Seminar talk, Jefferson Laboratory, 9 Sept 2015

Proton radius puzzle in Hamiltonian dynamics

Stanisław D. Głazek

Department of Physics, University of Warsaw

A non-perturbative proton-size correction to atomic ground state energy in QED is discussed using the renormalization group procedure for effective particles (RGPEP). The correction results from the scale-dependence of effective few-body theory. The correction magnitude appears relevant to the proton radius puzzle. Prospects of application of the RGPEP for calculating few-body effective approximations to quantum field theories, in particular to QCD for the purpose of deriving the quark model, are briefly mentioned in the context of duality.

stglazek@fuw.edu.pl

Selected references specifically for this talk content

1. R. Pohl, R. Gilman, G. Miller, K. Pachucki, *Muonic hydrogen and the proton radius puzzle,* Ann. Rev. Nucl. Part. Sci. **63**, 175 (2013).
2. K. Pachucki, K. Meissner, *Proton charge radius and the perturbative quantum electrodynamics*, arXiv:1405.6582 [hep- ph].
3. S. Głazek, *Calculation of size for bound-state constituents,* Phys. Rev. D **90**, 045020 (2014).
4. S. Głazek, *Proton Radius Puzzle in Hamiltonian Dynamics,* Few-Body Syst. **56**, 311 (2015).
5. M. Gómez-Rocha, S. Głazek, *Asymptotic freedom in the front-form Hamiltonian for quantum chromodynamics of gluons*, arXiv:1505.06688; Phys. Rev. D, in print.
6. G. de Teramond, S. Brodsky, *Hadronic spectrum of a holographic dual of QCD,* Phys. Rev. Lett. **94**, 201601 (2005).
7. S. Głazek, A. Trawiński, *Model of the AdS/QFT duality*, Phys. Rev. D **88**, 105025 (2013).
8. A. Trawiński, S. Głazek, S. Brodsky, G. de Tramond, H. Dosch, *Effective confining potentials for QCD,* Phys. Rev. D **90**, 074017 (2014).

Outline

spinoff from QCD studies

.

1. What is the proton radius puzzle? $r_p(H) - r_p(H_\mu) \sim 4\%$
2. Key question: QFT $\leftrightarrow$ Schrödinger QM, at precision $\sim \alpha^6$Ry ?
3. **R**enormalization **G**roup **P**rocedure for **E**ffective **P**articles (**RGPEP**)
4. The "true" Schrödinger equation involves a **scale** parameter, λ
5. The Schrödinger variables correspond to **AdS/QFT variables**
6. **Bound-state constituents** $\leftrightarrow$ **RGPEP effective particles**
7. **Scale** dependence $\rightarrow \sim 4\%$ lepton-mass effects in extracted r_p

stglazek@fuw.edu.pl

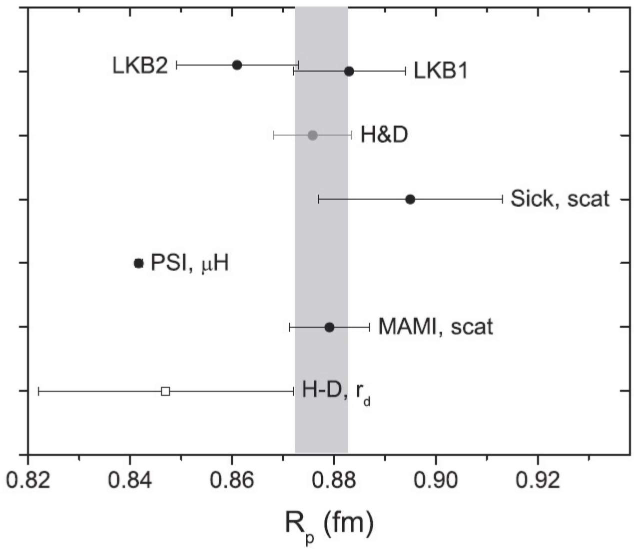
Proton radius puzzle

In the muon-proton atoms, proton appears $\sim$**4%** smaller than in hydrogen.

