Flavor Asymmetry of the Proton Sea in Chiral Effective Theory

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…

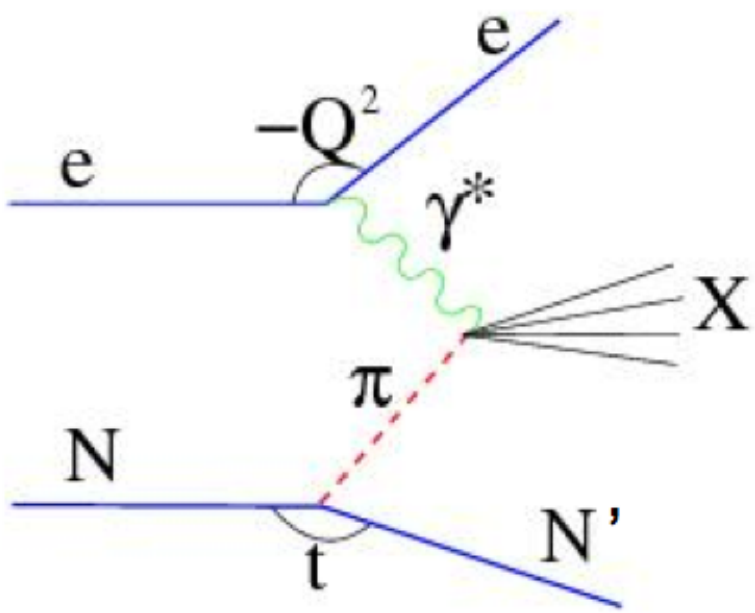
Y. Salamu, C. Ji, W. Melnitchouk, P. Wang, PRL 114, 122001 (2015)

Jefferson Lab

July 6, 2015

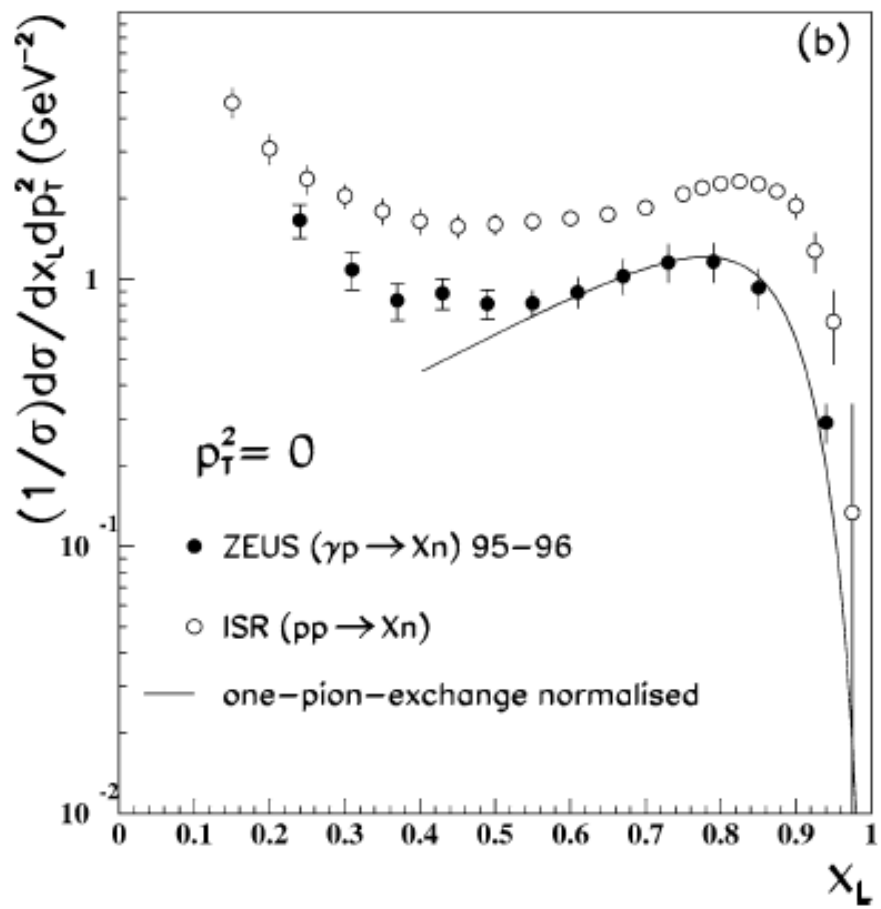
Measurement of Tagged Deep Inelastic Scattering (TDIS)

C.Keppel (Contact person)



$$e + p(or\ n) \rightarrow e' + p + X$$

$$e + D \rightarrow e' + p + p + X$$



Leading neutron production in e^+p collisions at HERA

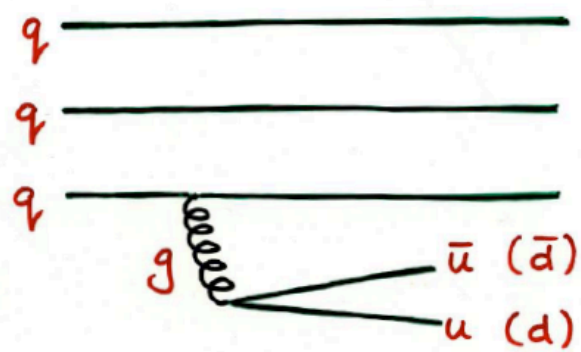
ZEUS Collaboration, NPB 637 (2002) 3–56

Outline

- Motivation: Flavor Asymmtery in Proton Sea
- Self-Energy: Treacherous Point
- Vertex Correction: “u-bar” – “d-bar”
- Summary and Works in progress

Flavor asymmetry

- Antiquarks in the proton "sea" produced predominantly by gluon radiation into quark-antiquark pairs, $g \to q\bar{q}$



→ since u and d quark masses are similar, expect flavor-symmetric sea, $\bar{d} \approx \bar{u}$

- Experimentally, one finds *large excess* of $\bar{d}$ over $\bar{u}$

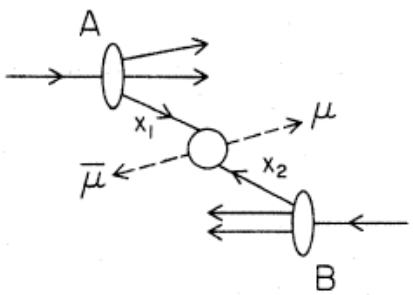
$$\int_0^1 dx\ (\bar{d}(x) - \bar{u}(x)) = 0.118 \pm 0.012$$

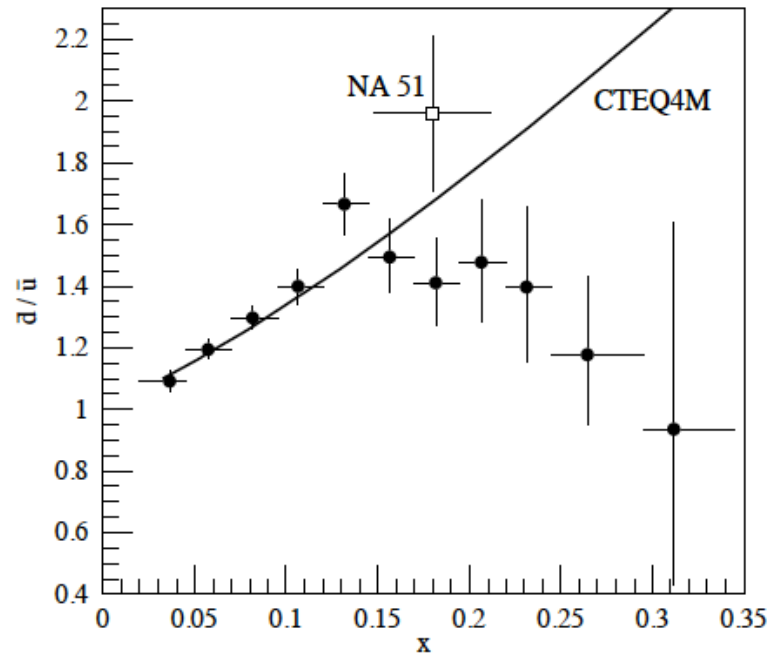
E866 (Fermilab), PRD ***64****, 052002 (2001)*

