DVCS and Conformal Symmetry

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In this talk:

- Eliminating gauge and frame dependence (target mass and finite-t corrections)

  V. Braun, A. Manashov, D. Müller, B. Pirnay, Phys.Rev. D89 (2014) 074022

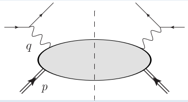
- Towards three-loop evolution equations for GPDs

  V. Braun, A. Manashov, S. Moch, M. Strohmaier, work in progress

Planar vs. non-planar kinematics

- **paradigm shift: finite t a "nuisance" $\longrightarrow$ important tool**

DIS

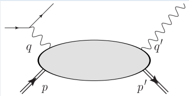


Define (p, q) as longitudinal plane:

$$p = (p_0, \vec{0}_\perp, p_z)$$
$$q = (q_0, \vec{0}_\perp, q_z)$$

$\Rightarrow$ parton fraction = Bjorken x

DVCS



Many choices possible:

$$p = (p_0, \vec{0}_\perp, p_z), \quad q = (q_0, \vec{0}_\perp, q_z)$$

or

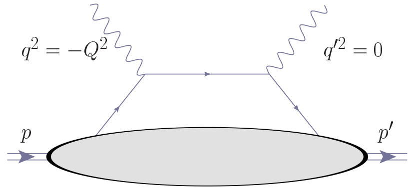
$$p + p' = (P_0, \vec{0}_\perp, P_z), \quad q = (q_0, \vec{0}_\perp, q_z)$$

etc.

$\Rightarrow$ parton fraction $2\xi = x_B[1+\mathcal{O}\left(\frac{t}{Q^2}\right)]$, redefinition of helicity amplitudes

- **Ambiguity is resolved by adding "kinematic" power corrections t/Q^2, m^2/Q^2**

Braun, Manashov, Pirnay: PRD **86** (2012) 014003



longitudinal plane (q, q')

$$n = q', \qquad \tilde{n} = -q + \frac{Q^2}{Q^2 + t}\, q'$$

with this choice $\Delta = q - q'$ is longitudinal and

$$|P_\perp|^2 = -m^2 - \frac{t}{4}\frac{1-\xi^2}{\xi^2} \;\sim\; t_{\min} - t$$

where

$$P = \frac{1}{2}(p + p')\,, \qquad \xi_{\rm BMP} = -\frac{(\Delta\cdot q')}{2(P\cdot q')} \;=\; \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)}$$

photon polarization vectors

$$\varepsilon^0_\mu = -\left(q_\mu - q'_\mu q^2/(qq')\right)/\sqrt{-q^2}\,,$$
$$\varepsilon^\pm_\mu = (P^\perp_\mu \pm i\bar{P}^\perp_\mu)/(\sqrt{2}|P_\perp|)\,, \qquad \bar{P}^\perp_\mu = \epsilon^\perp_{\mu\nu} P^\nu$$

Relating CFFs in the laboratory and photon reference frame

$$\mathcal{F}^{\text{lab}}_{++} = \mathcal{F}^{\text{phot}}_{++} + \frac{\varkappa}{2}\Big[\mathcal{F}^{\text{phot}}_{++} + \mathcal{F}^{\text{phot}}_{-+}\Big] - \varkappa_0\, \mathcal{F}^{\text{phot}}_{0+},$$

$$\mathcal{F}^{\text{lab}}_{0+} = -(1+\varkappa)\,\mathcal{F}^{\text{phot}}_{0+} + \varkappa_0\Big[\mathcal{F}^{\text{phot}}_{++} + \mathcal{F}^{\text{phot}}_{-+}\Big]$$

$$\boxed{\mathcal{F} \in \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}}$$

where

$$\varkappa_0 \sim \sqrt{(t_{\min} - t)/Q^2}\,, \qquad \varkappa \sim (t_{\min} - t)/Q^2$$

and different skewedness parameter

$$\xi^{\text{lab}} \simeq \frac{x_B}{2 - x_B} \quad \text{vs.} \quad \xi^{\text{phot}} = \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)}$$

Defining the Leading Twist approximation

Kumerički-Müller convention (KM)

