Decays of neutral pions

Electromagnetic form factors and radiative corrections

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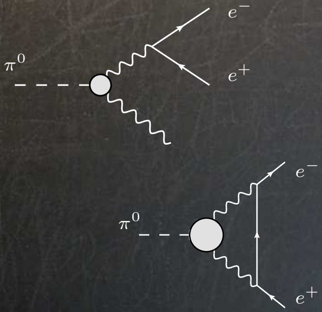
Introduction

Decay modes of neutral pion:

Process	Branching ratio
$\pi^0 \to \gamma\gamma$	$(98.823 \pm 0.034)\,\%$
$\pi^0 \to e^+e^-\gamma$	$(1.174 \pm 0.035)\,\%$
$\pi^0 \to e^+e^+e^-e^-$	$(3.34 \pm 0.16) \times 10^{-5}$
$\pi^0 \to e^-e^-$	$(6.46 \pm 0.33) \times 10^{-8}$



Rare decay $\pi^0 \to e^+e^-$

- interesting way to study low-energy (long-distance) dynamics in the SM
- systematic theoretical treatment dates back to *Drell*, NC (1959)
- suppressed in comparison to the decay $\pi^0 \to \gamma\gamma$ by a factor of $2(\alpha m_e/M_\pi)^2$

  → one-loop structure + approximate helicity conservation
  → may be sensitive to possible effects of new physics

KTeV measurement

KTeV-E799-II experiment at Fermilab (*Abouzaid et al.*, PRD 75 (2007))
→ precise measurements of branching ratio $\pi^0 \to e^+e^-$ (794 candidates)

$$\frac{\Gamma(\pi^0 \to e^+e^-(\gamma),\, x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma,\, x > 0.232)} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

Extrapolate the Dalitz decay branching ratio to full range of x

$$B^{\text{KTeV}}(\pi^0 \to e^+e^-(\gamma),\, x_{\text{D}} > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}$$

- PDG average value $(6.46 \pm 0.33) \times 10^{-8}$ mainly based on this result
- extrapolate full radiative tail beyond $x > 0.95$ (*Bergström*, Z.Ph.C 20 (1983))
- scale the result back by the overall radiative corrections

→ final result for lowest order (no final state radiation)

$$B^{\text{no-rad}}_{\text{KTeV}}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}$$

Comparison with SM prediction (*Dorokhov and Ivanov*, PRD 75 (2007))

$$B^{\text{no-rad}}_{\text{SM}}(\pi^0 \to e^+e^-) = (6.23 \pm 0.09) \times 10^{-8}$$

→ interpreted as 3.3 σ discrepancy between theory and experiment

New physics?

- very fashionable to ascribe eventual discrepancies to effects of new physics

Dark matter

Motivation

- theoretical models proposed and excluded due to various observations

- 511 keV signal coming from the center of the galaxy
  → corresponds to the annihilation of a great amount of e^+e^- pairs
  → cannot be explained by presently known astrophysical sources

- consistent with the scalar dark matter model
  → new light neutral gauge boson

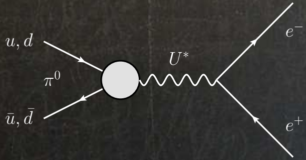
- one of the possible contributions of new physics to the process $\pi^0 \to e^+e^-$

U boson

Contribution to the $\pi^0 \to e^+e^-$ process

In this model (*Boehm and Fayet,* NPB 683 (2004)), we take simply

$$\mathcal{L}_{\text{int}}^{U,f} \overset{\text{eff}}{=} \sum_{\xi=u,d,e} \bar{\psi}_\xi \gamma_\mu (g_V^\xi + g_A^\xi \gamma_5) \psi_\xi U^\mu$$



To obtain the relevant informations, we assume (not excluded by observations)

- $M_U \simeq 10$ MeV (*Fayet,* PRD 74 (2006))
- $g_A^e \simeq (g_A^d - g_A^u)$

Final estimate of desired coupling in order to solve the discrepancy

$$|g_A^e| \simeq 2 \times 10^{-4}$$

What about radiative corrections?

- first, look for more conventional solution (i.e. within SM)

  → radiative corrections (usually very important)

  → form factor modeling

QCD at low energy

- QCD describes the dynamics of quarks and gluons

- at low energy we need to describe interactions of hadrons (e.g. pions) instead
  → the relevant degrees of freedom are different

- effective description
  → does not need to be necessarily less precise
  → in general, the structure at short distances (high energies) is ignored
  → many unknown parameters appear (so called LECs)
  → set from experiment or more fundamental theory (matching)

- the perturbation series is not based on the coupling
  → α_s grows with the distance
  → different power counting schemes - derivatives (momenta)

- χPT answers the question how to construct the Lagrangian monomials
  → Green functions satisfy the same Ward identities as in QCD

