Overview of Hadron Structure from Lattice QCD

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C Approach

D Important Physical Observables

- Nucleon Benchmark Quantities
- Nucleon Spin Puzzle
- Neutron EDM
- Dark Matter Searches

D Summary

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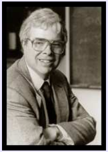
A

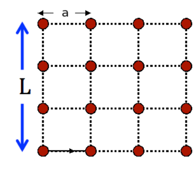
INTRODUCTION

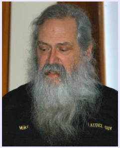
Lattice formulation of QCD



K. Wilson

formulation (1974)





M. Creutz

1^{st} numerical computation (1980)

- ★ **Space-time discretization on a finite-sized 4-D lattice**
  - **Quark fields on lattice points**
  - **Gluons on links**

Lattice results:

- ★ **Contact with well known experimental data**
- ★ **Input for quantities not easily accessible in experiments**
- ★ **New Physics searches**

QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f - \frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}$$

$$D_\mu = \partial_\mu - \tfrac{i}{2} g A^\mu_a \lambda^a$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu$$

Function of 7 parameters: $g,\ m_u,\ m_d,\ m_s,\ m_c,\ m_b,\ m_t$

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Function of 7 parameters: $g,\ m_u,\ m_d,\ m_s,\ m_c,\ m_b,\ m_t$

Discretization of $\mathcal{L}_{\mathrm{QCD}}$

- ★ **Clover improved Wilson**

  ALPHA, BMW, CLS, LHPC, NPQCD, PACS-CS, QCDSF

- ★ **Twisted Mass**

  ETMC

- ★ **Staggered**

  MILC, LHPC

- ★ **Overlap**

  JLQCD

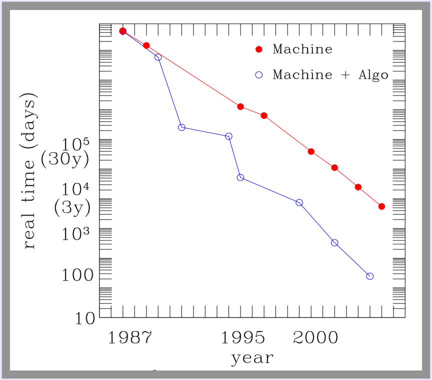
- ★ **Domain Wall**

  RBC-UKQCD

Huge computational power needed & Algorithmic improvements

$48^3 \times 96$ **lattice size: 8×4 × Volume ∼ 340 Million d.o.f !**

$32^3 \times 64$, 5000 configs



Cost of 1000 configurations at physical m_q is currently $\mathcal{O}(10)$ TFlops × year

Computational resources

Juelich Supercomputing Centre, Germany

Peak performance: 5.9 Petaflop/s

458 752 cores

Our time allocation: 65 Million core-h

Swiss National Supercomputing Centre, Switzerland

Peak performance: 7.8 PFlops/s

42 176 cores

Tesla Graphic cards

Our time allocation: 2 Million GPU node-h

Europe’s Fastest SuperComputer

HLRS, Stuttgart, Germany

Peak performance: 7.42 Petaflop/s

185 088 cores

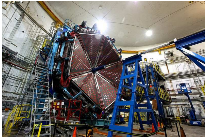
Our time allocation: 48 Million core-h

B

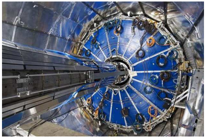
MOTIVATION

Lattice QCD meets Nature

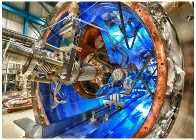
JLAB (12GeV Upgrade)



RHIC (BNL)



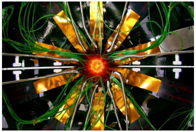
FERMILAB



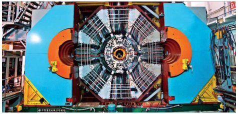
JPARC

Rich experimental activities in major facilities

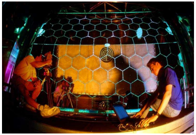
ALICE



BES III



COMPASS

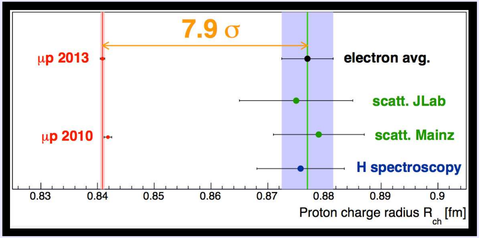


PSI

MAMI

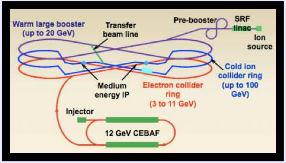
Proton Radius Puzzle

[R. Pohl et al. Nature 466, 213-217 (2010)]



A. Antognini, EINN15

- ★ **Different measurements of proton charge radius give different results**
  - **Several new planned experiments**
- ★ **Discrepancy not understood yet**
  - **no obvious way to connect different measurements**
- ★ **Lattice calculations can provide input**



Electron Ion Collider

The Next QCD Frontier

"Understanding the glue that binds us all"

[A. Accardi et al., EIC white paper, arXiv:1212.1701]

Lattice QCD necessary for EIC measurements

EIC program

structure & interactions of gluon-dominated matter

Measurements will probe the region of sea quarks

parton imaging with high statistics and with polarization in a wide range of small to moderate-x

Lattice QCD

Study of Gluon Observables is now feasible

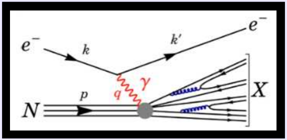
Simulations of the full theory with physical values of the m_q, larger volumes and small enough lattice spacings

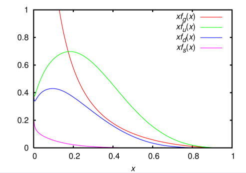
Unpolarized, Polarized and Transversity Distributions can be computed from first principles

C

HADRONS ON THE LATTICE

Probing Nucleon Structure





CTEQ6 PDFs

Generalized Parton Distribution Functions

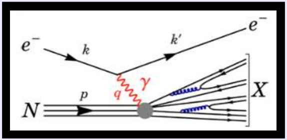
- ★ **Comprehensive description of hadron structure**
- ★ **Deep inelastic scattering (DIS) of leptons off nucleons**
- ★ **Parametrization of off-forward matrix of a bilocal quark operator (light-like)**

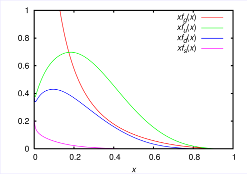
$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar\psi(-\lambda n/2)\, \mathcal{O}\, \underbrace{\mathcal{P}e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(n\alpha)}}_{\text{gauge invariance}} \psi(\lambda n/2)|p\rangle$$

- ★ **Until recently direct lattice calculation inaccessible**

Novel direct approach [X.Ji, arXiv:1305.1539]

Probing Nucleon Structure





CTEQ6 PDFs

Generalized Parton Distribution Functions

★ **Comprehensive description of hadron structure**

★ **Deep inelastic scattering (DIS) of leptons off nucleons**

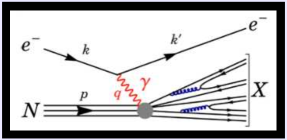
★ **Parametrization of off-forward matrix of a bilocal quark operator**

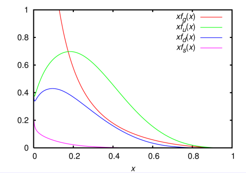
GPDs are central in the scientific program of JLab's 12GeV upgrade, e.g. :

Hall A: **electron-helicity dependent cross-sections of DVCS at 11 GeV**

Hall B: **DVCS with CLAS12 at 11 GeV**

Probing Nucleon Structure





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★ **Until recently direct lattice calculation inaccessible**

Novel direct approach **[X.Ji, arXiv:1305.1539]**

★ **On the lattice: moments of PDFs**

$$\boxed{f^n = \int_{-1}^{1} dx\, x^{n-1} f(x)}$$

★ **moments related to local operators**

A Unpolarized

$$\mathcal{O}^{\mu_1\cdots\mu_n} = \bar{q}\,\gamma^{\{\mu}\, iD^{\mu_1} \ldots iD^{\mu_{n-1}\}} q$$

DIS, Drell-Yan, W-asymmetry, γ^+ jet, ...

B Helicity (polarized)

$$\tilde{\mathcal{O}}^{\mu_1\cdots\mu_n} = \bar{q}\,\gamma_5\gamma^{\{\mu}\, iD^{\mu_1} \ldots iD^{\mu_{n-1}\}} q$$

polarized DIS, SIDIS, pp collisions,photo/electro production, ...

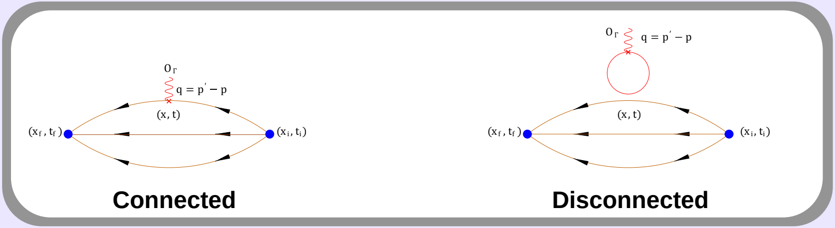
C Transversity

$$\mathcal{O}^{\mu_1\cdots\mu_{n-1}} = \bar{q}\,\sigma^{\mu\,\{\nu}\, iD^{\mu_1} \ldots iD^{\mu_{n-1}\}} q$$

single-spin asymmetry in SIDIS, ...

