm{s}} a_0\,\boldsymbol{U}_1\,\boldsymbol{L}\,\boldsymbol{U}_1 + a_0^2\,\boldsymbol{L}\left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right) \\
& + a_{\mathrm{s}}^3\,\boldsymbol{U}_3\,\boldsymbol{L} - a_{\mathrm{s}}^2 a_0\,\boldsymbol{U}_2\,\boldsymbol{L}\,\boldsymbol{U}_1 + a_{\mathrm{s}} a_0^2\,\boldsymbol{U}_1\,\boldsymbol{L}\left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right) \\
& - a_0^3\,\boldsymbol{L}\left(\boldsymbol{U}_1^3 - \boldsymbol{U}_1\,\boldsymbol{U}_2 - \boldsymbol{U}_1\,\boldsymbol{U}_2 + \boldsymbol{U}_3\right)\Big]\,\boldsymbol{q}(a_0)
\end{aligned}
$$

TRUNCATED AND ITERATED SOLUTION

TRUNCATED: Keep only terms up to α_s^m in the solution

$$
\begin{aligned}
\boldsymbol{q}_{\mathrm{N^3LO}}(a_s) \;=\; & \Big[\boldsymbol{L} + a_s \boldsymbol{U}_1 \boldsymbol{L} - a_0 \boldsymbol{L} \boldsymbol{U}_1 \\
& + a_s^2 \boldsymbol{U}_2 \boldsymbol{L} - a_s a_0 \boldsymbol{U}_1 \boldsymbol{L} \boldsymbol{U}_1 + a_0^2 \boldsymbol{L} \left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right) \\
& + a_s^3 \boldsymbol{U}_3 \boldsymbol{L} - a_s^2 a_0 \boldsymbol{U}_2 \boldsymbol{L} \boldsymbol{U}_1 + a_s a_0^2 \boldsymbol{U}_1 \boldsymbol{L} \left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right) \\
& - a_0^3 \boldsymbol{L} \left(\boldsymbol{U}_1^3 - \boldsymbol{U}_1 \boldsymbol{U}_2 - \boldsymbol{U}_1 \boldsymbol{U}_2 + \boldsymbol{U}_3\right)\Big] \boldsymbol{q}(a_0)
\end{aligned}
$$

- It solves the equation exactly only up to terms of order n > m

TRUNCATED AND ITERATED SOLUTION

ITERATED: Keep the all the m-terms generated from $\beta_{\mathrm{N^mLO}}$ and $\boldsymbol{P}_{\mathrm{N^mLO}}$

$$
\begin{aligned}
\boldsymbol{q}_{\mathrm{N^3LO}}(a_s) \;=\; & \Big[\boldsymbol{L} + a_s \boldsymbol{U}_1 \boldsymbol{L} - a_0 \boldsymbol{L} \boldsymbol{U}_1 \\
& + a_s^2 \boldsymbol{U}_2 \boldsymbol{L} - a_s a_0 \boldsymbol{U}_1 \boldsymbol{L} \boldsymbol{U}_1 + a_0^2 \boldsymbol{L} \left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right) \\
& + a_s^3 \boldsymbol{U}_3 \boldsymbol{L} - a_s^2 a_0 \boldsymbol{U}_2 \boldsymbol{L} \boldsymbol{U}_1 + a_s a_0^2 \boldsymbol{U}_1 \boldsymbol{L} \left(\boldsymbol{U}_1^2 - \boldsymbol{U}_2\right) \\
& - a_0^3 \boldsymbol{L} \left(\boldsymbol{U}_1^3 - \boldsymbol{U}_1 \boldsymbol{U}_2 - \boldsymbol{U}_1 \boldsymbol{U}_2 + \boldsymbol{U}_3\right)\Big] \boldsymbol{q}(a_0)
\end{aligned}
$$

- It corresponds to the solution done in x-Space
- It introduces more higher order scheme-dependent terms

TRUNCATED AND ITERATED SOLUTION

ITERATED-TRUNCATED = theoretical uncertainty of order $\mathcal{O}(\alpha_s^{m+1})$

THE NNLO EVOLUTION CODE "PEGASUS_FF"

Existing NNLO Evolution CODES:

X-SPACE APFEL(time-like version C/C++, Fortran77, Python)
Bertone1,Carrazza, Rojo (CERN-PH-TH/2013-209)

Mellin SPACE MELA(Fortran77)
Bertone1,Carrazza, Nocera (CERN-PH-TH-2014-265)

Newly born:

Mellin SPACE Pegasus_FF (Fortran77) → based on Pegasus(Fortran77)
Anderle, Ringer, Stratmann — Vogt (Comput.Phys.Commun.170:65-92,2005)

"PEGASUS_FF": HEAVY FLAVOURS

Parametrization of light patrons FF @ μ_0

$$D_i^h(z, Q_0) = \frac{N_i z^{\alpha_i}(1-z)^{\beta_i}[1+\gamma_i(1-z)^{\delta_i}]}{B[2+\alpha_i, \beta_i+1] + \gamma_i B[2+\alpha_i, \beta_i+\delta_i+1]}$$

So that $N_i = \int_0^1 z D_i^h dz$

"Pegasus_FF" OPTIONS

FIXED FLAVOUR SCHEME: the evolution is done for a fixed number of flavours for which the initial-scale functional form corresponds to the above one

NON PERTURBATIVE INPUT: at $\mu > m_q$ the evolution is set to evolve with $n_f + 1$ for flavours and for the q-heavy quark FF the same functional form as for the light quark is set at $\mu = m_q$

VARIABLE FLAVOUR SCHEME: at $\mu > m_q$ the evolution is set for $n_f + 1$ flavours and the q-heavy quark FF is fixed by matching-conditions at $\mu = m_q$

"PEGASUS_FF": HEAVY FLAVOURS

MATCHING CONDITION: computed by imposing the equality between the massive calculation and the massless (MS-bar) calculated cross section @ $\mu_f = m_q$

COMPUTED ONLY up to NLO: Cacciari, Nason, Oleari (JHEP 0510:034,2005)

$$D_{h/\bar{h}}^{(n)}(x,\mu) = \int_x^1 \frac{dy}{y} D_g(x/y,\mu) \times \frac{\alpha_S}{2\pi} C_F \frac{1+(1-y)^2}{y} \left[\log\frac{\mu^2}{m^2} - 1 - 2\log y\right]$$

$$D_g^{(n)}(x,\mu) = D_g^{(n_L)}(x,\mu)\left(1 - \frac{T_F \alpha_S}{3\pi}\log\frac{\mu^2}{m^2}\right)$$

$$D_{i/\bar{i}}^{(n)}(x,\mu) = D_{i/\bar{i}}^{(n_L)}(x,\mu) \qquad \text{for } i = q_1,\ldots,q_{n_L}$$

$$n_L = n_f + 1$$

TOWARDS A GLOBAL NNLO FF FIT

Ingredients needed to achieve the goal:

NNLO COEFFICINT FUNCTIONS:

SI-e^+e^- → x-Space

Rijken, van Neerven (Phys.Lett.B386(1996)422, Nucl.Phys.B488(1997)233,Phys.Lett.B392(1997)207)

Mellin-Space

Mitov, Moch (Nucl.Phys.B751 (2006) 18-52)
Blümlein, Ravindran (Nucl.Phys.B749 (2006) 1-24)

SIDIS → NOT COMPUTED YET but work in progress

$$\gamma q' \to q\bar{q}q'$$
$$\gamma g \to q\bar{q}q'$$

Anderle, de Florian, Rotstein, Vogelsang

SI- p(anti-)p → NOT COMPUTED YET

TOWARDS A GLOBAL NNLO FF FIT

Ingredients needed to achieve the goal:

NNLO COEFFICINT FUNCTIONS:

SI-e^+e^- → x-Space

Rijken, van Neerven (Phys.Lett.B386(1996)422, Nucl.Phys.B488(1997)233,Phys.Lett.B392(1997)207)

Mellin-Space

Mitov, Moch (Nucl.Phys.B751 (2006) 18-52)
Blümlein, Ravindran (Nucl.Phys.B749 (2006) 1-24)

@ NNLO Harmonic PolyLogs(HPL) appear in the coefficient functions

↓

Calculation of Mellin moments non trivial

TOWARDS A GLOBAL NNLO FF FIT

@NLO the moments of the coefficient functions contain at worst SINGLE HARMONIC SUMS, which can be consistently continued in the complex plane

$$
\begin{aligned}
S_k(N) &= (-1)^{k-1}\frac{1}{(k-1)!}\psi^{(k-1)}(N+1) + c_k^+ \\
S_{-k}(N) &= (-1)^{k-1+N}\frac{1}{(k-1)!}\beta^{(k-1)}(N+1) - c_k^-
\end{aligned}
$$

$\psi(z)$ first derivative of Euler Gamma Function

$$
\begin{aligned}
\beta(z) &= \frac{1}{2}\left[\psi\left(\frac{z+1}{2}\right) - \psi\left(\frac{z}{2}\right)\right] \\
c_1^+ &= \gamma_E \\
c_k^+ &= \zeta(k), \quad k \geq 2 \\
c_1^- &= \log(2) \\
c_k^+ &= \left(1 - \frac{1}{2^{k-1}}\right)\zeta(k), \quad k \geq 2
\end{aligned}
$$

TOWARDS A GLOBAL NNLO FF FIT

@NNLO → MULTIPLE HARMONIC SUMS from MT-HPLs

$$S_{k_1,\ldots,k_m}(N) = \sum_{n_1=1}^{N} \frac{[\mathrm{sign}(k_1)]^{n_1}}{n_1^{|k_1|}} \sum_{n_2=1}^{n_1} \frac{[\mathrm{sign}(k_2)]^{n_2}}{n_2^{|k_2|}} \cdots \sum_{n_m=1}^{n_{m-1}} \frac{[\mathrm{sign}(k_m)]^{n_m}}{n_m^{|k_m|}}$$

ANALITICAL CONTINUATIONS: provided by Blümlein,Kurth(Phys. Rev. D60 (1999) 014018)

also as FORTRAN77 routines Blümlein(Comput. Phys. Commun. 133 (2000) 76))