Chiral Perturbation Theory

QCD Lagrangian in massless limit

$$\mathcal{L}_{\mathrm{QCD}}^{(m_f=0)} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + i\bar{q}_{\mathrm{L}}\not{D}q_{\mathrm{L}} + i\bar{q}_{\mathrm{R}}\not{D}q_{\mathrm{R}}$$

where

$$q_{\mathrm{L,R}} = \frac{1}{2}(1 \mp \gamma_5)q\,, \qquad q = \begin{bmatrix} u \\ d \\ s \end{bmatrix}$$

→ invariant under chiral symmetry group $\mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}}$

$$\Downarrow$$

$$\mathcal{L}_{\chi\mathrm{PT}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots$$

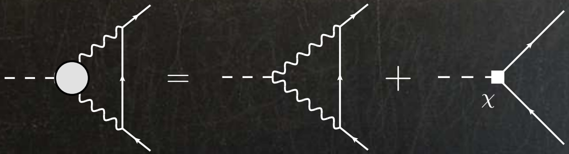
$$\mathcal{L}_2 = \tfrac{1}{4}F_0{}^2\langle D_\mu U(D^\mu U)^\dagger\rangle + \tfrac{1}{4}F_0{}^2\langle \chi U^\dagger + U\chi^\dagger\rangle\,, \quad \chi = 2B_0(s+ip)$$

$$\mathcal{L}_4 = \sum_{i=1}^{10} L_i O_4^i$$

$$\mathcal{L}_6 = \sum_{i=1}^{90} C_i O_6^i$$

Leading order

- pions are complicated composite objects
  → elementary interactions are not point-like
- electromagnetic pion transition form factor $F_{\pi^0\gamma^*\gamma^*}$ describes this complexity



LO contribution
in QED expansion

its representation
as the LO of χPT

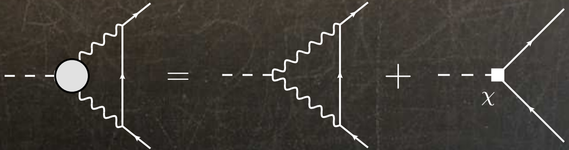
- free parameter $\chi^{(\mathrm{r})}(\mu)$ appears in the finite part of the counter term

$$\chi = [\text{UV-divergent part}] + \chi^{(\mathrm{r})}(\mu)$$

→ unique for every form factor, e.g. $\chi^{(\mathrm{r})}_{\mathrm{KTeV}}(M_\rho) = 6.0 \pm 1.0$

Leading order

Lagrangian



$$^{\text{tree}}\mathcal{L}_{\text{WZW}}^{\pi^0\to\gamma\gamma} = -\frac{e^2}{2}\frac{1}{4\pi^2 F}\epsilon^{\mu\nu\rho\sigma}(\partial_\mu A_\nu)(\partial_\rho A_\sigma)\pi^0 \quad\longrightarrow\quad F_{\pi^0\gamma^*\gamma^*}^{\text{LO}} = -\frac{1}{4\pi^2 F}$$

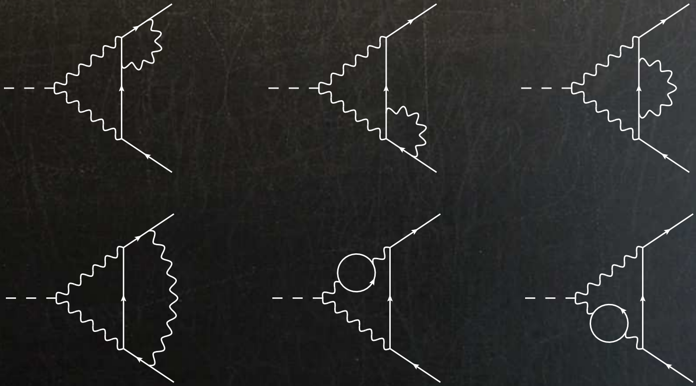
Local counter-term chiral Lagrangian à la *Savage et al.*, PLB 291 (1992)

$$\mathcal{L}_{\pi l\bar{l}} = \frac{3i}{2}\left(\frac{\alpha}{4\pi}\right)^2(\bar{l}\gamma^\mu\gamma_5 l)\times$$
$$\times\left\{\chi_1\operatorname{Tr}[Q^2\partial_\mu U U^\dagger - Q^2\partial_\mu U^\dagger U] + \chi_2\operatorname{Tr}[Q\partial_\mu U Q U^\dagger - Q\partial_\mu U^\dagger Q U]\right\}$$

$$\chi = -\frac{(\chi_1+\chi_2)}{4} \overset{\text{LO}}{=} \frac{3}{2}\left(\frac{1}{\epsilon} - \gamma_{\text{E}} + \log 4\pi\right) + \chi^{(\text{r})}(\mu)$$

