

# New mass relations for $L=1$ excited baryons

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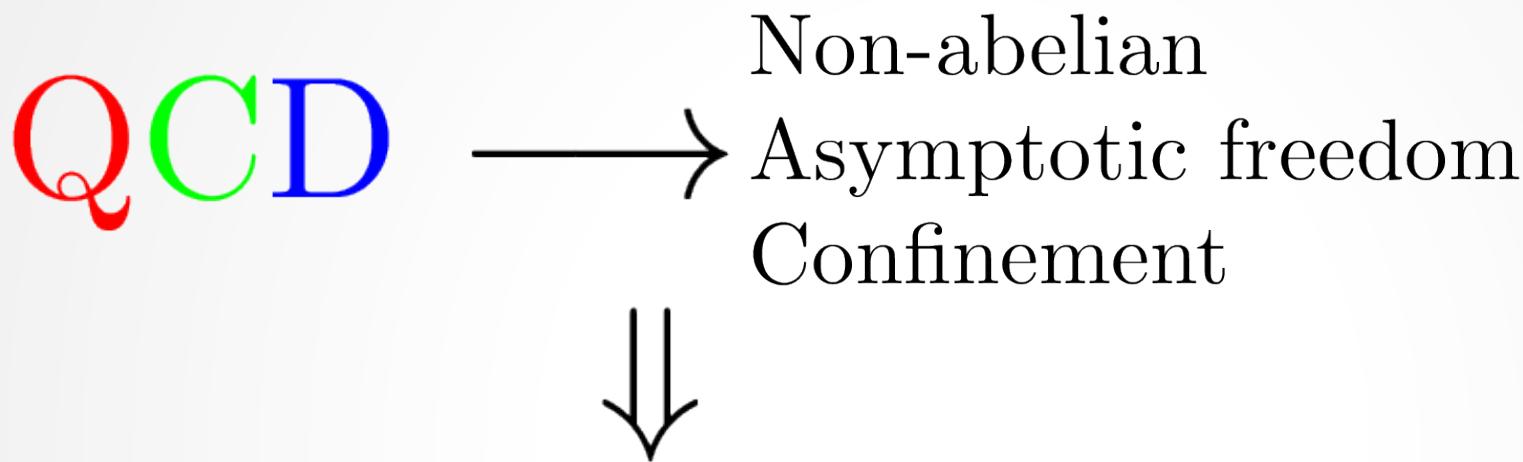
*Carlos SCHAT*

*Theory Center Seminar  
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# Outline

- Overview of Large  $N_c$
- Nonstrange L=1 baryons
- Non-relativistic quark model
- L=1 baryons in Large  $N_c$
- Matching the NR quark model to the  $1/N_c$  expansion
- Results
- Summary

# Overview of Large $N_c$ QCD



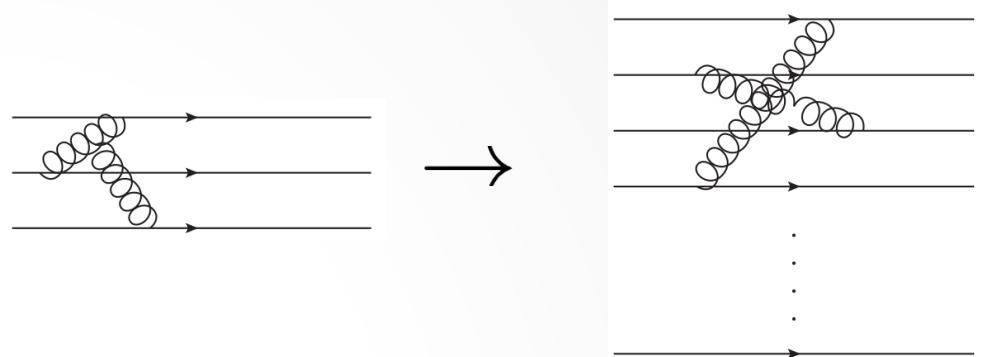
- Perturbative QCD
- EFT
- Lattice QCD
- ... Large  $N_c \rightarrow$  expansion of QCD in terms of  $1/N_c$

How does QCD generalized to  $N_c$  colors look like? ( $SU(3) \rightarrow SU(N_c)$ )

- Degrees of freedom increase

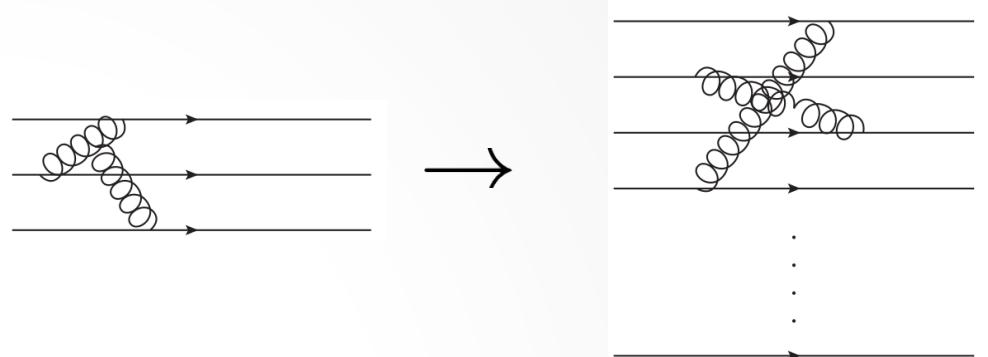
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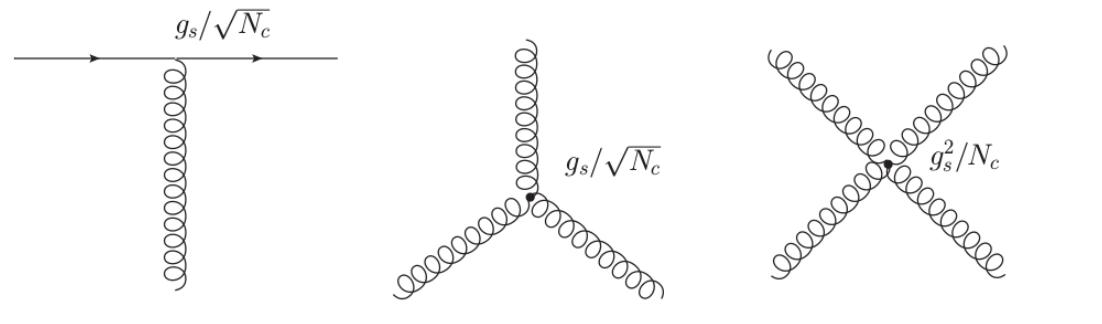


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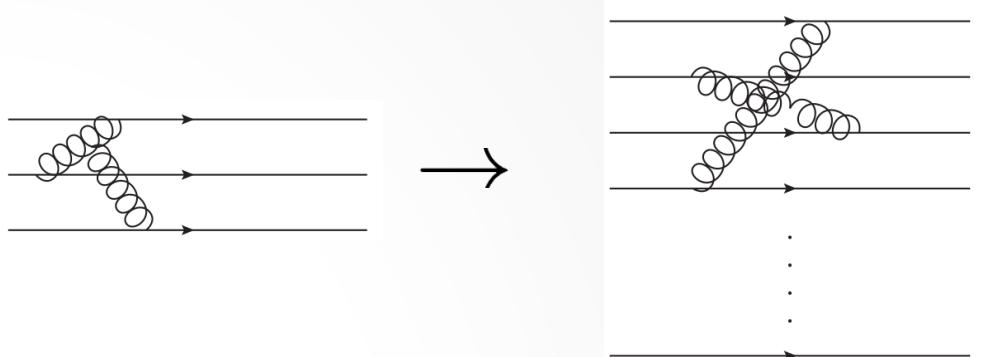
- Rescaling ( $g \rightarrow g/\sqrt{N_c}$ )