TOWARDS A GLOBAL NNLO FF FIT

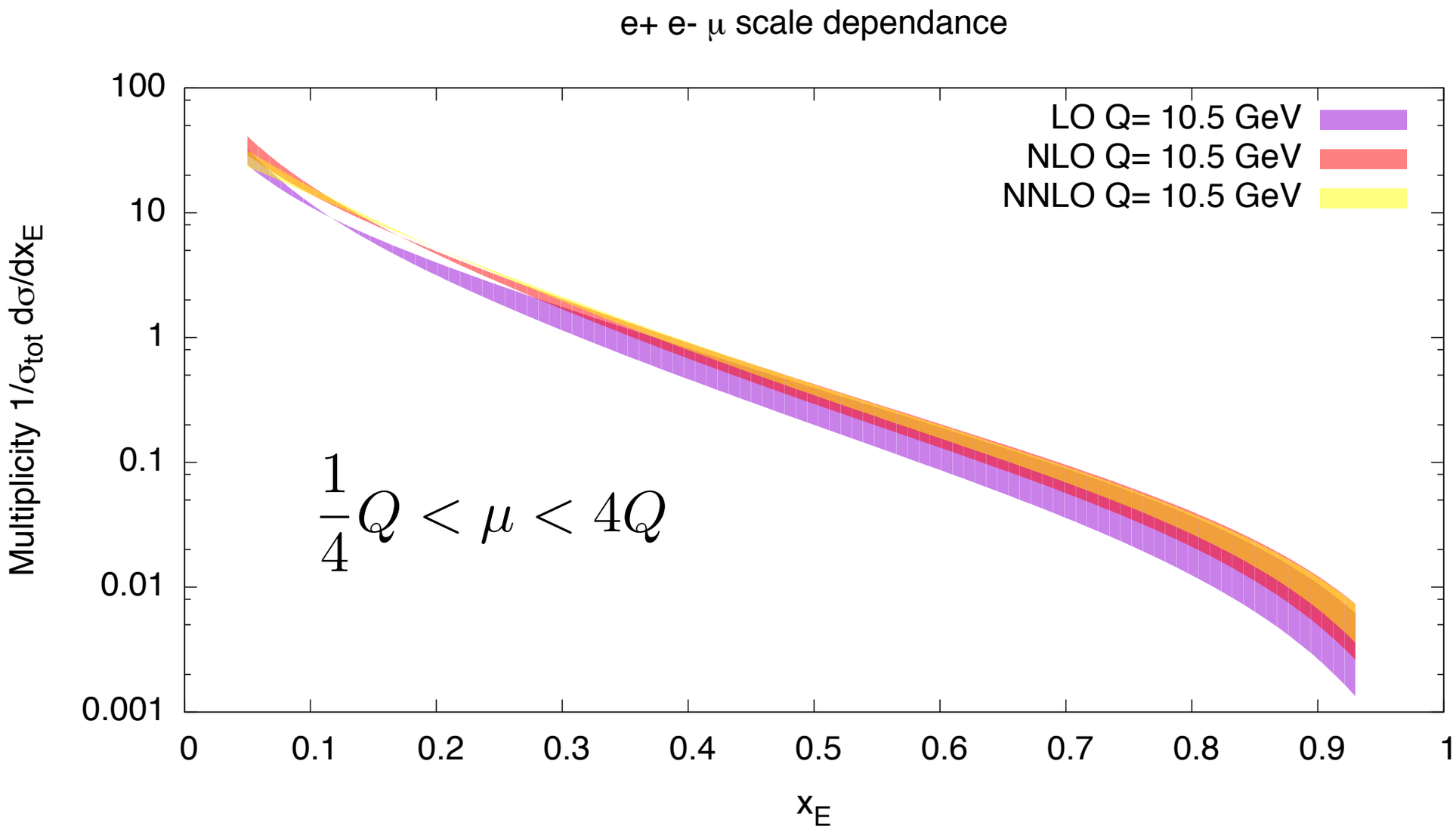
We have checked the Mellin moments calculation and the consistency between Mitov, Moch and Blümlein, Ravindran notation

NUMERICALLY and ANALITICALLY: making use of

- "HPL"-Mathematica package, D. Maître(Comput.Phys.Commun. 174 (2006) 222-240)

- "MT"-Mathematica package, Hoeschele,Hoff, Pak,Steinhauser, Ueda(arXiv:1307.6925)

NNLO e+e- WITH "PEGASUS_FF"

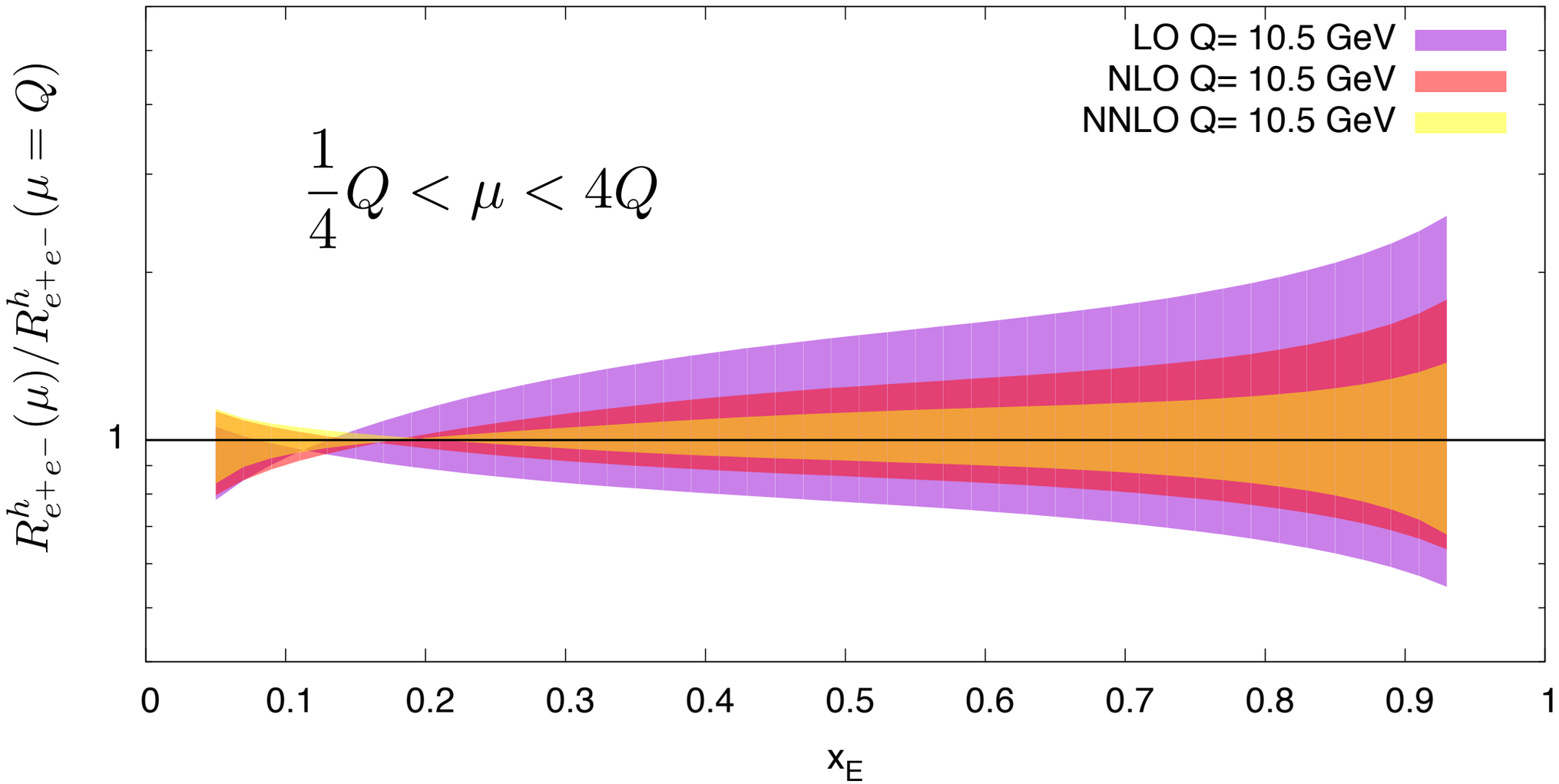


Multiplicity $R^h_{e^+e^-} \equiv \frac{1}{\sigma^{\rm tot}} \frac{d^2\sigma^h}{dx_E\, d\cos\theta}$

using input parameter for FF of Kretzer (Phys.Rev. D62 (2000) 054001) and truncated-solution

NNLO e+e- WITH "PEGASUS_FF"

