Challenges of the three-pion system

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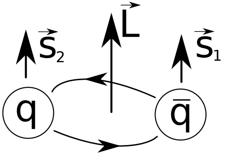
Acknowledgment

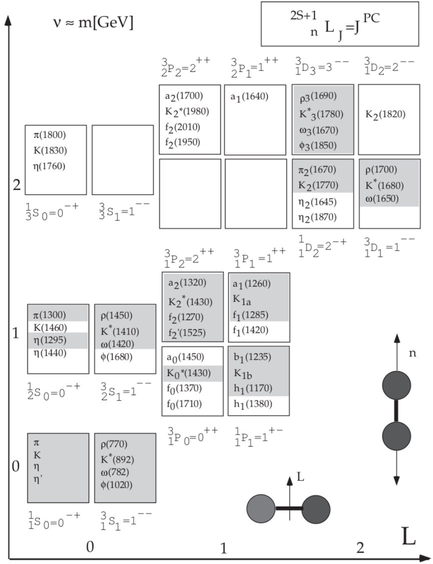
- Bernhard Ketzer, Adam Szczepaniak, Andrey Sarantsev;
- Andrew Jackura, JPAC Collaborators;
- Dima Riabchkov, Boris Grube, Fabian Krinner, Stefan Wallner, COMPASS Collaborators.

QCD and QED

- color-binding,



  - Radial excitation (n),
  - Orbital excitation (L),
- γ-emission tell us about energy levels,
- many strong transitions are possible, i.g. $\to 3\pi$



[Amsler et al., Phys. Rept. 389, 61 (2004)]

Meson spectrum

All mesons

```
{π0, π-, π+, K-, K+, K0, K_L, K_S, K0-bar, η, ρ+(770), ρ0(770), ρ-(770), ω(782), f_0(600), K_0^*0(800), K_0^*+(800), K_0^*-(800), K_0^*0-bar(800), K*-(892), K*+(892), K*0(892), K*0-bar(892),
η^'(958), f_0(980), a_0^-(980), a_0^0(980), a_0^+(980), ϕ(1020), X(1070), X(1110), h_1(1170), b_1^-(1235), b_1^0(1235), b_1^+(1235), a_1^-(1260), a_1^0(1260), a_1^+(1260), K_1^0(1270),
K_1^+(1270), K_1^-(1270), K_1^0-bar(1270), f_2(1270), f_1(1285), η(1295), π-(1300), π0(1300), π+(1300), a_2^-(1320), a_2^0(1320), a_2^+(1320), f_0(1370), π_1^-(1400), π_1^0(1400), π_1^+(1400),
h_1(1380), K_1^0(1400), K_1^+(1400), K_1^-(1400), K_1^0-bar(1400), η(1405), K_0^*0(1430), K_0^*+(1430), K*0(1410), K*+(1410), K_0^*-(1430), K_0^*0-bar(1430), K*-(1410), K*0-bar(1410), X(1420),
ω(1420), K_2^*+(1430), K_2^*-(1430), f_1(1420), K_2^*0(1430), K_2^*0-bar(1430), f_2(1430), ρ-(1450), ρ0(1450), ρ+(1450), K0(1460), K+(1460), K-(1460), K0-bar(1460), a_0^-(1450), a_0^0(1450),
a_0^+(1450), η(1475), f_0(1600), f_0(1500), f_1(1510), f_2^'(1525), f_2(1565), K_2^0(1580), K_2^+(1580), K_2^-(1580), K_2^0-bar(1580), h_1(1595), X(1600), η_2(1645), K0(1630), K+(1630),
K-(1630), K0-bar(1630), f_2(1640), a_1^-(1640), a_1^0(1640), a_1^+(1640), K_1^0(1650), K_1^+(1650), K_1^-(1650), K_1^0-bar(1650), X(1650), π_1^-(1600), π_1^0(1600), π_1^+(1600), ω_3(1670),
ω(1650), π_2^-(1670), π_2^0(1670), π_2^+(1670), ϕ(1680), ρ_3^-(1690), ρ_3^0(1690), ρ_3^+(1690), K*0(1680), K*+(1680), K*-(1680), K*0-bar(1680), f_0(1710), ρ-(1700), ρ0(1700), ρ+(1700),
a_2^+(1700), a_2^-(1700), a_2^0(1700), X(1750), η(1760), X(1775), K_2^0(1770), K_2^+(1770), K_2^-(1770), K_2^0-bar(1770), K_3^*0(1780), K_3^*+(1780), K_3^*-(1780), K_3^*0-bar(1780), π-(1800),
π0(1800), π+(1800), f_2(1810), K_2^0(1820), K_2^+(1820), K_2^-(1820), K_2^0-bar(1820), K0(1830), K+(1830), K-(1830), K0-bar(1830), X(1835), η_2(1870), ϕ_3(1850), X(1855), ρ-(1900), ρ0(1900),
ρ+(1900), D0, D0-bar, D-, D+, a_3(1875), X(1870), π_2(1880), f_2(1910), a_1(1930), X(1935), ρ_2(1940), f_2(1950), K_0^*0(1950), K_0^*+(1950), ω_3(1945), K_0^*-(1950), K_0^*0-bar(1950),
b_1(1960), h_1(1965), ω(1960), ρ(1965), D_s, D_s-bar, X(1970), f_1(1970), K_2^*0(1980), K_2^*+(1980), X(1975), K_2^*-(1980), K_2^*0-bar(1980), ω_2(1975), ρ_3^-(1990), ρ_3^0(1990),
ρ_3^+(1990), f_0(2020), X0(2000), ρ(2000), a_4^-(2040), a_4^0(2040), a_4^+(2040), f_2(2000), a_2(1990), π_2(2005), D*(2007), D*-bar(2007), D*-bar(2010), D*(2010), η(2010), f_2(2010),
π_1(2015), X(2020), b_3(2025), f_4(2050), h_3(2025), a_0(2020), η_2(2030), K_4^*0(2045), K_4^*+(2045), K_4^*-(2045), K_4^*0-bar(2045), f_3(2050), a_2(2080), f_0(2060), a_3(2070), π(2070),
X(2075), X0(2080), π_2^-(2100), π_2^0(2100), π_2^+(2100), a_1(2095), η(2100), X(2100), f_0(2100), X(2110), D_s^*, D_s^*-bar, f_2(2140), ρ-(2150), ρ0(2150), ρ+(2150), ω(2145), X(2150),
f_2(2150), a_2(2175), f_0(2200), η(2190), ω_2(2195), ω(2205), X0(2210), η(2225), h_1(2215), f_J(2220), b_1(2240), π_2(2245), ρ_2(2240), ρ_4(2240), K_2^0(2250), K_2^+(2250), K_2^-(2250),
K_2^0-bar(2250), η_2(2250), ω_3(2255), ω_4(2250), π_4(2250), ρ_3^-(2250), ρ_3^0(2250), ρ_3^+(2250), ρ(2265), X(2260), a_2(2270), a_1(2270), h_3(2275), ω_3(2285), ρ(2280), η(2280), X(2290),
f_2(2300), a_4(2280), ρ_3(2300), f_3(2300), a_3(2310), f_1(2310), D_s0^*(2317), D_s0^*-bar(2317), K_3^0(2320), K_3^+(2320), K_3^-(2320), K_3^0-bar(2320), η_4(2320), ω(2330), ρ_5^-(2350),
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D_0^*+(2400), D_0^*-(2400), D_1^0(2420), D_1^0-bar(2420), D_1^+(2420), D_1^-(2420), D_1(2430), D_1-bar(2430), X(2440), a_6^-(2450), a_6^0(2450), a_6^+(2450), D_s1(2460), D_s1-bar(2460),
D_2^*+(2460), D_2^*-(2460), f_6(2510), D_2^*0(2460), D_2^*0-bar(2460), K_4^0(2500), K_4^+(2500), K_4^-(2500), K_4^0-bar(2500), D_s1(2536), D_s1-bar(2536), D_s2(2573), D_s2-bar(2573), X(2632),
D*(2640), D*-bar(2640), X(2680), X(2710), X(2750), η_c(1S), K(3100), J/ψ(1S), f_6(3100), X0(3250), χ_c0(1P), χ_c1(1P), h_c(1P), χ_c2(1P), η_c(2S), ψ(2S), ψ(3770), X(3872), χ_c2(2P),
Y(3940), ψ(4040), ψ(4160), X(4260), ψ(4415), B-, B+, B0, B0-bar, B*-, B*+, B*0, B*0-bar, B_s, B_s-bar, B_s^*, B_s^*-bar, B_J^*0(5732), B_J^*+(5732), B_J^*-(5732), B_J^*0-bar(5732),
B_sJ^*(5850), B_sJ^*-bar(5850), B_c, B_c-bar, η_b(1S), Y(1S), χ_b0(1P), χ_b1(1P), χ_b2(1P), Y(2S), Y(1D), χ_b0(2P), χ_b1(2P), χ_b2(2P), Y(3S), Y(4S), Y(10860), Y(11020)}
```

Radial, **orbital** excitations + non-$q\bar{q}$-states ..