[1] G. 't Hooft, Nucl. Phys. **B72**, 461 (1974)

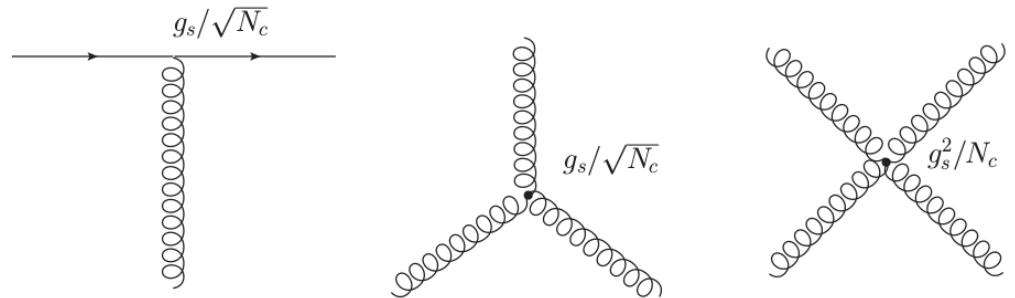
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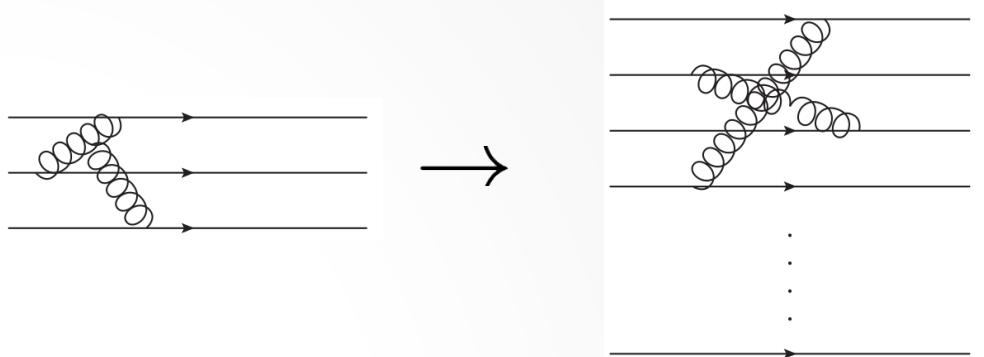
- Physics actually simplifies



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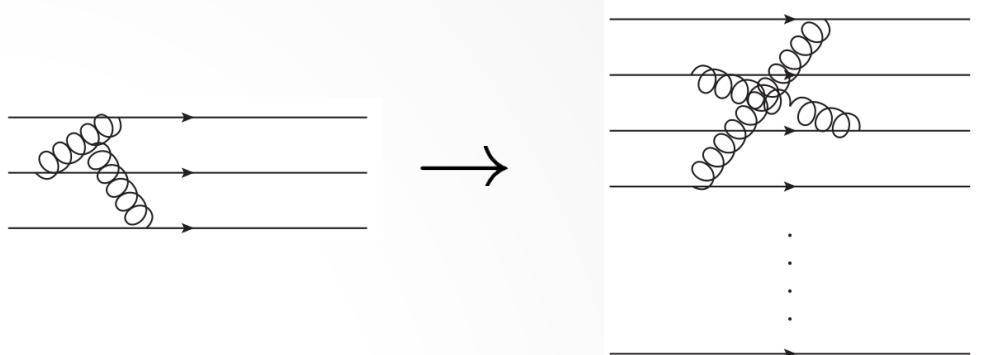
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- Physics actually simplifies

→ Planar diagrams are dominant  
→ Quark loops are suppressed

How does QCD generalized to  $N_c$  colors look like? ( $SU(3) \rightarrow SU(N_c)$ )

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- Rescaling ( $g \rightarrow g/\sqrt{N_c}$ )

- Physics actually simplifies

→ Planar diagrams are dominant  
→ Quark loops are suppressed

*$1/N_c$  is the only candidate for a perturbative expansion at all energies*

# Observables in Large $N_c$

The Large  $N_c$  expansion allows to organize operators by their effect on a given observable

$$\mathcal{O} = \sum_i c_i O_i$$

organized by powers of  $1/N_c$  in their matrix elements.

Large  $N_c$  was applied with great success to ground state baryons (masses, magnetic moments, axial vector current, etc..) and mass relations.

# Baryon Spectroscopy

Table 1. The status of the  $N$  resonances. Only those with an overall status of \*\*\* or \*\*\*\* are included in the main Baryon Summary Table.