R. Pohl, R. Gilman, G. A. Miller, K. Pachucki, Ann. Rev. Nucl. Part. Sci. **63**, 175 (2013):

"... [the proton radius] *should be a simple quantity to determine and understand, but that is not the case. Recent experimental results are not yet well understood, but future research may reveal the true value of this radius, lead to a better understanding of its structure, or demonstrate an unexpected aspect of its interactions."*

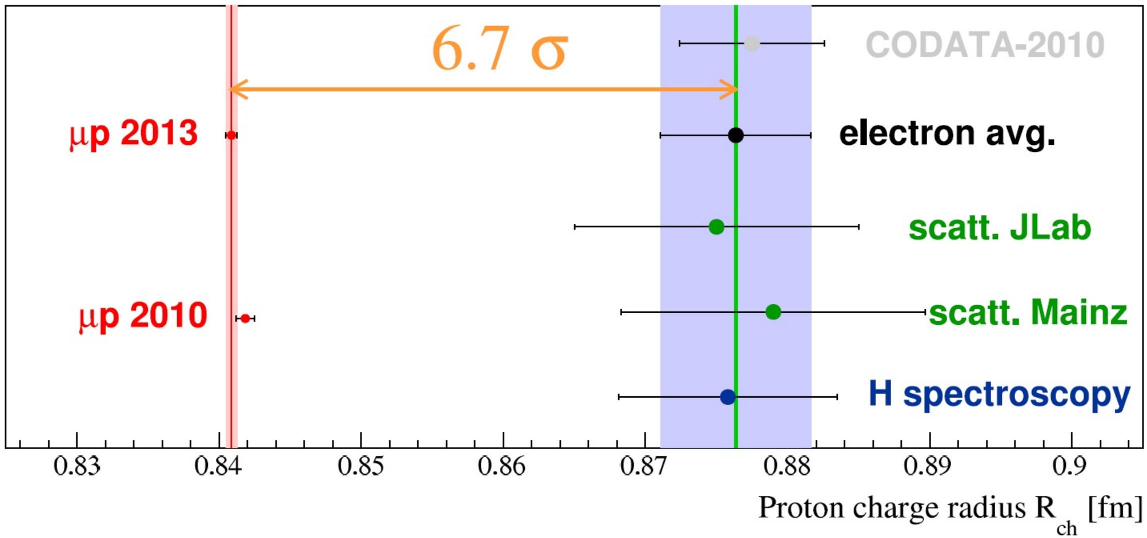
S. G. Karshenboim, Phys. Rev. A **91**, 012515 (30 January 2015):



J. Arrington, arXiv:1506.00873 (2 June 2015):

.

figure referenced as "courtesy of Randolf Pohl."



stglazek@fuw.edu.pl

Meaning of the puzzle

Schrödinger eigenvalue equation in lepton-proton relative motion

$$\frac{\vec{p}^{\,2}}{2\mu}\,\psi(\vec{p}) + \int \frac{d^3k}{(2\pi)^3}\, V(\vec{p},\vec{k}\,)\,\psi(\vec{k}\,) = -E\,\psi(\vec{p})$$

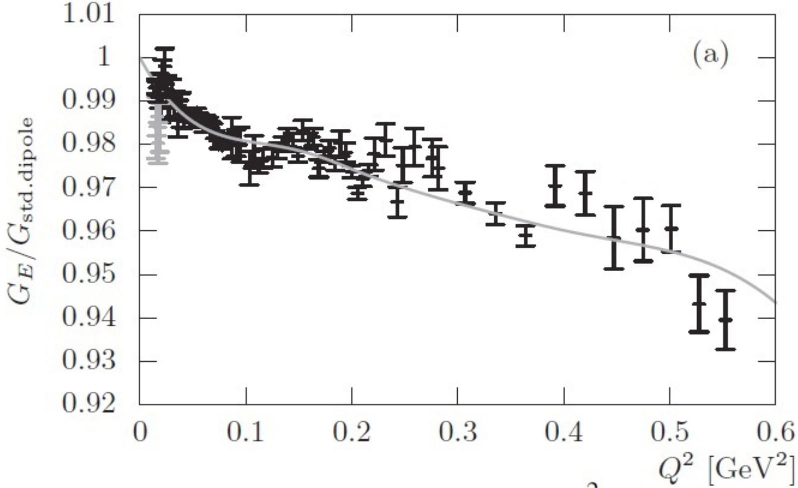
$$V(\vec{p},\vec{k}\,) = V_C^{\rm pt}(\vec{q}) = -\,\frac{4\pi\alpha}{\vec{q}^{\,2}} \qquad\qquad \vec{q} = \vec{p} - \vec{k}$$