Meson spectrum

All light mesons accessible by 3π system

```
{π0, π-, π+, ρ+(770), ρ0(770), ρ-(770), f_0(600),
f_0(980), a_0^-(980), a_0^0(980), a_0^+(980), a_1^-(1260), a_1^0(1260), a_1^+(1260),
π-(1300), π0(1300), π+(1300), a_2^-(1320), a_2^0(1320), a_2^+(1320), f_0(1370), π_1^-(1400), π_1^0(1400), π_1^+(1400),

ρ-(1450), ρ0(1450), ρ+(1450), a_0^0(1450),
a_0^+(1450), f_0(1600), f_0(1500),
a_1^-(1640), a_1^0(1640), a_1^+(1640),
π_2^-(1670), π_2^0(1670), π_2^+(1670), ρ_3^-(1690), ρ_3^0(1690), ρ_3^+(1690), f_0(1710), ρ-(1700), ρ0(1700), ρ+(1700),
a_2^+(1700), a_2^-(1700), a_2^0(1700), π-(1800),
π0(1800), π+(1800), ρ-(1900), ρ0(1900),
ρ+(1900), a_3(1875), π_2(1880), a_1(1930), ρ_2(1940),
ρ(1965), ρ_3^-(1990), ρ_3^0(1990),
ρ_3^+(1990), f_0(2020), ρ(2000), a_4^-(2040), a_4^0(2040), a_4^+(2040), f_2(2000), a_2(1990), π_2(2005), η(2010),
π_1(2015), a_0(2020), a_2(2080), f_0(2060), a_3(2070), π(2070),
π_2^-(2100), π_2^0(2100), π_2^+(2100), a_1(2095), f_0(2100), f_2(2140), ρ-(2150), ρ0(2150), ρ+(2150),
a_2(2175), f_0(2200), π_2(2245), ρ_2(2240), ρ_4(2240),
π_4(2250), ρ_3^-(2250), ρ_3^0(2250), ρ_3^+(2250), ρ(2265), a_2(2270), a_1(2270), ρ(2280),
a_4(2280), ρ_3(2300), a_3(2310), ρ_5^-(2350),
ρ_5^0(2350), ρ_5^+(2350), f_0(2330), a_1(2340), π(2360),
a_6^-(2450), a_6^0(2450), a_6^+(2450),

f_6(3100),

}
```

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- Scattering phases $3\pi \to 3\pi$, resonant poles positions,

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   decay physics `BES, LHCb, ...`

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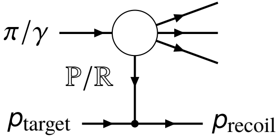
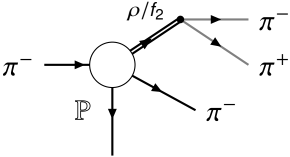
Data from scattering experiments

Study meson spectrum through peripheral resonance production

- High-energy beam,
- Pomeron/Reggeon t-channel exchange dominates,
- Recoil particle is kinematically decoupled
- Analysis at COMPASS
    - Large data sample with high purity
    - JPAC&COMPASS collaboration to perform theoretically advanced analysis on the complete data set
- Opportunities at GlueX

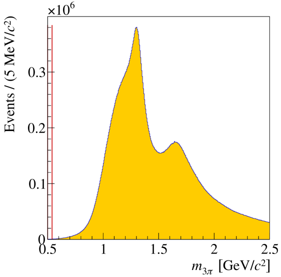


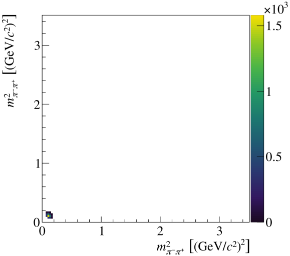


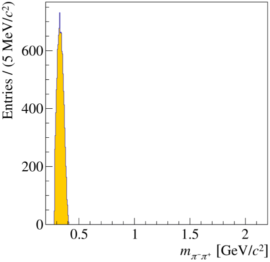
3π at COMPASS

Kinematical distributions

- The largest data set (50×10^6 events) on diffractively produced 3π systems.
- High-energy beam guaranties peripheral reaction $\sqrt{s} \approx 19\,$GeV.
- Many resonances are seen in the raw spectrum.





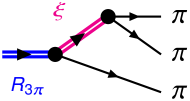


[Animation credit Boris Grube]

Isobar model

Isobar is just intermediate resonance

- Sequential decay $R_{3\pi} \to \xi\,\pi \to 3\pi$, e.g. $a_2^- \to \rho\pi \to 3\pi$.
- "Shape of isobar" does not depend on invariant mass of the system



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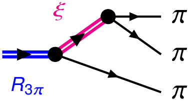

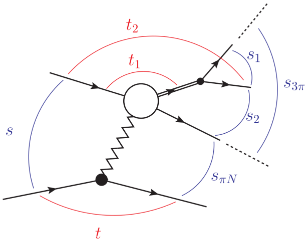

Partial-wave decomposition

- $J^{PC}M^{\epsilon}$ quantum numbers of system
- in case of three-body final state ξ is isobar state with spin S

$$A = \langle \text{final}|\,\hat{T}\,|\text{initial}\rangle = \sum_{JMLS\epsilon} F_{LS}^{JM\epsilon}\,\mathrm{PW}_{LS}^{JM\epsilon}(\tau)$$

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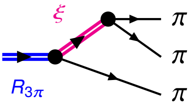

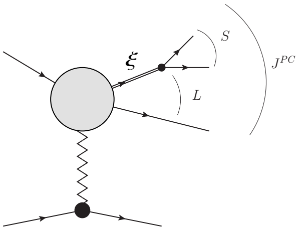

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$$PW_{LS}^{JM\epsilon}(\Omega,\Omega') = \left(\frac{2L+1}{2J+1}\right)^{1/2}\sum_{\lambda}\langle L0S\lambda|J\lambda\rangle\left(\frac{2J+1}{4\pi}\right)^{1/2} D_{M\lambda}^{J*}(\Omega)\left(\frac{2J+1}{4\pi}\right)^{1/2} D_{\lambda 0}^{S*}(\Omega')$$

Unitarity condition

$$\hat{S} = \hat{\mathbb{I}} + i\hat{T}, \quad \hat{S}\hat{S}^{\dagger} = \hat{\mathbb{I}} \quad \Rightarrow \quad \hat{T} - \hat{T}^{\dagger} = i\hat{T}\hat{T}^{\dagger},$$

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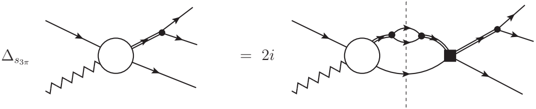
Constraints on the full amplitude: $\Delta A = i \int T^\dagger \, \mathrm{d}\Phi \, A$

$$A = \langle 3\pi | \, \hat{T} \, | \pi \mathbb{P} \rangle = \sum_{JMLS\epsilon} F_{LS}^{JM\epsilon} \, \mathrm{PW}_{LS}^{JM\epsilon}(\tau)$$



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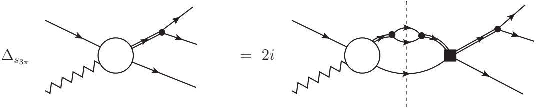
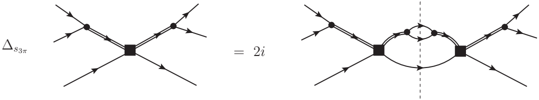


Elastic 3π unitarity: $\Delta T = i \int T^\dagger \, \mathrm{d}\Phi \, T$

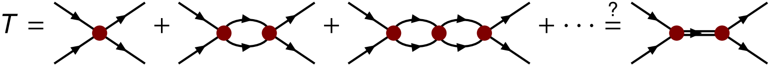
$$T = \langle 3\pi| \, \hat{T} \, |3\pi\rangle = \sum_{JMLSL'S'} T^{J\epsilon}_{LSL'S'} \, \mathrm{PW}^{JM\epsilon}_{LS}(\tau) \, \mathrm{PW}^{JM\epsilon}_{L'S'}(\tau')$$