Braun-Manashov-Pirnay convention (BMP)

$$\mathrm{LT}_{\mathrm{KM}}: \begin{cases} \mathcal{F}^{\text{lab}}_{++} = T_0 \otimes F, & \mathcal{F}^{\text{lab}}_{0+} = 0, \\ \mathcal{F}^{\text{lab}}_{-+} = 0, & \xi_{\mathrm{KM}} = \xi^{\text{lab}} \end{cases}$$

$$\mathrm{LT}_{\mathrm{BMP}}: \begin{cases} \mathcal{F}^{\text{phot}}_{++} = T_0 \otimes F, & \mathcal{F}^{\text{phot}}_{0+} = 0, \\ \mathcal{F}^{\text{phot}}_{-+} = 0, & \xi_{\mathrm{BMP}} = \xi^{\text{phot}} \end{cases}$$

$$\Downarrow$$

$$\mathrm{LT}_{\mathrm{BMP}}: \begin{cases} \mathcal{F}^{\text{lab}}_{++} = \left(1 + \frac{\varkappa}{2}\right) T_0 \otimes F, & \mathcal{F}_{0+} = \varkappa_0 \, T_0 \otimes F \\ \mathcal{F}^{\text{lab}}_{-+} = \frac{\varkappa}{2} T_0 \otimes F, & \xi = \xi_{\mathrm{BMP}}, \end{cases}$$

- **Changing frame of reference results in**
  - **Different skewedness parameter for a given x_B**
  - **Numerically significant excitation of helicity-flip CFFs**
- **Different results for experimental observables**

What is “the best” reference frame?

- **For many observables, “photon frame” LT calculation is very close to full twist-4**

Braun, Manashov, Müller, Pirnay: PRD89 (2014) 074022

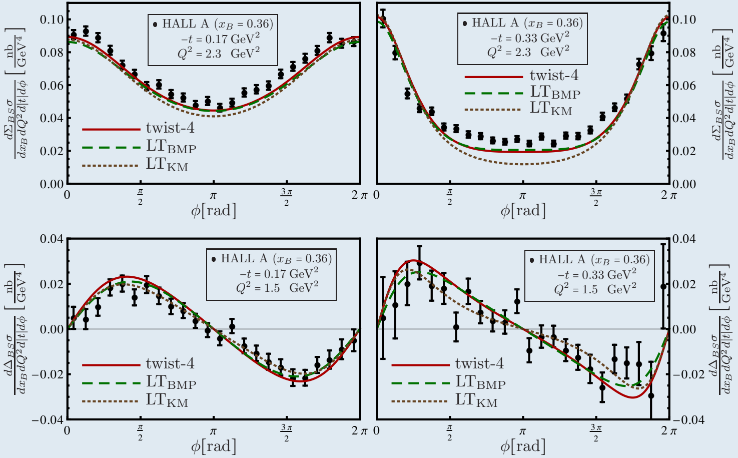
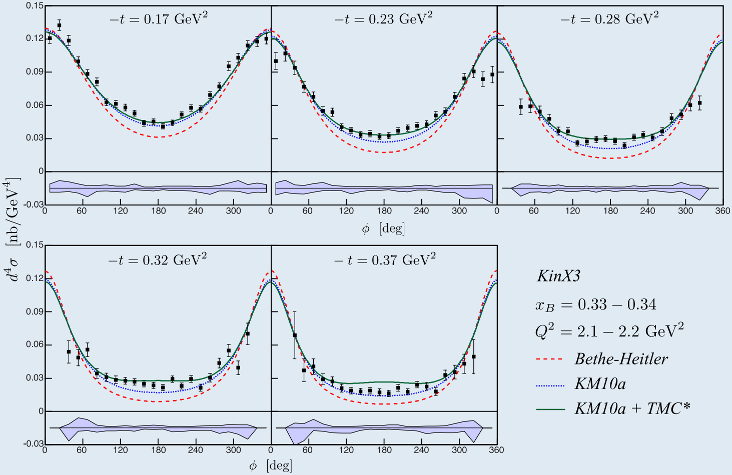


Figure: Unpolarized cross section [upper panels] and electron helicity dependent cross section difference [lower panels] from HALL A (old data) compared to the GK12 GPD model

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



- **TMC* refers to the calculation that includes full kinematic twist-4 corrections**

GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453

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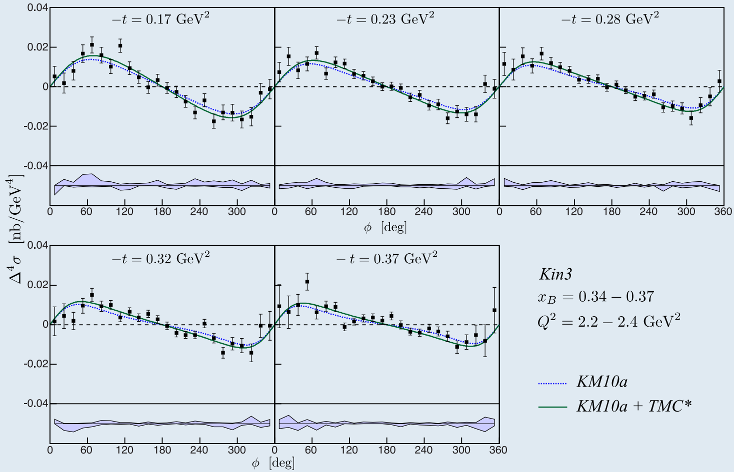

- **TMC* refers to the calculation that includes full kinematic twist-4 corrections**

GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

- **Early work:**

— Extension of Nachtmann's approach to target mass corrections in DIS
— Spin-rotation (Wandzura-Wilczek)

Blümlein, Robaschik: NPB581 (2000) 449
Radyushkin, Weiss: PRD63 (2001) 114012
Belitsky, Müller: NPB589 (2000) 611
...

— Results not gauge invariant
— Results not translation invariant

— A related discussion in a different context:
Ball, Braun: NPB543 (1999) 201

- **A conceptual problem?**

Contributions of different twist are intertwined by symmetries:

- Conservation of the electromagnetic current and translation invariance

$$\partial^\mu T\Big\{j_\mu^{em}(x)j_\nu^{em}(0)\Big\} = 0$$
$$T\Big\{j_\mu^{em}(2x)j_\nu^{em}(0)\Big\} = e^{-i\mathbf{P}\cdot x}\,T\Big\{j_\mu^{em}(x)j_\nu^{em}(-x)\Big\}\,e^{i\mathbf{P}\cdot x}$$

are valid in the sum of all twists but not for each twist separately

- Higher-twist contributions that restore gauge/translation invariance are due to descendants of leading-twist operators obtained by adding total derivatives

$$T\Big\{j_\mu^{em}(x)j_\nu^{em}(0)\Big\} = \underbrace{\sum_N a_N\mathcal{O}_N}_{\text{leading-twist}} + \sum_N\Big(b_N\partial^2\mathcal{O}_N + c_N(\partial\mathcal{O})_N\Big) + \text{ other operators}$$

- These operators contribute to finite-t and target mass corrections

Task: Find the contributions to the OPE of all descendants of leading-twist operators

Invisible operator?

- **The problem:**
  S. Ferrara, A. F. Grillo, G. Parisi and R. Gatto, Phys. Lett. **B38**, 333 (1972):

— matrix elements of $\partial^\mu \mathcal{O}_{\mu\mu_1\ldots\mu_N}$ over free quarks vanish

  - ? Go over to NLO
  - ? More complicated quark-gluon matrix elements
  - ? Off-shell OPE

— main difficulty is to disentangle the contribution of interest from "genuine" twist-4 operators

- Using EOM $\partial^\mu \mathcal{O}_{\mu\mu_1\ldots\mu_N}$ can be expressed in terms of quark-gluon operators. e.g.:

Kolesnichenko '84

$$\partial^\mu O_{\mu\nu} = 2i\bar{q}gF_{\nu\mu}\gamma^\mu q\,,$$

where

$$O_{\mu\nu} = (1/2)[\bar{q}\gamma_\mu \overset{\leftrightarrow}{D}_\nu q + (\mu \leftrightarrow \nu)]$$

- **One cannot ignore $\bar{q}Fq$ operators**

— Is it possible at all to define the separation between "kinematic" and "genuine" twist four?

Guidung principle:

Braun, Manashov, PRL 107 (2011) 202001

— "kinematic" approximation amounts to the assumption that genuine twist-four matrix elements are zero

— for consistency, they must remain zero at all scales

— they must not reappear at higher scales due to mixing with "kinematic" operators

- **"Kinematic" and "Dynamic" contributions must have autonomous scale-dependence**

Indeed, otherwise:

$$\Big(\langle\bar q F q\rangle + c\langle(\partial\mathcal{O})\rangle\Big)^{\mu^2} = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_4/\beta_0}\Big(\langle\bar q F q\rangle + c\langle(\partial\mathcal{O})\rangle\Big)^{\mu_0^2}$$

$$\Longrightarrow$$

$$\langle\bar q F q\rangle^{\mu^2} = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_4/\beta_0}\langle\bar q F q\rangle^{\mu_0^2} + c\left[\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_4/\beta_0} - \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_2/\beta_0}\right]\langle(\partial\mathcal{O})\rangle^{\mu_0^2}$$