Nucleon on the Lattice in a nutshell

1. Topologies:



2. Computation of 2pt- and 3pt-functions:

$$\text{2pt}: \quad G(\vec{q}, t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f \cdot \vec{q}}\, \mathbf{\Gamma}^{\mathbf{0}}_{\beta\alpha} \langle J_\alpha(\vec{x}_f, t_f) \overline{J}_\beta(0) \rangle$$

$$\text{3pt}: \quad G_{\mathcal{O}}(\mathbf{\Gamma}^\kappa, \vec{q}, t) = \sum_{\vec{x}_f, \vec{x}} e^{i\vec{x}\cdot\vec{q}} e^{-i\vec{x}_f \cdot \vec{p}'}\, \mathbf{\Gamma}^\kappa_{\beta\alpha} \langle J_\alpha(\vec{x}_f, t_f) \mathcal{O}(\vec{x}, t) \overline{J}_\beta(0) \rangle$$

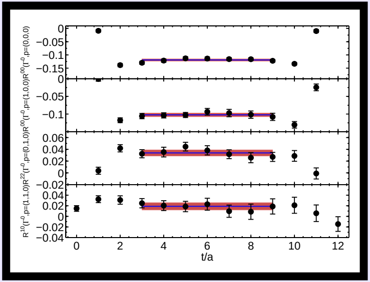
$\Gamma^0 \equiv \frac{1}{4}(1 + \gamma_0)$

$\Gamma^2 \equiv \Gamma^0 \cdot \gamma_5 \cdot \gamma_i$

and other variations

3. Construction of optimized ratio:

$$R_{\mathcal{O}}(\Gamma,\vec{q},t)=\frac{G_{\mathcal{O}}(\Gamma,\vec{q},t)}{G(\vec{0},t_f)}\times\sqrt{\frac{G(-\vec{q},t_f-t)G(\vec{0},t)G(\vec{0},t_f)}{G(\vec{0},t_f-t)G(-\vec{q},t)G(-\vec{q},t_f)}}$$



Plateau Method:

$$R_{\mathcal{O}}(\Gamma,\vec{q},t)\ \underset{t-t_i\to\infty}{\overrightarrow{t_f-t\to\infty}}\ \Pi(\Gamma,\vec{q})$$

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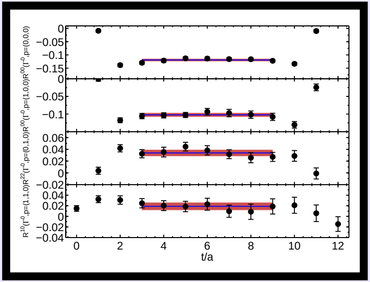

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$$R_{\mathcal{O}}(\Gamma,\vec{q},t)\underset{t-t_i\to\infty}{\xrightarrow{t_f-t\to\infty}}\Pi(\Gamma,\vec{q})$$

4. Renormalization:

connection to experiments

$$\Pi^R(\Gamma,\vec{q})=Z_{\mathcal{O}}\,\Pi(\Gamma,\vec{q})$$

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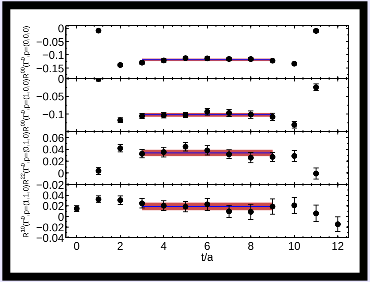

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$$\Pi^R(\Gamma,\vec{q})=Z_{\mathcal{O}}\,\Pi(\Gamma,\vec{q})$$

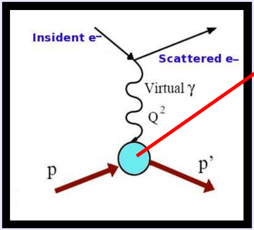
5. Extraction of form factors e.g. Axial current:

$$A^3_\mu\equiv\bar{\psi}\,\gamma_\mu\,\gamma_5\,\frac{\tau^3}{2}\,\psi\Rightarrow\bar{u}_N(p')\left[\mathbf{G_A(q^2)}\,\gamma_\mu\,\gamma_5+\mathbf{G_p(q^2)}\,\frac{q_\mu\,\gamma_5}{2\,m_N}\right]u_N(p)$$

D

EXAMPLES OF IMPORTANT OBSERVABLES

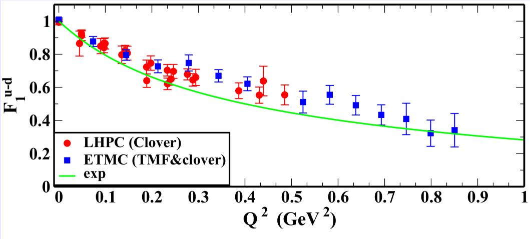
❶ ELECTROMAGNETIC FORM FACTORS



$F_1(Q^2)\&F_2(Q^2)$
Form Factors

★ **Describe how electric charge and current are distributed inside nucleon**

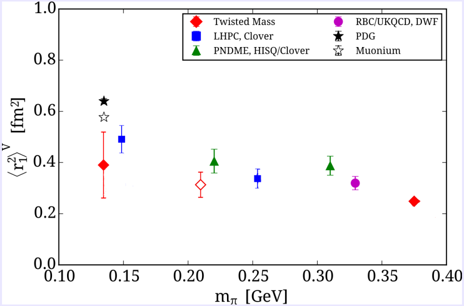
$$\langle N(p',s')|j_\mu(0)|N(p,s)\rangle \sim \bar{u}_N(p',s')\left[\mathbf{F_1(q^2)}\gamma_\mu + \mathbf{F_2(q^2)}\frac{i\,\sigma^{\mu\rho}\,q_\rho}{2m_N}\right]u_N(p,s)$$

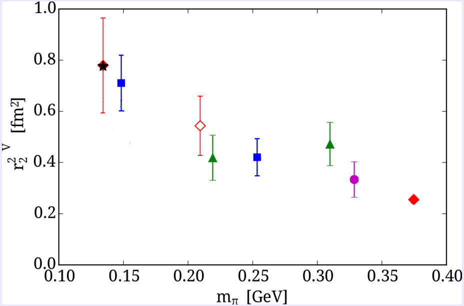


What is the size of a proton? Dirac & Pauli radii

Standard method: Extracted from fits on the form factors

$$\langle r_i^2 \rangle = -\frac{6}{F_i(Q^2)} \frac{dF_i(Q^2)}{dQ^2}\Big|_{Q^2=0}$$





- Estimation of radii strongly depends on small Q^2
- Need access for momenta close to zero $\Rightarrow$ larger volumes

Sachs FFs: $$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2} F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

$$\Pi_0\left(\vec{Q}, \Gamma^0\right) \propto \frac{E\left(\vec{Q}\right) + m_N}{2m_N} G_E\left(Q^2\right)$$

$$\Pi_i\left(\vec{Q}, \Gamma^k\right) \propto \frac{1}{4m_N} \epsilon_{ijk} Q_j G_M\left(Q^2\right)$$

Presence of Q_j: $G_M(0)$ cannot be extracted directly

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Nover Approaches to the radii

Avoid model dependence-fits

[C. Alexandrou et al. (ETMC), arXiv:1410.8818]

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Nover Approaches to the radii

Avoid model dependence-fits

1. direct application of a derivative

$$\lim_{Q^2 \to 0} \frac{\partial}{\partial Q_j} \Pi_i\left(\vec{Q}, \Gamma^k\right) \propto \frac{1}{2m_N} \epsilon_{ijk} G_M\left(Q^2\right)$$

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1. direct application of a derivative (finite L: residual t-dependence)

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2. Use of Fourier transform $F.T.[\Pi(Q)] \to \Pi(y) \underset{\text{average}}{\to} \bar{\Pi}(y)$

$\bar{\Pi}(y)$: **transformed back to momentum space**

$$G_M(\hat{k}^2) = i \sum_{n=1}^{N/2-1} P_n(\hat{k}^2)\, \bar{\Pi}(n) \qquad \hat{k} = 2\sin(k/2)$$

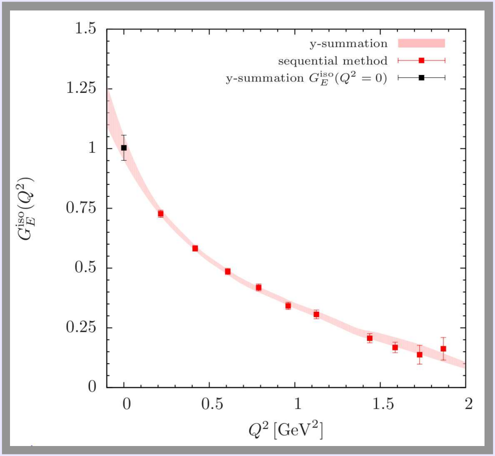
[C. Alexandrou et al. (ETMC), arXiv:1410.8818]

P_n : 2^{nd} **kind Chebyshev polynomials**

[C. Alexandrou et al. (ETMC), arXiv:1410.8818]

First Check: $G_E(0)$

$m_\pi = 375$ **MeV**

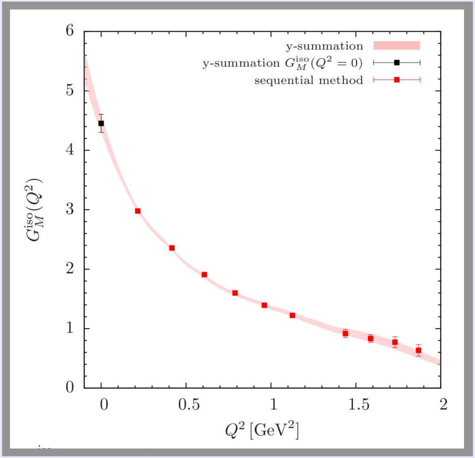


$$G_E(0) = 1$$

[C. Alexandrou et al. (ETMC), arXiv:1410.8818]

