Charmonium-like and other near-threshold mesons on the lattice

Sasa Prelovsek

JLab Theory Seminar, 23rd February, 2015

University of Ljubljana & Jozef Stefan Institute, Slovenia

February - August 2015
Theory & Computational Physics, JLab (office A206)

in collaboration with:
Christian B. Lang, Luka Leskovec, Daniel Mohler, Richard Woloshyn

Graz Ljubljana FERMILAB TRIUMF

Sasa Prelovsek, JLab 2015
Outline

• Experimental appetizer for states with heavy quarks:
  most interesting experimental states are located near threshold!

• I will discuss hadronic states that have to be address by
  simulating the scattering of two mesons on the lattice

• **States slightly below threshold**: $B_{s0}, B_{s1}, D_{s0}, D_{s1}, X(3872)$
  Flavor according to quark model: $s \ b \ s \ c \ c \ c$
  “deuterium-like” bound states: first lattice simulations of such states in mesonic system
  Only few such states in heavy-meson spectrum; none (to my knowledge) in light-meson spectrum

• **Search for flavor exotic states**: $Z_c^+, c\ c\ d\ u$

• **Resonances above threshold**: $K^*, a_1, b_1, D_0^*, D_1, \Psi(3770)$

• Conclusions
A charged charmonium-like state: \( Z_c^+(3900) \)

\[
Z_c^+(3900) \rightarrow J/\Psi \pi^+ \quad (\bar{c}c \quad du)
\]

confirmed by
BeSiI, Belle, Cleo-c

Sasa Prelovsek, JLab 2015
Charged bottomonium-like states: $Z_b^+(10610)$ and $Z_b^+(10650)$

$Z_b^+ \rightarrow Y(1S) \pi^+$
$Y(2S) \pi^+$
$Y(3S) \pi^+$
$h_b(1P) \pi^+$
$h_b(2P) \pi^+$
average

- both: $I^G(J^P)=1^+(1^+)$
- both: observed only by Belle [1105.4583]

$Z_b^+ \rightarrow Y(nS) \pi^+$

b b d u

Sasa Prelovsek, JLab 2015
Challenges for the theory community: quarkonium-like states at threshold

TABLE 10: Quarkonium-like states at the open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

<table>
<thead>
<tr>
<th>State</th>
<th>M, MeV</th>
<th>Γ, MeV</th>
<th>M, MeV</th>
<th>Process (mode)</th>
<th>Experiment (σ)</th>
<th>Year</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>X(3872)</td>
<td>3871.6 ± 0.17</td>
<td>&lt; 1.2</td>
<td>1±±</td>
<td>$B \to K(\pi^+\pi^− J/\psi)$</td>
<td>Belle [772, 992] (10), BaBar [993] (8.6)</td>
<td>2003</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$p\bar{p} \to (\pi^+\pi^− J/\psi)$</td>
<td>CDF [994, 995] (11.6), D0 [996] (5.2)</td>
<td>2003</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$p\bar{p} \to (\pi^+\pi^− J/\psi)$</td>
<td>LHCb [997, 998] (up)</td>
<td>2012</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$B \to K(\pi^+\pi^− \pi^0 J/\psi)$</td>
<td>Belle [999] (4.3), BaBar [1000] (4.0)</td>
<td>2005</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$B \to K(\gamma J/\psi)$</td>
<td>Belle [1001] (5.5), BaBar [1002] (3.5)</td>
<td>2005</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$B \to K(\gamma \psi (2S))$</td>
<td>LHCb [1003] (&gt; 10)</td>
<td>2008</td>
<td>NC</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$B \to K(D\bar{D}^*)$</td>
<td>BaBar [1002] (3.6), Belle [1001] (0.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$B \to K(D\bar{D}^*)$</td>
<td>LHCb [1003] (4.4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$Y(4260) \to \pi^−(D\bar{D}^*)$</td>
<td>Belle [1004] (6.4), Belle [1005] (4.9)</td>
<td>2006</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$Y(4260) \to \pi^−(\pi^+ J/\psi)$</td>
<td>BES II [1006] (np)</td>
<td>2013</td>
<td>NC!</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$Y(4260) \to \pi^−(\pi^+ J/\psi)$</td>
<td>BES III [1007] (8), Belle [1008] (5.2)</td>
<td>2013</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$Y(4260) \to \pi^−(\pi^+ J/\psi)$</td>
<td>T. Xiao et al. [CLEO data] [1009] (&gt;5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z(3885)+</td>
<td>3883.9 ± 4.5</td>
<td>25 ± 12</td>
<td>1±±</td>
<td>$Y(4260) \to \pi−(D\bar{D}^*)$</td>
<td>Belle [1004] (6.4), Belle [1005] (4.9)</td>
<td>2006</td>
<td>Ok</td>
</tr>
<tr>
<td>Z(3900)+</td>
<td>3891.2 ± 3.3</td>
<td>40 ± 8</td>
<td>1−−</td>
<td>$Y(4260) \to \pi−(\pi^+ J/\psi)$</td>
<td>BES II [1006] (np)</td>
<td>2013</td>
<td>NC!</td>
</tr>
<tr>
<td>Z(4020)+</td>
<td>4022.9 ± 2.8</td>
<td>7.9 ± 3.7</td>
<td>1−−</td>
<td>$Y(4260), 4360) \to \pi−(\pi^+ h_2)$</td>
<td>BES III [1010] (8.9)</td>
<td>2013</td>
<td>NC!</td>
</tr>
<tr>
<td>Z(4025)+</td>
<td>4026.3 ± 4.5</td>
<td>24 ± 8</td>
<td>1−−</td>
<td>$Y(4260), 4360) \to \pi−(\pi^+ h_2)$</td>
<td>BES III [1011] (10)</td>
<td>2013</td>
<td>NC!</td>
</tr>
<tr>
<td>Z(10610)+</td>
<td>10607.2 ± 2.0</td>
<td>18 ± 2</td>
<td>1−−</td>
<td>$Y(10860) \to \pi−(1S, 2S, 3S)$</td>
<td>Belle [1012–1014] (&gt;10)</td>
<td>2011</td>
<td>Ok</td>
</tr>
<tr>
<td>Z(10650)+</td>
<td>10652.2 ± 1.5</td>
<td>11.5 ± 2.2</td>
<td>1−−</td>
<td>$Y(10860) \to \pi−(1S, 2S, 3S)$</td>
<td>Belle [1013] (16)</td>
<td>2011</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$Y(10860) \to \pi−(1S, 2S, 3S)$</td>
<td>Belle [1013] (16)</td>
<td>2011</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$Y(10860) \to \pi−(1S, 2S, 3S)$</td>
<td>Belle [1015] (8)</td>
<td>2012</td>
<td>NC!</td>
</tr>
</tbody>
</table>