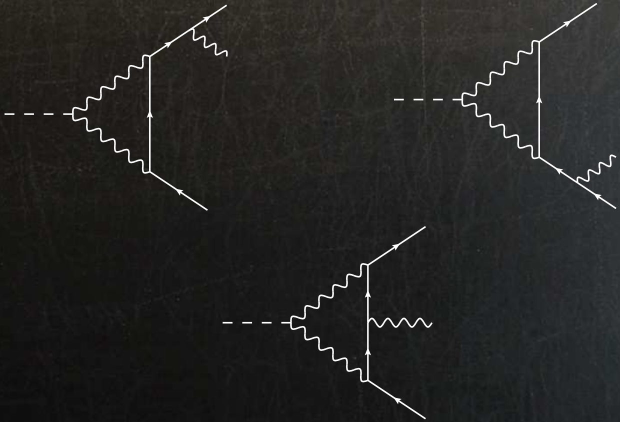
Two-loop virtual radiative corrections

- calculated by *Vaško and Novotný*, JHEP 1110 (2011)



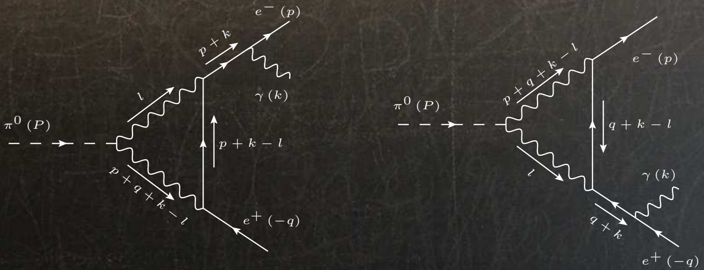
Bremsstrahlung

- compensation of infrared divergences in 2-loop contributions
  $\rightarrow$ *TH, Kampf and Novotný*, EPJC 74 (2014)



Bremsstrahlung

photon emission from the outer fermion line



- contain UV subdivergences → counter-term tree diagrams with couplig χ

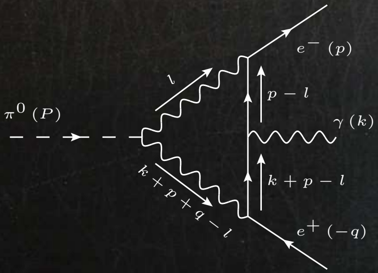
Bremsstrahlung

photon emission from the inner fermion line (propagator)

Do not forget the third, box diagram, necessary to satisfy the Ward identities

$$\mathcal{M}_{(\lambda)} = \varepsilon^{*\rho}_{(\lambda)}(k)\mathcal{M}^{\mathsf{BS}}_\rho \quad\longrightarrow\quad k^\rho \mathcal{M}^{\mathsf{BS}}_\rho = 0$$

- finite contribution to bremsstrahlung amplitude



Final matrix element

Contents

$$\begin{aligned}
i\mathcal{M}_{(\lambda)}(p,q,k) = {}& \frac{ie^5}{8\pi^2 F}\epsilon^{*\rho}_{(\lambda)}(k) \\
& \times\Big\{ P(x,y)\left[(k\cdot p)q_\rho - (k\cdot q)p_\rho\right]\left[\bar{u}(p,m)\gamma_5 v(q,m)\right] \\
& +A(x,y)\Big[\bar{u}(p,m)\left[\gamma_\rho(k\cdot p) - p_\rho \not{k}\right]\gamma_5 v(q,m)\Big] \\
& -A(x,-y)\Big[\bar{u}(p,m)\left[\gamma_\rho(k\cdot q) - q_\rho \not{k}\right]\gamma_5 v(q,m)\Big] \\
& +T(x,y)[\bar{u}(p,m)\gamma_\rho \not{k}\gamma_5 v(q,m)]\Big\}
\end{aligned}$$

$$\begin{aligned}
\overline{|\mathcal{M}^{\mathrm{BS}}(x,y)|^2} \equiv {}& \sum_\lambda \left|\mathcal{M}_{(\lambda)}(p,q,k)\right|^2 = \\
= {}& \frac{16\pi\alpha^5}{F^2}\frac{M^4(1-x)^2}{8}\Big\{M^2\left[x(1-y^2)-\nu^2\right]\Big[xM^2|P|^2 \\
& + 2\nu M\,\mathrm{Re}\left\{P^*\left[A(x,y)+A(x,-y)\right]\right\} - 4\,\mathrm{Re}\left\{P^*T\right\}\Big] \\
& + 2M^2(x-\nu^2)(1-y)^2|A(x,y)|^2 + (y\to -y) \\
& - 8\nu M y(1-y)\,\mathrm{Re}\left\{A(x,y)T^*\right\} + (y\to -y) \\
& - 4\nu^2 M^2 y^2\,\mathrm{Re}\left\{A(x,y)A(x,-y)^*\right\} + 8(1-y^2)|T|^2\Big\}
\end{aligned}$$

Size of the radiative corrections (newly calculated)

$$\delta^{\text{NLO}}(0.95) \equiv \delta^{\text{virt.}} + \delta^{\text{BS}}(0.95) = (-5.5 \pm 0.2)\,\%$$

- can be thought as model-independent
- differs significantly from previous approximate calculations

Bergström, Z.Ph.C 20 (1983): $\delta(0.95) = -13.8\,\%$
Dorokhov et al., EPJC 55 (2008): $\delta(0.95) = -13.3\,\%$