[The "kinematic" approximation corresponds to taking into account *all* operators with the same anomalous dimensions as the leading twist operators]

- Explicit diagonalization of the mixing matrix for twist-4 operators not feasible

- In a conformal theory

$$\Big(\mu\partial_\mu + \cancel{\beta(\alpha)\partial_\alpha} + \mathbb{H}\Big) O_j \;=\; 0 \quad\Longrightarrow\quad \langle T\{O_{j_1}(x) O_{j_2}(0)\}\rangle \sim \delta_{j_1 j_2}$$

therefore

$$T\Big\{j(x)j(0)\Big\} \;=\; \sum_N C_N(x,\partial)\mathcal{O}_N + \dots,$$

$$C(x,\partial)\mathcal{O}_N = a_N\mathcal{O}_N + b_N\partial^2\mathcal{O}_N + c_N(\partial\mathcal{O})_N + \dots$$

$\Longrightarrow$

$$\langle T\Big\{j(x)j(0)\mathcal{O}_N(y)\Big\}\rangle \;=\; C_N(x,\partial)\langle T\Big\{\mathcal{O}_N(0)\mathcal{O}_N(y)\Big\}\rangle + 0$$

— might work in QCD

- Orthogonality of eigenoperators suggests that $\mathbb{H}$ is a hermitian operator w.r.t. a certain scalar product

Braun, Manashov, Rohrwild, Nucl. Phys. **B807** (2009) 89; Nucl. Phys. **B826** (2010) 235.

Summary of this part:

- **noncomplanarity makes separation of collinear directions ambiguous**
  - — hence “leading twist approximation” ambiguous
  - — related to violation of translation invariance and EM Ward identities
- **Complete results available to t/Q^2, m^2/Q^2 accuracy**
  - — translation and gauge invariance restored
  - — for many observables, complete results close to LT in “photon frame”
- **Must be taken into account in all studies aiming at 3D proton structure**
- **Outlook:**
  - — all twists in LO, exact translation and gauge invariance
  - — small x, matching to the BFKL formalism
  - — NLO

V. Braun, A. Manashov, S. Moch, M. Strohmaier, work in progress

- **DIS: NNLO**

  Moch, Vermaseren, Vogt, NPB **688**, 101 (2004)
  Vogt, Moch, Vermaseren, NPB **691**, 129 (2004)

- **DVCS (GPDs): NLO**

  Belitsky, Müller, NPB **527**, 207 (1998); ibid. **537**, 397 (1999)
  Belitsky, Freund, Müller, NPB **574**, 347 (2000)

- **Can one close the gap?**

- **One loop:**

  anomalous dimensions $+$ conformal symmetry $\rightarrow$ full anomalous dimension matrix

$$\mathcal{O}_N \sim (\partial_{z_1} + \partial_{z_2})^N C_N^{3/2}\left(\frac{\partial_{z_1} - \partial_{z_2}}{\partial_{z_1} + \partial_{z_2}}\right) \bar{q}(z_1 n)\gamma_+ q(z_2 n)\Big|_{z_{1,2}\to 0}$$

Makeenko, 1980

- **D. Müller, '94-'00**

  — off-diagonal elements of the n-loop mixing matrix are determined by the $(n-1)$-loop conformal anomaly

- **Two loops:**

  Belitsky, Mueller, 2000

- **Three loops:**

  Braun, Manashov, Moch, Strohmaier, work in progress

Different approach:

Instead of studying consequences of conformal symmetry breaking in QCD we make use of *exact* conformal symmetry of a modified theory:

Large N_f QCD in $4-\epsilon$ dimension at critical coupling

QCD in $d = 4 - 2\epsilon$

Consider renormalized QCD in $d = 4 - 2\epsilon$ dimensions. In MS-like schemes

$$\beta^{QCD}(a_s) = 2a_s\,[-\epsilon - \beta_0 a_s + \ldots] \qquad Z = 1 + \sum_{j=1}^{\infty} \epsilon^{-j} \sum_{k=j}^{\infty} a_s^k \mathbb{Z}_{jk}$$