Extended proton charge distribution implies

$$V_C(\vec{q}) = V_C^{\rm pt}(\vec{q})\; G_E(\vec{q}^{\,2})$$

J. C. Bernauer *et al.*, Phys. Rev. C **90**, 015206 (2014):

.



$$G_{\text{standard dipole}}(Q^2) = \left(1 + \frac{Q^2}{0.71\ \text{GeV}^2}\right)^{-2}$$

$$G_E(\vec{q}^{\,2}) = 1 - \frac{1}{6}\, r_p^2\, \vec{q}^{\,2} + o(\vec{q}^{\,2})$$

$$\leftarrow \; e^{i\,\vec{q}\,\vec{r}}$$

$$V(\vec{p},\vec{k}\,) = -\,\frac{4\pi\alpha}{\vec{q}^{\,2}}\, G_E(\vec{q}^{\,2}) \;=\; V_C^{\rm pt}(\vec{p},\vec{k}\,) + \delta V(\vec{p},\vec{k}\,)$$

$$\delta V(\vec{p},\vec{k}\,) = \frac{2\pi\alpha}{3}\, r_p^2 \qquad\qquad \delta V(\vec{r}\,) \;=\; \frac{2\pi\alpha}{3}\, r_p^2\; \delta^3(\vec{r}\,)$$

stglazek@fuw.edu.pl

Correction to energy

$$\Delta E = \frac{2\pi\alpha}{3}\, r_p^2\, |\hat{\psi}(0)|^2 \qquad \hat{\psi}(0) = \int \frac{d^3k}{(2\pi)^3}\, \psi(\vec{k})$$

$$|\hat{\psi}(0)|^2 = (\alpha\mu)^3/\pi \qquad \Delta E = \frac{4}{3}\, (\alpha\mu\, r_p)^2\, \frac{\mu\alpha^2}{2}$$

$$\alpha\mu_H = 1/r_B \qquad \Delta E = \frac{4}{3}\, (r_p/r_B)^2\, \frac{\mu\alpha^2}{2} \qquad r_p/r_B \sim 10^{-5}$$

$$m_\mu \sim 200\; m_e \qquad \boxed{r_{p/\mu} \neq r_{p/e}}$$

stglazek@fuw.edu.pl

Key questions

.

How does the Schrödinger equation precisely emerge from QED?

Can it be accurate without any limitation on $|\vec{p}\,|$?

Is it correctable?

Answers suggested in this talk are based on S. D. Głazek, PRD90, 045020 (2014), *a la* RGPEP in QCD.

stglazek@fuw.edu.pl

From QFT to the Schrödinger equation

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{n=1}^{3} \bar{\psi}_n (i\partial\!\!\!/ - e_n A\!\!\!/ - m_n)\psi$$

$n = 1, 2, 3$ refer to electron, muon and proton

method developed for QCD

$$\mathcal{L} \to \mathcal{H} \to \hat{H} = \int d^3x \, \hat{\mathcal{H}} \to \hat{H}_R = \int d^3x \, \hat{\mathcal{H}}_R \to \hat{H}_R + \hat{H}_{CT} \to \hat{H}_\lambda$$

$\boxed{\text{Wilsonian RG} \leftrightarrow \text{family of effective Hamiltonians } \hat{H}_\lambda}$

$\lambda = \infty$ in canonical theory

Bound-state renormalization: $\boxed{\text{Wilsonian RG} \to \text{SRG} \to \text{RGPEP}}$

λ in the RGPEP $\boxed{\hat{H}_\lambda |\psi_\lambda\rangle = E |\psi_\lambda\rangle \quad \to \quad \hat{H}_{Sch} = ?}$