- original KTeV vs. SM discrepancy reduced to the $2\,\sigma$ level or less
- contact interaction coupling finite part set to

$$\chi^{(\text{r})}_{\text{LMD}}(M_\rho) = 2.2 \pm 0.9$$

New fit of the coupling $\chi^{(\mathrm{r})}$ value

LMD approximation to the large-N_c spectrum of vector meson resonances
→ theoretically modeled by *Knecht et al.*, PRL 83 (1999)

OR

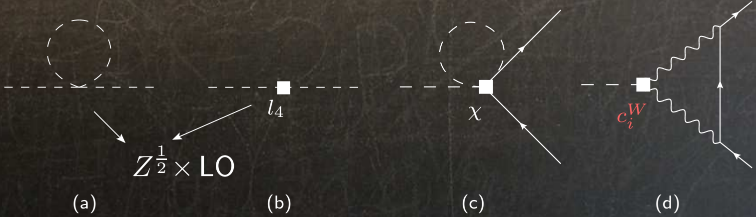
- numerically fit the coupling to KTeV result using all available corrections
- alternatively, use approximative formula → same result

$$\chi^{(\mathrm{r})}(M_\rho) \simeq \frac{5}{2} + \frac{3}{2}\log\left(\frac{M_\rho^2}{m^2}\right) - \frac{\pi^2}{12} - \frac{1}{4}\log^2\left(\frac{M^2}{m^2}\right)$$
$$+ \sqrt{\frac{1}{2}\left(\frac{\pi M}{\alpha m}\right)^2 \frac{B(\pi^0 \to e^+e^-(\gamma),\, x_\mathrm{D} > 0.95)}{B(\pi^0 \to \gamma\gamma)\left[1 + \delta^{(\text{2-loop})}(0.95)\right]} - \frac{\pi^2}{4}\log^2\left(\frac{M^2}{m^2}\right)}$$

Final model independent effective value

$$\chi^{(\mathrm{r})}(M_\rho) = 4.5 \pm 1.0$$

One-loop diagrams of order α^2/F^3 for process $\pi^0 \to e^+e^-$



- the leading log estimation, i.e. taking terms $\sim \log^2 \mu^2$ (up to two loops) → Weinberg consistency relation
- only the contribution from c_{13}^W diagram survives

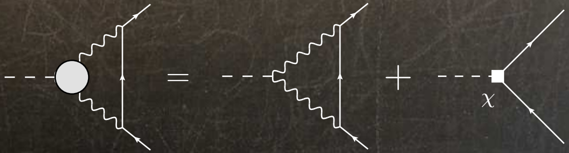
The final correction → stability in the strong sector

$$\Delta^{\mathsf{LL}}\chi^{(\mathsf{r})}(M_\rho) = \frac{1}{36}\left(\frac{M}{4\pi F}\right)^2\left(1-\frac{10m^2}{M^2}\right)\log^2\left(\frac{M_\rho^2}{m^2}\right) \doteq 0.081$$

End of story?

NLO radiative corrections in the QED sector did not solve the discrepancy
→ back to LO, but use different model

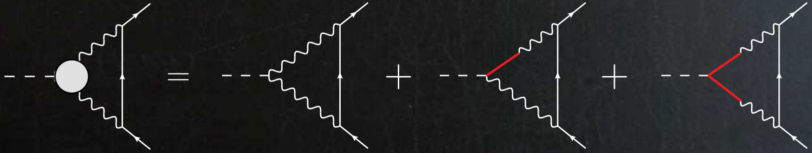
Resonances



Chiral Perturbation Theory (χPT)

$\Updownarrow$

Resonance Chiral Theory (RχT)



PVV correlator

Following *Knecht and Nyffeler*, EPJC 21 (2001)
→ introduce (using $r = p + q$) PVV correlator

$$d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \Pi(r^2; p^2, q^2) \equiv \int \mathrm{d}^4x\, \mathrm{d}^4y\, e^{ip\cdot x + iq\cdot y}\langle 0|T[P^a(0)V_\mu^b(x)V_\nu^c(y)]|0\rangle$$

vector currents and pseudoscalar densities defined by

$$V_\mu^a(x) \equiv \bar{q}(x)\gamma_\mu T^a q(x) \qquad P^a(x) \equiv \bar{q}(x)i\gamma_5 T^a q(x)$$

→ above we use

$$\mathrm{Tr}[T^a, T^b] = \frac{1}{2}\delta^{ab} \qquad d^{abc} \equiv 2\,\mathrm{Tr}[\{T^a, T^b\}T^c]$$

- for $a = 1, \dots, 8$ we have $T^a \equiv \lambda^a/2$
  → λ^a denote the Gell-Mann matrices in flavor space

THS model for PVV correlator

1) Ansatz for Pseudoscalar-Vector-Vector (PVV) correlator

- Two-Hadron-Saturation (THS) - 2 meson multiplets per channel

$$\Pi^{\text{THS}}(r^2;p^2,q^2) \sim \frac{1}{r^2(r^2-M_P^2)} \frac{P(r^2;p^2,q^2)}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)}$$