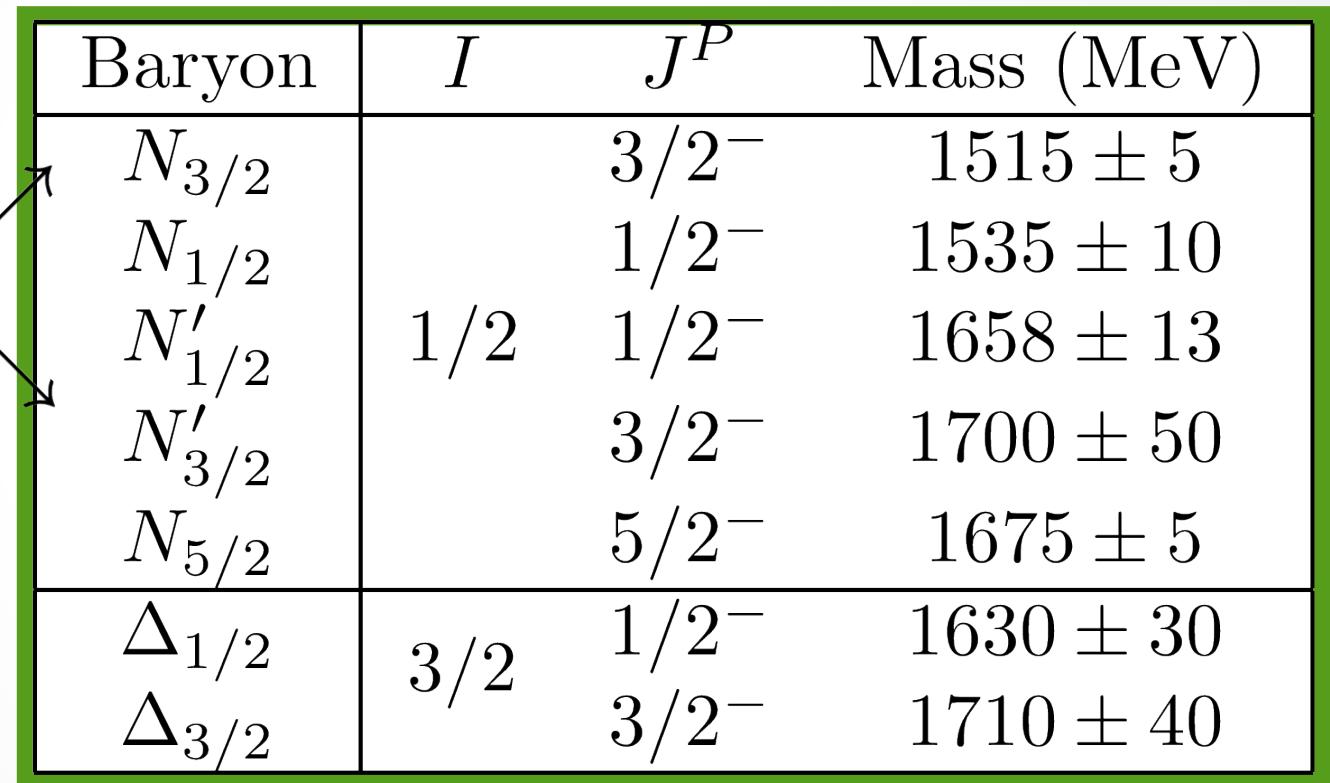
Particle $J^P$	Status	Status as seen in —									
		overall	$\pi N$	$\gamma N$	$N\eta$	$N\sigma$	$N\omega$	$\Lambda K$	$\Sigma K$	$N\rho$	$\Delta\pi$
$N$ $1/2^+$	****										
$N(1440)$ $1/2^+$	****	****	****	***				*	***		
$N(1520)$ $3/2^-$	****	****	****	***				***	***		
$N(1535)$ $1/2^-$	****	****	****	****				**	*		
$N(1650)$ $1/2^-$	****	****	***	***				***	**	**	***
$N(1675)$ $5/2^-$	****	****	***	*			*	*	***		
$N(1680)$ $5/2^+$	****	****	****	*	**			***	***		
$N(1685)$ ?	*										
$N(1700)$ $3/2^-$	***	***	**	*			*	*	*	***	
$N(1710)$ $1/2^+$	***	***	***	***		**	***	**	*	**	
$N(1720)$ $3/2^+$	****	****	***	***			**	**	**	*	
$N(1860)$ $5/2^+$	**	**						*	*		
$N(1875)$ $3/2^-$	***	*	***			**	***	**		***	
$N(1880)$ $1/2^+$	**	*	*		**			*			
$N(1895)$ $1/2^-$	**	*	**	**				**	*		
$N(1900)$ $3/2^+$	***	**	***	**		**	***	**	*	**	
$N(1990)$ $7/2^+$	**	**	**						*		
$N(2000)$ $5/2^+$	**	*	**	**		**	*	**		**	
$N(2040)$ $3/2^+$	*										
$N(2060)$ $5/2^-$	**	**	**	*				**			
$N(2100)$ $1/2^+$	*										
$N(2150)$ $3/2^-$	**	**	**			**			**		
$N(2190)$ $7/2^-$	****	****	***		*	**		*			
$N(2220)$ $9/2^+$	****	****									
$N(2250)$ $9/2^-$	****	****									
$N(2600)$ $11/2^-$	***	***									
$N(2700)$ $13/2^+$	**	**									

- \*\*\*\* Existence is certain, and properties are at least fairly well explored.
- \*\*\* Existence is very likely but further confirmation of quantum numbers and branching fractions is required.
- \*\* Evidence of existence is only fair.
- \* Evidence of existence is poor.

- At low energies, the strong coupling constant becomes large and perturbation theory can not be used to solve QCD
- Map the excited baryon spectrum to learn about the internal structure of nucleons

# L=1 baryons

All component spin and isospin multiplet for L=1 non-strange baryons have been observed



Baryon	$I$	$J^P$	Mass (MeV)
$N_{3/2}$		$3/2^-$	$1515 \pm 5$
$N_{1/2}$		$1/2^-$	$1535 \pm 10$
$N'_{1/2}$	$1/2$	$1/2^-$	$1658 \pm 13$
$N'_{3/2}$		$3/2^-$	$1700 \pm 50$
$N_{5/2}$		$5/2^-$	$1675 \pm 5$
$\Delta_{1/2}$	$3/2$	$1/2^-$	$1630 \pm 30$
$\Delta_{3/2}$		$3/2^-$	$1710 \pm 40$

The mixing angles are defined as

$$N_{J=1/2}(1535) = \cos \theta_1 {}^2 N_{1/2} + \sin \theta_1 {}^4 N_{1/2},$$

$$N'_{J=1/2}(1650) = -\sin \theta_1 {}^2 N_{1/2} + \cos \theta_1 {}^4 N_{1/2},$$

# Non-relativistic quark model

In the NR limit, the 2-body interactions contain three terms

$$H = H_0 + \sum_{i < j} V_{ij} = H_0 + V_{ss} + V_{so} + V_t .$$

three constituent quarks interacting  
by harmonic oscillator potentials

spin-spin      spin-orbit      quadrupole

# Non-relativistic quark model

In the NR limit, the 2-body interactions contain three terms

$$H = H_0 + \sum_{i < j} V_{ij} = H_0 + V_{ss} + V_{so} + V_t .$$

$$V_{ss} = V_{ss}^0 + V_{ss}^1 = \sum_{i < j=1}^N v_{ss}(r_{ij}) \vec{s}_i \cdot \vec{s}_j ,$$

$$v_\alpha = v_\alpha^0(r_{ij}) + v_\alpha^1(r_{ij}) \tau_i^a \tau_j^a$$

$$V_{so} = V_{so}^0 + V_{so}^1 = \sum_{i < j=1}^N v_{so}(r_{ij}) \left[ (\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_i - (\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j \right.$$

$$\left. + 2(\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_j - 2(\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_i \right] ,$$

$$V_t = V_t^0 + V_t^1 = \sum_{i < j=1}^N v_t(r_{ij}) \left[ 3(\hat{r}_{ij} \cdot \vec{s}_i)(\hat{r}_{ij} \cdot \vec{s}_j) - (\vec{s}_i \cdot \vec{s}_j) \right] .$$

Setting  $v_\alpha^1 = 0$  ( $v_\alpha^0 = 0$ ) we obtain the isospin independent (dependent) interactions labeled as OGE (OPE)

# Non-relativistic quark model

In the NR limit, the 2-body interactions contain three terms

$$H = H_0 + \sum_{i < j} V_{ij} = H_0 + V_{ss} + V_{so} + V_t .$$

The quark model states are given by

$$\begin{aligned} |N_J; J_3 I_3\rangle &= \frac{1}{2} \sum_{m, S_3} \left( \begin{array}{cc} 1 & \frac{1}{2} \\ m & S_3 \end{array} \middle| \begin{array}{c} J \\ J_3 \end{array} \right) [(\xi_{S_3}^\rho \varphi_{I_3}^\rho - \xi_{S_3}^\lambda \varphi_{I_3}^\lambda) \Psi_{1m}^\lambda + (\xi_{S_3}^\rho \varphi_{I_3}^\lambda - \xi_{S_3}^\lambda \varphi_{I_3}^\rho) \Psi_{1m}^\rho] , \\ |N'_J; J_3 I_3\rangle &= \frac{1}{\sqrt{2}} \sum_{m, S_3} \left( \begin{array}{cc} 1 & \frac{3}{2} \\ m & S_3 \end{array} \middle| \begin{array}{c} J \\ J_3 \end{array} \right) \xi_{S_3}^S (\varphi_{I_3}^\rho \Psi_{1m}^\rho + \varphi_{I_3}^\lambda \Psi_{1m}^\lambda) , \\ |\Delta_J; J_3 I_3\rangle &= \frac{1}{\sqrt{2}} \varphi_{I_3}^S \sum_{m, S_3} \left( \begin{array}{cc} 1 & \frac{3}{2} \\ m & S_3 \end{array} \middle| \begin{array}{c} J \\ J_3 \end{array} \right) (\xi_{S_3}^\rho \Psi_{1m}^\rho + \xi_{S_3}^\lambda \Psi_{1m}^\lambda) . \end{aligned}$$

where  $\Psi_{L,m}^{\rho,\lambda}$  are the eigenstates of the Hamiltonian  $H_0$  that describes three constituent quarks interacting by harmonic oscillator potentials