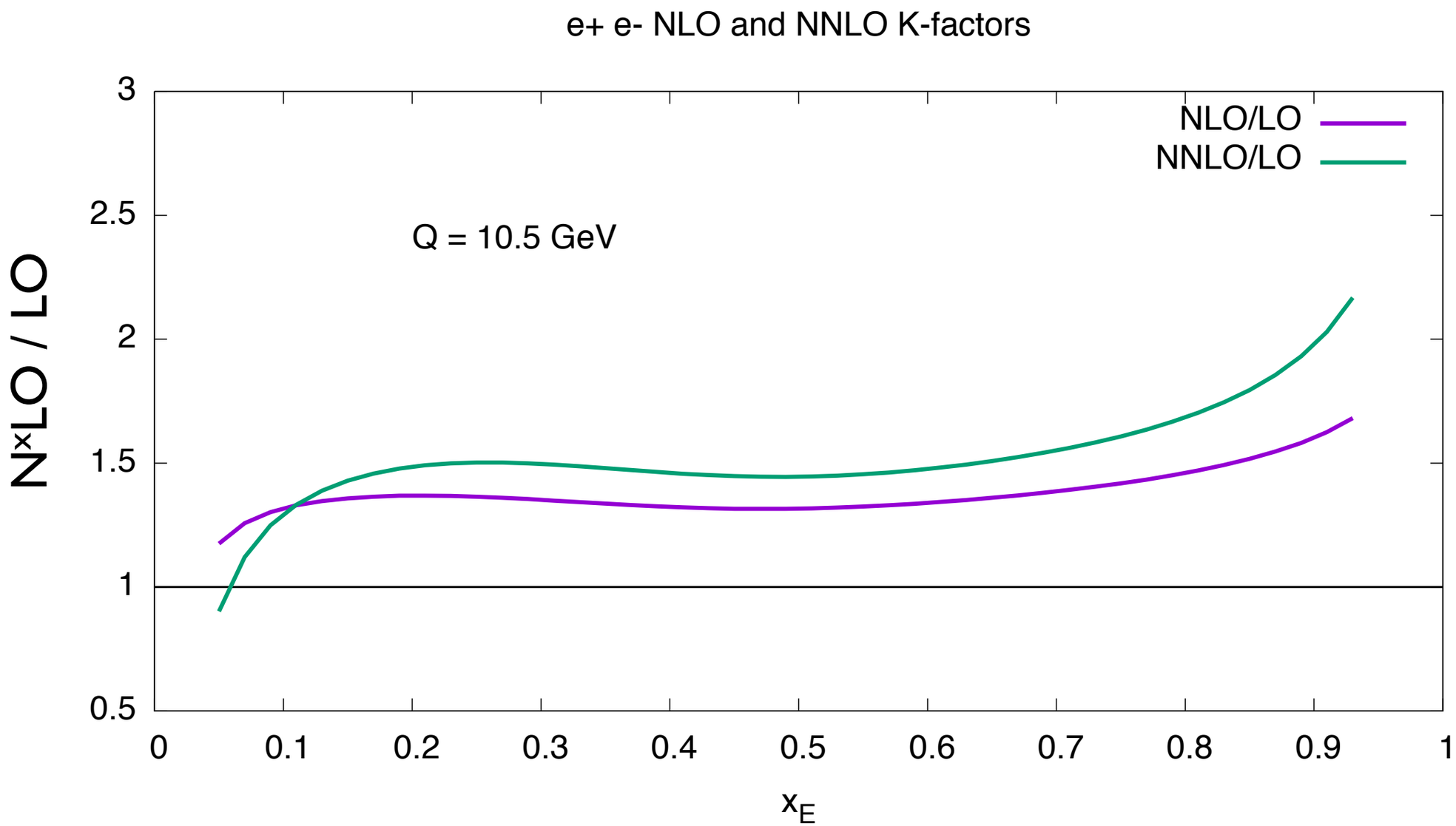
e+ e- μ scale dependance



Multiplicity $R^h_{e^+e^-} \equiv \frac{1}{\sigma^{\rm tot}} \frac{d^2\sigma^h}{dx_E\, d\cos\theta}$

using input parameter for FF of Kretzer (Phys.Rev. D62 (2000) 054001) and truncated-solution

NNLO e+e- WITH "PEGASUS_FF"



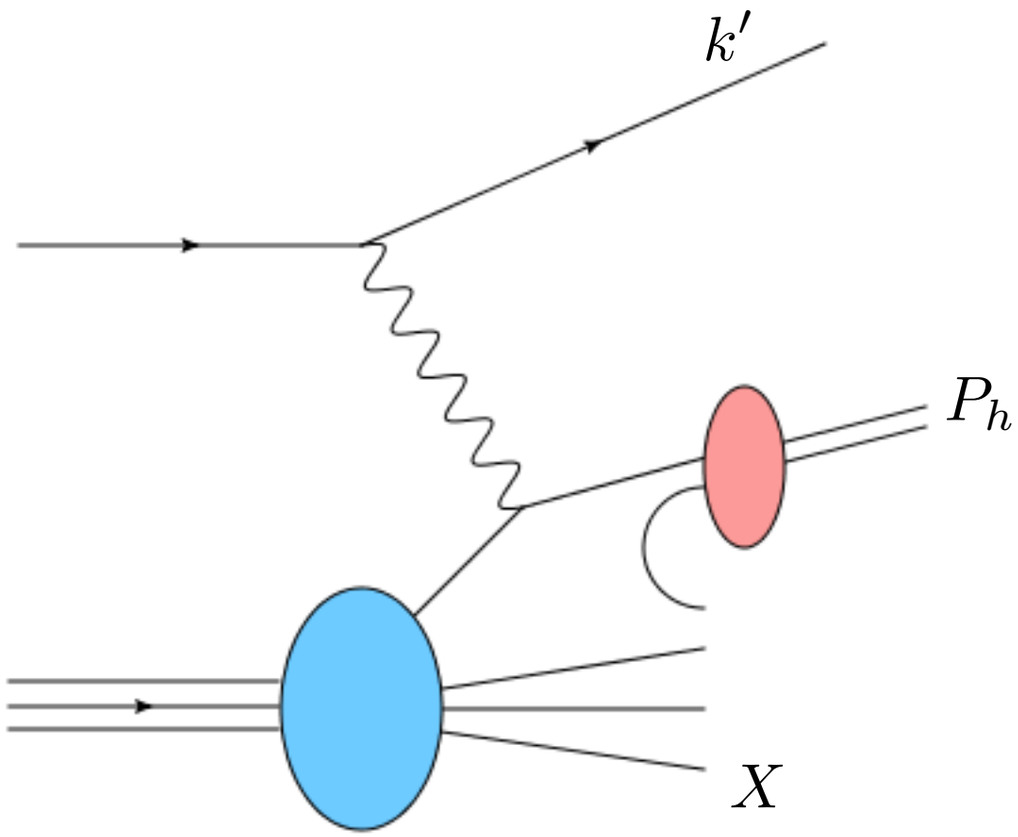
Multiplicity $R^h_{e^+e^-} \equiv \frac{1}{\sigma^{\rm tot}} \frac{d^2\sigma^h}{dx_E\, d\cos\theta}$

using input parameter for FF of Kretzer (Phys.Rev. D62 (2000) 054001) and truncated-solution

OUTLINE

- HMC + THRESHOLD RESUMMATION
- TOWARDS A GLOBAL NNLO FF FIT
- NEW CHANNELS IN SIDIS NNLO F_L
- CONCLUSIONS & OUTLOOK

SEMI-INCLUSIVE DIS



$$\ell(k)\, p(P) \to \ell(k')\, h(P_h)\, X$$

Define the usual variables:

$$Q^2 \equiv -q^2 = -(k-k')^2$$

$$y \equiv \frac{P \cdot q}{P \cdot k}$$

$$x \equiv \frac{Q^2}{2P \cdot q}$$

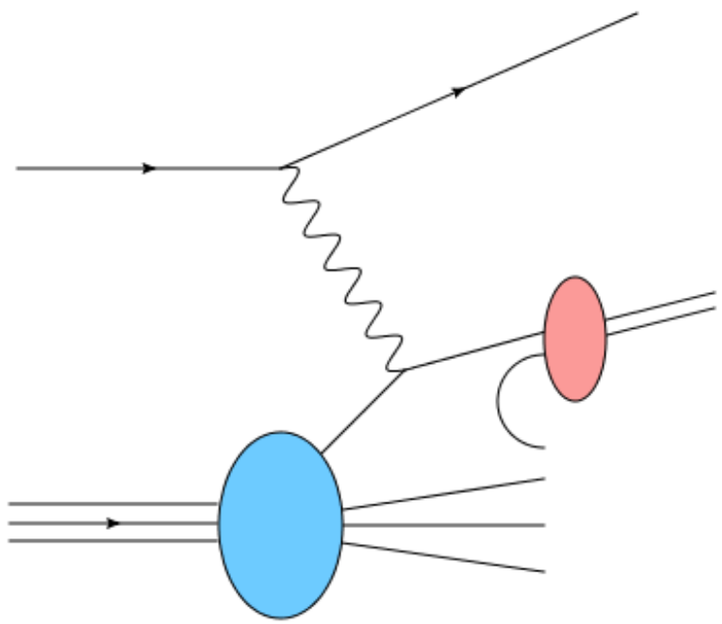
$$z \equiv \frac{P \cdot P_h}{P \cdot q}$$

Important for JLAB12 and EIC

SIDIS

$$\frac{d^3\sigma^h}{dxdydz} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}\mathcal{F}_T^h(x,z,Q^2) + \frac{1-y}{y}\mathcal{F}_L^h(x,z,Q^2)\right]$$

$$\mathcal{F}_i^h(x,z,Q^2) = \sum_{f,f'}\int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\, f\left(\frac{x}{\hat{x}},\mu^2\right) D_{f'}^h\left(\frac{z}{\hat{z}},\mu^2\right) \mathcal{C}_{f'f}^i\left(\hat{x},\hat{z},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)$$

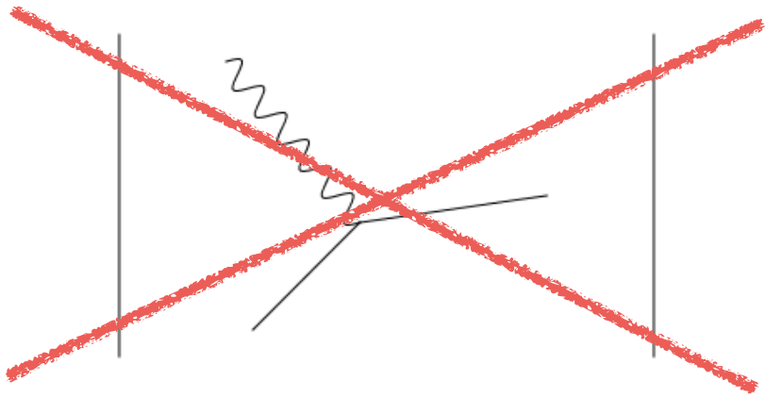


hard-scattering coefficient function:

$$\mathcal{C}_{f'f}^i = C_{f'f}^{i,(0)} + \frac{\alpha_s(\mu^2)}{2\pi}C_{f'f}^{i,(1)} + \mathcal{O}(\alpha_s^2)$$

TOWARDS NNLO F_L

LO:

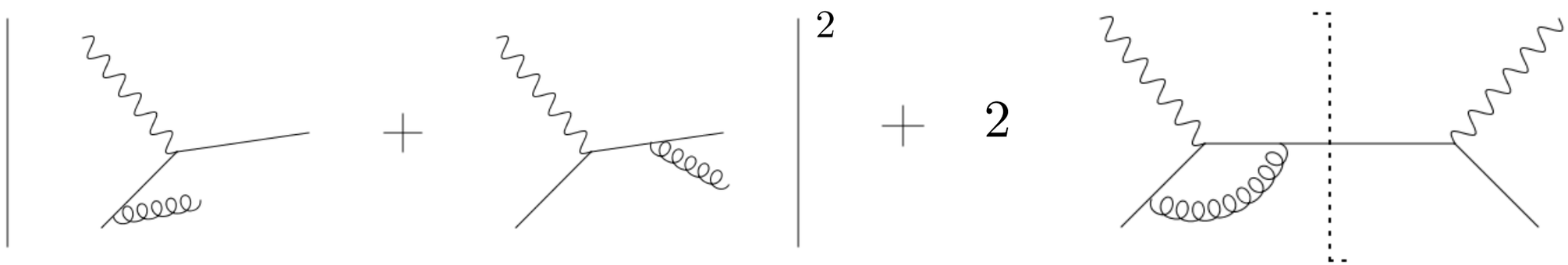


Parton Model:

$$C_{qq}^{T,(0)}(\hat{x},\hat{z}) = e_q^2\,\delta(1-\hat{x})\delta(1-\hat{z})$$

$$C_{qq}^{L,(0)}(\hat{x},\hat{z}) = 0$$

NLO:

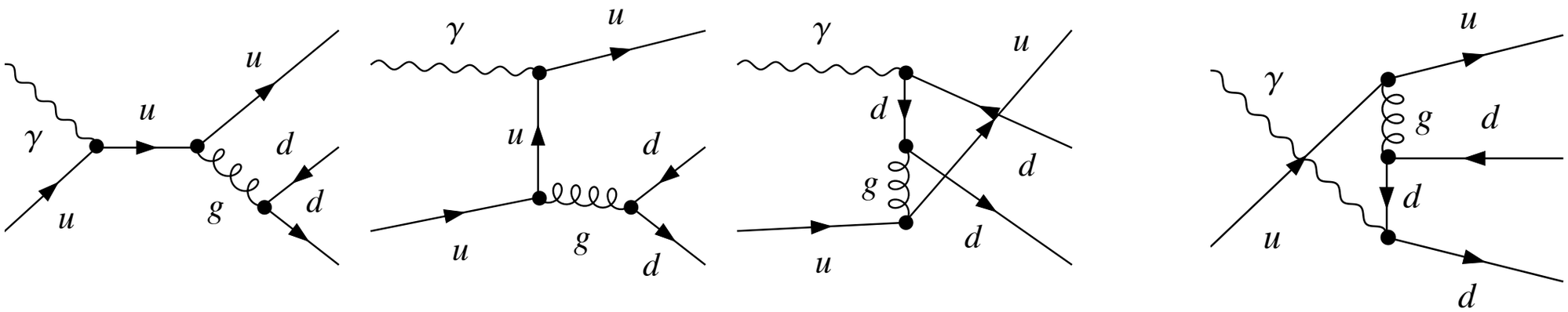


For the Longitudinal Structure Function at NLO, the quark scattering and the gluon-fusion are Tree-Level diagrams

THE NEW CHANNELS OF NNLO F_L

Tree Level diagrams at NNLO:

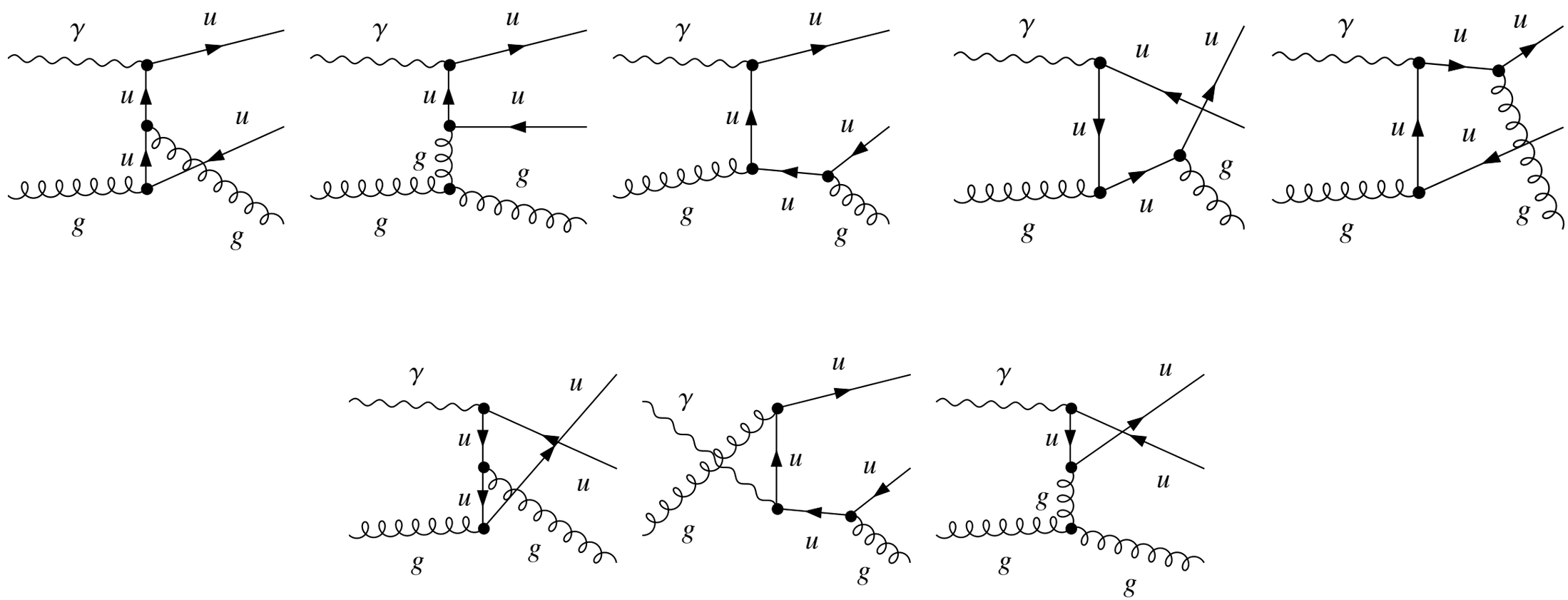
QUARK INITIATED $\gamma q \to q' \bar{q}' q \qquad q \neq q'$



THE NEW CHANNELS OF NNLO F_L

Tree Level diagrams at NNLO:

GLUON INITIATED $\gamma g \to q\bar{q}g \qquad q \neq q'$



THE CALCULATION

It is a BRUTE-FORCE calculation:

PHASE SPACE 2 to 3

$$\int \mathrm{d}\,\mathrm{PS}_3^{\mathrm{DI}} = \frac{1}{(4\pi)^n}\frac{(s-Q^2)^{n-3}}{\Gamma(n-3)}(1-x)^{n-3}\int_0^\pi \mathrm{d}\theta \int_0^\pi \mathrm{d}\phi\,(\sin\theta)^{n-3}(\sin\phi)^{n-4}$$
$$\times \int_0^1 \mathrm{d}y \int_0^1 \mathrm{d}z\, y^{(n/2)-2}(1-y)^{n-3}\{z(1-z)\}^{(n/2)-2}$$

- Angular part solvable using know integrals of type: Beenakker,Kuijf,van Neerven, Smith (Phys.Rev. D40 (1989) 54-82)

$$\int_0^\pi \mathrm{d}\theta \int_0^\pi \mathrm{d}\phi \frac{(\sin\theta)^{n-3}(\sin\phi)^{n-4}}{(a+b\cos\theta)^i(A+B\cos\theta+C\cos\phi\sin\theta)^j}$$