Application to $G_M(0)$

$m_\pi = 375$ **MeV**

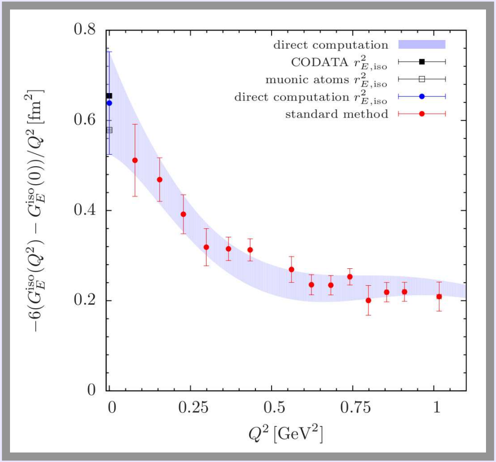


★ $G_M^{u-d}(0)$=4.45(15) **closer to exp. value (4.71)**

[C. Alexandrou et al. (ETMC), 2016]

Application to radii

$m_\pi = 130$ **MeV**

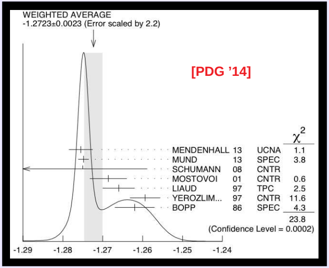


★ $\langle r_E^2 \rangle$ **between experimental points**

On-going work (1400 Measurements)

❷ AXIAL CHARGE

★ governs the rate of β-decay
Well-determined experimentally !



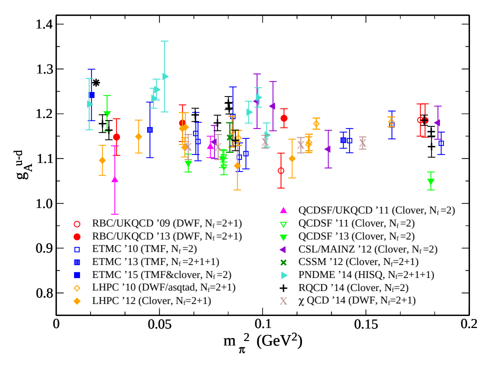
★ related to the intrinsic spin

Intrinsic spin: $\Delta\Sigma = g_A$

★ On the lattice: requires zero momentum transfer

★ Determined directly from lattice data (no fit necessary)

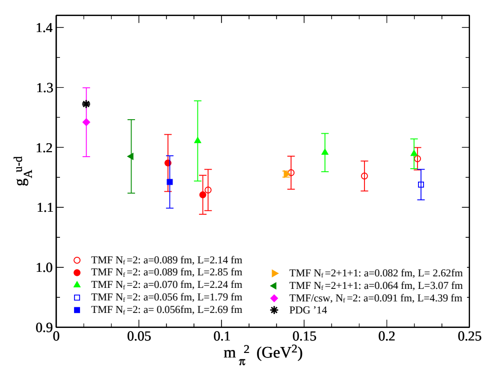
❷ AXIAL CHARGE



∼ 5 000 Measurements

(simulations at the physical point)

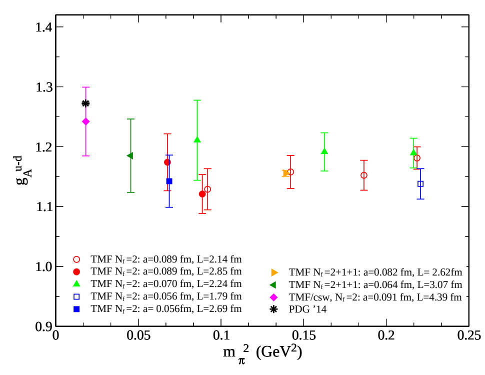
❷ AXIAL CHARGE



$\sim$ **5 000 Measurements**

(simulations at the physical point)

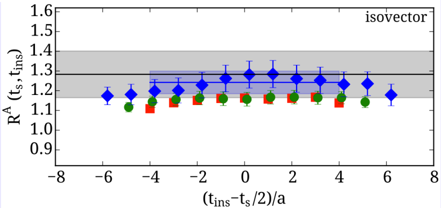
❷ AXIAL CHARGE



Reliable Contact With Experiments:

- Continuum extrapolation
- Infinite Volume extrapolation
  - ★ **Generation of new gauge configurations is required!**
- Excited states

m_π=**130 MeV**



~ 5 000 Measurements

(simulations at the physical point)

- Renormalization

Renormalization

Indispensable for ALL lattice discretizations

★ **Makes contact with experimental & phenomenological data**

$$\boxed{\langle \mathcal{O} \rangle^{\text{Lattice}} Z_{\mathcal{O}} = \langle \mathcal{O} \rangle^{\text{physical}}}$$

★ **perturbative or non-perturbative calculations**

Preferably: non-perturbatively

But: perturbation theory can be very useful

Renormalization

Indispensable for ALL lattice discretizations

★ Makes contact with experimental & phenomenological data

$$\boxed{\langle \mathcal{O} \rangle^{\rm Lattice} \, Z_{\mathcal{O}} = \langle \mathcal{O} \rangle^{\rm physical}}$$

★ perturbative or non-perturbative calculations

Preferably: non-perturbatively

But: perturbation theory can be very useful

Innovative method combines pert. & non-pert. data

Developed by M. Constantinou & H. Panagopoulos (Cyprus)

Synergy of perturbative and non-perturbative results

⇓

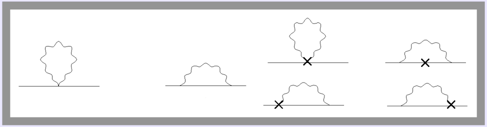
Crucial to control lattice artifacts

Perturbative Calculation on the Lattice

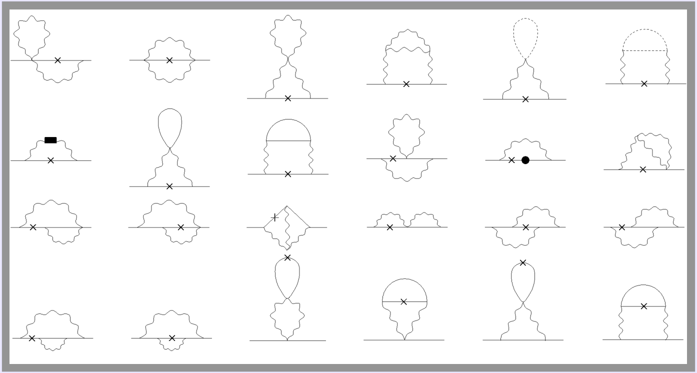
★ More diagrams than continuum

★ Extremely lengthy expressions

Significant human effort & expertise needed !!



1-loop

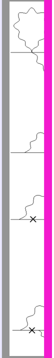


2-loop

Perturbative Calculation on the Lattice

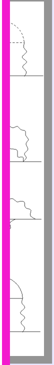
★ **More diagrams than continuum**

★ **Extremely lengthy expressions**



$$\bullet \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{\sin k_{\nu_1} \sin k_{\nu_2}}{\left(\hat{k}^2\right)^2 \left(\widehat{k+a\,p}^{\,2} + a^2M^2\right)} =$$

$$\begin{aligned}
&= \delta_{\nu_1\nu_2} \bigg(0.004327913823968648(1) - \frac{\ln[a^2M^2+a^2p2]}{64\pi^2} - \frac{M^2}{64\pi^2 p2} + \frac{M^4 \ln[1+\frac{p2}{M^2}]}{64\pi^2 p2^2}\bigg) \\
&\quad + \frac{p_{\nu_1}p_{\nu_2}}{p2}\bigg(\frac{1}{32\pi^2} + \frac{M^2}{16\pi^2 p2} - \Big(\frac{1}{16\pi^2} + \frac{M^2}{16\pi^2 p2}\Big)\frac{M^2\ln[1+\frac{p2}{M^2}]}{p2}\bigg) \\
&\quad + a^2 \Bigg(\delta_{\nu_1\nu_2}\bigg(0.00025539124(4)p2 - \frac{p2\ln[a^2M^2+a^2p2]}{1536\pi^2} + 0.00010358434(2)M^2 + \frac{5M^4}{3072\pi^2 p2} + \frac{M^6}{512\pi^2 p2^2} \\
&\qquad - \Big(\frac{1}{384\pi^2} + \frac{M^2}{512\pi^2 p2}\Big)\frac{M^6\ln[1+\frac{p2}{M^2}]}{p2^2}\bigg) + p_{\nu_1}p_{\nu_2}\bigg(-0.00037885376(9) + \frac{\ln[a^2M^2+a^2p2]}{768\pi^2} - \frac{M^2}{768\pi^2 p2} \\
&\qquad - \frac{23M^4}{1536\pi^2 p2^2} - \frac{3M^6}{256\pi^2 p2^3} + \Big(\frac{1}{128\pi^2} + \frac{M^2}{48\pi^2 p2} + \frac{3M^4}{256\pi^2 p2^2}\Big)\frac{M^4\ln[1+\frac{p2}{M^2}]}{p2^2}\bigg) \\
&\qquad + \delta_{\nu_1\nu_2}p_{\nu_1}^2\bigg(-0.00013565411323668763(1) + \frac{\ln[a^2M^2+a^2p2]}{768\pi^2} + \frac{5M^2}{768\pi^2 p2} + \frac{31M^4}{1536\pi^2 p2^2} + \frac{3M^6}{256\pi^2 p2^3} \\
&\qquad - \Big(\frac{1}{64\pi^2} + \frac{5M^2}{192\pi^2 p2} + \frac{3M^4}{256\pi^2 p2^2}\Big)\frac{M^4\ln[1+\frac{p2}{M^2}]}{p2^2}\bigg) + \frac{p_{\nu_1}p_{\nu_2}\left(p_{\nu_1}^2+p_{\nu_2}^2\right)}{p2}\bigg(-\frac{1}{384\pi^2} - \frac{M^2}{48\pi^2 p2} - \frac{3M^4}{64\pi^2 p2^2} \\
&\qquad - \frac{M^6}{32\pi^2 p2^3} + \Big(\frac{1}{96\pi^2} + \frac{M^2}{24\pi^2 p2} + \frac{M^4}{16\pi^2 p2^2} + \frac{M^6}{32\pi^2 p2^3}\Big)\frac{M^2\ln[1+\frac{p2}{M^2}]}{p2}\bigg) \\
&\qquad + \frac{\delta_{\nu_1\nu_2}p4}{p2}\bigg(\frac{1}{1536\pi^2} - \frac{M^2}{768\pi^2 p2} - \frac{M^4}{256\pi^2 p2^2} - \frac{M^6}{128\pi^2 p2^3} + \Big(\frac{1}{384\pi^2} + \frac{M^2}{128\pi^2 p2} + \frac{M^4}{128\pi^2 p2^2}\Big)\frac{M^4\ln[1+\frac{p2}{M^2}]}{p2^2}\bigg) \\
&\qquad + \frac{p_{\nu_1}p_{\nu_2}p4}{p2^2}\bigg(\frac{1}{768\pi^2} + \frac{5M^2}{192\pi^2 p2} + \frac{11M^4}{128\pi^2 p2^2} + \frac{5M^6}{64\pi^2 p2^3} \\
&\qquad - \Big(\frac{1}{96\pi^2} + \frac{M^2}{16\pi^2 p2} + \frac{M^4}{8\pi^2 p2^2} + \frac{5M^6}{64\pi^2 p2^3}\Big)\frac{M^2\ln[1+\frac{p2}{M^2}]}{p2}\bigg)\Bigg) + \mathcal{O}\left(a^4\right)
\end{aligned}$$