# Relevant operators to L=1 baryons

Physically observed baryons are assigned to the irreducible representations of the symmetry group:  $SU(6) \otimes O(3)$

$$S^i, T^a, G^{ia}$$

$$l^i$$

core operators

excited quark operators

Complete set of time reversal even,  
rotationally invariant, isosinglet  
operators for non-strange excited baryons

Order of matrix element	Operator
$N_c^1$	$N_c \mathbb{1}$
$N_c^0$	$ls, \frac{1}{N_c} ltG_c, \frac{1}{N_c} l^{(2)}gG_c,$
$N_c^{-1}$	$\frac{1}{N_c} tT_c, \frac{1}{N_c} lS_c, \frac{1}{N_c} lgT_c, \frac{1}{N_c} S_c^2, \frac{1}{N_c} sS_c,$ $\frac{1}{N_c} l^{(2)}sS_c, \frac{1}{N_c^2} l^{(2)}t\{S_c, G_c\}, \frac{1}{N_c^2} l^i g^{ja}\{S_c^j, G_c^{ia}\}$
$N_c^{-2}$	$\frac{1}{N_c^2} (lS_c)(tT_c), \frac{1}{N_c^2} gS_cT_c, \frac{1}{N_c^2} l^{(2)}S_cS_c,$ $\frac{1}{N_c^2} l^{(2)}gS_cT_c, \frac{1}{N_c^2}\{lS_c, sS_c\}, \frac{1}{N_c^2}(ls)S_c^2$

[3] C. E. Carlson, C. D. Carone, J. L. Goity and R. F. Lebed,  
Phys. Rev. D **59**, 114008 (1999)

# Relevant operators to L=1 baryons

The mass matrix of a L=1 baryon can be written as a linear combination of the CCGL<sup>[3]</sup> operators

$$M = \sum_{i=1}^{18} c_i O_i = O_{\ell=0} + O_{\ell=1} + O_{\ell=2}$$

organized by powers of  $1/N_c$  in their matrix elements.

For the content of states for  $N_c = 3$  there are seven possible masses and two mixing angles, which means at most nine independent mass operators

It can be shown that one of these operators is  $\sim 1/N_c^2$  (for a particular basis)

[3] C. E. Carlson, C. D. Carone, J. L. Goity and R. F. Lebed,  
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# The matching

The mass matrix of a L=1 baryon can be written as a linear combination of the CCGL<sup>[3]</sup> operators

$$M = \sum_{i=1}^9 c_i O_i = O_{\ell=0} + O_{\ell=1} + O_{\ell=2}$$



The matching was done considering:

- M.E. of the operators of the 1/N expansion as in [3]
- By explicitly calculating the ME for the quark model in the harmonic oscillator basis of eigenstates of  $H_0$

$$H = H_0 + \sum_{i < j} V_{ij} = H_0 + V_{ss} + V_{so} + V_t .$$

[3] C. E. Carlson, C. D. Carone, J. L. Goity and R. F. Lebed,  
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# The matching

	$V_{ss}^0$	$V_{ss}^1$	$V_{so-2B}^{0,1}$	$V_{so-3B}^0$	$V_{so-3B}^1$	$V_t^{0,1}$	$O_{\ell=0}$	$O_{\ell=1}$	$O_{\ell=2}$
${}^2N_{1/2}$	$S^0$	$S^1$	$2 P_{2B}^{0,1}$	0	$-4 P_{3B}^1$	0	$S_1$	$-2 P_1$	0
${}^4N_{1/2}$	$-S^0$	$S'^1$	$5 P_{2B}^{0,1}$	0	0	$5 D^{0,1}$	$S_2$	$5 P_2$	$5 D_1$
${}^2N_{1/2} - {}^4N_{1/2}$	0	0	$P_{2B}^{0,1}$	$P_{3B}^0$	$P_{3B}^1$	$5 D^{0,1}$	0	$P_3$	$-5 D_2$
${}^2N_{3/2}$	$S^0$	$S^1$	$-P_{2B}^{0,1}$	0	$2 P_{3B}^1$	0	$S_1$	$P_1$	0
${}^4N_{3/2}$	$-S^0$	$S'^1$	$2 P_{2B}^{0,1}$	0	0	$-4 D^{0,1}$	$S_2$	$2 P_2$	$-4 D_1$
${}^2N_{3/2} - {}^4N_{3/2}$	0	0	$\sqrt{\frac{5}{2}} P_{2B}^{0,1}$	$\sqrt{\frac{5}{2}} P_{3B}^0$	$\sqrt{\frac{5}{2}} P_{3B}^1$	$-\sqrt{\frac{5}{2}} D^{0,1}$	0	$\sqrt{\frac{5}{2}} P_3$	$\sqrt{\frac{5}{2}} D_2$
${}^4N_{5/2}$	$-S^0$	$S'^1$	$-3 P_{2B}^{0,1}$	0	0	$D^{0,1}$	$S_2$	$-3 P_2$	$D_1$
${}^2\Delta_{1/2}$	$S'^0$	$S'^1$	0	$-2 P_{3B}^0$	$2 P_{3B}^1$	0	$S_3$	$-2 P_4$	0
${}^2\Delta_{3/2}$	$S'^0$	$S'^1$	0	$P_{3B}^0$	$-P_{3B}^1$	0	$S_3$	$P_4$	0