- in numerator stands general polynomial symmetrical in p^2 and q^2
  → correlator must drop at large momenta
  → 22 free parameters

$$P(r^2;p^2,q^2) = c_0 p^2 q^2 + c_1[(p^2)^3 q^2 + (q^2)^3 p^2] + c_2 (r^2)^2 p^2 q^2 + \ldots$$

2) Use high- and low-energy limits to constrain the parameters

- Operator product expansion (OPE)
- Brodsky–Lepage (BL) quark counting rules
- chiral anomaly

THS model for PVV correlator

High-energy constraints: OPE

Leading-order OPE constraints (*Knecht and Nyffeler*, EPJC 21 (2001)):

$$\Pi\big((\lambda r)^2;(\lambda p)^2,(\lambda q)^2\big) = \frac{1}{2}B_0F^2\frac{1}{\lambda^4}\frac{r^2+p^2+q^2}{r^2p^2q^2}+\mathcal{O}\left(\frac{1}{\lambda^6}\right)$$

$$\Pi\big(r^2;(\lambda p)^2,(r-\lambda p)^2\big) = B_0F^2\frac{1}{\lambda^2}\frac{1}{r^2p^2}+\mathcal{O}\left(\frac{1}{\lambda^3}\right)$$

- third OPE constraint automatically fulfilled

$$\Pi\big((q+\lambda p)^2;(\lambda p)^2,q^2\big) = \frac{1}{\lambda^2}\frac{1}{p^2}f(q^2)+\mathcal{O}\left(\frac{1}{\lambda^3}\right)$$

THS model for PVV correlator

High-energy constraints: B–L

Define the πVV correlator:

$$\mathcal{F}_{\pi VV}(p^2,q^2) \equiv \frac{1}{\mathcal{Z}_\pi} \lim_{r^2\to 0} r^2 \Pi(r^2;p^2,q^2)\,,$$

where

$$\mathcal{Z}_\pi \equiv \frac{i}{2}\langle 0|(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)|\pi^0\rangle = B_0 F$$

→ overlap between the pion field and the pseudoscalar quark density

Instead of involving subleading orders in the high-energy expansion
→ Brodsky–Lepage (B–L) constraint (*Brodsky and Lepage*, PRD 24 (1981)):

$$\frac{\mathcal{F}_{\pi VV}(0,q^2)}{\mathcal{F}_{\pi VV}(0,0)} \to -\frac{24\pi^2 F^2}{N_c}\frac{1}{q^2}\,,\quad q^2\to-\infty$$

THS model for PVV correlator

Low-energy constraints: chiral anomaly

Next, define the pion transition form factor (only rescaling)

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(p^2,q^2)=\frac{2}{3}\mathcal{F}_{\pi VV}(p^2,q^2)$$

and match it at photon point to chiral anomaly:

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,0)=-\frac{N_c}{12\pi^2F}$$

$${}^{\text{tree}}\mathcal{L}_{\text{WZW}}^{\pi^0\to\gamma\gamma}=-\frac{e^2}{2}\frac{1}{4\pi^2F}\epsilon^{\mu\nu\rho\sigma}(\partial_\mu A_\nu)(\partial_\rho A_\sigma)\pi^0 \quad\longrightarrow\quad F_{\pi^0\gamma^*\gamma^*}^{\text{LO}}=-\frac{1}{4\pi^2F}$$

THS and $\mathcal{F}_{\pi^0\gamma^*\gamma^*}$ form factor

Form factor is in general related to PVV correlator as

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) \sim \lim_{r^2\to 0} r^2\Pi(r^2;p^2,q^2)$$

→ in our case complicated, but with only one free parameter

$$\mathcal{F}^{\mathrm{THS}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) = -\frac{N_c}{12\pi^2 F}\left[\frac{M_{V_1}^4 M_{V_2}^4}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)}\right]$$
$$\times\left\{1+\frac{\kappa}{2N_c}\frac{p^2q^2}{(4\pi F)^4}-\frac{4\pi^2F^2(p^2+q^2)}{N_cM_{V_1}^2M_{V_2}^2}\left[6+\frac{p^2q^2}{M_{V_1}^2M_{V_2}^2}\right]\right\}$$

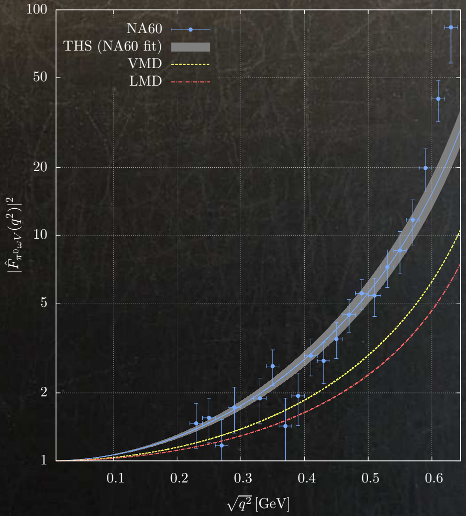
κ determined from fit to ω-π transition form factor measurements
→ *NA60*, PLB 677 (2009)

$$\kappa = 21 \pm 3$$

$M_{V_1} \sim \rho,\omega$ vector-meson mass
$M_{V_2} \sim$ between physical masses of first and second vector-meson excitations

$$M_{V_2} \in [1400, 1740]\ \text{MeV}$$

Fit of κ to NA60 data



VMD and LMD models

Examples of other approaches

- Vector-Meson Dominance (VMD)

$$\mathcal{F}^{\text{VMD}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) = -\frac{N_c}{12\pi^2 F}\left[\frac{M_{V_1}^4}{(p^2-M_{V_1}^2)(q^2-M_{V_1}^2)}\right]$$