- scale and conformal invariance at the critical point — Banks, Zaks, '82

$$a_s^* = -4\pi\epsilon/\beta_0 + \ldots \qquad \beta^{QCD}(a_s^*) = 0$$

- Z_{jk} do not depend on ϵ by construction, thus

$$\boxed{\mathbb{H}(a_s^*) = a_s^*\,\mathbb{H}^{(1)} + (a_s^*)^2\,\mathbb{H}^{(2)} + \ldots} \qquad \Big(\mu\partial_\mu \; + \; \mathbb{H}(a_s^*)\Big)[\mathcal{O}](z_1, z_2) = 0$$

$$\Longleftrightarrow$$

$$\boxed{\mathbb{H}(a_s) = a_s\,\mathbb{H}^{(1)} + a_s^2\,\mathbb{H}^{(2)} + \ldots} \qquad \Big(\mu\partial_\mu + \beta(g)\partial_g + \mathbb{H}(a_s)\Big)[\mathcal{O}(z_1, z_2)] = 0$$

$$\Big(\mu\partial_\mu + \beta(g)\partial_g + \mathbb{H}(a_s)\Big)[\mathcal{O}(z_1,z_2)] = 0$$

- Conformal symmetry implies existence of three generators that satisfy usual SL(2) relations and commute with the renormalization kernel

$$[S_k, \mathbb{H}] = 0$$

$$[S_+, S_-] = 2S_0 \qquad [S_0, S_+] = S_+ \qquad [S_0, S_-] = -S_-$$

- True to all orders in perturbation theory (in MS-like schemes)
- Complete RG kernel in $d = 4$, a digression to $d = 4 - \epsilon$ is an intermediate step

In a free theory, in coordinate representation

generators $j = 1$ for quarks

$$\begin{aligned} S_+ &= z^2\partial_z + 2jz \\ S_0 &= z\partial_z + j \\ S_- &= -\partial_z \end{aligned}$$

$SL(2)$ algebra

$$\begin{aligned} [S_+, S_-] &= 2S_0 \\ [S_0, S_+] &= S_+ \\ [S_0, S_-] &= -S_- \end{aligned}$$

- In the interacting theory the generators are modified by quantum corrections

$$\begin{aligned} S_+ &= S_+^{(0)} + a_s^* S_+^{(1)} + (a_s^*)^2 S_+^{(2)} + \dots \\ S_0 &= S_0^{(0)} + a_s^* S_0^{(1)} + (a_s^*)^2 S_0^{(2)} + \dots \\ S_- &= S_-^{(0)} \end{aligned}$$

- General structure

$$
\begin{aligned}
S_- &= S_-^{(0)}\,,\\
S_0 &= S_0^{(0)} - \epsilon + \frac{1}{2}\mathbb{H}(a_s^*)\,,\\
S_+ &= S_+^{(0)} + (z_1+z_2)\Big(-\epsilon + \frac{1}{2}\mathbb{H}\Big) + (z_1-z_2)\Delta_+\,,
\end{aligned}
$$

$$
a_s = \frac{\alpha_s^*}{4\pi}
$$

where

$$
\mathbb{H}(a_s^*) = a_s^*\,\mathbb{H}^{(1)} + (a_s^*)^2\,\mathbb{H}^{(2)} + (a_s^*)^3\,\mathbb{H}^{(3)} + \ldots
$$

$$
\Delta_+(a_s^*) = a_s^*\,\Delta_+^{(1)} + (a_s^*)^2\,\Delta_2^{(2)} + (a_s^*)^3\,\Delta_3^{(3)} + \ldots
$$

- Modification of S_0 can be written in terms of the evolution kernel

…but $\Delta_+(a_s^*)$ requires explicit calculation

- Light-ray operator representation

Balitsky, Braun '89

$$[\mathcal{O}](z_1,z_2) \quad\equiv\quad [\bar q(z_1 n)\not n q(z_2 n)] \;\equiv\; \sum_{m,k}\frac{z_1^m z_2^k}{m!k!}[(D_n^m\bar q)(0)\not n(D_n^k q)(0)]$$

$$\mathbb{H}[\mathcal{O}](z_1,z_2) = \int_0^1 d\alpha\int_0^1 d\beta\, h(\alpha,\beta)\,[\mathcal{O}](z_{12}^\alpha, z_{21}^\beta)$$

$$z_{12}^\alpha \equiv z_1\bar\alpha + z_2\alpha \qquad \bar\alpha = 1-\alpha$$

one-loop result

Braun, Manashov, PLB **734**, 137 (2014)

$$\Delta_+^{(1)}[\mathcal{O}](z_1,z_2) = -2C_F\int_0^1 d\alpha\left(\frac{\bar\alpha}{\alpha}+\ln\alpha\right)\Big[[\mathcal{O}](z_{12}^\alpha,z_2) - [\mathcal{O}](z_1,z_{21}^\alpha)\Big]$$