- z-Integration remaining can be solved analytically with many tricks

$$
\begin{aligned}
\mathrm{eq} = {} & -\frac{4}{3} a\, as^2\, CF\, EI^2 \left(-6\left(-2+z+z^2\right)+x\left(\pi^2\,(1+2z)+21\left(-1+z^2\right)\right)\right)+8\, a\, as^2\, CF\, EI^2\, x\left(-2+z+z^2\right)\mathrm{Log}[1-x]-16\, a\, as^2\, CF\, EI^2\, x\left(-2+z+z^2\right)\mathrm{Log}[x]+8\, a\, as^2\, CF\, EI^2\, x\left(-2+z+z^2\right)\mathrm{Log}[1-z]+8\, a\, as^2\, CF\, EI^2\left(-1-2z+x\left(-1+4z+z^2\right)\right)\mathrm{Log}[z]-8\, a\, as^2\, CF\, EI^2\, x\,(1+2z)\,\mathrm{Log}[1-x]\,\mathrm{Log}[z]+ \\
& 16\, a\, as^2\, CF\, EI^2\, x\,(1+2z)\,\mathrm{Log}[x]\,\mathrm{Log}[z]-8\, a\, as^2\, CF\, EI^2\, x\,(1+2z)\,\mathrm{Log}[z]^2+8\, a\, as^2\, CF\, EI^2\, x\,(1+2z)\,\mathrm{PolyLog}[2,z];
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{eqp} = {} & \frac{8\, a\, as^2\, CF\, EII^2\left(4+32x^3-3x^2\left(-7+\pi^2+30z\right)+x\,(-57+90z)\right)}{9x}+\frac{16\, a\, as^2\, CF\, EII^2\left(1-3x+2x^3\right)\mathrm{Log}[1-x]}{3x}-\frac{8\, a\, as^2\, CF\, EII^2\left(4x^4+x^3\,(15-22z)-4z+2x^2\left(9-20z+10z^2\right)+x\left(7-18z+20z^2\right)\right)\mathrm{Log}[x]}{1+x^2+x\,(2-4z)}+ \\
& \frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}16\, a\, as^2\, CF\, EII^2\, x\left(-1-3x^2\,(1-2z)^2+6z-6z^2+x^3\,(-1+2z)+x\left(-3+16z-30z^2+20z^3\right)\right)\mathrm{Log}[2]\,\mathrm{Log}[x]+ \\
& \frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}8\, a\, as^2\, CF\, EII^2\, x\left(-1+6z-6z^2+x^3\,(-1+2z)+2\sqrt{1+2x+x^2-4xz}+x\,(-1+2z)\left(3-10z+10z^2-4\sqrt{1+2x+x^2-4xz}\right)+x^2\left(-3+12z-12z^2+2\sqrt{1+2x+x^2-4xz}\right)\right)\mathrm{Log}[x]^2+\frac{16\, a\, as^2\, CF\, EII^2\left(1-3x+2x^3\right)\mathrm{Log}[1-z]}{3x}-16\, a\, as^2\, CF\, EII^2\, x\,\mathrm{Log}[x]\,\mathrm{Log}[1-z]+ \\
& \frac{8\, a\, as^2\, CF\, EII^2\,(-1+x)\left(-2+4x^4+x^3\,(15-34z)+x\,(3-4z)+2x^2\left(8-29z+30z^2\right)\right)\mathrm{Log}[z]}{3x\left(1+x^2+x\,(2-4z)\right)}+\frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}32\, a\, as^2\, CF\, EII^2\, x\left(-1-3x^2\,(1-2z)^2+6z-6z^2+x^3\,(-1+2z)+x\left(-3+16z-30z^2+20z^3\right)\right)\mathrm{Log}[2]\,\mathrm{Log}[z]- \\
& \frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}8\, a\, as^2\, CF\, EII^2\, x\left(2+x^3\,(2-4z)-12z+12z^2+\sqrt{1+2x+x^2-4xz}-2x\,(-1+2z)\left(3-10z+10z^2+\sqrt{1+2x+x^2-4xz}\right)+x^2\left(6-24z+24z^2+\sqrt{1+2x+x^2-4xz}\right)\right)\mathrm{Log}[x]\,\mathrm{Log}[z]+ \\
& \frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}16\, a\, as^2\, CF\, EII^2\, x\left(-1-3x^2\,(1-2z)^2+6z-6z^2+x^3\,(-1+2z)+x\left(-3+16z-30z^2+20z^3\right)\right)\mathrm{Log}[1-z]\,\mathrm{Log}[z]-\frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}32\, a\, as^2\, CF\, EII^2\, x\left(-1-3x^2\,(1-2z)^2+6z-6z^2+x^3\,(-1+2z)+x\left(-3+16z-30z^2+20z^3\right)\right)\mathrm{Log}[z]\,\mathrm{Log}\left[1-x+\sqrt{(1+x)^2-4xz}\right]- \\
& \frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}16\, a\, as^2\, CF\, EII^2\, x\left(-1-3x^2\,(1-2z)^2+6z-6z^2+x^3\,(-1+2z)+x\left(-3+16z-30z^2+20z^3\right)\right)\mathrm{Log}[x]\,\mathrm{Log}\left[1+x+\sqrt{(1+x)^2-4xz}\right]+16\, a\, as^2\, CF\, EII^2\, x\,\mathrm{PolyLog}[2,x]- \\
& \frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}16\, a\, as^2\, CF\, EII^2\, x\left(-1-3x^2\,(1-2z)^2+6z-6z^2+x^3\,(-1+2z)+x\left(-3+16z-30z^2+20z^3\right)\right)\mathrm{PolyLog}\left[2,\frac{1}{2}\left(1+x-\sqrt{(1+x)^2-4xz}\right)\right]+\frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}8\, a\, as^2\, CF\, EII^2\, x\left(-1-3x^2\,(1-2z)^2+6z-6z^2+x^3\,(-1+2z)+x\left(-3+16z-30z^2+20z^3\right)\right)\mathrm{PolyLog}\left[2,\frac{1+x-\sqrt{(1+x)^2-4xz}}{2x}\right]+ \\
& \frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}8\, a\, as^2\, CF\, EII^2\, x\left(-1-3x^2\,(1-2z)^2+6z-6z^2+x^3\,(-1+2z)+x\left(-3+16z-30z^2+20z^3\right)\right)\mathrm{PolyLog}\left[2,\frac{1+x-\sqrt{(1+x)^2-4xz}}{2z}\right]-\frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}16\, a\, as^2\, CF\, EII^2\, x\left(-1-3x^2\,(1-2z)^2+6z-6z^2+x^3\,(-1+2z)+x\left(-3+16z-30z^2+20z^3\right)\right)\mathrm{PolyLog}\left[2,\frac{2}{1+x+\sqrt{(1+x)^2-4xz}}\right]+ \\
& \frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}8\, a\, as^2\, CF\, EII^2\, x\left(-1-3x^2\,(1-2z)^2+6z-6z^2+x^3\,(-1+2z)+x\left(-3+16z-30z^2+20z^3\right)\right)\mathrm{PolyLog}\left[2,\frac{2x}{1+x+\sqrt{(1+x)^2-4xz}}\right]+\frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}8\, a\, as^2\, CF\, EII^2\, x\left(-1-3x^2\,(1-2z)^2+6z-6z^2+x^3\,(-1+2z)+x\left(-3+16z-30z^2+20z^3\right)\right)\mathrm{PolyLog}\left[2,\frac{2z}{1+x+\sqrt{(1+x)^2-4xz}}\right];
\end{aligned}
$$