$\rightarrow$ violates OPE: $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q^2,q^2) \not\sim \frac{1}{q^2}\,,\ q^2 \to -\infty$

- Lowest-Meson Dominance (LMD)

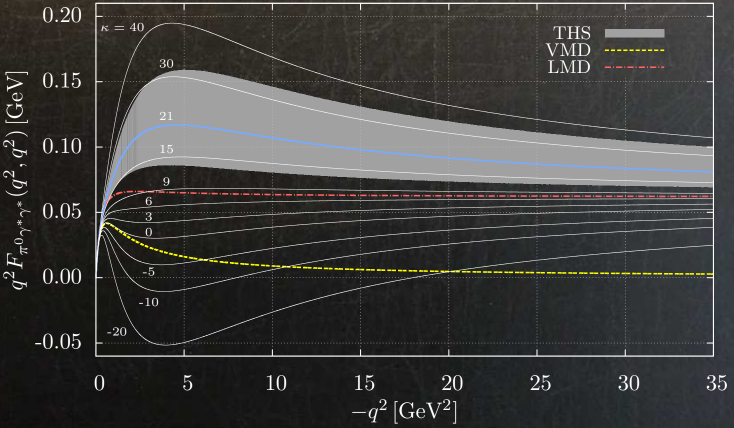
$$\mathcal{F}^{\text{LMD}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) = \mathcal{F}^{\text{VMD}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2)\left\{1-\frac{4\pi^2F^2(p^2+q^2)}{N_c M_{V_1}^4}\right\}$$

$\rightarrow$ violates BL: $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,q^2) \not\sim \frac{1}{q^2}\,,\ q^2 \to -\infty$

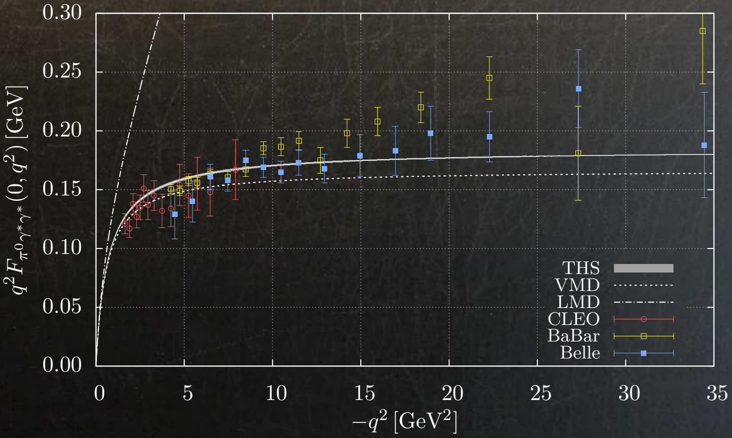
- none of the models used two meson multiplets in both channels

  $\rightarrow$ vector and pseudoscalar

Doubly off-shell form factor



Form factor data



Results

Theoretical prediction within THS model

$$B^{\mathsf{THS}}(\pi^0 \to e^+e^-(\gamma),\, x_{\mathrm{D}} > 0.95) = (5.8 \pm 0.2) \times 10^{-8}$$

- recall experimental value: $B^{\mathsf{KTeV}} = (6.44 \pm 0.33) \times 10^{-8}$
  → disagreement at the level of only 1.8 σ
- matching on LO χPT gives $\chi^{(\mathsf{r})}_{\mathsf{THS}}(M_\rho) = 2.2 \pm 0.7$
- if KTeV result confirmed (e.g. by NA62) → two scenarios are conceivable:
  a) some aspects of the THS approach not well-suited for $\pi^0 \to e^+e^-$
  b) beyond-Standard Model physics influences the rare pion decay significantly
- under the present circumstances the current discrepancy is inconclusive

Quantity really measured by KTeV

$$\left.\frac{\Gamma(\pi^0 \to e^+e^-(\gamma)\,,\; x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma(\gamma)\,,\; x > 0.2319)}\right|_{\mathsf{KTeV}} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

→ Dalitz decay comes into play

Dalitz decay

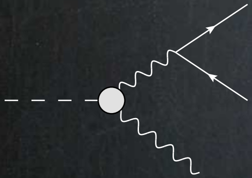
- second most important decay channel of a neutral pion
  → branching ratio $(1.174 \pm 0.035)\,\%$
- first studied by *Richard H. Dalitz*, PPSA 64 (1951), whose name it carries
- experimental data of this process provide the information about the singly off-shell pion transition form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,q^2)$
  → in particular about its slope parameter a_π