Evolution equations from operator algebra

- **Expanding the commutation relations in powers of a_s^***

$$[S_+^{(0)}, \mathbb{H}^{(1)}] = 0\,,$$
$$[S_+^{(0)}, \mathbb{H}^{(2)}] = [\mathbb{H}^{(1)}, S_+^{(1)}]\,,$$
$$[S_+^{(0)}, \mathbb{H}^{(3)}] = [\mathbb{H}^{(1)}, S_+^{(2)}] + [\mathbb{H}^{(2)}, S_+^{(1)}]\,, \qquad \text{etc.}$$

- A nested set of inhomogenious first order differential equations for $\mathbb{H}^{(k)}$
  Their solution determines $\mathbb{H}^{(k)}$ up to an $SL(2)$-invariant term
- The r.h.s. involves $\mathbb{H}^{(k)}$ and $S_+^{(m)}$ at one oder less compared to the l.h.s. D.Müller

$$\mathbb{H}[\mathcal{O}](z_1,z_2) = \int_0^1 d\alpha \int_0^1 d\beta\, h(\alpha,\beta)\, [\mathcal{O}](z_{12}^{\alpha}, z_{21}^{\beta})$$

$$z_{12}^{\alpha} \equiv z_1\bar{\alpha} + z_2\alpha \qquad \bar{\alpha} = 1-\alpha$$

$$\boxed{[S_+^{(0)}, \mathbb{H}^{(1)}] = 0} \qquad \Longrightarrow \qquad h^{(1)}(\alpha,\beta) = h^{(1)}\left(\frac{\alpha\beta}{\bar{\alpha}\bar{\beta}}\right) = h^{(1)}(\tau)$$

i.e. function of two variables reduces to a function of one variable

$\longrightarrow$ can be restored from anomalous dimensions

$$\gamma_N = \int_0^1 d\alpha \int_0^1 d\beta\, (1-\alpha-\beta)^{N-1} h(\alpha,\beta)$$

Balitsky, Braun, 1989

$$h^{(1)}(\alpha,\beta) = -4C_F\left[\delta_+(\tau) + \theta(1-\tau) - \frac{1}{2}\delta(\alpha)\delta(\beta)\right]$$

- **Combined DGLAP, ERBL and GPD evolution equations in a compact form**

Two loops (I)

Braun, Manashov, PLB734 (2014) 137

have to solve

$$[S_+^{(0)}, \mathbb{H}^{(2)}] = \; [\mathbb{H}^{(1)}, S_+^{(1)}] = [\mathbb{H}^{(1)}, z_1 + z_2] \left(\beta_0 + \frac{1}{2}\mathbb{H}^{(1)}\right) + [\mathbb{H}^{(1)}, \Delta_+^{(1)}]$$

$$\mathbb{H}^{(2)} = \mathbb{T}^{(1)} \left(\beta_0 + \frac{1}{2}\mathbb{H}^{(1)}\right) + [\mathbb{H}^{(1)}, \mathbb{X}^{(1)}] + \mathbb{H}^{(2)}_{inv}$$

$$\mathbb{X}^{(1)} f(z_1, z_2) = 2C_F \int_0^1 \frac{d\alpha}{\alpha} \ln\alpha \Big[2f(z_1, z_2) - f(z_1, z_{21}^\alpha) - f(z_{12}^\alpha, z_2)\Big]$$

$$\mathbb{T}^{(1)} f(z_1, z_2) = 4C_F \Big\{ \int_0^1 \frac{d\alpha}{\alpha} \bar\alpha \ln\bar\alpha \Big[2f(z_1, z_2) - f(z_1, z_{21}^\alpha) - f(z_{12}^\alpha, z_2)\Big] - \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta \, \ln(1-\alpha-\beta) f(z_{12}^\alpha, z_{21}^\beta) \Big\}$$