QCD and strongly coupled gauge theories: challenges and perspectives

[review: Brambilla et al., 1404.3723]
More challenges: quarkonium-like states above threshold

[review: Brambilla et al., 1404.3723]

All these believed NOT to be $QQ$!
Wilon-clover quarks

Fermilab method for c and b:
\[ E_M(p) = M_1 + \frac{p^2}{2M_2} - \frac{a^3 W_4}{6} \sum_i p_i^4 - \frac{(p^2)^2}{8M_4^3} + \ldots \]

[El Khadra, Kronfeld et al, 1997]

Rest hadron energies have sizable discretization errors but these largely cancel in splittings.

Only splittings with respect to a chosen reference mass are compared to experiment.

- evaluating Wick contr.: distillation (Ensemble 1) [Peardon et. al., HSC, 2009]
- stochastic distillation (Ensemble 2) [Morningstar et al., 2011]
Discrete energy spectrum from correlators

Example: meson channel with given $J^{PC}$

$$\Theta = \bar{q} \Gamma q, \quad (\bar{q} \Gamma_1 q)(\bar{q} \Gamma_2 q), \quad [\bar{q} q][q q]$$

$$C_{ij}(t) = \langle 0 | \Theta_i(t) \Theta_j^+(0) | 0 \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t} \quad Z_i^n = \langle 0 | \Theta | n \rangle$$

Energies and overlaps extracted using GEVP

$$C(t) u^{(n)}(t) = \lambda^{(n)}(t) C(t_0) u^{(n)}(t)$$

All physical states with given $J^{PC}$ appear as energy levels $E_n$ in principle: single particle, two-particle,...

channel : "eigenstates"

$J^{PC} = 0^+, \bar{s} b : \quad B_{s0}, BK$

$J^{PC} = 1^{++}, \bar{c} c : \quad \chi_{c1}, X(3872), DD^*$,

$J^{PC} = 1^{+-}, \bar{c} c d u : \quad Z^+_c, J / \psi \pi^*,...$

$J^{PC} = 1^{--}, \bar{s} u : \quad K^*, K \pi$

$J^{PC} = 1^{++}, \bar{u} d : \quad a_1, \rho \pi$

Two-meson states:

- In experiment: two-meson decay products with continuous E.
- On lattice: discrete E due to finite L and periodic BC: p=n 2π/L

Bound state and narrow resonance:

typically lead to extra energy level (in addition to two-meson levels)
Elastic scattering of two mesons

at total momentum $P=0$

$E_n(L) \rightarrow p \quad \text{Luscher's eq.} \quad \delta(p)$

Scattering matrix for partial wave $l$: 

$S(p) = e^{2i\delta(p)}$, $S(p) = 1 + 2iT(p)$, $T(p) = \frac{1}{\cot(\delta_l(p)) - i}$

Bound state:

$\cot[\delta(p_B)] = i$, $p_B^2 < 0$

$m_B = E_{M_1}(p_B) + E_{M_2}(-p_B)$

Resonance (of Breit-Wigner type):

$T(p) = -\sqrt{s} \frac{\Gamma(p)}{s - m_R^2 + i\sqrt{s} \Gamma(p)}$

$\Gamma(p) = g^2 \frac{p^{2l+1}}{s}$

$\frac{p^{2l+1}}{\sqrt{s}} \cot(p) = \frac{1}{g^2} (m_R^2 - s)$

Sasa Prelovsek, JLab 2015
Bound states slightly below threshold
Mass prediction for missing $B_{s0}$ below BK threshold

- Scalar Bs state has not been experimentally found yet.
- It is expected slightly below BK threshold.
- We simulated BK scattering on PACS-CS Ensemble (2).

\[ J^P = 0^+ : \quad \mathcal{O} = \bar{s}b, \quad B(0) K(0), \quad B(1) K(-1) \]

\[ |p| \quad -|p_B| \]

\[ \delta \text{ interpolated using eff. range approx} \]
\[ p \cot \delta = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 \]

\[ T(p) = \frac{1}{\cot (\delta_f(p)) - i} \]

The scattering matrix has a pole at the position of the bound state (on the first Riemann sheet)

\[ p_B = i |p_B| \quad \cot \delta(p_B) = i \]
\[ p_B \cot \delta(p_B) = -|p_B| \]

\[ m_{lat}^B = E_B(p_B) + E_K(-p_B) \]

[C. Lang, D. Mohler, S.P., R. Woloshyn: 1501.0164]

Sasa Prelovsek, JLab 2015
Mass prediction for missing $B_{s0}$ and $B_{s1}$

Quantities shown:

for two bound states:

$$m_B = (m_B - E_{th})^{lat} + E_{th}^{exp}$$

for other states:

$$m = (m - \bar{m})^{lat} + \bar{m}^{lat}$$

for dotted lattice thresholds:

$$E_{th} = (E_{th} - \bar{m})^{lat} + \bar{m}^{lat}$$

$$\bar{m} \equiv \frac{1}{4}(m_{Bs} + 3m_{Bs^*})$$

- $B_{s1}'$ and $B_{s2}$ agree well with exp
- $B_{s0}$ and $B_{s1}$ are predictions for yet unobserved states (errors contain statistical and several sources of systematical uncertainties)