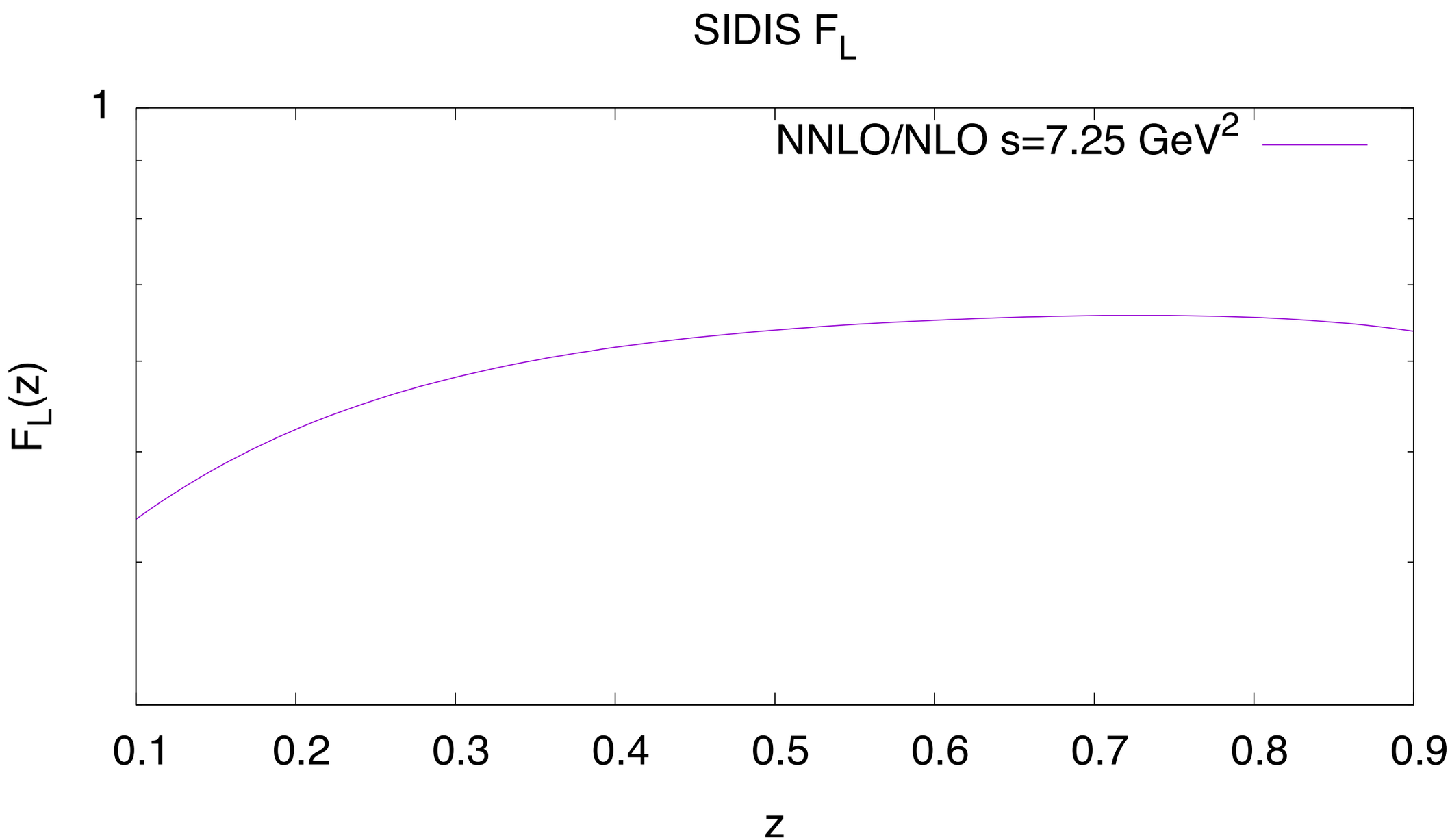
$$
\begin{aligned}
\mathrm{eqeqp} = {} & \frac{8\, a\, as^2\, CF\, EI\, EII}{1+x}-\frac{8\, a\, as^2\, CF\, EI\, EII\, x^2}{1+x}-\frac{8\, a\, as^2\, CF\, EI\, EII\, z}{1+x}-\frac{8\, a\, as^2\, CF\, EI\, EII\, \pi^2\, x\, z}{3\,(1+x)}+\frac{8\, a\, as^2\, CF\, EI\, EII\, x^2\, z}{1+x}-\frac{8\, a\, as^2\, CF\, EI\, EII\, \pi^2\, x^2\, z}{3\,(1+x)}+\frac{16\, a\, as^2\, CF\, EI\, EII\, \pi^2\, x\, z^2}{3\,(1+x)}+\frac{16\, a\, as^2\, CF\, EI\, EII\, \pi^2\, x^2\, z^2}{3\,(1+x)}-32\, a\, as^2\, CF\, EI\, EII\, x\sqrt{1+(-2+4x)\,z+z^2}\,\mathrm{Log}[2]-64\, a\, as^2\, CF\, EI\, EII\, x\, z\,(1+2z)\,\mathrm{Log}[2]^2- \\
& 32\, a\, as^2\, CF\, EI\, EII\, x\, z\,(1+2z)\,\mathrm{Log}[2]\,\mathrm{Log}[1-x]-\frac{a\, as^2\, CF\, EI\, EII\left(16x\sqrt{xz}\sqrt{1+(-2+4x)\,z+z^2}+\pi\left(x-z+3x^2\,z-23x\,z^2\right)\right)\mathrm{Log}[x]}{\sqrt{xz}}- \\
& \frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}16\, a\, as^2\, CF\, EI\, EII\, x\left(1+x^3\,(1-2z)+2z\left(-1+\sqrt{1+2x+x^2-4xz}\right)+z^2\left(2+4\sqrt{1+2x+x^2-4xz}\right)+x^2\left(3+2z\left(-6+\sqrt{1+2x+x^2-4xz}\right)+4z^2\left(3+\sqrt{1+2x+x^2-4xz}\right)\right)-x\,(-1+2z)\left(3+z\left(-6+4\sqrt{1+2x+x^2-4xz}\right)+z^2\left(6+8\sqrt{1+2x+x^2-4xz}\right)\right)\right)\mathrm{Log}[2]\,\mathrm{Log}[x]- \\
& 16\, a\, as^2\, CF\, EI\, EII\, x\left(-1+3z+2z^2\right)\mathrm{Log}[1-x]\,\mathrm{Log}[x]+8\, a\, as^2\, CF\, EI\, EII\, x\left((1-2z)^2+\frac{-1-3x^2\,(1-2z)^2+2z-2z^2+x^3\,(-1+2z)+3x\left(-1+4z-6z^2+4z^3\right)}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}\right)\mathrm{Log}[x]^2+16\, a\, as^2\, CF\, EI\, EII\, x\left(-1+z+2z^2\right)\mathrm{Log}[x]\,\mathrm{Log}[1+x]+2\,i\, a\, as^2\, CF\, EI\, EII\left(\sqrt{\frac{x}{z}}+3x^{3/2}\sqrt{z}-23\sqrt{x}\,z^{3/2}-\sqrt{\frac{z}{x}}\right)\mathrm{Log}[x]\,\mathrm{Log}\left[\sqrt{x}-i\sqrt{z}\right]- \\
& 2\,i\, a\, as^2\, CF\, EI\, EII\left(\sqrt{\frac{x}{z}}+3x^{3/2}\sqrt{z}-23\sqrt{x}\,z^{3/2}-\sqrt{\frac{z}{x}}\right)\mathrm{Log}[x]\,\mathrm{Log}\left[\sqrt{x}+i\sqrt{z}\right]+\left(4\, a\, as^2\, CF\, EI\, EII\left(3x^4\,(-1+z)+(-1+z)\,z+x^3\left(-5+16z-11z^2\right)+x\left(1+4z-9z^2+4z^3\right)+x^2\left(-1+18z-37z^2+20z^3\right)\right)\mathrm{Log}\left[\frac{x}{z}\right]\right)\Big/\left(\left(1+x^2+x\,(2-4z)\right)\left(1-\frac{x}{z}\right)z\right)+ \\
& \left(a\, as^2\, CF\, EI\, EII\left(\pi\left(-z+3x^4\,z+x^3\left(1+6z-35z^2\right)+x\left(1-2z-19z^2\right)+x^2\left(2-2z-46z^2+92z^3\right)\right)-8\sqrt{xz}\left(-z+x^3\left(-3+2\sqrt{1-2z+4xz+z^2}\right)+x\left(-3+6z-4z^2+2\sqrt{1-2z+4xz+z^2}\right)+x^2\left(-6+11z+4\sqrt{1-2z+4xz+z^2}-8z\sqrt{1-2z+4xz+z^2}\right)\right)\right)\mathrm{Log}[z]\right)\Big/\left(\left(1+x^2+x\,(2-4z)\right)\sqrt{xz}\right)- \\
& \frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}32\, a\, as^2\, CF\, EI\, EII\, x\left(1+x^3\,(1-2z)+2z\left(-1+\sqrt{1+2x+x^2-4xz}\right)+z^2\left(2+4\sqrt{1+2x+x^2-4xz}\right)+x^2\left(3+2z\left(-6+\sqrt{1+2x+x^2-4xz}\right)+4z^2\left(3+\sqrt{1+2x+x^2-4xz}\right)\right)-x\,(-1+2z)\left(3+z\left(-6+4\sqrt{1+2x+x^2-4xz}\right)+z^2\left(6+8\sqrt{1+2x+x^2-4xz}\right)\right)\right)\mathrm{Log}[2]\,\mathrm{Log}[x]+ \\
& \frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}16\, a\, as^2\, CF\, EI\, EII\, x\left(-1-3x^2\,(1-2z)^2+2z-2z^2+x^3\,(-1+2z)+3x\left(-1+4z-6z^2+4z^3\right)\right)\mathrm{Log}[x]\,\mathrm{Log}[z]-2\,i\, a\, as^2\, CF\, EI\, EII\left(\sqrt{\frac{x}{z}}+3x^{3/2}\sqrt{z}-23\sqrt{x}\,z^{3/2}-\sqrt{\frac{z}{x}}\right)\mathrm{Log}\left[\sqrt{x}-i\sqrt{z}\right]\mathrm{Log}[z]+2\,i\, a\, as^2\, CF\, EI\, EII\left(\sqrt{\frac{x}{z}}+3x^{3/2}\sqrt{z}-23\sqrt{x}\,z^{3/2}-\sqrt{\frac{z}{x}}\right)\mathrm{Log}\left[\sqrt{x}+i\sqrt{z}\right]\mathrm{Log}[z]+ \\
& \frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}16\, a\, as^2\, CF\, EI\, EII\, x\left(-1-2z^2+x^3\,(-1+2z)-\sqrt{1+2x+x^2-4xz}+2z\left(1+\sqrt{1+2x+x^2-4xz}\right)+x^2\,(-1+2z)\left(3-6z+\sqrt{1+2x+x^2-4xz}\right)+x\,(-1+2z)\left(3+6z^2+2\sqrt{1+2x+x^2-4xz}-2z\left(3+2\sqrt{1+2x+x^2-4xz}\right)\right)\right)\mathrm{Log}[1-z]\,\mathrm{Log}[z]+ \\
& 8\, a\, as^2\, CF\, EI\, EII\, x\,(1-2z)\,\mathrm{Log}[z]^2+32\, a\, as^2\, CF\, EI\, EII\, x\, z\,\mathrm{Log}[x]\,\mathrm{Log}[x+z]-32\, a\, as^2\, CF\, EI\, EII\, x\, z\,\mathrm{Log}[z]\,\mathrm{Log}[x+z]-64\, a\, as^2\, CF\, EI\, EII\, x\, z^2\,\mathrm{Log}[x]\,\mathrm{Log}[1+x\,z]-64\, a\, as^2\, CF\, EI\, EII\, x\, z^2\,\mathrm{Log}[z]\,\mathrm{Log}[1+x\,z]-2\,i\, a\, as^2\, CF\, EI\, EII\left(\sqrt{\frac{x}{z}}+3x^{3/2}\sqrt{z}-23\sqrt{x}\,z^{3/2}-\sqrt{\frac{z}{x}}\right)\mathrm{Log}[x]\,\mathrm{Log}\left[-i+\sqrt{x\,z}\right]- \\
& 2\,i\, a\, as^2\, CF\, EI\, EII\left(\sqrt{\frac{x}{z}}+3x^{3/2}\sqrt{z}-23\sqrt{x}\,z^{3/2}-\sqrt{\frac{z}{x}}\right)\mathrm{Log}[z]\,\mathrm{Log}\left[-i+\sqrt{x\,z}\right]+2\,i\, a\, as^2\, CF\, EI\, EII\left(\sqrt{\frac{x}{z}}+3x^{3/2}\sqrt{z}-23\sqrt{x}\,z^{3/2}-\sqrt{\frac{z}{x}}\right)\mathrm{Log}[x]\,\mathrm{Log}\left[i+\sqrt{x\,z}\right]+2\,i\, a\, as^2\, CF\, EI\, EII\left(\sqrt{\frac{x}{z}}+3x^{3/2}\sqrt{z}-23\sqrt{x}\,z^{3/2}-\sqrt{\frac{z}{x}}\right)\mathrm{Log}[z]\,\mathrm{Log}\left[i+\sqrt{x\,z}\right]- \\
& \frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}32\, a\, as^2\, CF\, EI\, EII\, x\left(-1-3x^2\,(1-2z)^2+2z-2z^2+x^3\,(-1+2z)+3x\left(-1+4z-6z^2+4z^3\right)\right)\mathrm{Log}[z]\,\mathrm{Log}\left[1-x+\sqrt{(1+x)^2-4xz}\right]-\frac{1}{\left(1+x^2+x\,(2-4z)\right)^{3/2}}16\, a\, as^2\, CF\, EI\, EII\, x\left(-1-3x^2\,(1-2z)^2+2z-2z^2+x^3\,(-1+2z)+3x\left(-1+4z-6z^2+4z^3\right)\right)\mathrm{Log}[x]\,\mathrm{Log}\left[1+x+\sqrt{(1+x)^2-4xz}\right]+ \\
& 32\, a\, as^2\, CF\, EI\, EII\, x\sqrt{1+(-2+4x)\,z+z^2}\,\mathrm{Log}\left[1-z+\sqrt{(1-z)^2+4xz}\right]+64\, a\, as^2\, CF\, EI\, EII\, x\, z\,(1+2z)\,\mathrm{Log}[2]\,\mathrm{Log}\left[1-z+\sqrt{(1-z)^2+4xz}\right]+32\, a\, as^2\, CF\, EI\, EII\, x\, z\,(1+2z)\,\mathrm{Log}[1-x]\,\mathrm{Log}\left[1-z+\sqrt{(1-z)^2+4xz}\right]+32\, a\, as^2\, CF\, EI\, EII\, x\, z\,(1+2z)\,\mathrm{Log}[z]\,\mathrm{Log}\left[1-z+\sqrt{(1-z)^2+4xz}\right]+ \\
& 64\, a\, as^2\, CF\, EI\, EII\, x\, z\,(1+2z)\,\mathrm{Log}[2]\,\mathrm{Log}\left[1+z+\sqrt{(1-z)^2+4xz}\right]+32\, a\, as^2\, CF\, EI\, EII\, x\, z\,(1+2z)\,\mathrm{Log}[x]\,\mathrm{Log}\left[1+z+\sqrt{(1-z)^2+4xz}\right]+32\, a\, as^2\, CF\, EI\, EII\, x\, z\,(1+2z)\,\mathrm{Log}[z]\,\mathrm{Log}\left[1+z+\sqrt{(1-z)^2+4xz}\right]-64\, a\, as^2\, CF\, EI\, EII\, x\, z\,(1+2z)\,\mathrm{Log}\left[1-z+\sqrt{(1-z)^2+4xz}\right]\mathrm{Log}\left[1+z+\sqrt{(1-z)^2+4xz}\right]-
\end{aligned}
$$