Braun, Manashov, PLB **734** (2014) 137

invariant part

$$\mathbb{H}^{(2)}_{inv}(N) = \mathbb{H}^{(2)}(N) - T_1(N)\Big(\beta_0 + \frac{1}{2}\mathbb{H}^{(1)}(N)\Big)$$

$$\mathbb{H}^{(2)}_{inv} = 4C_F\Big\{\beta_0\left(\frac{13}{12} + \frac{5}{3}\mathcal{H}_{\langle+\rangle} - \frac{11}{3}\mathcal{H}_{\langle1\rangle}\right) + 2C_A\left(\frac{19}{48} - \frac{1}{3}\mathcal{H}_{\langle+\rangle} - \frac{2}{3}\mathcal{H}_{\langle1\rangle} - \frac{1}{4}\mathcal{H}^2_{\langle1\rangle}\right)$$
$$+ \frac{2}{N_c}\Big[\Big(3\zeta(3) - \frac{\pi^2}{3} + \frac{11}{16}\Big) - \frac{\pi^2-6}{6}\left(\mathcal{H}_{\langle+\rangle} - \mathcal{H}_{\langle1\rangle}\right) + \mathcal{H}_{\langle\frac{1}{\tau}\ln\bar\tau\rangle} - \frac{3}{4}\mathcal{H}^2_{\langle1\rangle} - \mathcal{H}^3_{\langle1\rangle}$$
$$- \frac{1}{2}\mathbb{P}_{12}\Big(\mathcal{H}_{\langle\ln^2\bar\tau\rangle} - 2\mathcal{H}_{\langle\tau\ln\bar\tau\rangle}\Big)\Big]\Big\}.$$

compare LO

$$\mathbb{H}^{(1)} = 4C_F\Big\{\mathcal{H}_{\langle+\rangle} - \mathcal{H}_{\langle1\rangle} + \frac{1}{2}\Big\}$$

Two-loop conformal anomaly

- two-loop result

Braun, Manashov, Moch, Strohmaier: JHEP **1603** (2016) 142

$$[\Delta_+^{(2)}\mathcal{O}](z_1,z_2) = \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta \Big[\omega(\alpha,\beta)+\omega^{\mathbb{P}}(\alpha,\beta)\mathbb{P}_{12}\Big]\Big[\mathcal{O}(z_{12}^{\alpha},z_{21}^{\beta})-\mathcal{O}(z_{12}^{\beta},z_{21}^{\alpha})\Big] + \int_0^1 du \int_0^1 dt\, \varkappa(t)\Big[\mathcal{O}(z_{12}^{ut},z_2)-\mathcal{O}(z_1,z_{21}^{ut})\Big].$$

$$\omega^{\mathbb{P}} = 2\Big[C_F^2-\frac{1}{2}C_F C_A\Big]\Big[\Big(\bar\alpha-\frac{1}{\bar\alpha}\Big)\Big[\mathrm{Li}_2\Big(\frac{\alpha}{\bar\beta}\Big)-\mathrm{Li}_2(\alpha)-\ln\bar\alpha\ln\bar\beta\Big]+\alpha\bar\tau\ln\bar\tau+\frac{\beta^2}{\bar\beta}\ln\bar\alpha\Big]$$

Two-loop conformal anomaly (II)

- two-loop result — Braun, Manashov, Moch, Strohmaier: JHEP **1603** (2016) 142

$$\omega(\alpha,\beta) = C_F^2\,\omega_{FF}(\alpha,\beta) + C_F C_A\,\omega_{FA}(\alpha,\beta)$$

$$\begin{aligned}
\omega_{FF} = 4\Big\{&\Big(\alpha-\frac{1}{\alpha}\Big)\Big[\mathrm{Li}_2\Big(\frac{\beta}{\bar\alpha}\Big)-\mathrm{Li}_2(\beta)-\mathrm{Li}_2(\alpha)-\frac{1}{4}\ln^2\bar\alpha\Big]-\alpha\Big[\mathrm{Li}_2(\alpha)-\mathrm{Li}_2(1)\Big]\\
&-\frac{\alpha+\beta}{2}\ln\alpha\ln\bar\alpha+\frac{1}{4}\Big[\beta\ln^2\bar\alpha-\alpha\ln^2\alpha\Big]-\frac{\alpha}{\tau}\Big[\tau\ln\tau+\bar\tau\ln\bar\tau\Big]\\
&+\frac{1}{4}\Big[\beta-2\bar\alpha+\frac{2\beta}{\alpha}\Big]\ln\bar\alpha+\frac{1}{2}\Big[\bar\alpha-\frac{\alpha}{\bar\alpha}-3\beta\Big]\ln\alpha-\frac{15}{4}\alpha\Big\},
\end{aligned}$$