Parametrization of the scattering matrix

Find parametrization of T which satisfies unitarity by construction.

$$T = \frac{K}{1 - i\tilde{\rho}K} = K + K\,[i\tilde{\rho}]\,K + K\,[i\tilde{\rho}]\,K\,[i\tilde{\rho}]\,K\,[i\tilde{\rho}]\,K + \dots$$



Fit T-parameters to data and extract resonance information

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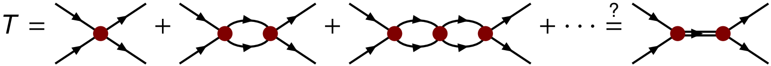
Find parametrization of T which satisfies unitarity by construction.

$$T = \frac{K}{1 - i\tilde{\rho}K} = K + K\,[i\tilde{\rho}]\,K + K\,[i\tilde{\rho}]\,K\,[i\tilde{\rho}]\,K\,[i\tilde{\rho}]\,K + \dots$$



Fit T-parameters to data and extract resonance information

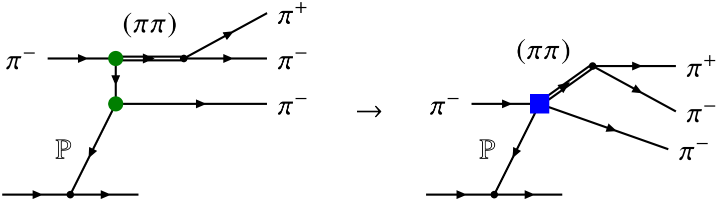
K-matrix approach

$$K_{ij}(s) = \sum_r \frac{g_i^r g_j^r}{m_r^2 - s} + \sum_n \gamma_{ij}^n s^n$$

CDD-poles approach

$$K_{ij}^{-1}(s) = M_{ij}(s) = c_0 + c_1 s + \sum_r \frac{g_i^r g_j^r}{c_r - s}$$

Production process

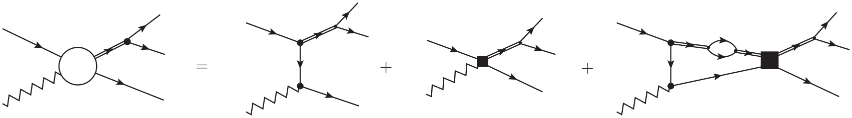


Long-range(only LHC) and Short-range production amplitudes.

- Consider $\pi + \mathbb{P} \rightarrow (\pi\pi)\,\pi$ scattering via t-exchanges.
- Interaction range is determined by the mass of the exchange particle
- Pion is lightest exchange particle with range ~ 1 fm.

Unitarized model [Basdevant, Berger, 1967]

Everything which is produced is supposed to scatter



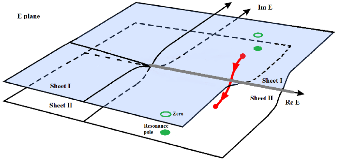
- Production process via an exchange alone does not satisfy probability conservation.
- Rescattering (unitarisation) term has to be added.
- In the limit of short range the production amplitude is approximated by a constant c_{LS}.
- Amplitude has correct threshold behavior

$$F_{LS}(s) = b_{LS}(s) + T_{LSL'S'}(s)c_{L'S'} + \frac{T_{LSL'S'}(s)}{\pi}\int \frac{\rho_Q(s')b_{L'S'}(s')}{s'-s}\mathrm{d}s'$$

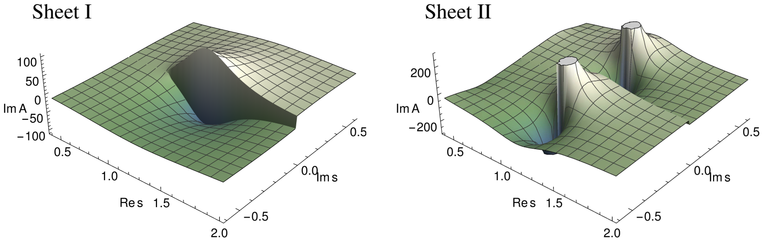
Analytic structure

General note

- We consider the amplitude as complex function of invariant mass squared w^2 and explore the structure.
- The physical region is $A(s + i\epsilon)$



Imaginary part of Breit-Wigner amplitude on the complex plane

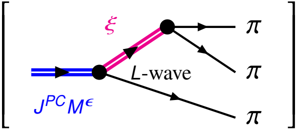

Sheet I

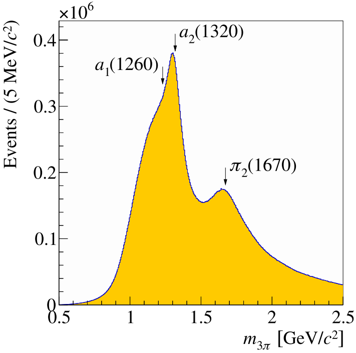
Sheet II

3π at COMPASS

Step 1: mass-independent analysis

$$A(m_{3\pi}, m_{2\pi}, \Omega, \Omega') = \sum_{J^{PC}M^{\epsilon}\ \xi\ L\text{−wave}}^{88} c_{LS}^{JM}(m_{3\pi})$$





[C. Adolph et al. [COMPASS Collaboration], arXiv:1509.00992]

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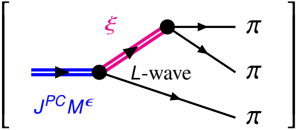

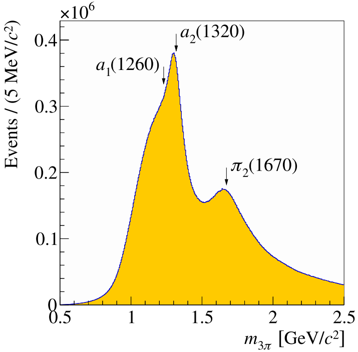

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COMPASS 3π PWA:

- $\pi^-\pi^+\pi^-$ final state,
  $m_{3\pi} < 2.5\,\text{GeV}$, $0.1 < t' < 1\,\text{GeV}^2$,

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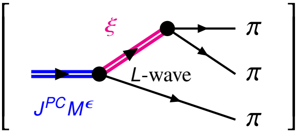

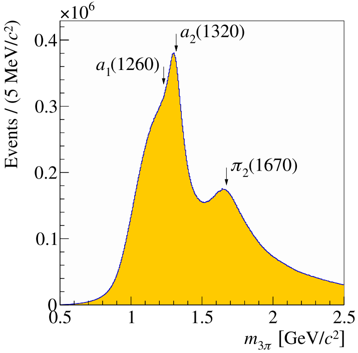

[C. Adolph et al. [COMPASS Collaboration], arXiv:1509.00992]

COMPASS 3π PWA:

- $\pi^-\pi^+\pi^-$ final state, $m_{3\pi} < 2.5\,\text{GeV}, 0.1 < t' < 1\,\text{GeV}^2$,
- Independent PWA in $m_{3\pi} \times t'$ bins $(100 \times 11\text{ bins})$,
- $\pi^+\pi^-$-resonances: $f_0(500), \rho, f_0(980), f_2, \rho_3(1670)$.
- PWA model consists of 88 waves $J^{PC} = 0^{-+}, 1^{++}, 1^{-+}, 2^{++}, 2^{-+}, \dots$

Partial waves in the 2^{-+} sector

Partial wave
$2^{-+}0^+$
$f_2\pi$ S
$f_2\pi$ D
$\rho\pi$ P
$\rho\pi$ F
$(\pi\pi)_S$ D
$f_0\pi$ D
$\rho_3\pi$ P
$f_2\pi$ G
$2^{-+}1^+$
$\rho\pi$ P
$f_2\pi$ S
$\rho\pi$ F
$(\pi\pi)_S$ D
$\rho_3\pi$ P
$f_2\pi$ D
$2^{-+}2^+$
$\rho\pi$ P
$f_2\pi$ S
$f_2\pi$ D