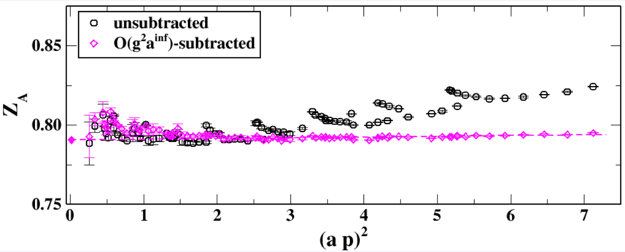


1-loop

2-loop

Example: Improvement of Z_A

ETMC ($N_f=2$, TMF & clover)



[M. Constantinou et al. (ETMC), arXiv:1509.00213]

- **Lattice artifacts computed perturbatively**
- **Subtraction from non-perturbative estimates**

★ **Results useful for any discretizations used by various groups**

★ **employed by ETM Collaboration, QCDSF Collaboration**

❸ QUARK MOMENTUM FRACTION

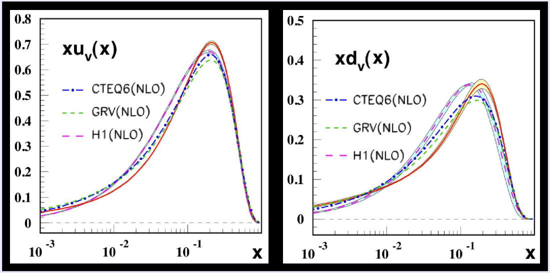
★ Distribution of nucleon momentum among its constituents

★ First non-trivial moment

(moment fixed by the number of valence quarks)

★ Measured in DIS experiments

Value uses input from phenomenological models



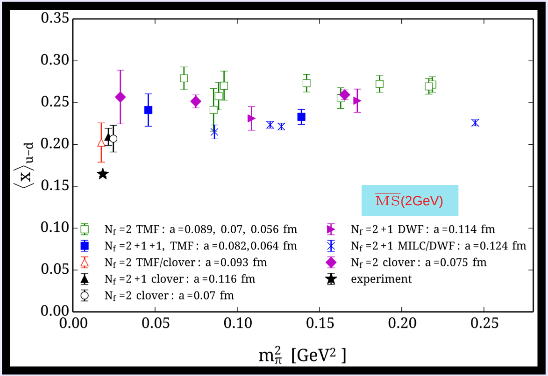
[J. Blumlein et al., arXiv:hep-ph/0607200]

★ Benchmark quantity for lattice QCD calculations

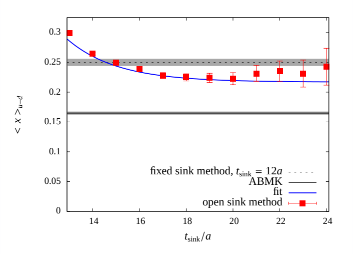
❸ QUARK MOMENTUM FRACTION

1-derivative vector current: $\mathcal{O}^{\mu\nu}_{\rm DV} \equiv \bar\psi\,\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\psi$

$$\langle N(p',s')|\mathcal{O}^{\mu\nu}_{\rm DV}|N(p,s)\rangle = \bar u_N(p',s')\Big[\mathbf{A_{20}(q^2)}\,\gamma^{\{\mu}P^{\nu\}} + \mathbf{B_{20}(q^2)}\,\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + \mathbf{C_{20}(q^2)}\,\frac{1}{m}q^{\{\mu}q^{\nu\}}\Big]u_N(p,s)$$



- $\langle x\rangle^{\rm phen} = 0.1646(27)\ (\overline{\rm MS}(2{\rm GeV}))$
[S. Alekhin et al., arXiv:0908.2766]
- **Scheme and scale dependence**
- **All lattice results overestimate phen. value**



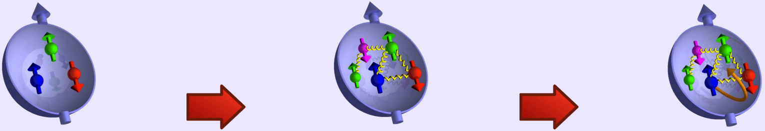
~ 5 000 Measurements

NUCLEON

SPIN PUZZLE

Nucleon Spin Puzzle

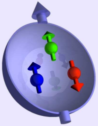
Evolution of understanding the proton spin:



Simple parton model	Sea quarks & Gluons contributions	Parton orbital angular momentum
$\frac{1}{2}\left(\Delta u_v + \Delta d_v\right) = \frac{1}{2}$	$\frac{1}{2}\left(\Delta q + \bar{\Delta} q\right) + \Delta G = \frac{1}{2}$	$\frac{1}{2}\left(\Delta q + \bar{\Delta} q\right) + \Delta G + L_z = \frac{1}{2}$

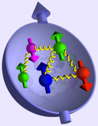
Nucleon Spin Puzzle

Evolution of understanding the proton spin:



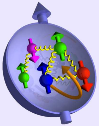
Simple parton model

$\frac{1}{2}\left(\Delta u_v + \Delta d_v\right) = \frac{1}{2}$



Sea quarks & Gluons contributions

$\frac{1}{2}\left(\Delta q + \Delta\bar{q}\right) + \Delta G = \frac{1}{2}$



Parton orbital angular momentum

$\frac{1}{2}\left(\Delta q + \Delta\bar{q}\right) + \Delta G + L_z = \frac{1}{2}$

Spin Structure from First Principles

Quark orbital angular momentum

Spin Sum Rule:

$$\boxed{\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left(L^q + \frac{1}{2}\Delta\Sigma^q\right) + J^G}$$

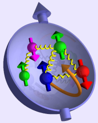
Intrinsic Spin

Gluon part

Extraction from LQCD:

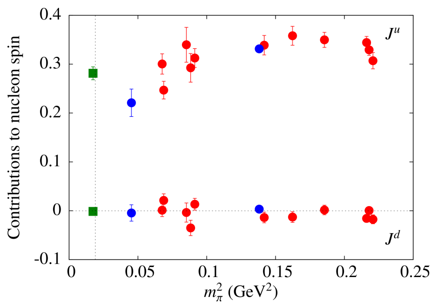
$$\boxed{J^q = \frac{1}{2}\left(A_{20}^q + B_{20}^q\right)\,, \quad L^q = J^q - \Sigma^q\,, \quad \Sigma^q = g_A^q}$$

Quark Contributions to Spin

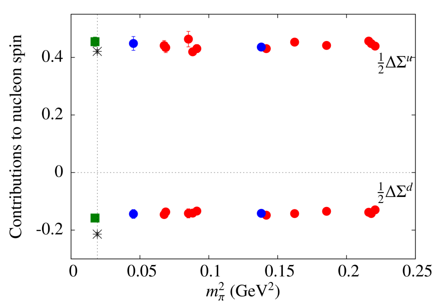


Valence Quarks Contributions

Total Spin



Intrinsic Spin

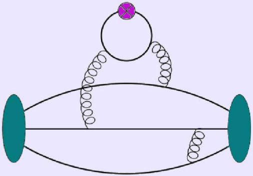


★ **Valence Quark carry** $\sim$ **half of the proton spin**

Where does the rest of the spin comes from ?