Flavor asymmetry

- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

→ Drell-Yan process





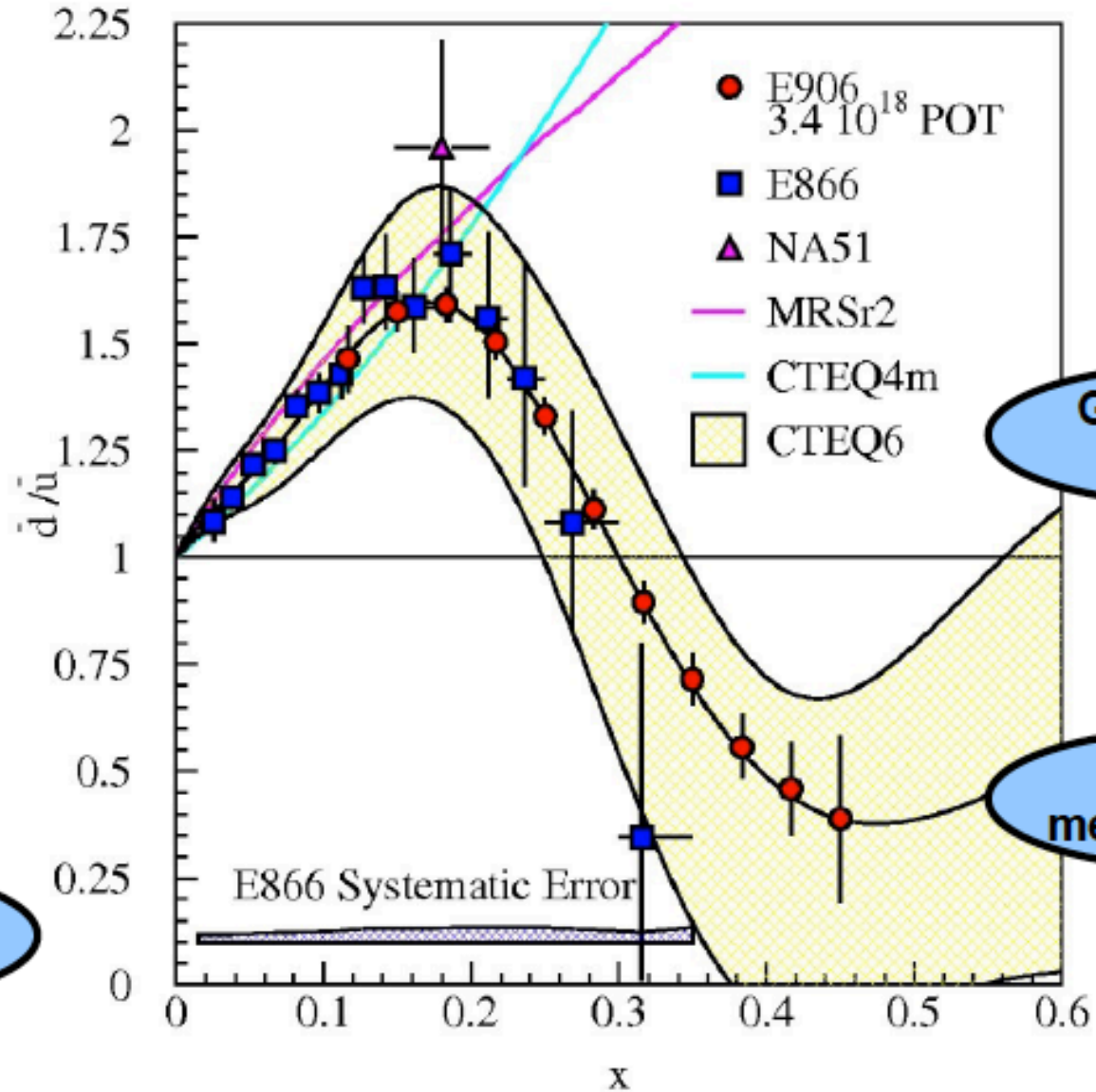
*Fermilab E866, PRL **80**, 3715 (1998)*

$$\frac{d^2\sigma}{dx_b dx_t} = \frac{4\pi\alpha^2}{9Q^2}\sum_q e_q^2\left(q(x_b)\bar{q}(x_t) + \bar{q}(x_b)q(x_t)\right)$$

→ for $x_b \gg x_t$

$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2}\left(1 + \frac{\bar{d}(x_t)}{\bar{u}(x_t)}\right) \quad \rightarrow \quad \int_0^1 dx\,(\bar{d} - \bar{u}) = 0.118 \pm 0.012$$

SeaQuest probing the proton sea



Global NLO PDF fit

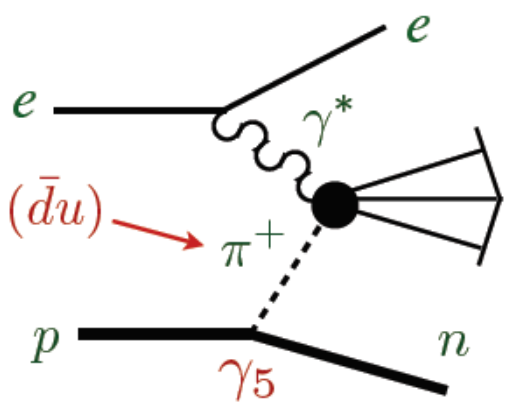
extend measurement

SeaQuest: Syst. ~ 1%

Flavor asymmetry

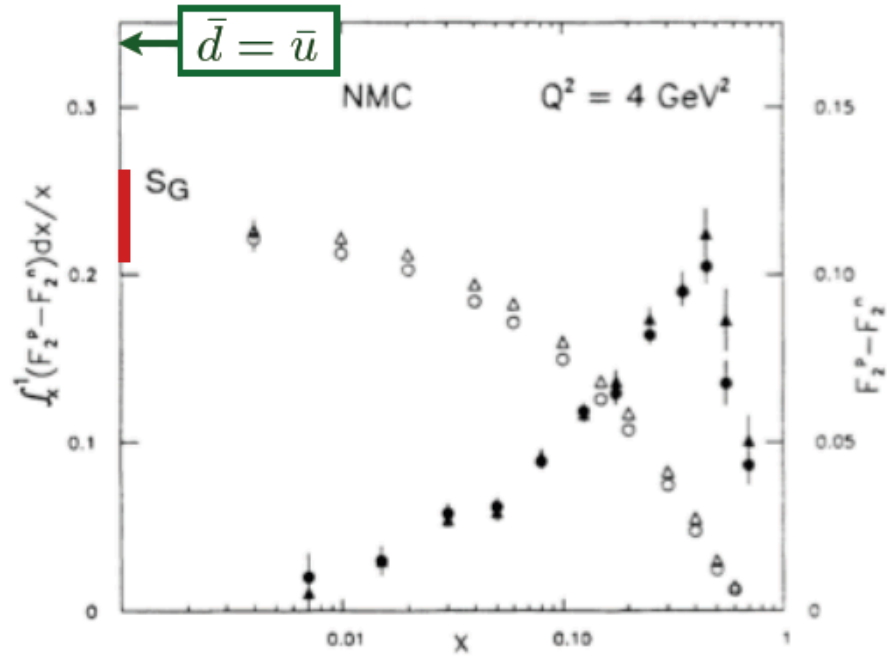
- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

→ Sullivan process



Sullivan, PRD ***5****, 1732 (1972)*
Thomas, PLB ***126****, 97 (1983)*

$$\boxed{\bar{d} > \bar{u}}$$



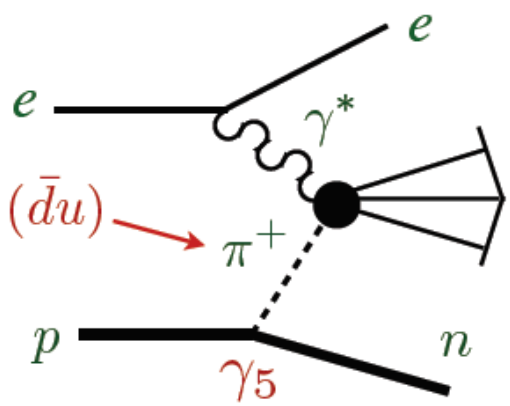
$$\int_0^1 \frac{dx}{x}(F_2^p - F_2^n) = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\,(\bar{d} - \bar{u})$$
$$= 0.235(26)$$

New Muon Collaboration, PRD ***50****, 1 (1994)*

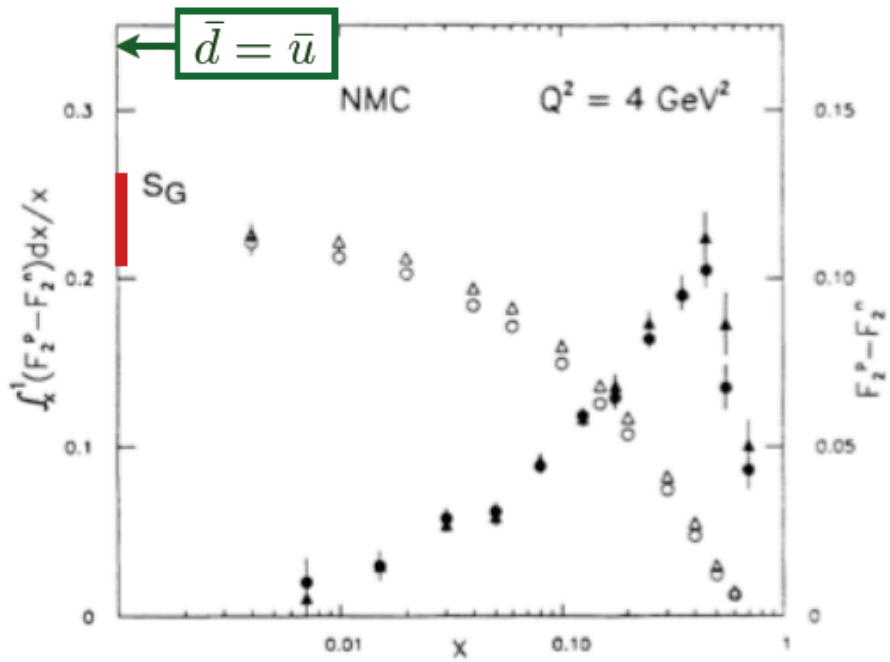
Flavor asymmetry

- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

→ Sullivan process



Sullivan, PRD ***5****, 1732 (1972)*
Thomas, PLB ***126****, 97 (1983)*



$$(\bar{d} - \bar{u})(x) = \frac{2}{3} \int_x^1 \frac{dy}{y} f_\pi(y)\, \bar{q}^\pi(x/y)$$