Dalitz decay

Leading order

Lorentz-invariant matrix element and its manifestly gauge-invariant form

$$\begin{aligned} i\mathcal{M}_{\mathrm{D}}^{\mathrm{LO}}(p,q,k) = {} & \frac{e^3}{M^2 x}\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,M^2x)\epsilon^{*\rho}(k) \\ & \times \Big\{ 2m\left[\bar{u}(p,m)\gamma_\rho \not{k}\gamma_5 v(q,m)\right] \\ & + \Big[\bar{u}(p,m)\left[\gamma_\rho\,(k\cdot p) - p_\rho\not{k}\right]\gamma_5 v(q,m)\Big] \\ & - \Big[\bar{u}(p,m)\left[\gamma_\rho\,(k\cdot q) - q_\rho\not{k}\right]\gamma_5 v(q,m)\Big]\Big\} \end{aligned}$$



Kinematic variables

- normalized square of the total energy of e^+e^- pair in their CMS

$$x = \frac{(p+q)^2}{M^2}$$

- rescaled cosine of the angle between the directions of outgoing photon and positron in the e^+e^- CMS

$$y = -\frac{2}{M^2}\left[\frac{k\cdot(p-q)}{1-x}\right]$$

Introduce $\nu = 2m/M$ and $\beta(x) = \sqrt{1-\frac{\nu^2}{x}}$

→ limits on x and y

$$x \in [\nu^2, 1]\,, \quad y \in [-\beta(x), \beta(x)]$$

Dalitz decay

Decay width

Contents

LO expression for the decay rate of the neutral pion main decay mode

$$\Gamma^{\text{LO}}_{\pi^0\to\gamma\gamma} = \frac{e^4 M^3}{64\pi}\left|\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,0)\right|^2$$

In general in terms of variables x and y

$$\mathrm{d}\Gamma(x,y) = \frac{M}{(8\pi)^3}\overline{|\mathcal{M}(x,y)|^2}(1-x)\,\mathrm{d}x\,\mathrm{d}y$$

Differential decay rate then reads

$$\frac{\mathrm{d}^2\Gamma^{\text{LO}}_{\text{D}}(x,y)}{\mathrm{d}x\mathrm{d}y} = \left(\frac{\alpha}{\pi}\right)|f(x)|^2\,\Gamma^{\text{LO}}_{\pi^0\to\gamma\gamma}\frac{(1-x)^3}{4x}\left[1+y^2+\frac{\nu^2}{x}\right]$$

$$\frac{\mathrm{d}\Gamma^{\text{LO}}_{\text{D}}(x)}{\mathrm{d}x} = \left(\frac{\alpha}{\pi}\right)|f(x)|^2\,\Gamma^{\text{LO}}_{\pi^0\to\gamma\gamma}\frac{(1-x)^3}{4x}\frac{8\beta(x)}{3}\left[1+\frac{\nu^2}{2x}\right]$$

→ normalized singly off-shell form factor related directly to slope

$$f(x) \equiv \frac{\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,M^2x)}{\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,0)} \simeq 1 + a_\pi x$$

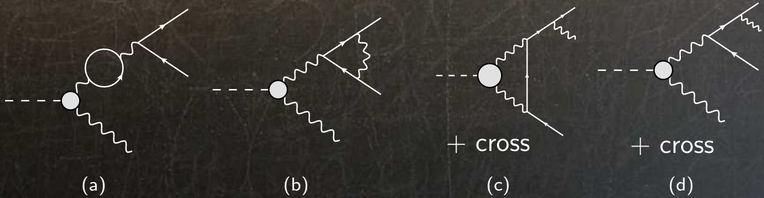
- radiative corrections to the total decay rate of the Dalitz decay
  → first addressed by *Joseph*, NC 16 (1960)

- pioneering study of corrections to the differential decay rate
  → *Lautrup and Smith*, PRD 3 (1971)
  → soft-photon approximation

- extended by *Mikaelian and Smith*, PRD 5 (1972)
  → hard-photon corrections
  → whole range of bremsstrahlung photon energy
  → table of values

Dalitz decay

Radiative corrections



(a) (b) (c) (d)

- new calculations motivated by needs of NA48/NA62 experiments at CERN
  $\rightarrow$ measure the slope a_π of $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,q^2)$
- unlike before no approximation was used
  $\rightarrow$ can be used also for related decays $\eta \rightarrow \ell^+\ell^-\gamma$ etc.
- C++ code returns the correction for any given x and y
  $\rightarrow$ propagated into simulation software of NA62 experiment
- *TH, Kampf and Novotný,* PRD 92 (2015)