- ★ **Sea Quark Contributions**
- ★ **Gluon Contributions**

Sea Quark Contribution



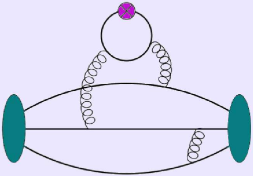
Disconnected Diagram

★ **very noisy and very expensive computationally**

★ **We've come far in development of techniques**

- Truncated Solver Method
- One-end-trick
- All-Mode-Averaging
- Hierarchical probing

Sea Quark Contribution



Disconnected Diagram

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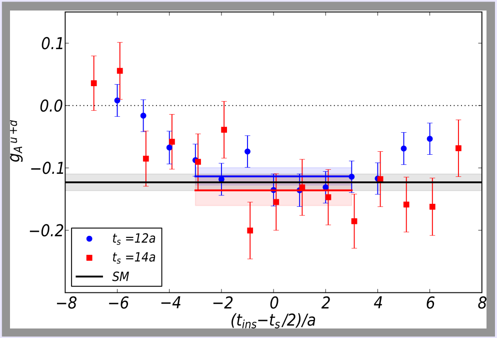
- ★ We've come far in development of techniques
  - Truncated Solver Method
  - One-end-trick
  - All-Mode-Averaging
  - Hierarchical probing

(up + down sea quarks)

Axial charge

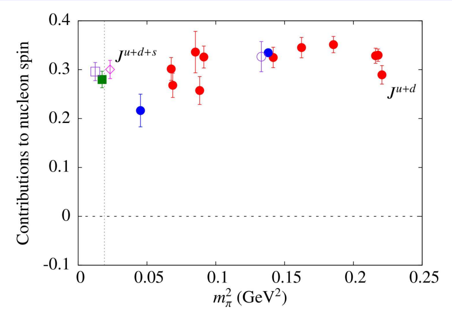
$\sim$ 180 000 Measurements !

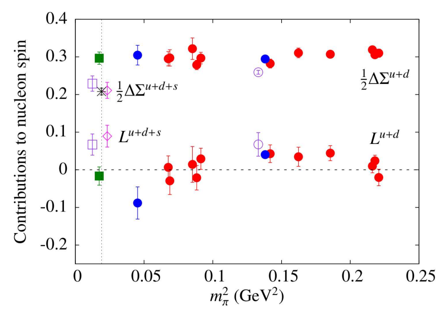


Nucleon Spin at the Physical Point

★ **Valence Quarks** **(up, down)**

★ **Sea Quarks** **(up, down, strange)**





Presented for the very first time at the physical point

★ **Sea Quark contribution bring data in agreement with experiment !**

❹ GLUON MOMENTUM FRACTION

Direct computation: **Disconnected contribution**

$$\boxed{\mathcal{O}^g_{\mu\nu} = -\mathrm{Tr}\left[G_{\mu\rho}G_{\nu\rho}\right]}$$

$$\langle N(0)|\mathcal{O}_{44} - \frac{1}{3}\sum_{j=1}^{3}\mathcal{O}_{jj}|N(0)\rangle = m_N \, \langle x\rangle_g$$

First Lattice Calculation:

- ★ **from full QCD**
- ★ **at the physical point**

❹ GLUON MOMENTUM FRACTION

Direct computation: **Disconnected contribution**

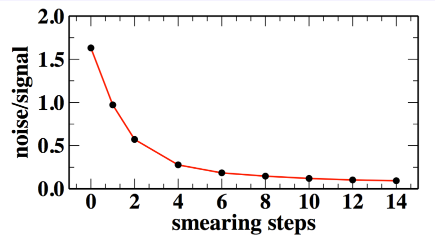
$$\boxed{\mathcal{O}^g_{\mu\nu} = -\mathrm{Tr}\left[G_{\mu\rho}G_{\nu\rho}\right]} \qquad \langle N(0)|\mathcal{O}_{44} - \tfrac{1}{3}\textstyle\sum_{j=1}^{3}\mathcal{O}_{jj}|N(0)\rangle = m_N\,\langle x\rangle_g$$

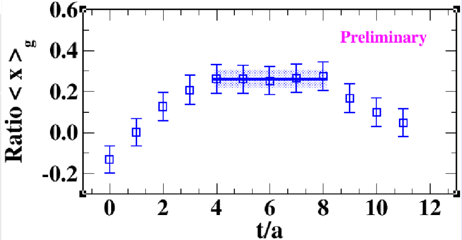
First Lattice Calculation:

- ★ **from full QCD**
- ★ **at the physical point**

$N_f=2$ **TM fermions,** $m_\pi=130$**MeV**

[C. Alexandrou et al. (ETMC), 2015]





Smearing: improves signal

Challenges

★ **Disconnected $\Rightarrow$ Small signal-to-noise ratio**

★ **Renormalization**

- **Mixing with operator for $\langle x \rangle_{u+d}$**

  Unavoidable

- **Mixing with other Operators** Gauge Invariant, BRS transformation, vanish by e.o.m.

  Vanish in physical matrix elements

Challenges

★ **Disconnected $\Rightarrow$ Small signal-to-noise ratio**

★ **Renormalization**

- **Mixing with operator for $\langle x \rangle_{u+d}$**

  Unavoidable

- **Mixing with other Operators** **Gauge Invariant, BRS transformation, vanish by e.o.m.**

  Vanish in physical matrix elements

MUST compute mixing coefficients and subtract contributions

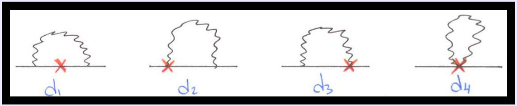
$\Downarrow$

Perturbation Theory

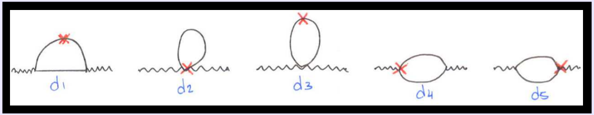
Perturbative computation

(2 years of intensive human and computational work!!)

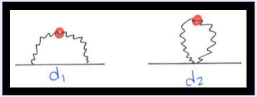
$\times Z_{qq}$: $\Lambda_{qq} = \langle q|\mathcal{O}_q|q\rangle$



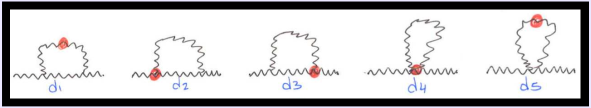
$\times Z_{qg}$: $\Lambda_{qg} = \langle g|\mathcal{O}_q|g\rangle$



$\bullet Z_{gq}$: $\Lambda_{gq} = \langle q|\mathcal{O}_g|q\rangle$



$\bullet Z_{gg}$: $\Lambda_{gg} = \langle g|\mathcal{O}_g|g\rangle$



Perturbative computation

(2 years of intensive human and computational work!!)

× Z_{qq} : $\Lambda_{qq} = \langle q|\mathcal{O}_q|q\rangle$

$$\boxed{Z_{gg}{=}1{+}\frac{g^2}{16\pi^2}\left(1.0574\,N_f + \frac{-13.5627}{N_c} - \frac{2\,N_f}{3}\log(a^2\bar{\mu}^2)\right)}$$

× Z_{qg} : $\Lambda_{qg} = \langle g|\mathcal{O}_q|g\rangle$

$$\boxed{Z_{gq}{=}0{+}\frac{g^2\,C_f}{16\pi^2}\left(0.8114 + 0.4434\,c_{SW} - 0.2074\,c_{SW}^2 + \frac{4}{3}\log(a^2\bar{\mu}^2)\right)}$$

• Z_{gq} : $\Lambda_{gq} = \langle q|\mathcal{O}_g|q\rangle$

$$\boxed{Z_{qq}{=}1{+}\frac{g^2}{16\pi^2}\left(-1.8557 + 2.9582\,c_{SW}{+}0.3984\,c_{SW}^2 - \frac{8}{3}\log(a^2\bar{\mu}^2)\right)}$$

• Z_{gg} : $\Lambda_{gg} = \langle g|\mathcal{O}_g|g\rangle$

$$\boxed{Z_{qg}{=}0{+}\frac{g^2\,N_f}{16\pi^2}\left(0.2164 + 0.4511\,c_{SW} + 1.4917\,c_{SW}^2 - \frac{4}{3}\log(a^2\bar{\mu}^2)\right)}$$

Elimination of mixing

★ **Results** (in progress): with 2 dynamical TM fermions at physical point

$$\boxed{\langle x \rangle_g = 0.283(41)}$$

$\boxed{\sim 156\,000 \text{ Measurements !}}$

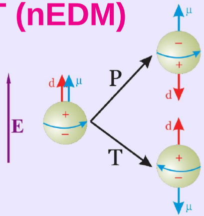
[C. Alexandrou et al. (ETMC), 2015]

Momentum Conservation

$$\boxed{\sum_q \langle x \rangle_q^R + \langle x \rangle_G^R \underset{u,d,s}{=} \langle x \rangle_{u+d}^{CI,R} + \langle x \rangle_{u+d+s}^{DI,R} + \langle x \rangle_G^R = 0.929(64)}$$

PHYSICS BEYOND THE STANDARD MODEL

❺ NEUTRON ELECTRIC DIPOLE MOMENT (nEDM)



- ★ distribution of positive & negative charge in neutron
- ★ nonzero EDM violates P, T symmetry (and also CP)
- ★ probe for BSM physics
- ★ no finite nEDM has been reported
- ★ **Experiments:**

  change in spin presession frequency of UCN in weak B when a strong background E flips sign

- ★ first report in 1950 (ORNL)

THE NEWS
OAK RIDGE NATIONAL LABORATORY

A Publication by and for the ORNL Employees of Carbide and Carbon Chemicals Division, Union Carbide and Carbon Corporation

Vol. 3—No. 13 OAK RIDGE, TENNESSEE Friday, September 29, 1950

HARVARD UNIVERSITY SPONSORS PROGRAM HERE — James H. Smith, Harvard University graduate student in physics, is shown as he adjusts a neutron beam apparatus at the south face of the Oak Ridge Pile. Using the Pile as a source of neutrons, Mr. Smith is engaged in a project jointly sponsored by Harvard University and Oak Ridge National Laboratory for the purpose of determining if neutrons have permanent electric dipole moments.