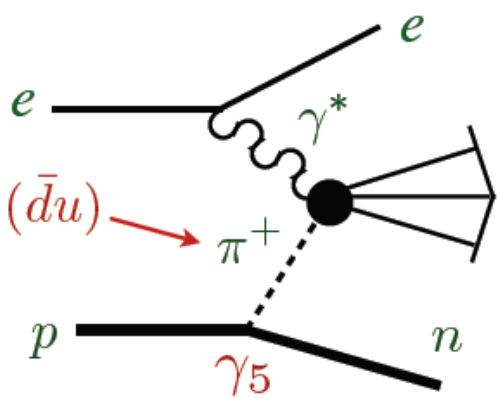
$$f_\pi(y) = \frac{3 g_{\pi NN}^2}{16\pi^2}\, y \int dt \frac{-t\ \mathcal{F}_{\pi NN}^2(t)}{(t - m_\pi^2)^2}$$

pion light-cone momentum distribution in nucleon

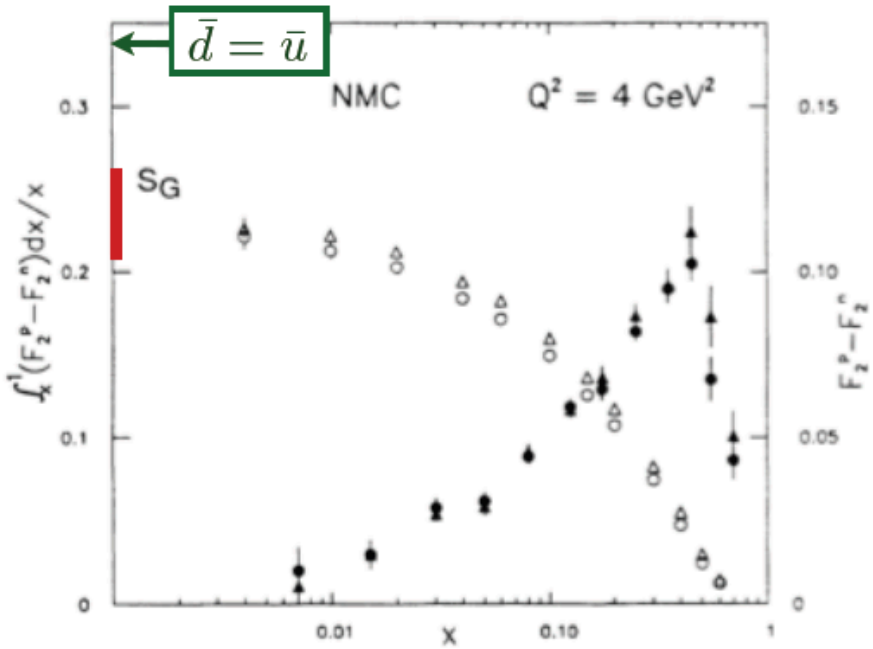
Flavor asymmetry

- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

→ Sullivan process



Sullivan, PRD **5**, *1732 (1972)*
Thomas, PLB **126**, *97 (1983)*



$$(\bar{d} - \bar{u})(x) = \frac{2}{3} \int_x^1 \frac{dy}{y} f_\pi(y)\, \bar{q}^\pi(x/y)$$

$$f_\pi(y) = \frac{3 g_{\pi NN}^2}{16\pi^2}\, y \int dt \frac{-t\, \mathcal{F}_{\pi NN}^2(t)}{(t - m_\pi^2)^2}$$

connection to QCD?

Connection with QCD

- $(\bar{d}-\bar{u})(x) = \frac{2}{3}\int_x^1 \frac{dy}{y} f_\pi(y)\ \bar{q}^\pi(x/y)$ $\quad f_\pi(y) = \frac{3g^2_{\pi NN}}{16\pi^2}\, y \int dt \frac{-t\ \mathcal{F}^2_{\pi NN}(t)}{(t-m_\pi^2)^2}$

  → *model-independent* leading nonanalytic (LNA) behavior *consistent with Chiral Symmetry of QCD*.

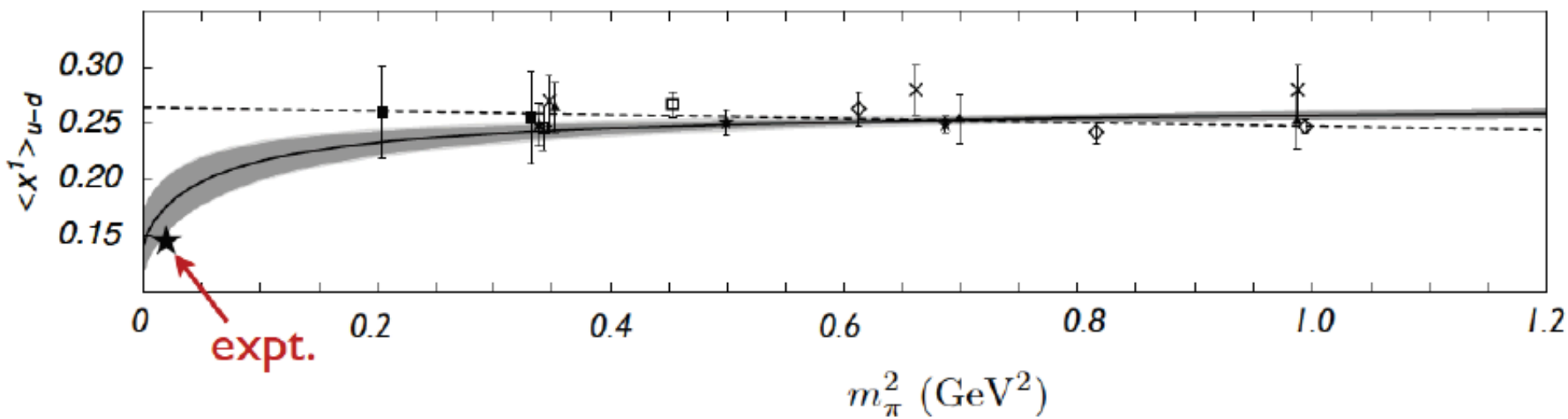
$$\langle x^0\rangle_{\bar{d}-\bar{u}} \equiv \int_0^1 dx(\bar{d}-\bar{u})$$

$$m_\pi^2 f_\pi^2 = -2m_q < \bar{q}q >$$

$$= \frac{2}{3}\int_0^1 dy f_\pi(y) = \frac{2g_A^2}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) \ + \ \text{analytic terms}$$

- Nonanalytic behavior vital for chiral extrapolation of lattice data

Thomas, Melnitchouk, Steffens PRL ***85****, 2892 (2000)*



Connection with QCD?

- Direct calculation of matrix elements of local twist-2 operators in ChPT disagrees with "Sullivan" result

$$\langle x^n \rangle_{u-d} = a_n \left(1 + \frac{(3g_A^2 + 1)}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) \right) + \mathcal{O}(m_\pi^2)$$

Chen, X. Ji, PLB ***523****, 107 (2001)*
Arndt, Savage, NPA ***692****, 429 (2002)*

cf. $4g_A^2$ in "Sullivan", via moments of $f_\pi(y)$

→ is there a problem with application of ChPT or "Sullivan process" to DIS?

→ is light-front treatment of pion loops problematic?

→ investigate relation between *covariant*, *instant-form*, and *light-front* formulations

→ consider simple test case: nucleon self-energy

πN Lagrangian

- Chiral (pseudovector) Lagrangian

$$\mathcal{L}_{\pi N} = \frac{g_A}{2f_\pi}\,\bar{\psi}_N \gamma^\mu \gamma_5\, \vec{\tau}\cdot\partial_\mu \vec{\pi}\,\psi_N - \frac{1}{(2f_\pi)^2}\,\bar{\psi}_N \gamma^\mu\, \vec{\tau}\cdot(\vec{\pi}\times\partial_\mu\vec{\pi})\,\psi_N$$

$g_A = 1.267$
$f_\pi = 93$ MeV

→ lowest order approximation of chiral perturbation theory Lagrangian

→ *cf.* pseudoscalar Lagrangian

$$\mathcal{L}^{\rm PS}_{\pi N} = -g_{\pi NN}\,\bar{\psi}_N\, i\gamma_5 \vec{\tau}\cdot\vec{\pi}\,\psi_N \;+\; \sigma NN \text{ term}$$

Weinberg, PRL ***18****, 88 (1967)*

Relation between PV and PS Theories

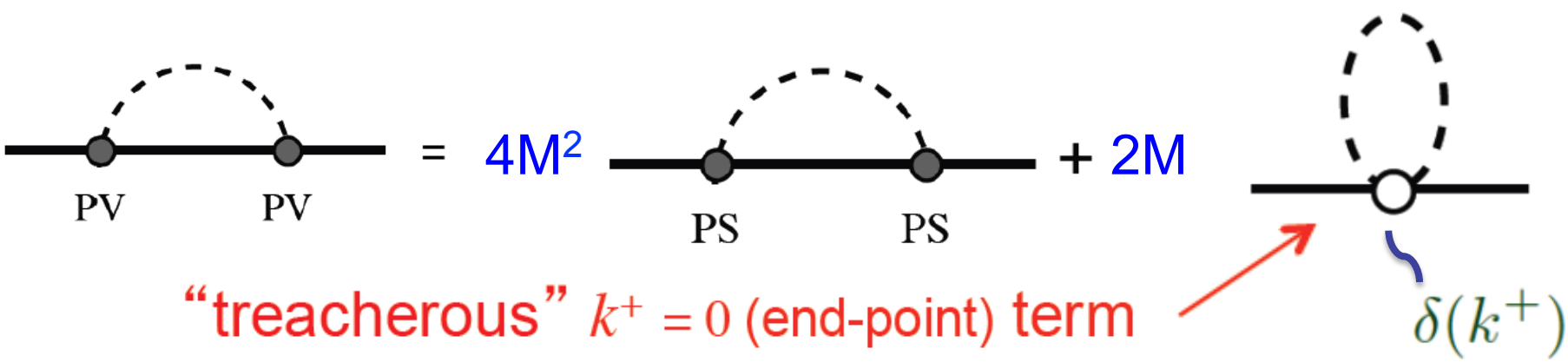
Self-Energy

$$\Sigma^{PV} = \frac{1}{2}\sum_s \bar{u}(p,s)\hat{\Sigma}^{PV} u(p,s) \qquad \hat{\Sigma}^{PV} = -i\left(\frac{2g_A}{f_\pi}\right)^2 \vec{\tau}\cdot\vec{\tau}\int \frac{d^4k}{(2\pi)^4}\frac{\not{k}\gamma_5(\not{p}-\not{k}+M)\gamma_5\not{k}}{D_\pi D_N}$$