Table 1: General structure of the mass operator

# The matching

We found that only 7 operators  $O_i$  are needed

	$V_{ss}^0$	$V_{ss}^1$	$V_{so-2B}^{0,1}$	$V_{so-3B}^0$	$V_{so-3B}^1$	$V_t^{0,1}$	$O_{\ell=0}$	$O_{\ell=1}$	$O_{\ell=2}$
${}^2N_{1/2}$	$S^0$	$S^1$	$2 P_{2B}^{0,1}$	0	$-4 P_{3B}^1$	0	$S_1$	$-2 P_1$	0
${}^4N_{1/2}$	$-S^0$	$S'^1$	$5 P_{2B}^{0,1}$	0	0	$5 D^{0,1}$	$S_2$	$5 P_2$	$5 D_1$
${}^2N_{1/2} - {}^4N_{1/2}$	0	0	$P_{2B}^{0,1}$	$P_{3B}^0$	$P_{3B}^1$	$5 D^{0,1}$	0	$P_3$	$-5 D_2$
${}^2N_{3/2}$	$S^0$	$S^1$	$-P_{2B}^{0,1}$	0	$2 P_{3B}^1$	0	$S_1$	$P_1$	0
${}^4N_{3/2}$	$-S^0$	$S'^1$	$2 P_{2B}^{0,1}$	0	0	$-4 D^{0,1}$	$S_2$	$2 P_2$	$-4 D_1$
${}^2N_{3/2} - {}^4N_{3/2}$	0	0	$\sqrt{\frac{5}{2}} P_{2B}^{0,1}$	$D^0 = -\frac{2}{5} \frac{\alpha^5}{\sqrt{\pi}} J_4$		$-\sqrt{\frac{5}{2}} D^{0,1}$	0	$\sqrt{\frac{5}{2}} P_3$	$\sqrt{\frac{5}{2}} D_2$
${}^4N_{5/2}$	$-S^0$	$S'^1$	$-3 P_{2B}^{0,1}$			$D$	Linear combination of $c_i$		
${}^2\Delta_{1/2}$	$S'^0$	$S'^1$	0	$-2 P_{3B}^0$	$2 P_{3B}^1$		0	$S_3$	$-2 P_4$
${}^2\Delta_{3/2}$	$S'^0$	$S'^1$	0	$P_{3B}^0$	$-P_{3B}^1$	$D$	0	$S_3$	$P_4$

Table 1: General structure of the mass operator

$$\text{where } J_4^0 = \int_0^\infty \rho^4 v_t^0(\sqrt{2}\rho) e^{-\alpha^2 \rho^2} d\rho$$

# The matching

	$V_{ss}^0$	$V_{ss}^1$	$V_{so-2B}^{0,1}$	$V_{so-3B}^0$	$V_{so-3B}^1$	$V_t^{0,1}$	$O_{\ell=0}$	$O_{\ell=1}$	$O_{\ell=2}$
${}^2N_{1/2}$	$S^0$	$S^1$	$2 P_{2B}^{0,1}$	0	$-4 P_{3B}^1$	0	$S_1$	$-2 P_1$	0
${}^4N_{1/2}$	$-S^0$	$S'^1$	$5 P_{2B}^{0,1}$	0	0	$5 D^{0,1}$	$S_2$	$5 P_2$	$5 D_1$
${}^2N_{1/2} - {}^4N_{1/2}$	0	0	$P_{2B}^{0,1}$	$P_{3B}^0$	$P_{3B}^1$	$5 D^{0,1}$	0	$P_3$	$-5 D_2$
${}^2N_{3/2}$	$S^0$	$S^1$	$-P_{2B}^{0,1}$	0	$2 P_{3B}^1$	0	$S_1$	$P_1$	0
${}^4N_{3/2}$	$-S^0$	$S'^1$	$2 P_{2B}^{0,1}$	0	0	$-4 D^{0,1}$	$S_2$	$2 P_2$	$-4 D_1$
${}^2N_{3/2} - {}^4N_{3/2}$	0	0	$\sqrt{\frac{5}{2}} P_{2B}^{0,1}$	$D^0 = -\frac{2}{5} \frac{\alpha^5}{\sqrt{\pi}} J_4$		$-\sqrt{\frac{5}{2}} D^{0,1}$	0	$\sqrt{\frac{5}{2}} P_3$	$\sqrt{\frac{5}{2}} D_2$
${}^4N_{5/2}$	$-S^0$	$S'^1$	$-3 P_{2B}^{0,1}$	$D$		Linear combination of $c_i$			
${}^2\Delta_{1/2}$	$S'^0$	$S'^1$	0	$-2 P_{3B}^0$	$2 P_{3B}^1$	0	$S_3$	$-2 P_4$	0
${}^2\Delta_{3/2}$	$S'^0$	$S'^1$	0	$P_{3B}^0$	$-P_{3B}^1$	0	$S_3$	$P_4$	0

Table 1: General structure of the mass operator



$$D_1 + D_2 = 0$$

where  $J_4^0 = \int_0^\infty \rho^4 v_t^0(\sqrt{2}\rho) e^{-\alpha^2 \rho^2} d\rho$

# The matching

	$V_{ss}^0$	$V_{ss}^1$	$V_{so-2B}^{0,1}$	$V_{so-3B}^0$	$V_{so-3B}^1$	$V_t^{0,1}$	$O_{\ell=0}$	$O_{\ell=1}$	$O_{\ell=2}$
${}^2N_{1/2}$	$S^0$	$S^1$	$2 P_{2B}^{0,1}$	0	$-4 P_{3B}^1$	0	$S_1$	$-2 P_1$	0
${}^4N_{1/2}$	$-S^0$	$S'^1$	$5 P_{2B}^{0,1}$	0	0	$5 D^{0,1}$	$S_2$	$5 P_2$	$5 D_1$
${}^2N_{1/2} - {}^4N_{1/2}$	0	0	$P_{2B}^{0,1}$	$P_{3B}^0$	$P_{3B}^1$	$5 D^{0,1}$	0	$P_3$	$-5 D_2$
${}^2N_{3/2}$	$S^0$	$S^1$	$-P_{2B}^{0,1}$	0	$2 P_{3B}^1$	0	$S_1$	$P_1$	0
${}^4N_{3/2}$	$-S^0$	$S'^1$	$2 P_{2B}^{0,1}$	0	0	$-4 D^{0,1}$	$S_2$	$2 P_2$	$-4 D_1$
${}^2N_{3/2} - {}^4N_{3/2}$	0	0	$\sqrt{\frac{5}{2}} P_{2B}^{0,1}$	$D^0 = -\frac{2}{5} \frac{\alpha^5}{\sqrt{\pi}} J_4$		$-\sqrt{\frac{5}{2}} D^{0,1}$	0	$\sqrt{\frac{5}{2}} P_3$	$\sqrt{\frac{5}{2}} D_2$
${}^4N_{5/2}$	$-S^0$	$S'^1$	$-3 P_{2B}^{0,1}$			$D$	Linear combination of $c_i$		
${}^2\Delta_{1/2}$	$S'^0$	$S'^1$	0	$-2 P_{3B}^0$	$2 P_{3B}^1$	0	$S_3$	$-2 P_4$	0
${}^2\Delta_{3/2}$	$S'^0$	$S'^1$	0	$P_{3B}^0$	$-P_{3B}^1$	0	$S_3$	$P_4$	0

Table 1: General structure of the mass operator

We found two relations



$$D_1 + D_2 = 0$$

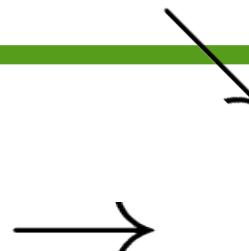
$$P_1 + 2P_2 - P_3 + P_4 = 0$$

# The matching

We found that only 7 operators  $O_i$  are needed

$$M_{matched} = \sum_{i=1}^7 c_i O_i = O_{\ell=0} + O_{\ell=1} + O_{\ell=2}$$

7 baryons masses  
2 mixing angles  
for L=1, non-strange



2 Mass-angles  
relations

# Mass-angle relations

$$\begin{aligned} & \frac{1}{2}(N_{1/2} - N'_{1/2})(3 \cos 2\theta_1 + \sin 2\theta_1) + (N_{3/2} - N'_{3/2}) \left( -\frac{3}{5} \cos 2\theta_3 + \sqrt{\frac{5}{2}} \sin 2\theta_3 \right) \\ &= -\frac{1}{2}(N_{1/2} + N'_{1/2}) + \frac{7}{5}(N_{3/2} + N'_{3/2}) - \frac{9}{5}N_{5/2} - 2\Delta_{1/2} + 2\Delta_{3/2}. \end{aligned}$$

$$\begin{aligned} & 5(N_{1/2} - N'_{1/2})(\cos 2\theta_1 + 2 \sin 2\theta_1) - 4(N_{3/2} - N'_{3/2}) \left( 2 \cos 2\theta_3 + \sqrt{\frac{5}{2}} \sin 2\theta_3 \right) \\ &= 5(N_{1/2} + N'_{1/2}) - 8(N_{3/2} + N'_{3/2}) + 6N_{5/2}. \end{aligned}$$