[Mass predic@on for missing $B_{s0}$ and $B_{s1}$

Ensemble (2), $m_\pi=156$ MeV

Quantities shown:

for two bound states:

$$m_B = (m_B - E_{th})^{lat} + E_{th}^{exp}$$

for other states:

$$m = (m - \bar{m})^{lat} + \bar{m}^{lat}$$

for dotted lattice thresholds:

$$E_{th} = (E_{th} - \bar{m})^{lat} + \bar{m}^{lat}$$

$$\bar{m} \equiv \frac{1}{4}(m_{Bs} + 3m_{Bs^*})$$

- $B_{s1}'$ and $B_{s2}$ agree well with exp
- $B_{s0}$ and $B_{s1}$ are predictions for yet unobserved states (errors contain statistical and several sources of systematical uncertainties)

[C. Lang, D. Mohler, S.P., R. Woloshyn: 1501.0164]
$D_{s0}$ and $D_{s1}$ below DK and $D^*K$ thresholds

- Analogy, just $b$ replaced with $c$
- In this case $D_{s0}$ and $D_{s1}$ have been observed experimentally
- Quark models expected them above thresholds but they were found below them
- Our postdictions agree with measured masses

[D. Mohler, C. Lang, L. Leskovec, S.P. , R. Woloshyn:
1308.3175, Phys. Rev. Lett 2013
1403.8103, PRD 2014]
**D_{s0} from indirect simulation**

(1) Five channels that do not include Wick contractions simulated

(2) Scattering lengths $a = \lim_{p \to 0} \frac{\tan \delta(p)}{p}$ for four $m_\pi$ extracted

<table>
<thead>
<tr>
<th>Channel</th>
<th>SU(3)</th>
<th>LEC's</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D\bar{K}$ ($S = 1, I = 0$)</td>
<td>$D\bar{K}$ ($-1, 0$)</td>
<td>$DK$ ($S = 2, I = \frac{1}{2}$)</td>
</tr>
</tbody>
</table>

(3) simultaneous fit using SU(3) unitarized ChPT is performed and LEC's are determined

(4) using these LEC's indirect predictions for $D_{s0}$ channel with DK ($S=1, I=0$):

- pole in the first Riemann sheet found

<table>
<thead>
<tr>
<th>$D_{s0}^*$ (2317)</th>
<th>$m$</th>
<th>$\Gamma [D_{s0}^* \rightarrow D_s \pi]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>indirect lat</td>
<td>2315 $^{+18}_{-28}$ MeV</td>
<td>133$^{\pm 22}$ keV</td>
</tr>
<tr>
<td>exp</td>
<td>2317.8 $\pm 0.6$ MeV</td>
<td>$&lt; 3.8$ MeV</td>
</tr>
</tbody>
</table>

L. Liu, Orginos, Guo, Hanhart, Meissner, 1208.4535, PRD, $m_\pi \approx 300-620$ MeV, $N_f=2+1$
**X(3872), J^{PC}=1^{++}, charmonium-like**

- First charmonium-like state discovered [Belle, PRL, 2003]
- sits within 1 MeV of $D^0D^{0*}$ threshold
  8 MeV below $D^+D^{*-}$ threshold
- believed to have a large molecular $D^0D^{0*}$ Fock component
- $\Gamma < 1.2$ MeV
- decays to $I=0, 1$ equally important
  
  $X(3872) \rightarrow J/\Psi \omega$ ( $I=0$ )
  
  $X(3872) \rightarrow J/\Psi \rho$ ( $I=1$ )

Sasa Prelovsek, JLab 2015
$X(3872), \ 1^{++}, l=0$

$\mathcal{O}: \bar{c}c, \ D\bar{D}^*= (\bar{c}u)(\bar{u}c) + (\bar{c}d)(\bar{d}c), \quad J/\psi \omega = (\bar{c}c)(\bar{u}u + \bar{d}d)$

- all Wick contractions calculated using distillation method
  [Peardon et al. 2009]

- charm annihilation contractions not used in analysis

X(3872) below DD* threshold, l=0

\[ \Theta : \bar{c}c, \quad DD^*, \quad J/\psi \omega \]

- \( \delta \) for DD* scattering in s-wave extracted using Luscher's relation
- \( \delta \) interpolated near threshold
- pole found in the scattering matrix

\[ T \propto (\cot \delta - i)^{-1} = \infty, \quad \cot \delta(p_{BS}) = i \]

\[ m_{D^*_0}^{\text{lat}, L \to \infty} = E_D(p_{BS}) + E_K(p_{BS}) \]

Assumptions/approximations:
- charm Wick annihilation omitted
- DD* scattering analyzed assuming \( J/\psi \omega \) is decoupled (good evidence for that from the lattice data)
- \( m_u = m_d \)

<table>
<thead>
<tr>
<th>X(3872)</th>
<th>m - (m_{D^0} + m_{D^{*0}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>lat</td>
<td>-11 ± 7 MeV</td>
</tr>
<tr>
<td>exp</td>
<td>-0.14 ± 0.22 MeV</td>
</tr>
</tbody>
</table>

X(3872) appears only if both cc and DD* interp. used.