Partial waves in the 2^{-+} sector

Partial wave
$2^{-+}0^{+}$
$f_2\pi$ S
$f_2\pi$ D
$\rho\pi$ P
$\rho\pi$ F
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$f_0\pi$ D
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$f_2\pi$ S
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$\rho_3\pi$ P
$f_2\pi$ D
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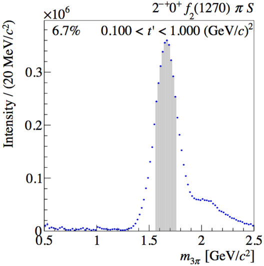
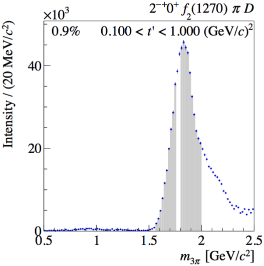
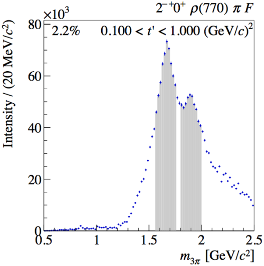
$\rho\pi$ P
$f_2\pi$ S
$f_2\pi$ D





- intensity peak for $f_2\pi$ S- and $f_2\pi$ D-waves appear at different places
- $\rho\pi$ F-wave shows two separated peaks

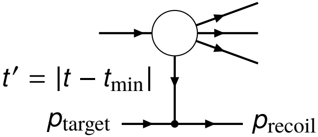


[C. Adolph et al. [COMPASS Collaboration], arXiv:1509.00992]

Fit of all t' slices

Simultaneous fit of

- 5 intensities & 4 phases in 11 t'-bins

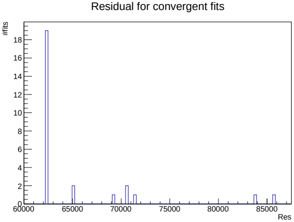


Model

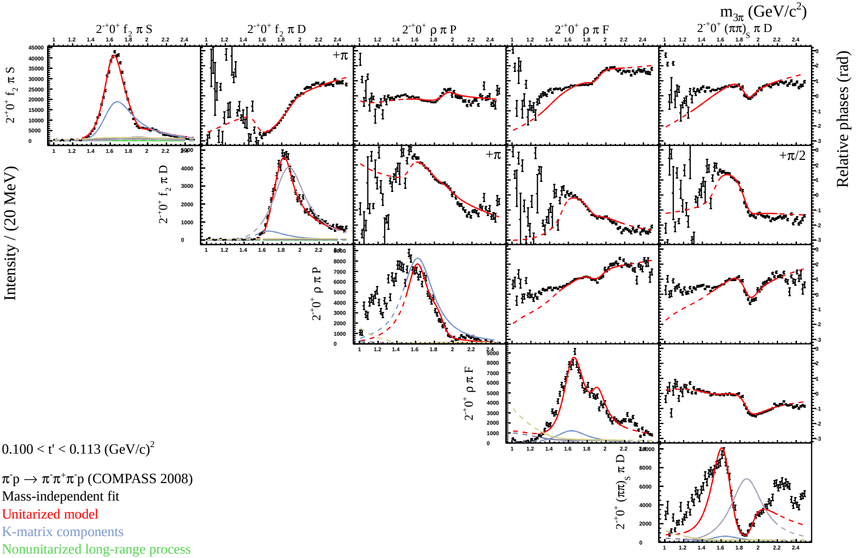
1. Scattering matrix has 5 channels. It does not depend on t'.
2. K-matrix with 4 poles is used for the parametrization.
3. Production includes short- and long-range processes.
   A new set of the coupling parameters is used for every t'-bin.

Fit

1. 145 independent parameters.
2. 12 steps fit. MC sampling of starting values.



Fit over all t' slices



0.100 < t' < 0.113 $(\mathrm{GeV}/c)^2$

$\pi^- p \to \pi^-\pi^+\pi^- p$ (COMPASS 2008)

Mass-independent fit

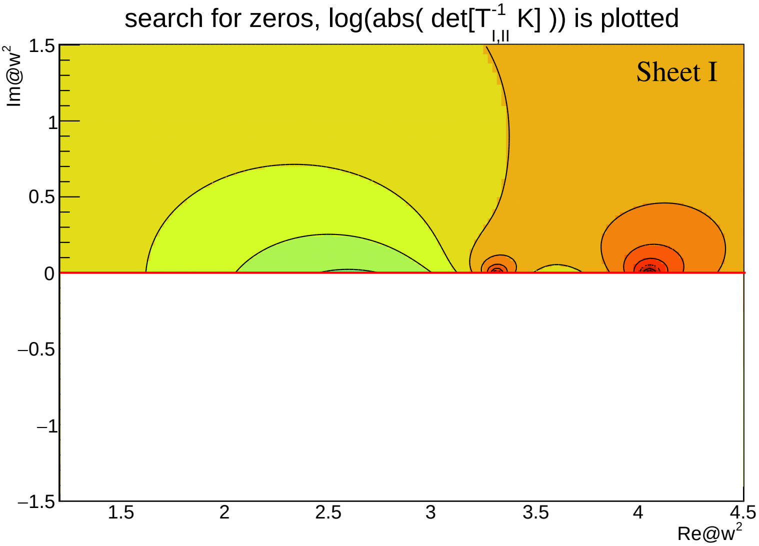
Unitarized model

K-matrix components

Nonunitarized long-range process

2^{-+} resonances

poles on the second sheet



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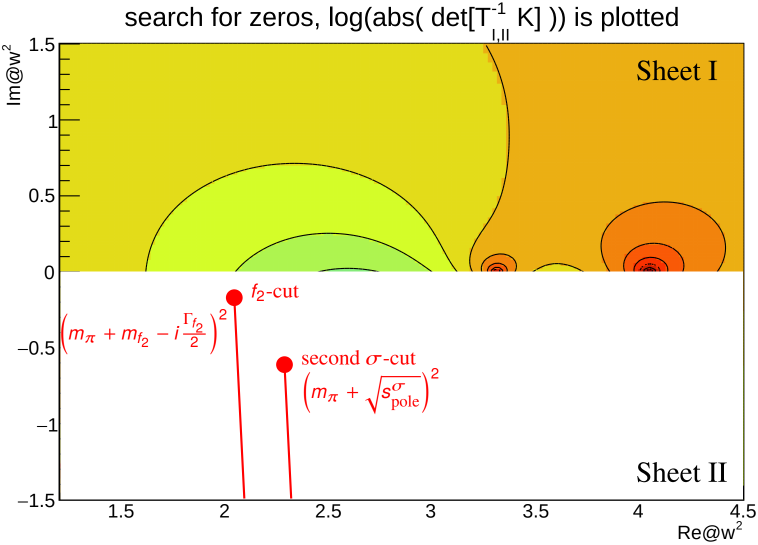

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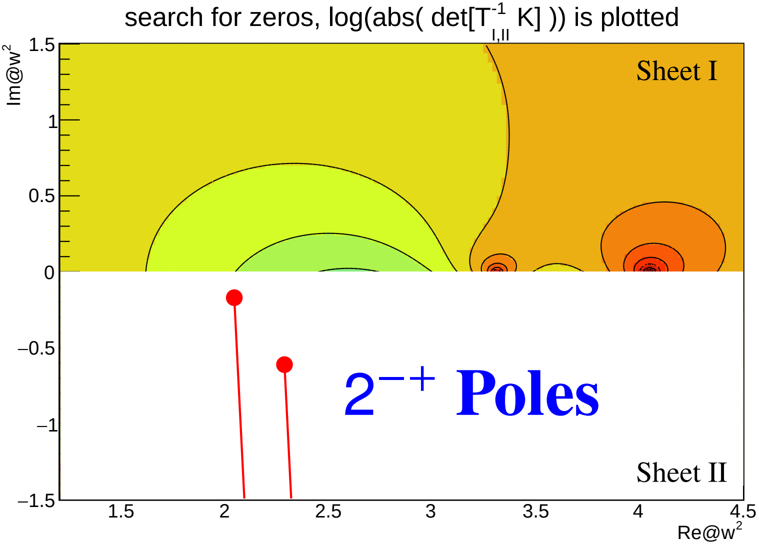