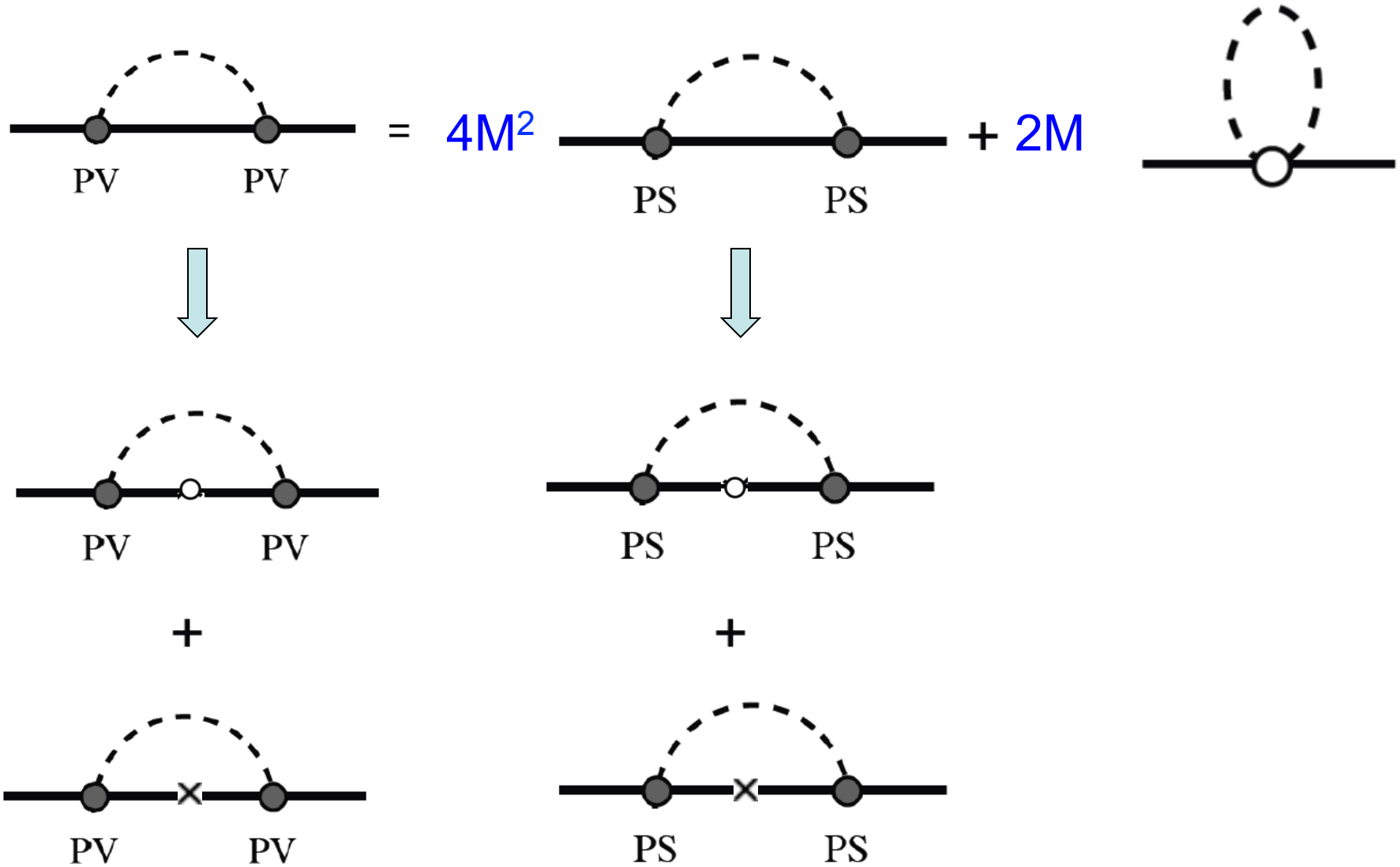
$$D_\pi = k^2 - m_\pi^2 + i\varepsilon \qquad D_N = (p-k)^2 - M^2 + i\varepsilon$$

$$\bar{u}(p)\not{k}\gamma^5 \frac{1}{\not{p}-\not{k}-M}\gamma^5\not{k}u(p) = \bar{u}(p)[\not{k}-\not{p}+M]\gamma^5\frac{1}{\not{p}-\not{k}-M}\gamma^5[\not{k}-\not{p}+M]u(p)$$

$$= 4M^2\,\bar{u}(p)\gamma^5\frac{1}{\not{p}-\not{k}-M}\gamma^5 u(p) + 2M\,\bar{u}(p)u(p) + \cancel{\bar{u}(p)\not{k}u(p)}$$

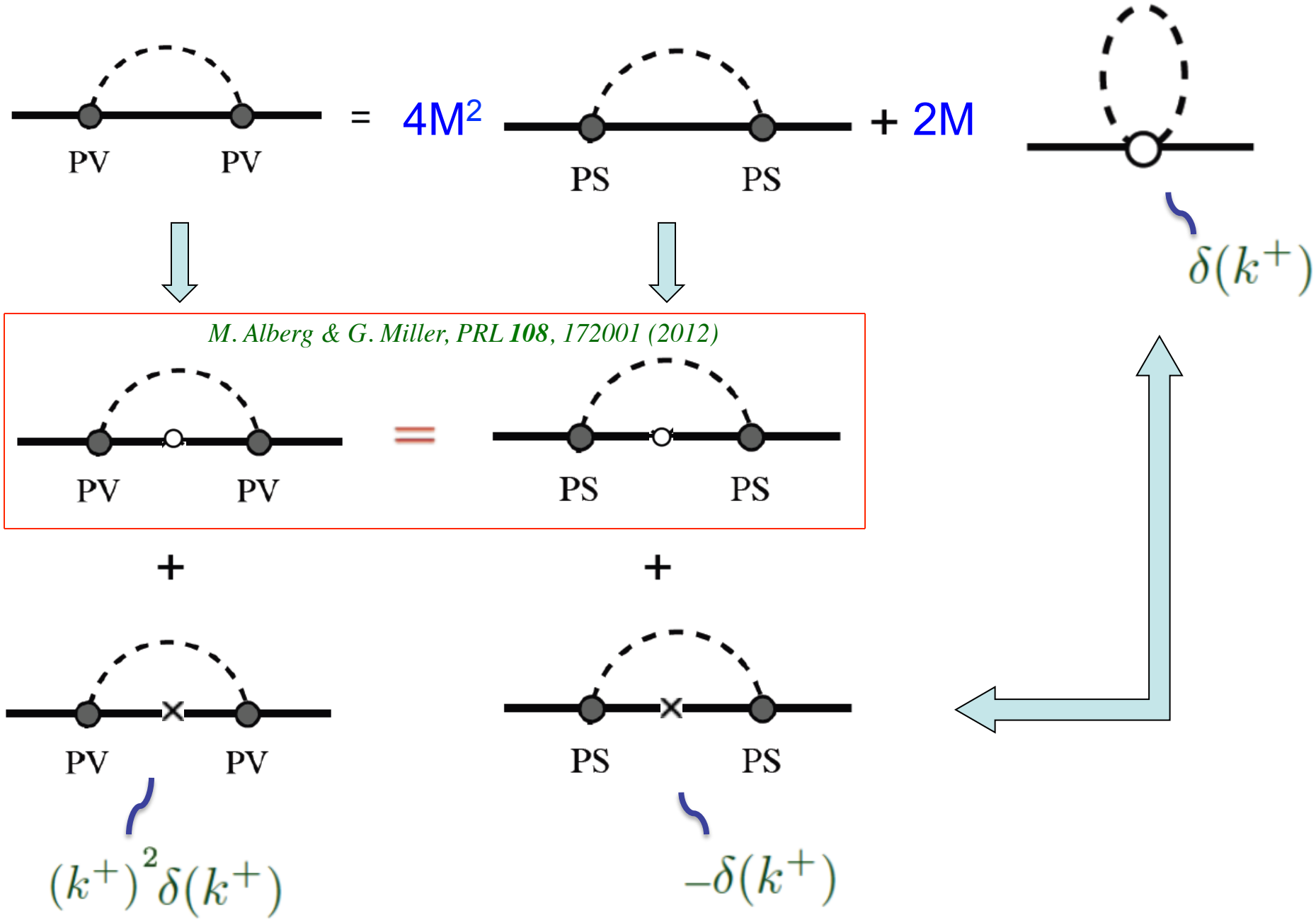


"treacherous" $k^+ = 0$ (end-point) term



$$\frac{1}{\not{p} - M} = \frac{\sum_s u(p,s)\bar{u}(p,s)}{p^2 - M^2} + \frac{\gamma^+}{2p^+}$$