Harvard University Conducts Important Research at ORNL

The growing importance of Oak Ridge National Laboratory as a research center is manifested particularly in its assistance to universities and technical schools on various projects in which nuclear research is involved. An example of such relationship is its present collaboration with Harvard University in an investigation to determine if neutrons have permanent electric dipole moments.

The work of the project is under the direction of Professors E. M. Purcell and Norman F. Ramsey of the Harvard University Physics Department and is being conducted on the Laboratory area by James H. Smith, a

DR. TAYLOR

ACS Lectureship Set For October 26, 27

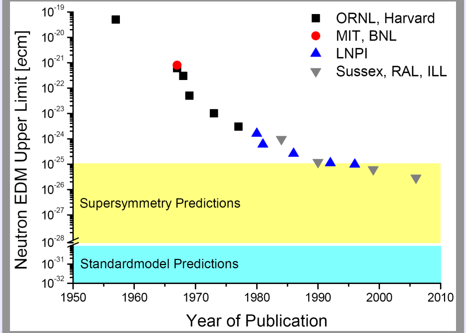
The East Tennessee Section of the American Chemical Society will have its Annual East Tennessee Lectureship this year in two sessions, according to plans re-

Dr. Ellison Taylor Appointed Chem. Division Director

Effective October 1, Dr. Ellison H. Taylor will assume the duties of Director of the Chemistry Division. In this capacity he will succeed Dr. John A. Swartout, who was recently elevated to the position of Assistant Research Director of Oak Ridge National Laboratory.

Dr. Taylor's present connection with the Chemistry Division is that of Associate Director of the Division and Group Leader of the Radiation Chemistry Group, in which capacities he has served since June, 1948. Previously, he had been Assistant Director of the Division, from June, 1946, to February, 1948, and was Acting Di-

Upper limits of nEDM



Source: A. Knecht

- **current best exp. upper limit:**

$\boxed{|d_n| < 2.9 \times 10^{-26}\, e \cdot cm}$ **(90 % C.L.)**

(ILL Grenoble)

- **EFT calculations:**

$$|d_n| \sim \sum \theta \cdot \mathcal{O}(10^{-2} \sim 10^{-3})\, e \cdot \mathrm{fm} \Rightarrow \theta \lesssim \mathcal{O}(10^{-10} \sim 10^{-11})$$

θ: **strength of the CP-breaking**

nEDM on the Lattice

CP violation from QCD

$$\boxed{\mathcal{L}(x) \to \mathcal{L}_{QCD}(x) - \theta\, \frac{i}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\left[G_{\mu\nu}(x) G_{\rho\sigma}(x)\right]}$$

does not modify the e.o.m.

• θ: strength of the CP-breaking

topological charge density: $q(x) \equiv \frac{i}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\left[G_{\mu\nu}(x) G_{\rho\sigma}(x)\right]$

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Methods for extracting the nEDM

★ **External electric field**

★ **Imaginary** θ

★ **matrix element of nEDM from** $\mathcal{O}(\theta)$**:** $F_3(Q^2)$

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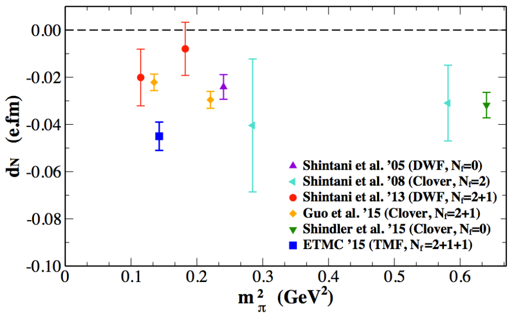
★ **matrix element of nEDM from** $\mathcal{O}(\theta)$**:** $F_3(Q^2)$

$$\boxed{\langle N(p', s')|j_\mu(0)|N(p,s)\rangle \propto \theta\, Q_k \left[\frac{\alpha F_1(Q^2)}{2m_N} + \frac{(E_N + 3m_N)\alpha F_2(Q^2)}{4m_N^2} + \frac{(E_N + m_N) F_3(Q^2)}{4m_N^2}\right]}$$

F_3: **CP-odd form factor**

$$\boxed{|\vec{d}_N| = \lim_{\vec{q}\to 0} \theta\, \frac{F_3(q^2)}{2m_N}}$$

Collection of lattice results



[C. Alexandrou et al. (ETMC), arXiv:1510.05823]

Data from different methods:

1. external electric field: ◀
2. imaginary θ: ♦
3. F_3 from $\mathcal{O}(\theta^1)$: ● ▲ ▼ ■

(**ETMC data:** include direct computation of $F_3(0)$!)

DARK MATTER SEARCHES

❻ NUCLEON σ-TERMS

★ **Role in direct search of dark matter** *(enters cross-section of DM-nuclei elastic scattering)*

★ **sensitivity of m_N to m_q:**

$$\boxed{m_q \langle N|\bar{q}q|N\rangle = m_q \frac{\partial m_N}{\partial m_q}}$$

★ **no direct experimental measurements**

★ **Indirect measurements:**

6 NUCLEON σ-TERMS

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★ $\pi - N$ **scattering amplitutes**

χ**PT:** $\sigma_{\pi N} \sim 45\mathrm{MeV}$ **[J. Gasser et al., PLB 253(1991) 252]**

partial wave analysis: $\sigma_{\pi N} \sim 80\mathrm{MeV}$ **[M. Pavan et al., hep-ph/0111066]**

★ $K - N$ **scattering phase shift or** $\sigma_{\pi N}$ & m_s/m_{ud} & $SU(3)$ χ**PT**

$\sigma_s = 27(27)\mathrm{MeV}$ **[X.L. Ren et al., arXiv:1404.4799]** $\sigma_s = 84^{+28}_{-4}\mathrm{MeV}$ **[M. Lutz et al., arXiv:1401.7805]**

large uncertainties in WIMP-nucleon cross-section

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Lattice calculations

Direct Method

$\langle N|\bar{q}q|N\rangle$ **(3pt CI $\&$ DI)**

discussed in this talk

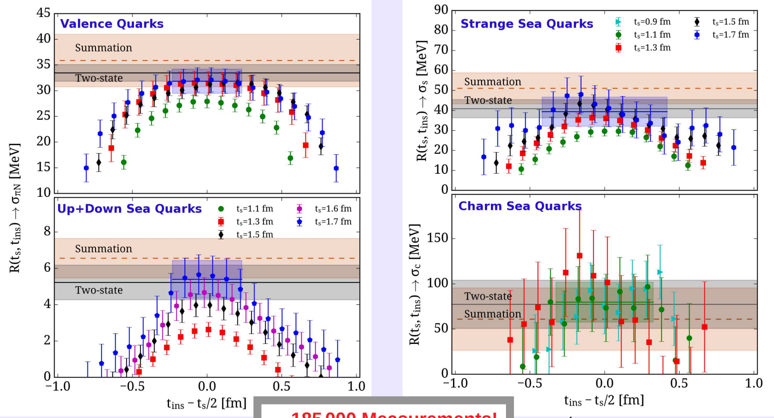
Spectrum Method

Feynman-Hellmann on $\frac{\partial m_N}{\partial m_q}$

R. Young, [arXiv:1301.1765]

σ - terms at the physical point

[A.Abdel-Rehim et al. (ETMC), arXiv:1601.01624] (Submitted to PRL)



$\sim$ 185 000 Measurements!

sea sector

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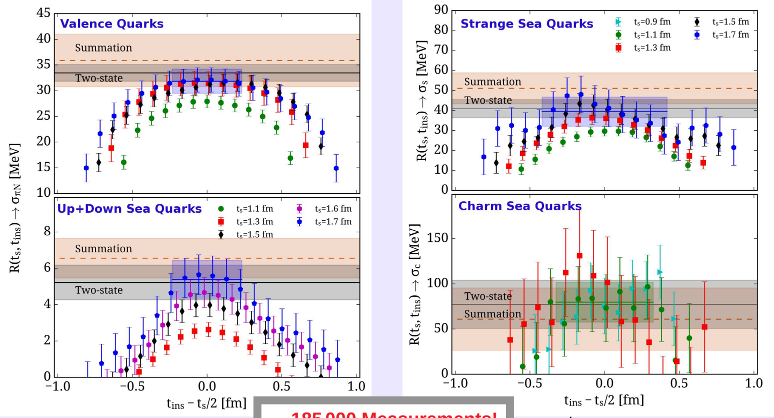

$\sim$ 185 000 Measurements!

sea sector

Our calculation yields:

$$\sigma_{\pi N} = 37.22\,(2.57)\,\left(^{+0.99}_{-0.63}\right)\ \text{MeV}$$

$$\sigma_{\text{strange}} = 41.05\,(8.25)\,\left(^{+1.09}_{-0.69}\right)\ \text{MeV} \qquad \sigma_{\text{charm}} = 79\,(21)\,\left(^{+2.1}_{-1.3}\right)\ \text{MeV}$$

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- **Physical parameters**
- **Control of statistical and systematic uncertainties**
- **Disconnected contributions at physical m_π!**

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SUMMARY

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★ **New physics BSM (scalar & tensor charges, σ-terms)**

- Lattice QCD provides predictions

SUMMARY

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★ **New physics BSM (scalar & tensor charges, σ-terms)**

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★ **Lattice Perturbation Theory**