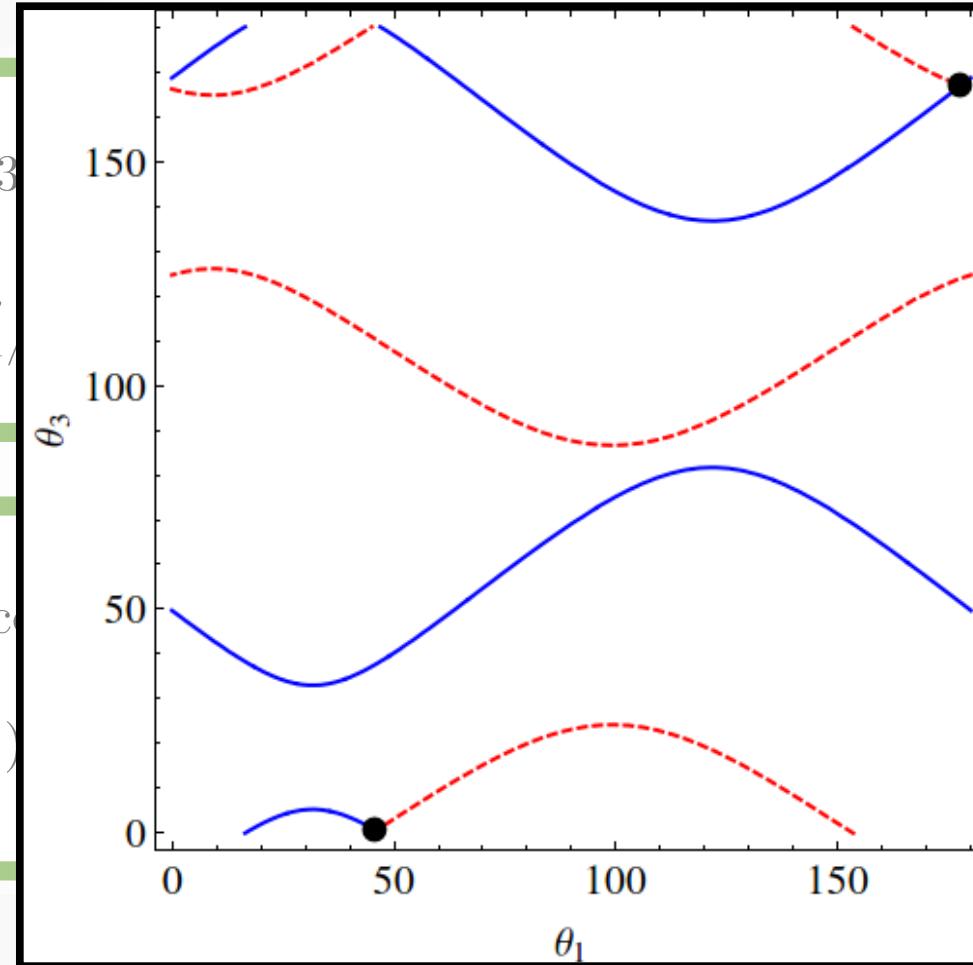
# Mass-angle relations

$$\frac{1}{2}(N_{1/2} - N'_{1/2})(3)$$

$$= -\frac{1}{2}(N_{1/2} + N'_{1/2})$$

$$5(N_{1/2} - N'_{1/2})(c)$$

$$= 5(N_{1/2} + N'_{1/2})$$



$$2\theta_3 + \sqrt{\frac{5}{2}} \sin 2\theta_3 \Big)$$

$\Delta_{3/2} \cdot$

$$2\theta_3 + \sqrt{\frac{5}{2}} \sin 2\theta_3 \Big)$$

Correlations in the  $(\theta_1, \theta_3)$  plane in the quark model with a general 2-body quark interaction.

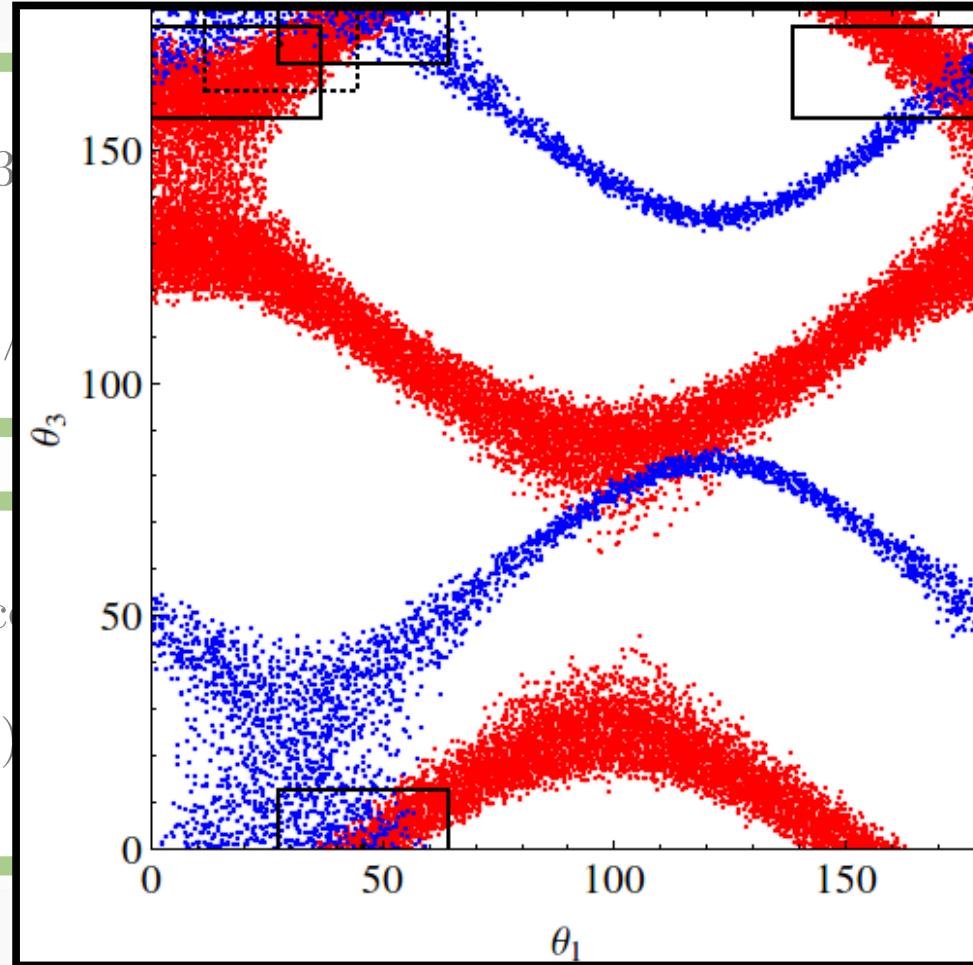
# Mass-angle relations

$$\frac{1}{2}(N_{1/2} - N'_{1/2})(3\theta_3 + \sqrt{\frac{5}{2}} \sin 2\theta_3)$$

$$= -\frac{1}{2}(N_{1/2} + N'_{1/2})\Delta_{3/2} \cdot (2\theta_3 + \sqrt{\frac{5}{2}} \sin 2\theta_3)$$

$$5(N_{1/2} - N'_{1/2})(c\theta_3 + \sqrt{\frac{5}{2}} \sin 2\theta_3)$$

$$= 5(N_{1/2} + N'_{1/2})\Delta_{3/2} \cdot (2\theta_3 + \sqrt{\frac{5}{2}} \sin 2\theta_3)$$