S.-J.Chang and T.-M.Yan, PRD, 1147 (1973)



*C.Ji, W.Melnitchouk, A.W.Thomas, PRL **110**, 179191 (2013)*

Self-energy

- From lowest order PV Lagrangian

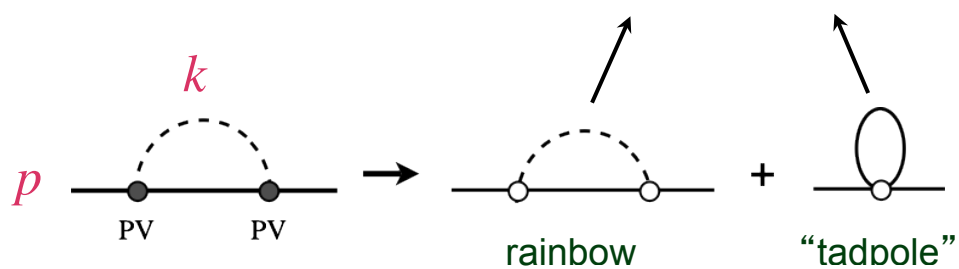
$$\Sigma = i\left(\frac{g_{\pi NN}}{2M}\right)^2 \bar{u}(p)\int\frac{d^4k}{(2\pi)^4}\,(\not{k}\gamma_5\vec{\tau})\,\frac{i\,(\not{p}-\not{k}+M)}{D_N}(\gamma_5\not{k}\vec{\tau})\,\frac{i}{D_\pi}\,u(p)$$

Goldberger-Treiman $\dfrac{g_A}{f_\pi} = \dfrac{g_{\pi NN}}{M}$

$$D_\pi \equiv k^2 - m_\pi^2 + i\varepsilon$$
$$D_N \equiv (p-k)^2 - M^2 + i\varepsilon$$

→ rearrange in more transparent "reduced" form

$$\Sigma = -\frac{3ig_A^2}{4f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{2M}\left[4M^2\left(\frac{m_\pi^2}{D_\pi D_N}+\frac{1}{D_N}\right)+\cancel{\frac{2p\cdot k}{D_\pi}}\right]$$



LNA terms

*C.Ji, W.Melnitchouk, A.W.Thomas, PRD **80**, 054018 (2009)*

Self-energy

- Covariant (equal-time + dimensional regularization)

$$\int d^{4-2\varepsilon}k \frac{1}{D_\pi D_N} = -i\pi^2 \left(\gamma + \log\pi - \frac{1}{\varepsilon} + \int_0^1 dx \log\frac{(1-x)^2M^2 + xm_\pi^2}{\mu^2} + \mathcal{O}(\varepsilon)\right)$$

$$\int d^{4-2\varepsilon}k \frac{1}{D_N} = -i\pi^2 M^2 \left(\gamma + \log\pi - \frac{1}{\varepsilon} + \log\frac{\mu^2}{M^2} + \mathcal{O}(\varepsilon)\right)$$

non-analytic

$$\rightarrow \quad \Sigma_{\rm cov} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4}{M}\log m_\pi^2 + \mathcal{O}(m_\pi^5)\right)$$

$$-\frac{3g_A^2 M}{32\pi^2 f_\pi^2}\left[R(M^2+m_\pi^2) - m_\pi^2 + (M^2+m_\pi^2)\log\frac{M^2}{\mu^2}\right]$$

analytic

$$R = -\frac{1}{\varepsilon} - \log 4\pi + \gamma - 1$$

$\rightarrow$ in $\widetilde{\rm MS}$ scheme, absorb R in counter-terms, set $\mu = M$

$\rightarrow$ $\Sigma_{\rm cov}^{\rm analytic} \longrightarrow 0$ in chiral limit

S.Scherer and M.Schindler, Lect.Notes Phys.830, pp.1-338 (2012);arXiv:hep-ph/0505265v1(2005)

Self-energy

- Light-front

$$\int dk^+ dk^- d^2k_\perp \frac{1}{D_\pi D_N} = \frac{1}{p^+}\int_{-\infty}^{\infty}\frac{dx}{x(x-1)} d^2k_\perp \int dk^- \left(k^- - \frac{k_\perp^2+m_\pi^2}{xp^+} + \frac{i\varepsilon}{xp^+}\right)^{-1}$$
$$\times \left(k^- - \frac{M^2}{p^+} - \frac{k_\perp^2+M^2}{(x-1)p^+} + \frac{i\varepsilon}{(x-1)p^+}\right)^{-1}$$
$$= 2\pi^2 i \int_0^1 dx\, dk_\perp^2\, \frac{1}{k_\perp^2+(1-x)m_\pi^2+x^2M^2}$$

$x = k^+/p^+$

→ identical nonanalytic results as covariant & instant form

$$\Sigma_{\rm LF}^{\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4}{M}\log m_\pi^2 + \mathcal{O}(m_\pi^5)\right)$$

Self-energy

- Pseudoscalar interaction

$$\Sigma^{\rm PS} = i g_{\pi NN}^2 \, \overline{u}(p) \int \frac{d^4k}{(2\pi)^4} (\gamma_5 \vec{\tau}) \frac{i\,(\not{p} - \not{k} + M)}{D_N} (\gamma_5 \vec{\tau}) \frac{i}{D_\pi} u(p)$$

$$= -\frac{3 i g_A^2 M}{2 f_\pi^2} \int \frac{d^4k}{(2\pi)^4} \left[\frac{m_\pi^2}{D_\pi D_N} + \frac{1}{D_N} - \frac{1}{D_\pi} \right]$$

→ contains additional ("treacherous") pion "tadpole" term

$\sim \delta(k^+)$

→ similar evaluation as for $1/D_N$ term

$$\Sigma_{\rm LNA}^{\rm PS} = \frac{3 g_A^2}{32\pi f_\pi^2} \left(\frac{M}{\pi} m_\pi^2 \log m_\pi^2 - m_\pi^3 - \frac{m_\pi^4}{2\pi M^2} \log \frac{m_\pi^2}{M^2} + \mathcal{O}(m_\pi^5) \right)$$

additional *lower order* term in PS theory!

Self-energy

- Alberg & Miller claim on light-front $\Sigma^{\rm PS} = \Sigma^{\rm PV}$
  – "form factor removes $k^+ = 0$ contribution"

  *M. Alberg & G. Miller, PRL **108**, 172001 (2012)*

- In practice, AM drop "treacherous" $k^+ = 0$ (end-point) term

$$\Sigma^{\rm PS} = \Sigma^{\rm PV} + \Sigma^{\rm PS}_{\rm end-pt}$$

after which PS result happens to coincide with PV

→ but, *even with* form factors, end-point term is non-zero

$$\Sigma^{\rm PS}_{\rm end-pt} = \frac{3g_A^2 M}{16\pi^2 f_\pi^2}\int_0^\infty dt\,\frac{\sqrt{t}\,F^2(m_\pi^2,-t)}{\sqrt{t+m_\pi^2}} \xrightarrow{\rm LNA} \frac{3g_A^2}{32\pi f_\pi^2}\frac{M}{\pi}m_\pi^2\log m_\pi^2$$

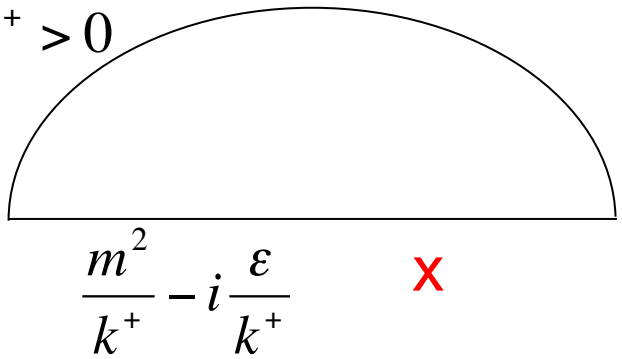
*C.Ji, W.Melnitchouk, A.W.Thomas, PRL **110**, 179191 (2013)*

→ *ansatz* does not work for other quantities
e.g. vertex renormalization

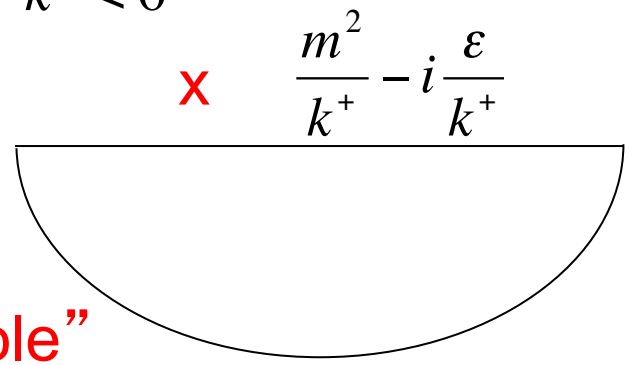
LFD

$$I = \frac{1}{2}\int dk^+ dk^- \frac{1}{k^+k^- - m^2 + i\varepsilon} = \frac{1}{2}\int \frac{dk^+}{k^+}\int dk^- \frac{1}{k^- - \frac{m^2}{k^+} + i\frac{\varepsilon}{k^+}}$$

$k^+ > 0$



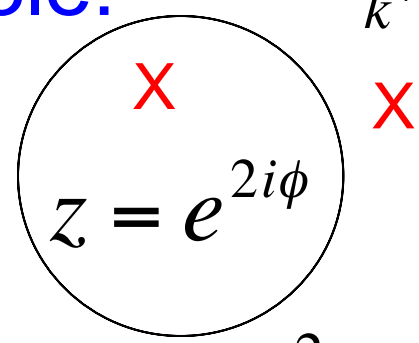
$k^+ < 0$



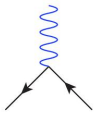
``Moving Pole”

Capture the pole!