- identification of mixing
- control of lattice artifacts

F

FUTURE

PERSPECTIVES

Future Directions

Addressing open questions

★ **Proton spin**

- **Individual quark contributions**
- **Gluon contribution**

Need of State-of-the-art simulations

$\rightsquigarrow$ **Include strange & charm quarks in simulations**

$\rightsquigarrow$ **Continuum and infinite volume limits**

Improvements needed

$\rightsquigarrow$ **Algorithmic improvements & Noise reduction techniques**

$\rightsquigarrow$ **Advance Lattice Perturbation Theory**

Approved JLab experiments:

1. **Longitudinal spin structure of nucleon at moderate-large x**
2. **EMC effect in spin structure functions**

Future Directions

Addressing open questions

★ **E/M form factors at low and high Q^2**

★ **Proton radius puzzle**

★ **Strangeness of nucleon**

Improvements needed

★ **Better approach for extracting data at $Q^2=0$**

$\rightsquigarrow$ **Algorithmic improvements & Noise reduction techniques**

Approved JLab experiments:

1. **Measurements of G_E^p, G_M^p, G_E^n, G_M^n at high Q^2 (6 experiments in Halls A-C)**
2. **High precision measurement of proton charge radius**

Future Directions

Exploration of other approaches

★ **Direct calculation of PDFs on the lattice** [X.Ji, arXiv:1305.1539]

Only 2 pilot studies

[H-W. Lin et al., arXiv:1402.1462] [C. Alexandrou et al., arXiv:1504.07455]

Renormalization missing $\Rightarrow$ **No connection to physics!**

Plans

★ **development of renormalization scheme**

M. Constantinou et al. (Cyprus)

★ **Computation of all 3 types of PDFs**

unpolarized, helicity, transversity

Approved JLab experiments:

1. Proton’s Quark Dynamics in Semi-Inclusive Pion Production (TMDs)

Future Directions

Probing beyond the Standard Model Physics

- ★ **scalar & tensor interactions**

- ★ **Neutron Electric Dipole Moment**
  Preparatory study on methodology:

  [C. Alexandrou et al. (ETMC), arXiv:1510.05823]

Future Directions

Probing beyond the Standard Model Physics

★ **scalar & tensor interactions**

★ **Neutron Electric Dipole Moment**
Preparatory study on methodology:

[C. Alexandrou et al. (ETMC), arXiv:1510.05823]

Understanding Hadron Structure

★ **Methods developed for nucleon can be utilized in**

- **Form Factors of other hadrons**

STAY TUNED !

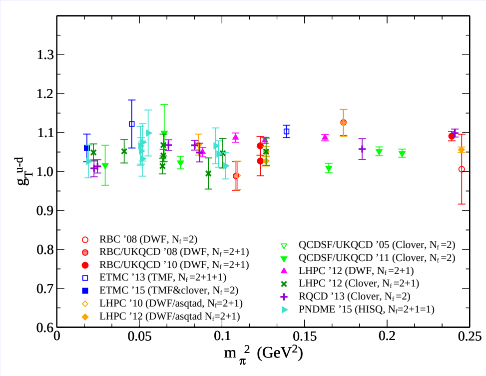
THANK YOU

BACKUP SLIDES

Tensor Charge

$$\langle N(p',s')|\,\sigma^{\mu\nu}\,|N(p,s)\rangle \;\Rightarrow\; \mathbf{A_{T10}(q^2)},\ \mathbf{B_{T10}(q^2)},\ \mathbf{C_{T10}(q^2)}$$

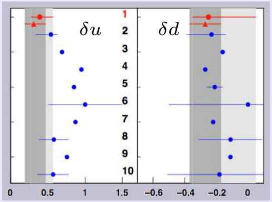
$$g_T \equiv \langle 1\rangle_{\delta q} = A_{T10}(0)$$



SIDIS results (HERMES, COMPASS) and BELLE e+ e- analysis

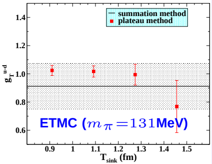
- $g_T^{\mathrm{exp}}(0.8\mathrm{GeV}^2)=0.77^{+0.13}_{-0.27}$

[M.Anselmino et al., arXiv:0812.4366]



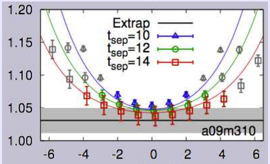
[M.Anselmino et al., arXiv:1303.3822]

★ Agreement among most lattice points
★ Mild m_π dependence



ETMC (m_π=131MeV)

PNDME (m_π=310MeV) ⇒



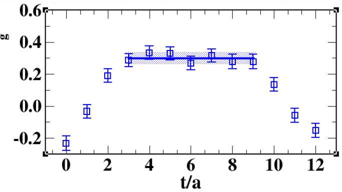
[T. Bhattacharya (PNDME), arXiv:1306.5435]

Gluon unpolarized disctribution

- $\mathcal{O}^g_{\mu\nu} = -\mathrm{Tr}\,[G_{\mu\rho}G_{\nu\rho}]$ $\quad \langle N(p)|\mathcal{O}_{44} - \frac{1}{3}\sum_{j=1}^{3}\mathcal{O}_{jj}|N(p)\rangle = \left(m_N + \frac{2}{3\,E_N}\vec{p}^{\,2}\right)\langle x\rangle_g$

- Direct lattice computation of gluon moment $\langle x\rangle_g$: disconnected diagram

Dynamical



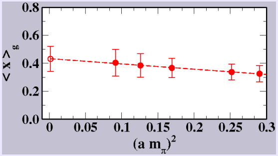
[C. Alexandrou et al. (ETMC), 2015]

$N_f=2+1+1$ **ETM,** $m_\pi=375$ **MeV**

$\langle x\rangle_g = 0.309(25)$

Smearing: improves signal

Quenched

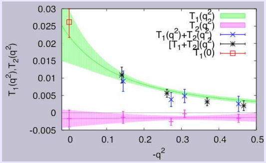


[R. Horsley (QCDSF), 2012, arXiv:1205.6410]

$N_f=0$ **Clover,** $m_\pi=314-555$ **MeV**

$\langle x\rangle_g = 0.43(7)(5)$

Quenched

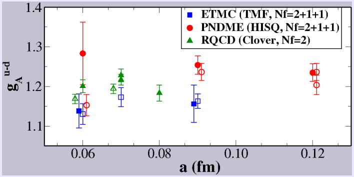


[M. Deka (χQCD), 2013, arXiv:1312.4816]

$N_f=0$ **Wilson,** $m_\pi=478-650$ **MeV**

$\langle x\rangle_g = 0.313(56)$

1. Cut-off effects



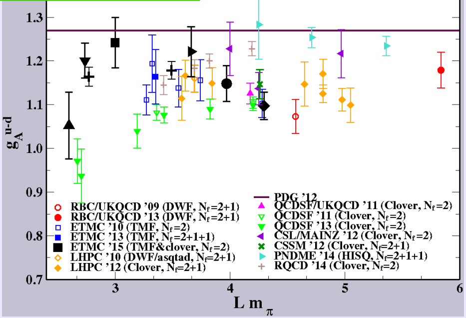
Continuum extrapolation requires 3 lattice spacings

[C. Alexandrou et al. (ETMC), arXiv:1012.0857]
[G. Bali et al. (RQCD), 2014]
[R. Gupta et al. (PNDME), 2014]

$a < 0.1$ **fm is sufficient**

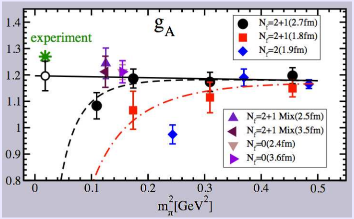
2. Finite Volume Effects

$128\ \text{MeV} \leq m_\pi \leq 300\ \text{MeV}$



Lattice data for plateau method
No volume corrections

[T. Yamazaki et al. (RBC/UKQCD) arXiv:0801.4016]

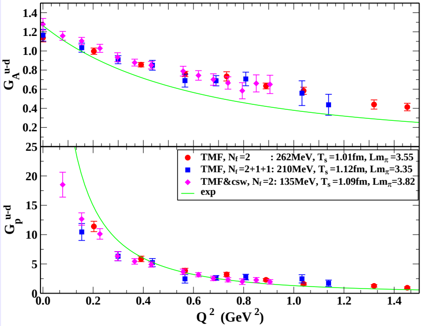


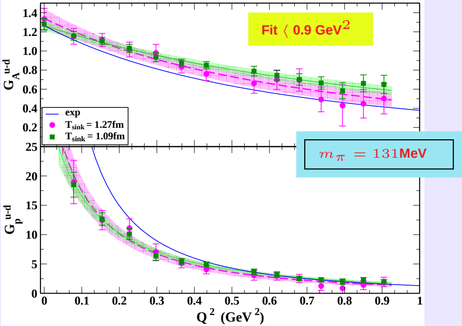
FVE not fully understood:

Group	m_π (MeV)	Lm_π	g_A
QCDSF	158	2.7	1.052(76)
ETMC	131	3.0	1.242(57)
PNDME	125	3.7	1.222(57)
LHPC	149	4.0	1.097(32)

Nucleon Axial form factors

ETM, $N_f = 2$, $N_f = 2+1+1$ and ETM & clover , $N_f = 2$





★ **Dipole fits:**

$$G_A(Q^2) = \frac{g_A}{\left(1+Q^2/m_A^2\right)^2} \qquad m_A^{exp} = 1.069\text{GeV}^{\dagger}$$

$$1.2\text{GeV} < m_A^{lattice} < 1.45\text{GeV}^{\star}$$

$$G_p(Q^2) = \frac{G_A(Q^2)\,G_p(0)}{\left(Q^2+m_p^2\right)} \qquad 0.3\text{GeV} < m_p^{lattice} < 0.5\text{GeV}^{\star}$$

† [V.Bernard et al., hep-ph/0607200]