How does the parameter λ enter in $\hat{H}_{Sch}$ in the RGPEP?

stglazek@fuw.edu.pl

The RGPEP interpretation of bound-state constituents

$$\begin{bmatrix} \dots \\ .. \\ e\gamma e\bar{e}p\gamma \\ e\gamma p\gamma \\ e\gamma p \\ \boxed{ep} \end{bmatrix} \rightarrow \begin{bmatrix} \dots \\ .. \\ e\gamma e\bar{e} \\ e\gamma\gamma \\ e\gamma \\ e \end{bmatrix} \times \begin{bmatrix} \dots \\ .. \\ pe\bar{e} \\ p\gamma\gamma \\ p\gamma \\ p \end{bmatrix} + \dots \quad \sim \quad |e_\lambda p_\lambda\rangle + \dots$$

$$\psi_2(e_{can}, p_{can}) + \infty \quad \rightarrow \quad \psi_{2\lambda}(e_\lambda, p_\lambda) + corrections$$

A. Trawiński, S. Głazek, *Model of the AdS/QFT duality,* Phys. Rev. D **88**, 105025 (2013)

stglazek@fuw.edu.pl

RGPEP $\mathcal{L} \to \mathcal{H} \to \hat{H} = \int d^3x \, \hat{\mathcal{H}}$

quantization $\psi, A \to \hat{\psi}_{can}, \hat{A}_{can}$ $H \to \hat{H}$

New step: change of bare quanta to effective quanta

$\lambda = \infty \leftrightarrow$ pointlike bare canonical quanta

$\lambda = \lambda_0 \leftrightarrow$ effective quanta of size $1/\lambda_0$

$\hat{\psi}_\lambda = \mathcal{U}_\lambda \, \hat{\psi}_{can} \mathcal{U}_\lambda^\dagger$ $\hat{A}_\lambda = \mathcal{U}_\lambda \, \hat{A}_{can} \mathcal{U}_\lambda^\dagger$ $\mathcal{U}_\infty = 1$

Hamiltonian as an operator kept unchanged

$$H_\lambda(\hat{\psi}_\lambda, \hat{A}_\lambda) = H_\infty(\hat{\psi}_{can}, \hat{A}_{can}) \quad \to \quad H_\lambda(\hat{\psi}_{can}, \hat{A}_{can}) = \mathcal{U}_\lambda^\dagger \, H_\infty(\hat{\psi}_{can}, \hat{A}_{can}) \, \mathcal{U}_\lambda$$

RGPEP equation for H_λ

.

$$\mathcal{H}_\lambda = H_\lambda(\hat\psi_{can}, \hat A_{can}) \ = \mathcal{U}_\lambda^\dagger \, H_\infty(\hat\psi_{can}, \hat A_{can}) \, \mathcal{U}_\lambda$$

differentiation with respect to λ (actually, wrt $1/\lambda^4$)

$$\mathcal{H}'_\lambda = [\mathcal{G}_\lambda, \mathcal{H}_\lambda]$$

generator $\mathcal{G}_\lambda = -\mathcal{U}_\lambda^\dagger \mathcal{U}'_\lambda$ and $\mathcal{U}_\lambda = T_\rho \exp\left(- \int_\infty^\lambda d\rho \, \mathcal{G}_\rho \right)$

$$\boxed{\text{Non-perturbative definition of the effective theory } H_\lambda = H_0 + H_{I\lambda}}$$

$$\mathcal{G}_\lambda \sim [\mathcal{H}_0, \mathcal{H}_\lambda]$$

Similar to Wegner's equation. **Why the double commutator?**

stglazek@fuw.edu.pl

RGPEP vertex form factor

$$H_\lambda = H_0 + H_{I\lambda}$$

$$\mathcal{H}'_\lambda = [\mathcal{G}_\lambda, \mathcal{H}_\lambda] \qquad \mathcal{G}_\lambda = [\mathcal{H}_0, \mathcal{H}_{I\lambda}]$$