Ensemble (1), \( m_\pi \approx 266 \) MeV, Nf=2

Sasa Prelovsek, JLab 2015
New evidence for $X(3872)$: $J^{PC}=1^{++}$, $I=0$

$\mathcal{O}$: $\bar{c}c$, $DD^*$

**Interpretation**

- $D(0)D^*(0)$ state is not found although $O$ incorporated
- $D(1)D^*(-1)$ state is not found although $O$ incorporated

<table>
<thead>
<tr>
<th>$X(3872)$</th>
<th>$m - (m_{D^0} + m_{D^0^*})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>lat</td>
<td>$-13 \pm 6$ MeV</td>
</tr>
<tr>
<td>exp</td>
<td>$-0.14 \pm 0.22$ MeV</td>
</tr>
</tbody>
</table>

HISQ quarks, $m_u = m_d = m_s/5$, $16^3 \times 48$, $a=0.15$ fm

[C. DeTar, Song-haeng Lee, et al., 1411.1389, Lat14 proceedings]

Sasa Prelovsek, JLab 2015
Search for hadrons with manifestly exotic flavor:

$$\text{ccud} \quad (I=1)$$
\[ Z_c^{+} \text{ channel: } I^G=1^+, J^{PC}=1^{++} \]

\[ E = E[ M_1(p_1) ] + E[ M_2(p_2) ] \]

in non-interacting case

Extracting 13 two-meson states is a huge challenge!

[S.P., Lang, Leskovec, Mohler, 1405.7612v2, PRD 2015]

Ensemble (2), \( m_R \approx 266 \text{ MeV}, L=2 \text{ fm}, N_f=2 \)
**$Z_c^+$ channel:** \( |G=1^+, J^{PC}=1^{+-} | \)

**Interpolating fields**

\[
\begin{align*}
\mathcal{O}_1^{\psi(0)\pi(0)} &= \bar{c}\gamma_i c(0) \, \bar{d}\gamma_5 u(0), \\
\mathcal{O}_1^{\psi(1)\pi(-1)} &= \sum_{e_k=\pm x, y, z} \bar{c}\gamma_i c(e_k) \, \bar{d}\gamma_5 u(-e_k), \\
\mathcal{O}_{\eta c}(0)^{\rho(0)} &= \bar{c}\gamma_5 c(0) \, \bar{d}\gamma_i u(0), \\
\mathcal{O}_{D(0)D^*(0)} &= \bar{c}\gamma_5 u(0) \, \bar{d}\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}, \\
\mathcal{O}_{D^*(0)D^*(0)} &= \epsilon_{ijk} \, \bar{c}\gamma_j u(0) \, \bar{d}\gamma_k c(0),
\end{align*}
\]

and 13 others..

**Wick contractions**

\[
C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^{\dagger}(0) | 0 \rangle
\]

\[
\begin{align*}
\mathcal{O}_1^{4q} &\approx \left[ \bar{c} \, C \gamma_5 \bar{d} \right]_{3c} \left[ c \, \gamma_i C \, u \right]_{\bar{3}c} \\
\mathcal{O}_2^{4q} &\approx \left[ \bar{c} \, C \, \bar{d} \right]_{3c} \left[ c \, \gamma_i \gamma_5 C \, u \right]_{\bar{3}c}
\end{align*}
\]

and 2 others..

---

18 two-meson (MM)

Aiming at 9 two-meson states listed in previous slide

4 diquark-antidiquark (4Q)

Aiming to find additional state related to exotic $Z_c^+$
Conclusion:

- we find 13 two-meson states (black circles) as expected
- we find no additional state below 4.2 GeV
- we find no candidate for $Z_c$ below 4.2 GeV

[S.P., Lang, Leskovec, Mohler, 1405.7612v2, PRD 2015]

Ensemble (1), $m_\pi \approx 266$ MeV, $L=2$ fm, $N_f=2$
Results from the extended basis:

- based on $E_n$ and $Z_i^n = \langle 0 | \Phi | n \rangle$
  - lowest 13 states (black):
    - two-meson states
  - no extra state below 4.2 GeV
  - no extra state at 4.16 GeV
    - (extended basis gives an extra state at 4.4 GeV)
  - attributing a state at 4.16 GeV to $Z_c^+$ (green) was a premature conclusion
  - we can not exclude that state at 4.16 GeV was a linear combination of omitted two-meson states, induced via $O^{4q}$

**Conclusion:** we do not find $Z_c^+$ candidate below 4.2 GeV
Similar conclusion concerning $Z_c^+$ channel

$Z_c$ channel, $I=1$, $J^{PC}=1^{+-}$

- $\psi_{2S}(0) + \pi(0)$
- $D(-1)D^*(1)$
- $D(0)D^*(0)$
- $\psi_{2S}(0) + \pi(0)$
- $J/\psi(1) + \pi(-1)$
- $J/\psi(0) + \pi(0)$

Only expected two-meson states, no candidate for $Z_c$

S.-H. Lee, C. DeTar, H. Na,
Lattice 2014 proc.: 1411.1389

$m_u=m_s/5;\ N_f=2+1+1;\ \text{HISQ}$
Puzzle: why there is no additional eigenstate related to $Z_c^+(3900)$ on the lattice?

Why does such large basis of creation operators not excite observed $Z_c^+$ (in addition to two-meson states)?

✧ Even more interpolators needed?

✧ Is $m_\pi=266$ MeV to high?

✧ Could the neglect of charm annihilation contribution significantly modify the conclusion?

✧ Is something wrong with working assumption that $Zc^+$ should lead to an additional eigenstate?

Note: we have observed additional levels for resonances: $\rho$, $K^*$, $D_{0^*}$, $D_1$, $a_1$, $b_1$, $\Psi(3770)$ bound states: $D_{50^*}$, $D_{51^*}$, $B_{50^*}$, $B_{51}$, $X(3872)$

Is $Z_c$ is of different origin and does not lead to additional level?

✧ Perhaps an additional eigenstate does not have to arise if it is a coupled channel effect: more analytical work is needed

Sasa Prelovsek, JLab 2015
Is $Z_c$ related to coupled-channel effect?