$k^+ = r\cos\phi \quad k^- = r\sin\phi$



$$I = \int_0^\infty \frac{dr}{r}\oint dz \frac{2}{\left[z - (i\alpha + \sqrt{1-\alpha^2} + \varepsilon)\right]\left[z - (i\alpha - \sqrt{1-\alpha^2} + \varepsilon)\right]} \Rightarrow i\pi \log m^2$$



Tree Level

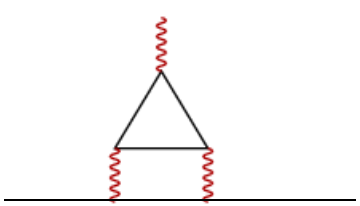
$$\partial_\mu J^\mu = 0\,;\; q_\mu \bar{u}(p')\gamma^\mu u(p)$$
$$= \bar{u}(p')[\not{p}' - \not{p}]u(p)$$
$$= \bar{u}(p')[m - m]u(p)$$
$$= 0$$

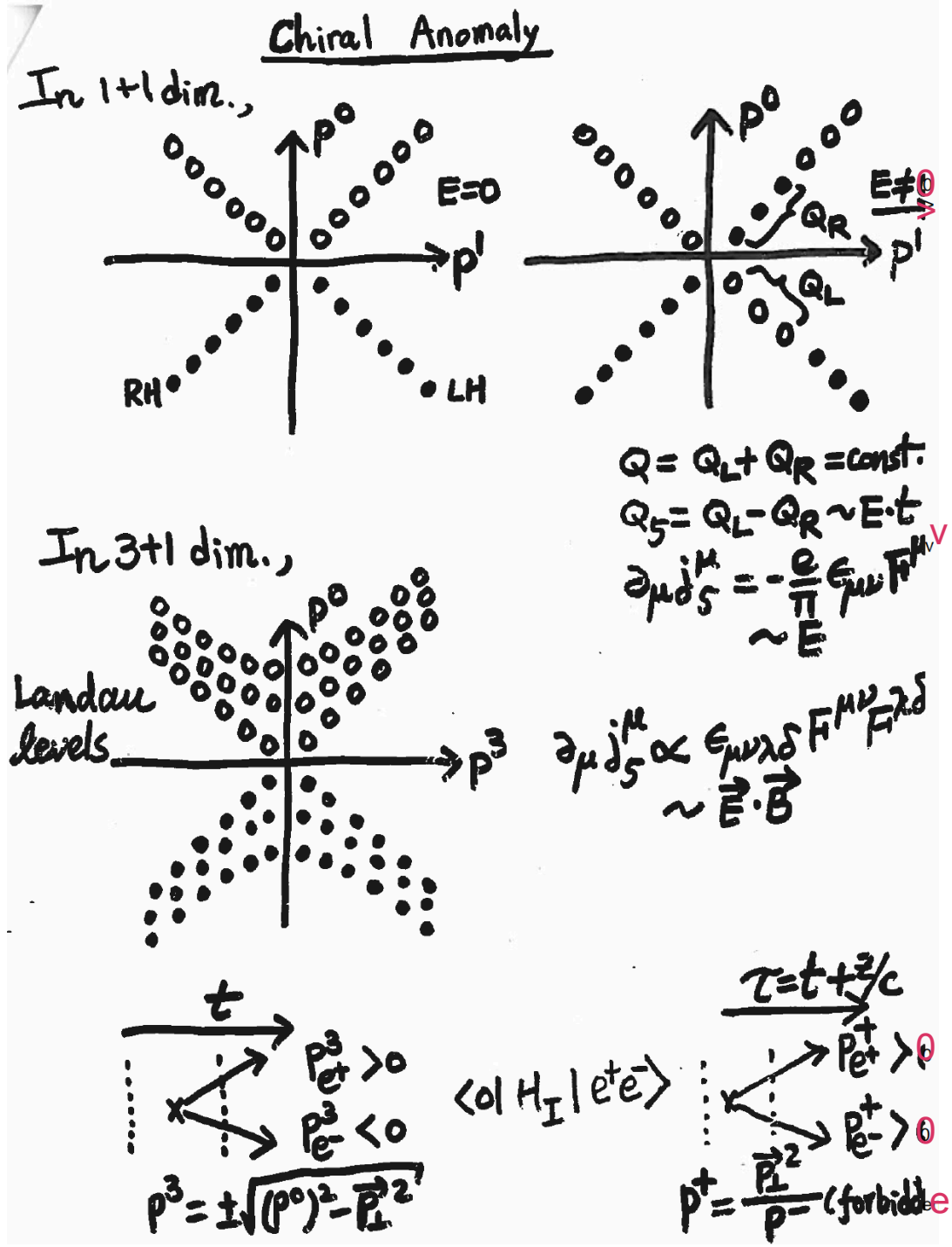
$$\partial_\mu J_5^\mu = 0\,;\; q_\mu \bar{u}(p')\gamma^\mu \gamma_5 u(p)$$
$$= \bar{u}(p')[\not{p}' - \not{p}]\gamma_5 u(p)$$
$$= \bar{u}(p')[\not{p}'\gamma_5 + \gamma_5 \not{p}]u(p)$$
$$= 2m\bar{u}(p')\gamma_5 u(p)$$
$$= 0 \quad if \quad m = 0$$

Loop Level

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J_5^\mu = \frac{e^2}{16\pi^2}\varepsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}$$



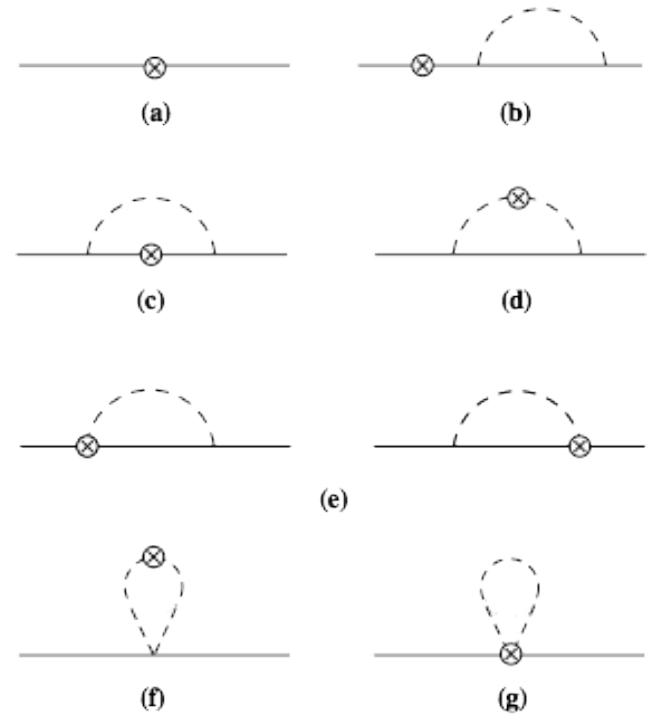


Classical symmetry is broken due to infinite degrees of freedom in quantum fields.

Vertex corrections

- Pion cloud corrections to electromagnetic N coupling

  → N rainbow **(c)**, π rainbow **(d)**, Kroll-Ruderman **(e)**, π tadpole **(f)**, N tadpole **(g)**



- Vertex renormalization

$$(Z_1^{-1} - 1)\,\bar{u}(p)\,\gamma^\mu\,u(p) = \bar{u}(p)\,\Lambda^\mu\,u(p)$$

→ taking "+" components: $Z_1^{-1} - 1 \approx 1 - Z_1 = \dfrac{M}{p^+}\,\bar{u}(p)\,\Lambda^+\,u(p)$

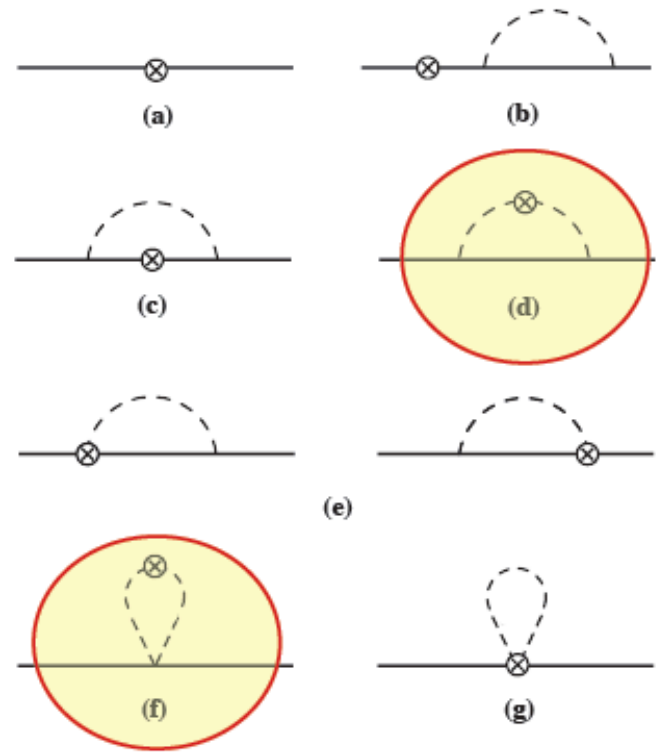
→ $e.g.$ for N rainbow contribution,

$$\Lambda_\mu^N = -\frac{\partial\hat{\Sigma}}{\partial p^\mu}$$

Pion distribution functions

- Pion cloud corrections to electromagnetic N coupling

  → N rainbow (**c**), π rainbow (**d**), Kroll-Ruderman (**e**), π bubble (**f**), π tadpole (**g**)