Correlations in the  $(\theta_1, \theta_3)$  plane in the quark model with a general 2-body quark interaction.

# Mass-angle relations: OPE and OGE

In the NR limit, the 2-body interactions contain three terms

$$H = H_0 + \sum_{i < j} V_{ij} = H_0 + V_{ss} + V_{so} + V_t .$$

$$V_{ss} = V_{ss}^0 + V_{ss}^1 = \sum_{i < j=1}^N v_{ss}(r_{ij}) \vec{s}_i \cdot \vec{s}_j ,$$

$$v_\alpha = v_\alpha^0(r_{ij}) + v_\alpha^1(r_{ij}) \tau_i^a \tau_j^a$$

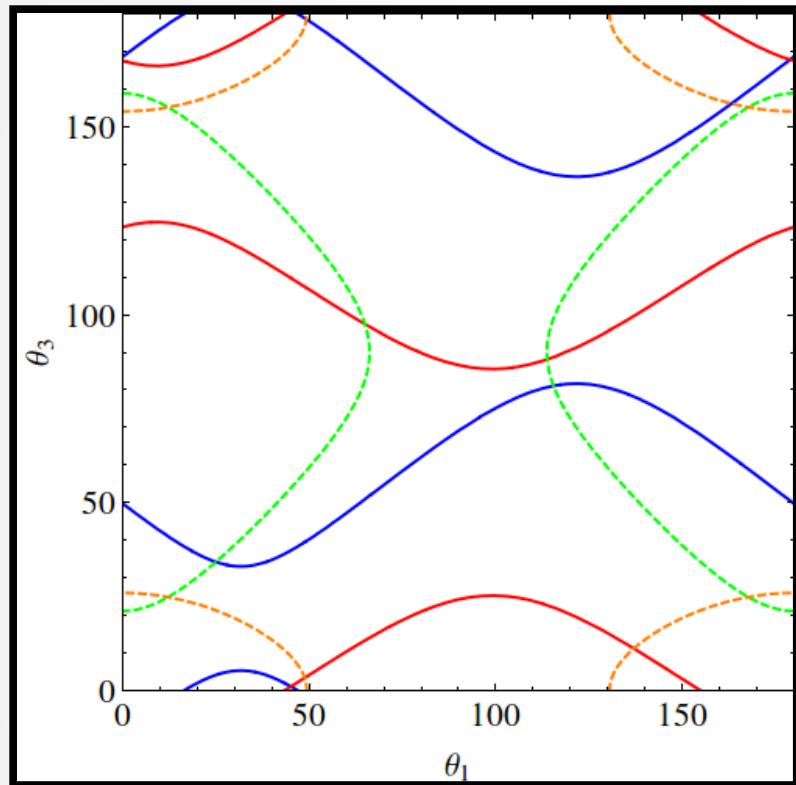
$$V_{so} = V_{so}^0 + V_{so}^1 = \sum_{i < j=1}^N v_{so}(r_{ij}) \left[ (\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_i - (\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j \right.$$

$$\left. + 2(\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_j - 2(\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_i \right] ,$$

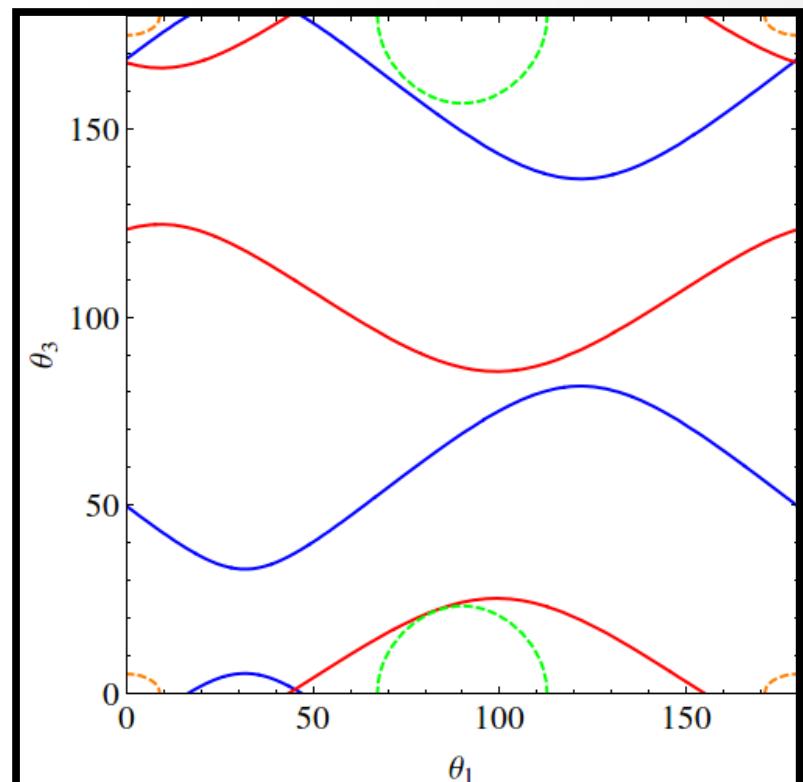
$$V_t = V_t^0 + V_t^1 = \sum_{i < j=1}^N v_t(r_{ij}) \left[ 3(\hat{r}_{ij} \cdot \vec{s}_i)(\hat{r}_{ij} \cdot \vec{s}_j) - (\vec{s}_i \cdot \vec{s}_j) \right] .$$

Setting  $v_\alpha^1 = 0$  ( $v_\alpha^0 = 0$ ) we obtain a OGE (OPE) interaction and we find **two additional mass-angle relations**