NLO correction δ to the LO differential decay width

$$\delta(x,y) = \frac{\mathrm{d}^2\Gamma^{\mathsf{NLO}}}{\mathrm{d}x\mathrm{d}y} \Big/ \frac{\mathrm{d}^2\Gamma^{\mathsf{LO}}}{\mathrm{d}x\mathrm{d}y}$$

$$\delta(x) = \frac{\mathrm{d}\Gamma^{\mathsf{NLO}}}{\mathrm{d}x} \Big/ \frac{\mathrm{d}\Gamma^{\mathsf{LO}}}{\mathrm{d}x}$$

Can be separated into three parts emphasizing its origin

$$\delta = \delta^{\mathsf{virt}} + \delta^{1\gamma\mathsf{IR}} + \delta^{\mathsf{BS}}$$

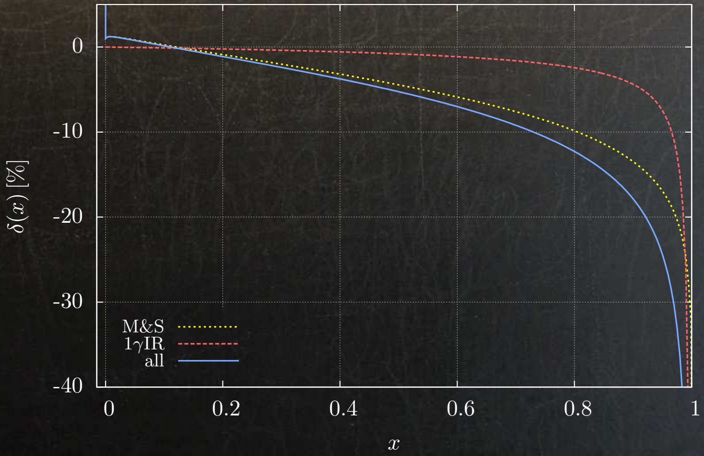
- $\delta^{\mathsf{virt}} \leftrightarrow$ virtual radiative corrections
- $\delta^{1\gamma\mathsf{IR}} \leftrightarrow$ one-photon-irreducible contribution (treated separately from δ^{virt})
- $\delta^{\mathsf{BS}} \leftrightarrow$ bremsstrahlung

Knowledge of $\delta(x,y) \to \delta(x)$

$$\delta(x) = \frac{3}{8\beta(x)} \frac{1}{(1+\frac{\nu^2}{2x})} \int_{-\beta(x)}^{\beta(x)} \delta(x,y) \left[1 + y^2 + \frac{\nu^2}{x}\right] \mathrm{d}y$$

Dalitz decay

Radiative corrections



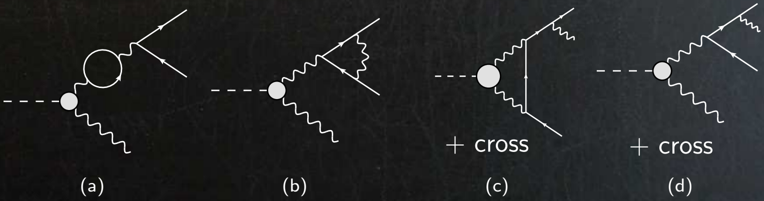
Dalitz decay of $\eta^{(\prime)}$

Radiative corrections

Unlike before no approximation was used
→ results can be used also for related decays $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ etc.

Additional contributions needed

- muon loop contribution to δ^{virt}
- model of $\eta - \eta'$ mixing
  → the model dependency of $\delta^{1\gamma\text{IR}}$ becomes non-negligible
- novel contribution to δ^{BS} due to non-negligible slope $a_{\eta^{(\prime)}}$



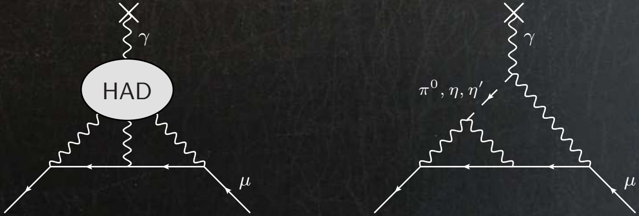
Outlook

Pseudoscalar decays

- $\chi^{(\mathrm{r})}$ universal for $P \to l^+ l^-$ processes up to $\mathcal{O}(m_l^2/\Lambda_{\chi\mathrm{PT}}^2)$

Muon $g-2$: hadronic light-by-light scattering

- pseudoscalar meson exchange contribution requires hadron-physics input



(a) HLbL scattering general contribution

(b) Pseudoscalar meson exchange

Summary

All NLO QED radiative corrections for discussed processes are now available
→ can be taken into account in future experimental analyses

- $\pi^0 \to e^+e^-$
  Vaško and Novotný, JHEP 1110 (2011)
  TH, Kampf and Novotný, EPJC 74 (2014)
- $\pi^0 \to e^+e^-\gamma$
  TH, Kampf and Novotný, PRD 92 (2015)

THS model for $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(p^2,q^2)$

- phenomenologically successful
- satisfies all main theoretical constraints
- *TH and S. Leupold*, EPJC 75 (2015)

Altogether, we get reasonable SM prediction
→ differs from KTeV by 1.8 σ

Goodbye

Thank you for listening!