- Vertex renormalization

$$(Z_1^{-1} - 1)\, \bar{u}(p)\, \gamma^\mu\, u(p) = \bar{u}(p)\, \Lambda^\mu\, u(p)$$

→ taking "+" components: $Z_1^{-1} - 1 \approx 1 - Z_1 = \dfrac{M}{p^+}\, \bar{u}(p)\, \Lambda^+\, u(p)$

→ $e.g.$ for N rainbow contribution,

$$\Lambda_\mu^N = -\frac{\partial \hat{\Sigma}}{\partial p^\mu}$$

Moments of PDFs

- PDF moments related to nucleon matrix elements of local twist-2 operators

$$\langle N|\widehat{\mathcal{O}}_q^{\mu_1\cdots\mu_n}|N\rangle \;=\; 2\,\langle x^{n-1}\rangle_q\; p^{\{\mu_1}\cdots p^{\mu_n\}}$$

$\longrightarrow$ n-th moment of (spin-averaged) PDF $q(x)$

$$\langle x^{n-1}\rangle_q \;=\; \int_0^1 dx\, x^{n-1}\,(q(x)+(-1)^n\bar{q}(x))$$

$\longrightarrow$ operator is $\widehat{\mathcal{O}}_q^{\mu_1\cdots\mu_n} \;=\; \bar{\psi}\gamma^{\{\mu_1}\, iD^{\mu_2}\cdots iD^{\mu_n\}}\psi \;-\; \text{traces}$

- Lowest $(n{=}1)$ moment $\langle x^0\rangle_q \equiv \mathcal{M}_N + \mathcal{M}_\pi$ given by vertex renormalization factors $\sim 1 - Z_1^i$

Vertex corrections

- Define light-cone momentum distributions $f_i(y)$

$$1 - Z_1^i = \int dy\, f_i(y)$$

for isovector $(p-n)$ distribution

where

$$f_\pi(y) = 4f^{(\mathrm{on})}(y) + 4f^{(\delta)}(y)$$
$$f_N(y) = -f^{(\mathrm{on})}(y) - f^{(\mathrm{off})}(y) + f^{(\delta)}(y)$$
$$f_{\mathrm{KR}}(y) = 4f^{(\mathrm{off})}(y) - 8f^{(\delta)}(y)$$
$$f_{\pi(\mathrm{tad})}(y) = -f_{N(\mathrm{tad})}(y) \;=\; 2f^{(\mathrm{tad})}(y)$$

with components

$$f^{(\mathrm{on})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{y(k_\perp^2 + y^2 M^2)}{[k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2]^2}$$
$$f^{(\mathrm{off})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{y}{k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2}$$
$$f^{(\delta)}(y) = \frac{g_A^2}{4(4\pi f_\pi)^2} \int dk_\perp^2 \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y)$$
$$f^{(\mathrm{tad})}(y) = -\frac{1}{(4\pi f_\pi)^2} \int dk_\perp^2 \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y)$$

Burkardt, Hendricks, Ji, Melnitchouk, Thomas, PRD 87, 056009 (2013)

Nonanalytic behavior of vertex renormalization factors

	$1/D_\pi D_N^2$	$1/D_\pi^2 D_N$	$1/D_\pi D_N$	$1/D_\pi$ or $1/D_\pi^2$	sum (PV)	sum (PS)
$1-Z_1^N$	g_A^2 *	0	$-\frac{1}{2}g_A^2$	$\frac{1}{4}g_A^2$	$\frac{3}{4}g_A^2$	g_A^2
$1-Z_1^\pi$	0	g_A^2 *	0	$-\frac{1}{4}g_A^2$	$\frac{3}{4}g_A^2$	g_A^2
$1-Z_1^{\mathrm{KR}}$	0	0	$-\frac{1}{2}g_A^2$	$\frac{1}{2}g_A^2$	0	0
$1-Z_1^{N\,\mathrm{tad}}$	0	0	0	$-1/2$	$-1/2$	0
$1-Z_1^{\pi\,\mathrm{tad}}$	0	0	0	$1/2$	$1/2$	0

* also in PS

in units of $\frac{1}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$

$\longrightarrow$ origin of ChPT *vs*. Sullivan process difference clear!

$$\left(1-Z_1^{N\,(\mathrm{PV})}\right)_{\mathrm{LNA}} = \frac{3}{4}\left(1-Z_1^{N\,(\mathrm{PS})}\right)_{\mathrm{LNA}}$$

Nonanalytic behavior

$$\mathcal{M}_N^{(p)} \xrightarrow{\text{LNA}} 1 - \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2 \qquad \mathcal{M}_\pi^{(p)} \xrightarrow{\text{LNA}} \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_N^{(n)} \xrightarrow{\text{LNA}} \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2 \qquad \mathcal{M}_\pi^{(n)} \xrightarrow{\text{LNA}} -\frac{(3g_A^2+1)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

→ no pion corrections to isosclar moments

→ isovector correction agrees with ChPT calculation

$$\mathcal{M}_N^{(p-n)} \xrightarrow{\text{LNA}} 1 - \frac{(4g_A^2 + [1-g_A^2])}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_\pi^{(p-n)} \xrightarrow{\text{LNA}} \frac{(4g_A^2 + [1-g_A^2])}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

PS ("on-shell") contribution

δ-function contribution

Flavor asymmetry

- Contributions to PDFs related to matrix elements of non-local operators, in terms of convolutions

→ $$q(x) = Z_2\, q_0(x) + \left([f_N + f_{\rm tad}] \otimes q_0\right)(x)$$
$$+ \left([f_\pi + f_{\rm bub}] \otimes q_\pi\right)(x) + \left(f_{\rm KR} \otimes \Delta q_0\right)(x)$$

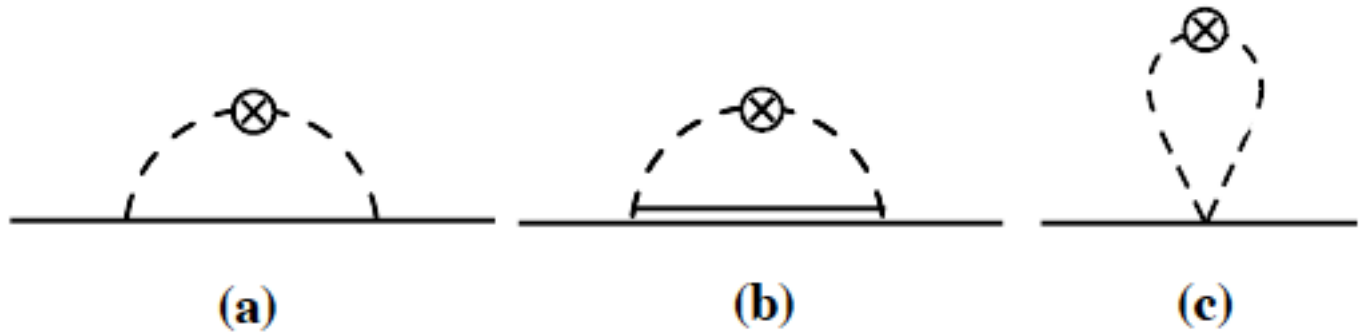
Moiseeva, Vladimirov
EPJA ***49****, 23 (2013)*

→ if "bare" nucleon has symmetric sea, $\bar{d} = \bar{u}$
then only "pion" term contributes

$$(\bar{d} - \bar{u})(x) = \left([f_{\pi^+} + f_{\rm bub}] \otimes \bar{q}_\pi\right)(x)$$

$$f_{\pi^+} = 2f^{(\rm on)} + 2f^{(\delta)}$$

C. Ji, W. Melnitchouk, A. Thomas,
PRD ***88****, 076005 (2013)*



V.Pascalutsa and M.Vanderhaeghen,Phys.Lett.B636, 31 (2006)

$$f_{\pi^+\Delta^0}(y) = f_\Delta^{(\mathrm{on})}(y) + f_\Delta^{(\mathrm{end\ point})}(y) + f_\Delta^{(\delta)}(y)$$