$$\mathcal{H}'_\lambda = [[\mathcal{H}_0, \mathcal{H}_\lambda], \mathcal{H}_\lambda] = [[\mathcal{H}_0, \mathcal{H}_{I\lambda}], \mathcal{H}_0] + O(\mathcal{H}^2_{I\lambda})$$

$$\langle m|\mathcal{H}'_{I\lambda}|n\rangle = -(E_m - E_n)^2 \, \langle m|\mathcal{H}_{I\infty}|n\rangle + O(\mathcal{H}^2_I)$$

$$\langle m|\mathcal{H}_{I\lambda}|n\rangle = e^{-(E_m-E_n)^2/\lambda^2}\langle m|\mathcal{H}_{I\infty}|n\rangle + O(\mathcal{H}^2_{I\infty})$$

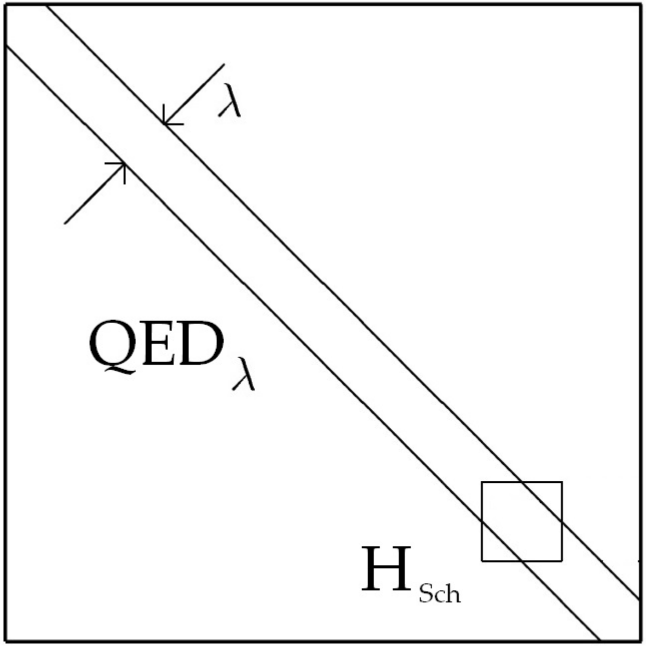
Additional dynamical features of the RGPEP lead to the Lorentz symmetry:

$$E_m \quad \to \quad \mathcal{M}^2_m \quad \text{and} \quad e^{-(E_m-E_n)^2/\lambda^2} \quad \to \quad e^{-(\mathcal{M}^2_m-\mathcal{M}^2_n)^2/\lambda^4}$$

stglazek@fuw.edu.pl

Effective Hamiltonian

.



stglazek@fuw.edu.pl

"True" Schrödinger equation, corresponding to $\hat{H}_\lambda|\psi_\lambda\rangle = E|\psi_\lambda\rangle$

$$\frac{\vec{p}^{\,2}}{2\mu}\,\psi(\vec{p}) + \int \frac{d^3k}{(2\pi)^3}\, V_\lambda(\vec{p},\vec{k}\,)\,\psi(\vec{k}\,) = -E_B\,\psi(\vec{p})$$

$$V_\lambda(\vec{p},\vec{k}\,) = f_\lambda(\vec{p},\vec{k}\,)\; V_C^{\rm pt}(\vec{q}\,)\; G_E(\vec{q}^{\,2})$$

definitions of $\vec{p}$ and $\vec{k}$ match **AdS/QFT** SDG, Acta Phys. Pol. B **42**, 1933 (2011)

$$\beta = m_l/(m_p + m_l)\ , \qquad c = \sqrt{m_l m_p}/(m_l + m_p)\ , \qquad x = p_l^+/(p_l^+ + p_p^+)$$

$$p^\perp = c\ \left[(1-x)\,p_l^\perp - x\,p_p^\perp\right]/\sqrt{x(1-x)} \qquad \text{Brodsky} - \text{de Teramond}$$

$$p^z = c\,(m_l + m_p)\,(x-\beta)/\sqrt{x(1-x)} \qquad \text{holography}$$

stglazek@fuw.edu.pl

$$\mathcal{M}_{lp}^2 = \frac{m_l^2 + p^{\perp\,2}}{x} + \frac{m_p^2 + p^{\perp\,2}}{1-x}$$