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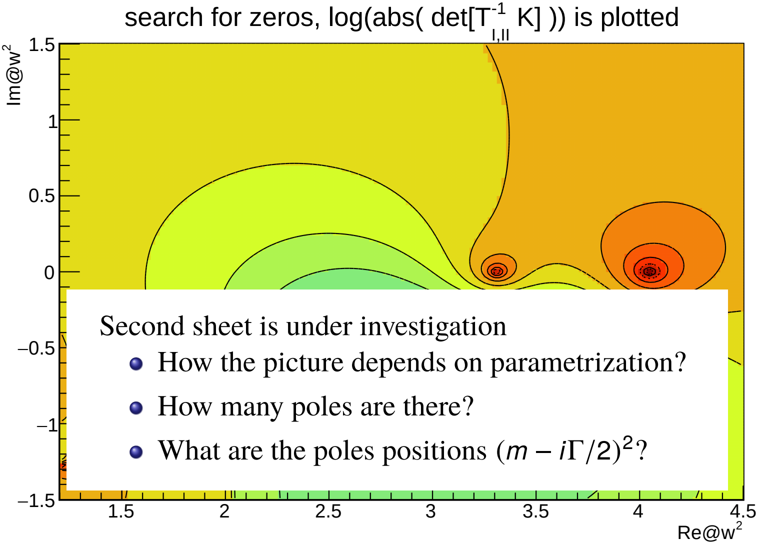


Second sheet is under investigation

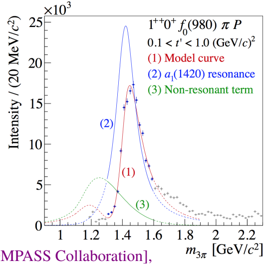
- How the picture depends on parametrization?
- How many poles are there?
- What are the poles positions $(m - i\Gamma/2)^2$?

Future developments for COMPASS Analysis

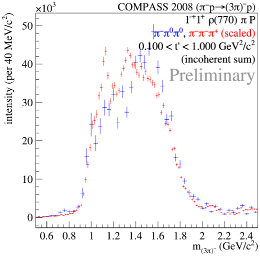
Many ideas to continue:

- Extend 5-waves-fit to available data for 2^{-+} sector, extract pole positions
- Apply the formalism to other $J^{PC}M^{\epsilon}$ sectors of 3π data. Several interesting cases along the way:
    - 2^{++} sector: a_2 resonances
    - 0^{-+} sector: π resonances
    - 1^{-+} sector: exotics.
- make 3π scattering amplitudes available for use in other experiments, MC generators
- 3π scattering matrices to be compared to lattice calculation

[COMPASS Collaboration], Phys. Rev. Lett. 115, 082001 (2015)



[COMPASS Collaboration], AIP Conf. Proc. **1735**, 020007 (2016)



Beyond the isobar model

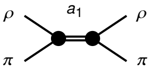
Cross-channel rescattering effects

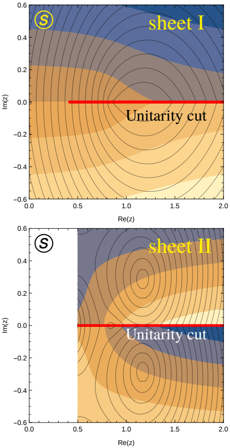
Complex structure

- Amplitude $t(s)$ of scattering procces is complex function of s.
- Every resonance is a pole in the amplitude
- Every one channel for decay produces cut along the real axis

$t^{-1}(s)$ is shown in s-plane, color code is $\textsc{Im}[t^{-1}(s)]$, equipotential lines are $\textsc{Abs}[t^{-1}(s)]$.

Two-body resonance





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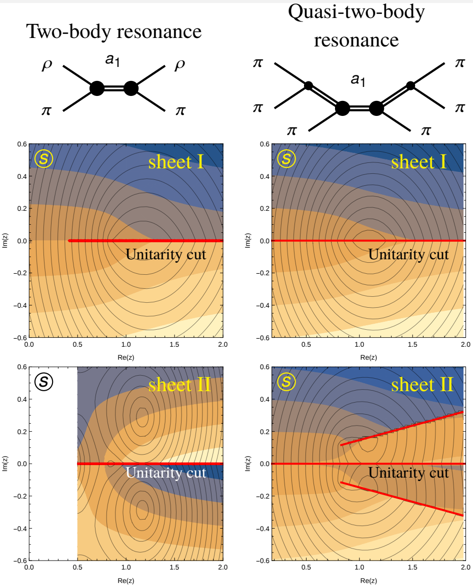

Two-body resonance

Quasi-two-body resonance

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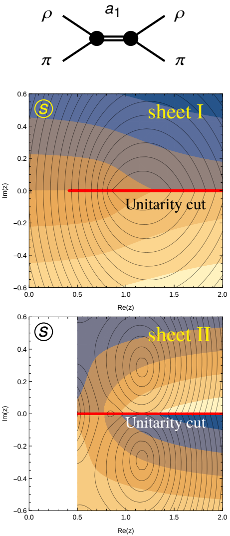
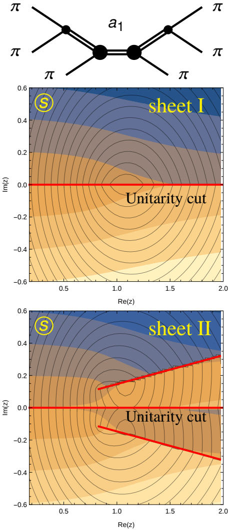
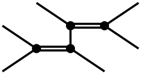
Two-body resonance



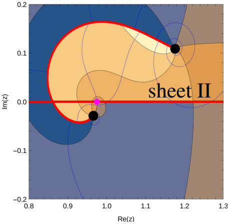
Quasi-two-body resonance



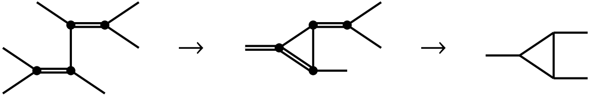
Cross-channel exchange



- Additional structure appears on the second sheet.
- log branching point can be close to the physical region.



Triangle diagram



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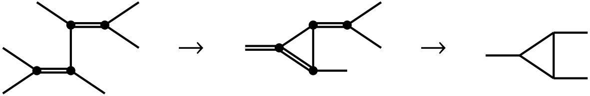
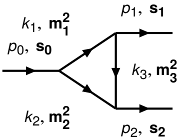
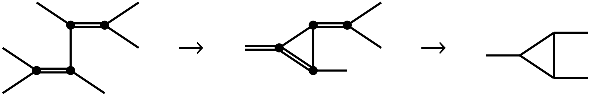


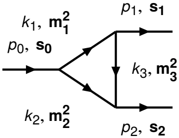


$$= g^3 \int \frac{d^4 k_1}{(2\pi)^4 i} \frac{1}{\Delta_1 \Delta_2 \Delta_3} = \frac{g^3}{16\pi^2} \int_0^1 \frac{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z}{D} \delta(1-x-y-z),$$

$$D = x\, m_1^2 + y\, m_2^2 + z\, m_3^2 - x\, y\, s_0 - z\, x\, s_1 - y\, z\, s_2.$$

Triangle diagram





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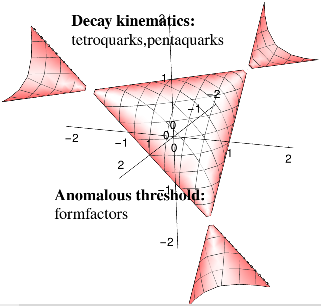
$$D = x\, m_1^2 + y\, m_2^2 + z\, m_3^2 - x\, y\, s_0 - z\, x\, s_1 - y\, z\, s_2 .$$

- $\Delta_i = m_i^2 - k_i^2$ is propagators of the particles in the loop,
- Positions of singularities are given by Landau equations. [Landau, Nucl. Phys. **13**, 181 (1959)]
- Landau surface is represented in normalized invariants (y_0, y_1, y_2), $y_i = \frac{s_i - m_{i1}^2 - m_{i2}^2}{2 m_{i1} m_{i2}}$



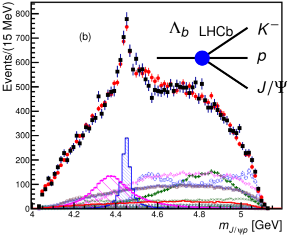
Decay kinematics:
tetroquarks,pentaquarks

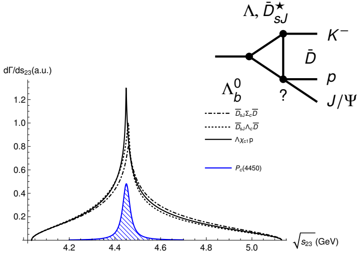
Anomalous threshold:
formfactors

Examples

hypotheses for an explanation of exotics

LHCb: pentaquark $P_c(4450)$





[MM, arXiv: 1507.06552]