W.Melnitchouk,J.Speth,A.W.Thomas,Phys.Rev.D59,014033(1998)

$$f_\Delta^{(\mathrm{Sul})}(y) = C_\Delta \int dk_\perp^2 y \times \frac{[k_\perp^2 + (\Delta + yM)^2][k_\perp^2 + (\bar{M} - yM)^2]^2}{(1-y)^4 D_{\pi\Delta}^2}$$

$$D_{\pi\Delta} = -[k_\perp^2 - y(1-y)M^2 + yM_\Delta^2 + (1-y)m_\pi^2]/(1-y)$$

LNA of $\bar{D} - \bar{U} \equiv \int_0^1 dx(\bar{d} - \bar{u})$

$$(\bar{D} - \bar{U})_{\rm LNA} = \frac{3g_A^2 + 1}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2 - \frac{g_{\pi N\Delta}^2}{12\pi^2} J_1$$

$$J_1 = (m_\pi^2 - 2\Delta^2)\log m_\pi^2 + 2\Delta r \log[(\Delta - r)/(\Delta + r)],$$

$$\Delta \equiv M_\Delta - M \qquad\qquad r = \sqrt{\Delta^2 - m_\pi^2}$$

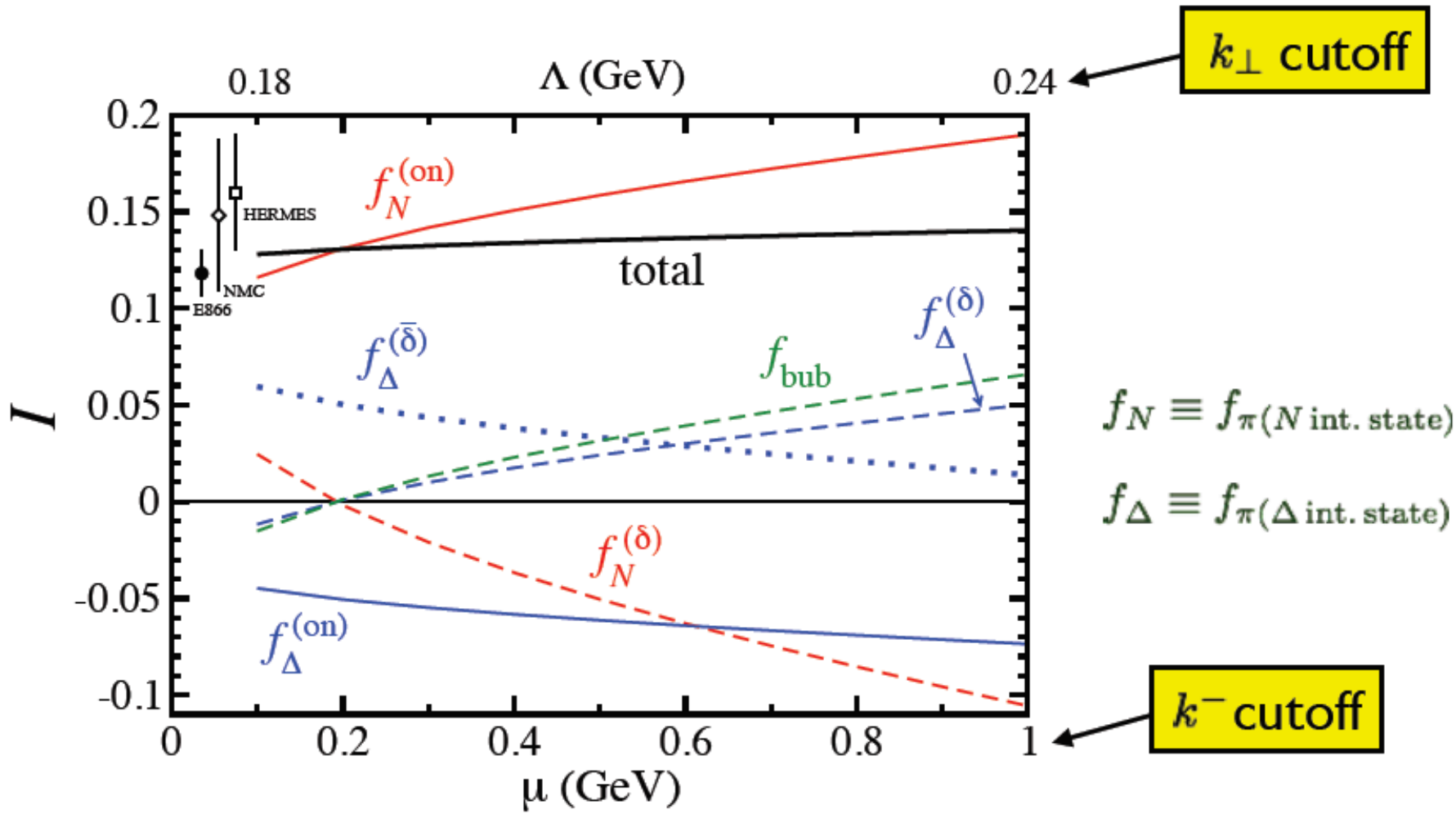
D.Arndt and M.Savage,Nucl.Phys.A697, 429 (2002)

$\Delta \to 0$ limit $\quad [(27/50)g_A^2 + 1/2]/(4\pi f_\pi)^2$

SU(6) couplings $\quad$ vs. $\quad (18/25)g_A^2/(4\pi f_\pi)^2$

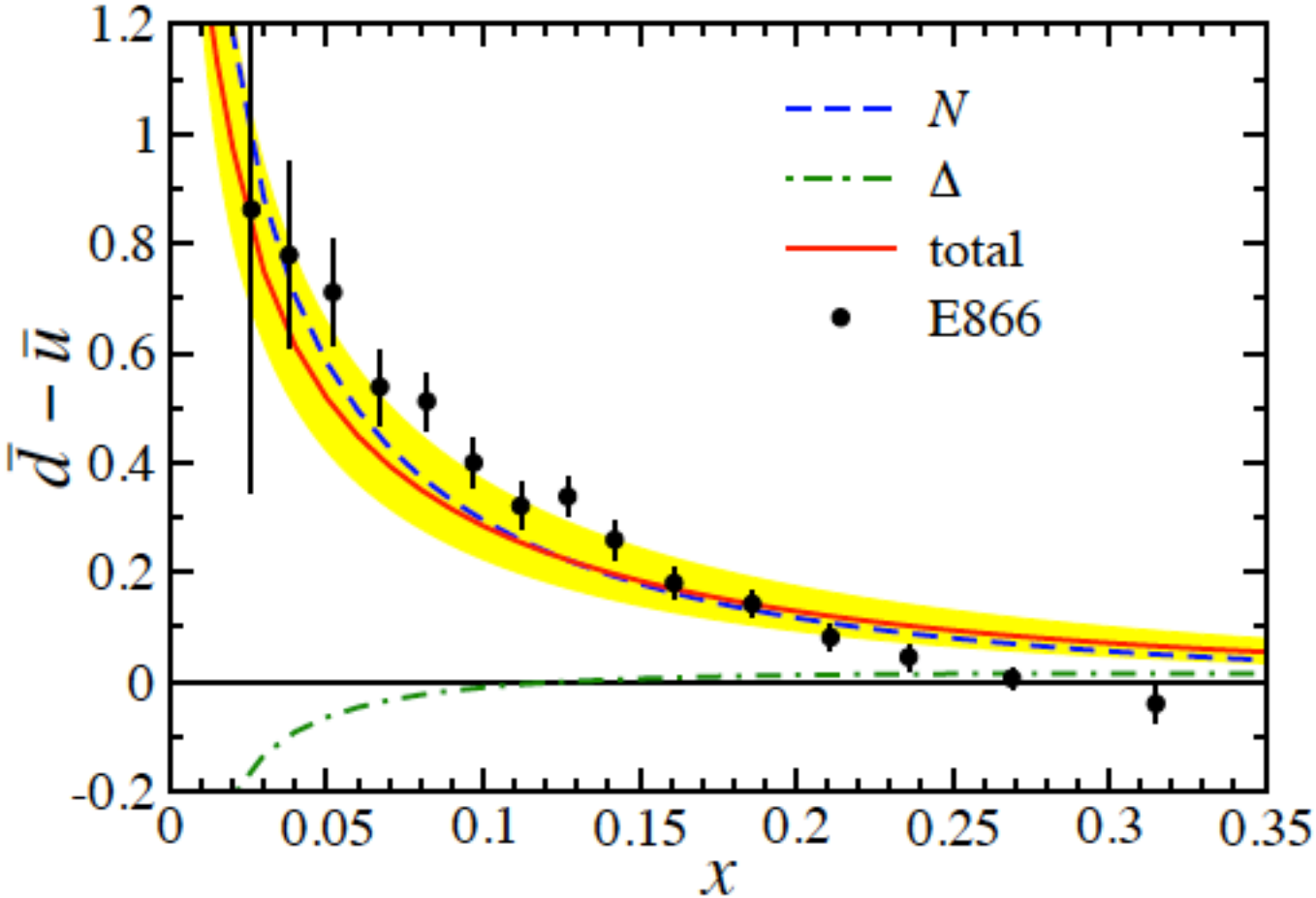
Flavor asymmetry

- Integrated asymmetry $I = \int_0^1 dx\,(\bar{d} - \bar{u})(x)$



→ N on-shell contribution ≈ total!

Y. Salamu, C. Ji, W. Melnitchouk, P. Wang, PRL 114, 122001 (2015)



Small x region: N.Kivel and M.Polyakov,Phys.Lett.B664, 64 (2008)

Summary

- No problem calculating π loop corrections to PDFs in LFD (if symmetries respected and $k^+ \to 0$ treated correctly)
- LNA provides a unique constraint on theoretical prediction
- EFT approach puts "Sullivan process" in proper context

  → on-shell (pole) approximation

  → $\delta(k^+)$ contributions from "rainbow" & "tadpole" diagrams affect integrated distributions

- First estimate for "d-bar"-"u-bar" phenomenology

  → illustrated for $k_\perp$ and k^- cutoffs

Works in progress

- DR_4 and DR_2 (scheme & scale dependence explicit) along with PVR and other regularization method such as FFs
- Analysis of HERA data for the future JLab TDIS experiment
- SU(3) extension for the “s-sbar” phenomenology