$$\begin{aligned}
\omega_{FA} = 2\Big\{&\Big(\frac{1}{\alpha}-\alpha\Big)\Big[\mathrm{Li}_2\Big(\frac{\beta}{\bar\alpha}\Big)-\mathrm{Li}_2(\beta)-2\,\mathrm{Li}_2(\alpha)-\ln\alpha\ln\bar\alpha\Big]\\
&+\frac{\alpha}{\tau}\Big[\tau\ln\tau+\bar\tau\ln\bar\tau\Big]-\bar\beta\ln\alpha-\frac{\bar\alpha}{\alpha}\ln\bar\alpha\Big\},
\end{aligned}$$

$$\tau = \frac{\alpha\beta}{\bar\alpha\bar\beta}$$

- result for $\varkappa(t)$ of similar complexity

Braun, Manashov, Moch, Strohmaier: work in progress

- have to solve

$$[S_+^{(0)}, \mathbb{H}^{(3)}] = [\mathbb{H}^{(1)}, S_+^{(2)}] + [\mathbb{H}^{(2)}, S_+^{(1)}]$$

$\Rightarrow$ three-loop evolution equation for light-ray operators

- **Alternative: find a transformation**

$$\mathcal{O}'(z_1, z_2) = U\,\mathcal{O}(z_1, z_2)\,, \qquad S_k' = U\, S_k U^{-1}\,, \qquad \mathbb{H}' = U\,\mathbb{H}\,U^{-1}\,,$$
$$S_0' = S_0^{(0)} - \epsilon + \frac{1}{2}\mathbb{H}'(a_s^*)\,,$$
$$S_+' = S_+^{(0)} + (z_1 + z_2)\Big(-\epsilon + \frac{1}{2}\mathbb{H}'\Big) + \cancel{(z_1 - z_2)\Delta_+}\,,$$

$\Rightarrow$ the generators acquire their canonical form

For conformal operators $\mathbb{H}'(a_s^*) \to \gamma_N^*$

$$S_0' = z_1 \frac{\partial}{\partial z_1} + z_2 \frac{\partial}{\partial z_2} + j_N$$

$$S_+' = z_1^2 \frac{\partial}{\partial z_1} + z_2^2 \frac{\partial}{\partial z_2} + (z_1 + z_2) j_N$$

$$\boxed{j_N = 2 - \epsilon + N + \frac{1}{2}\gamma_N^*}$$

$\Rightarrow$ multiplicatively renormalizable light-ray operators

Summary for this part

- QCD evolution equations in $\overline{MS}$ schemes have "hidden" conformal symmetry

$$[S_k, \mathbb{H}] = 0 \qquad [S_+, S_-] = 2S_0 \qquad [S_0, S_+] = S_+ \qquad [S_0, S_-] = -S_-$$

- The generators are deformed by quantum corrections; known to two-loop accuracy
- QCD at $d = 4 - \epsilon$ at critical coupling can be used to define

$$\mathcal{Q} = \mathcal{Q}^{\text{conformal}} + \frac{\beta(g)}{g}\Delta\mathcal{Q}$$

- Possible applications:
  - — **Three-loop evolution equations in non-forward kinematics**
  - — **Generalized Crewther relation**
  - — **Large-N_f QCD in conformal window**
  - — $N = 4$ **SYM**

Outlook

- Studies of hard exclusive (and semiinclusive) reactions have a long history.
  The problem is that the theory description remains semiquantitative, with an ever longer list of reactions described with increasingly sophisticated nonperturbative input.
  This has to end for the field to have a future.

- Arguing for EIC as "exclusive" machine, need "gold plated" processes where QCD description can and will match the accuracy achieved in DIS and jet physics.
  DVCS can play this role.

- **Present:**
  — **Full NLO results available**
  — **Gauge and Lorentz invariance to t/Q^2 and m^2/Q^2 accuracy (kinematic twist-4)**
  — **Exact expressions for observables in terms of CFFs**
  — **Need standard (open source?) NLO+ analysis code**

- **Future:**
  — **NNLO**
  — **Gauge and Lorentz invariant LT at LO level**
  — **Gauge and Lorentz invariant LT beyond LO**
  — **Lattice calculations of the first two GPD moments**