Examples

hypotheses for an explanation of exotics

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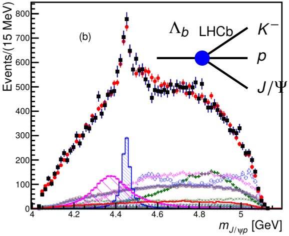
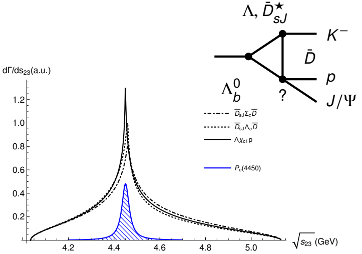
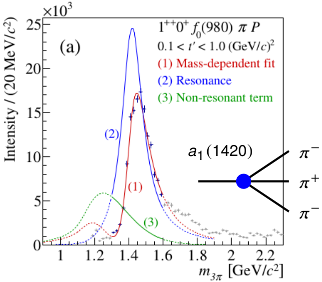
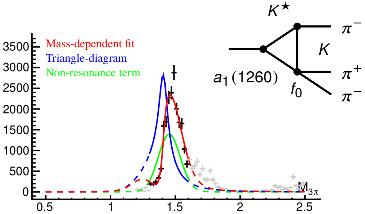




[MM, arXiv: 1507.06552]

COMPASS: exotic $a_1(1420)$



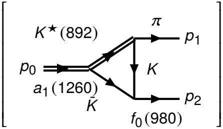


[MM *et al.*, Phys. Rev. D91, 094015 (2015)]

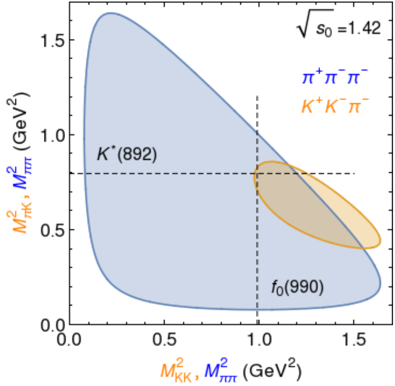
Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0\pi$

For the realistic decay, the amplitude is similar to the scalar case.

$\mathbb{M}^{(\mathrm{vpp})}_{a_1\to\pi f_0} =$

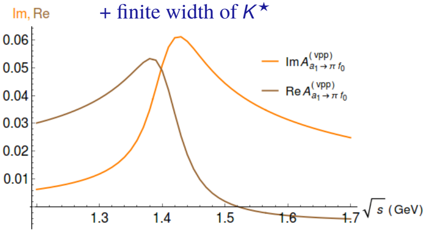


- Spin-Parity of particles.
- Width of $K^\star$



If one fixes mass of f_0, i.e. $p^2_{f_0} = m^2_{f_0}$, then only $p^2_0 = s$ is variable.

+ finite width of $K^\star$



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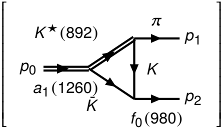

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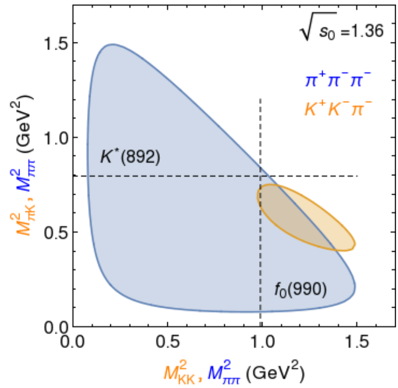

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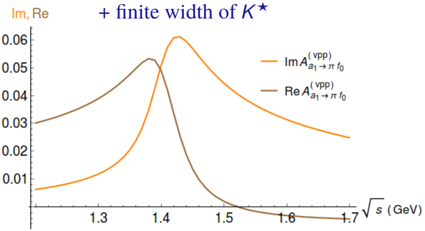
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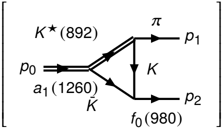

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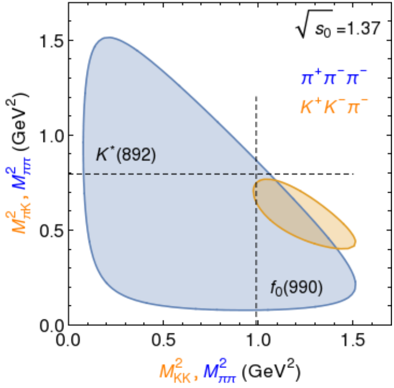

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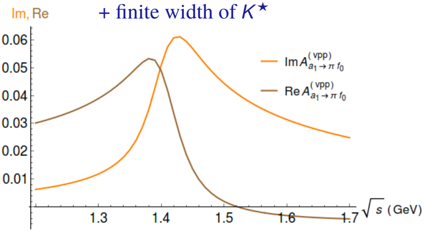
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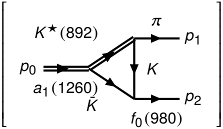

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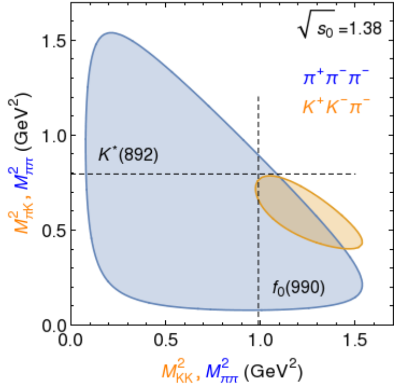

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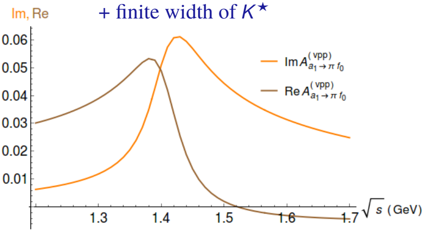
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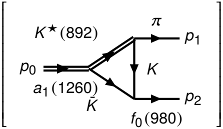

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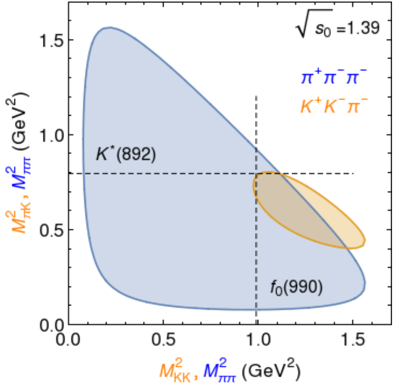

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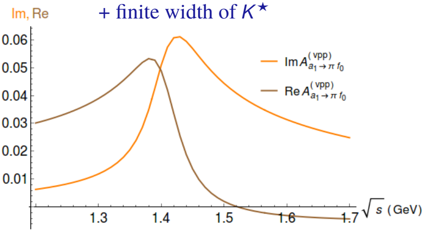
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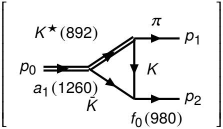

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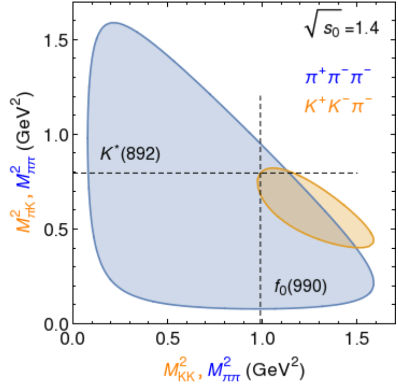

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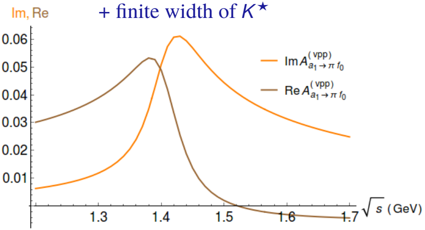

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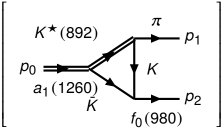

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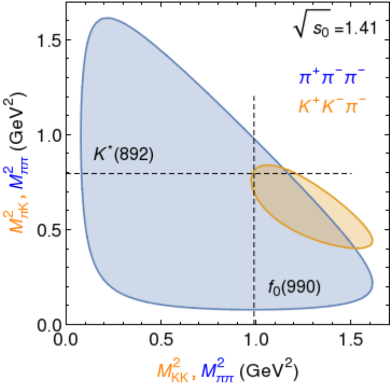