50%

$$
\begin{aligned}
&32\, a\, as^2\, CA\, EI^2\, x\, (1+x)\, \mathrm{Log}[x]\, \mathrm{Log}[1+x] + \\
&\frac{1}{32\, x^3\, z^5}\, a\, as^2\, CA\, EI^2 \Big(\pi\, x^{3/2} \Big(15\, z^{9/2} + 15\, x^4\, z^{9/2} + 128\, x^{7/2}\, z^2 \sqrt{x\, z^5} + 16 \sqrt{x}\, z \sqrt{x\, z^5}\, (5 - 15\, z + 7\, z^2) - 30\, x\, z^{7/2} (11 - 24\, z + 11\, z^2) - 30\, x^3\, z^{7/2} (11 - 24\, z + 11\, z^2) - 16\, x^{5/2}\, z \sqrt{x\, z^5}\, (-13 + 15\, z + 17\, z^2) + 16\, x^{3/2} \sqrt{x\, z^5}\, (15 - 45\, z + 64\, z^2 - 45\, z^3 + 13\, z^4) - 5\, x^2\, z^{5/2} (125 - 368\, z + 468\, z^2 - 368\, z^3 + 125\, z^4)\Big) + \\
&\quad 4\, (-1+x)\, x^2 \Big(-15\, z^4 - 15\, z^5 + 240 \sqrt{x\, z}\, \sqrt{x\, z^5} - 800\, z \sqrt{x\, z}\, \sqrt{x\, z^5} + 256\, z^2 \sqrt{x\, z}\, \sqrt{x\, z^5} + 1168\, z^3 \sqrt{x\, z}\, \sqrt{x\, z^5} - 128\, x^3\, z^4\, (1+z) + x\, z\, (1+z) \Big(735\, z^2 + 1020\, z^3 - 225\, z^4 + 64 \sqrt{x\, z}\, \sqrt{x\, z^5}\Big) + x^2\, z \Big(49\, z^3 + 305\, z^4 + 128 \sqrt{x\, z}\, \sqrt{x\, z^5} + 128\, z \sqrt{x\, z}\, \sqrt{x\, z^5}\Big)\Big)\Big)\, \mathrm{Log}[z] - \\
&32\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{Log}[1-x]\, \mathrm{Log}[z] - 24\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{Log}[z]^2 - \frac{8\, a\, as^2\, CA\, EI^2\, (-1+z) \big(10\, (-1+z)^2 + 3\, x^2\, z + x\, (10 - 17\, z + 4\, z^2)\big)\, \mathrm{Log}[x+z]}{z^2} - 32\, a\, as^2\, CA\, EI^2\, x^2\, \mathrm{Log}[x]\, \mathrm{Log}[x+z] + 32\, a\, as^2\, CA\, EI^2\, x^2\, \mathrm{Log}[z]\, \mathrm{Log}[x+z] + \frac{8\, a\, as^2\, CA\, EI^2\, (-1+z) \big(10\, (-1+z)^2 + 3\, x^2\, z + x\, (10 - 17\, z + 4\, z^2)\big)\, \mathrm{Log}[1+x\, z]}{z^2} - \\
&32\, a\, as^2\, CA\, EI^2\, x\, \mathrm{Log}[x]\, \mathrm{Log}[1+x\, z] - 32\, a\, as^2\, CA\, EI^2\, x\, \mathrm{Log}[z]\, \mathrm{Log}[1+x\, z] + \frac{1}{1792\, x^{3/2}\, z^{5/2}}\, a\, as^2\, CA\, EI^2 \big(-2161\, z^2 - 2161\, x^4\, z^2 + 2\, x\, z\, (5883 - 16088\, z + 5883\, z^2) + 2\, x^3\, z\, (5883 - 16088\, z + 5883\, z^2) + x^2\, (30415 - 98128\, z + 122460\, z^2 - 98128\, z^3 + 30415\, z^4)\big)\, \mathrm{Log}\big[1 + x\, z - 2 \sqrt{x\, z}\big] + \\
&\frac{1}{1792\, x^{3/2}\, z^{5/2}}\, a\, as^2\, CA\, EI^2 \big(2161\, z^2 + 2161\, x^4\, z^2 - 2\, x\, z\, (5883 - 16088\, z + 5883\, z^2) - 2\, x^3\, z\, (5883 - 16088\, z + 5883\, z^2) + x^2\, (-30415 + 98128\, z - 122460\, z^2 + 98128\, z^3 - 30415\, z^4)\big)\, \mathrm{Log}\big[1 - \sqrt{x\, z}\big] + \frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[2]\, \mathrm{Log}\big[i\, x - \sqrt{x\, z}\big] + \\
&\frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[x]\, \mathrm{Log}\big[i\, x - \sqrt{x\, z}\big] + \frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[2]\, \mathrm{Log}\big[i\, z - \sqrt{x\, z}\big] + \\
&\frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[z]\, \mathrm{Log}\big[i\, z - \sqrt{x\, z}\big] - \frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[2]\, \mathrm{Log}\big[-i + \sqrt{x\, z}\big] + \\
&\frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[2]\, \mathrm{Log}\big[i + \sqrt{x\, z}\big] + \frac{1}{1792\, x^{3/2}\, z^{5/2}}\, a\, as^2\, CA\, EI^2 \big(-2161\, z^2 - 2161\, x^4\, z^2 + 2\, x\, z\, (5883 - 16088\, z + 5883\, z^2) + 2\, x^3\, z\, (5883 - 16088\, z + 5883\, z^2) + x^2\, (30415 - 98128\, z + 122460\, z^2 - 98128\, z^3 + 30415\, z^4)\big)\, \mathrm{Log}\big[1 + \sqrt{x\, z}\big] - \\
&\frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[2]\, \mathrm{Log}\big[i\, x + \sqrt{x\, z}\big] - \frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[x]\, \mathrm{Log}\big[i\, x + \sqrt{x\, z}\big] - \\
&\frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[2]\, \mathrm{Log}\big[i\, z + \sqrt{x\, z}\big] - \frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[z]\, \mathrm{Log}\big[i\, z + \sqrt{x\, z}\big] + \\
&\frac{1}{1792\, x^{3/2}\, z^{5/2}}\, a\, as^2\, CA\, EI^2 \big(2161\, z^2 + 2161\, x^4\, z^2 - 2\, x\, z\, (5883 - 16088\, z + 5883\, z^2) - 2\, x^3\, z\, (5883 - 16088\, z + 5883\, z^2) + x^2\, (-30415 + 98128\, z - 122460\, z^2 + 98128\, z^3 - 30415\, z^4)\big)\, \mathrm{Log}\big[-x\, z + \sqrt{x\, z}\big] - \frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[2]\, \mathrm{Log}\big[-i\, x\, z + \sqrt{x\, z}\big] - \\
&\frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[x]\, \mathrm{Log}\big[-i\, x\, z + \sqrt{x\, z}\big] - \frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[z]\, \mathrm{Log}\big[-i\, x\, z + \sqrt{x\, z}\big] + \\
&\frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[2]\, \mathrm{Log}\big[i\, x\, z + \sqrt{x\, z}\big] + \frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[x]\, \mathrm{Log}\big[i\, x\, z + \sqrt{x\, z}\big] + \\
&\frac{1}{16\, x^{3/2}\, z^{5/2}}\, i\, a\, as^2\, CA\, EI^2 \big(15\, z^2 + 143\, x^4\, z^2 + 2\, x\, z\, (-5 + 11\, z^2) + x^3\, (118\, z - 362\, z^3) + x^2\, (15 + 124\, z^2 - 17\, z^4)\big)\, \mathrm{Log}[z]\, \mathrm{Log}\big[i\, x\, z + \sqrt{x\, z}\big] + \frac{1}{1792\, x^{3/2}\, z^{5/2}}\, a\, as^2\, CA\, EI^2 \big(-2161\, z^2 - 2161\, x^4\, z^2 + 2\, x\, z\, (5883 - 16088\, z + 5883\, z^2) + 2\, x^3\, z\, (5883 - 16088\, z + 5883\, z^2) + x^2\, (30415 - 98128\, z + 122460\, z^2 - 98128\, z^3 + 30415\, z^4)\big)\, \mathrm{Log}\big[x\, z + \sqrt{x\, z}\big] + \\
&\frac{1}{1792\, x^{3/2}\, z^{5/2}}\, a\, as^2\, CA\, EI^2 \big(2161\, z^2 + 2161\, x^4\, z^2 - 2\, x\, z\, (5883 - 16088\, z + 5883\, z^2) - 2\, x^3\, z\, (5883 - 16088\, z + 5883\, z^2) + x^2\, (-30415 + 98128\, z - 122460\, z^2 + 98128\, z^3 - 30415\, z^4)\big)\, \mathrm{Log}\big[1 + x\, z + 2 \sqrt{x\, z}\big] - \frac{16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x \sqrt{1 + (-2+4\, x)\, z + z^2}\, \mathrm{Log}\big[1 - 2\, x - z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big]}{z} - \\
&16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{Log}\big[1 + z - \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big]\, \mathrm{Log}\big[1 - 2\, x - z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big] + 16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{Log}\big[1 - 2\, x - z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big]\, \mathrm{Log}\big[1 + z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big] + \frac{16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x \sqrt{1 + (-2+4\, x)\, z + z^2}\, \mathrm{Log}\big[-1 + 2\, x + z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big]}{z} + \\
&16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{Log}\big[1 + z - \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big]\, \mathrm{Log}\big[-1 + 2\, x + z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big] - 16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{Log}\big[1 + z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big]\, \mathrm{Log}\big[-1 + 2\, x + z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big] + \frac{16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x \sqrt{1 + (-2+4\, x)\, z + z^2}\, \mathrm{Log}\big[-1 + z - 2\, x\, z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big]}{z} + \\