- lattice, HALQCD method, $m_\pi \approx 410$ MeV, $N_f=2+1+1$

HALQCD coll, Iketa et al [not available on arXiv yet, slides from Quarkonium 2014]
Is $Z_c$ related to coupled-channel effect?

$\sigma \propto \text{Im}[T]$ 
$J/\psi \pi$

m$_{\pi}$≈410 MeV, Nf=2+1+1

lattice (HALQCD method)  

experiment

$\sigma \propto \text{Im}[T]$ 
$D \bar{D}^*$

HALQCD is investigating if extracted scattering matrix has a pole.
It needs to be investigated whether this scenario would lead to an additional energy level or not.
It is not clear yet whether this scenario could be compatible with absence of the $Z_c$ energy level in our spectrum.
Mesons above threshold – resonances

\[ O = \bar{q} \Gamma q, \]

\[ (\bar{q} \Gamma_1 q) \bar{p}_1 (\bar{q} \Gamma_2 q) \bar{p}_2 = M_1(\bar{p}_1) M_2(\bar{p}_2) \]

\( E_n(L) \rightarrow \delta \rightarrow m, \Gamma \) or coupling

Sasa Prelovsek, JLab 2015
Almost all hadrons are resonances above threshold

<table>
<thead>
<tr>
<th>( \bar{c}c )</th>
<th>( \bar{u}u )</th>
<th>( \bar{s}u )</th>
<th>( \bar{c}u )</th>
<th>( uud )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_c(1S) )</td>
<td>( \eta^+ )</td>
<td>( K^\pm )</td>
<td>( D^\pm )</td>
<td>( p )</td>
</tr>
<tr>
<td>( J/\psi(1S) )</td>
<td>( \eta^0 )</td>
<td>( K^0 )</td>
<td>( D^0 )</td>
<td>( n )</td>
</tr>
<tr>
<td>( \chi_{c0}(1P) )</td>
<td>( \eta ' )</td>
<td>( K_{0}^{*} )</td>
<td>( D_{0}^{*}(2070) ) ( 0 )</td>
<td>( N(1440) ) ( 1/2^+ )</td>
</tr>
<tr>
<td>( \chi_{c1}(1P) )</td>
<td>( f_0(980) )</td>
<td>( K_{S}^{0} )</td>
<td>( D_{0}^{*}(2400) ) ( 0 )</td>
<td>( N(1520) ) ( 3/2^- )</td>
</tr>
<tr>
<td>( \chi_{c2}(1P) )</td>
<td>( a_0(980) )</td>
<td>( K_{L}^{0} )</td>
<td>( D_{0}^{*}(2400) ) ( \pm )</td>
<td>( N(1535) ) ( 1/2^- )</td>
</tr>
<tr>
<td>( \eta_c(2S) )</td>
<td>( \phi(1020) )</td>
<td>( \eta_c(800) ) or ( \kappa )</td>
<td>( D_{1}(2420) ) ( 0 )</td>
<td>( N(1650) ) ( 1/2^- )</td>
</tr>
<tr>
<td>( \psi(2S) )</td>
<td>( h_1(1170) )</td>
<td>( K_{0}^{*}(892) )</td>
<td>( D_{1}(2420) ) ( \pm )</td>
<td>( N(1675) ) ( 5/2^- )</td>
</tr>
<tr>
<td>( \psi(3770) )</td>
<td>( b_1(1235) )</td>
<td>( K_{1}(1270) )</td>
<td>( D_{1}(2420) ) ( 0 )</td>
<td>( N(1680) ) ( 5/2^+ )</td>
</tr>
<tr>
<td>( X(3872) )</td>
<td>( a_1(1260) )</td>
<td>( K_{1}(1400) )</td>
<td>( D_{1}(2430) ) ( 0 )</td>
<td>( N(1685) ) ( ? )</td>
</tr>
<tr>
<td>( \chi_{c0}(2P)_{w} )</td>
<td>( f_2(1270) )</td>
<td>( K_{2}^{*}(1410) )</td>
<td>( D_{2}^{*}(2460) ) ( 0 )</td>
<td>( N(1700) ) ( 3/2^- )</td>
</tr>
<tr>
<td>( \chi_{c2}(2P) )</td>
<td>( f_1(1285) )</td>
<td>( K_{0}^{*}(1440) )</td>
<td>( D_{2}^{*}(2460) ) ( \pm )</td>
<td>( N(1710) ) ( 1/2^+ )</td>
</tr>
<tr>
<td>( X(3940) )</td>
<td>( \eta_{c}(1295) )</td>
<td>( K_{2}(1460) )</td>
<td>( D_{2}^{*}(2460) ) ( \pm )</td>
<td>( N(1720) ) ( 3/2^+ )</td>
</tr>
<tr>
<td>( \psi(4040) )</td>
<td>( \rho(1300) )</td>
<td>( K_{2}(1580) )</td>
<td>( D_{0}^{*}(2400) ) ( \pm )</td>
<td>( N(1860) ) ( 5/2^+ )</td>
</tr>
<tr>
<td>( X(4050) ) ( \pm )</td>
<td>( a_2(1320) )</td>
<td>( K_{0}(1600) )</td>
<td>( D_{1}(2420) ) ( \pm )</td>
<td>( N(1875) ) ( 3/2^- )</td>
</tr>
<tr>
<td>( X(4140) )</td>
<td>( a_1(1320) )</td>
<td>( K_{1}(1650) )</td>
<td>( D_{1}(2430) ) ( 0 )</td>
<td>( N(1880) ) ( 1/2^+ )</td>
</tr>
<tr>
<td>( \psi(4160) )</td>
<td>( f_0(1370) )</td>
<td>( K_{1}(1650) )</td>
<td>( D_{2}^{*}(2460) ) ( 0 )</td>
<td>( N(1895) ) ( 1/2^- )</td>
</tr>
<tr>
<td>( X(4160) )</td>
<td>( h_{1}(1380) )</td>
<td>( K'_{1}(1680) )</td>
<td>( D_{2}^{*}(2460) ) ( \pm )</td>
<td>( N(1900) ) ( 3/2^+ )</td>
</tr>
<tr>
<td>( X(4250) ) ( \pm )</td>
<td>( m_{1}(1400) )</td>
<td>( K_{2}(1770) )</td>
<td>( D_{0}^{*}(2400) ) ( \pm )</td>
<td>( N(1990) ) ( 7/2^+ )</td>
</tr>
<tr>
<td></td>
<td>( n_{1}(1400) )</td>
<td>( K_{2}(1820) )</td>
<td>( D_{0}^{*}(2400) ) ( \pm )</td>
<td>( N(2000) ) ( 5/2^+ )</td>
</tr>
<tr>
<td></td>
<td>( \eta(1405) )</td>
<td>( K_{3}^{*}(1780) )</td>
<td>( D_{0}^{*}(2400) ) ( \pm )</td>
<td>( N(2040) ) ( 3/2^+ )</td>
</tr>
<tr>
<td></td>
<td>( f_{1}(1420) )</td>
<td>( K_{2}(1820) )</td>
<td>( D_{0}^{*}(2400) ) ( \pm )</td>
<td>( N(2060) ) ( 5/2^+ )</td>
</tr>
<tr>
<td></td>
<td>( \omega(1420) )</td>
<td>( K'_{2}(1830) )</td>
<td>( D_{0}^{*}(2400) ) ( \pm )</td>
<td>( N(2100) ) ( 1/2^+ )</td>
</tr>
</tbody>
</table>