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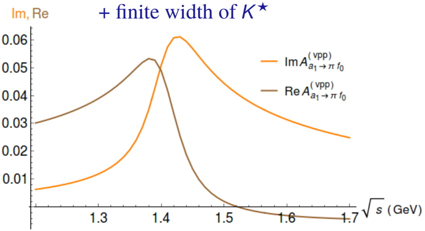
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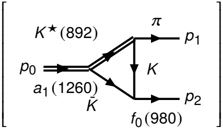

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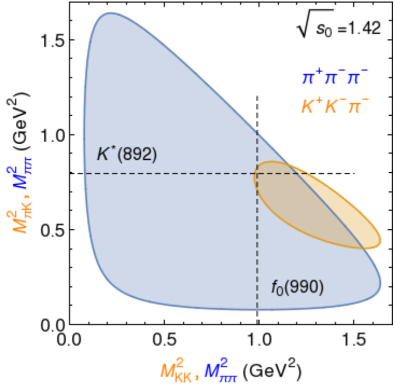

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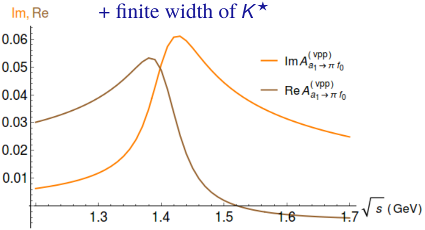

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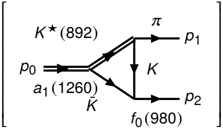

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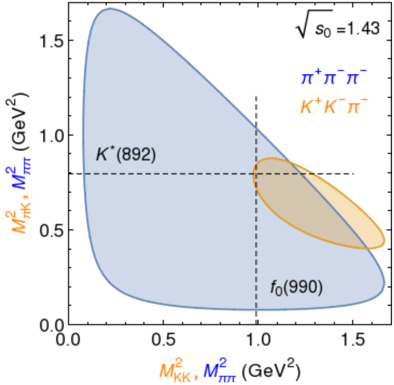

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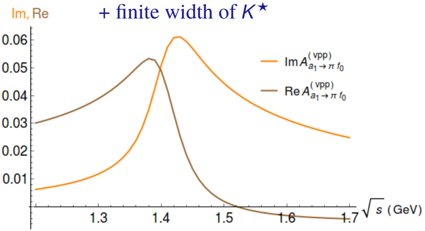

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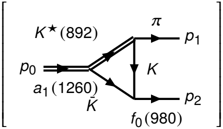

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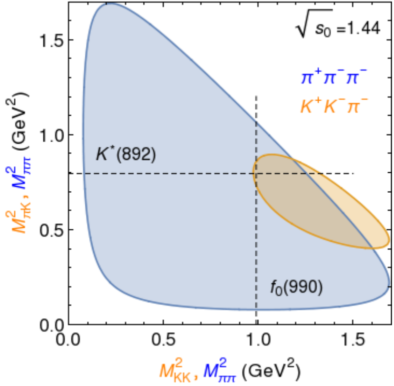

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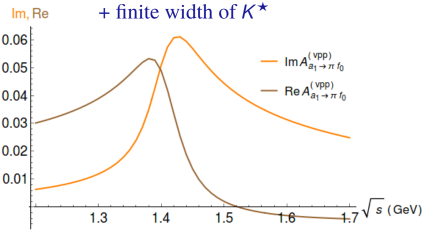
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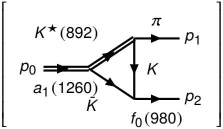

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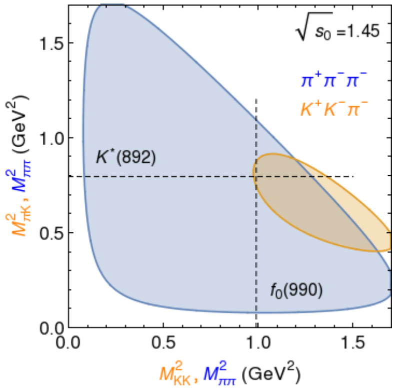

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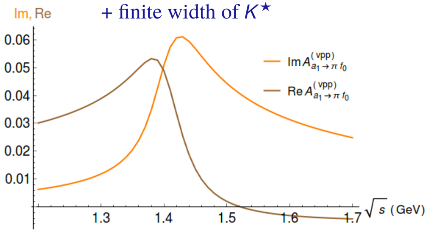
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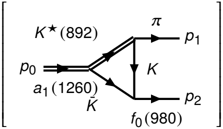

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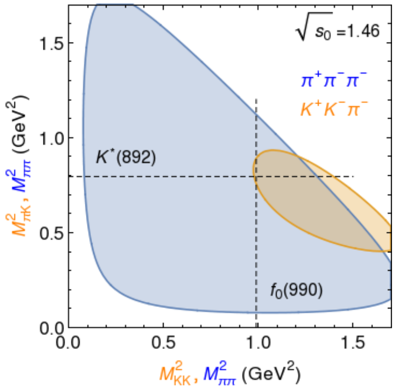

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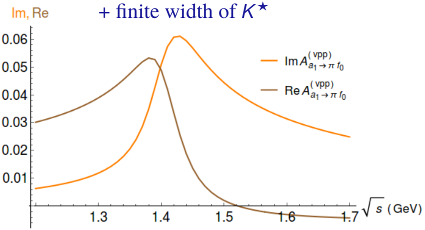

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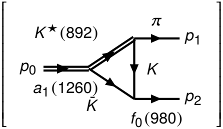

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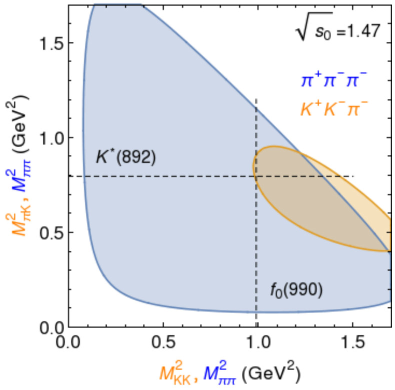

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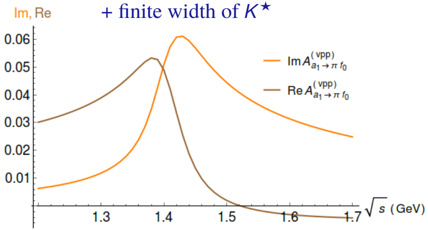

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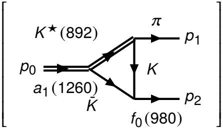

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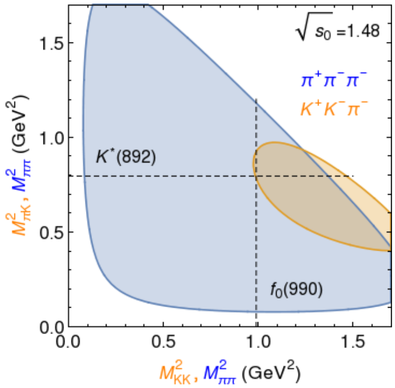

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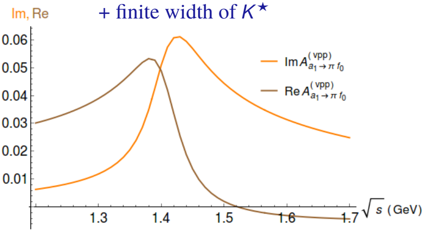
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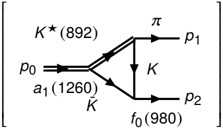

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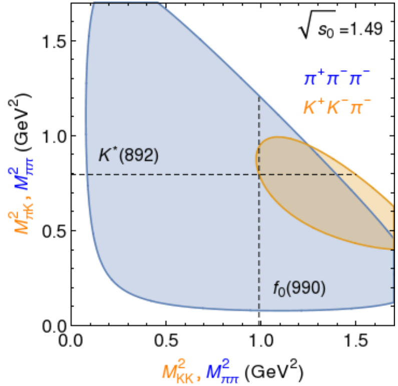