# Mass-angle relations: OPE and OGE



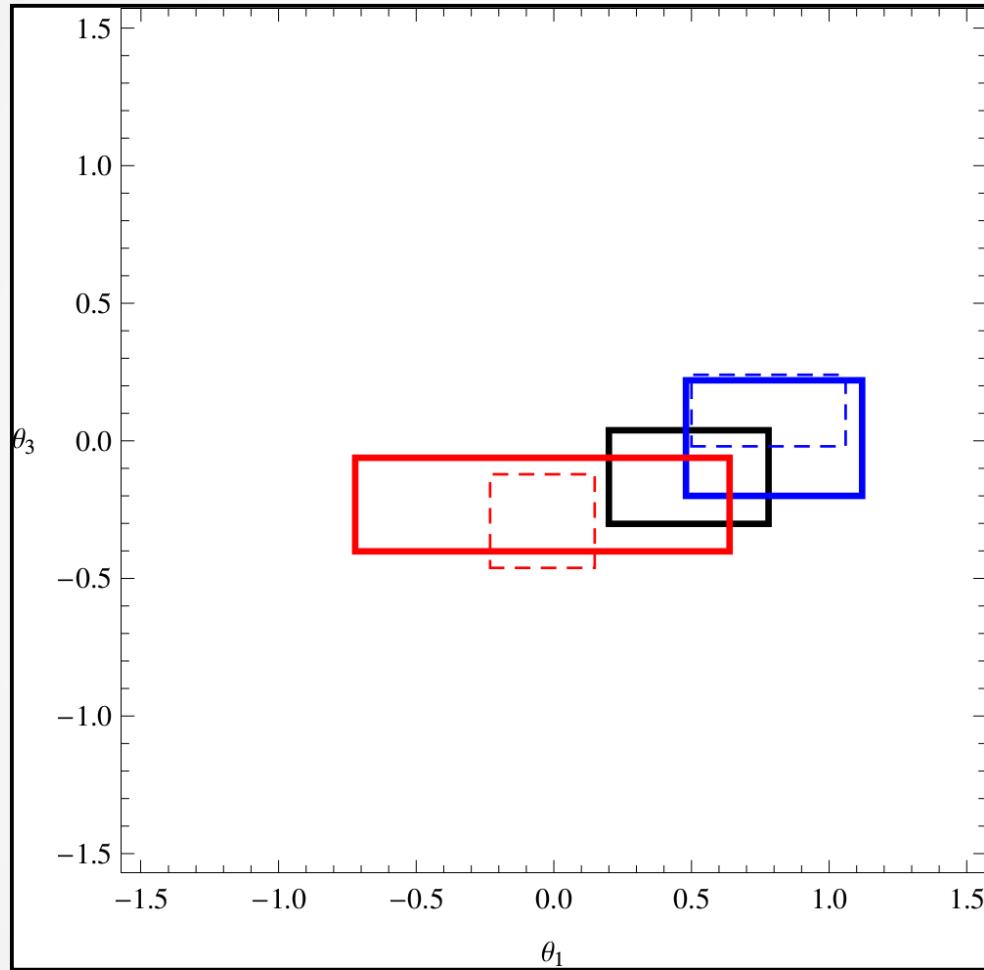
Four mass-angle relations in the  $(\theta_1, \theta_3)$  with an OGE interaction



Four mass-angle relations in the  $(\theta_1, \theta_3)$  with an OPE interaction

Setting  $v_\alpha^1 = 0$  ( $v_\alpha^0 = 0$ ) we obtain a OGE (OPE) interaction and we find **two additional mass-angle relations**

# Mass-angle relations



Correlations in the  $(\theta_1 \theta_3)$  plane for:  
a general 2-body quark interaction  
(filled red and blue lines)  
OPE model (red dashed line)  
OGE model (blue dashed line)  
Global fit [4] (filled black lines)



$1/N_c$  analysis,  $c_i$  are fitted  
to the experimental data

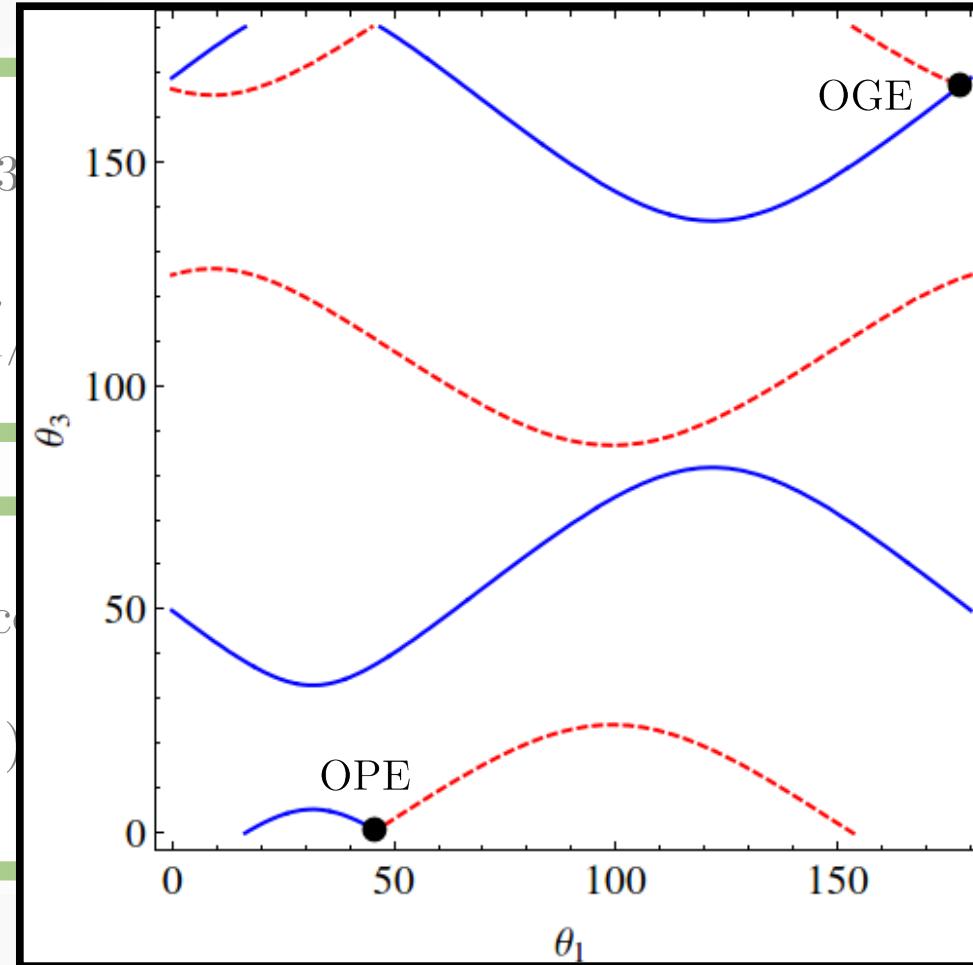
# Mass-angle relations

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$$2\theta_3 + \sqrt{\frac{5}{2}} \sin 2\theta_3 \Big)$$

$\Delta_{3/2} \cdot$

$$2\theta_3 + \sqrt{\frac{5}{2}} \sin 2\theta_3 \Big)$$

Correlations in the  $(\theta_1, \theta_3)$  plane in the quark model with a general 2-body quark interaction.

# Summary

- Large  $N_c$  is a tool that can be used to describe QCD phenomenology at non-perturbative energies
- Large  $N_c$  allows to do a global fit of the coefficients of the  $1/N_c$  expansion as was done in [4]
- Tool to compare models in a systematic way ( $c_i$  can be calculated explicitly)

In our work..

- We matched different versions of the NR quark model to the  $1/N_c$  quark operator basis. And we obtained analytical and parameter free relations between the masses and angles of the L=1 baryons

[4] E. Gonzalez de Urreta, J. L. Goity, N. N. Scoccola, Phys.Rev. **D89** (2014) 3, 034024

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