well below strong decay th.

well below open charm decay th.

rigorous treatment attempted
$K^*(892)$ resonance $K\pi$

$I=1/2$: p-wave phase shift

\[ \Gamma[K^* \rightarrow K\pi] = \frac{g^2 p^3}{6\pi} \]

$K(\rho) \rightarrow \bar{s}u$

$p^3 \cot \delta / s^{1/2}$

Irreps where p-wave does not mix with s-wave, Lusher-type rel.: [Lekovec, S.P. PRD 2012]

\[ \frac{p^3}{\sqrt{s}} \cot \delta = \frac{6\pi}{g^2} (m_R^2 - s) \]

<table>
<thead>
<tr>
<th>$m_{K^*}(892)$ [MeV]</th>
<th>$g_{K^*}(892)$ [no unit]</th>
</tr>
</thead>
<tbody>
<tr>
<td>lat</td>
<td>891 ± 14</td>
</tr>
<tr>
<td>exp</td>
<td>891.66 ± 0.26</td>
</tr>
</tbody>
</table>

[S.P. ,Lang, Leskovec, Mohler, 1307.0736, PRD 2013] $m_\pi \approx 266$ MeV
Resonances in Kπ, Kη coupled channels

- qq, Kπ, Kη interpolators
- a number of different 0<P≤2
- for each E_n: one determinant equation for many unknowns
- T-matrix parametrized to get around this problem
- the location of poles of T-matrix in complex plain is given below
- K*(892) and κ are below threshold for this m_π
- K_0^*, K_2^* are resonances
- m_π=391 MeV, N_L=16, 20, 24

[Dudek, Edwards, Thomas, Wilson, HSC, 1406.4158, PRL; 1411.2004]

\[
\det \left[ \delta_{ij} \delta_{JJ'} + i\rho_i t_{ij}^{(j)}(E_{cm}) \left( \delta_{JJ'} + i\mathcal{M}_{JJ'}(p_iL) \right) \right] = 0.
\]

location of poles in T matrix in complex plane
Simulating scattering:

\( \rho \pi \) in \( 1^{++} \) channel to extract \( a_1(1260) \)

\( \omega \pi \) in \( 1^{+} \) channel to extract \( b_1(1235) \)

One motivation: COMPASS claim for \( a_1'(1420) \) from \( f_0(980) \pi \) [1312.3678]

Our spectrum supports only one \( a_1 \) below 1.8 GeV (not two)

\( m_\pi \approx 266 \text{ MeV}, L \approx 2 \text{ fm}, N_f = 2, P = 0 \) [Lang, Leskovec, Mohler, S.P., 1401.2088, JHEP]

\( \Gamma(E) \equiv g^2 \frac{P}{E^2} \)

<table>
<thead>
<tr>
<th>resonance</th>
<th>( a_1(1260) )</th>
<th>( b_1(1235) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>quantity</td>
<td>( m_{a_1}^{\text{res}} ) [GeV]</td>
<td>( g_{a_1 \rho \pi} ) [GeV]</td>
</tr>
<tr>
<td>lat</td>
<td>1.435(53)(^{+0}_{-109})</td>
<td>1.71(39)</td>
</tr>
<tr>
<td>exp</td>
<td>1.230(40)</td>
<td>1.35(30)</td>
</tr>
</tbody>
</table>

[PDG] [Basdevant, Berger, 1501.04643]

\( \rho \) and \( \omega \) assumed to be stable, good approx. for given simulation parameters: [Roca, Oset, 1201.0438]

going beyond that approximation will be very challenging

3-particles: [Hansen, Sharpe 1311.4848; Polejaeva, Rusetsky, 1203.1241; Briceno, Davoudi, 1212.3398,....]
**D-meson resonances in Dπ and D*π**

\[ \Gamma(E) = g^2 \frac{p}{E^2} \]

\( g \) is compared to \( \exp \) instead of \( \Gamma \) (\( \Gamma \) depends on phase sp. and \( m \)).