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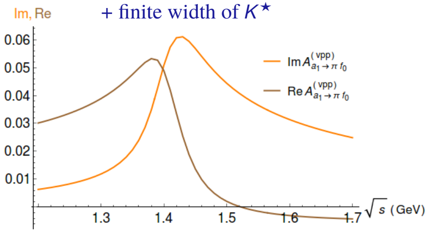
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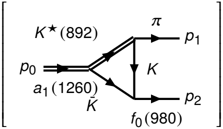

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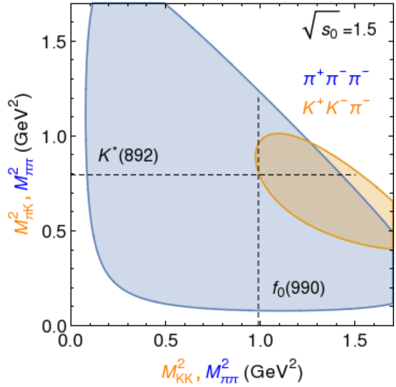

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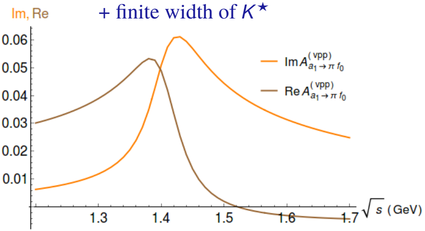
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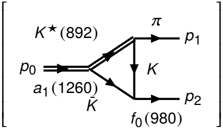

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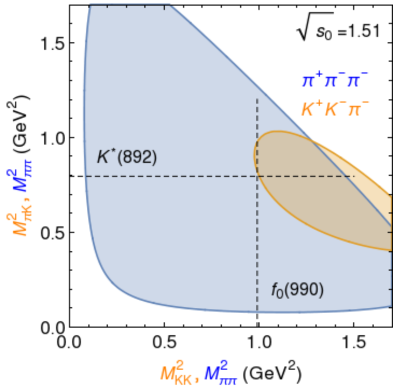

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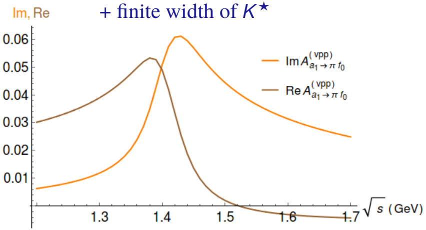

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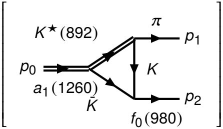

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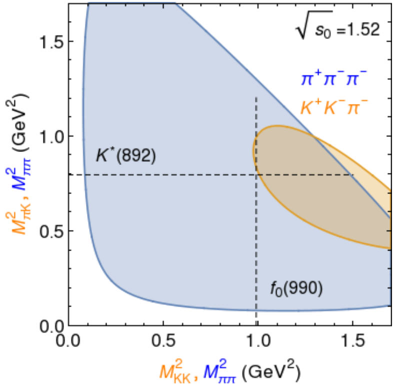

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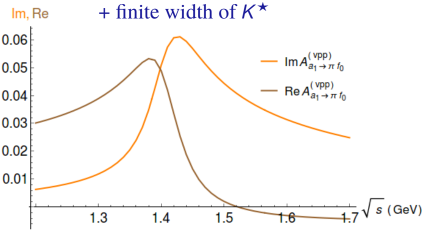
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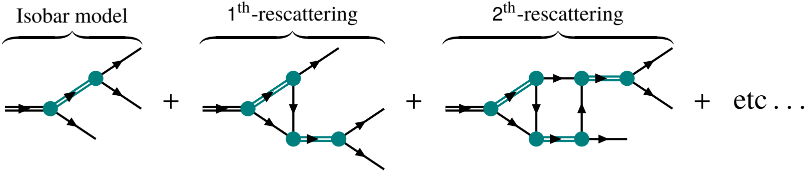
Rescattering series

How a further rescattering changes an amplitude?

- Isobar model is common tool of the data analysis (PWA at VES,COMPASS,BES,CLEO,LHCb,. . .).
- 1^{th} order rescattering is triangle diagram. One loop, four integrals can be reduced to one.
- 2^{th} is two-loop diagram. Very complecated to evaluate!



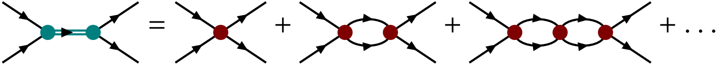
There is alternative method.

Two body unitarity

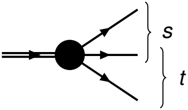
Unitarity = propability conservation. $\hat{S} \cdot \hat{S}^{\dagger} = \mathbb{I}$.

$$\hat{S} = \hat{\mathbb{I}} + i\hat{T} \quad \Rightarrow \quad \hat{T} - \hat{T}^{\dagger} = i\hat{T}\hat{T}^{\dagger}, \ \Rightarrow \ \Delta t = it\,\rho\,t^{\star}.$$

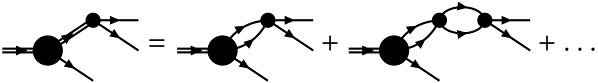
Two-body unitarity and resonance



$$t = \frac{g^2}{m^2 - s - im\Gamma} = \frac{1}{K^{-1} - i\rho} = K + K\,i\rho\,K + K\,i\rho\,K\,i\rho\,K + \ldots, \qquad K = \frac{g^2}{m^2 - s}.$$



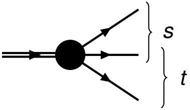
Subchannel resonances



Complete only if interaction with particle 3 is negligible

Formalism

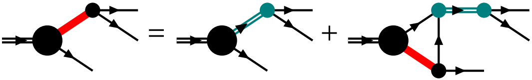
Khuri-Treiman equation



$$= A^{(s)} + A^{(t)} + A^{(u)}, \quad A^{(j)} = \sum_l (2l+1) a_l{}^{(j)}(s_j) P_l(\cos\theta^{(j)})$$

Due to unitarity: $$a_l{}^{(s)} = \underbrace{t_l{}^{(s)} c_l^{(s)}}_{\text{Isobar model}} + \underbrace{\frac{t_l{}^{(s)}}{2\pi} \int \frac{b_l^{(s)} \rho}{s' - s} \mathrm{d}s'}_{\text{rescattering corrections}} .$$

- $a_l^{(s)}$ is a corrected two-body amplitude,
- $b_l^{(s)}$ is a projetion of cross channel waves.
- We get a system of integral equations.



Model ingredients and result

Model is given by

- Set of waves up to L_j for every channel s, t, u.

  Example $\pi^-\pi^+\pi^-$: s,t-channel isobars are σ, $f_0 (I=0)$, $\rho (I=1)$

- Parameterizations of elastic amplitudes $t_l^{(j)}$

  Example $\pi^-\pi^+\pi^-$: $(\pi\pi)_S$-wave, $(\pi\pi)_P$-wave from K-function/phase shift

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- The solution is linear on production constants, thus we can rescatter every wave **independently** [F. Niecknig, B. Kubis, JHEP **1510**, 142 (2015)].

Example $\pi^-\pi^+\pi^-$: final shape and strength of ρ-siobar is

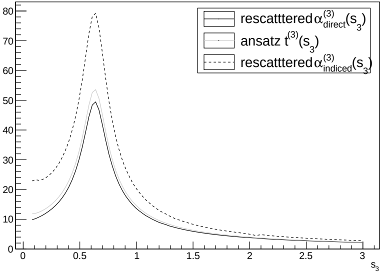
$$A_\rho^{(s)} = (c_\rho a_\rho^{\text{direct}} + c_\sigma a_\rho^{\text{induces}}) P_l(\cos\theta_s)$$

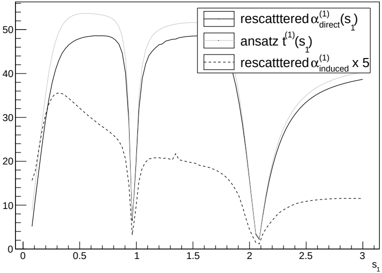
Numerical example

Rescattered and induced “isobar shape“

- anzatz is measurements of $\pi\pi$ phase shift from other experiments
- rescattered and induced shapes are solutions of KT-equation.

[PoS BORMIO2016, MM et al.]





- Chance of narrow resonances is small
- Wide resonances induces high sigmal in cross channels
- Modification depends on the invariant mass of the system (here $\sqrt{s} = 1.3\,\text{GeV}$)

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- Three-body unitarity approach is in progress.

Thank you for the attention