$$= (m_l + m_p)^2 + \frac{m_l + m_p}{\mu}\,\vec{p}^{\,2}$$

stglazek@fuw.edu.pl

Scale-sensitive potential in quantum mechanics $\alpha_\lambda = \alpha$

$$V_\lambda(\vec{p},\vec{k}\,) = f_\lambda(\vec{p},\vec{k}\,)\, V_C^{\rm pt}(\vec{q}\,)\, G_E(\vec{q}^{\,2})$$

$$f_\lambda(\vec{p},\vec{k}\,) = e^{-(\vec{p}^2-\vec{k}^2)^2(m_l+m_p)^4/(\lambda^2 m_l m_p)^2}$$

To match universal α-scaling of the atomic Schrödinger equation

$$\lambda = a\sqrt{\mu\,(m_l+m_p)/2} \qquad\qquad a \sim 1$$

$$f_\lambda(\vec{p},\vec{k}\,) = e^{-(\vec{p}^{\,2}-\vec{k}^{\,2})^2/(a\mu)^4}$$

stglazek@fuw.edu.pl

Energy correction due to the proton radius

$$\delta V(\vec{p},\vec{k}\,) = f_\lambda(\vec{p},\vec{k}\,)\,\frac{2\pi\alpha}{3}\,r_p^2$$

$$\Delta E = d_a\,\frac{2\pi\alpha}{3}\,r_p^2\,|\hat{\psi}(0)|^2$$

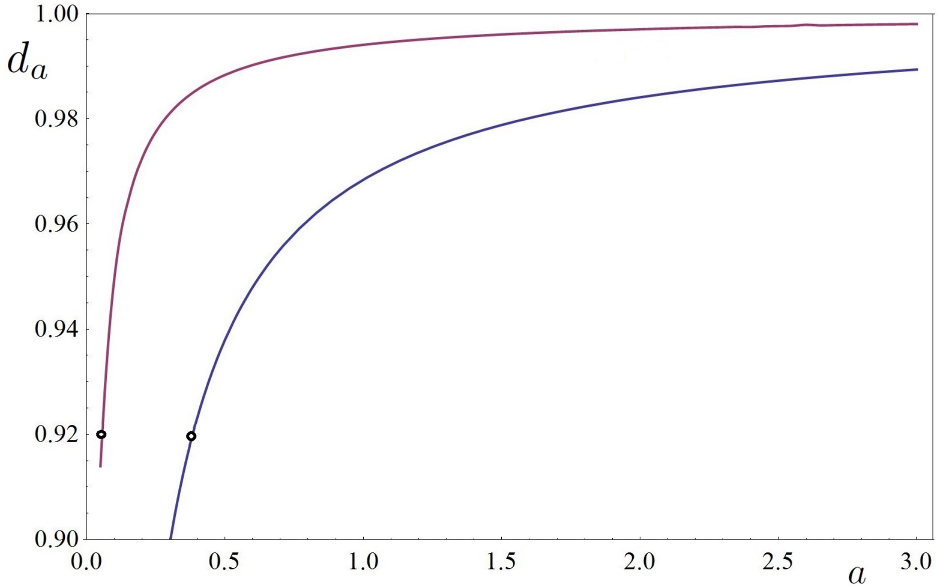
$$d_a = \frac{1}{|\hat{\psi}(0)|^2}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\,\psi(\vec{p}\,)\,f_\lambda(\vec{p},\vec{k}\,)\,\psi(\vec{k}\,)$$

$$f_\lambda(\vec{p},\vec{k}\,) = e^{-(\vec{p}^{\,2}-\vec{k}^{\,2})^2/(a\mu)^4}$$

Despite $e^{-c\alpha^4}$, **the correction is not pertubative,**

i.e., not expandable in powers of α.

stglazek@fuw.edu.pl



$$\lambda = a\sqrt{\mu\,(m_l + m_p)/2}$$

$$\boxed{a_e \sim (\sqrt{m_\mu/m_e} \sim 14)\ a_\mu}$$

Why two curves?

Conclusion

.