### J^P=0^+ : D π

<table>
<thead>
<tr>
<th>( D^*_0 (2400) )</th>
<th>( m - 1/4(mD+3mD^*) )</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>lat</td>
<td>351 ± 21 MeV</td>
<td>2.55 ± 0.21 GeV</td>
</tr>
<tr>
<td>exp</td>
<td>347 ± 29 MeV</td>
<td>1.92 ± 0.14 GeV</td>
</tr>
</tbody>
</table>

### J^P=1^+ : D* π

(analysis of spectrum in this case is based on an assumption given in paper below)

<table>
<thead>
<tr>
<th>( D_1(2430) )</th>
<th>( m - 1/4(mD+3mD^*) )</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>lat</td>
<td>381 ± 20 MeV</td>
<td>2.01 ± 0.15 GeV</td>
</tr>
<tr>
<td>exp</td>
<td>456 ± 40 MeV</td>
<td>2.50 ± 0.40 GeV</td>
</tr>
</tbody>
</table>

First lattice result for strong decay width of a hadron containing charm quark

[D. Mohler, S.P., R. Woloshyn: 1208.4059, PRD]

- \( m_π \approx 266 \) MeV, \( L \approx 2 \) fm, \( N_f = 2 \)

Sasa Prelovsek, JLab 2015
Resonance $\psi(3770)$ in $p$-wave $DD$ scattering

$\bar{c}c, J^{PC} = 1^{--}: J/\psi, \psi(2S)$ below $D\bar{D}$ threshold

$\psi(3770)$ lowest state above threshold $\Gamma^{\exp} \sim 27$ MeV

<table>
<thead>
<tr>
<th></th>
<th>Mass [MeV]</th>
<th>$\mathcal{g}_{\psi(3770)DD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ensemble(1)</td>
<td>3785(7)(8)</td>
<td>11.4 (0.8)</td>
</tr>
<tr>
<td>Ensemble(2)</td>
<td>3783(49)(10)</td>
<td>21.6(14.9)</td>
</tr>
<tr>
<td>Experiment</td>
<td>3773.15(33)</td>
<td>$\approx 18.7$</td>
</tr>
</tbody>
</table>

\[
\frac{p^3}{\sqrt{s}} \cot \delta = \frac{6\pi}{g^2} (m_R^2 - s)
\]

\[
\Gamma(s) = \frac{g^2 p^3}{6\pi s}
\]
Conclusions

Status of meson spectrum from lattice simulations (in brief):

- Evidence found for states with **non-exotic** flavor:
  - states well below th.: charmonium, D, π, K ... and all the others
  - resonances via BW: ρ, K⁺, K₀⁺(1430), K₂, D₀⁺, D₁, a₁, b₁, Ψ(3770)
  - shallow bound states: D₃₀, D₅₁, B₃₀, B₅₁, Χ(3872) with I=0

  All these manifest themselves via an additional energy level (in our experience so far)!

- No evidence for manifestly **exotic** states (yet), at least by searching an additional energy level
  - Z⁺ₐ = ccud

Theory is facing a serious challenge to establish whether exotic states arise from QCD or not.

Only after this is settled, theory can claim structure (mesonic molecules, diquark antidiquark, coupled channel effects, ... )
Backup slides
Charged charmonium $Z_c^+$: experimental status

candidate with preferred $I^G=1^+, J^{PC}=1^{+-}$

<table>
<thead>
<tr>
<th>particle</th>
<th>C</th>
<th>J$^P$</th>
<th>decay</th>
<th>year</th>
<th>coll</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z^+(4430)$</td>
<td>-</td>
<td>1$^+$</td>
<td>$\psi(2S)\pi^+$</td>
<td>2008</td>
<td>Belle, BABAR, LHCb</td>
</tr>
<tr>
<td>$Z_c^+(3900)$</td>
<td>-</td>
<td>?</td>
<td>$J/\psi\pi^+$</td>
<td>2013</td>
<td>BESIII, Belle, CLEOc</td>
</tr>
<tr>
<td>$Z_c^+(3885)$</td>
<td>-</td>
<td>1$^+$</td>
<td>$(D\bar{D^*})^+$</td>
<td>2013</td>
<td>BESIII</td>
</tr>
<tr>
<td>$Z_c^+(4020)$</td>
<td>-</td>
<td>?</td>
<td>$h_c(1P)\pi^+$</td>
<td>2013</td>
<td>BESIII</td>
</tr>
<tr>
<td>$Z_c^+(4025)$</td>
<td>-</td>
<td>?</td>
<td>$(D^<em>\bar{D^</em>})^+$</td>
<td>2013</td>
<td>BES III</td>
</tr>
<tr>
<td>$Z^+(4200)$</td>
<td>-</td>
<td>1$^+$</td>
<td>$J/\psi\pi^+$</td>
<td>2014</td>
<td>Belle</td>
</tr>
<tr>
<td>$Z^+(4050)$</td>
<td>+</td>
<td>?</td>
<td>$\chi_{c1}\pi^+$</td>
<td>2008</td>
<td>Belle</td>
</tr>
<tr>
<td>$Z^+(4250)$</td>
<td>+</td>
<td>?</td>
<td>$\chi_{c1}\pi^+$</td>
<td>2008</td>
<td>Belle</td>
</tr>
</tbody>
</table>

[review: Brambilla et al., 1404.3723]

[BESIII, 2013, 1303.5949, PRL]

$Z_c^+(3900) \rightarrow J/\Psi \pi^+$

Sasa Prelovsek, JLab 2015
Challenge: precision simulation of $Z_c^+$

On larger volume: more two particle states

Rigorous treatment very challenging: at least 6 two-particle channels coupled!!

Physical masses used

$E(L) = \sqrt{m_1^2 + \tilde{p}_1^2 + \sqrt{m_2^2 + \tilde{p}_2^2 + \Delta E}}$

$\tilde{p}_1 = \frac{2\pi}{L} \hat{n}_1 \quad \tilde{p}_2 = \frac{2\pi}{L} \hat{n}_2$
Another challenge: $Z_{b^+}$

On larger volume: more two-particle states

Rigorous treatment very challenging: at least 6 two-particle channels coupled!!