&16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{Log}\big[1 + z - \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big]\, \mathrm{Log}\big[-1 + z - 2\, x\, z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big] - 16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{Log}\big[1 + z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big]\, \mathrm{Log}\big[-1 + z - 2\, x\, z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big] - \frac{16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x \sqrt{1 + (-2+4\, x)\, z + z^2}\, \mathrm{Log}\big[1 - z + 2\, x\, z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big]}{z} - \\
&16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{Log}\big[1 + z - \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big]\, \mathrm{Log}\big[1 - z + 2\, x\, z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big] + 16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{Log}\big[1 + z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big]\, \mathrm{Log}\big[1 - z + 2\, x\, z + \sqrt{1 - 2\, z + 4\, x\, z + z^2}\big] + 16\, a\, as^2\, CA\, EI^2\, x\, (1+x)\, \mathrm{PolyLog}\Big[2, \frac{1-z}{1+x}\Big] + 16\, a\, as^2\, CA\, EI^2\, x\, (1+x)\, \mathrm{PolyLog}\Big[2, \frac{-1+z}{(1+x)\, z}\Big] - \\
&16\, a\, as^2\, CA\, EI^2\, x\, (1+x)\, \mathrm{PolyLog}\Big[2, \frac{x\, (-1+z)}{(1+x)\, z}\Big] - \frac{5\, i\, a\, as^2\, CA\, EI^2\, (-1+z)^2 \big(3\, z + 3\, x^2\, z + x\, (5 - 4\, z + 5\, z^2)\big)\, \mathrm{PolyLog}\big[2, -\frac{i}{\sqrt{x\, z}}\big]}{\sqrt{x\, z^5}} + \frac{5\, i\, a\, as^2\, CA\, EI^2\, (-1+z)^2 \big(3\, z + 3\, x^2\, z + x\, (5 - 4\, z + 5\, z^2)\big)\, \mathrm{PolyLog}\big[2, \frac{i}{\sqrt{x\, z}}\big]}{\sqrt{x\, z^5}} - \\
&\frac{1}{16\, x^{3/2}\, z^5}\, i\, a\, as^2\, CA\, EI^2 \Big(15\, z^{9/2} + 15\, x^4\, z^{9/2} + 128\, x^{7/2}\, z^2 \sqrt{x\, z^5} - 30\, x\, z^{7/2} (3 - 8\, z + 3\, z^2) - 30\, x^3\, z^{7/2} (3 - 8\, z + 3\, z^2) + 16 \sqrt{x}\, z \sqrt{x\, z^5}\, (5 - 15\, z + 7\, z^2) - 16\, x^{5/2}\, z \sqrt{x\, z^5}\, (-13 + 15\, z + 17\, z^2) - 45\, x^2\, z^{5/2} (5 - 16\, z + 20\, z^2 - 16\, z^3 + 5\, z^4) + 16\, x^{3/2} \sqrt{x\, z^5}\, (15 - 45\, z + 64\, z^2 - 45\, z^3 + 13\, z^4)\Big)\, \mathrm{PolyLog}\Big[2, -\frac{i\, x}{\sqrt{x\, z}}\Big] + \\
&\frac{1}{16\, x^{3/2}\, z^5}\, i\, a\, as^2\, CA\, EI^2 \Big(15\, z^{9/2} + 15\, x^4\, z^{9/2} + 128\, x^{7/2}\, z^2 \sqrt{x\, z^5} - 30\, x\, z^{7/2} (3 - 8\, z + 3\, z^2) - 30\, x^3\, z^{7/2} (3 - 8\, z + 3\, z^2) + 16 \sqrt{x}\, z \sqrt{x\, z^5}\, (5 - 15\, z + 7\, z^2) - 16\, x^{5/2}\, z \sqrt{x\, z^5}\, (-13 + 15\, z + 17\, z^2) - 45\, x^2\, z^{5/2} (5 - 16\, z + 20\, z^2 - 16\, z^3 + 5\, z^4) + 16\, x^{3/2} \sqrt{x\, z^5}\, (15 - 45\, z + 64\, z^2 - 45\, z^3 + 13\, z^4)\Big)\, \mathrm{PolyLog}\Big[2, \frac{i\, x}{\sqrt{x\, z}}\Big] - \\
&\frac{1}{16\, x^{3/2}\, z^5}\, i\, a\, as^2\, CA\, EI^2 \Big(15\, z^{9/2} + 15\, x^4\, z^{9/2} + 128\, x^{7/2}\, z^2 \sqrt{x\, z^5} - 30\, x\, z^{7/2} (3 - 8\, z + 3\, z^2) - 30\, x^3\, z^{7/2} (3 - 8\, z + 3\, z^2) + 16 \sqrt{x}\, z \sqrt{x\, z^5}\, (5 - 15\, z + 7\, z^2) - 16\, x^{5/2}\, z \sqrt{x\, z^5}\, (-13 + 15\, z + 17\, z^2) - 45\, x^2\, z^{5/2} (5 - 16\, z + 20\, z^2 - 16\, z^3 + 5\, z^4) + 16\, x^{3/2} \sqrt{x\, z^5}\, (15 - 45\, z + 64\, z^2 - 45\, z^3 + 13\, z^4)\Big)\, \mathrm{PolyLog}\Big[2, -\frac{i\, z}{\sqrt{x\, z}}\Big] + \\
&\frac{1}{16\, x^{3/2}\, z^5}\, i\, a\, as^2\, CA\, EI^2 \Big(15\, z^{9/2} + 15\, x^4\, z^{9/2} + 128\, x^{7/2}\, z^2 \sqrt{x\, z^5} - 30\, x\, z^{7/2} (3 - 8\, z + 3\, z^2) - 30\, x^3\, z^{7/2} (3 - 8\, z + 3\, z^2) + 16 \sqrt{x}\, z \sqrt{x\, z^5}\, (5 - 15\, z + 7\, z^2) - 16\, x^{5/2}\, z \sqrt{x\, z^5}\, (-13 + 15\, z + 17\, z^2) - 45\, x^2\, z^{5/2} (5 - 16\, z + 20\, z^2 - 16\, z^3 + 5\, z^4) + 16\, x^{3/2} \sqrt{x\, z^5}\, (15 - 45\, z + 64\, z^2 - 45\, z^3 + 13\, z^4)\Big)\, \mathrm{PolyLog}\Big[2, \frac{i\, z}{\sqrt{x\, z}}\Big] + \\
&\frac{1}{8\, x^{3/2}\, z^5}\, i\, a\, as^2\, CA\, EI^2 \Big(15\, z^{9/2} + 15\, x^4\, z^{9/2} + 128\, x^{7/2}\, z^2 \sqrt{x\, z^5} + 30\, x\, z^{7/2} (1 + z^2) + 30\, x^3\, z^{7/2} (1 + z^2) + 16 \sqrt{x}\, z \sqrt{x\, z^5}\, (5 - 15\, z + 7\, z^2) - 16\, x^{5/2}\, z \sqrt{x\, z^5}\, (-13 + 15\, z + 17\, z^2) - 5\, x^2\, z^{5/2} (5 - 32\, z + 36\, z^2 - 32\, z^3 + 5\, z^4) + 16\, x^{3/2} \sqrt{x\, z^5}\, (15 - 45\, z + 64\, z^2 - 45\, z^3 + 13\, z^4)\Big)\, \mathrm{PolyLog}\big[2, -i \sqrt{x\, z}\big] - \\
&\frac{1}{8\, x^{3/2}\, z^5}\, i\, a\, as^2\, CA\, EI^2 \Big(15\, z^{9/2} + 15\, x^4\, z^{9/2} + 128\, x^{7/2}\, z^2 \sqrt{x\, z^5} + 30\, x\, z^{7/2} (1 + z^2) + 30\, x^3\, z^{7/2} (1 + z^2) + 16 \sqrt{x}\, z \sqrt{x\, z^5}\, (5 - 15\, z + 7\, z^2) - 16\, x^{5/2}\, z \sqrt{x\, z^5}\, (-13 + 15\, z + 17\, z^2) - 5\, x^2\, z^{5/2} (5 - 32\, z + 36\, z^2 - 32\, z^3 + 5\, z^4) + 16\, x^{3/2} \sqrt{x\, z^5}\, (15 - 45\, z + 64\, z^2 - 45\, z^3 + 13\, z^4)\Big)\, \mathrm{PolyLog}\big[2, i \sqrt{x\, z}\big] - \\
&16\, a\, as^2\, CA\, EI^2\, x\, (1+x)\, \mathrm{PolyLog}\Big[2, \frac{x - x\, z}{1+x}\Big] - 16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{PolyLog}\Big[2, \frac{1}{2} \Big(1 - z - \sqrt{1 + z\, (-2 + 4\, x + z)}\Big)\Big] + 16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{PolyLog}\Big[2, \frac{-1 + z - \sqrt{1 + z\, (-2 + 4\, x + z)}}{2\, z}\Big] - 16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{PolyLog}\Big[2, \frac{1}{2} \Big(1 - z + \sqrt{1 + z\, (-2 + 4\, x + z)}\Big)\Big] + \\
&16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{PolyLog}\Big[2, \frac{-1 + z + \sqrt{1 + z\, (-2 + 4\, x + z)}}{2\, z}\Big] - 16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{PolyLog}\Big[2, \frac{1 - 2\, x - z + \sqrt{1 + z\, (-2 + 4\, x + z)}}{1 + z + \sqrt{1 + z\, (-2 + 4\, x + z)}}\Big] - 16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{PolyLog}\Big[2, \frac{-1 + 2\, x + z + \sqrt{1 + z\, (-2 + 4\, x + z)}}{-1 - z + \sqrt{1 + z\, (-2 + 4\, x + z)}}\Big] + 16\, a\, as^2\, CA\, EI^2\, (-1+x)\, x\, \mathrm{PolyLog}\Big[2, \frac{-1 + z - 2\, x\, z + \sqrt{1 + z\, (-2 + 4\, x + z)}}{1 + z + \sqrt{1 + z\, (-2 + 4\, x + z)}}\Big] + \\
&\quad 1 + (-1 + 2\, x)\, z + \sqrt{1 + z\, (-2 + 4\, x + z)}
\end{aligned}
$$

50%

PRELIMINARY PLOT



Using MSTW PDF and DSS FF for HERMES Kinematics

CONCLUSIONS & OUTLOOK

- We have presented a framework for combined HMC with Resummation. Future extension to SIDIS
- Work in progress for e+e- only FF NNLO fit and extension to a global fit
- Future resummed FF fit including Log(N)/N
- Work in progress for NNLO SIDIS

Thanks for Your Attention

Any questions?