⋆ ETM, m_π=131MeV (ETMC 2014)

- G_p strongly dependent on the lowest values of Q^2

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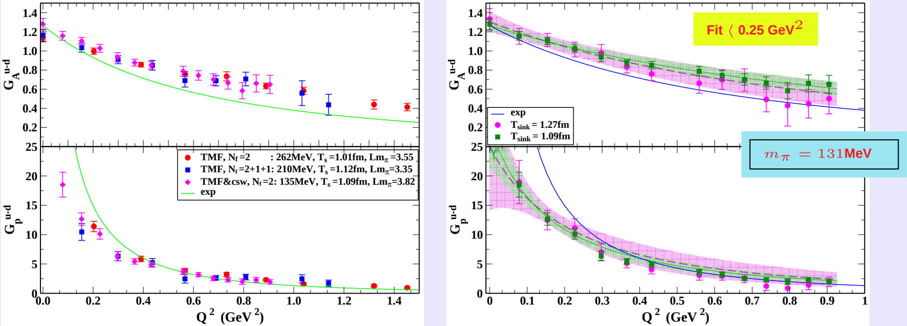

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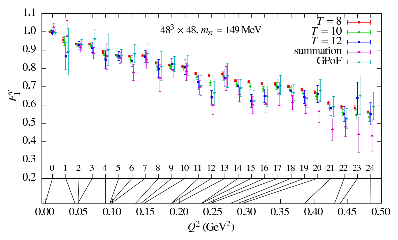
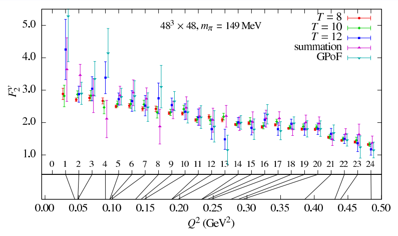
Nucleon EM form factors

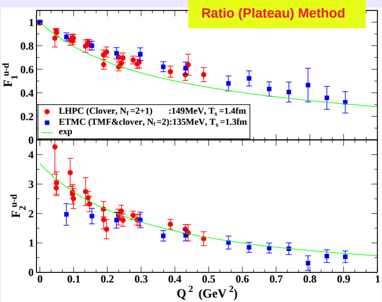
$$\langle N(p',s')|\gamma_\mu|N(p,s)\rangle \sim \bar{u}_N(p',s')\left[\mathbf{F_1(q^2)}\gamma_\mu + \mathbf{F_2(q^2)}\frac{i\,\sigma^{\mu\rho}\,q_\rho}{2m_N}\right]u_N(p,s)$$

LHPC: $m_\pi = 149$MeV, $a = 0.116$fm, $\mathcal{O}(7\,800)$ stat.

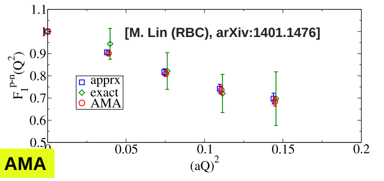




[J.R.Green et al. (LHPC), arXiv:1211.0253]



- **ETMC:** $a = 0.094$ fm, $L = 4.5$ fm, $L\,m_\pi = 3.1$
- **LHPC:** $a = 0.116$ fm, $L = 5.6$ fm, $L\,m_\pi = 4.2$



AMA

- **Summation method goes either direction**
- **errors are large**

PDFs on the Lattice

- ★ characterize the dynamics of quarks and gluons inside hadrons
- ★ predictions for collision experiments
- ★ non-perturbative nature $\Rightarrow$ hard to compute
- ★ non-local light-cone correpators, time dependent and intrinsically Minkowskian, requires $t^2 + \vec{x}^2 \sim 0 \Rightarrow$ difficult on lattice

On the lattice we study Mellin moments of PDFs:

$$\langle x \rangle_q = \int_{-1}^{1} dx\, x^{n-1}\, q(x)\,, \qquad \langle x \rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1}\, \Delta q(x)\,, \qquad \langle x \rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\, \delta q(x)$$

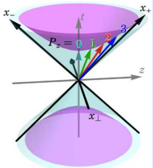
However, reconstruction of PDFs seems unfeasible:

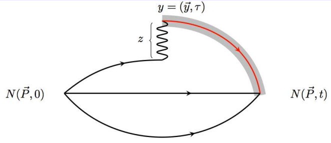
- ★ signal-to-noise is bad for higher moments
- ★ n $>$ 3: operator mixing (unavoidable!)
- ★ gluon moments: limited progress (discon. diagram, signal quality, operator mixing)

Novel direct approach [X.Ji, arXiv:1305.1539]

- ★ compute a P*quasi*-DF (accessible on the lattice)
- ★ contact with physical PDFs via a matching procedure

Access of PDFs on Euclidean lattice





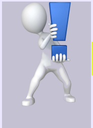
- ★ rest frame: parton physics correspond to light-cone correlation **BUT**:
- ★ same physics obtained from t-independent spatial correlation in the IMF
- ★ P*quasi*-DF ($\tilde{q}$) purely spatial for nucleons with finite momentum (e.g. in z-direction)

$$\tilde{q}(x, \mu^2, P_3) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \langle N(P_3)|\bar{\Psi}(z)\,\gamma^z\,\mathcal{A}(z,0)\Psi(0)|N(P_3)\rangle_{\mu^2}$$

- $\mathcal{A}(z, 0)$: Wilson line from $0 \to z$
- z: distance in any spatial direction (momentum boost in z direction)

- ★ At finite but feasibly large momenta on the lattice: [X.Ji, arXiv:1305.1539]

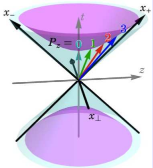
  a large momentum EFT can relate Euclidean $\tilde{q}$ to PDFs through a factorization theorem

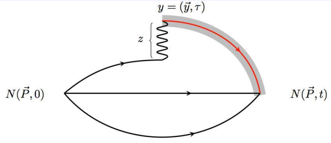


- ★ use of Perturbation Theory for the matching

Computation is difficult and costly

Access of PDFs on Euclidean lattice





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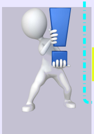


- ★ **use of Perturbation Theory for the matching**

Computation is difficult and costly

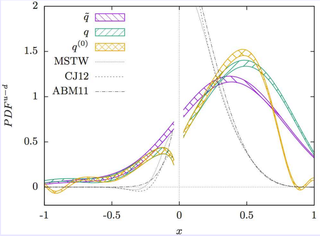
Cyprus Group

Lattice results on unpolarized PDFs

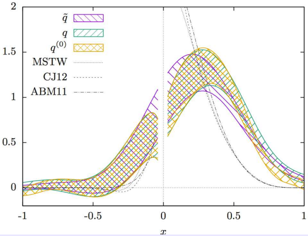
$$u(x) - d(x)$$

N_f=2+1+1 TM fermions, m_π=375 Mev

[C. Alexandrou et al. (ETMC), arXiv:1504.07455]



$P_3 = 2\pi/L * 2$, 5 HYP steps



$P_3 = 2\pi/L * 3$, 5 HYP steps

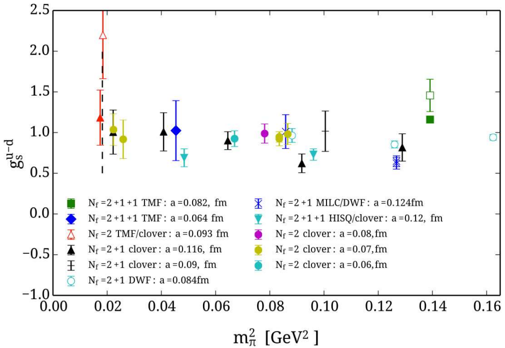
Sea-quark distribution: $\tilde{q}(x) = -q(-x)$

Phenomenological data: **MSDW: A. Martin, [arXiv:0901.0002]** **CTEQ-JLab: J. Owens, [arXiv:1212.1702]**

★ **Renormalization of $\tilde{q}$: still in progress!**

SCALAR CHARGE: The Squiggly One

$$g_S \equiv \langle N|\bar{u}u - \bar{d}d|N\rangle|_{Q^2=0}$$



Challenging calculation:

- ★ smallest signal-to-noise ratio
- ★ systematics are not well-controlled
- ★ disconnected contributions not negligible
- ★ requires vacuum subtraction

[A.Abdel-Rehim et al. (ETMC), arXiv:1507.04936]

$\sim$ 23 000 Measurements

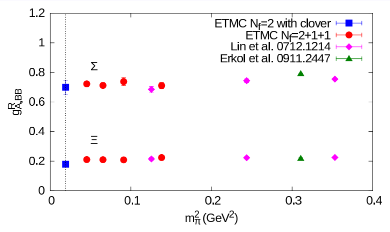
(simulations at the physical point)

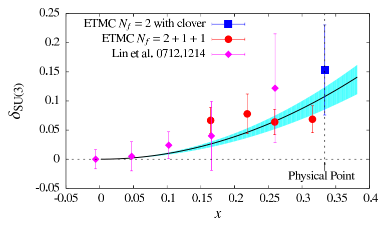
Swiss National Supercomputing Centre

Axial charges of hyperons

Axial matrix element: $\langle B(p')|\bar{\psi}(x)\,\gamma_\mu\,\gamma_5\,\psi(x)|B(p)\rangle\Big|_{q^2=0}$

- **Connected part**





[C. Alexandrou et al. (ETMC), arXiv:1411.3494]

- **First promising results at the physical point**
- **SU(3) breaking** $\delta_{SU(3)} = g_A^N - g_A^\Sigma + g_A^\Xi$ **versus** $x = \left(m_K^2 - m_\pi^2\right)/(4\pi^2 f_\pi^2)$