$E(L) = \sqrt{m_1^2 + \vec{p}_1^2} + \sqrt{m_2^2 + \vec{p}_2^2} + \Delta E$

$\vec{p}_1 = \frac{2\pi}{L} \vec{n}_1$

$\vec{p}_2 = \frac{2\pi}{L} \vec{n}_2$
$D_{s0}^*(2317)$: bound state below DK threshold, $J^P=0^+$

- $\delta$ for DK scattering in s-wave
  extracted using Luscher's relation
- $\delta$ interpolated near threshold
- pole found in the scattering matrix

$$T \propto [\cot \delta - i]^{-1} = \infty, \quad \cot \delta(p_{BS}) = i$$

$$m_{D_{s0}^{lat.}, L=\infty}^{lat} = E_D(p_{BS}) + E_K(p_{BS})$$

[D. Mohler, C. Lang, L. Leskovec, S.P., R. Woloshyn:
1308.3175, Phys. Rev. Lett 2013
1403.8103, PRD 2014]
**X(3872) channel: I=1, J^{PC}=1^{++}**

Only expected two-particle states observed. No candidate for X(3872) with I=1 found.

In agreement with experiment that does not find charged X either.

The simulation is done in the isospin limit $m_u=m_d$. The absence of I=1 state for $m_u=m_d$ is in agreement with two interpretations:

1. $X(3872) = a_{I=0}|DD^*\rangle_{I=0} + a_{I=1}|DD^*\rangle_{I=1}$
   
   $a_{I=1}(m_u = m_d) = 0$
   
   $a_{I=1}(m_u \neq m_d) \ll a_{I=0}$

2. X(3872) pure I=0 state

   isospin breaking decay $X(3872) \rightarrow J/\psi \rho$ (I=1)

   is due to isospin splitting $D^0D^{0*}, D^+D^*$

---

S. P. and L. Leskovec: 1307.5172

PRL 2013, $m_\pi=266$ MeV, Nf=2

Sasa Prelovsek, JLab 2015
Puzzle from experiment: $Z_c^+(3900)$ seen only in $Y$ decays

What about other channels in experiment and other experiments?

- $Z_c(3900)$ was found in $J/\psi \pi$ inv. mass only in
  - [BESIII, Belle, CLEOc, 2013] $Y(4260) \rightarrow (J/\psi \pi^+)\pi^-$

- $Z_c(3900)$ was NOT found in $J/\psi \pi$ inv. mass in
  - Belle 2014, 1408.6457 $\bar{B}^0 \rightarrow (J/\psi \pi^+)K^-$
  - LHCb, 2014, 1404.5673 $\bar{B}^0 \rightarrow (J/\psi \pi^+)\pi^-$
  - COMPASS, 2014, 1407.6186 $\gamma p \rightarrow (J/\psi \pi^+)n$
Checking our implementation of Fermilab method: splittings on Ensemble (2)

<table>
<thead>
<tr>
<th>Mass splitting</th>
<th>This work</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_{B^*} - m_B )</td>
<td>46.8(7.0)(0.7)</td>
<td>45.78(35)</td>
</tr>
<tr>
<td>( m_{B_s^*} - m_{B_s} )</td>
<td>47.1(1.5)(0.7)</td>
<td>48.7^{+2.3}_{-2.1}</td>
</tr>
<tr>
<td>( m_{B_s} - m_B )</td>
<td>81.5(4.1)(1.2)</td>
<td>87.35(23)</td>
</tr>
<tr>
<td>( m_Y - m_{\eta_b} )</td>
<td>44.2(0.3)(0.6)</td>
<td>62.3(3.2)</td>
</tr>
<tr>
<td>( 2m_{\bar{B}} - m_{\bar{b}b} )</td>
<td>1190(11)(17)</td>
<td>1182.7(1.0)</td>
</tr>
<tr>
<td>( 2m_{\bar{B}<em>s} - m</em>{\bar{b}b} )</td>
<td>1353(2)(19)</td>
<td>1361.7(3.4)</td>
</tr>
<tr>
<td>( 2m_{B_c} - m_{\eta_b} - m_{\eta_c} )</td>
<td>169.4(0.4)(2.4)</td>
<td>167.3(4.9)</td>
</tr>
</tbody>
</table>
**cc spectrum: single-hadron approximation**

[HSC, L. Liu et al: 1204.5425, JHEP]

- $m_n \approx 400$ MeV, $L \approx 2.9$ fm, $N_f=2+1$
- Reliable $J^P_C$ determination
- Identification with $n^{2S+1}L_J$ multiplets using $\langle O | n \rangle$
- Green: lat, black: exp

**Hybrids:**
Some of them have exotic $J^{PC}$
Large overlap with $O= q F_{ij} q$
D spectrum: single-hadron approximation

G. Moir et al, HSC (Hadron Spectrum Coll.): 1301.7670, JHEP:
- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, $N_f=2+1$
- reliable $J^P$ determination; many excited states
- identification with $n^{2S+1}L_J$ multiplets using $\langle O | n \rangle$
- green: lat, black: exp

Hybrids:
large overlap with $O = q F_{ij} q$
gluonic tensor $F_{ij} = [D_i, D_j]$
**D_s spectrum: single-hadron approximation**

G. Moir et al., HSC : 1301.7670, JHEP:
- \( m_n \approx 400 \text{ MeV}, \ L \approx 2.9 \text{ fm}, \ N_f=2+1 \)
- reliable \( J^{PC} \) determination
- identification with \( n^{2S+1L_J} \) multiplets using \( \langle O \mid n \rangle \)
- green: lat, black: exp

Hybrids:
large overlap with \( O= q F_{ij} q \)
gluonic tensor \( F_{ij} = [D_i, D_j] \)