- The RGPEP yields an effective Schrödinger equation
- for bound states made of effective, low-energy leptons and protons.
- The Coulomb potential in the effective theory develops
- a nonlocality at short distances that matters in extraction of

  the proton radius with % accuracy.
- The RGPEP suggests a quantum mechanism for AdS/QFT duality.
- The RGPEP is available for studying QCD and other theories.

stglazek@fuw.edu.pl

$$P^- = \int dx^- d^2x^\perp \left\{ \frac{1}{2} A_\mu \, \partial^{\perp\,2} A^\mu + \sum_{n=1}^{3} \left[\bar\psi_n \gamma^+ \frac{-\partial^{\perp\,2} + m_n^2}{2i\partial^+} \, \psi_n + e_n \, \bar\psi_n \not{A} \, \psi_n \right.\right.$$

$$\left.\left. + \; e_n^2 \, \bar\psi_n \not{A} \, \frac{\gamma^+}{2i\partial^+} \, \not{A} \, \psi_n + e_n \bar\psi_n \gamma^+ \psi_n \, \frac{1}{2(i\partial^+)^2} \, \sum_{k=1}^{3} e_k \, \bar\psi_k \gamma^+ \psi_k \right] \right\}$$

stglazek@fuw.edu.pl

$$\hat{\psi}_{n+}(x) = \sum_s \int_p \Delta^{1/2}(p) \sqrt{p^+} \, \left[b_{nps} - d^\dagger_{nps} \, \sigma^1 \right] \begin{bmatrix} \chi_s \\ 0 \end{bmatrix} e^{-ic_p p \, x - \epsilon |x|}$$

$$\hat{A}^\perp(x) = \sum_s \int_p \Delta^{1/2}(p) \left[a_{ps} \, \varepsilon^\perp_s + a^\dagger_{ps} \, \varepsilon^{\perp *}_s \right] e^{-ic_p p \, x - \epsilon |x|}$$

integration measure

$$\int_p = \int \frac{dp^+ \; d^2 p^\perp}{2p^+ (2\pi)^3} \, \theta(p^+)$$

regularization factor

$$\Delta(p) \; = \Delta \left(|p^+|, |p^\perp| \right)$$

stglazek@fuw.edu.pl

$$\mathcal{H}'_\lambda = [[\mathcal{H}_0, \tilde{\mathcal{H}}_\lambda], \mathcal{H}_\lambda] \tag{1}$$

$$\mathcal{H}_\lambda = \sum_{n=2}^{\infty} \sum_{i_1, i_2, ..., i_n} c_\lambda(i_1, ..., i_n) \; q^\dagger_{\infty i_1} \cdots q_{0 i_n}$$

$$\tilde{\mathcal{H}}_\lambda = \sum_{n=2}^{\infty} \sum_{i_1, i_2, ..., i_n} c_\lambda(i_1, ..., i_n) \; \left(\frac{1}{2} \sum_{k=1}^{n} p^+_{i_k} \right)^2 \; q^\dagger_{\infty i_1} \cdots q_{0 i_n}$$

This ensures $\mathcal{H}_\lambda$ to possess 7 kinematical symmetries of the FF dynamics.

Perturbative expansion of the RGPEP in charge e

.

$$\mathcal{H}_t = \mathcal{H}_f + e\mathcal{H}_t^{(1)} + e^2\mathcal{H}_t^{(2)} + \ldots$$

$$\mathcal{H}_{t\,ab}^{(1)} = f_{ab}\,\mathcal{H}_{0\,ab}^{(1)}$$

$$\mathcal{H}_{t\,ab}^{(2)} = f_{ab}\left[\mathcal{H}_{0\,ab}^{(2)} + \sum_x \mathcal{F}_{axb}\mathcal{H}_{0\,ax}^{(1)}\mathcal{H}_{0\,xb}^{(1)}\right]$$

$$f_{ab} = \exp\left(-t\,ab^2\right)$$

$$\mathcal{F}_{axb} = \frac{p_{ax}\,ax + p_{bx}\,bx}{ax^2 + xb^2 - ab^2}\left[1 - e^{-t(ax^2+xb^2-ab^2)}\right]$$